Title: Standard Model Lecture

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Gnd state with lower energy can lead to non-manifest Symmetry
(not preserving sym trans)

$[H, U^+] = 0 \qquad H \to U H U^+ = H$

<u>U sym trans</u> $\quad |A\rangle, |B\rangle$ 2 states related by U: $\; U|A\rangle = |B\rangle$

$U a_A^+ |0\rangle \qquad E_A = \langle A|H|A\rangle$

Spontaneous Sym Breaking?

Sym can be non-manifest in Lagrangian when expanded about a vacuum state.

$$\left(H^\dagger H - v^2/2\right)$$

$|H\rangle$

then $|A\rangle = a_A^\dagger |0\rangle$

$|B\rangle = a_B^\dagger |0\rangle$

and $U a_A^\dagger U^\dagger = a_B^\dagger$

$$U|A\rangle = U a_A^\dagger |0\rangle$$

$$= \underbrace{U a_A^\dagger U^\dagger}_{a_B^\dagger} U |0\rangle$$

$$= a_B^\dagger U |0\rangle$$

So long as $U|0\rangle = |0\rangle$

$$U|A\rangle = U a_A^{\dagger} |0\rangle$$

$$= \underbrace{U a_A^{\dagger} U^{\dagger}}_{a_B^{\dagger}} U |0\rangle$$

$$= a_B^{\dagger} U |0\rangle$$

So long as $U|0\rangle = |0\rangle$

$$E_A = \langle A|H|A\rangle$$

$$= \langle B|H|B\rangle$$

$$= E_B$$

Need $U|0\rangle = |0\rangle$

But this can be not satisfied

$$V(H^\dagger H) = \frac{\lambda}{4}\left(H^\dagger H - v^2/2\right)^2$$

$SU_L(2)$;

Y ;

$$\langle H_I^\dagger H^I \rangle = v^2/2$$

$$|H^I\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

$SU_L(2) \longleftrightarrow$ Tadpoles

$SU_L(2)$: $T_a = \sigma_a/2$

Y : $Y_h \mathbb{1} = \frac{1}{2}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

$SU_L(2) \times U_Y(1)$

$$\sigma_1 \quad \frac{1}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}0\\ v/\sqrt{2}\end{pmatrix} = \frac{1}{2}\begin{pmatrix}v/\sqrt{2}\\ 0\end{pmatrix}$$

$$\sigma_2 \quad \frac{1}{2}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}0\\ v/\sqrt{2}\end{pmatrix} = \frac{1}{2}\begin{pmatrix}-iv/\sqrt{2}\\ 0\end{pmatrix}$$

$$\sigma_3 \quad \frac{1}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}0\\ v/\sqrt{2}\end{pmatrix} = \frac{1}{2}\begin{pmatrix}0\\ -v/\sqrt{2}\end{pmatrix}$$

$$Y \quad \frac{1}{2}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}0\\ v/\sqrt{2}\end{pmatrix}$$

$$Q = T_3 + Y_h \mathbb{1}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$SU_L(2) \times U_Y(1) \to U_{em}(1)$$

$$Q\,|H\rangle = |0\rangle$$

$\hookrightarrow$ $U_{em}(1) \to$ photon

$$\boxed{m_A = 0}$$

$SU(2)$ $\qquad\qquad\qquad\qquad$ $\alpha = (1,2,3)$

$$\mathcal{L}_{Higgs} = (D^\mu H)^\dagger (D_\mu H) - \frac{\lambda}{4}\left(H^\dagger H - v^2/2\right)^2 - \Big[H^{\dagger I}\, \bar{d}_R^{i\alpha}\, Y_D^{ij}\, Q_{L,\alpha}^{j,I} + \tilde{H}_I^{\dagger}\, \bar{u}_R^{i\alpha}\, Y_u^{ij}\, Q_{L,\alpha}^{j,I} + \dots$$

$$\hookrightarrow \tilde{H}^I = \varepsilon^{IJ} H^{\dagger,J}$$

$$H(x) = \begin{pmatrix} h^+(x) \\ v/\sqrt{2} + h^0(x) \end{pmatrix}$$

$$\hookrightarrow \mathrm{Re}(h^0(x))$$

$$\mathrm{Im}(h^0(x))$$

$$V(H^\dagger H) = \frac{\lambda}{4}\left(|h^+|^2 + |h^0|^2 + \sqrt{2}\, v\, \mathrm{Re}[h^0] \right)^2$$

Goldstone's $\underline{\text{thm}}$: Broken gen $\to$ mass

$SU(2)$ $\qquad\qquad\qquad\qquad$ $\alpha = (1,2,3)$

$$\ldots/2)^2 - \left[H^{\dagger I} \bar{d}_R^{i\alpha} Y_D^{ij} Q_{L,\alpha}^{j,I} + \tilde{H}^{\dagger}_I \bar{u}_R^{i\alpha} Y_u^{ij} Q_{L,\alpha}^{j,I} + H^{\dagger j,I} \bar{e}_R^{i} Y_L^{ij} l_L^{j} + h.c. \right]$$

$$L \to \tilde{H}^I = \varepsilon^{IJ} H^{\dagger,J}$$

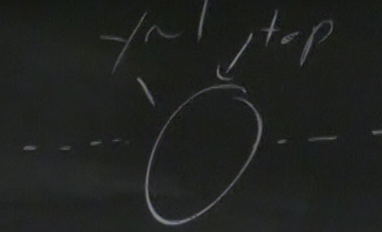
$$V(H^\dagger H) = \frac{\lambda}{4}\left(|h^+|^2 + |h'|^2 + \sqrt{2}\, v\, \mathrm{Re}[h'] \right)^2 \qquad m_H^2 = \frac{\lambda}{2} v^2 = (\ldots$$

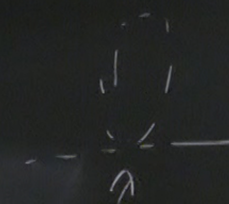
Goldstone's <u>thm</u>: Broken gen $\to$ massless scalar

(3)

$\alpha = (1,2,3)$

$$\bar{u}_R^{i\alpha} Y_u^{ij} Q_{L,\alpha}^{j,I} + H^{\dagger j,I}\, \bar{e}_R^{i} Y_L^{ij} l_L^{j} + h.c. \Big]$$





$$\tilde{H}^I = \epsilon^{IJ} H^{\dagger,J}$$

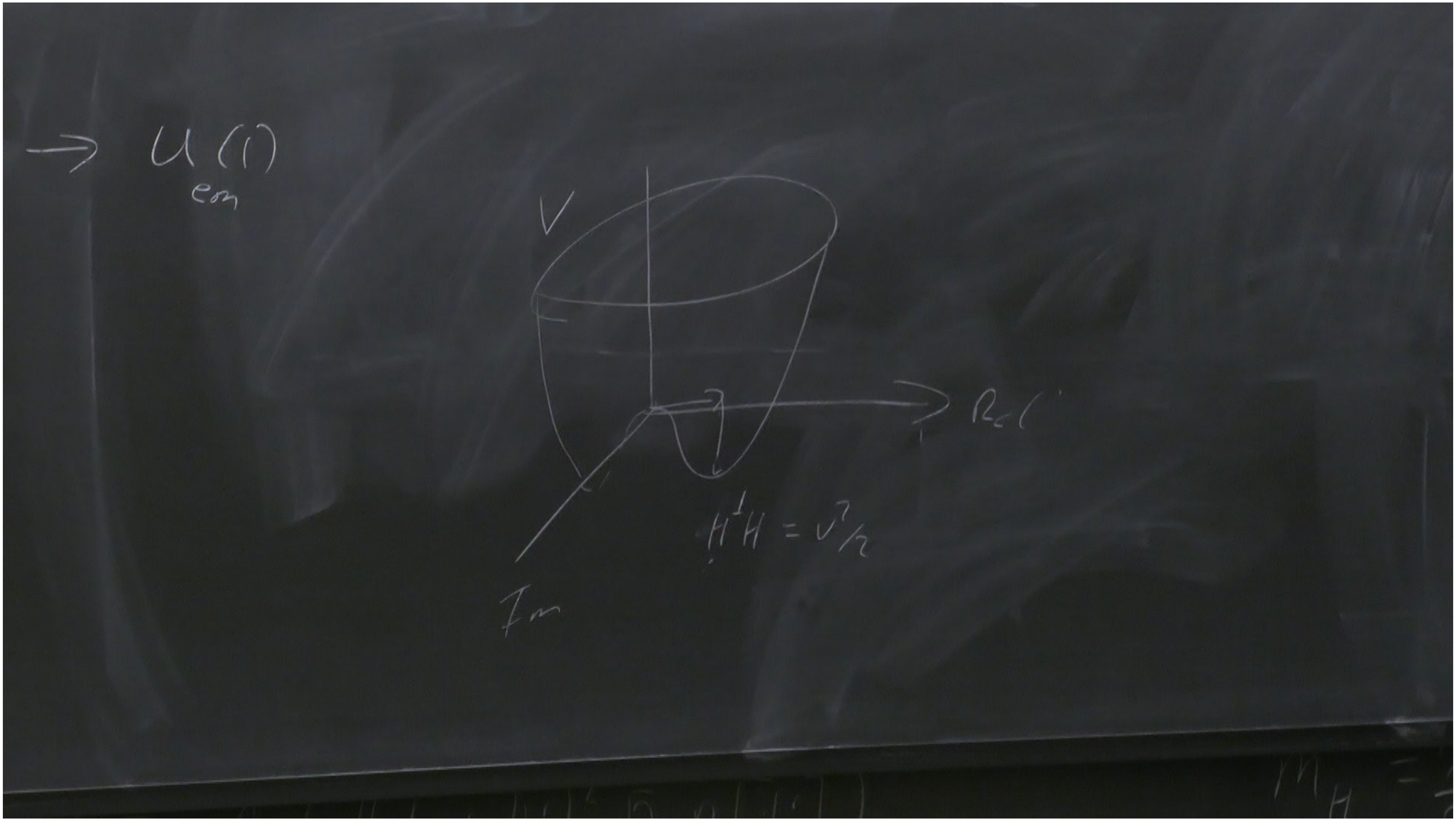
$$R_c[h]\Big)^2$$

$$m_H^2 = \frac{\lambda}{2} v^2 = (125\ \text{GeV})^2$$

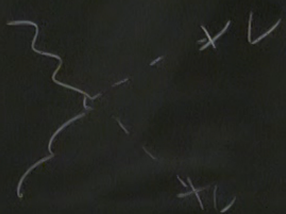
$$v \to (246\ \text{GeV})$$

$\to$ massless scalar

(3)


→ U(1)
em
V
Re
Im
H†H = v²/2

$$(D^\mu H)^\dagger (D_\mu H) \supseteq \frac{g_2^2 v^2}{8}\underbrace{\left(W_1^\mu W_{1\mu} + W_2^\mu W_{2\mu}\right)}_{2W_\mu^+ W_\mu^-} + \frac{v^2}{8}\left(g_2 W_\mu^3 - g_1 B_\mu\right)^2 + \dots$$



$$W_\mu^\pm = \frac{W_\mu^1 \mp W_\mu^2 i}{\sqrt{2}}$$

$$m_W^2 = \frac{g_2^2 v^2}{4}$$

$Z^\mu Z_\mu$

$$\left(\frac{g_2 W_\mu^3}{\sqrt{g_1^2+g_2^2}} - \frac{g_1 B_\mu}{\sqrt{g_1^2+g_2^2}}\right)^2$$

$$\cos\theta = \frac{g_2}{\sqrt{g_1^2+g_2^2}}$$

$$\sin\theta = \frac{g_1}{\sqrt{g_1^2+g_2^2}}$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B \end{pmatrix}$$

$$Z^\mu = \cos\theta\, W_3^\mu - \sin\theta\, B_\mu$$

$$A = \sin\theta\, W_3 + \cos\theta\, B_\mu$$

$$m_Z^2 = \frac{v^2}{4}(g_1^2+g_2^2)$$

$$m_A^2 = 0$$

$$\rho \equiv \frac{m_W^2}{\cos^2\theta \, m_Z^2} = 1$$

<u>Yukawa's</u>

$$-\left[H_I^\dagger \, \bar{e}_R^i \, Y^{ij} L_L^{jI} + h.c. \right]$$

$$\left[(0, v/\sqrt{2}) \sim \right.$$

$$\Rightarrow \frac{v}{\sqrt{2}} Y^{ij} \bar{e}_R^i e_L^j + h.c. \left. \right]$$

$$e_L^i \to U(L,e) \, e_L'$$

$$e_R^i \to U(R,e) \, e_R'$$

$$[H_I \, \bar{e}_R Y^{ij} L_L^{j\dagger} + h.c.] \Rightarrow m_i \bar{e}^i P_L e^i + h.c. \Rightarrow m_i \bar{e}_i e_i$$

$$[(0, v/\sqrt{2}) \sim$$

$$P_R + P_L = 1$$

$$\Rightarrow \frac{v}{\sqrt{2}} Y^{ij} \bar{e}_R^i e_L^j + h.c.]$$

$$U^\dagger Y v/\sqrt{2}\, U \equiv \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_i \end{pmatrix}$$

$$e_L \to U(L,e)\, e'_L$$

$$e_R \to U(R,e)\, e'_R$$

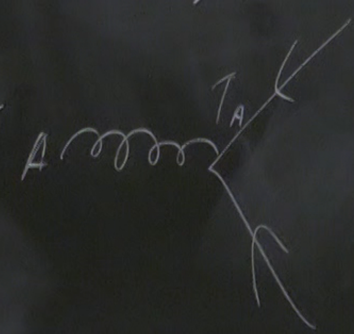
$$-\left[H^{\dagger}_{I,} \, \bar{d}_R^{\,i,a} \, Y_b^{ij} \, Q_L^{j,a} + h.c. \right] \qquad i,a \leftarrow 1,2,3$$

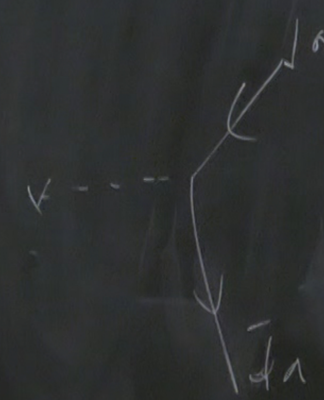
$$\Rightarrow \quad m_d^i \, \bar{d}_a d^a$$

$$\bar{Q}_L^{a,F} \, i \not{D} \, Q_L^{b,F}$$

$$\downarrow$$

$$i g_3 T_A^{ab} A^A$$





$$\mathcal{L}_{gauge} = -\frac{1}{2\xi}\sum_a \left(\partial^\mu W^a_\mu + i g_2 \xi \left(\langle H\rangle^\dagger T^a H - H^\dagger T^a \langle H\rangle\right)\right]^2$$

R_ξ

$$-\frac{1}{2\xi}\left[\partial^\mu B_\mu + i g_1 \xi \left(\langle H\rangle^\dagger Y_h H - H^\dagger Y_h \langle H\rangle\right)\right]^2$$

Mass $\propto \xi \frac{v^2}{2}|h^+|^2$ $\quad \xi \to \infty$ unitary gauge

$Z^\mu Z_\mu$

$\left(\frac{\ldots W_\mu^3 - g_1 B_\mu}{\sqrt{g_1^2+g_2^2}} \right)^2$

$\cos\theta = \frac{g_2}{\sqrt{g_1^2+g_2^2}}$

$\sin\theta = \frac{g_1}{\sqrt{g_1^2+g_2^2}}$

$(\xi-1)\, \partial_\mu W_L^+ \, h^+$ ~~~ ....

$\xi \to 1$ Feynman Gauge

$(\xi-1)\, \frac{k^\mu k^\nu}{m^2}$