Title: Standard Model Lecture

Speakers: Michael Trott

Collection: Standard Model 2023/24

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B ... (0) $\varepsilon \to$

QED running:

renormalized $\rightarrow$ bare

$$A_\mu^{(r)} = A_\mu^{(0)} / \sqrt{Z_A}$$

$$\psi^{(r)} = \psi^{(0)} / \sqrt{Z_\psi}$$

$$e^{(r)} = e^{(0)} \mu^{-\varepsilon} / Z_e$$

$$m_e^{(r)} = m_e^{(0)} / Z_m$$

Use $\overline{MS}$ subtraction, dim. reg.

$Z = 1 + \frac{\#}{16\pi^2 \varepsilon}$, only subtract

M.S. $\to$ minimal subtraction

$\overline{MS}$ also $\mu^2 \to \mu^2 e^{\gamma}/4\pi$, $\gamma =$

this cleans up loop calcs (

dim. reg. $d = 4 - 2\varepsilon$ ← small

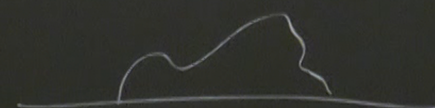
nly subtract $\frac{1}{\varepsilon}$ bits

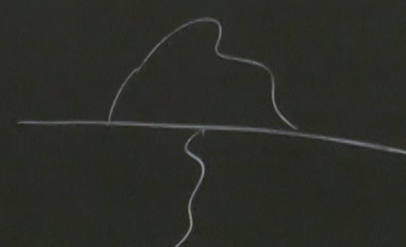
ction

$/4\pi$, $\gamma = 0.577\ldots$

p calcs (no $\log(4\pi)$ etc)

 $\rightarrow Z_A$

 $\rightarrow Z_\psi, Z_m$

 $\rightarrow Z_e$

Running $\quad e^{(0)} = \mu^{\epsilon} e Z_e$

$$Z_A = 1 - \frac{4}{3}\left(\frac{e^2}{16\pi^2 \epsilon}\right) + \dots$$

$$Z_\psi = 1 - \left(\frac{e^2}{16\pi^2 \epsilon}\right) + \dots$$

$$Z_m = 1 - 3\left(\frac{e^2}{16\pi^2 \epsilon}\right) + \dots$$

Z_e

$1/\sqrt{Z_A}$ at one loop

identity ✓

exp in $\frac{\alpha}{4\pi}$, $\alpha \equiv \frac{g^2}{4\pi}$ $\qquad \alpha(m_e) \sim \frac{1}{137}$

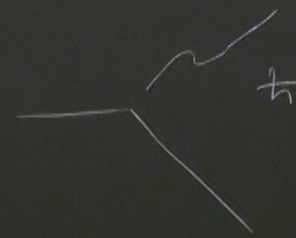
$\# + \frac{\alpha}{4\pi} \#^2 + \ldots$

$\sqrt{Z_A}$ at one loop

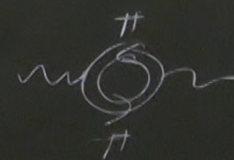
t, t_y

exp in $\frac{\alpha}{4\pi}$, $\alpha \equiv \frac{g^2}{4\pi}$

$+ \frac{\alpha}{4\pi} \cancel{\alpha}^2 + \dots$



$p^2 \to 0 \to \alpha(0)$



$\alpha(m_Z) \sim \frac{1}{137}$

$\downarrow$

$\alpha(m_Z)$

S_0

$$\beta(e) = \mu \frac{de}{d\mu}$$

$$= -\varepsilon e - e \frac{d \ln Z_c}{d \ln(\mu)}$$

$$= -\varepsilon e + \frac{e^3}{12\pi^2} + \ldots$$

$$\frac{d \ln Z_c}{d \ldots}$$

$$\frac{e^3}{\ldots\pi^2} + \ldots$$

$$\mu \frac{d}{d\mu} e = \frac{e^3}{12\pi^2} + \ldots$$

$$\frac{1}{e^2(\mu_2)} = \frac{1}{e^2(\mu_1)} - \frac{1}{12\pi^2} \ln\left[\mu_2^2 / \mu_1^2\right]$$

$$e \frac{d \ln Z_c}{d \ln(\mu)}$$

$$= \frac{e^3}{12\pi^2} + \dots$$

$$\mu \frac{d}{d\mu} e = \frac{e^3}{12\pi^2} + \dots$$

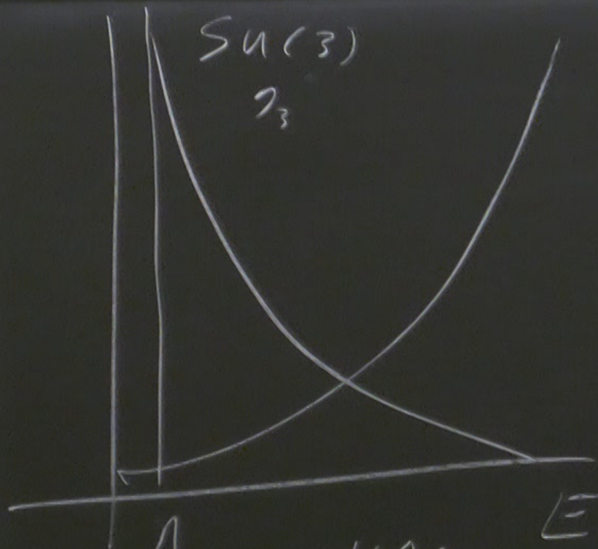
$$\frac{1}{e^2(\mu_2)} = \frac{1}{e^2(\mu_1)} - \frac{1}{12\pi^2} \ln\left[\mu_2^2/\mu_1^2\right]$$

$$= \frac{e^3}{12\pi^2} + \cdots$$

$$\langle O \rangle = \# + \frac{\alpha(\mu)}{4\pi} \# + \cdots$$

$$= \frac{1}{e^2(\mu_1)} - \frac{1}{12\pi^2} \ln\left[\mu_2^2/\mu_1^2\right]$$

$$\log\left[\frac{\mu_2}{\mu_1}\right]$$



$\Lambda_{QCD} \sim 100\,\mathrm{MeV}$

$v \sim 246$

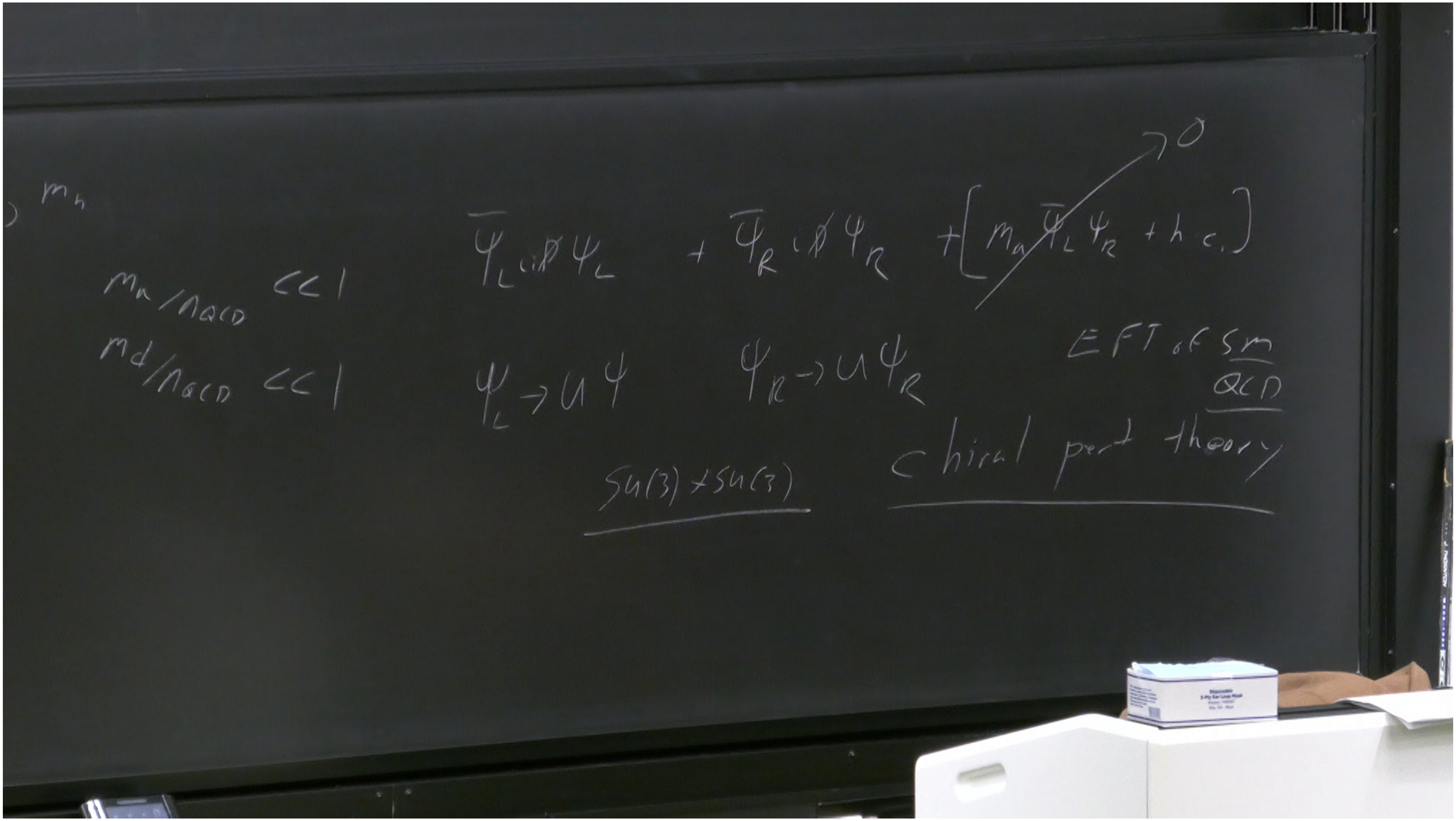
SU(2) g_2

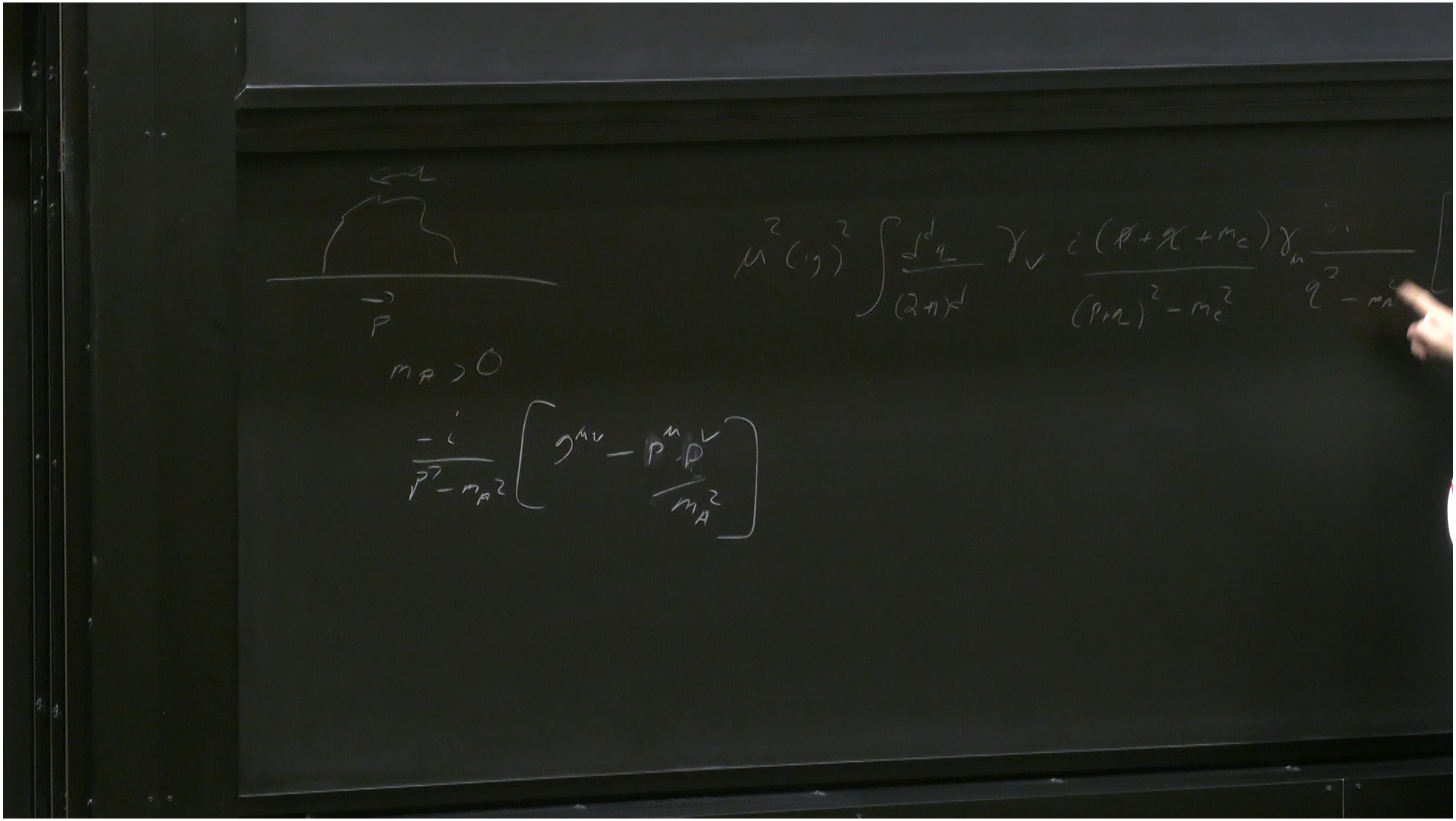
U(1) g_1

$m_u, m_d < \Lambda_{QCD} <<< m_t, m_W, m_Z$

$\sim m_c, m_s$

$\Lambda_{QCD} << m_b, m_t$


m_u
$m_u/\Lambda_{QCD} \ll 1$
$m_d/\Lambda_{QCD} \ll 1$
$\bar{\psi}_L i\partial\!\!\!/ \psi_L + \bar{\psi}_R i\partial\!\!\!/ \psi_R + [\cancel{m_u \bar{\psi}_L \psi_R + h.c.}] \to 0$
$\psi_L \to U\psi \qquad \psi_R \to U\psi_R$
EFT of SM QCD
SU(3) × SU(3)
chiral pert theory



q
$\vec{p}$
$\mu^2 (ig)^2 \int \frac{d^d q}{(2\pi)^d} \gamma_\nu \frac{i(\not p + \not q + m_e)}{(p+q)^2 - m_e^2} \gamma_\mu \frac{\cdots}{q^2 - m_A^2}$
$m_A > 0$
$\frac{-i}{p^2 - m_A^2}\left[g^{\mu\nu} - \frac{p^\mu p^\nu}{m_A^2} \right]$

$$ {}^2 (ig)^2 \int \frac{d^d q}{(2\pi)^d} \gamma_\nu \frac{i(\not p + \not q + m_c)}{(p+q)^2 - m_c^2} \gamma_\mu \frac{-i}{q^2 - m_A^2} \left[g^{\mu\nu} - q^\mu q^\nu / m_A^2 \right] \qquad Z_0 \frac{Z_\psi}{m_A^2} \overset{0}{\boxed{\bar\psi^{(n)} i \not\partial\, \partial^2 \psi^{(n)}}} $$

$$ \not q (\not p + \not q) \not q \qquad \gamma\gamma\gamma \to g\gamma + g\gamma + g\gamma - i\epsilon\gamma_5 $$

$$ \int \frac{d^d q}{(2\pi)^d} \frac{q^2}{(q^2 - m_A^2)\left((p+q)^2 - m_c^2\right)} \Rightarrow \frac{i}{16\pi^2 \epsilon} \frac{m_c^2}{m_A^2} \not p $$

$Z_e \, d\mu \,)$

$\Sigma = \exp[i\sigma_a \pi^a / v]$ $\qquad$ $\Sigma \to L \Sigma R^\dagger$

$$+ \frac{v^2}{4} \mathrm{Tr}[D_\mu \Sigma^\dagger D^\mu \Sigma] - \frac{v}{\sqrt{2}} (\bar{u}_L^i \ \bar{d}_L^i) \, \Sigma \begin{pmatrix} Y^u_{ij} u_R^j \\ Y^d_{ij} d_R^j \end{pmatrix} + h.c. + \frac{\text{higher D}}{\text{Ops}}$$

$\downarrow$

$$m_W^2 W_L^\mu W_L^\mu + m_Z Z_L^\mu Z_L^\mu$$

E.W. Chiral Lagrangian

$$\Sigma = \exp$$

$$\mathcal{L} = \frac{1}{4} \underbrace{W_I^{\mu\nu} W_I^{\mu\nu}}_{SU(2)} - \frac{1}{4} \underbrace{B_{\mu\nu} B^{\mu\nu}}_{U(1)_Y} - \frac{1}{4} \underbrace{G_A^{\mu\nu} G_A^{\mu\nu}}_{SU(3)} + \sum_{\psi} \bar{\psi}\, i \not{D} \psi + \frac{v^2}{4}$$

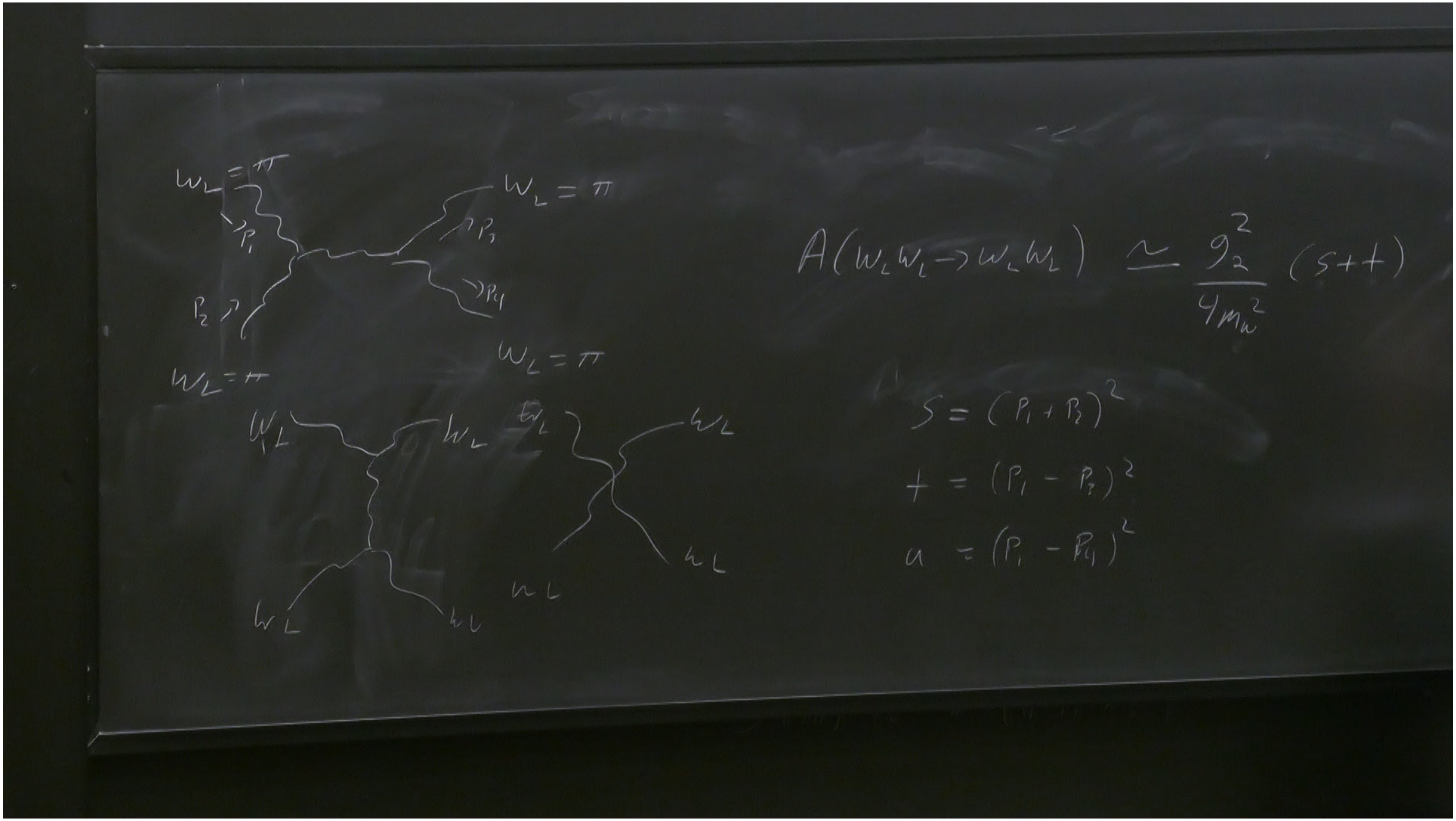
$$m_h \sim 125\ \mathrm{GeV}$$

$$\frac{\Lambda}{m_b} \ll 1$$

$\langle b \rangle$ Λ_{QCD}

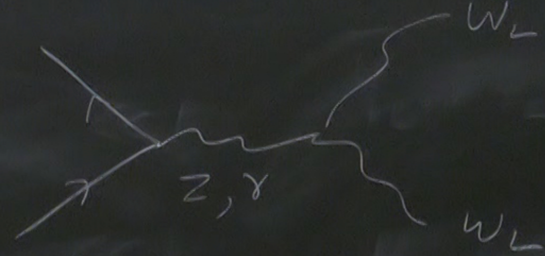
'heavy Quark physics'

H.Q.E.T.



$W_L = \pi$
p_1
p_2
$W_L = \pi$
p_3
p_4
$W_L = \pi$
$W_L = \pi$
$A(W_L W_L \to W_L W_L) \simeq \frac{g_2^2}{4 m_W^2}(s+t)$
$s = (p_1 + p_2)^2$
$t = (p_1 - p_3)^2$
$u = (p_1 - p_4)^2$
W_L

$$A(W_L W_L \to W_L W_L) \simeq \frac{g_2^2}{4m_W^2}(s+t)$$



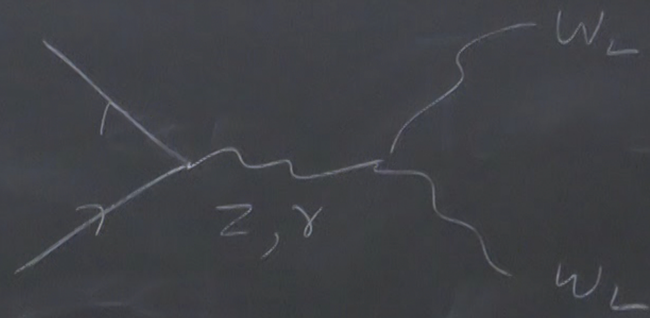
$$s = (p_1 + p_2)^2$$

$$t = (p_1 - p_3)^2$$

$$u = (p_1 - p_4)^2$$

$$A(\bar{\psi}\psi \to W_L W_L) \simeq \frac{m_\psi \sqrt{s}}{v^2}$$

s grows $\to$ Amp grows $\sigma \propto A^2$ $\nearrow$ bigger



$$A(\bar{\psi}\psi \to W_L W_L) \simeq \frac{m_\psi \sqrt{s}}{v^2}$$

$p > 1$

grows $\to$ Amp grows $\quad \sigma \propto A^2 \nearrow$ bigger

Unitarity violation

$\sigma_a \pi^a / v \big]$ $\qquad \Sigma \to L \Sigma R^\dagger$

$\big[D_\mu \Sigma^\dagger D^\mu \Sigma \big] \quad - \frac{v}{\sqrt{2}} \left(\bar{u}_L^i \; \bar{d}_L^i \right) \Sigma \begin{pmatrix} Y_{ij}^u \, u_R^j \\ Y_{ij}^d \, d_R^j \end{pmatrix} + h.c. + \underline{\frac{\text{higher D}}{\text{Ops}}}$

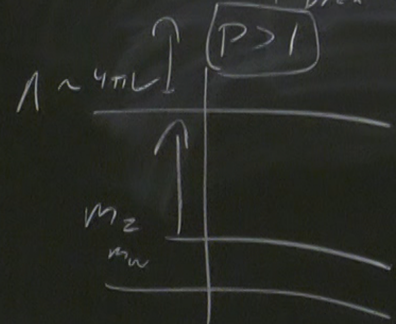
$\boxed{m_n^2 \gg s}$

$\frac{1}{}$

$w^2 W_L^\mu W_L^\mu \quad + m_Z Z_L^\mu Z_L^\mu$

$$\hbar \frac{d}{dt}\psi = H\psi$$

$$\Lambda_{break} \sim 4\pi v$$



$$\Sigma = \exp[i \sigma_a \pi^a / v] \qquad \Sigma \to L \Sigma R^\dagger$$

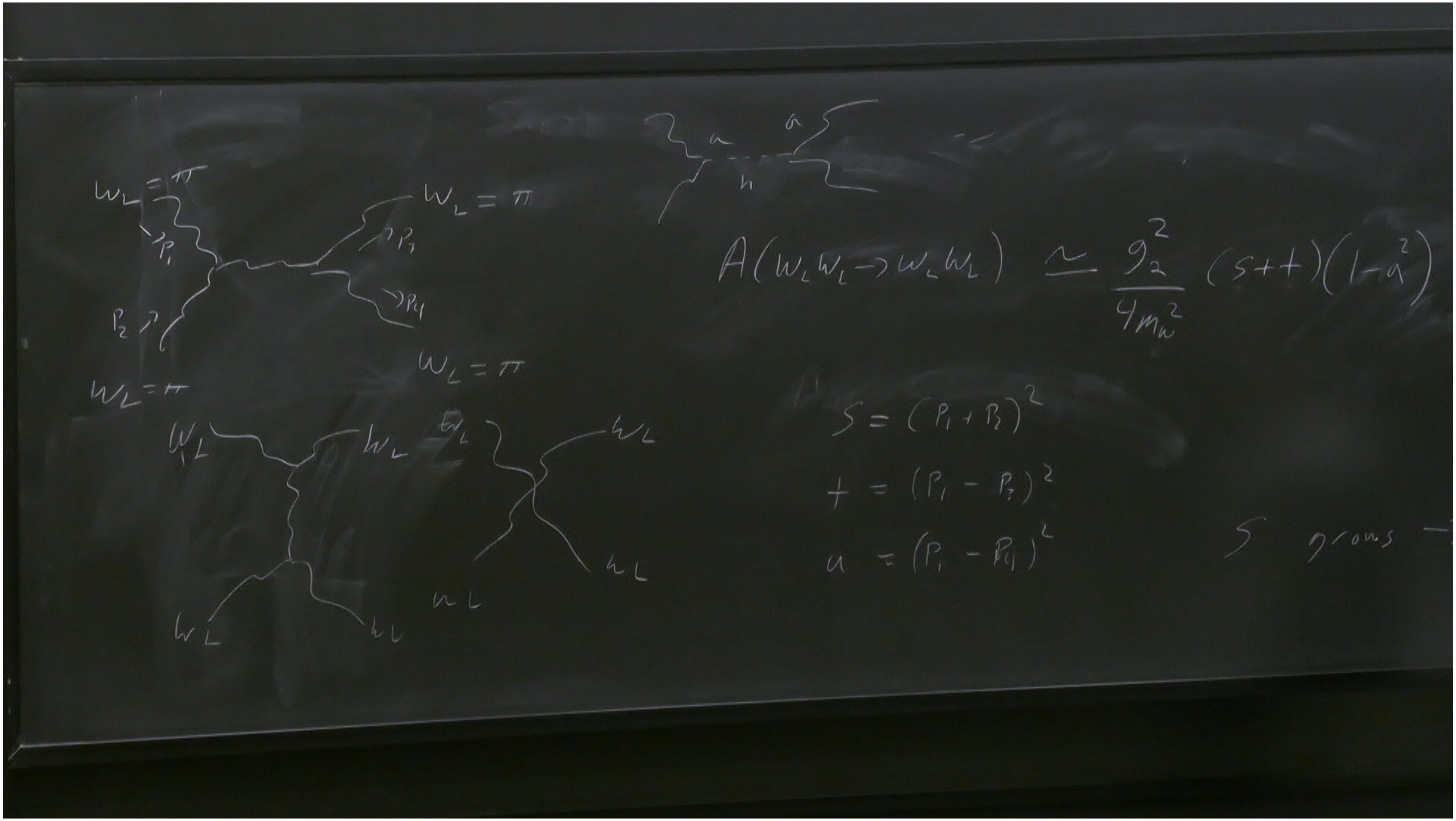
$$\boxed{\left[1 + 2a\tfrac{h}{v} + \cdots\right]} \qquad \left[1 + c\tfrac{h}{v} + \cdots\right]$$

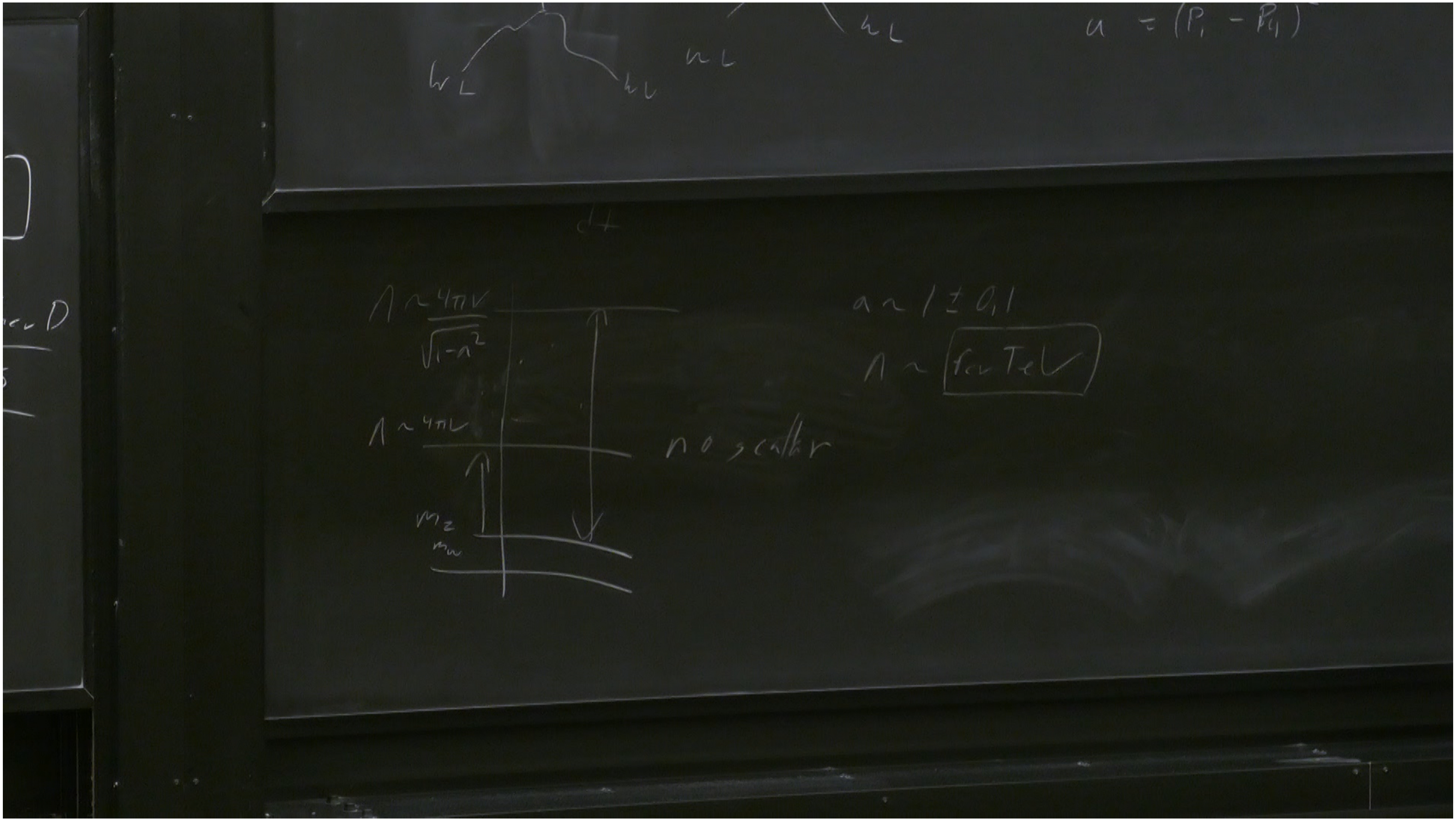
$$\not{D}\psi + \frac{v^2}{4} \mathrm{Tr}\left[D_\mu \Sigma^\dagger D^\mu \Sigma\right] - \frac{v}{\sqrt{2}} \left(\bar{u}_L^i \; \bar{d}_L^i\right) \Sigma \begin{pmatrix} Y^u_{ij} u_R^j \\ Y^d_{ij} d_R^j \end{pmatrix} + h.c. + \frac{\text{higher D}}{\text{Ops}}$$

DSGeL

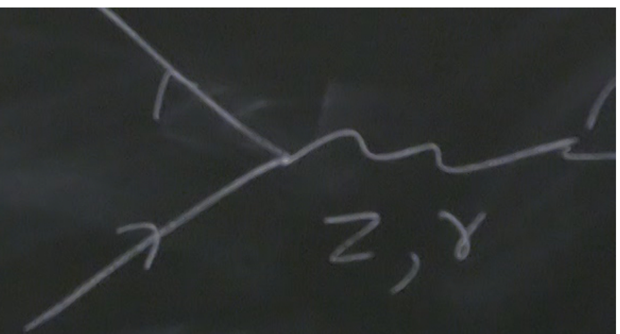
$$m_W^2 W_L^\mu W_L^\mu + m_Z Z_L^\mu Z_L^\mu$$

$$\boxed{m_h^2 \gg s}$$


W_L = π
W_L = π
p_1
p_2
p_3
p_4
W_L = π
W_L = π
W_L
W_L
W_L
W_L
W_L
W_L
W_L
W_L
a
a
h
A(W_L W_L → W_L W_L) ≃ g_2^2 / 4m_W^2 (s+t)(1−a^2)
s = (p_1 + p_2)^2
t = (p_1 − p_3)^2
u = (p_1 − p_4)^2
s grows


Λ ~ 4πv / √(1-a²)
Λ ~ 4πv
m_Z
m_W
no scalar
a ~ 1 ± 0.1
Λ ~ TeV

$$W_L W_L) \simeq \frac{g_2^2}{4m_W^2}(s+t)(1-a^2)$$



$(p_1 + p_2)^2$

$(p_1 - p_3)^2$

$(p_1 - p_4)^2$

$$\boxed{\begin{matrix} a = 1 \\ c = 1 \end{matrix}} = \underline{\underline{\text{SM Higgs}}}$$

$A(\bar{\psi}\psi \to W_L$

s grows $\to$ Amp grows