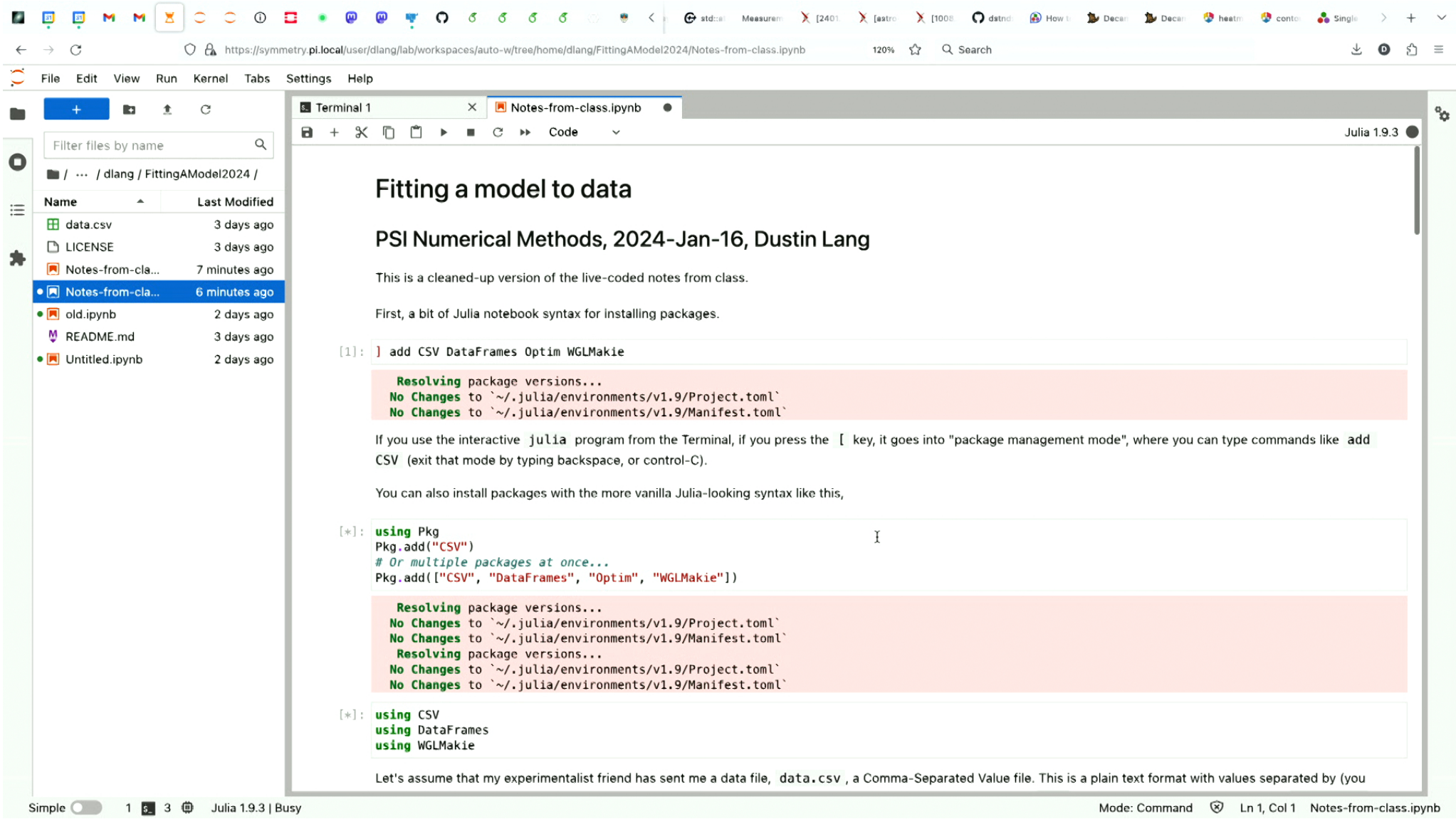
Title: Numerical Methods Lecture

Speakers: Dustin Lang

Collection: Numerical Methods 2023/24

Date: January 18, 2024 - 10:15 AM

URL: https://pirsa.org/24010022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Code
Julia 1.9.3
Fitting a model to data
PSI Numerical Methods, 2024-Jan-16, Dustin Lang
This is a cleaned-up version of the live-coded notes from class.
First, a bit of Julia notebook syntax for installing packages.
[1]:] add CSV DataFrames Optim WGLMakie
Resolving package versions...
No Changes to `~/.julia/environments/v1.9/Project.toml`
No Changes to `~/.julia/environments/v1.9/Manifest.toml`
If you use the interactive julia program from the Terminal, if you press the [key, it goes into "package management mode", where you can type commands like add CSV (exit that mode by typing backspace, or control-C).
You can also install packages with the more vanilla Julia-looking syntax like this,
[*]: using Pkg
Pkg.add("CSV")
# Or multiple packages at once...
Pkg.add(["CSV", "DataFrames", "Optim", "WGLMakie"])
Resolving package versions...
No Changes to `~/.julia/environments/v1.9/Project.toml`
No Changes to `~/.julia/environments/v1.9/Manifest.toml`
Resolving package versions...
No Changes to `~/.julia/environments/v1.9/Project.toml`
No Changes to `~/.julia/environments/v1.9/Manifest.toml`
[*]: using CSV
using DataFrames
using WGLMakie
Let's assume that my experimentalist friend has sent me a data file, data.csv , a Comma-Separated Value file. This is a plain text format with values separated by (you
Simple 1 3 Julia 1.9.3 | Busy
Mode: Command Ln 1, Col 1 Notes-from-class.ipynb

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Code | Julia 1.9.3

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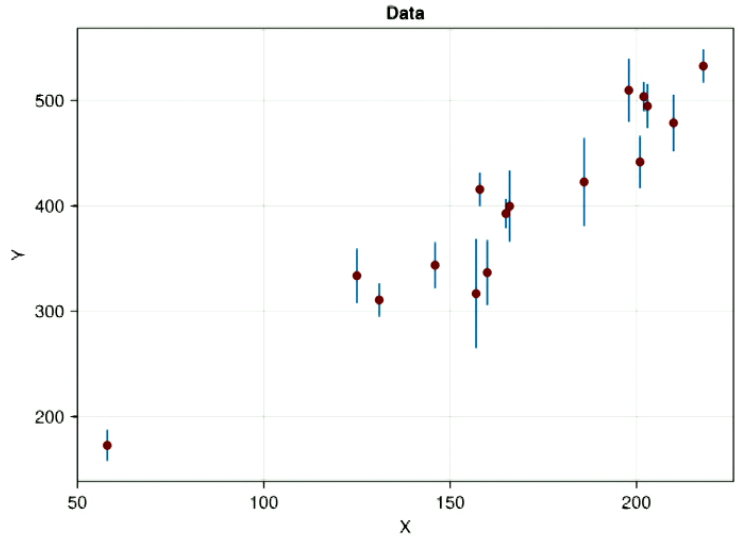
```
errorbars!(data.x, data.y, data.sigma_y)
# You can plot multiple things on the same plot using these commands ending with "!".
# The "!" is a common thing in Julia that means "this function modifies the state".
scatter!(data.x, data.y, markersize=10, color=:maroon)
f
```

[7]:

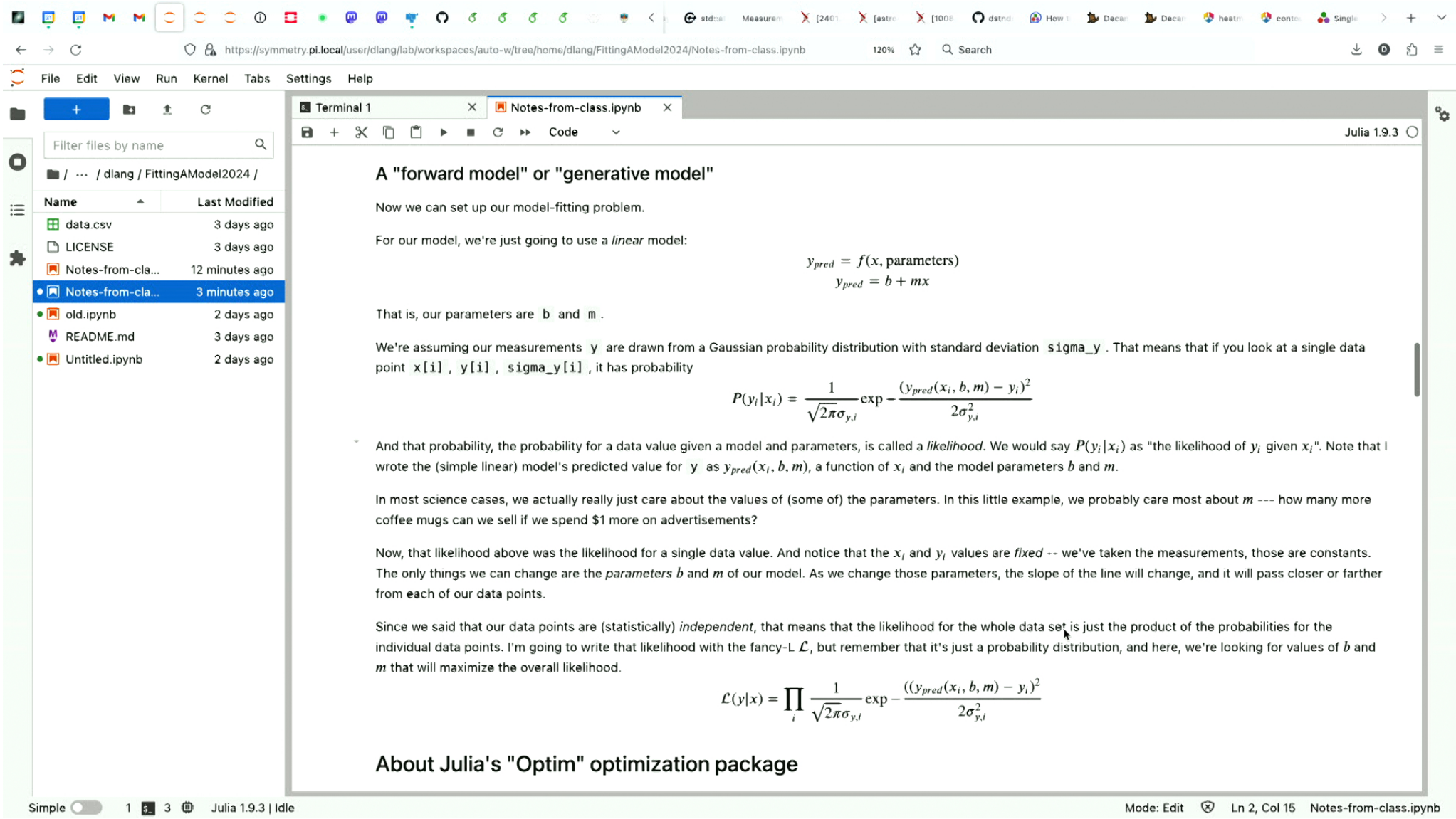


This is a pretty typical kind of setup for a model-fitting task. But if you think about it a bit, there are lot of really strong assumptions built in:

- no uncertainties on the `x` values (maybe we can measure `x` so well compared to `y` that the `x` uncertainties are irrelevant -- this does actually happen a lot in astrophysics)
- the measurements on `y` are distributed according to a Gaussian probability distribution with the given standard deviations
- the data points are *independent* -- if one data point is high or low, it has no effect on whether another data point is high or low

Given all of this, we can set up a *forward model* for the data. That is, we can come up with a function `y_model` that takes an `x` value and some parameters, and produces

Simple 1 3 Julia 1.9.3 | Idle Mode: Edit Ln 2, Col 15 Notes-from-class.ipynb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A "forward model" or "generative model"
Now we can set up our model-fitting problem.
For our model, we're just going to use a linear model:
$$y_{pred} = f(x, \text{parameters})$$
$$y_{pred} = b + mx$$
That is, our parameters are b and m .
We're assuming our measurements y are drawn from a Gaussian probability distribution with standard deviation sigma_y . That means that if you look at a single data point x[i] , y[i] , sigma_y[i] , it has probability
$$P(y_i|x_i) = \frac{1}{\sqrt{2\pi}\sigma_{y,i}} \exp - \frac{(y_{pred}(x_i, b, m) - y_i)^2}{2\sigma_{y,i}^2}$$
And that probability, the probability for a data value given a model and parameters, is called a likelihood. We would say $P(y_i|x_i)$ as "the likelihood of y_i given x_i". Note that I wrote the (simple linear) model's predicted value for y as $y_{pred}(x_i, b, m)$, a function of x_i and the model parameters b and m.
In most science cases, we actually really just care about the values of (some of) the parameters. In this little example, we probably care most about m --- how many more coffee mugs can we sell if we spend $1 more on advertisements?
Now, that likelihood above was the likelihood for a single data value. And notice that the x_i and y_i values are fixed -- we've taken the measurements, those are constants. The only things we can change are the parameters b and m of our model. As we change those parameters, the slope of the line will change, and it will pass closer or farther from each of our data points.
Since we said that our data points are (statistically) independent, that means that the likelihood for the whole data set is just the product of the probabilities for the individual data points. I'm going to write that likelihood with the fancy-L $\mathcal{L}$, but remember that it's just a probability distribution, and here, we're looking for values of b and m that will maximize the overall likelihood.
$$\mathcal{L}(y|x) = \prod_i \frac{1}{\sqrt{2\pi}\sigma_{y,i}} \exp - \frac{((y_{pred}(x_i, b, m) - y_i)^2}{2\sigma_{y,i}^2}$$
About Julia's "Optim" optimization package
Simple 1 3 Julia 1.9.3 | Idle
Mode: Edit Ln 2, Col 15 Notes-from-class.ipynb

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Julia 1.9.3

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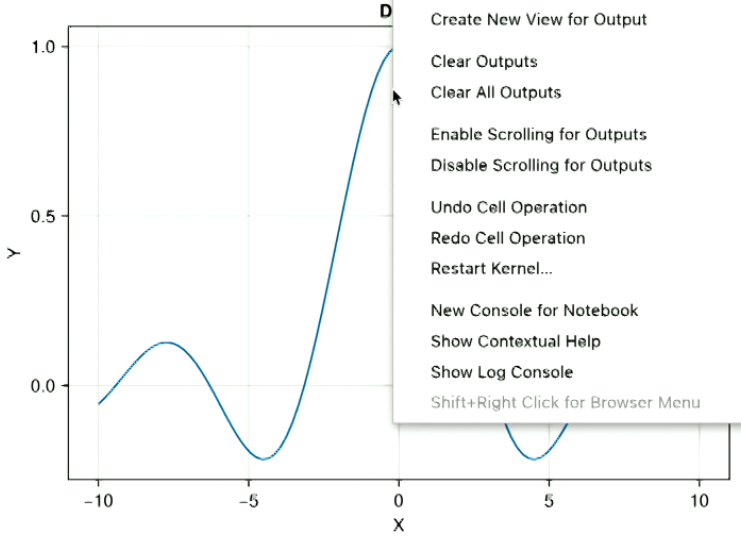
Code

[9]:
```
x_grid = LinRange(-10, +10, 100);
```

[10]:
```
f = Figure()
Axis(f[1,1], title="Data", xlabel="X", y
lines!(x_grid, my_func(x_grid))
f
```

[10]:



Copy Output to Clipboard

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Create New View for Output

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Disable Scrolling for Outputs

Undo Cell Operation	Z
Redo Cell Operation	⇧ Z

Restart Kernel...

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Show Log Console

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[11]:
```
using Optim
```

We want to use the "Optim.optimize" function. This takes a function and an initial guess, and finds a local minimum of the function. It assumes the function takes a single argument, a *vector* of numbers, and returns a *scalar*.

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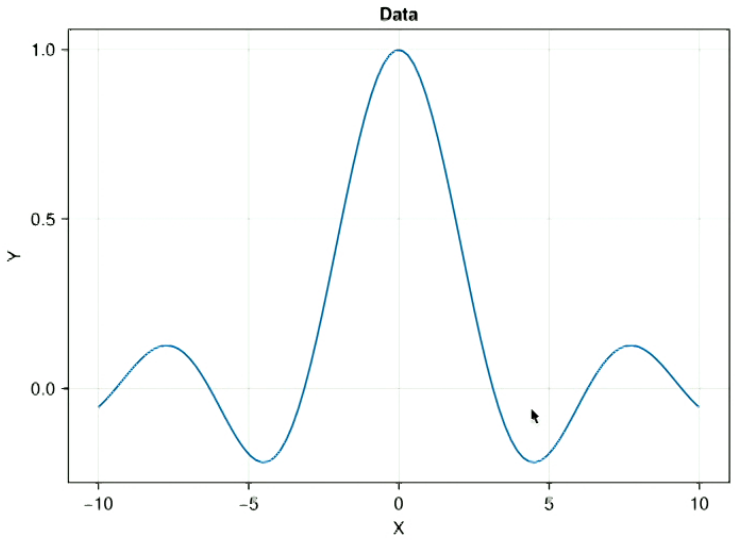
Now create a vector of x values where we want to plot the function;

[9]:
```
x_grid = LinRange(-10, +10, 100);
```

[10]:
```
f = Figure()
Axis(f[1,1], title="Data", xlabel="X", ylabel="Y")
lines!(x_grid, my_func(x_grid))
f
```

[10]:



[11]:
```
using Optim
```

We want to use the "Optim.optimize" function. This takes a function and an initial guess, and finds a local minimum of the function. It assumes the function takes a single argument, a *vector* of numbers, and returns a *scalar*.

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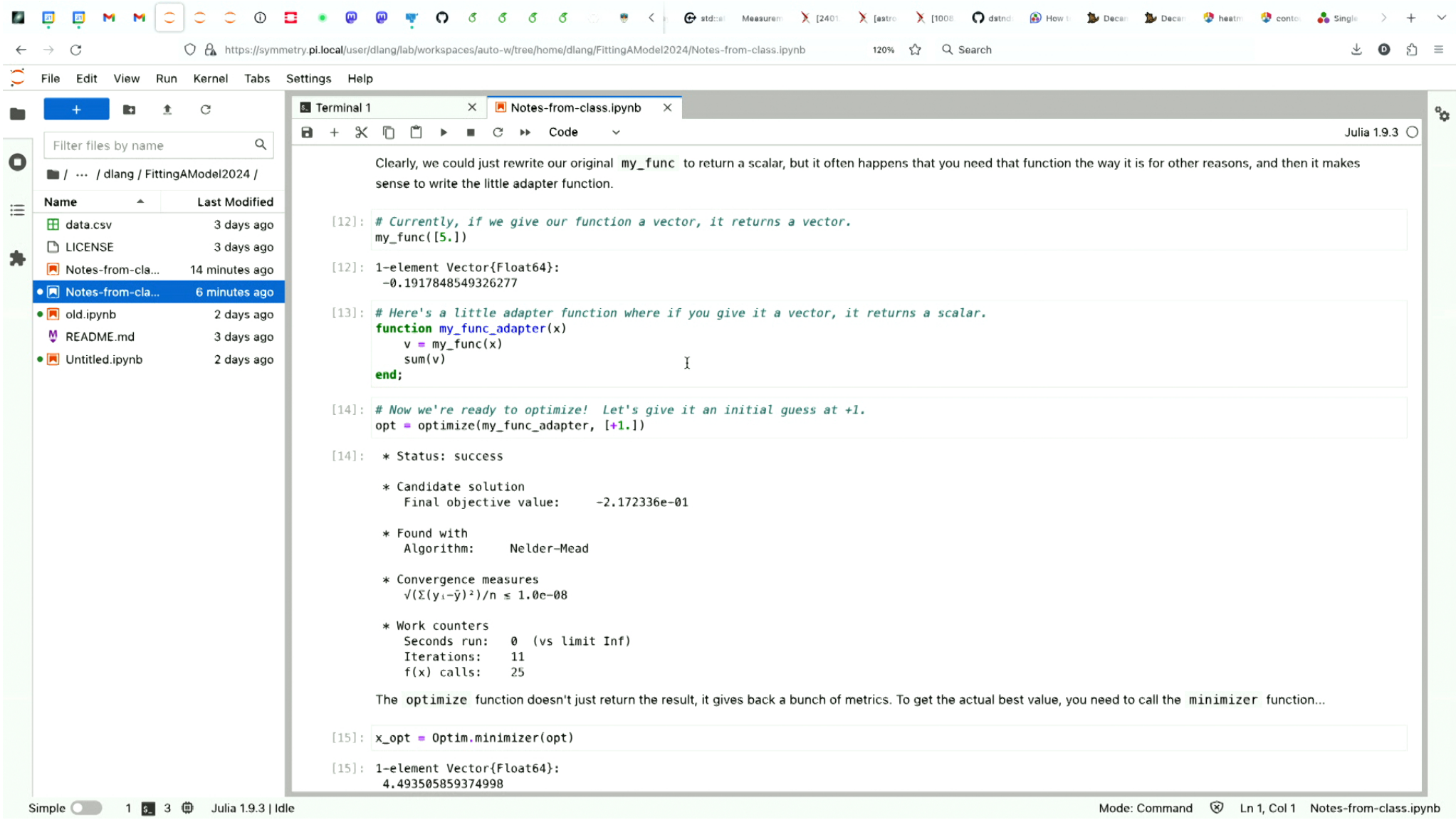
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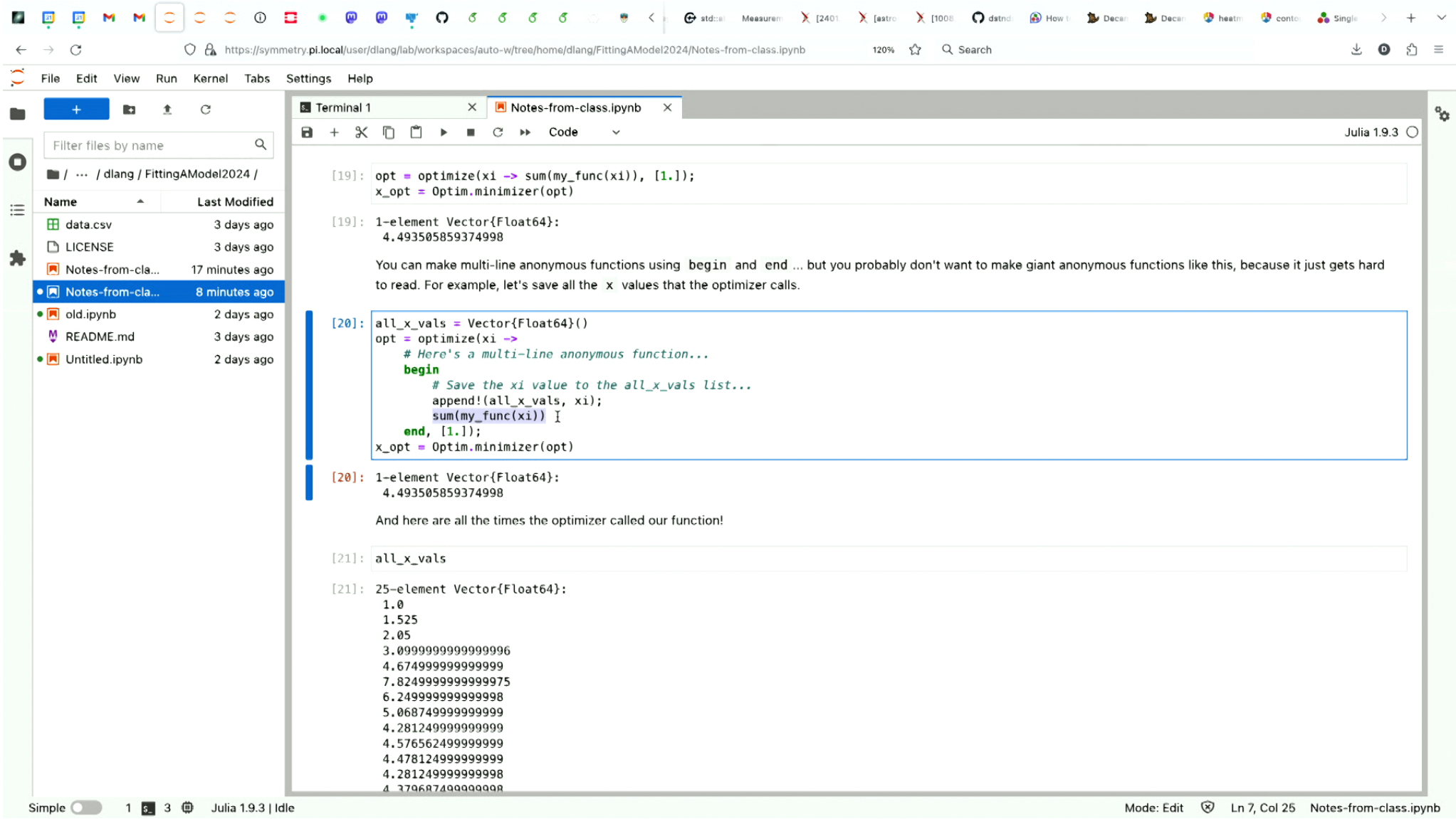
Clearly, we could just rewrite our original my_func to return a scalar, but it often happens that you need that function the way it is for other reasons, and then it makes sense to write the little adapter function.
[12]: # Currently, if we give our function a vector, it returns a vector.
my_func([5.])
[12]: 1-element Vector{Float64}:
-0.1917848549326277
[13]: # Here's a little adapter function where if you give it a vector, it returns a scalar.
function my_func_adapter(x)
v = my_func(x)
sum(v)
end;
[14]: # Now we're ready to optimize! Let's give it an initial guess at +1.
opt = optimize(my_func_adapter, [+1.])
[14]: * Status: success
* Candidate solution
Final objective value: -2.172336e-01
* Found with
Algorithm: Nelder-Mead
* Convergence measures
√(Σ(yᵢ−ȳ)²)/n ≤ 1.0e-08
* Work counters
Seconds run: 0 (vs limit Inf)
Iterations: 11
f(x) calls: 25
The optimize function doesn't just return the result, it gives back a bunch of metrics. To get the actual best value, you need to call the minimizer function...
[15]: x_opt = Optim.minimizer(opt)
[15]: 1-element Vector{Float64}:
4.493505859374998
Simple 1 3 Julia 1.9.3 | Idle
Mode: Command Ln 1, Col 1 Notes-from-class.ipynb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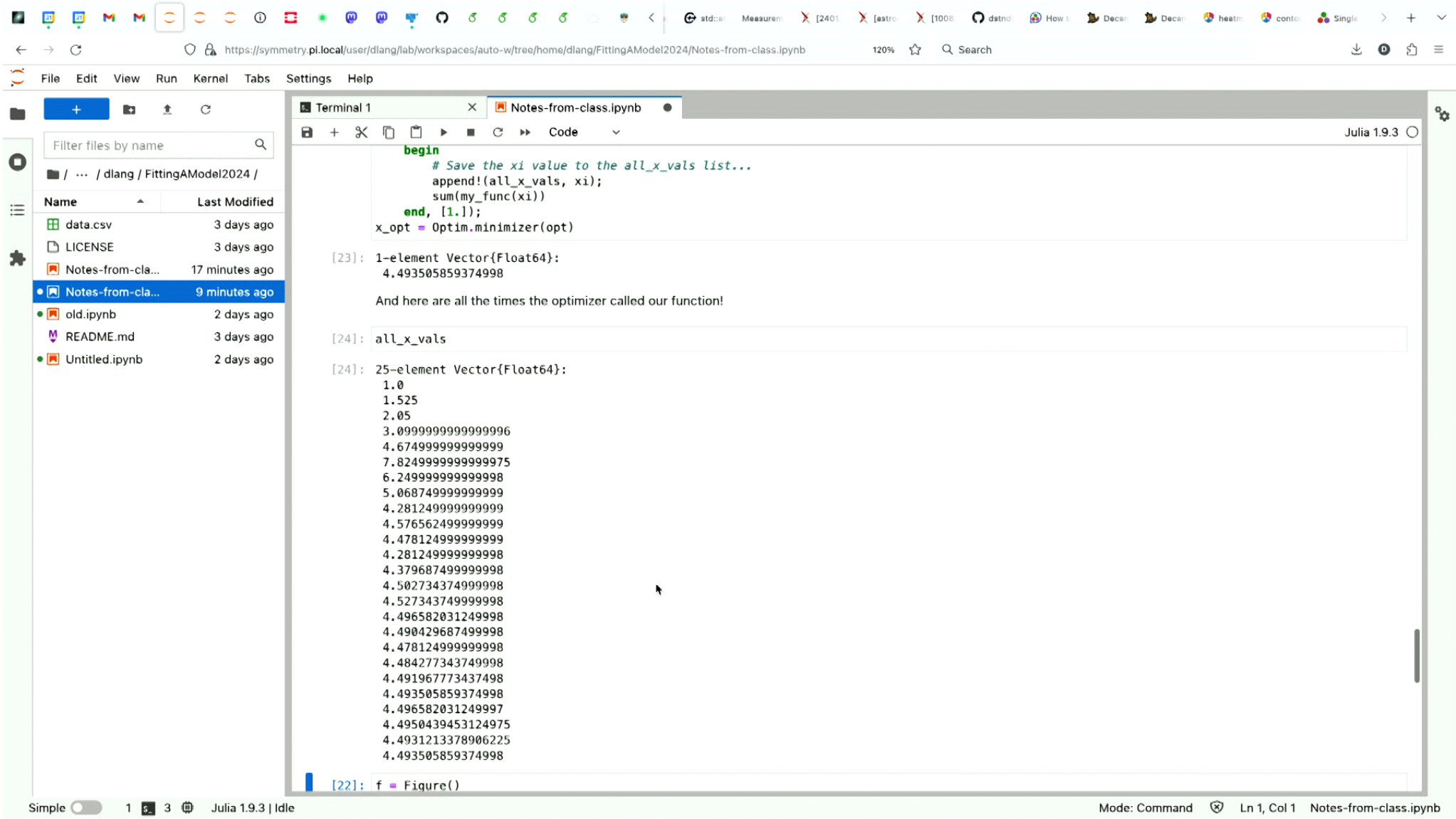
[19]: opt = optimize(xi -> sum(my_func(xi)), [1.]);
x_opt = Optim.minimizer(opt)
[19]: 1-element Vector{Float64}:
4.493505859374998
You can make multi-line anonymous functions using begin and end ... but you probably don't want to make giant anonymous functions like this, because it just gets hard to read. For example, let's save all the x values that the optimizer calls.
[20]: all_x_vals = Vector{Float64}()
opt = optimize(xi ->
# Here's a multi-line anonymous function...
begin
# Save the xi value to the all_x_vals list...
append!(all_x_vals, xi);
sum(my_func(xi))
end, [1.]);
x_opt = Optim.minimizer(opt)
[20]: 1-element Vector{Float64}:
4.493505859374998
And here are all the times the optimizer called our function!
[21]: all_x_vals
[21]: 25-element Vector{Float64}:
1.0
1.525
2.05
3.0999999999999996
4.674999999999999
7.8249999999999975
6.249999999999998
5.068749999999999
4.281249999999999
4.576562499999999
4.478124999999999
4.281249999999998
4.379687499999998
Simple 1 3 Julia 1.9.3 | Idle
Mode: Edit Ln 7, Col 25 Notes-from-class.ipynb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begin
# Save the xi value to the all_x_vals list...
append!(all_x_vals, xi);
sum(my_func(xi))
end, [1.]);
x_opt = Optim.minimizer(opt)
[23]: 1-element Vector{Float64}:
4.493505859374998
And here are all the times the optimizer called our function!
[24]: all_x_vals
[24]: 25-element Vector{Float64}:
1.0
1.525
2.05
3.0999999999999996
4.674999999999999
7.8249999999999975
6.249999999999998
5.068749999999999
4.281249999999999
4.576562499999999
4.478124999999999
4.281249999999998
4.379687499999998
4.502734374999998
4.527343749999998
4.496582031249998
4.490429687499998
4.478124999999998
4.484277343749998
4.491967773437498
4.493505859374998
4.496582031249997
4.4950439453124975
4.4931213378906225
4.493505859374998
[22]: f = Figure()
Simple 1 3 Julia 1.9.3 | Idle
Mode: Command Ln 1, Col 1 Notes-from-class.ipynb

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Code | Julia 1.9.3

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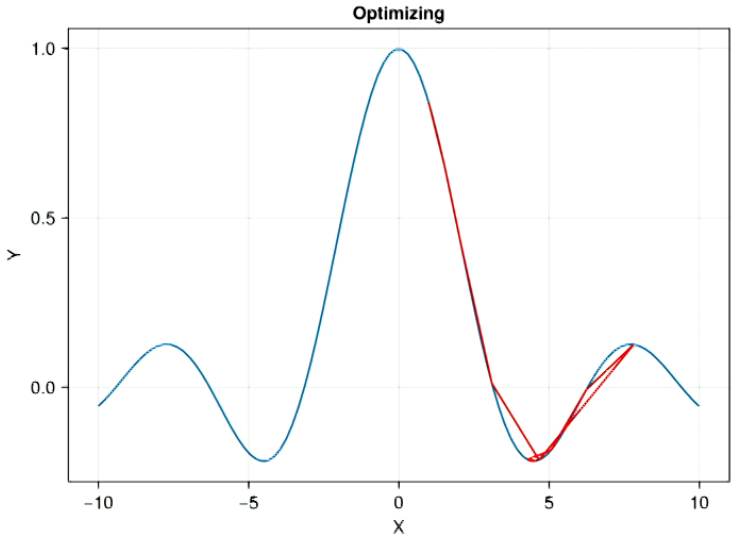
```
4.4842773437499998
4.491967773437498
4.493505859374998
4.496582031249997
4.4950439453124975
4.4931213378906225
4.493505859374998
```

[25]:
```
f = Figure()
Axis(f[1,1], title="Optimizing", xlabel="X", ylabel="Y")
lines!(x_grid, my_func(x_grid))
#scatter!(x_opt, my_func(x_opt), color=:orange, markersize=5)
lines!(all_x_vals, my_func(all_x_vals), color=:red)
f
```

[25]:



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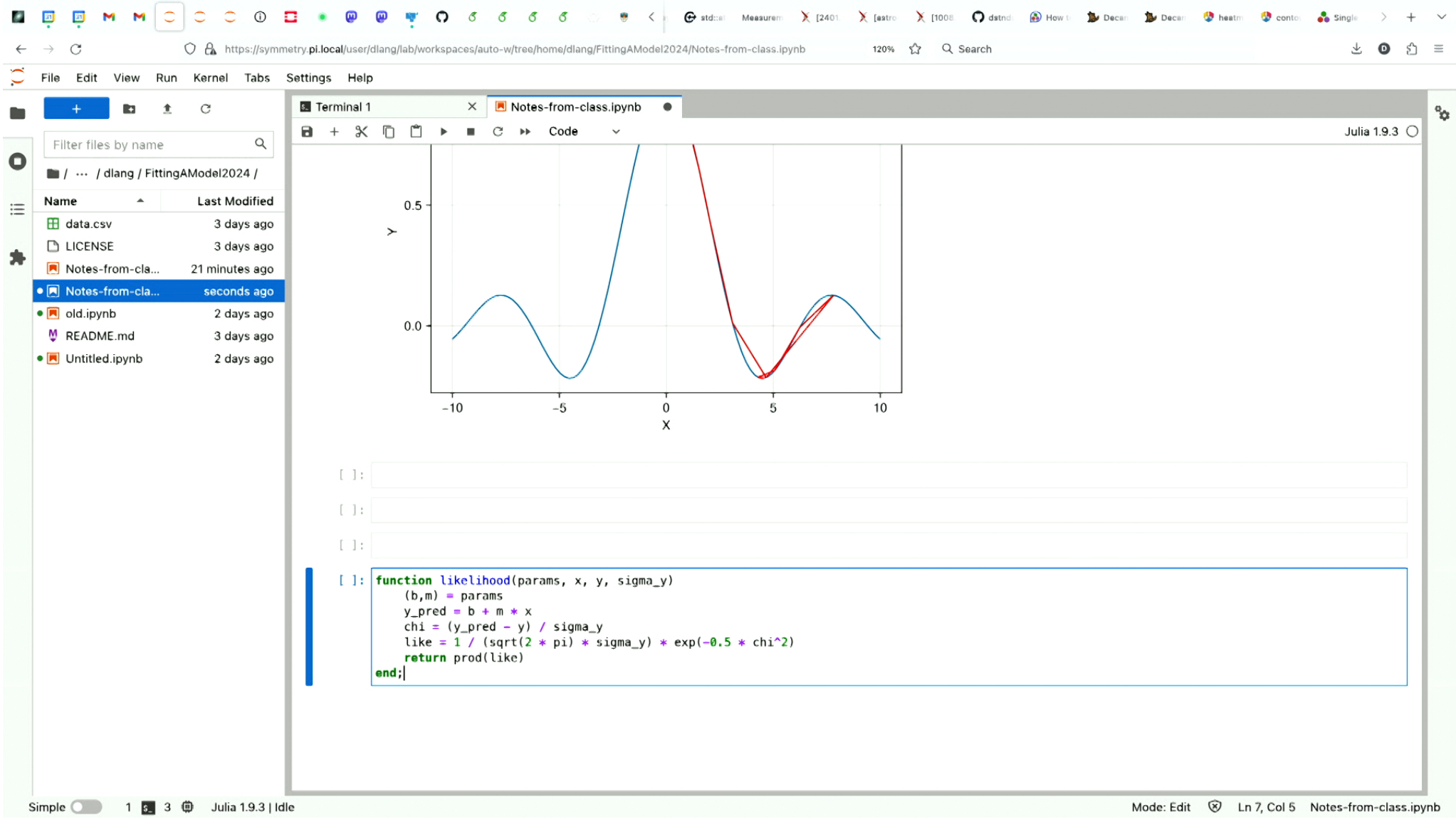
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[ ]:
[ ]:
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[ ]:
function likelihood(params, x, y, sigma_y)
(b,m) = params
y_pred = b + m * x
chi = (y_pred - y) / sigma_y
like = 1 / (sqrt(2 * pi) * sigma_y) * exp(-0.5 * chi^2)
return prod(like)
end;
Simple
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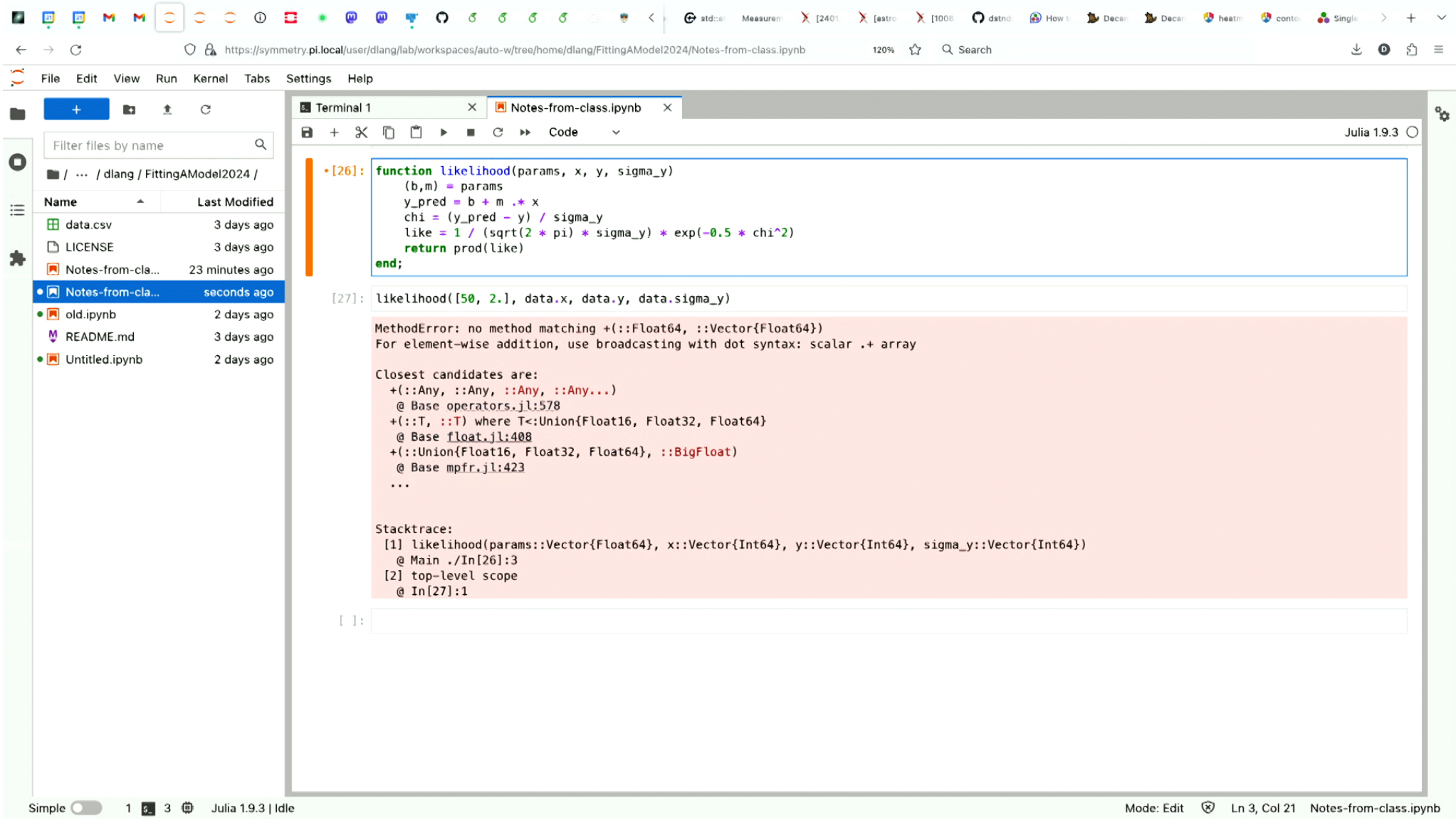
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[26]:
function likelihood(params, x, y, sigma_y)
(b,m) = params
y_pred = b + m .* x
chi = (y_pred - y) / sigma_y
like = 1 / (sqrt(2 * pi) * sigma_y) * exp(-0.5 * chi^2)
return prod(like)
end;
[27]:
likelihood([50, 2.], data.x, data.y, data.sigma_y)
MethodError: no method matching +(::Float64, ::Vector{Float64})
For element-wise addition, use broadcasting with dot syntax: scalar .+ array
Closest candidates are:
+(::Any, ::Any, ::Any, ::Any...)
@ Base operators.jl:578
+(::T, ::T) where T<:Union{Float16, Float32, Float64}
@ Base float.jl:408
+(::Union{Float16, Float32, Float64}, ::BigFloat)
@ Base mpfr.jl:423
...
Stacktrace:
[1] likelihood(params::Vector{Float64}, x::Vector{Int64}, y::Vector{Int64}, sigma_y::Vector{Int64})
@ Main ./In[26]:3
[2] top-level scope
@ In[27]:1
[ ]:
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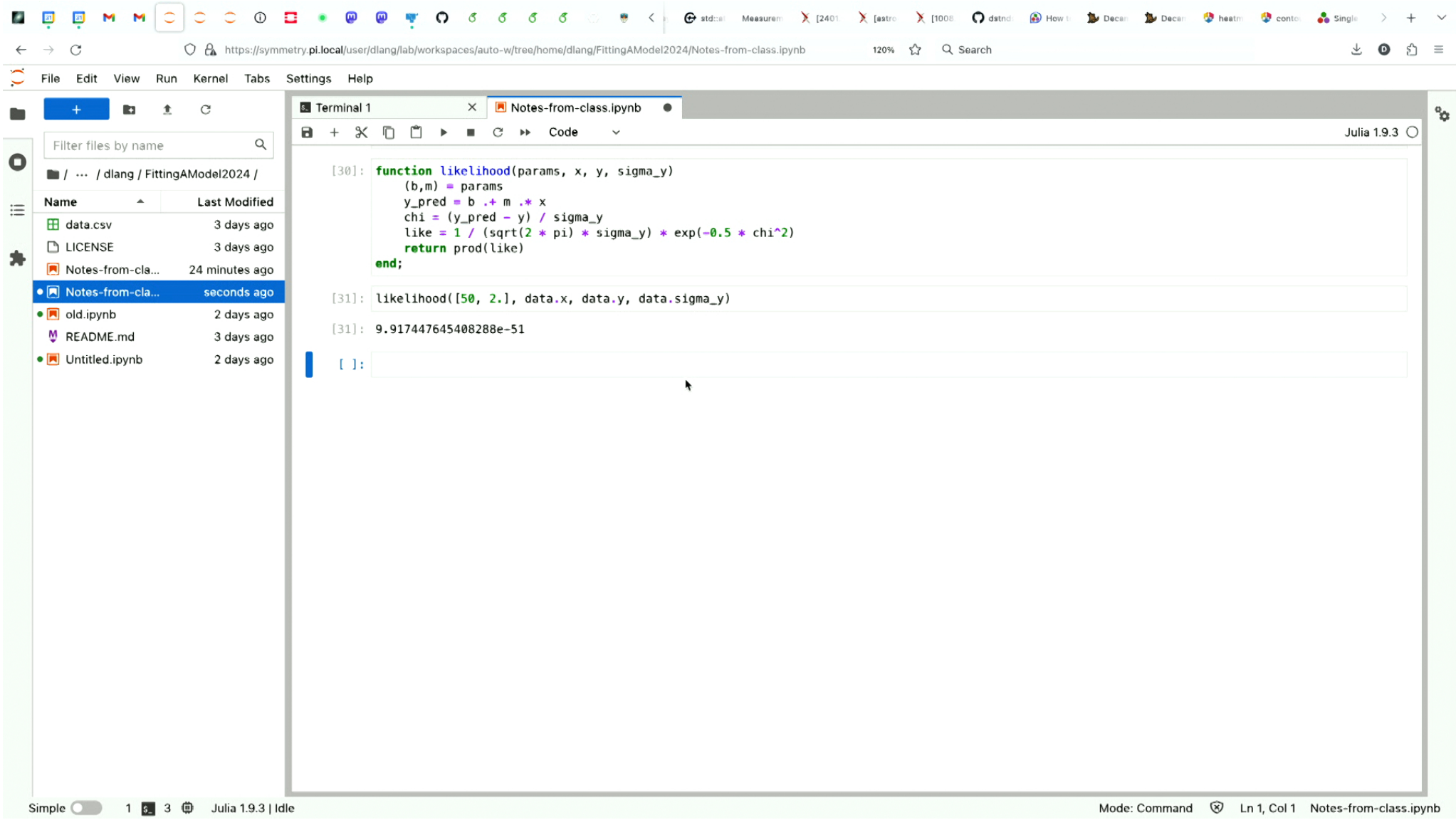
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[30]:
function likelihood(params, x, y, sigma_y)
(b,m) = params
y_pred = b .+ m .* x
chi = (y_pred - y) / sigma_y
like = 1 / (sqrt(2 * pi) * sigma_y) * exp(-0.5 * chi^2)
return prod(like)
end;
[31]:
likelihood([50, 2.], data.x, data.y, data.sigma_y)
[31]:
9.917447645408288e-51
[ ]:
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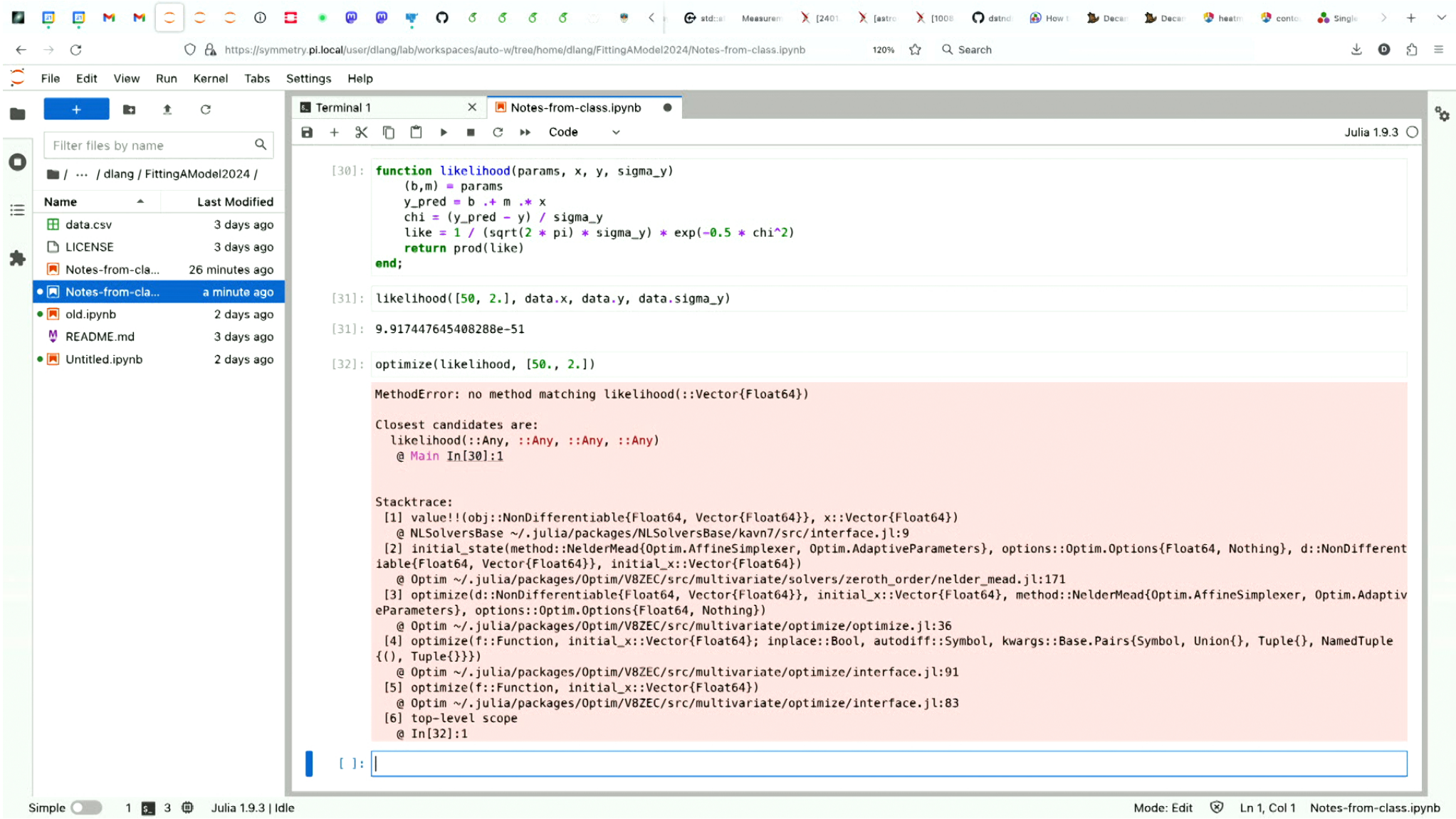
Notes-from-class.ipynb


https://symmetry.pi.local/user/dlang/lab/workspaces/auto-w/tree/home/dlang/FittingAModel2024/Notes-from-class.ipynb
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Terminal 1
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Code
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Notes-from-cla... 26 minutes ago
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Untitled.ipynb 2 days ago
[30]:
function likelihood(params, x, y, sigma_y)
(b,m) = params
y_pred = b .+ m .* x
chi = (y_pred - y) / sigma_y
like = 1 / (sqrt(2 * pi) * sigma_y) * exp(-0.5 * chi^2)
return prod(like)
end;
[31]:
likelihood([50, 2.], data.x, data.y, data.sigma_y)
[31]:
9.917447645408288e-51
[32]:
optimize(likelihood, [50., 2.])
MethodError: no method matching likelihood(::Vector{Float64})
Closest candidates are:
likelihood(::Any, ::Any, ::Any, ::Any)
@ Main In[30]:1
Stacktrace:
[1] value!!(obj::NonDifferentiable{Float64, Vector{Float64}}, x::Vector{Float64})
@ NLSolversBase ~/.julia/packages/NLSolversBase/kavn7/src/interface.jl:9
[2] initial_state(method::NelderMead{Optim.AffineSimplexer, Optim.AdaptiveParameters}, options::Optim.Options{Float64, Nothing}, d::NonDifferentiable{Float64, Vector{Float64}}, initial_x::Vector{Float64})
@ Optim ~/.julia/packages/Optim/V8ZEC/src/multivariate/solvers/zeroth_order/nelder_mead.jl:171
[3] optimize(d::NonDifferentiable{Float64, Vector{Float64}}, initial_x::Vector{Float64}, method::NelderMead{Optim.AffineSimplexer, Optim.AdaptiveParameters}, options::Optim.Options{Float64, Nothing})
@ Optim ~/.julia/packages/Optim/V8ZEC/src/multivariate/optimize/optimize.jl:36
[4] optimize(f::Function, initial_x::Vector{Float64}; inplace::Bool, autodiff::Symbol, kwargs::Base.Pairs{Symbol, Union{}, Tuple{}, NamedTuple{(), Tuple{}}})
@ Optim ~/.julia/packages/Optim/V8ZEC/src/multivariate/optimize/interface.jl:91
[5] optimize(f::Function, initial_x::Vector{Float64})
@ Optim ~/.julia/packages/Optim/V8ZEC/src/multivariate/optimize/interface.jl:83
[6] top-level scope
@ In[32]:1
[ ]:
Simple
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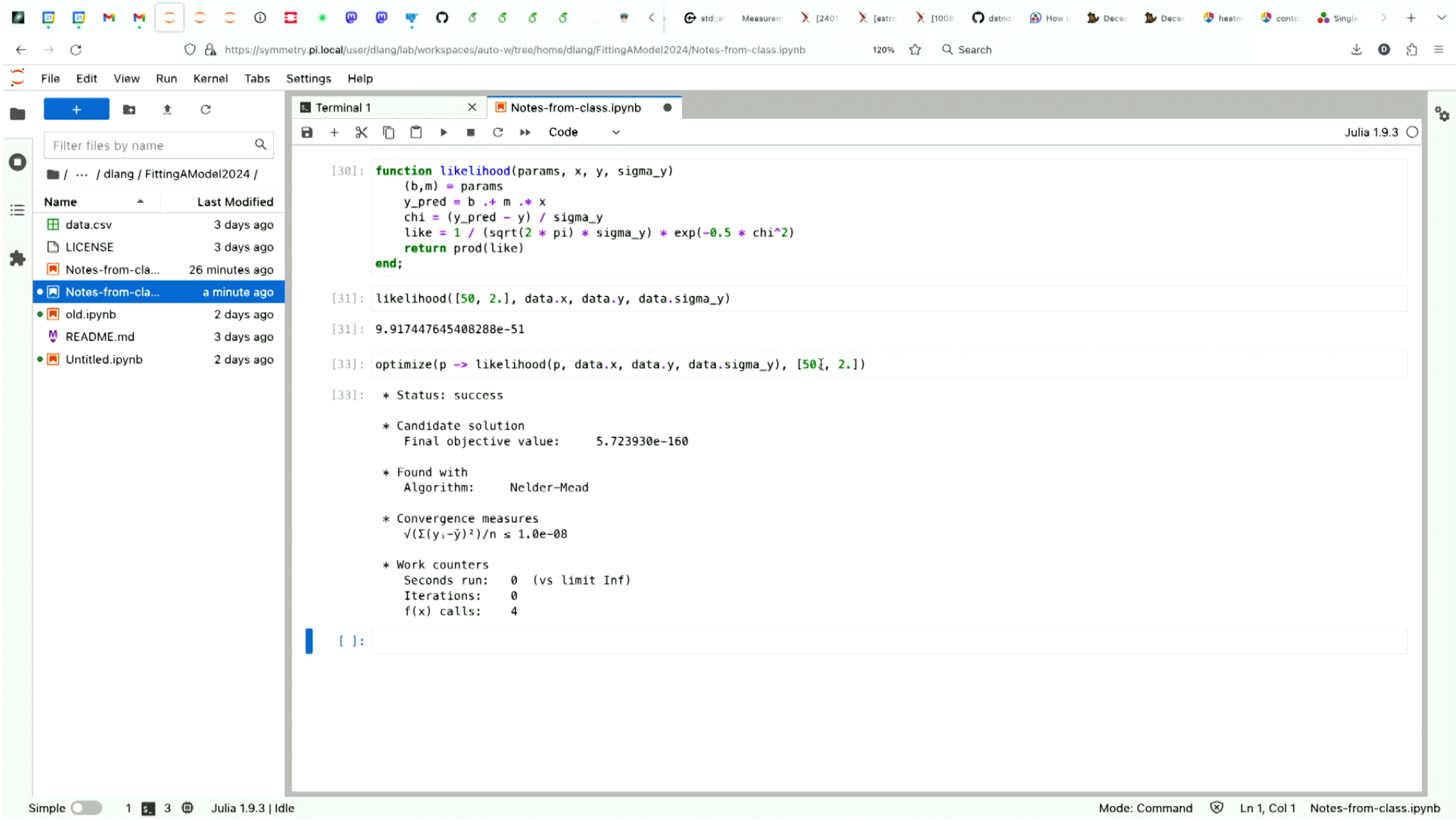
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[30]:
function likelihood(params, x, y, sigma_y)
(b,m) = params
y_pred = b .+ m .* x
chi = (y_pred - y) / sigma_y
like = 1 / (sqrt(2 * pi) * sigma_y) * exp(-0.5 * chi^2)
return prod(like)
end;
[31]:
likelihood([50, 2.], data.x, data.y, data.sigma_y)
[31]:
9.917447645408288e-51
[33]:
optimize(p -> likelihood(p, data.x, data.y, data.sigma_y), [50, 2.])
[33]:
* Status: success
* Candidate solution
Final objective value: 5.723930e-160
* Found with
Algorithm: Nelder-Mead
* Convergence measures
√(Σ(yᵢ-ȳ)²)/n ≤ 1.0e-08
* Work counters
Seconds run: 0 (vs limit Inf)
Iterations: 0
f(x) calls: 4
[ ]:
Simple
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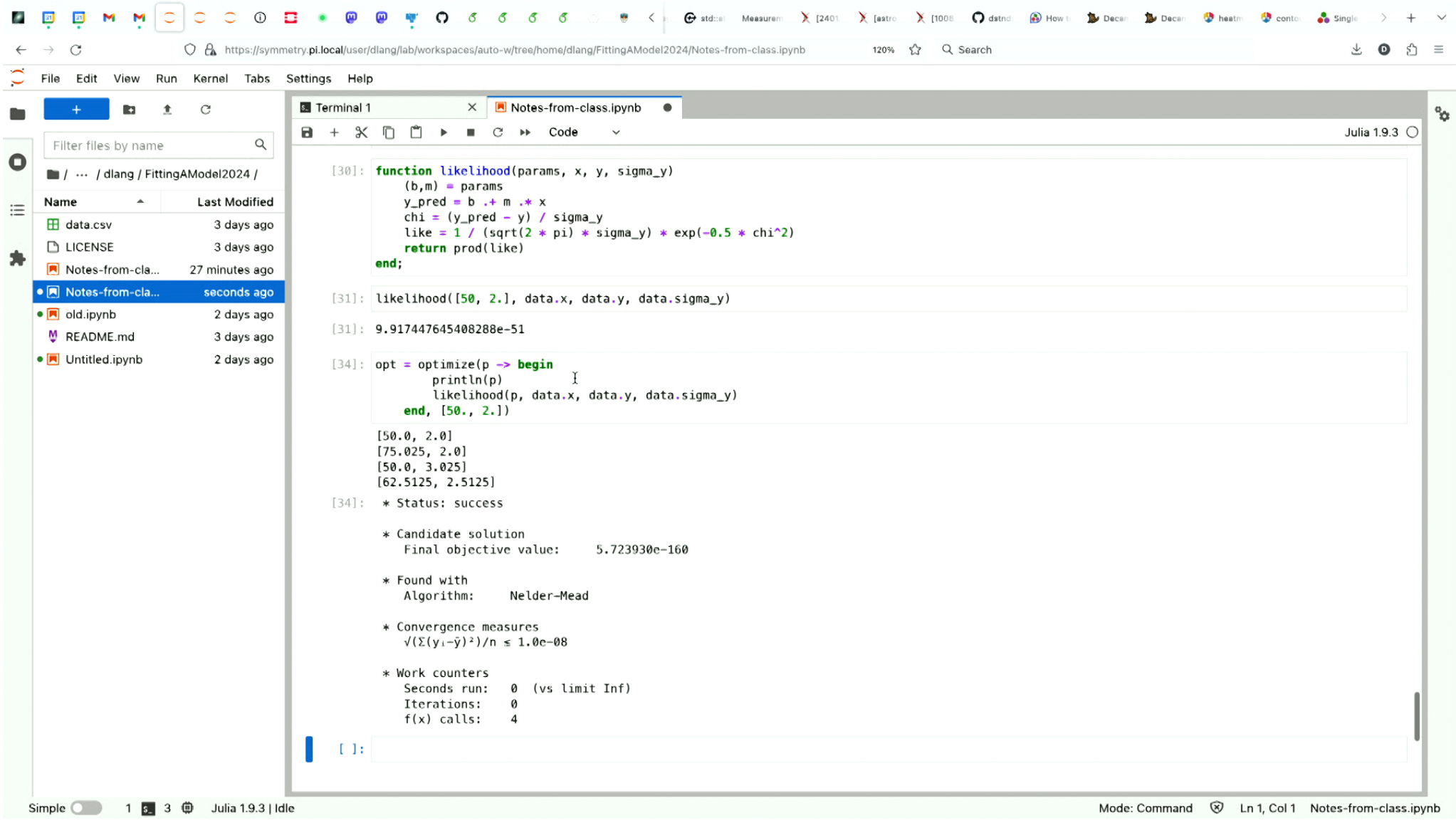
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[31]:
likelihood([50, 2.], data.x, data.y, data.sigma_y)
[31]:
9.917447645408288e-51
[34]:
opt = optimize(p -> begin
println(p)
likelihood(p, data.x, data.y, data.sigma_y)
end, [50., 2.])
[50.0, 2.0]
[75.025, 2.0]
[50.0, 3.025]
[62.5125, 2.5125]
[34]:
* Status: success
* Candidate solution
Final objective value: 5.723930e-160
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Algorithm: Nelder-Mead
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Seconds run: 0 (vs limit Inf)
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[ ]:
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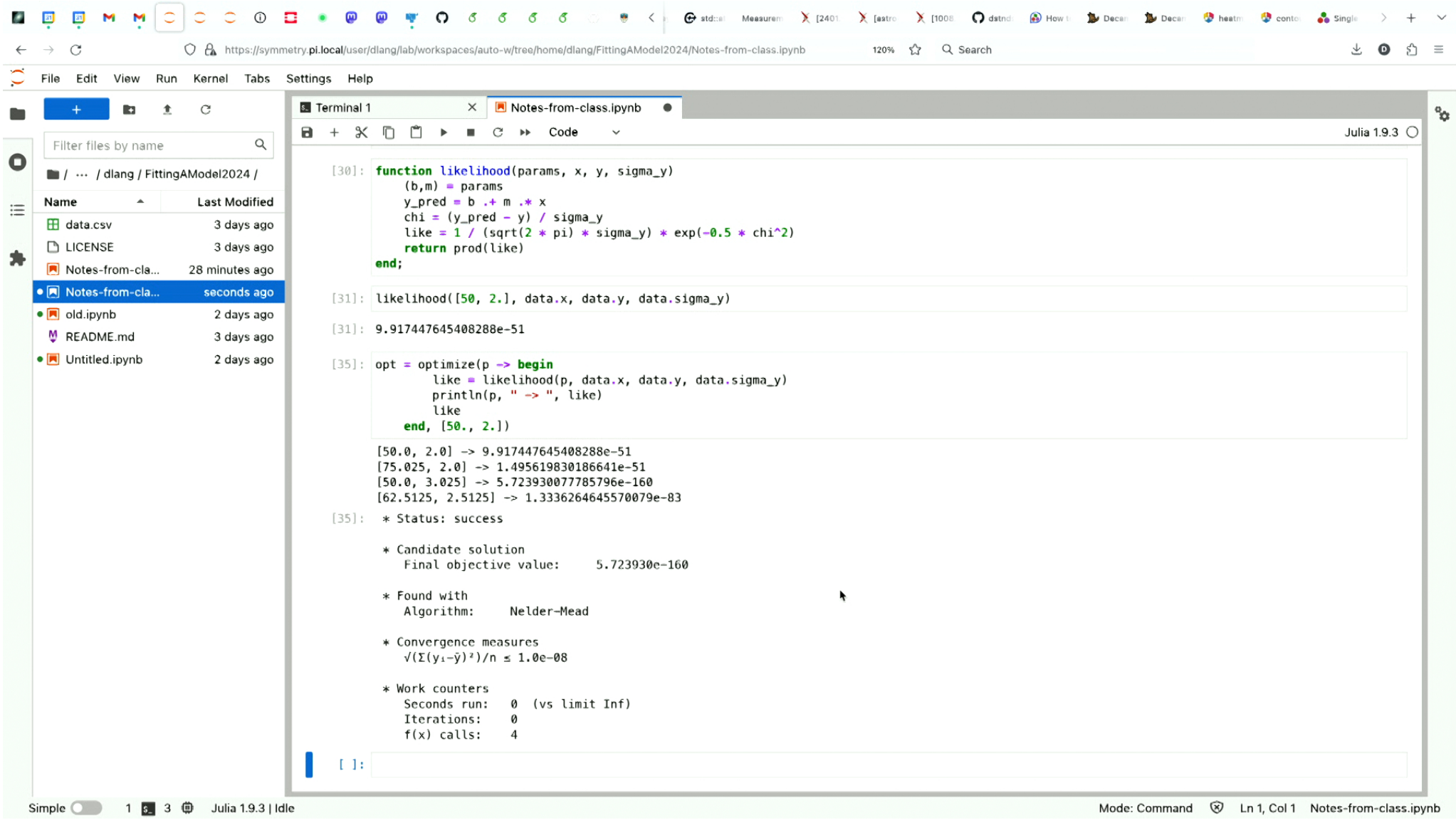
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[31]:
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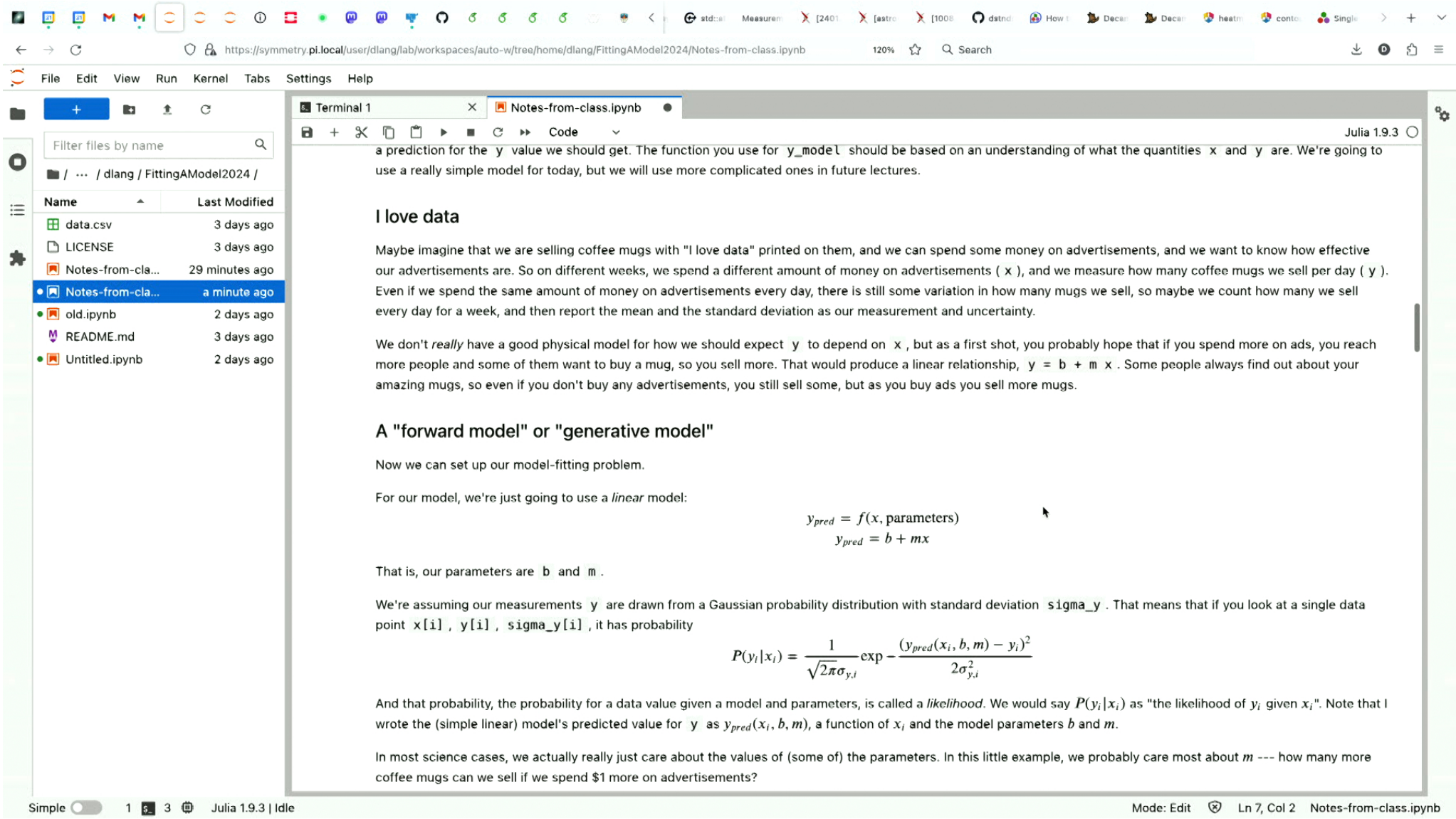
[35]:
opt = optimize(p -> begin
like = likelihood(p, data.x, data.y, data.sigma_y)
println(p, " -> ", like)
like
end, [50., 2.])
[50.0, 2.0] -> 9.917447645408288e-51
[75.025, 2.0] -> 1.495619830186641e-51
[50.0, 3.025] -> 5.723930077785796e-160
[62.5125, 2.5125] -> 1.3336264645570079e-83
[35]:
* Status: success
* Candidate solution
Final objective value: 5.723930e-160
* Found with
Algorithm: Nelder-Mead
* Convergence measures
√(Σ(yᵢ-ȳ)²)/n ≤ 1.0e-08
* Work counters
Seconds run: 0 (vs limit Inf)
Iterations: 0
f(x) calls: 4
[ ]:
Simple 1 3 Julia 1.9.3 | Idle
Mode: Command Ln 1, Col 1 Notes-from-class.ipynb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a prediction for the y value we should get. The function you use for y_model should be based on an understanding of what the quantities x and y are. We're going to use a really simple model for today, but we will use more complicated ones in future lectures.
I love data
Maybe imagine that we are selling coffee mugs with "I love data" printed on them, and we can spend some money on advertisements, and we want to know how effective our advertisements are. So on different weeks, we spend a different amount of money on advertisements (x), and we measure how many coffee mugs we sell per day (y). Even if we spend the same amount of money on advertisements every day, there is still some variation in how many mugs we sell, so maybe we count how many we sell every day for a week, and then report the mean and the standard deviation as our measurement and uncertainty.
We don't really have a good physical model for how we should expect y to depend on x , but as a first shot, you probably hope that if you spend more on ads, you reach more people and some of them want to buy a mug, so you sell more. That would produce a linear relationship, y = b + m x . Some people always find out about your amazing mugs, so even if you don't buy any advertisements, you still sell some, but as you buy ads you sell more mugs.
A "forward model" or "generative model"
Now we can set up our model-fitting problem.
For our model, we're just going to use a linear model:
$$y_{pred} = f(x, \text{parameters})$$
$$y_{pred} = b + mx$$
That is, our parameters are b and m .
We're assuming our measurements y are drawn from a Gaussian probability distribution with standard deviation sigma_y . That means that if you look at a single data point x[i] , y[i] , sigma_y[i] , it has probability
$$P(y_i|x_i) = \frac{1}{\sqrt{2\pi}\sigma_{y,i}} \exp - \frac{(y_{pred}(x_i, b, m) - y_i)^2}{2\sigma_{y,i}^2}$$
And that probability, the probability for a data value given a model and parameters, is called a likelihood. We would say $P(y_i|x_i)$ as "the likelihood of y_i given x_i". Note that I wrote the (simple linear) model's predicted value for y as $y_{pred}(x_i, b, m)$, a function of x_i and the model parameters b and m.
In most science cases, we actually really just care about the values of (some of) the parameters. In this little example, we probably care most about m --- how many more coffee mugs can we sell if we spend $1 more on advertisements?
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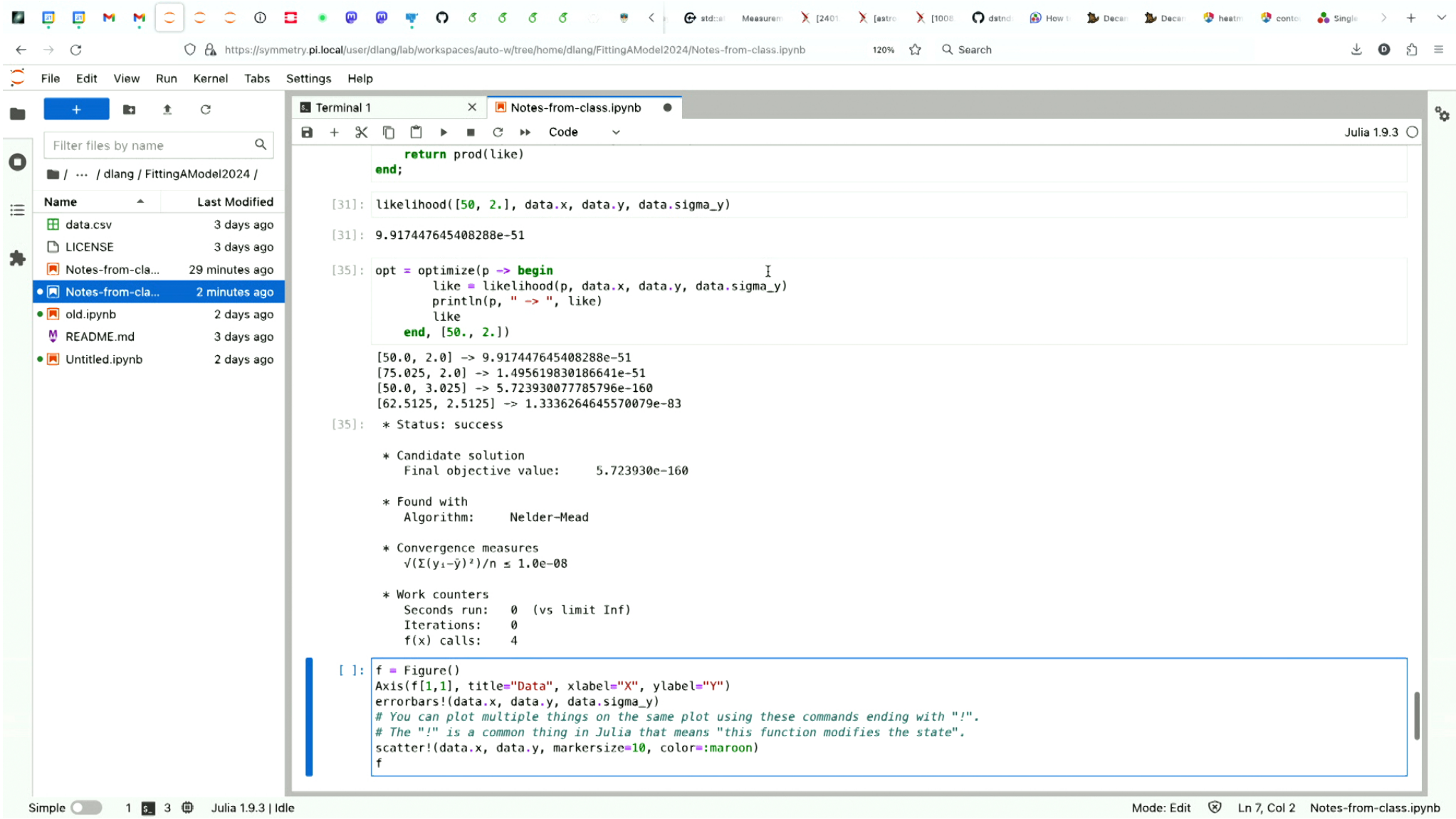
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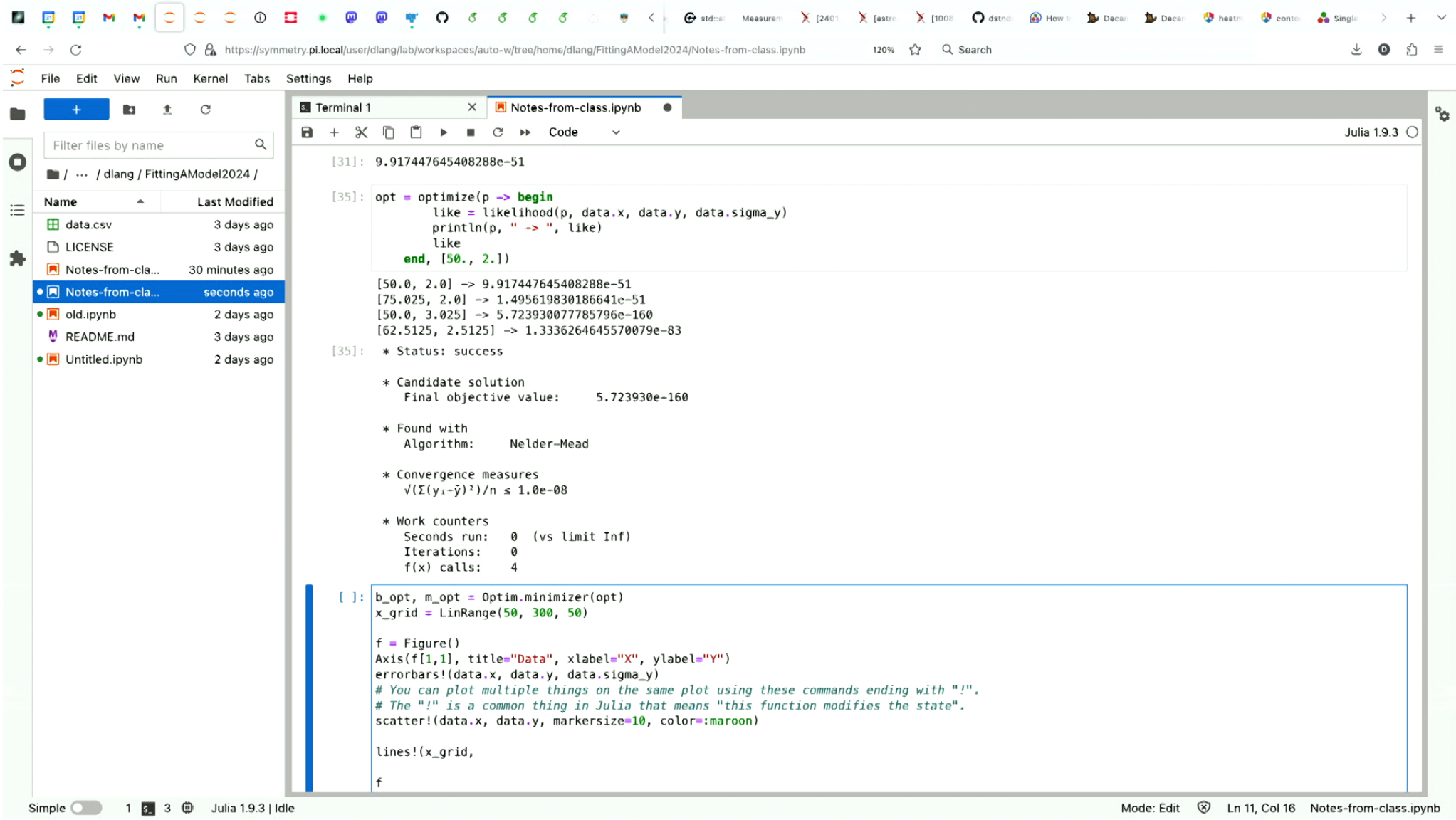
return prod(like)
end;
[31]: likelihood([50, 2.], data.x, data.y, data.sigma_y)
[31]: 9.917447645408288e-51
[35]: opt = optimize(p -> begin
like = likelihood(p, data.x, data.y, data.sigma_y)
println(p, " -> ", like)
like
end, [50., 2.])
[50.0, 2.0] -> 9.917447645408288e-51
[75.025, 2.0] -> 1.495619830186641e-51
[50.0, 3.025] -> 5.723930077785796e-160
[62.5125, 2.5125] -> 1.3336264645570079e-83
[35]: * Status: success
* Candidate solution
Final objective value: 5.723930e-160
* Found with
Algorithm: Nelder-Mead
* Convergence measures
√(Σ(yᵢ−ȳ)²)/n ≤ 1.0e-08
* Work counters
Seconds run: 0 (vs limit Inf)
Iterations: 0
f(x) calls: 4
[ ]: f = Figure()
Axis(f[1,1], title="Data", xlabel="X", ylabel="Y")
errorbars!(data.x, data.y, data.sigma_y)
You can plot multiple things on the same plot using these commands ending with "!".
The "!" is a common thing in Julia that means "this function modifies the state".
scatter!(data.x, data.y, markersize=10, color=:maroon)
f
Simple 1 3 Julia 1.9.3 | Idle
Mode: Edit Ln 7, Col 2 Notes-from-class.ipynb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[31]: 9.917447645408288e-51
[35]: opt = optimize(p -> begin
like = likelihood(p, data.x, data.y, data.sigma_y)
println(p, " -> ", like)
like
end, [50., 2.])
[50.0, 2.0] -> 9.917447645408288e-51
[75.025, 2.0] -> 1.495619830186641e-51
[50.0, 3.025] -> 5.723930077785796e-160
[62.5125, 2.5125] -> 1.3336264645570079e-83
[35]: * Status: success
* Candidate solution
Final objective value: 5.723930e-160
* Found with
Algorithm: Nelder-Mead
* Convergence measures
√(Σ(yᵢ-ȳ)²)/n ≤ 1.0e-08
* Work counters
Seconds run: 0 (vs limit Inf)
Iterations: 0
f(x) calls: 4
[ ]: b_opt, m_opt = Optim.minimizer(opt)
x_grid = LinRange(50, 300, 50)
f = Figure()
Axis(f[1,1], title="Data", xlabel="X", ylabel="Y")
errorbars!(data.x, data.y, data.sigma_y)
You can plot multiple things on the same plot using these commands ending with "!".
The "!" is a common thing in Julia that means "this function modifies the state".
scatter!(data.x, data.y, markersize=10, color=:maroon)
lines!(x_grid,
f
Simple 1 3 Julia 1.9.3 | Idle
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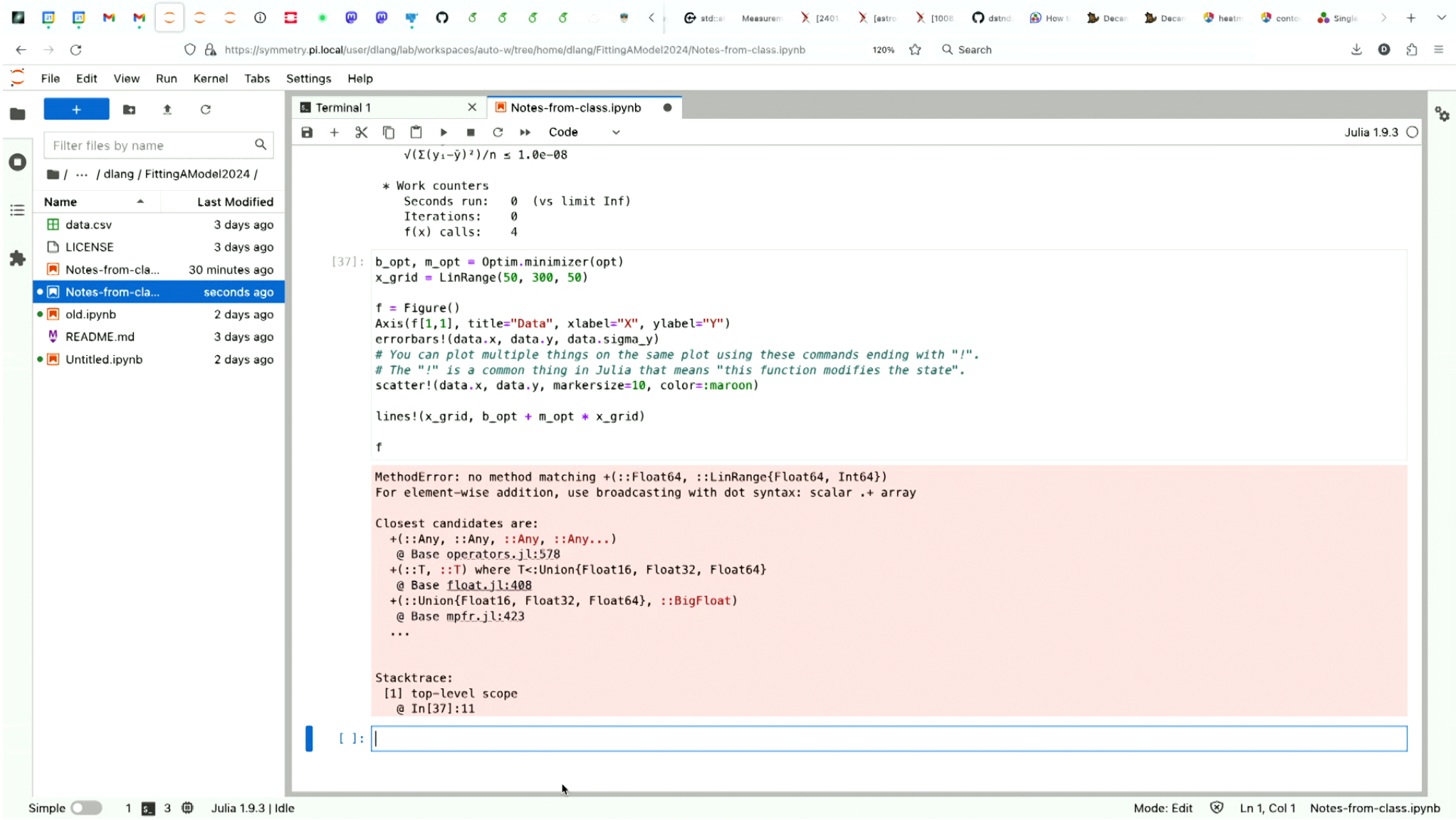
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* Work counters
Seconds run: 0 (vs limit Inf)
Iterations: 0
f(x) calls: 4
[37]:
b_opt, m_opt = Optim.minimizer(opt)
x_grid = LinRange(50, 300, 50)
f = Figure()
Axis(f[1,1], title="Data", xlabel="X", ylabel="Y")
errorbars!(data.x, data.y, data.sigma_y)
You can plot multiple things on the same plot using these commands ending with "!".
The "!" is a common thing in Julia that means "this function modifies the state".
scatter!(data.x, data.y, markersize=10, color=:maroon)
lines!(x_grid, b_opt + m_opt * x_grid)
f
MethodError: no method matching +(::Float64, ::LinRange{Float64, Int64})
For element-wise addition, use broadcasting with dot syntax: scalar .+ array
Closest candidates are:
+(::Any, ::Any, ::Any, ::Any...)
@ Base operators.jl:578
+(::T, ::T) where T<:Union{Float16, Float32, Float64}
@ Base float.jl:408
+(::Union{Float16, Float32, Float64}, ::BigFloat)
@ Base mpfr.jl:423
···
Stacktrace:
[1] top-level scope
@ In[37]:11
[ ]:
Simple 1 3 Julia 1.9.3 | Idle
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Notes-from-cla...	a minute ago
old.ipynb	2 days ago
README.md	3 days ago
Untitled.ipynb	2 days ago

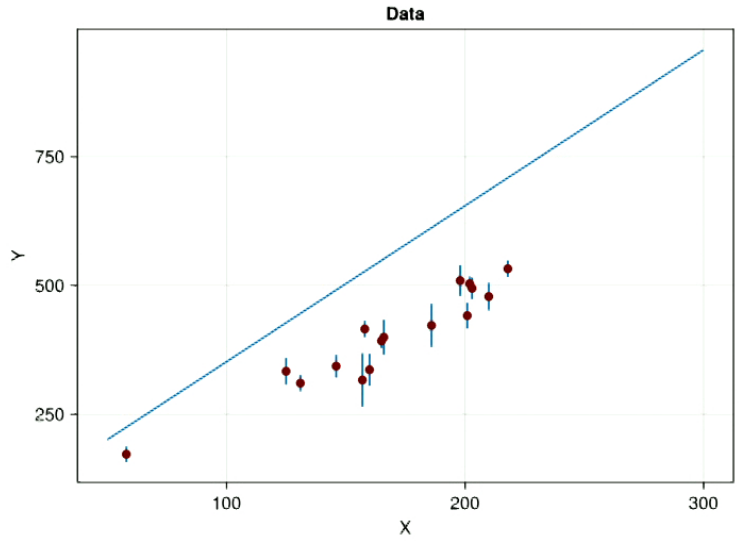
[38]:
```
b_opt, m_opt = Optim.minimizer(opt)
x_grid = LinRange(50, 300, 50)

f = Figure()
Axis(f[1,1], title="Data", xlabel="X", ylabel="Y")
errorbars!(data.x, data.y, data.sigma_y)
# You can plot multiple things on the same plot using these commands ending with "!".
# The "!" is a common thing in Julia that means "this function modifies the state".
scatter!(data.x, data.y, markersize=10, color=:maroon)

lines!(x_grid, b_opt .+ m_opt .* x_grid)

f
```

[38]:



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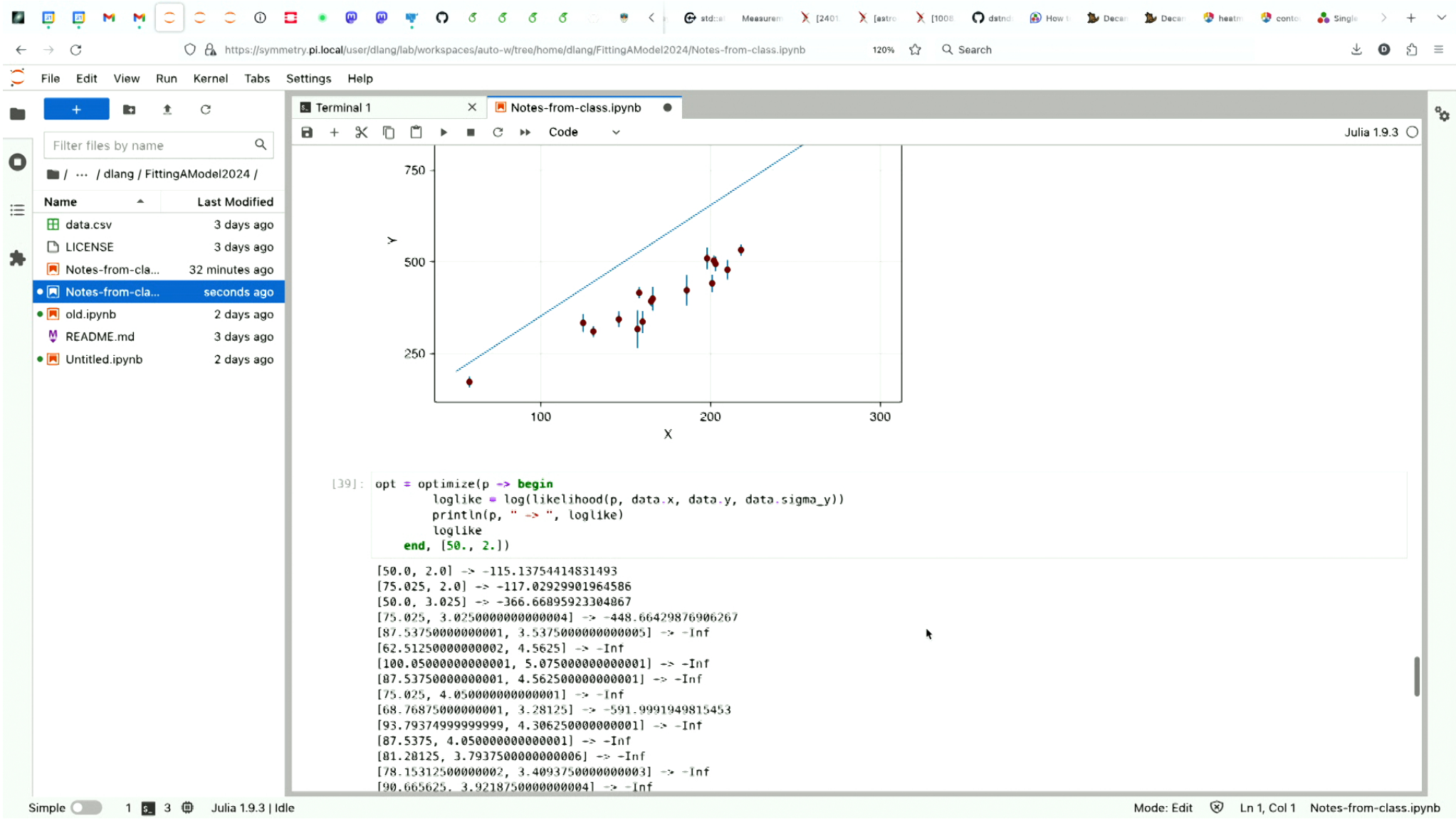
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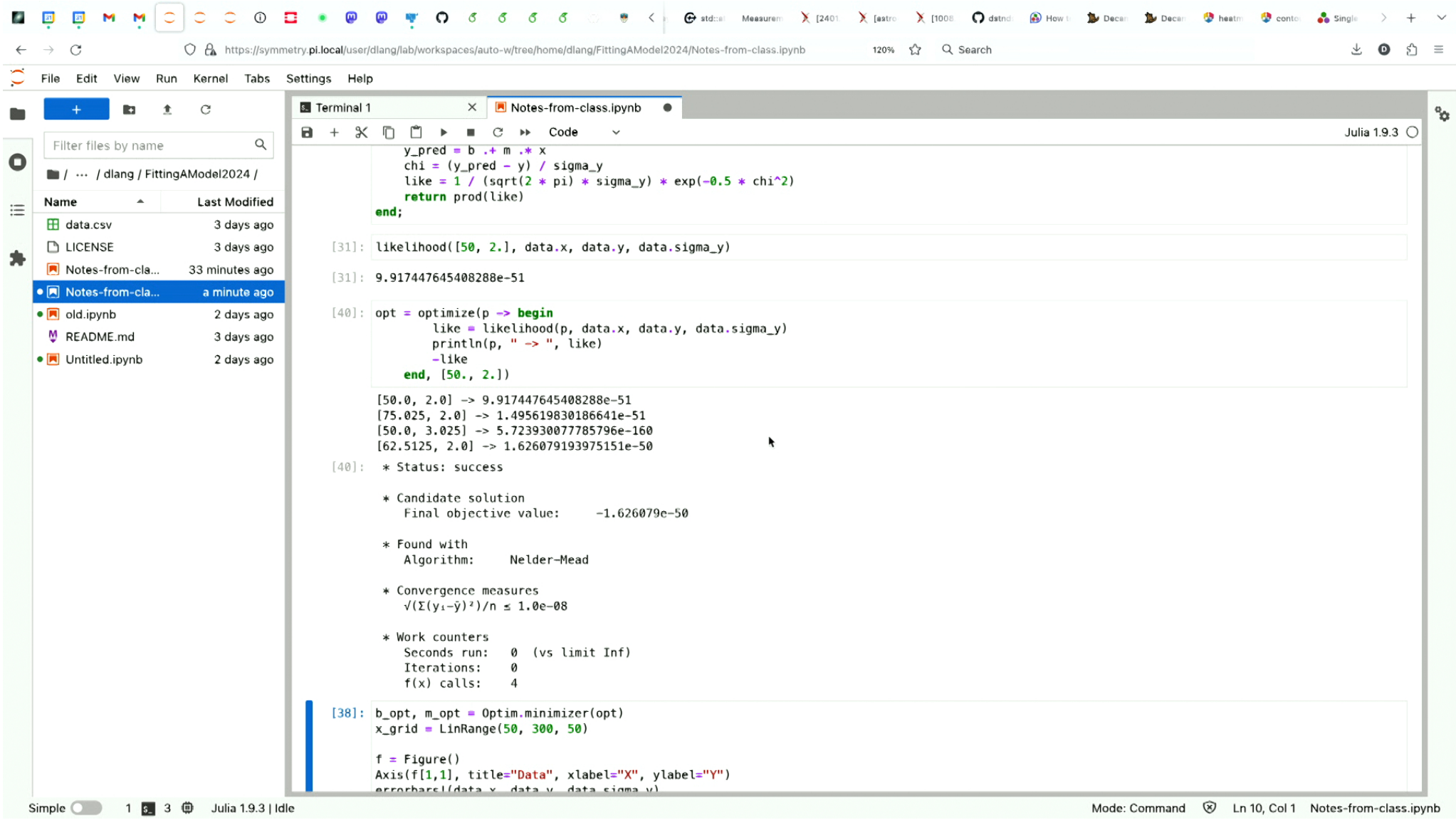
750
500
250
Y
100
200
300
X
[39]:
opt = optimize(p -> begin
loglike = log(likelihood(p, data.x, data.y, data.sigma_y))
println(p, " -> ", loglike)
loglike
end, [50., 2.])
[50.0, 2.0] -> -115.13754414831493
[75.025, 2.0] -> -117.02929901964586
[50.0, 3.025] -> -366.66895923304867
[75.025, 3.0250000000000004] -> -448.66429876906267
[87.53750000000001, 3.5375000000000005] -> -Inf
[62.51250000000002, 4.5625] -> -Inf
[100.05000000000001, 5.075000000000001] -> -Inf
[87.53750000000001, 4.562500000000001] -> -Inf
[75.025, 4.050000000000001] -> -Inf
[68.76875000000001, 3.28125] -> -591.9991949815453
[93.79374999999999, 4.306250000000001] -> -Inf
[87.5375, 4.050000000000001] -> -Inf
[81.28125, 3.7937500000000006] -> -Inf
[78.15312500000002, 3.4093750000000003] -> -Inf
[90.665625, 3.9218750000000004] -> -Inf
Simple 1 3 Julia 1.9.3 | Idle
Mode: Edit Ln 1, Col 1 Notes-from-class.ipynb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like = 1 / (sqrt(2 * pi) * sigma_y) * exp(-0.5 * chi^2)
return prod(like)
end;
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[31]: 9.917447645408288e-51
[40]: opt = optimize(p -> begin
like = likelihood(p, data.x, data.y, data.sigma_y)
println(p, " -> ", like)
-like
end, [50., 2.])
[50.0, 2.0] -> 9.917447645408288e-51
[75.025, 2.0] -> 1.495619830186641e-51
[50.0, 3.025] -> 5.723930077785796e-160
[62.5125, 2.0] -> 1.626079193975151e-50
[40]: * Status: success
* Candidate solution
Final objective value: -1.626079e-50
* Found with
Algorithm: Nelder-Mead
* Convergence measures
√(Σ(yᵢ-ȳ)²)/n ≤ 1.0e-08
* Work counters
Seconds run: 0 (vs limit Inf)
Iterations: 0
f(x) calls: 4
[38]: b_opt, m_opt = Optim.minimizer(opt)
x_grid = LinRange(50, 300, 50)
f = Figure()
Axis(f[1,1], title="Data", xlabel="X", ylabel="Y")
Simple 1 3 Julia 1.9.3 | Idle
Mode: Command Ln 10, Col 1 Notes-from-class.ipynb

https://symmetry.pi.local/user/dlang/lab/workspaces/auto-w/tree/home/dlang/FittingAModel2024/Notes-from-class.ipynb

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Terminal 1 | Notes-from-class.ipynb

Code | Julia 1.9.3

Filter files by name

/ ··· / dlang / FittingAModel2024 /

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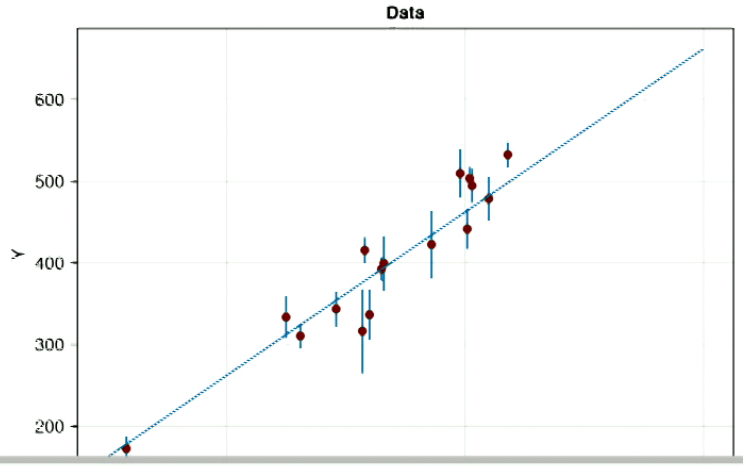
LICENSE	3 days ago
Notes-from-cla...	33 minutes ago
Notes-from-cla...	2 minutes ago
old.ipynb	2 days ago
README.md	3 days ago
Untitled.ipynb	2 days ago

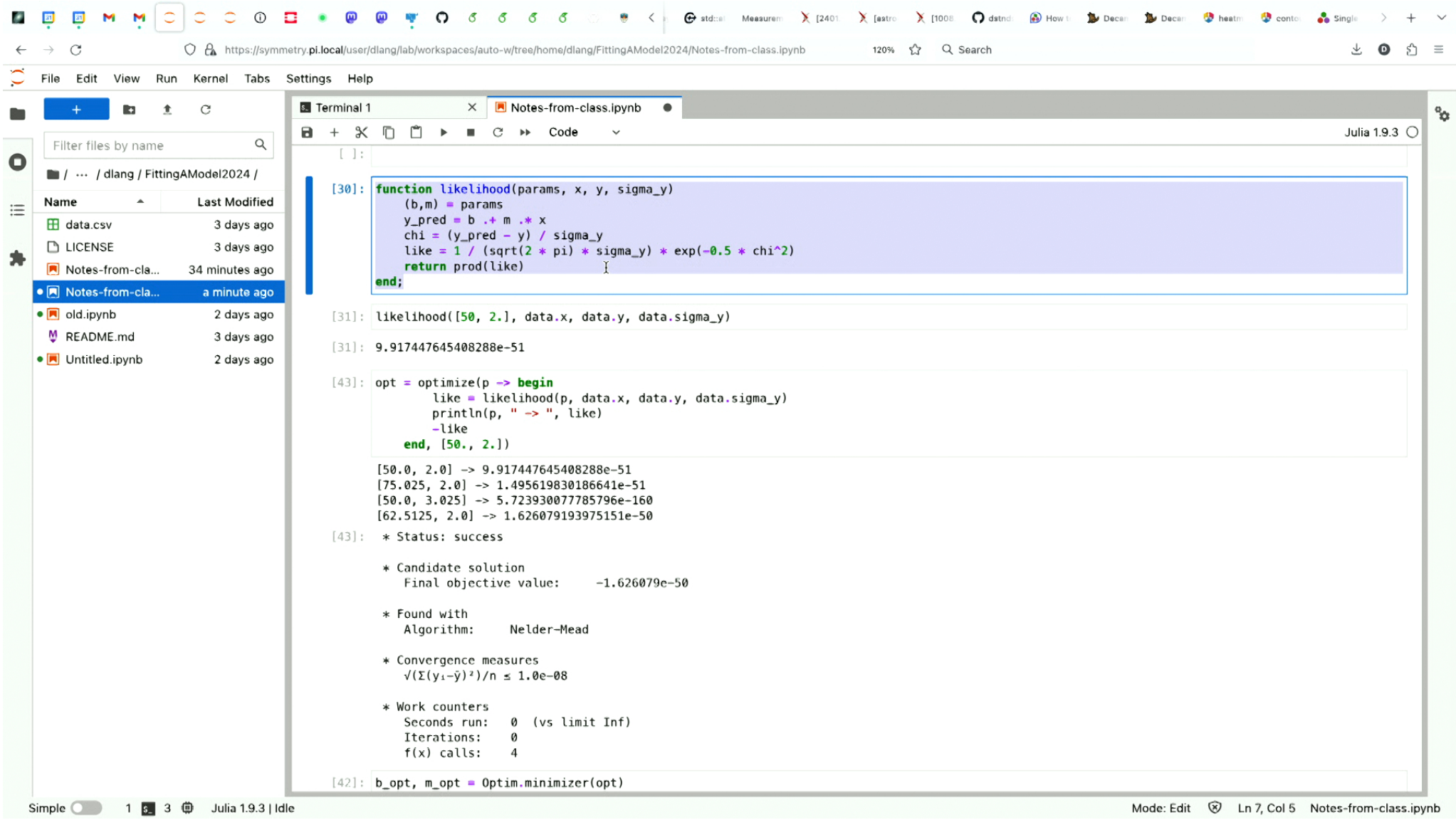
```
 * Work counters
    Seconds run:   0  (vs limit Inf)
    Iterations:    0
    f(x) calls:    4
```

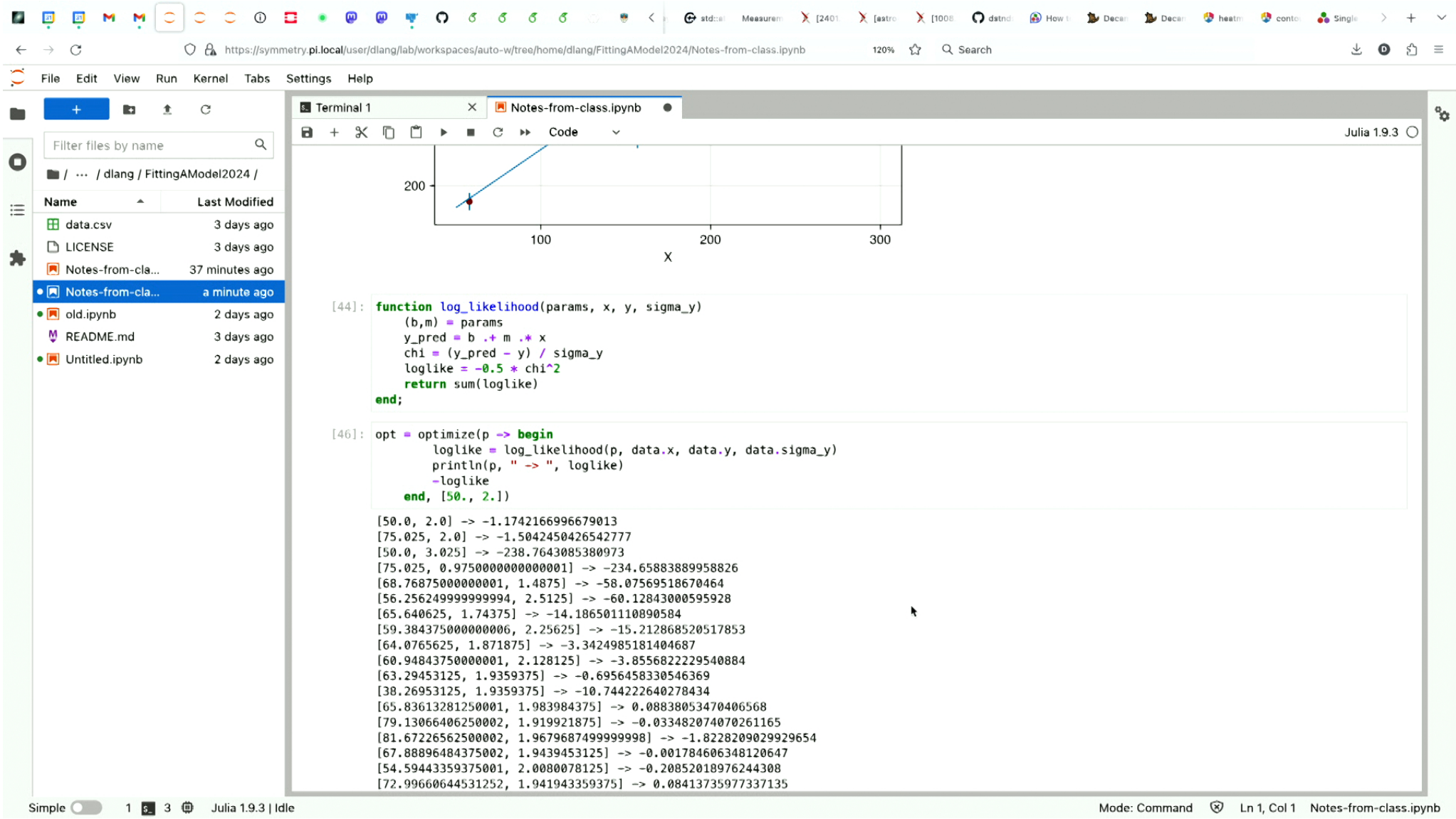
[41]:
```
b_opt, m_opt = Optim.minimizer(opt)
x_grid = LinRange(50, 300, 50)

f = Figure()
Axis(f[1,1], title="Data", xlabel="X", ylabel="Y")
errorbars!(data.x, data.y, data.sigma_y)
# You can plot multiple things on the same plot using these commands ending with "!".
# The "!" is a common thing in Julia that means "this function modifies the state".
scatter!(data.x, data.y, markersize=10, color=:maroon)

lines!(x_grid, b_opt .+ m_opt .* x_grid)

f
```

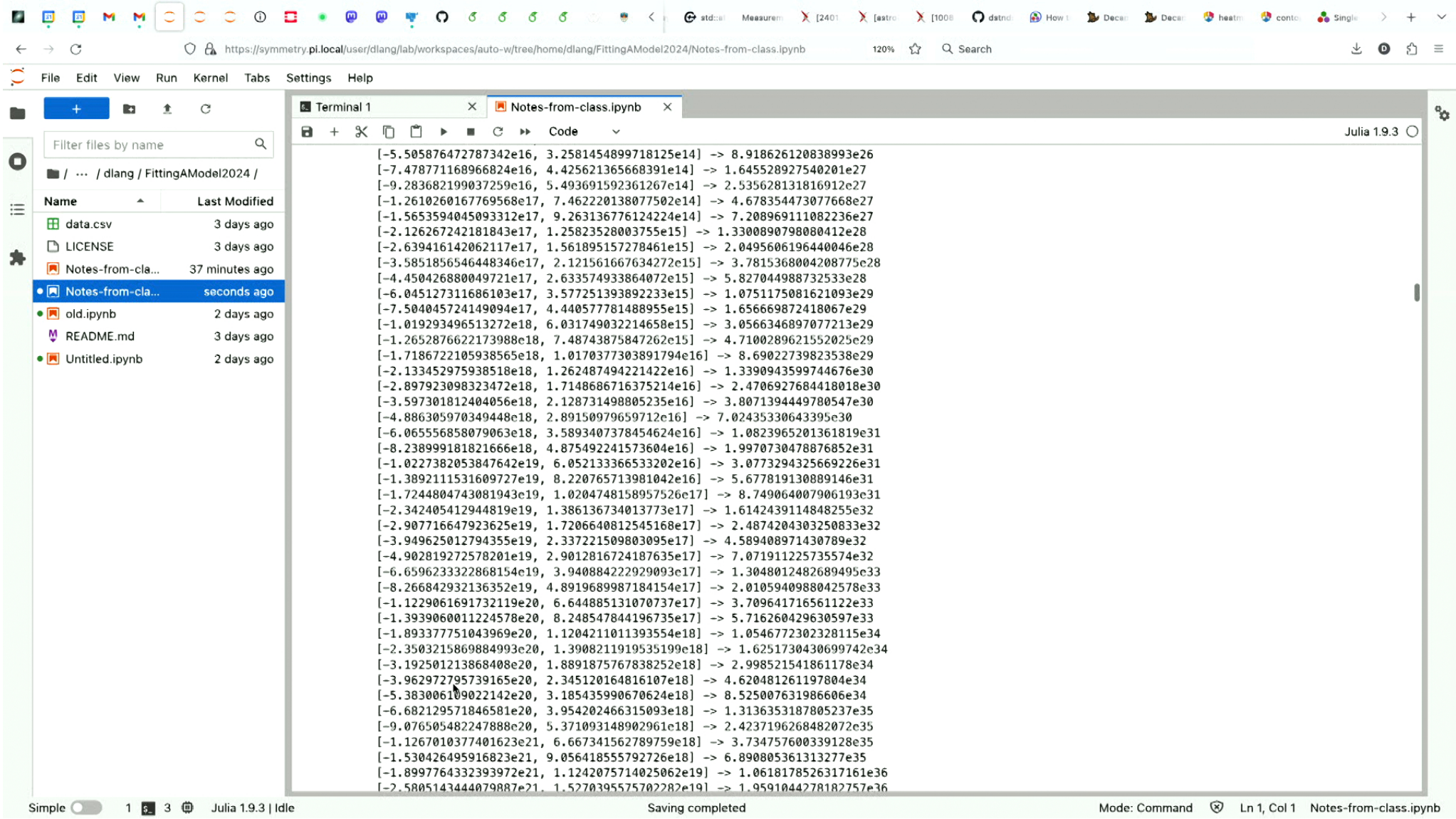
[41]:



Simple 1 3 Julia 1.9.3 | Idle Mode: Command Ln 5, Col 20 Notes-from-class.ipynb


https://symmetry.pi.local/user/dlang/lab/workspaces/auto-w/tree/home/dlang/FittingAModel2024/Notes-from-class.ipynb
File Edit View Run Kernel Tabs Settings Help
Terminal 1
Notes-from-class.ipynb
Code
Julia 1.9.3
Filter files by name
/ ··· / dlang / FittingAModel2024 /
Name
Last Modified
data.csv 3 days ago
LICENSE 3 days ago
Notes-from-cla... 34 minutes ago
Notes-from-cla... a minute ago
old.ipynb 2 days ago
README.md 3 days ago
Untitled.ipynb 2 days ago
[ ]:
[30]:
function likelihood(params, x, y, sigma_y)
(b,m) = params
y_pred = b .+ m .* x
chi = (y_pred - y) / sigma_y
like = 1 / (sqrt(2 * pi) * sigma_y) * exp(-0.5 * chi^2)
return prod(like)
end;
[31]:
likelihood([50, 2.], data.x, data.y, data.sigma_y)
[31]:
9.917447645408288e-51
[43]:
opt = optimize(p -> begin
like = likelihood(p, data.x, data.y, data.sigma_y)
println(p, " -> ", like)
-like
end, [50., 2.])
[50.0, 2.0] -> 9.917447645408288e-51
[75.025, 2.0] -> 1.495619830186641e-51
[50.0, 3.025] -> 5.723930077785796e-160
[62.5125, 2.0] -> 1.626079193975151e-50
[43]:
* Status: success
* Candidate solution
Final objective value: -1.626079e-50
* Found with
Algorithm: Nelder-Mead
* Convergence measures
√(Σ(yᵢ-ȳ)²)/n ≤ 1.0e-08
* Work counters
Seconds run: 0 (vs limit Inf)
Iterations: 0
f(x) calls: 4
[42]:
b_opt, m_opt = Optim.minimizer(opt)
Simple 1 3 Julia 1.9.3 | Idle
Mode: Edit Ln 7, Col 5 Notes-from-class.ipynb


https://symmetry.pi.local/user/dlang/lab/workspaces/auto-w/tree/home/dlang/FittingAModel2024/Notes-from-class.ipynb
File Edit View Run Kernel Tabs Settings Help
Terminal 1
Notes-from-class.ipynb
Code
Julia 1.9.3
Filter files by name
/ ··· / dlang / FittingAModel2024 /
Name
Last Modified
data.csv 3 days ago
LICENSE 3 days ago
Notes-from-cla... 37 minutes ago
Notes-from-cla... a minute ago
old.ipynb 2 days ago
README.md 3 days ago
Untitled.ipynb 2 days ago
200
100
200
300
X
[44]:
function log_likelihood(params, x, y, sigma_y)
(b,m) = params
y_pred = b .+ m .* x
chi = (y_pred - y) / sigma_y
loglike = -0.5 * chi^2
return sum(loglike)
end;
[46]:
opt = optimize(p -> begin
loglike = log_likelihood(p, data.x, data.y, data.sigma_y)
println(p, " -> ", loglike)
-loglike
end, [50., 2.])
[50.0, 2.0] -> -1.1742166996679013
[75.025, 2.0] -> -1.5042450426542777
[50.0, 3.025] -> -238.7643085380973
[75.025, 0.9750000000000001] -> -234.65883889958826
[68.76875000000001, 1.4875] -> -58.07569518670464
[56.256249999999994, 2.5125] -> -60.12843000595928
[65.640625, 1.74375] -> -14.186501110890584
[59.384375000000006, 2.25625] -> -15.212868520517853
[64.0765625, 1.871875] -> -3.3424985181404687
[60.94843750000001, 2.128125] -> -3.8556822229540884
[63.29453125, 1.9359375] -> -0.6956458330546369
[38.26953125, 1.9359375] -> -10.744222640278434
[65.83613281250001, 1.983984375] -> 0.08838053470406568
[79.13066406250002, 1.919921875] -> -0.033482074070261165
[81.67226562500002, 1.9679687499999998] -> -1.8228209029929654
[67.88896484375002, 1.9439453125] -> -0.001784606348120647
[54.59443359375001, 2.0080078125] -> -0.20852018976244308
[72.99660644531252, 1.941943359375] -> 0.08413735977337135
Simple 1 3 Julia 1.9.3 | Idle
Mode: Command Ln 1, Col 1 Notes-from-class.ipynb


https://symmetry.pi.local/user/dlang/lab/workspaces/auto-w/tree/home/dlang/FittingAModel2024/Notes-from-class.ipynb
File Edit View Run Kernel Tabs Settings Help
Terminal 1
Notes-from-class.ipynb
Code
Julia 1.9.3
Filter files by name
/ ··· / dlang / FittingAModel2024 /
Name
Last Modified
data.csv 3 days ago
LICENSE 3 days ago
Notes-from-cla... 37 minutes ago
Notes-from-cla... seconds ago
old.ipynb 2 days ago
README.md 3 days ago
Untitled.ipynb 2 days ago
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[-7.478771168966824e16, 4.425621365668391e14] -> 1.645528927540201e27
[-9.283682199037259e16, 5.493691592361267e14] -> 2.535628131816912e27
[-1.2610260167769568e17, 7.462220138077502e14] -> 4.678354473077668e27
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[-2.126267242181843e17, 1.25823528003755e15] -> 1.3300890798080412e28
[-2.639416142062117e17, 1.561895157278461e15] -> 2.0495606196440046e28
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[-1.718672210593856 5e18, 1.0170377303891794e16] -> 8.690227398235386e29
[-2.133452975938518e18, 1.262487494221422e16] -> 1.3390943599744676e30
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[-3.597301812404056e18, 2.128731498805235e16] -> 3.8071394449780547e30
[-4.886305970349448e18, 2.89150979659712e16] -> 7.02435330643395e30
[-6.065556858079063e18, 3.589340737845462 4e16] -> 1.0823965201361819e31
[-8.238999181821666e18, 4.875492241573604e16] -> 1.9970730478876852e31
[-1.022738205384764 2e19, 6.052133366533202e16] -> 3.0773294325669226e31
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[-4.902819272578201e19, 2.901281672418763 5e17] -> 7.071911225735574e32
[-6.659623332286815 4e19, 3.940884222929093e17] -> 1.3048012482689495e33
[-8.266842932136352e19, 4.891968998718415 4e17] -> 2.0105940988042578e33
[-1.122906169173211 9e20, 6.644885131070737e17] -> 3.709641716561122e33
[-1.393906001122457 8e20, 8.248547844196735e17] -> 5.716260429630597e33
[-1.893377751043969e20, 1.1204211011393554e18] -> 1.0546772302328115e34
[-2.350321586988499 3e20, 1.3908211919535199e18] -> 1.6251730430699742e34
[-3.192501213868408e20, 1.8891875767838252e18] -> 2.9985215418611 78e34
[-3.962972795739165e20, 2.345120164816107e18] -> 4.620481261197804e34
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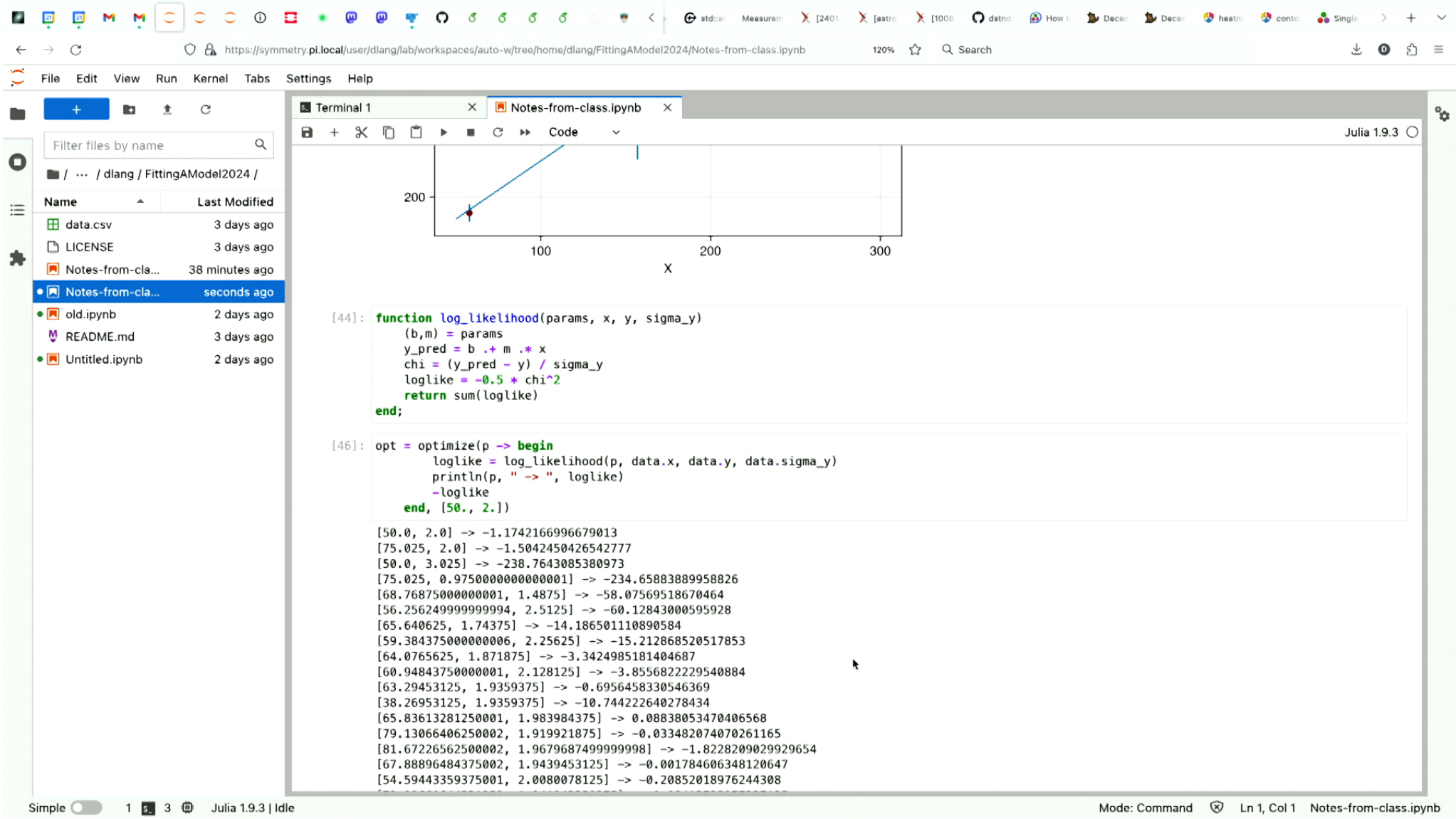
[-6.682129571846581e20, 3.954202466315093e18] -> 1.3136353187805237e35
[-9.076505482247888e20, 5.371093148902961e18] -> 2.4237196268482072e35
[-1.126701037740162 3e21, 6.667341562789759e18] -> 3.7347576003391 28e35
[-1.530426495916823e21, 9.056418555792726e18] -> 6.890805361313277e35
[-1.899776433239397 2e21, 1.1242075714025062e19] -> 1.0618178526317161e36
[-2.580514344407988 7e21, 1.5270395575702282e19] -> 1.9591044278182757e36
Simple 1 3 Julia 1.9.3 | Idle
Saving completed
Mode: Command Ln 1, Col 1 Notes-from-class.ipynb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Terminal 1
Notes-from-class.ipynb
Code
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Last Modified
data.csv 3 days ago
LICENSE 3 days ago
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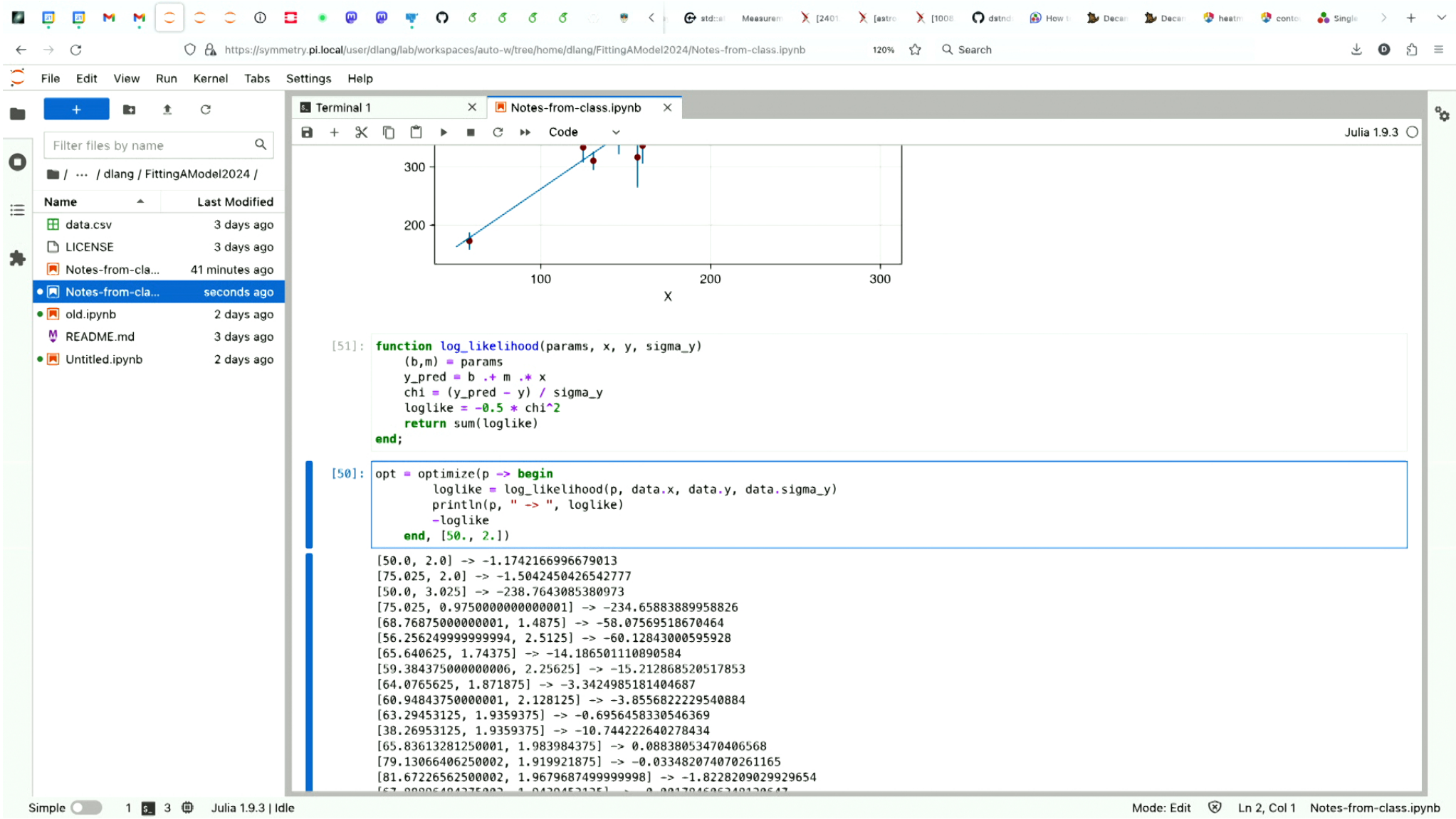
Notes-from-cla... seconds ago
old.ipynb 2 days ago
README.md 3 days ago
Untitled.ipynb 2 days ago
200
100
200
300
X
[44]:
function log_likelihood(params, x, y, sigma_y)
(b,m) = params
y_pred = b .+ m .* x
chi = (y_pred - y) / sigma_y
loglike = -0.5 * chi^2
return sum(loglike)
end;
[46]:
opt = optimize(p -> begin
loglike = log_likelihood(p, data.x, data.y, data.sigma_y)
println(p, " -> ", loglike)
-loglike
end, [50., 2.])
[50.0, 2.0] -> -1.1742166996679013
[75.025, 2.0] -> -1.5042450426542777
[50.0, 3.025] -> -238.7643085380973
[75.025, 0.9750000000000001] -> -234.65883889958826
[68.76875000000001, 1.4875] -> -58.07569518670464
[56.256249999999994, 2.5125] -> -60.12843000595928
[65.640625, 1.74375] -> -14.186501110890584
[59.384375000000006, 2.25625] -> -15.212868520517853
[64.0765625, 1.871875] -> -3.3424985181404687
[60.94843750000001, 2.128125] -> -3.8556822229540884
[63.29453125, 1.9359375] -> -0.6956458330546369
[38.26953125, 1.9359375] -> -10.744222640278434
[65.83613281250001, 1.983984375] -> 0.08838053470406568
[79.13066406250002, 1.919921875] -> -0.033482074070261165
[81.67226562500002, 1.9679687499999998] -> -1.8228209029929654
[67.88896484375002, 1.9439453125] -> -0.001784606348120647
[54.59443359375001, 2.0080078125] -> -0.20852018976244308
Simple 1 3 Julia 1.9.3 | Idle
Mode: Command Ln 1, Col 1 Notes-from-class.ipynb


https://symmetry.pi.local/user/dlang/lab/workspaces/auto-w/tree/home/dlang/FittingAModel2024/Notes-from-class.ipynb
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Terminal 1
Notes-from-class.ipynb
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LICENSE 3 days ago
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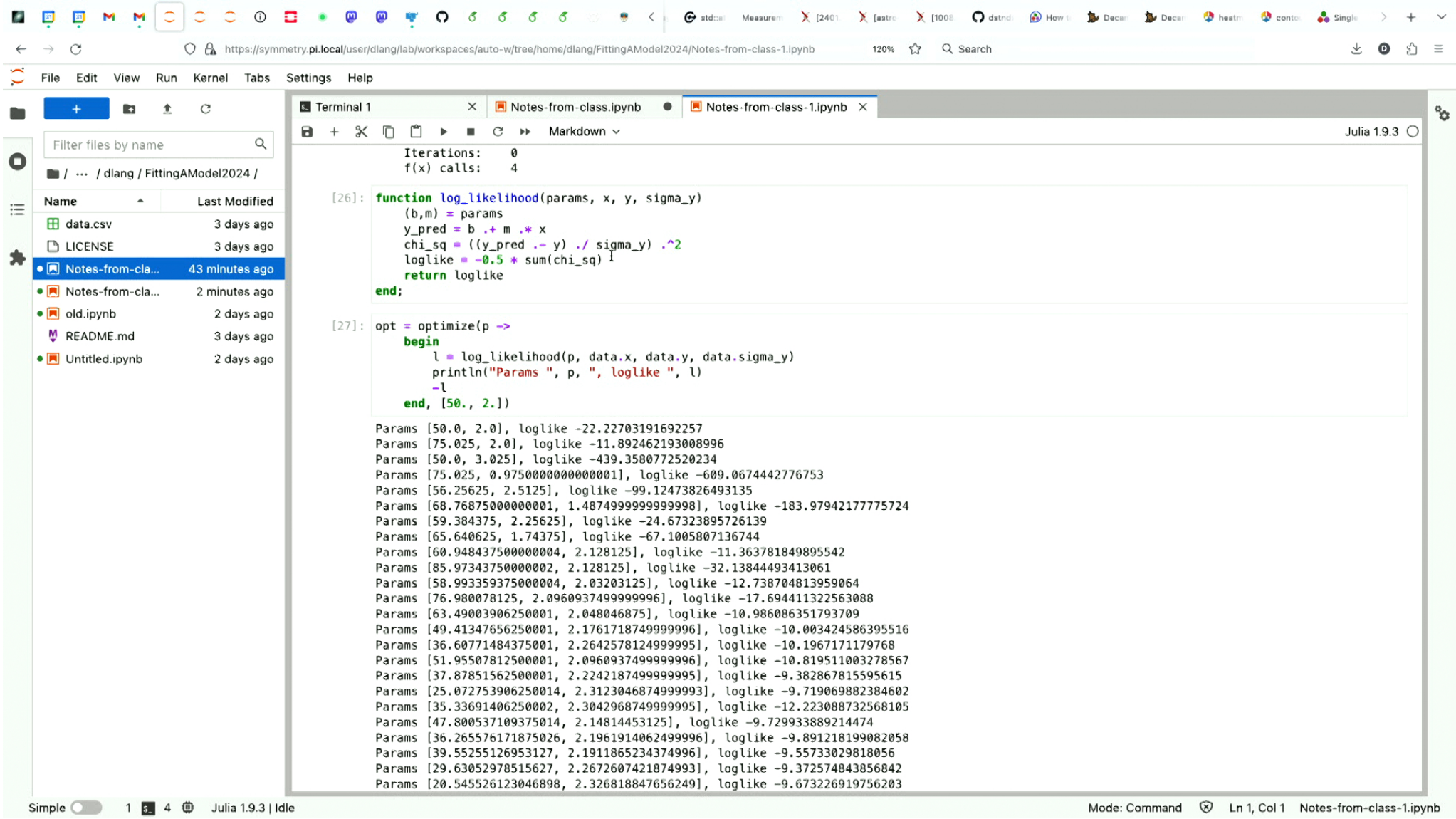
Notes-from-cla... seconds ago
old.ipynb 2 days ago
README.md 3 days ago
Untitled.ipynb 2 days ago
300
200
100
200
300
X
[51]:
function log_likelihood(params, x, y, sigma_y)
(b,m) = params
y_pred = b .+ m .* x
chi = (y_pred - y) / sigma_y
loglike = -0.5 * chi^2
return sum(loglike)
end;
[50]:
opt = optimize(p -> begin
loglike = log_likelihood(p, data.x, data.y, data.sigma_y)
println(p, " -> ", loglike)
-loglike
end, [50., 2.])
[50.0, 2.0] -> -1.1742166996679013
[75.025, 2.0] -> -1.5042450426542777
[50.0, 3.025] -> -238.7643085380973
[75.025, 0.9750000000000001] -> -234.65883889958826
[68.76875000000001, 1.4875] -> -58.07569518670464
[56.256249999999994, 2.5125] -> -60.12843000595928
[65.640625, 1.74375] -> -14.186501110890584
[59.384375000000006, 2.25625] -> -15.212868520517853
[64.0765625, 1.871875] -> -3.3424985181404687
[60.94843750000001, 2.128125] -> -3.8556822229540884
[63.29453125, 1.9359375] -> -0.6956458330546369
[38.26953125, 1.9359375] -> -10.744222640278434
[65.83613281250001, 1.983984375] -> 0.08838053470406568
[79.13066406250002, 1.919921875] -> -0.033482074070261165
[81.67226562500002, 1.9679687499999998] -> -1.8228209029929654
Simple 1 3 Julia 1.9.3 | Idle
Mode: Edit Ln 2, Col 1 Notes-from-class.ipynb


https://symmetry.pi.local/user/dlang/lab/workspaces/auto-w/tree/home/dlang/FittingAModel2024/Notes-from-class-1.ipynb
File Edit View Run Kernel Tabs Settings Help
Terminal 1
Notes-from-class.ipynb
Notes-from-class-1.ipynb
Markdown
Julia 1.9.3
Filter files by name
/ ··· / dlang / FittingAModel2024 /
Name
Last Modified
data.csv 3 days ago
LICENSE 3 days ago
Notes-from-cla... 43 minutes ago
Notes-from-cla... 2 minutes ago
old.ipynb 2 days ago
README.md 3 days ago
Untitled.ipynb 2 days ago
Iterations: 0
f(x) calls: 4
[26]:
function log_likelihood(params, x, y, sigma_y)
(b,m) = params
y_pred = b .+ m .* x
chi_sq = ((y_pred .- y) ./ sigma_y) .^2
loglike = -0.5 * sum(chi_sq)
return loglike
end;
[27]:
opt = optimize(p ->
begin
l = log_likelihood(p, data.x, data.y, data.sigma_y)
println("Params ", p, ", loglike ", l)
-l
end, [50., 2.])
Params [50.0, 2.0], loglike -22.22703191692257
Params [75.025, 2.0], loglike -11.892462193008996
Params [50.0, 3.025], loglike -439.3580772520234
Params [75.025, 0.9750000000000001], loglike -609.0674442776753
Params [56.25625, 2.5125], loglike -99.12473826493135
Params [68.76875000000001, 1.4874999999999998], loglike -183.97942177775724
Params [59.384375, 2.25625], loglike -24.67323895726139
Params [65.640625, 1.74375], loglike -67.1005807136744
Params [60.948437500000004, 2.128125], loglike -11.363781849895542
Params [85.97343750000002, 2.128125], loglike -32.13844493413061
Params [58.993359375000004, 2.03203125], loglike -12.738704813959064
Params [76.980078125, 2.0960937499999996], loglike -17.694411322563088
Params [63.49003906250001, 2.048046875], loglike -10.986086351793709
Params [49.41347656250001, 2.1761718749999996], loglike -10.003424586395516
Params [36.60771484375001, 2.2642578124999995], loglike -10.1967171179768
Params [51.95507812500001, 2.0960937499999996], loglike -10.819511003278567
Params [37.87851562500001, 2.2242187499999995], loglike -9.382867815595615
Params [25.072753906250014, 2.3123046874999993], loglike -9.719069882384602
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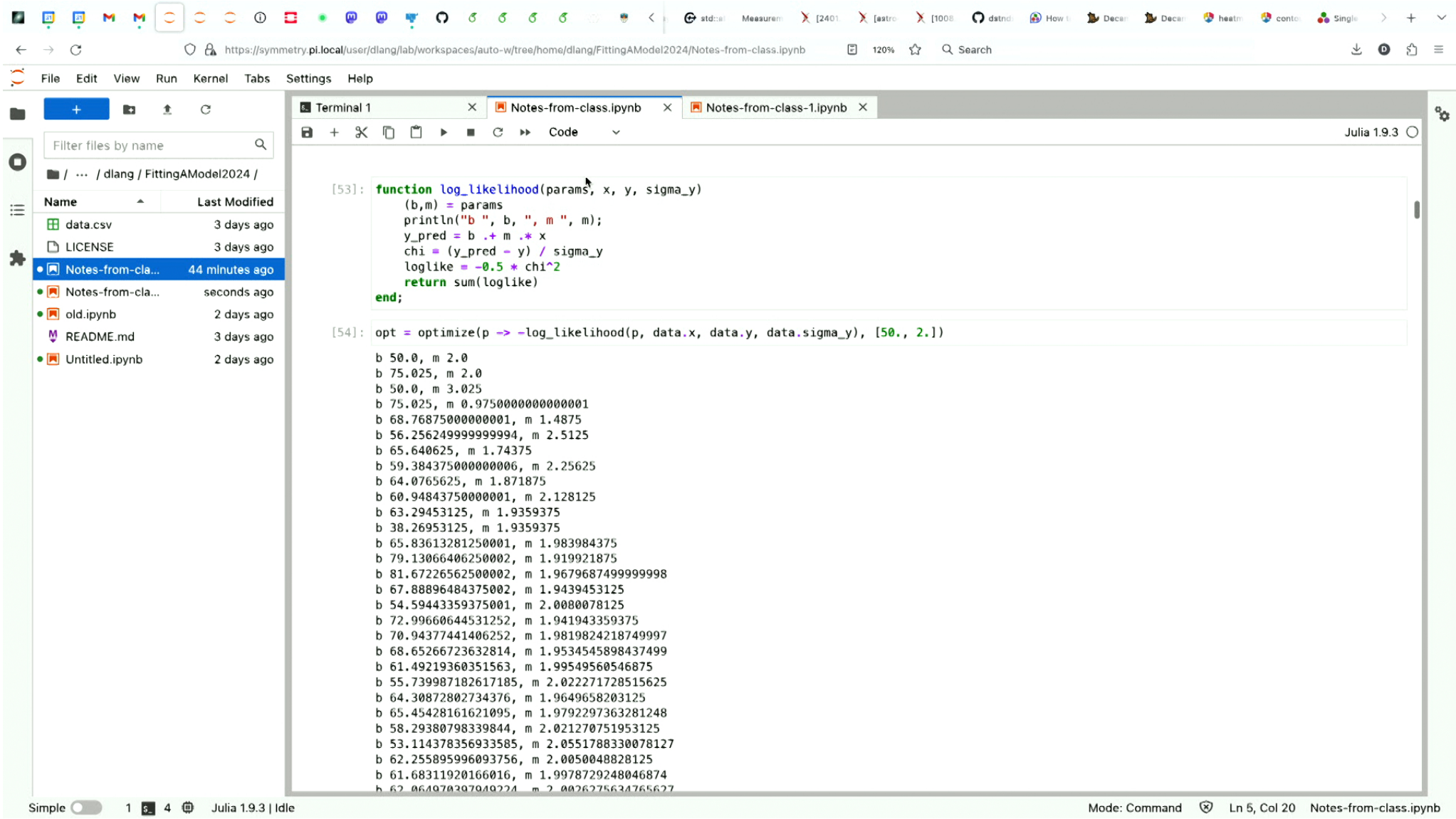
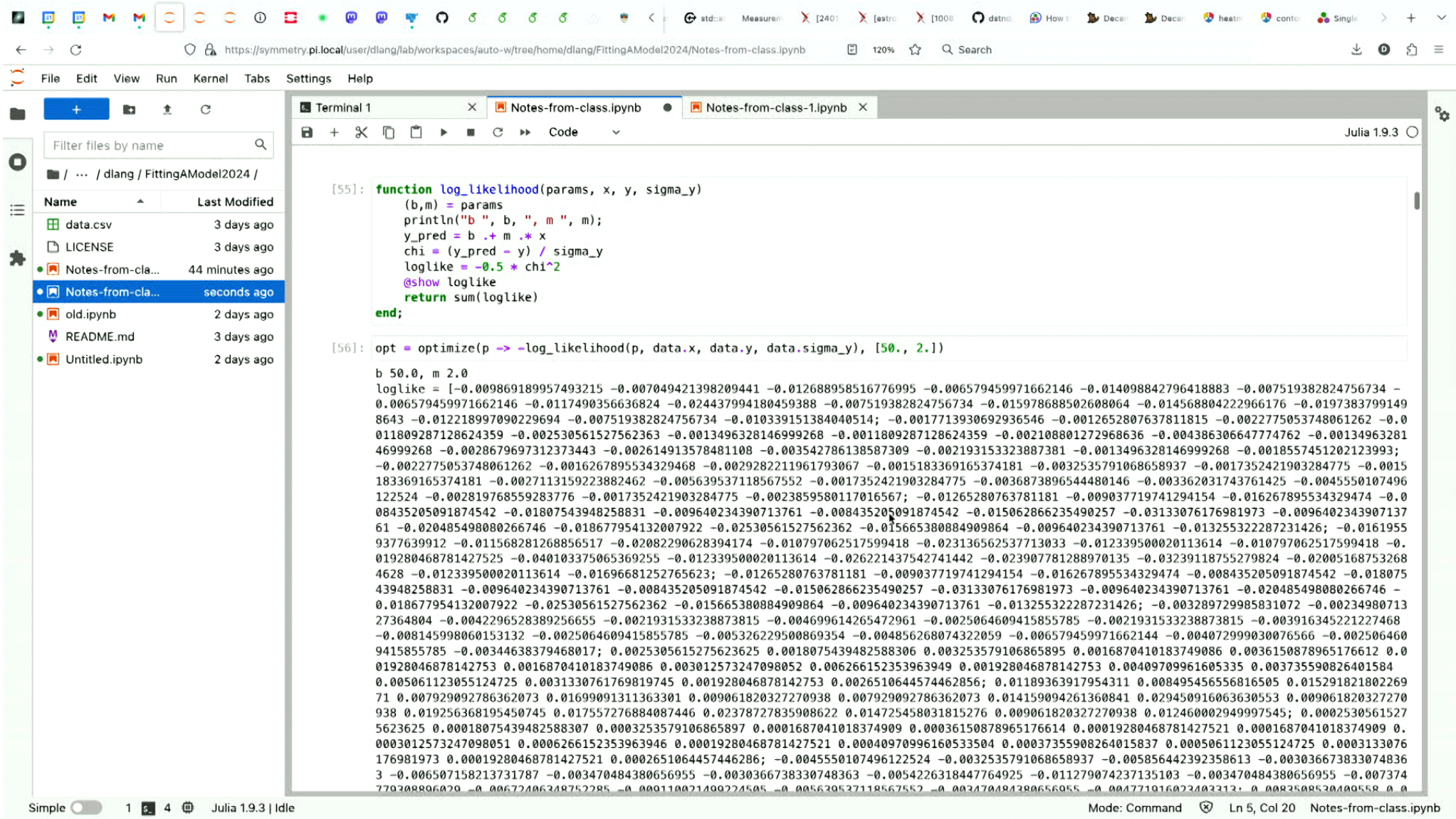
Params [47.800537109375014, 2.14814453125], loglike -9.729933889214474
Params [36.265576171875026, 2.1961914062499996], loglike -9.891218199082058
Params [39.55255126953127, 2.1911865234374996], loglike -9.55733029818056
Params [29.63052978515627, 2.2672607421874993], loglike -9.372574843856842
Params [20.545526123046898, 2.326818847656249], loglike -9.673226919756203
Simple 1 4 Julia 1.9.3 | Idle
Mode: Command Ln 1, Col 1 Notes-from-class-1.ipynb


https://symmetry.pi.local/user/dlang/lab/workspaces/auto-w/tree/home/dlang/FittingAModel2024/Notes-from-class.ipynb
File Edit View Run Kernel Tabs Settings Help
Terminal 1
Notes-from-class.ipynb
Notes-from-class-1.ipynb
Code
Julia 1.9.3
Filter files by name
/ ··· / dlang / FittingAModel2024 /
Name
Last Modified
data.csv 3 days ago
LICENSE 3 days ago
Notes-from-cla... 44 minutes ago
Notes-from-cla... seconds ago
old.ipynb 2 days ago
README.md 3 days ago
Untitled.ipynb 2 days ago
[53]:
function log_likelihood(params, x, y, sigma_y)
(b,m) = params
println("b ", b, ", m ", m);
y_pred = b .+ m .* x
chi = (y_pred - y) / sigma_y
loglike = -0.5 * chi^2
return sum(loglike)
end;
[54]:
opt = optimize(p -> -log_likelihood(p, data.x, data.y, data.sigma_y), [50., 2.])
b 50.0, m 2.0
b 75.025, m 2.0
b 50.0, m 3.025
b 75.025, m 0.9750000000000001
b 68.76875000000001, m 1.4875
b 56.256249999999994, m 2.5125
b 65.640625, m 1.74375
b 59.384375000000006, m 2.25625
b 64.0765625, m 1.871875
b 60.94843750000001, m 2.128125
b 63.29453125, m 1.9359375
b 38.26953125, m 1.9359375
b 65.83613281250001, m 1.983984375
b 79.13066406250002, m 1.919921875
b 81.67226562500002, m 1.9679687499999998
b 67.88896484375002, m 1.9439453125
b 54.59443359375001, m 2.0080078125
b 72.99660644531252, m 1.941943359375
b 70.94377441406252, m 1.9819824218749997
b 68.65266723632814, m 1.9534545898437499
b 61.49219360351563, m 1.99549560546875
b 55.739987182617185, m 2.022271728515625
b 64.30872802734376, m 1.9649658203125
b 65.45428161621095, m 1.9792297363281248
b 58.29380798339844, m 2.021270751953125
b 53.114378356933585, m 2.0551788330078127
b 62.255895996093756, m 2.0050048828125
b 61.68311920166016, m 1.9978729248046874
Simple 1 4 Julia 1.9.3 | Idle
Mode: Command Ln 5, Col 20 Notes-from-class.ipynb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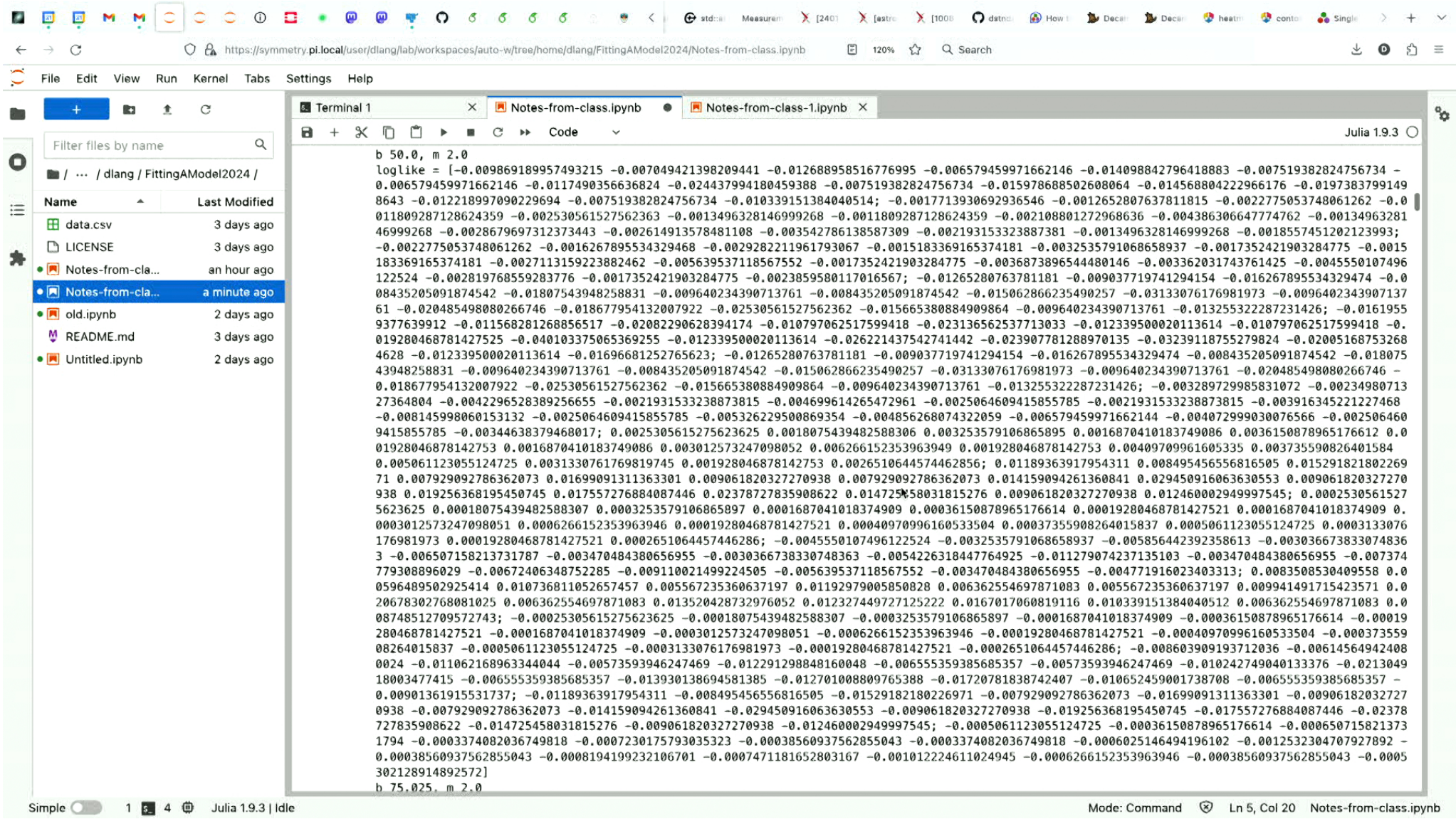
Untitled.ipynb 2 days ago
Terminal 1
Notes-from-class.ipynb
Notes-from-class-1.ipynb
Code
Julia 1.9.3
[55]:
function log_likelihood(params, x, y, sigma_y)
(b,m) = params
println("b ", b, ", m ", m);
y_pred = b .+ m .* x
chi = (y_pred - y) / sigma_y
loglike = -0.5 * chi^2
@show loglike
return sum(loglike)
end;
[56]:
opt = optimize(p -> -log_likelihood(p, data.x, data.y, data.sigma_y), [50., 2.])
b 50.0, m 2.0
loglike = [-0.009869189957493215 -0.007049421398209441 -0.012688958516776995 -0.006579459971662146 -0.014098842796418883 -0.007519382824756734 -0.006579459971662146 -0.0117490356636824 -0.024437994180459388 -0.007519382824756734 -0.01597868850268064 -0.014568804222966176 -0.019738379914986643 -0.012218997090229694 -0.007519382824756734 -0.010339151384040514; -0.0017713930692936546 -0.0012652807637811815 -0.0022775053748061262 -0.0011809287128624359 -0.00253056152756236 3 -0.0013496328146999268 -0.0011809287128624359 -0.002108801272968636 -0.004386306647774762 -0.00134963281469992 68 -0.0028679697312373443 -0.0026149135784811 08 -0.003542786138587309 -0.002193153323887381 -0.0013496328146999268 -0.0018557451202123993; -0.0022775053748061262 -0.0016267895534329468 -0.0029282211961793067 -0.0015183369165374181 -0.0032535791068658937 -0.0017352421903284775 -0.0015183369165374181 -0.0027113159223882462 -0.005639537118567552 -0.0017352421903284775 -0.0036873896544480146 -0.003362031743761425 -0.0045550107496122524 -0.002819768559283776 -0.0017352421903284775 -0.0023859580117016567; -0.012652807637811 81 -0.009037719741294154 -0.016267895534329474 -0.008435205091874542 -0.018078543948258831 -0.009640234390713761 -0.008435205091874542 -0.015062866235490257 -0.03133076176981973 -0.009640234390713761 -0.020485498080266746 -0.018677954132007922 -0.02530561527562362 -0.015665380884909864 -0.009640234390713761 -0.013255322287231426; -0.016195593776396 99 -0.011568281268856517 -0.02082290628394174 -0.010797062517599418 -0.023136562537713033 -0.012339500020113614 -0.010797062517599418 -0.019280468781427525 -0.040103375065369255 -0.012339500020113614 -0.026221437542741442 -0.023907781288970135 -0.03239118755279824 -0.020051687532684628 -0.012339500020113614 -0.016966812527656 23; -0.012652807637811 81 -0.009037719741294154 -0.016267895534329474 -0.008435205091874542 -0.018075439482588311 -0.009640234390713761 -0.008435205091874542 -0.015062866235490257 -0.03133076176981973 -0.009640234390713761 -0.020485498080266746 -0.018677954132007922 -0.02530561527562362 -0.015665380884909864 -0.009640234390713761 -0.013255322287231426; -0.003289729985831072 -0.002349807132736480 4 -0.004229652838925665 5 -0.002193153323887381 5 -0.004696142654729 61 -0.002506460941585578 5 -0.002506460941585578 5 -0.003446383794680 17; 0.0025305615275623625 0.0018075439482588306 0.003253579106865895 0.0016870410183749086 0.0036150878965176612 0.0019280468781427 53 0.0016870410183749086 0.003012573247098052 0.006266152353963949 0.001928046878142753 0.004097099616053 35 0.003735590826401584 0.005061123055124725 0.0031330761769819745 0.001928046878142753 0.0026510644574462856; 0.01189363917954311 0.008495456556816505 0.01529182180226971 0.007929209278636207 3 0.01699091311363301 0.009061820327270938 0.007929209278636207 3 0.014159094261360841 0.029450916063630553 0.009061820327270938 0.019256368195450745 0.017557276884087446 0.02378727835908622 0.014725458031815276 0.009061820327270938 0.012460002949997545; 0.0002530561527562362 5 0.0001807543948258831 0.0003253579106865897 0.0001687041018374909 0.0003615087896517661 4 0.0001928046878142752 1 0.0001687041018374909 0.0003012573247098051 0.0006266152353963946 0.0001928046878142752 1 0.0004097099616053350 4 0.0003735590826401583 7 0.005061123055124725 0.0003133076176981973 0.0001928046878142752 1 0.0002651064457446286; -0.0045550107496122524 -0.0032535791068658937 -0.005856442392358613 -0.0030366738330748363 -0.006507158213731787 -0.003470484380656955 -0.0030366738330748363 -0.005422631844776492 5 -0.011279074237135103 -0.003470484380656955 -0.007374779308896029 -0.006724063487522 85 -0.009110021499224505 -0.005639537118567552 -0.003470484380656955 -0.004771916022403313; 0.0083508530409558 0.0
Simple 1 4 Julia 1.9.3 | Idle
Mode: Command Ln 5, Col 20 Notes-from-class.ipynb


https://symmetry.pi.local/user/dlang/lab/workspaces/auto-w/tree/home/dlang/FittingAModel2024/Notes-from-class.ipynb
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Terminal 1
Notes-from-class.ipynb
Notes-from-class-1.ipynb
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old.ipynb 2 days ago
README.md 3 days ago
Untitled.ipynb 2 days ago
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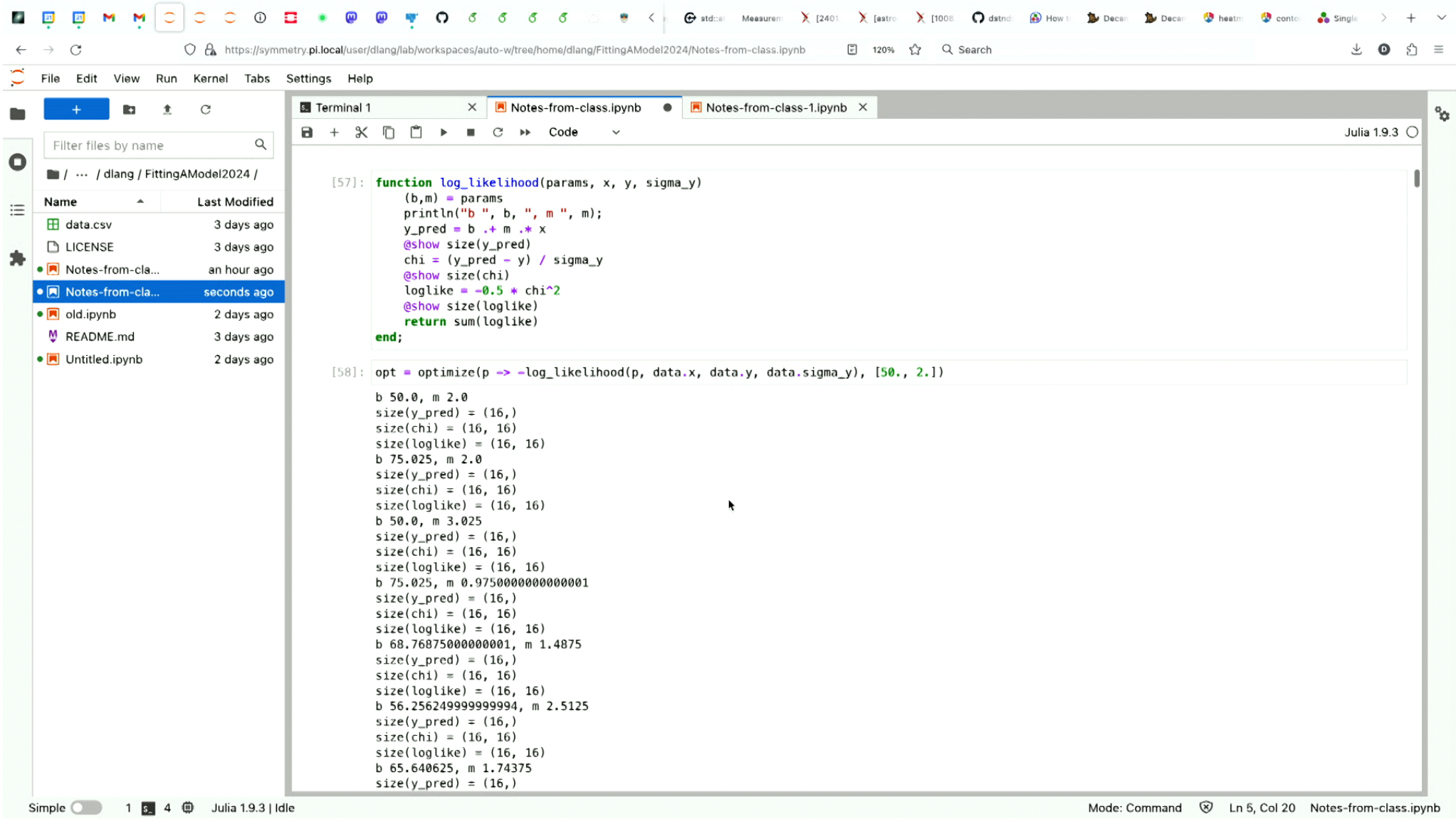
b 75.025, m 2.0
Simple 1 4 Julia 1.9.3 | Idle
Mode: Command Ln 5, Col 20 Notes-from-class.ipynb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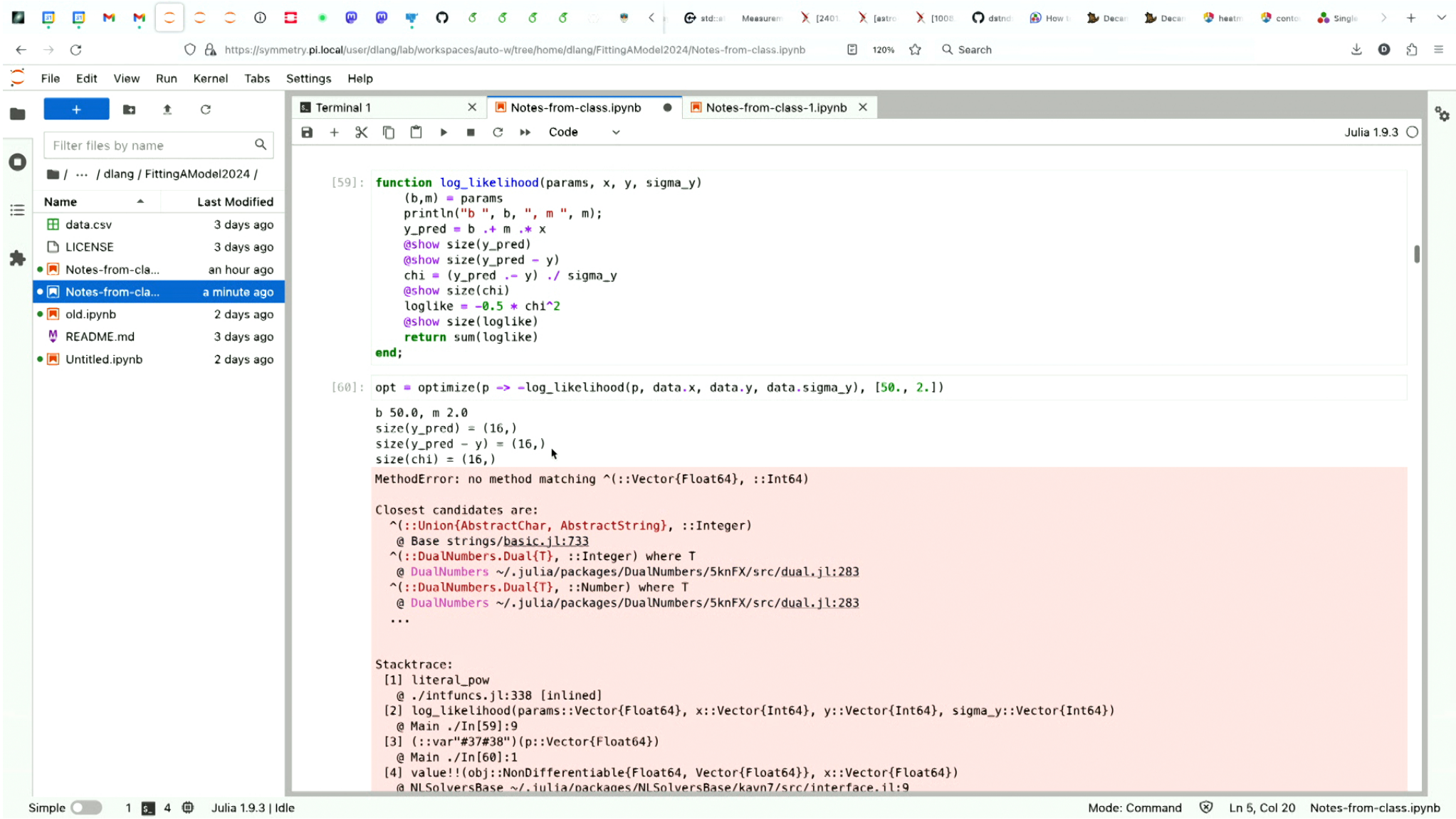
Untitled.ipynb 2 days ago
Terminal 1
Notes-from-class.ipynb
Notes-from-class-1.ipynb
Code
Julia 1.9.3
[57]:
function log_likelihood(params, x, y, sigma_y)
(b,m) = params
println("b ", b, ", m ", m);
y_pred = b .+ m .* x
@show size(y_pred)
chi = (y_pred - y) / sigma_y
@show size(chi)
loglike = -0.5 * chi^2
@show size(loglike)
return sum(loglike)
end;
[58]:
opt = optimize(p -> -log_likelihood(p, data.x, data.y, data.sigma_y), [50., 2.])
b 50.0, m 2.0
size(y_pred) = (16,)
size(chi) = (16, 16)
size(loglike) = (16, 16)
b 75.025, m 2.0
size(y_pred) = (16,)
size(chi) = (16, 16)
size(loglike) = (16, 16)
b 50.0, m 3.025
size(y_pred) = (16,)
size(chi) = (16, 16)
size(loglike) = (16, 16)
b 75.025, m 0.9750000000000001
size(y_pred) = (16,)
size(chi) = (16, 16)
size(loglike) = (16, 16)
b 68.76875000000001, m 1.4875
size(y_pred) = (16,)
size(chi) = (16, 16)
size(loglike) = (16, 16)
b 56.256249999999994, m 2.5125
size(y_pred) = (16,)
size(chi) = (16, 16)
size(loglike) = (16, 16)
b 65.640625, m 1.74375
size(y_pred) = (16,)
Simple 1 4 Julia 1.9.3 | Idle
Mode: Command Ln 5, Col 20 Notes-from-class.ipynb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Notes-from-class.ipynb
Notes-from-class-1.ipynb
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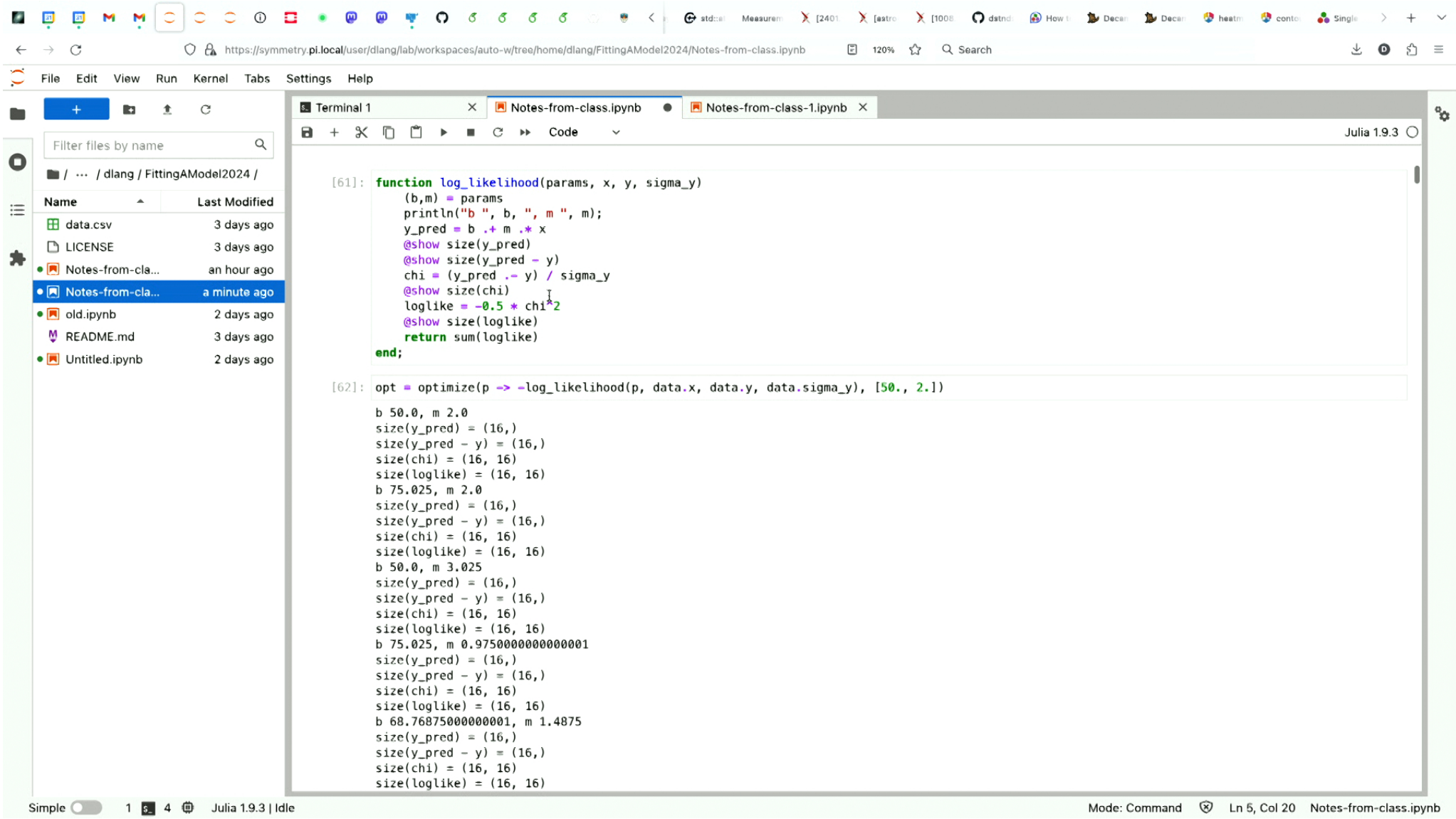
Notes-from-cla... an hour ago
Notes-from-cla... a minute ago
old.ipynb 2 days ago
README.md 3 days ago
Untitled.ipynb 2 days ago
[59]:
function log_likelihood(params, x, y, sigma_y)
(b,m) = params
println("b ", b, ", m ", m);
y_pred = b .+ m .* x
@show size(y_pred)
@show size(y_pred - y)
chi = (y_pred .- y) ./ sigma_y
@show size(chi)
loglike = -0.5 * chi^2
@show size(loglike)
return sum(loglike)
end;
[60]:
opt = optimize(p -> -log_likelihood(p, data.x, data.y, data.sigma_y), [50., 2.])
b 50.0, m 2.0
size(y_pred) = (16,)
size(y_pred - y) = (16,)
size(chi) = (16,)
MethodError: no method matching ^(::Vector{Float64}, ::Int64)
Closest candidates are:
^(::Union{AbstractChar, AbstractString}, ::Integer)
@ Base strings/basic.jl:733
^(::DualNumbers.Dual{T}, ::Integer) where T
@ DualNumbers ~/.julia/packages/DualNumbers/5knFX/src/dual.jl:283
^(::DualNumbers.Dual{T}, ::Number) where T
@ DualNumbers ~/.julia/packages/DualNumbers/5knFX/src/dual.jl:283
...
Stacktrace:
[1] literal_pow
@ ./intfuncs.jl:338 [inlined]
[2] log_likelihood(params::Vector{Float64}, x::Vector{Int64}, y::Vector{Int64}, sigma_y::Vector{Int64})
@ Main ./In[59]:9
[3] (::var"#37#38")(p::Vector{Float64})
@ Main ./In[60]:1
[4] value!!(obj::NonDifferentiable{Float64, Vector{Float64}}, x::Vector{Float64})
@ NLSolversBase ~/.julia/packages/NLSolversBase/kavn7/src/interface.jl:9
Simple 1 4 Julia 1.9.3 | Idle
Mode: Command Ln 5, Col 20 Notes-from-class.ipynb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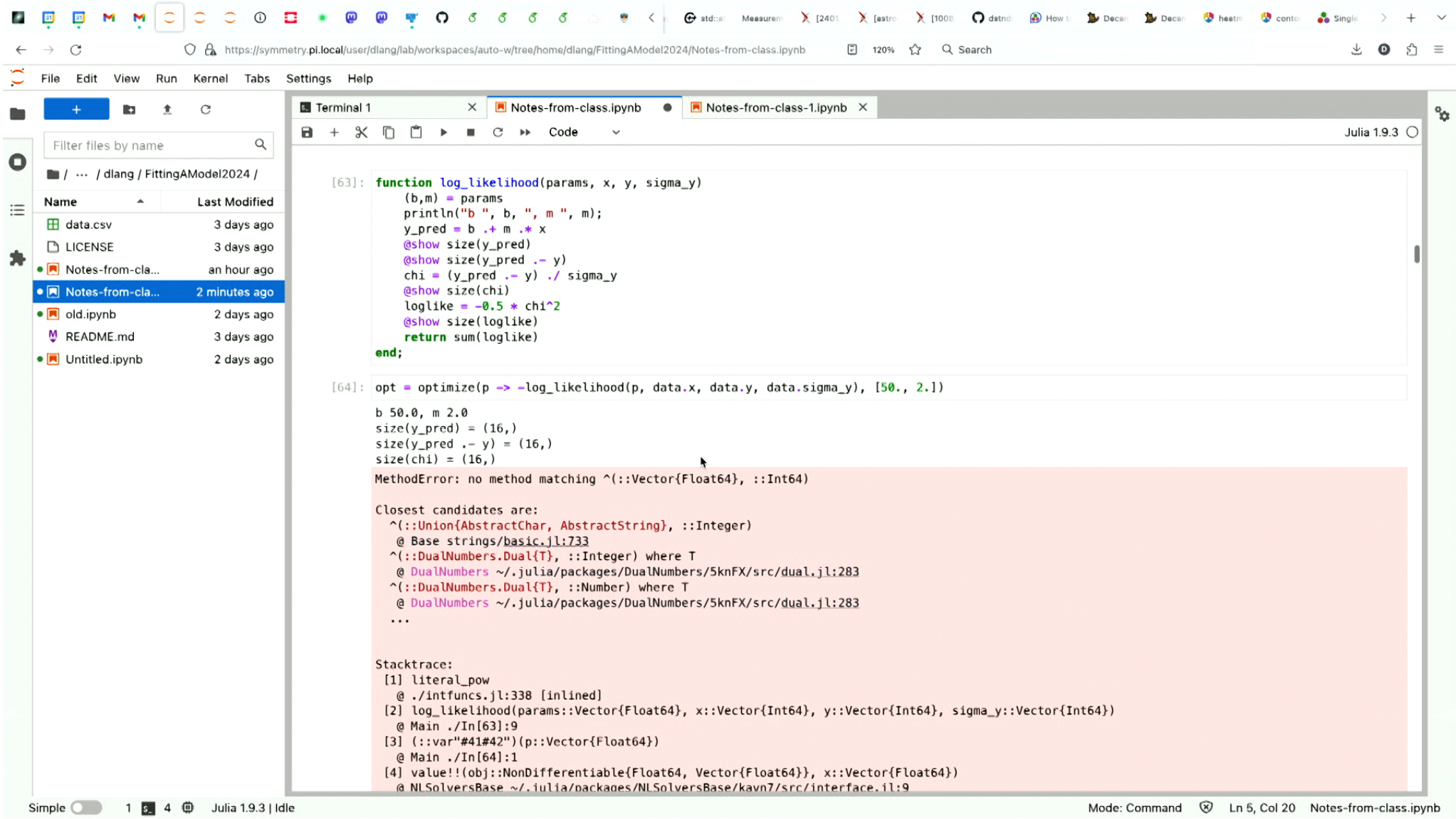
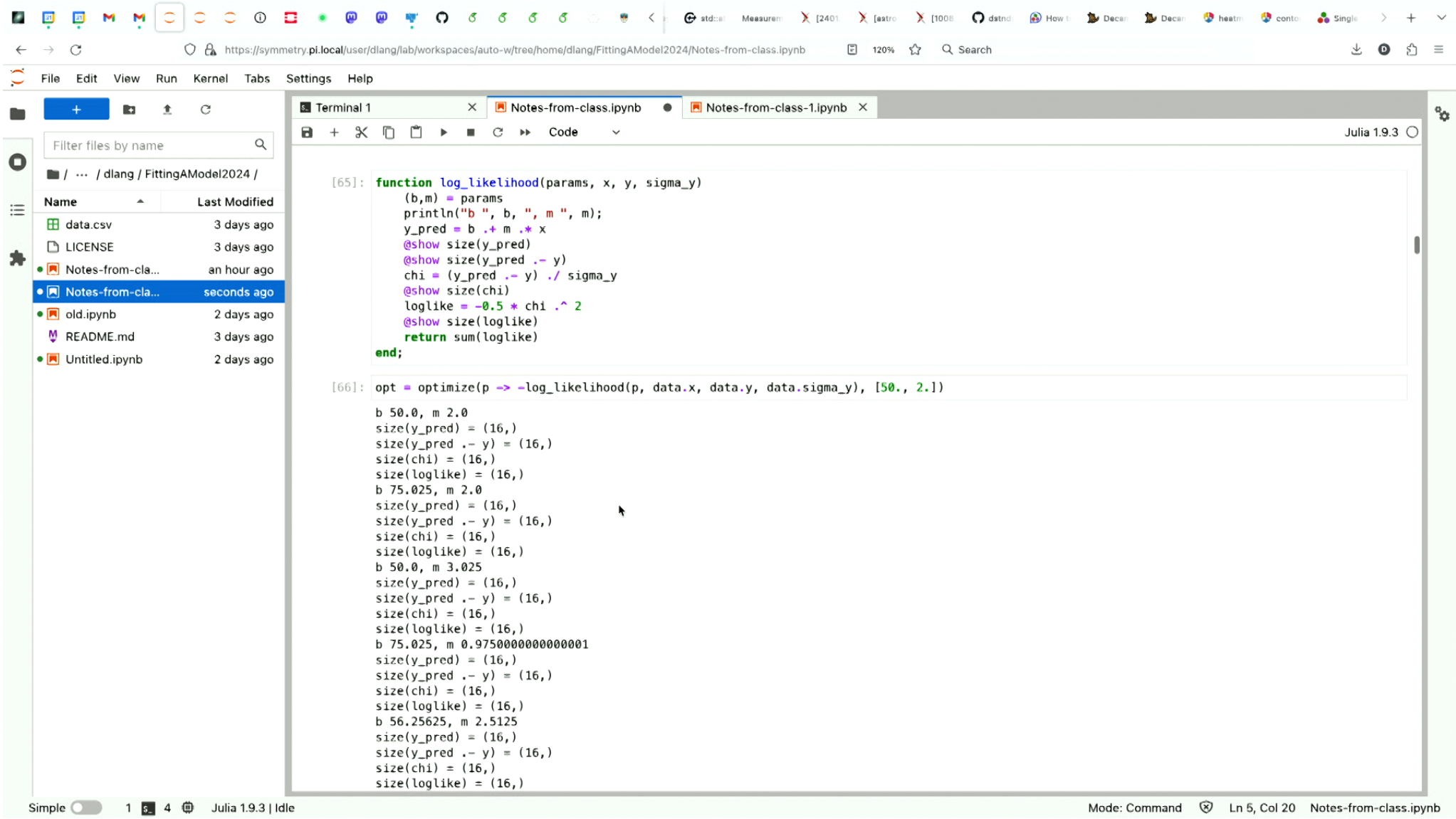
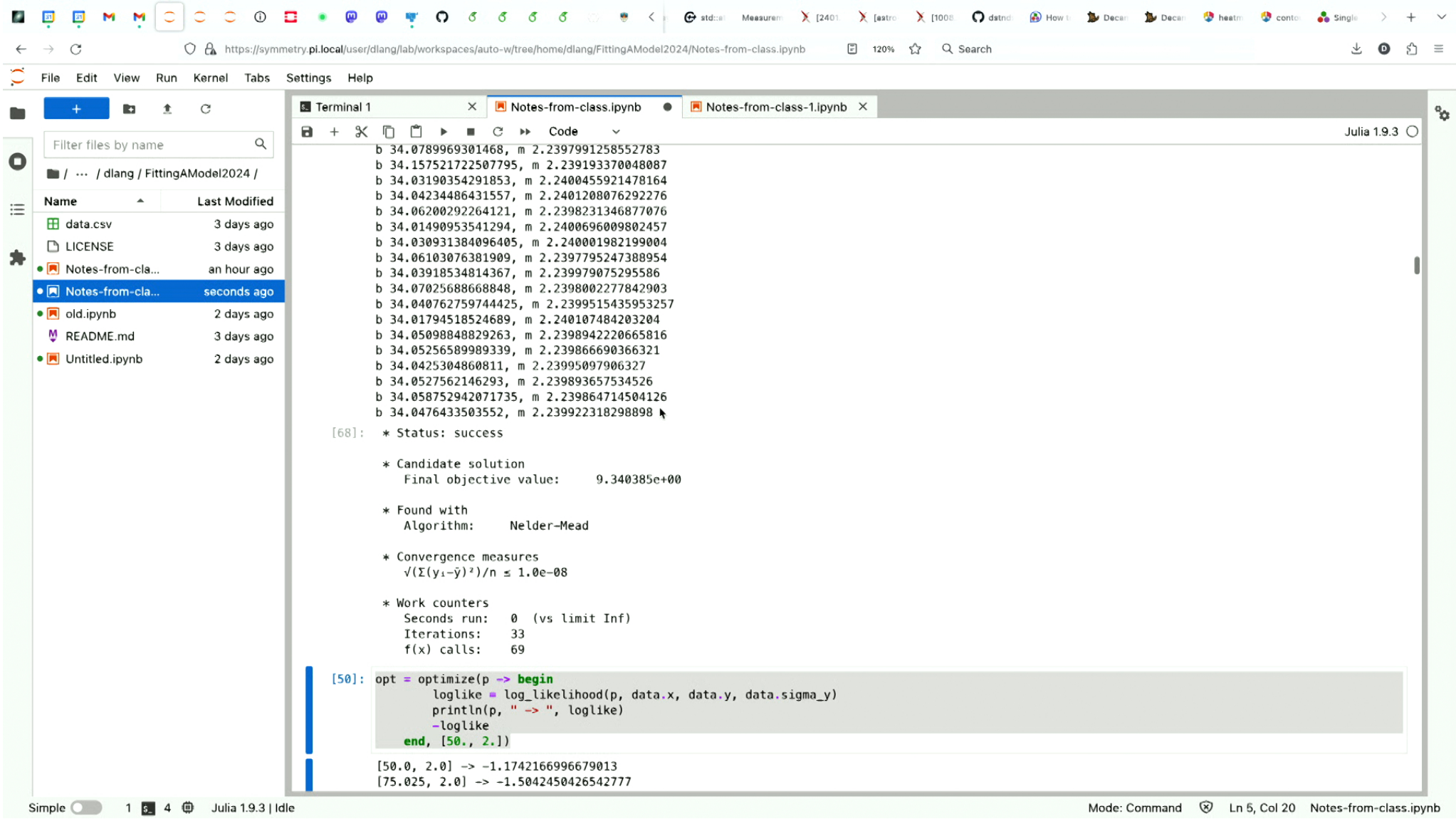
Terminal 1
Notes-from-class.ipynb
Notes-from-class-1.ipynb
Code
Julia 1.9.3
[61]:
function log_likelihood(params, x, y, sigma_y)
(b,m) = params
println("b ", b, ", m ", m);
y_pred = b .+ m .* x
@show size(y_pred)
@show size(y_pred - y)
chi = (y_pred .- y) / sigma_y
@show size(chi)
loglike = -0.5 * chi^2
@show size(loglike)
return sum(loglike)
end;
[62]:
opt = optimize(p -> -log_likelihood(p, data.x, data.y, data.sigma_y), [50., 2.])
b 50.0, m 2.0
size(y_pred) = (16,)
size(y_pred - y) = (16,)
size(chi) = (16, 16)
size(loglike) = (16, 16)
b 75.025, m 2.0
size(y_pred) = (16,)
size(y_pred - y) = (16,)
size(chi) = (16, 16)
size(loglike) = (16, 16)
b 50.0, m 3.025
size(y_pred) = (16,)
size(y_pred - y) = (16,)
size(chi) = (16, 16)
size(loglike) = (16, 16)
b 75.025, m 0.9750000000000001
size(y_pred) = (16,)
size(y_pred - y) = (16,)
size(chi) = (16, 16)
size(loglike) = (16, 16)
b 68.76875000000001, m 1.4875
size(y_pred) = (16,)
size(y_pred - y) = (16,)
size(chi) = (16, 16)
size(loglike) = (16, 16)
Simple
Julia 1.9.3 | Idle
Mode: Command
Ln 5, Col 20
Notes-from-class.ipynb


https://symmetry.pi.local/user/dlang/lab/workspaces/auto-w/tree/home/dlang/FittingAModel2024/Notes-from-class.ipynb
File Edit View Run Kernel Tabs Settings Help
Terminal 1
Notes-from-class.ipynb
Notes-from-class-1.ipynb
Code
Julia 1.9.3
Filter files by name
/ ··· / dlang / FittingAModel2024 /
Name
Last Modified
data.csv 3 days ago
LICENSE 3 days ago
Notes-from-cla... an hour ago
Notes-from-cla... 2 minutes ago
old.ipynb 2 days ago
README.md 3 days ago
Untitled.ipynb 2 days ago
[63]:
function log_likelihood(params, x, y, sigma_y)
(b,m) = params
println("b ", b, ", m ", m);
y_pred = b .+ m .* x
@show size(y_pred)
@show size(y_pred .- y)
chi = (y_pred .- y) ./ sigma_y
@show size(chi)
loglike = -0.5 * chi^2
@show size(loglike)
return sum(loglike)
end;
[64]:
opt = optimize(p -> -log_likelihood(p, data.x, data.y, data.sigma_y), [50., 2.])
b 50.0, m 2.0
size(y_pred) = (16,)
size(y_pred .- y) = (16,)
size(chi) = (16,)
MethodError: no method matching ^(::Vector{Float64}, ::Int64)
Closest candidates are:
^(::Union{AbstractChar, AbstractString}, ::Integer)
@ Base strings/basic.jl:733
^(::DualNumbers.Dual{T}, ::Integer) where T
@ DualNumbers ~/.julia/packages/DualNumbers/5knFX/src/dual.jl:283
^(::DualNumbers.Dual{T}, ::Number) where T
@ DualNumbers ~/.julia/packages/DualNumbers/5knFX/src/dual.jl:283
...
Stacktrace:
[1] literal_pow
@ ./intfuncs.jl:338 [inlined]
[2] log_likelihood(params::Vector{Float64}, x::Vector{Int64}, y::Vector{Int64}, sigma_y::Vector{Int64})
@ Main ./In[63]:9
[3] (::var"#41#42")(p::Vector{Float64})
@ Main ./In[64]:1
[4] value!!(obj::NonDifferentiable{Float64, Vector{Float64}}, x::Vector{Float64})
@ NLSolversBase ~/.julia/packages/NLSolversBase/kavn7/src/interface.jl:9
Simple 1 4 Julia 1.9.3 | Idle
Mode: Command Ln 5, Col 20 Notes-from-class.ipynb


https://symmetry.pi.local/user/dlang/lab/workspaces/auto-w/tree/home/dlang/FittingAModel2024/Notes-from-class.ipynb
File Edit View Run Kernel Tabs Settings Help
Terminal 1
Notes-from-class.ipynb
Notes-from-class-1.ipynb
Code
Julia 1.9.3
Filter files by name
/ ··· / dlang / FittingAModel2024 /
Name
Last Modified
data.csv 3 days ago
LICENSE 3 days ago
Notes-from-cla... an hour ago
Notes-from-cla... seconds ago
old.ipynb 2 days ago
README.md 3 days ago
Untitled.ipynb 2 days ago
[65]:
function log_likelihood(params, x, y, sigma_y)
(b,m) = params
println("b ", b, ", m ", m);
y_pred = b .+ m .* x
@show size(y_pred)
@show size(y_pred .- y)
chi = (y_pred .- y) ./ sigma_y
@show size(chi)
loglike = -0.5 * chi .^ 2
@show size(loglike)
return sum(loglike)
end;
[66]:
opt = optimize(p -> -log_likelihood(p, data.x, data.y, data.sigma_y), [50., 2.])
b 50.0, m 2.0
size(y_pred) = (16,)
size(y_pred .- y) = (16,)
size(chi) = (16,)
size(loglike) = (16,)
b 75.025, m 2.0
size(y_pred) = (16,)
size(y_pred .- y) = (16,)
size(chi) = (16,)
size(loglike) = (16,)
b 50.0, m 3.025
size(y_pred) = (16,)
size(y_pred .- y) = (16,)
size(chi) = (16,)
size(loglike) = (16,)
b 75.025, m 0.9750000000000001
size(y_pred) = (16,)
size(y_pred .- y) = (16,)
size(chi) = (16,)
size(loglike) = (16,)
b 56.25625, m 2.5125
size(y_pred) = (16,)
size(y_pred .- y) = (16,)
size(chi) = (16,)
size(loglike) = (16,)
Simple
Julia 1.9.3 | Idle
Mode: Command
Ln 5, Col 20
Notes-from-class.ipynb


https://symmetry.pi.local/user/dlang/lab/workspaces/auto-w/tree/home/dlang/FittingAModel2024/Notes-from-class.ipynb
File Edit View Run Kernel Tabs Settings Help
Terminal 1
Notes-from-class.ipynb
Notes-from-class-1.ipynb
Code
Julia 1.9.3
Filter files by name
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Name
Last Modified
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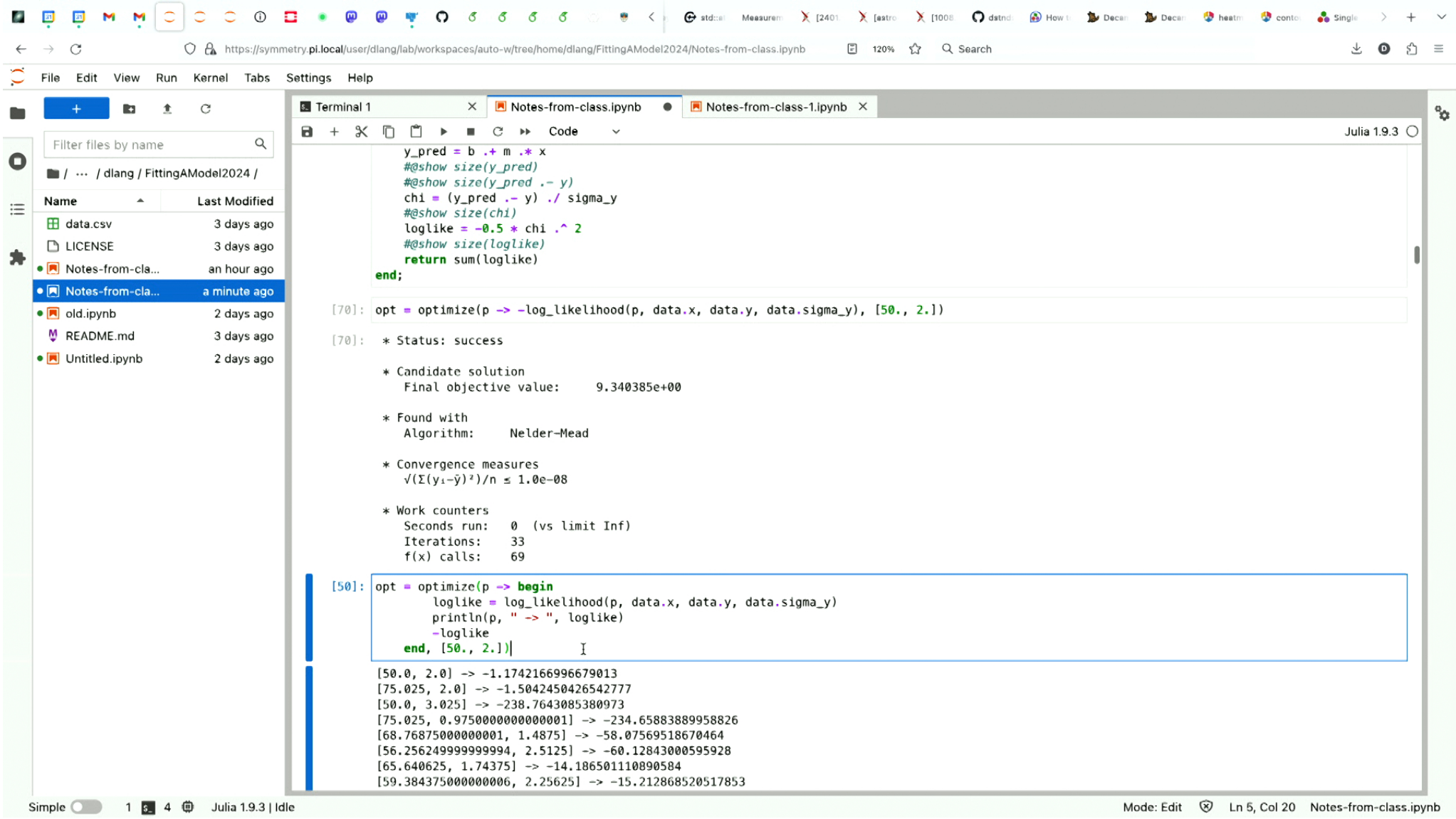
Notes-from-cla... an hour ago
Notes-from-cla... seconds ago
old.ipynb 2 days ago
README.md 3 days ago
Untitled.ipynb 2 days ago
b 34.0789969301468, m 2.2397991258552783
b 34.157521722507795, m 2.239193370048087
b 34.03190354291853, m 2.2400455921478164
b 34.04234486431557, m 2.2401208076292276
b 34.06200292264121, m 2.2398231346877076
b 34.01490953541294, m 2.2400696009802457
b 34.030931384096405, m 2.240001982199004
b 34.06103076381909, m 2.2397795247388954
b 34.03918534814367, m 2.239979075295586
b 34.07025688668848, m 2.2398002277842903
b 34.040762759744425, m 2.2399515435953257
b 34.01794518524689, m 2.240107484203204
b 34.05098848829263, m 2.2398942220665816
b 34.05256589989339, m 2.239866690366321
b 34.0425304860811, m 2.23995097906327
b 34.0527562146293, m 2.239893657534526
b 34.058752942071735, m 2.239864714504126
b 34.0476433503552, m 2.239922318298898
[68]:
* Status: success
* Candidate solution
Final objective value: 9.340385e+00
* Found with
Algorithm: Nelder-Mead
* Convergence measures
√(Σ(yᵢ−ȳ)²)/n ≤ 1.0e-08
* Work counters
Seconds run: 0 (vs limit Inf)
Iterations: 33
f(x) calls: 69
[50]:
opt = optimize(p -> begin
loglike = log_likelihood(p, data.x, data.y, data.sigma_y)
println(p, " -> ", loglike)
-loglike
end, [50., 2.])
[50.0, 2.0] -> -1.1742166996679013
[75.025, 2.0] -> -1.5042450426542777
Simple 1 4 Julia 1.9.3 | Idle
Mode: Command Ln 5, Col 20 Notes-from-class.ipynb


https://symmetry.pi.local/user/dlang/lab/workspaces/auto-w/tree/home/dlang/FittingAModel2024/Notes-from-class.ipynb
File Edit View Run Kernel Tabs Settings Help
Terminal 1
Notes-from-class.ipynb
Notes-from-class-1.ipynb
Code
Julia 1.9.3
Filter files by name
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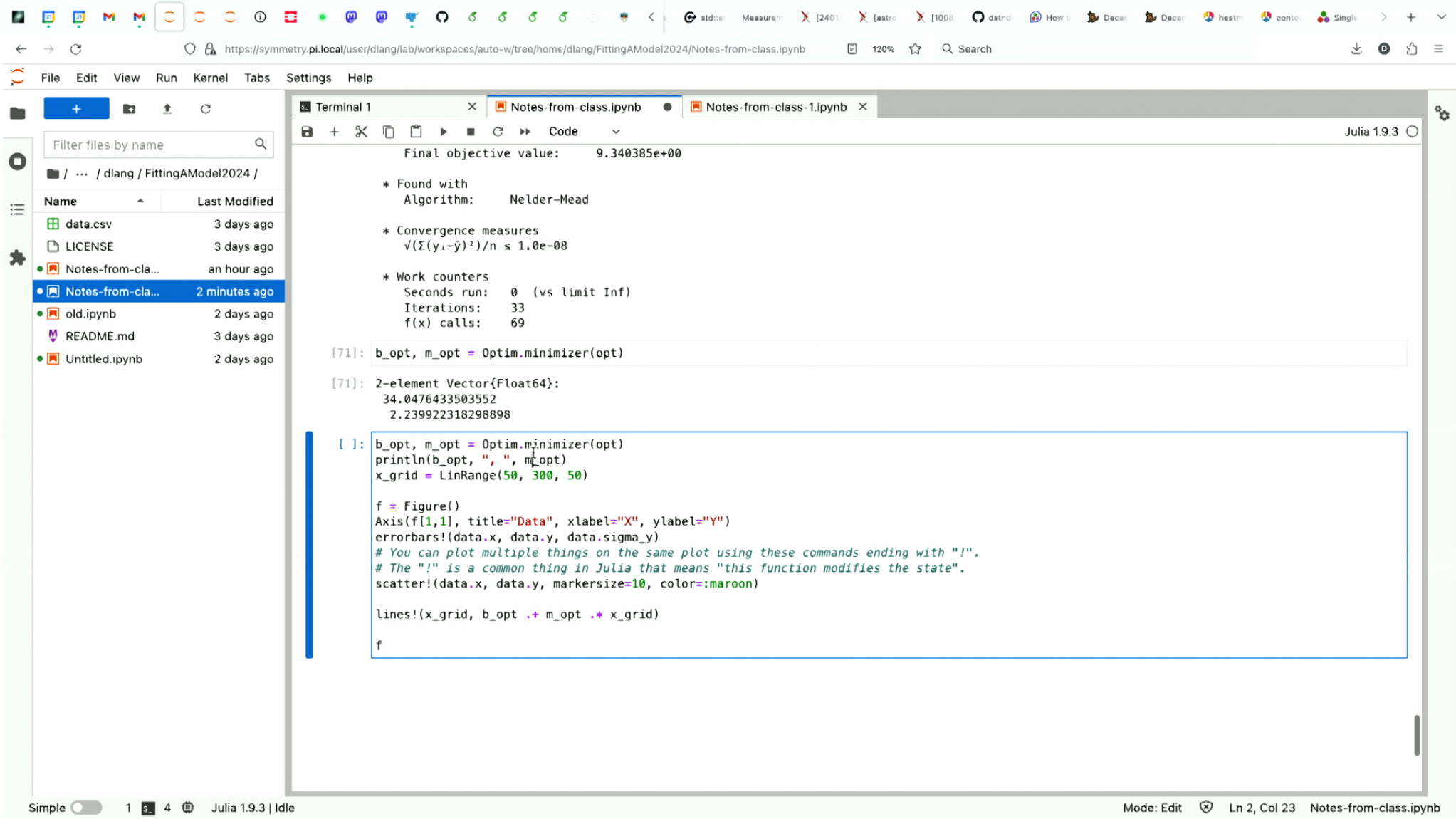
Name
Last Modified
data.csv 3 days ago
LICENSE 3 days ago
Notes-from-cla... an hour ago
Notes-from-cla... a minute ago
old.ipynb 2 days ago
README.md 3 days ago
Untitled.ipynb 2 days ago
y_pred = b .+ m .* x
#@show size(y_pred)
#@show size(y_pred .- y)
chi = (y_pred .- y) ./ sigma_y
#@show size(chi)
loglike = -0.5 * chi .^ 2
#@show size(loglike)
return sum(loglike)
end;
[70]: opt = optimize(p -> -log_likelihood(p, data.x, data.y, data.sigma_y), [50., 2.])
[70]: * Status: success
* Candidate solution
Final objective value: 9.340385e+00
* Found with
Algorithm: Nelder-Mead
* Convergence measures
√(Σ(yᵢ-ȳ)²)/n ≤ 1.0e-08
* Work counters
Seconds run: 0 (vs limit Inf)
Iterations: 33
f(x) calls: 69
[50]: opt = optimize(p -> begin
loglike = log_likelihood(p, data.x, data.y, data.sigma_y)
println(p, " -> ", loglike)
-loglike
end, [50., 2.])
[50.0, 2.0] -> -1.1742166996679013
[75.025, 2.0] -> -1.5042450426542777
[50.0, 3.025] -> -238.7643085380973
[75.025, 0.9750000000000001] -> -234.65883889958826
[68.76875000000001, 1.4875] -> -58.07569518670464
[56.256249999999994, 2.5125] -> -60.12843000595928
[65.640625, 1.74375] -> -14.186501110890584
[59.384375000000006, 2.25625] -> -15.212868520517853
Simple 1 4 Julia 1.9.3 | Idle
Mode: Edit Ln 5, Col 20 Notes-from-class.ipynb


https://symmetry.pi.local/user/dlang/lab/workspaces/auto-w/tree/home/dlang/FittingAModel2024/Notes-from-class.ipynb
File Edit View Run Kernel Tabs Settings Help
Terminal 1
Notes-from-class.ipynb
Notes-from-class-1.ipynb
Code
Julia 1.9.3
Filter files by name
/ ··· / dlang / FittingAModel2024 /
Name
Last Modified
data.csv 3 days ago
LICENSE 3 days ago
Notes-from-cla... an hour ago
Notes-from-cla... 2 minutes ago
old.ipynb 2 days ago
README.md 3 days ago
Untitled.ipynb 2 days ago
Final objective value: 9.340385e+00
* Found with
Algorithm: Nelder-Mead
* Convergence measures
√(Σ(yᵢ-ȳ)²)/n ≤ 1.0e-08
* Work counters
Seconds run: 0 (vs limit Inf)
Iterations: 33
f(x) calls: 69
[71]: b_opt, m_opt = Optim.minimizer(opt)
[71]: 2-element Vector{Float64}:
34.0476433503552
2.239922318298898
[ ]: b_opt, m_opt = Optim.minimizer(opt)
println(b_opt, ", ", m_opt)
x_grid = LinRange(50, 300, 50)
f = Figure()
Axis(f[1,1], title="Data", xlabel="X", ylabel="Y")
errorbars!(data.x, data.y, data.sigma_y)
You can plot multiple things on the same plot using these commands ending with "!".
The "!" is a common thing in Julia that means "this function modifies the state".
scatter!(data.x, data.y, markersize=10, color=:maroon)
lines!(x_grid, b_opt .+ m_opt .* x_grid)
f
Simple 1 4 Julia 1.9.3 | Idle
Mode: Edit Ln 2, Col 23 Notes-from-class.ipynb

https://symmetry.pi.local/user/dlang/lab/workspaces/auto-w/tree/home/dlang/FittingAModel2024/Notes-from-class.ipynb

File Edit View Run Kernel Tabs Settings Help

Terminal 1 | Notes-from-class.ipynb | Notes-from-class-1.ipynb

Code | Julia 1.9.3

Filter files by name

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Notes-from-cla...	seconds ago
old.ipynb	2 days ago
README.md	3 days ago
Untitled.ipynb	2 days ago

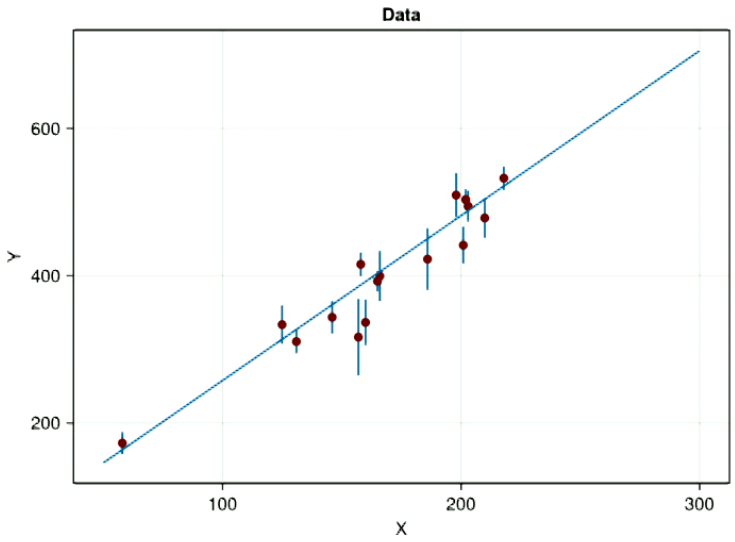
```
f = Figure()
Axis(f[1,1], title="Data", xlabel="X", ylabel="Y")
errorbars!(data.x, data.y, data.sigma_y)
# You can plot multiple things on the same plot using these commands ending with "!".
# The "!" is a common thing in Julia that means "this function modifies the state".
scatter!(data.x, data.y, markersize=10, color=:maroon)

lines!(x_grid, b_opt .+ m_opt .* x_grid)

f
```

```
34.0476433503552, 2.239922318298898
```

[72]:



[ ]:

Simple 1 4 Julia 1.9.3 | Idle Mode: Command Ln 4, Col 1 Notes-from-class.ipynb

https://symmetry.pi.local/user/dlang/lab/workspaces/auto-w/tree/home/dlang/FittingAModel2024/Notes-from-class.ipynb

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Terminal 1 | Notes-from-class.ipynb | Notes-from-class-1.ipynb

Code | Julia 1.9.3

Filter files by name

/ ··· / dlang / FittingAModel2024 /

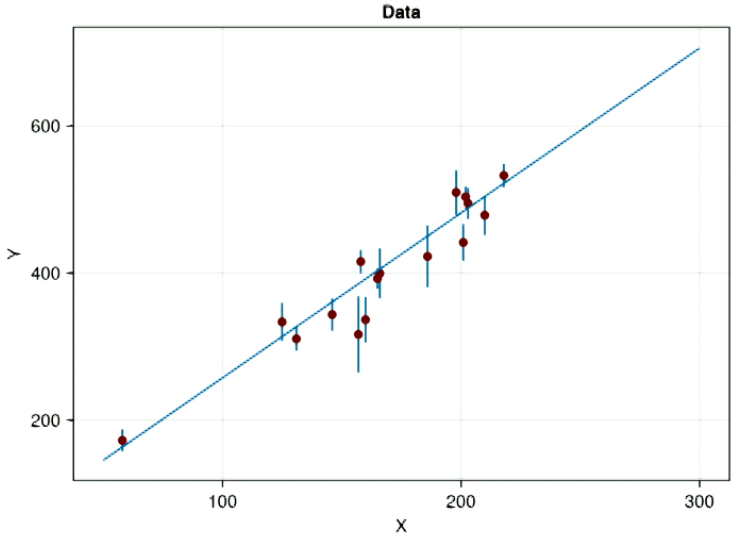
Name	Last Modified
data.csv	3 days ago
LICENSE	3 days ago
Notes-from-cla...	an hour ago
Notes-from-cla...	a minute ago
old.ipynb	2 days ago
README.md	3 days ago
Untitled.ipynb	2 days ago

```
lines!(x_grid, b_opt .+ m_opt .* x_grid)

f
```

```
34.0476433503552, 2.239922318298898
```

[72]:



[ ]:

```
n_b, n_m = 30,40
b_grid = LinRange(0, 100, n_b)
m_grid = LinRange(1.5, 3.5, n_m)
ll_grid = zeros(n_b, n_m)
```

Simple 1 4 Julia 1.9.3 | Idle Mode: Edit Ln 4, Col 26 Notes-from-class.ipynb

https://symmetry.pi.local/user/dlang/lab/workspaces/auto-w/tree/home/dlang/FittingAModel2024/Notes-from-class.ipynb

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Terminal 1 | Notes-from-class.ipynb | Notes-from-class-1.ipynb

Code | Julia 1.9.3

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Name	Last Modified
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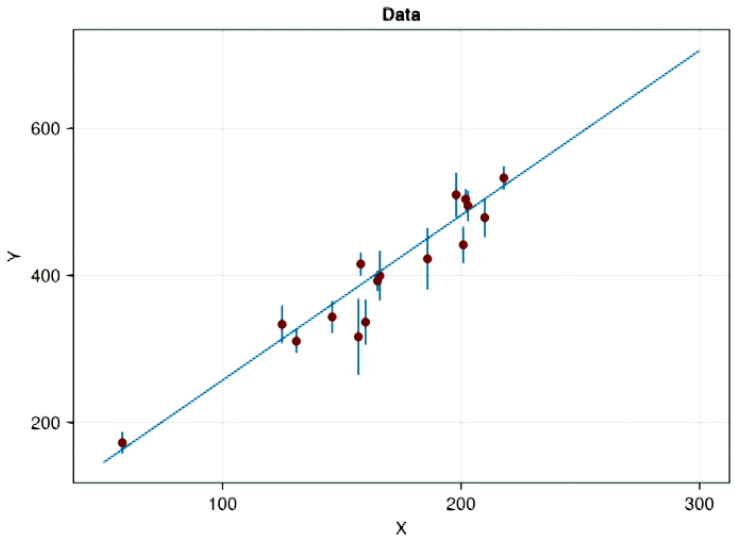
Notes-from-cla...	2 minutes ago
old.ipynb	2 days ago
README.md	3 days ago
Untitled.ipynb	2 days ago

```
f
```

```
34.0476433503552, 2.239922318298898
```

[72]:



[ ]:

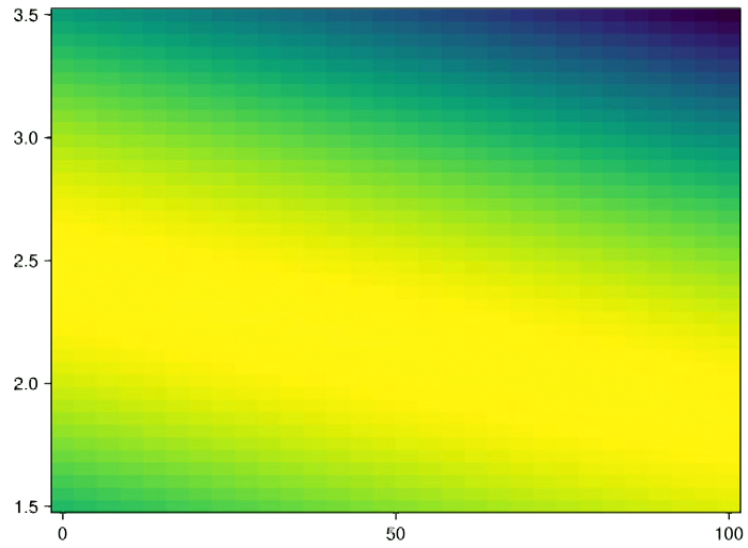
```
n_b, n_m = 30,40
b_grid = LinRange(0, 100, n_b)
m_grid = LinRange(1.5, 3.5, n_m)
ll_grid = zeros(n_b, n_m)
for i in 1:n_b
    for j in 1:n_m
        ll_grid[i
```

Simple 1 4 Julia 1.9.3 | Idle Mode: Edit Ln 7, Col 18 Notes-from-class.ipynb

```
ll_grid = zeros(n_b, n_m)
for i in 1:n_b
    for j in 1:n_m
        ll_grid[i, j] = log_likelihood([b_grid[i], m_grid[j]], data.x, data.y, data.sigma_y)
    end
end
```

[74]:
```
fig, ax, hm = heatmap(b_grid, m_grid, ll_grid)
```

[74]:



[ ]:

https://symmetry.pi.local/user/dlang/lab/workspaces/auto-w/tree/home/dlang/FittingAModel2024/Notes-from-class.ipynb

File Edit View Run Kernel Tabs Settings Help

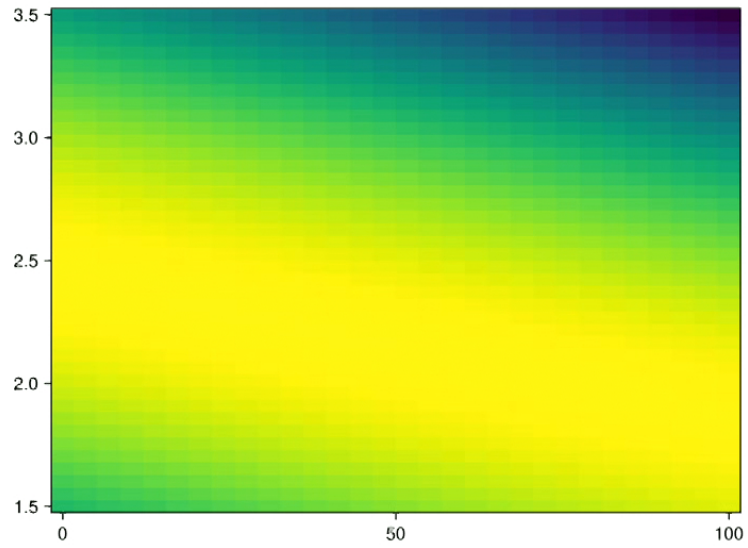
Terminal 1 | Notes-from-class.ipynb | Notes-from-class-1.ipynb

Code Julia 1.9.3

```
ll_grid = zeros(n_b, n_m)
for i in 1:n_b
    for j in 1:n_m
        ll_grid[i, j] = log_likelihood([b_grid[i], m_grid[j]], data.x, data.y, data.sigma_y)
    end
end
```

[75]:
```
fig, ax, hm = heatmap(b_grid, m_grid, ll_grid, xlabel="B")
```

[75]:



[ ]:

Filter files by name

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data.csv	3 days ago
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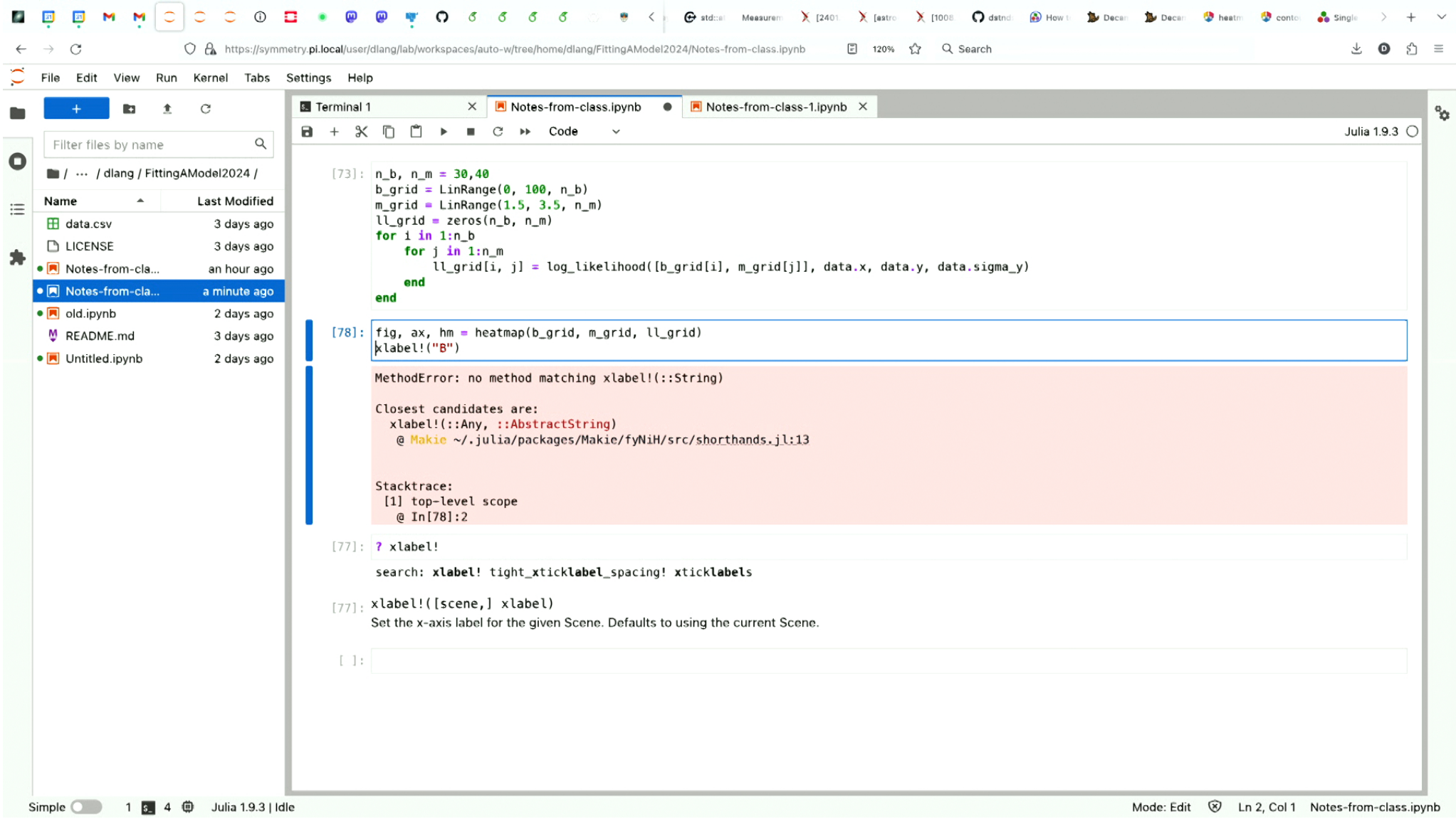
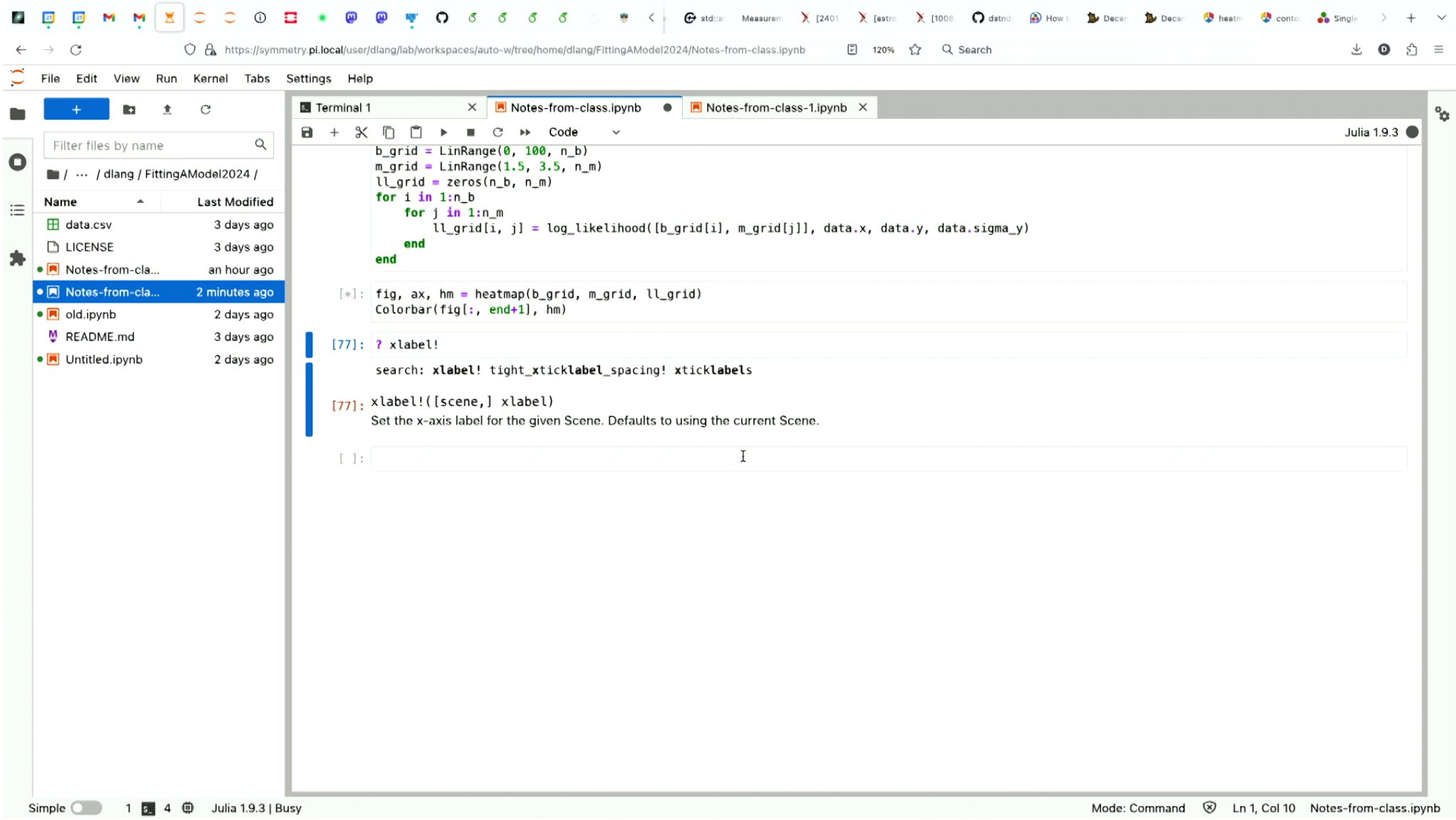
Notes-from-cla...	an hour ago
Notes-from-cla...	a minute ago
old.ipynb	2 days ago
README.md	3 days ago
Untitled.ipynb	2 days ago

Simple 1 4 Julia 1.9.3 | Idle Mode: Edit Ln 1, Col 58 Notes-from-class.ipynb


https://symmetry.pi.local/user/dlang/lab/workspaces/auto-w/tree/home/dlang/FittingAModel2024/Notes-from-class.ipynb
File Edit View Run Kernel Tabs Settings Help
Terminal 1
Notes-from-class.ipynb
Notes-from-class-1.ipynb
Code
Julia 1.9.3
Filter files by name
/ ··· / dlang / FittingAModel2024 /
Name
Last Modified
data.csv 3 days ago
LICENSE 3 days ago
Notes-from-cla... an hour ago
Notes-from-cla... a minute ago
old.ipynb 2 days ago
README.md 3 days ago
Untitled.ipynb 2 days ago
[73]:
n_b, n_m = 30,40
b_grid = LinRange(0, 100, n_b)
m_grid = LinRange(1.5, 3.5, n_m)
ll_grid = zeros(n_b, n_m)
for i in 1:n_b
for j in 1:n_m
ll_grid[i, j] = log_likelihood([b_grid[i], m_grid[j]], data.x, data.y, data.sigma_y)
end
end
[78]:
fig, ax, hm = heatmap(b_grid, m_grid, ll_grid)
xlabel!("B")
MethodError: no method matching xlabel!(::String)
Closest candidates are:
xlabel!(::Any, ::AbstractString)
@ Makie ~/.julia/packages/Makie/fyNiH/src/shorthands.jl:13
Stacktrace:
[1] top-level scope
@ In[78]:2
[77]:
? xlabel!
search: xlabel! tight_xticklabel_spacing! xticklabels
[77]:
xlabel!([scene,] xlabel)
Set the x-axis label for the given Scene. Defaults to using the current Scene.
[ ]:
Simple
1 4
Julia 1.9.3 | Idle
Mode: Edit
Ln 2, Col 1
Notes-from-class.ipynb


https://symmetry.pi.local/user/dlang/lab/workspaces/auto-w/tree/home/dlang/FittingAModel2024/Notes-from-class.ipynb
File Edit View Run Kernel Tabs Settings Help
Terminal 1
Notes-from-class.ipynb
Notes-from-class-1.ipynb
Code
Julia 1.9.3
Filter files by name
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Name
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LICENSE 3 days ago
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Notes-from-cla... 2 minutes ago
old.ipynb 2 days ago
README.md 3 days ago
Untitled.ipynb 2 days ago
b_grid = LinRange(0, 100, n_b)
m_grid = LinRange(1.5, 3.5, n_m)
ll_grid = zeros(n_b, n_m)
for i in 1:n_b
for j in 1:n_m
ll_grid[i, j] = log_likelihood([b_grid[i], m_grid[j]], data.x, data.y, data.sigma_y)
end
end
[*]:
fig, ax, hm = heatmap(b_grid, m_grid, ll_grid)
Colorbar(fig[:, end+1], hm)
[77]:
? xlabel!
search: xlabel! tight_xticklabel_spacing! xticklabels
[77]:
xlabel!([scene,] xlabel)
Set the x-axis label for the given Scene. Defaults to using the current Scene.
[ ]:
Simple
1 4
Julia 1.9.3 | Busy
Mode: Command
Ln 1, Col 10
Notes-from-class.ipynb

https://symmetry.pi.local/user/dlang/lab/workspaces/auto-w/tree/home/dlang/FittingAModel2024/Notes-from-class.ipynb

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Terminal 1 | Notes-from-class.ipynb | Notes-from-class-1.ipynb

Code | Julia 1.9.3

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old.ipynb	2 days ago
README.md	3 days ago
Untitled.ipynb	2 days ago

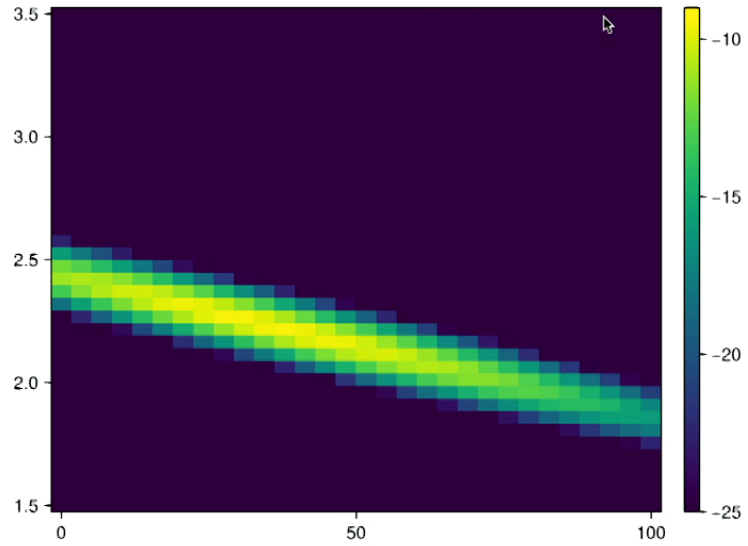
```
for i in 1:n_b
    for j in 1:n_m
        ll_grid[i, j] = log_likelihood([b_grid[i], m_grid[j]], data.x, data.y, data.sigma_y)
    end
end
```

[82]:
```
maximum(ll_grid)
```

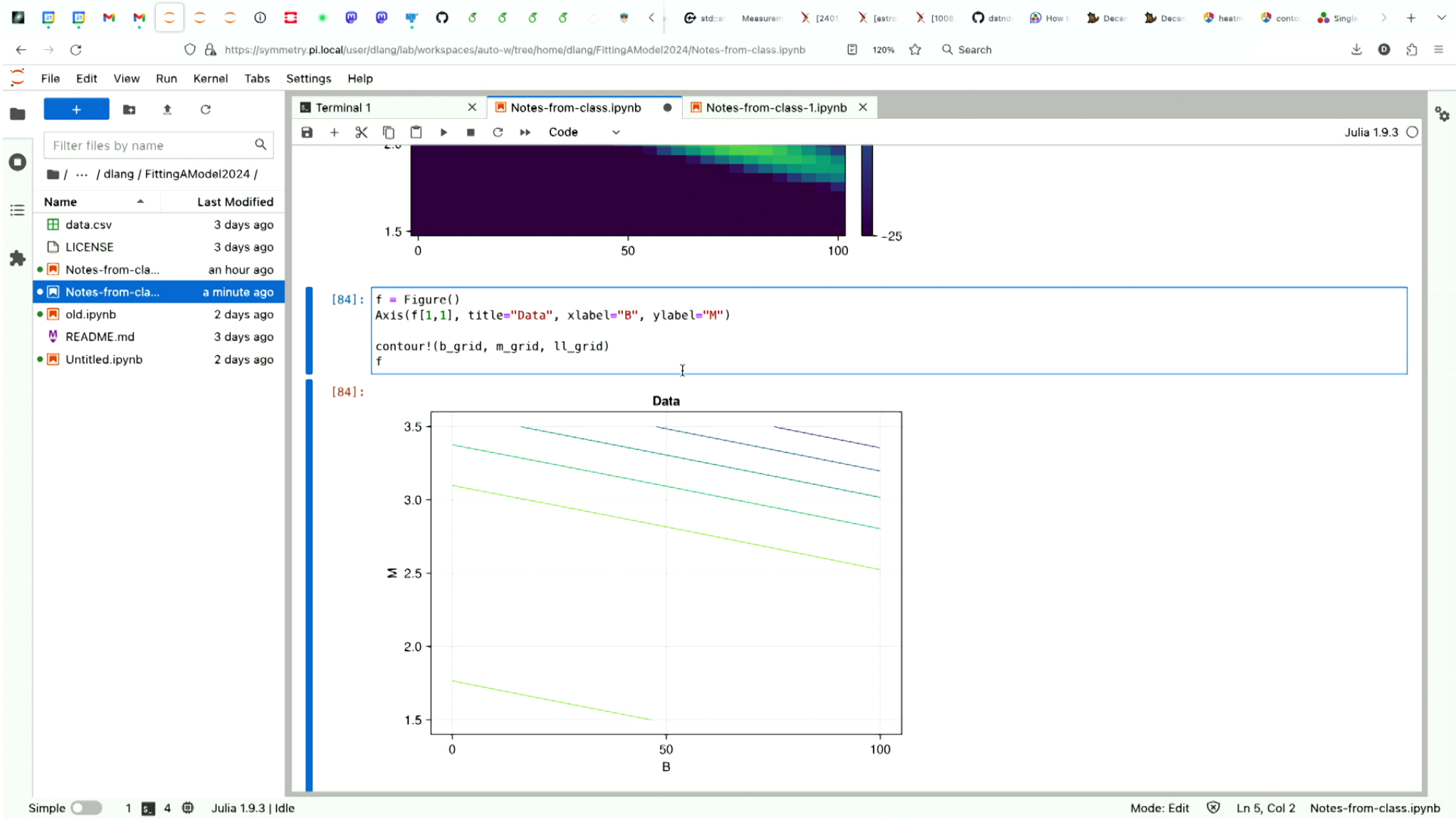
[82]:
```
-9.363035567226113
```

[83]:
```
fig, ax, hm = heatmap(b_grid, m_grid, ll_grid, colorrange=(-25, -9))
Colorbar(fig[:, end+1], hm)
fig
```

[83]:



Simple 1 4 Julia 1.9.3 | Idle Mode: Command Ln 1, Col 1 Notes-from-class.ipynb


https://symmetry.pi.local/user/dlang/lab/workspaces/auto-w/tree/home/dlang/FittingAModel2024/Notes-from-class.ipynb
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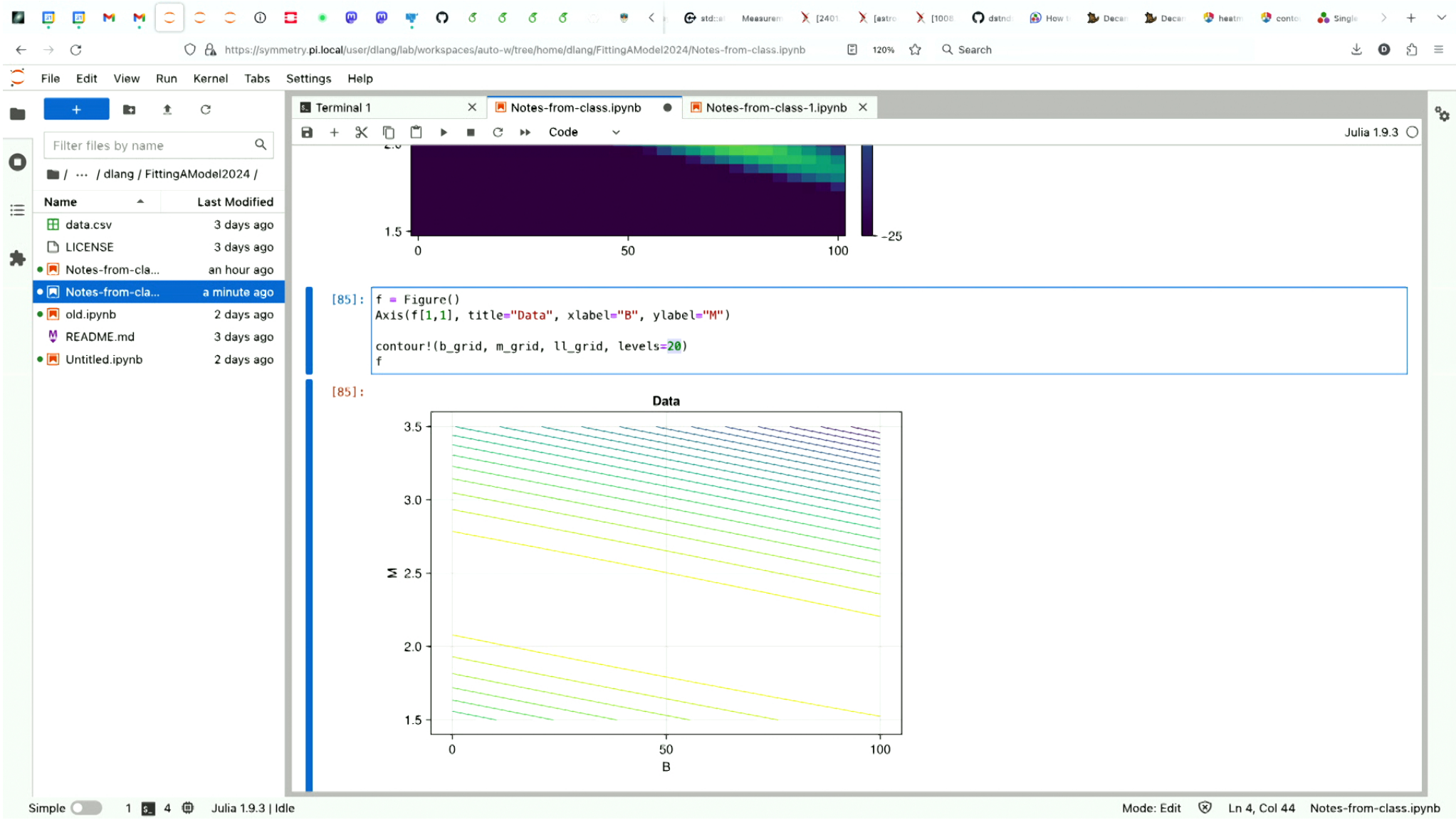
old.ipynb 2 days ago
README.md 3 days ago
Untitled.ipynb 2 days ago
[84]:
f = Figure()
Axis(f[1,1], title="Data", xlabel="B", ylabel="M")
contour!(b_grid, m_grid, ll_grid)
f
[84]:
Data
M
B
Simple
Julia 1.9.3 | Idle
Mode: Edit
Ln 5, Col 2
Notes-from-class.ipynb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Terminal 1
Notes-from-class.ipynb
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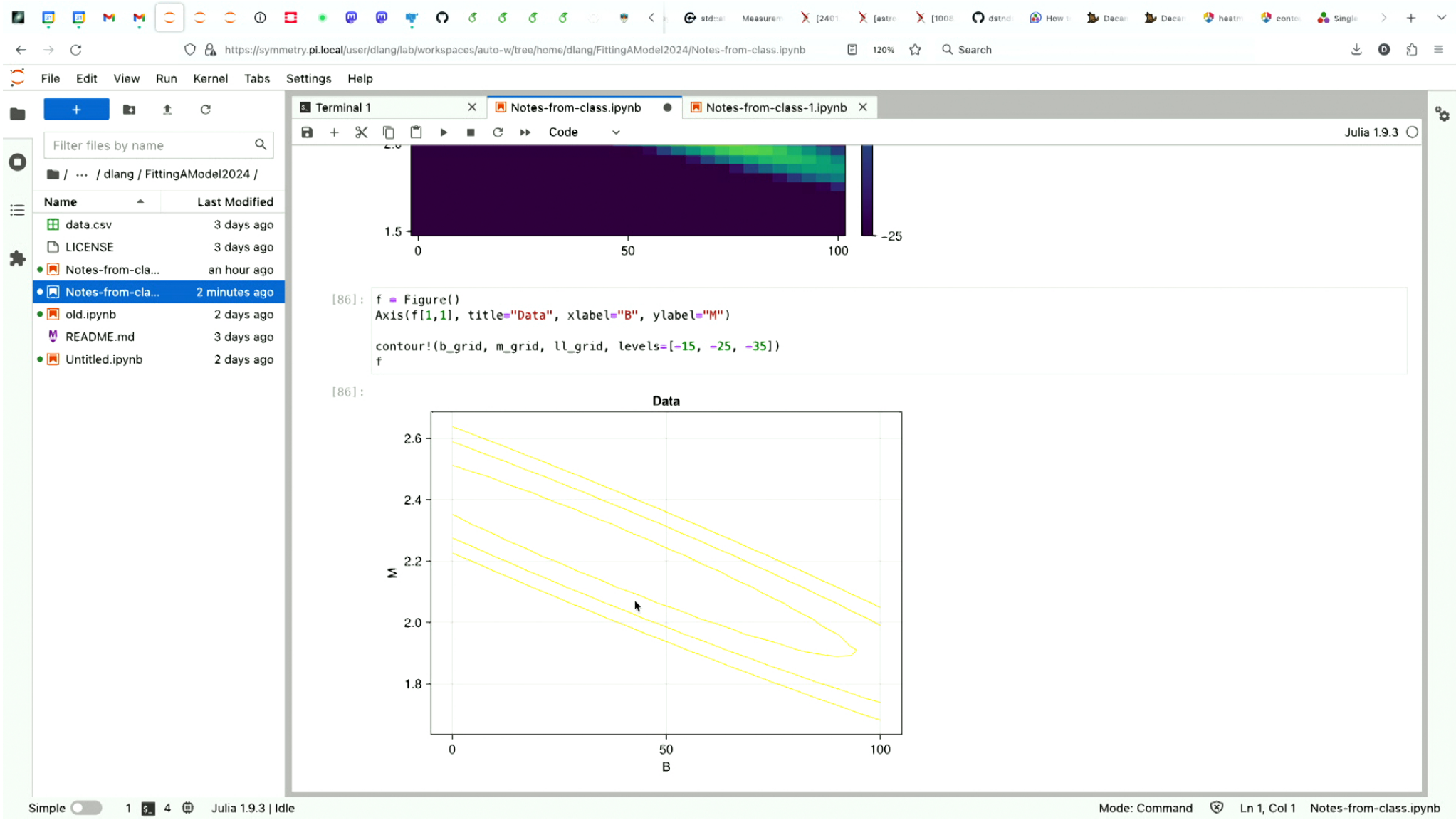
old.ipynb 2 days ago
README.md 3 days ago
Untitled.ipynb 2 days ago
[85]:
f = Figure()
Axis(f[1,1], title="Data", xlabel="B", ylabel="M")
contour!(b_grid, m_grid, ll_grid, levels=20)
f
[85]:
Data
M
B
Simple
Julia 1.9.3 | Idle
Mode: Edit
Ln 4, Col 44
Notes-from-class.ipynb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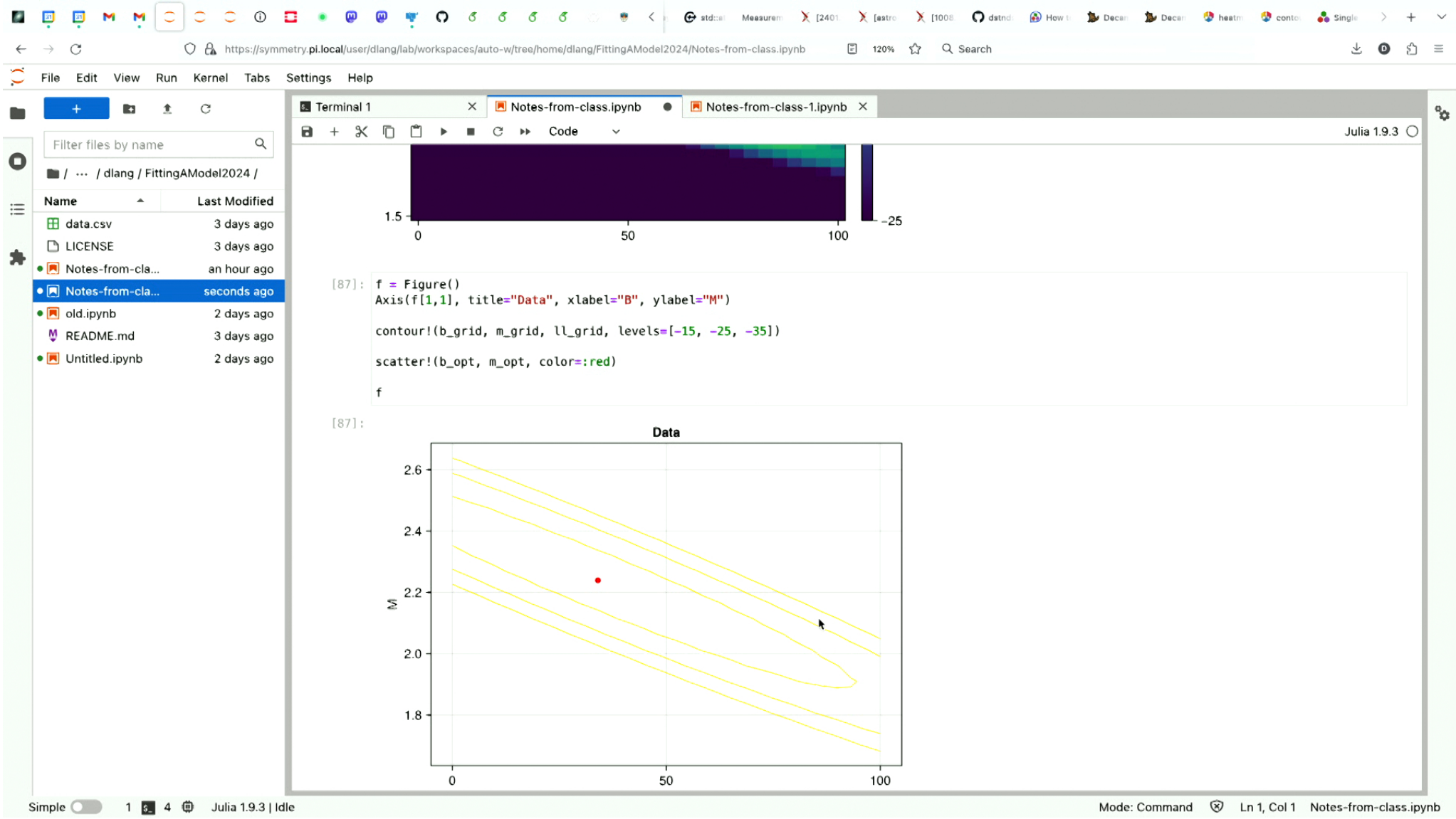
Untitled.ipynb 2 days ago
1.5
0
50
100
-25
[86]:
f = Figure()
Axis(f[1,1], title="Data", xlabel="B", ylabel="M")
contour!(b_grid, m_grid, ll_grid, levels=[-15, -25, -35])
f
[86]:
Data
2.6
2.4
2.2
2.0
1.8
M
0
50
100
B
Simple
1 4
Julia 1.9.3 | Idle
Mode: Command
Ln 1, Col 1
Notes-from-class.ipynb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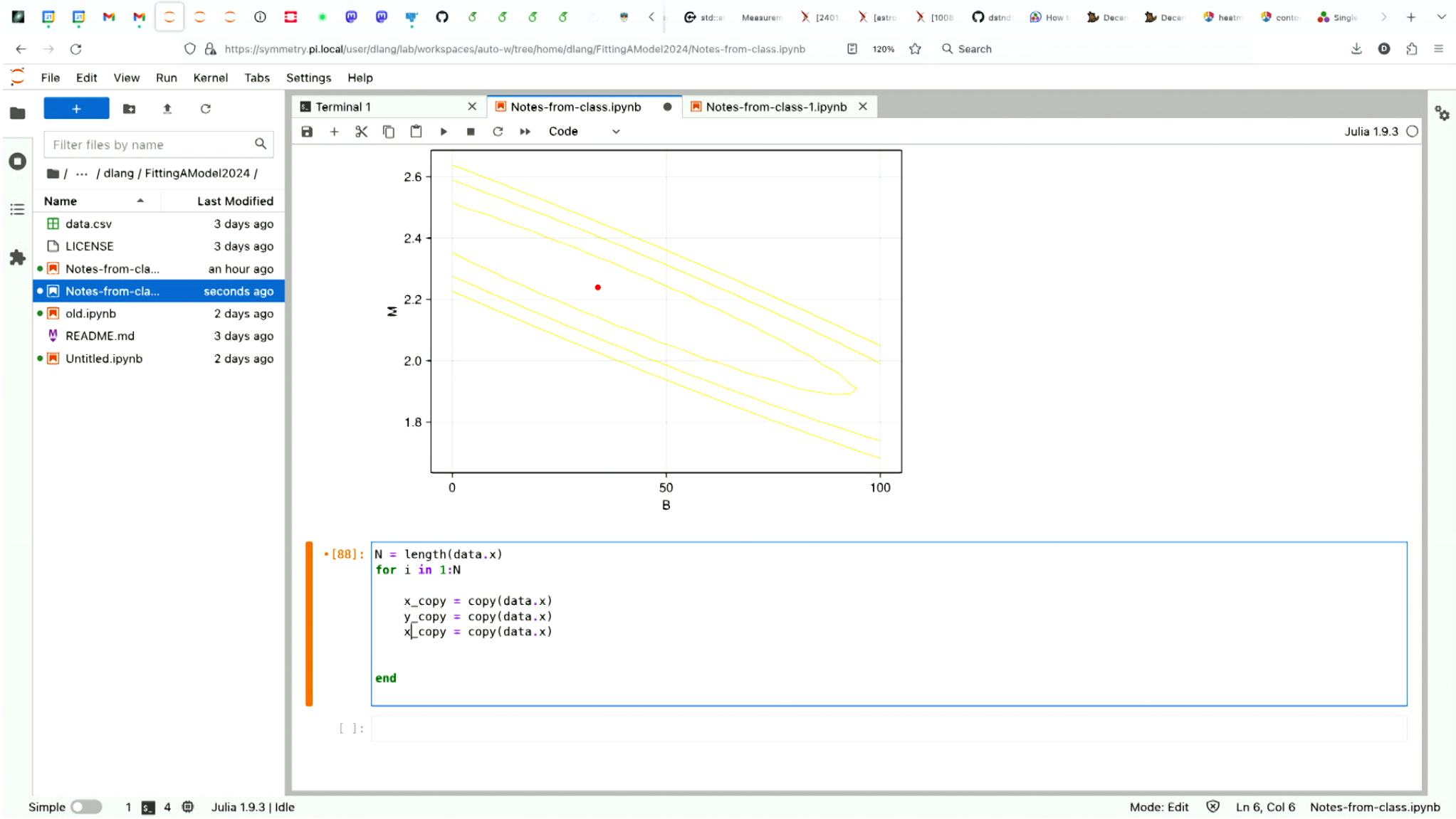
1.5
0
50
100
-25
[87]:
f = Figure()
Axis(f[1,1], title="Data", xlabel="B", ylabel="M")
contour!(b_grid, m_grid, ll_grid, levels=[-15, -25, -35])
scatter!(b_opt, m_opt, color=:red)
f
[87]:
Data
2.6
2.4
2.2
2.0
1.8
M
0
50
100
Simple
1
4
Julia 1.9.3 | Idle
Mode: Command
Ln 1, Col 1
Notes-from-class.ipynb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Terminal 1
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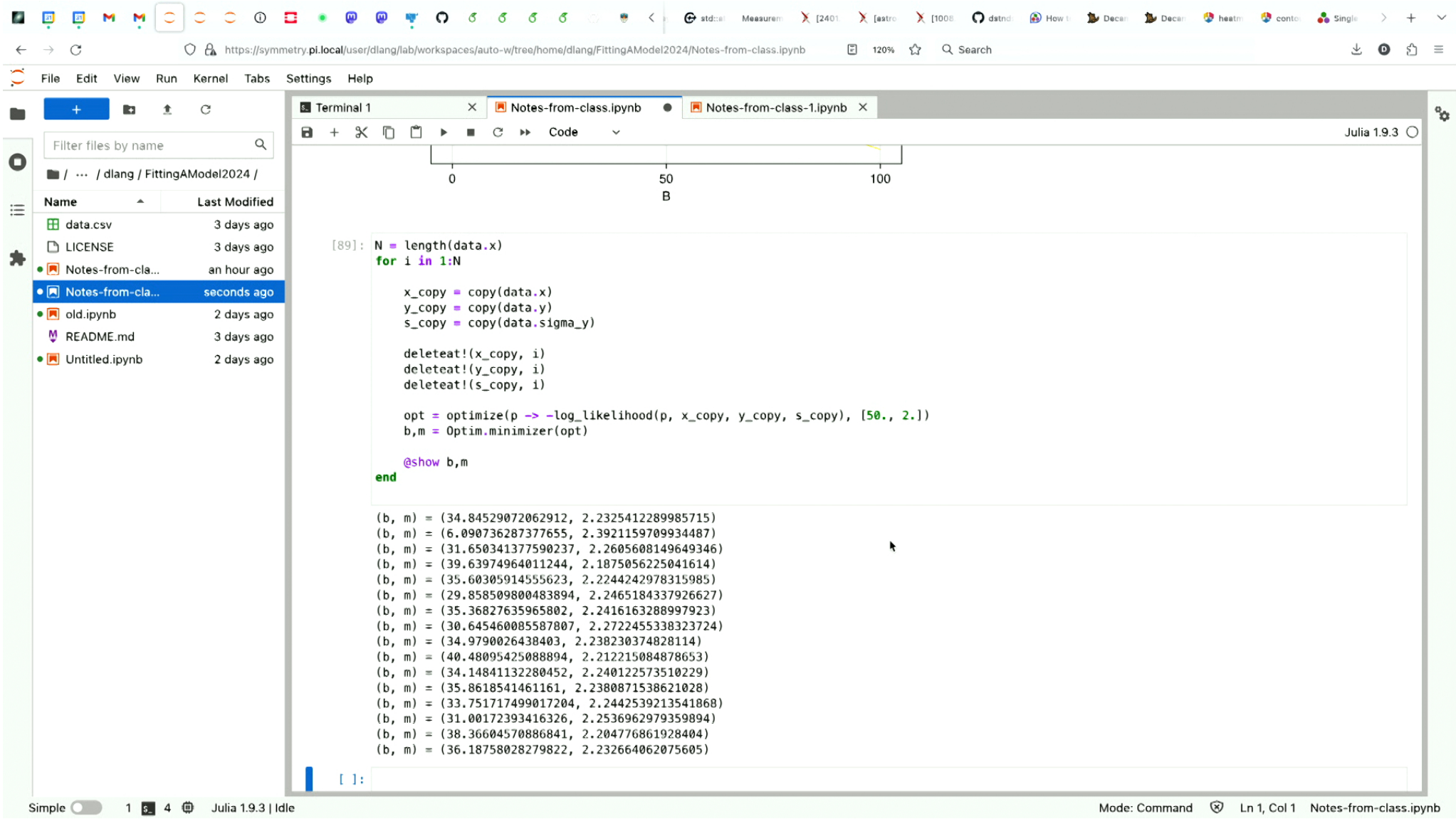
old.ipynb 2 days ago
README.md 3 days ago
Untitled.ipynb 2 days ago
2.6
2.4
2.2
2.0
1.8
M
0
50
100
B
[88]:
N = length(data.x)
for i in 1:N
x_copy = copy(data.x)
y_copy = copy(data.x)
x_copy = copy(data.x)
end
[ ]:
Simple
1 4
Julia 1.9.3 | Idle
Mode: Edit
Ln 6, Col 6
Notes-from-class.ipynb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Terminal 1
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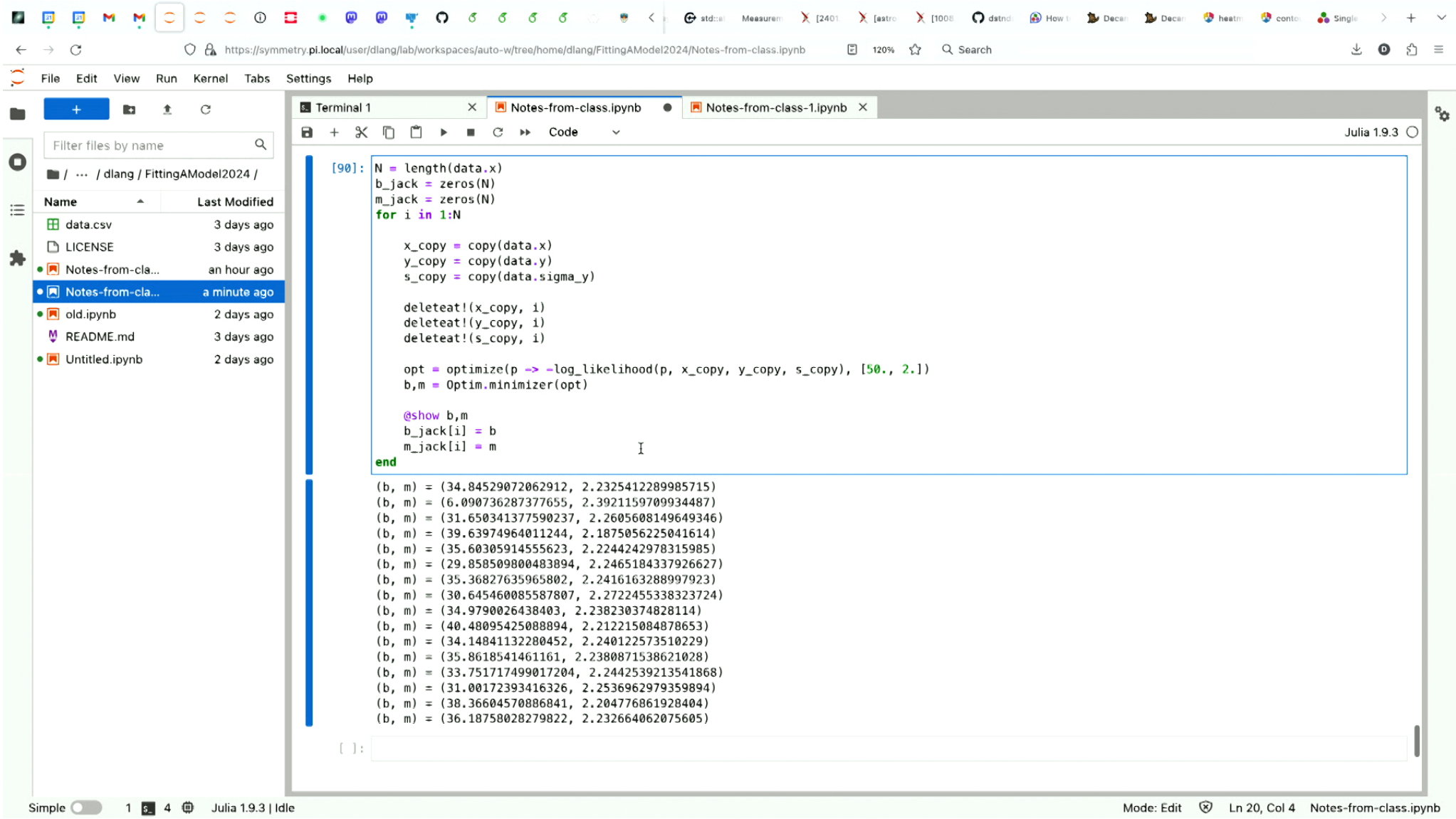
Untitled.ipynb 2 days ago
0
50
100
B
[89]:
N = length(data.x)
for i in 1:N
x_copy = copy(data.x)
y_copy = copy(data.y)
s_copy = copy(data.sigma_y)
deleteat!(x_copy, i)
deleteat!(y_copy, i)
deleteat!(s_copy, i)
opt = optimize(p -> -log_likelihood(p, x_copy, y_copy, s_copy), [50., 2.])
b,m = Optim.minimizer(opt)
@show b,m
end
(b, m) = (34.84529072062912, 2.2325412289985715)
(b, m) = (6.090736287377655, 2.3921159709934487)
(b, m) = (31.650341377590237, 2.2605608149649346)
(b, m) = (39.63974964011244, 2.1875056225041614)
(b, m) = (35.60305914555623, 2.2244242978315985)
(b, m) = (29.858509800483894, 2.2465184337926627)
(b, m) = (35.36827635965802, 2.2416163288997923)
(b, m) = (30.645460085587807, 2.2722455338323724)
(b, m) = (34.9790026438403, 2.238230374828114)
(b, m) = (40.48095425088894, 2.212215084878653)
(b, m) = (34.14841132280452, 2.240122573510229)
(b, m) = (35.8618541461161, 2.2380871538621028)
(b, m) = (33.751717499017204, 2.2442539213541868)
(b, m) = (31.00172393416326, 2.2536962979359894)
(b, m) = (38.36604570886841, 2.204776861928404)
(b, m) = (36.18758028279822, 2.232664062075605)
[ ]:
Simple 1 4 Julia 1.9.3 | Idle
Mode: Command Ln 1, Col 1 Notes-from-class.ipynb


https://symmetry.pi.local/user/dlang/lab/workspaces/auto-w/tree/home/dlang/FittingAModel2024/Notes-from-class.ipynb
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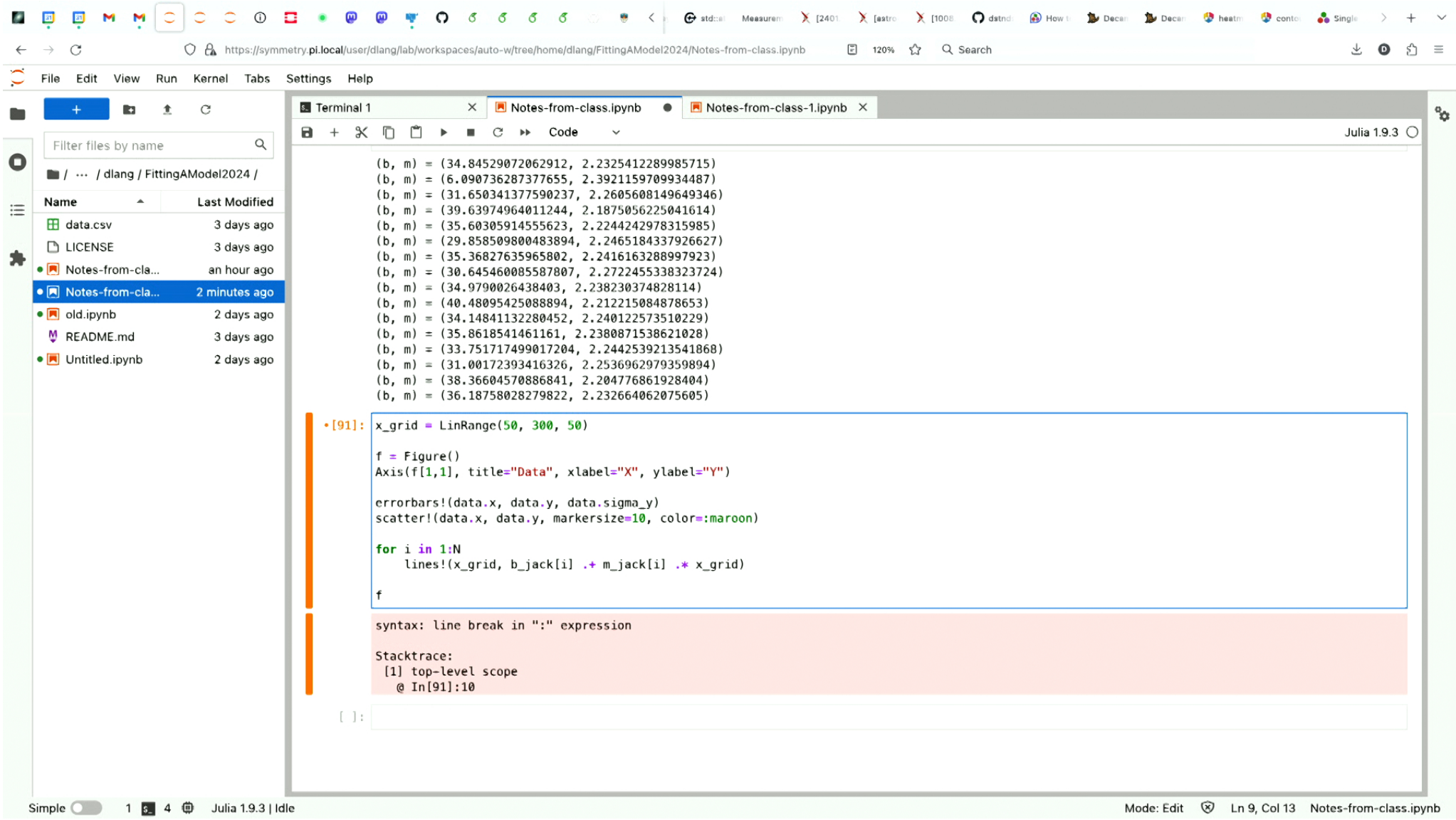
Terminal 1
Notes-from-class.ipynb
Notes-from-class-1.ipynb
Code
Julia 1.9.3
[90]:
N = length(data.x)
b_jack = zeros(N)
m_jack = zeros(N)
for i in 1:N
x_copy = copy(data.x)
y_copy = copy(data.y)
s_copy = copy(data.sigma_y)
deleteat!(x_copy, i)
deleteat!(y_copy, i)
deleteat!(s_copy, i)
opt = optimize(p -> -log_likelihood(p, x_copy, y_copy, s_copy), [50., 2.])
b,m = Optim.minimizer(opt)
@show b,m
b_jack[i] = b
m_jack[i] = m
end
(b, m) = (34.84529072062912, 2.2325412289985715)
(b, m) = (6.090736287377655, 2.3921159709934487)
(b, m) = (31.650341377590237, 2.2605608149649346)
(b, m) = (39.63974964011244, 2.1875056225041614)
(b, m) = (35.60305914555623, 2.2244242978315985)
(b, m) = (29.858509800483894, 2.2465184337926627)
(b, m) = (35.36827635965802, 2.2416163288997923)
(b, m) = (30.645460085587807, 2.2722455338323724)
(b, m) = (34.9790026438403, 2.238230374828114)
(b, m) = (40.48095425088894, 2.212215084878653)
(b, m) = (34.14841132280452, 2.240122573510229)
(b, m) = (35.8618541461161, 2.2380871538621028)
(b, m) = (33.751717499017204, 2.2442539213541868)
(b, m) = (31.00172393416326, 2.2536962979359894)
(b, m) = (38.36604570886841, 2.204776861928404)
(b, m) = (36.18758028279822, 2.232664062075605)
[ ]:
Simple 1 4 Julia 1.9.3 | Idle
Mode: Edit Ln 20, Col 4 Notes-from-class.ipynb


https://symmetry.pi.local/user/dlang/lab/workspaces/auto-w/tree/home/dlang/FittingAModel2024/Notes-from-class.ipynb
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old.ipynb 2 days ago
README.md 3 days ago
Untitled.ipynb 2 days ago
(b, m) = (34.84529072062912, 2.2325412289985715)
(b, m) = (6.090736287377655, 2.3921159709934487)
(b, m) = (31.650341377590237, 2.2605608149649346)
(b, m) = (39.63974964011244, 2.1875056225041614)
(b, m) = (35.60305914555623, 2.2244242978315985)
(b, m) = (29.858509800483894, 2.2465184337926627)
(b, m) = (35.36827635965802, 2.2416163288997923)
(b, m) = (30.645460085587807, 2.2722455338323724)
(b, m) = (34.9790026438403, 2.238230374828114)
(b, m) = (40.48095425088894, 2.212215084878653)
(b, m) = (34.14841132280452, 2.240122573510229)
(b, m) = (35.8618541461161, 2.2380871538621028)
(b, m) = (33.751717499017204, 2.2442539213541868)
(b, m) = (31.00172393416326, 2.2536962979359894)
(b, m) = (38.36604570886841, 2.204776861928404)
(b, m) = (36.18758028279822, 2.232664062075605)
•[91]:
x_grid = LinRange(50, 300, 50)
f = Figure()
Axis(f[1,1], title="Data", xlabel="X", ylabel="Y")
errorbars!(data.x, data.y, data.sigma_y)
scatter!(data.x, data.y, markersize=10, color=:maroon)
for i in 1:N
lines!(x_grid, b_jack[i] .+ m_jack[i] .* x_grid)
f
syntax: line break in ":" expression
Stacktrace:
[1] top-level scope
@ In[91]:10
[ ]:
Simple
1 4
Julia 1.9.3 | Idle
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Ln 9, Col 13
Notes-from-class.ipynb

https://symmetry.pi.local/user/dlang/lab/workspaces/auto-w/tree/home/dlang/FittingAModel2024/Notes-from-class.ipynb

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Code | Julia 1.9.3

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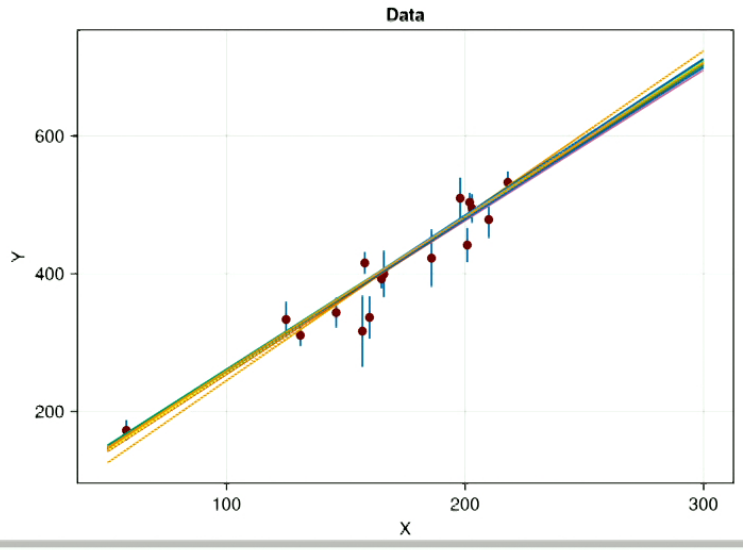
Notes-from-cla...	2 minutes ago
old.ipynb	2 days ago
README.md	3 days ago
Untitled.ipynb	2 days ago

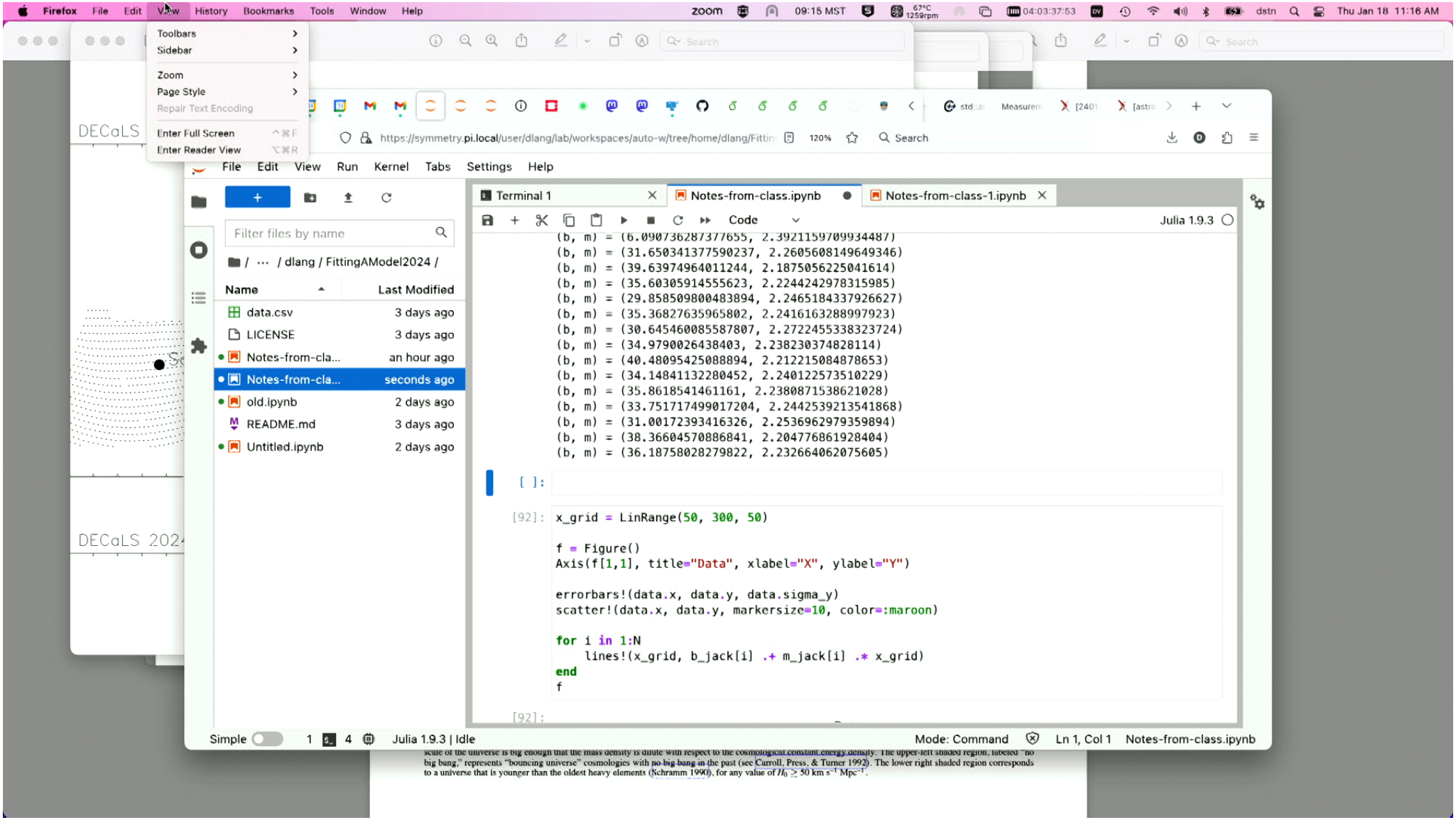
```
(b, m) = (31.001723934163 26, 2.25369629793598 94)
(b, m) = (38.36604570886841, 2.204776861928404)
(b, m) = (36.18758028279822, 2.232664062075605)
```

[92]:
```
x_grid = LinRange(50, 300, 50)

f = Figure()
Axis(f[1,1], title="Data", xlabel="X", ylabel="Y")

errorbars!(data.x, data.y, data.sigma_y)
scatter!(data.x, data.y, markersize=10, color=:maroon)

for i in 1:N
    lines!(x_grid, b_jack[i] .+ m_jack[i] .* x_grid)
end
f
```

[92]:



Simple 1 4 Julia 1.9.3 | Idle Mode: Command Ln 1, Col 1 Notes-from-class.ipynb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Terminal 1
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Notes-from-class-1.ipynb
Code
Julia 1.9.3
(b, m) = (31.650341377590237, 2.2605608149649346)
(b, m) = (39.63974964011244, 2.187505622504161 4)
(b, m) = (35.60305914555623, 2.2244242978315985)
(b, m) = (29.858509800483894, 2.2465184337926627)
(b, m) = (35.36827635965802, 2.2416163288997923)
(b, m) = (30.645460085587807, 2.2722455338323724)
(b, m) = (34.9790026438403, 2.238230374828114)
(b, m) = (40.48095425088894, 2.212215084878653)
(b, m) = (34.14841132280452, 2.240122573510229)
(b, m) = (35.8618541461161, 2.2380871538621028)
(b, m) = (33.751717499017204, 2.2442539213541868)
(b, m) = (31.00172393416326, 2.2536962979359894)
(b, m) = (38.36604570886841, 2.204776861928404)
(b, m) = (36.18758028279822, 2.232664062075605)
[ ]:
[92]:
x_grid = LinRange(50, 300, 50)
f = Figure()
Axis(f[1,1], title="Data", xlabel="X", ylabel="Y")
errorbars!(data.x, data.y, data.sigma_y)
scatter!(data.x, data.y, markersize=10, color=:maroon)
for i in 1:N
lines!(x_grid, b_jack[i] .+ m_jack[i] .* x_grid)
end
f
[92]:
Simple 1 4 Julia 1.9.3 | Idle
Mode: Command Ln 1, Col 1 Notes-from-class.ipynb
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https://symmetry.pi.local/user/dlang/lab/workspaces/auto-w/tree/home/dlang/FittingAModel2024/Notes-from-class.ipynb

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Code | Julia 1.9.3

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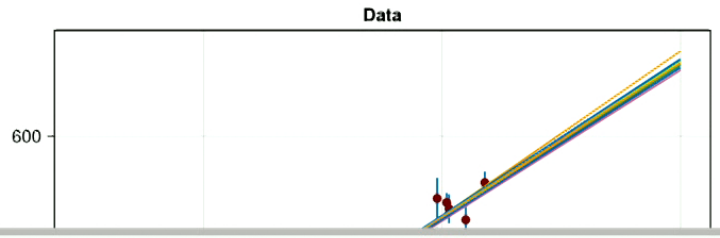
README.md	3 days ago
Untitled.ipynb	2 days ago

```
(b, m) = (6.090736287377655, 2.3921159709934487)
(b, m) = (31.650341377590237, 2.2605608149649346)
(b, m) = (39.63974964011244, 2.1875056225041614)
(b, m) = (35.60305914555623, 2.2244242978315985)
(b, m) = (29.858509800483894, 2.2465184337926627)
(b, m) = (35.36827635965802, 2.2416163288997923)
(b, m) = (30.645460085587807, 2.2722455338323724)
(b, m) = (34.9790026438403, 2.238230374828114)
(b, m) = (40.48095425088894, 2.212215084878653)
(b, m) = (34.14841132280452, 2.240122573510229)
(b, m) = (35.8618541461161, 2.2380871538621028)
(b, m) = (33.751717499017204, 2.2442539213541868)
(b, m) = (31.00172393416326, 2.2536962979359894)
(b, m) = (38.36604570886841, 2.204776861928404)
(b, m) = (36.18758028279822, 2.232664062075605)
```

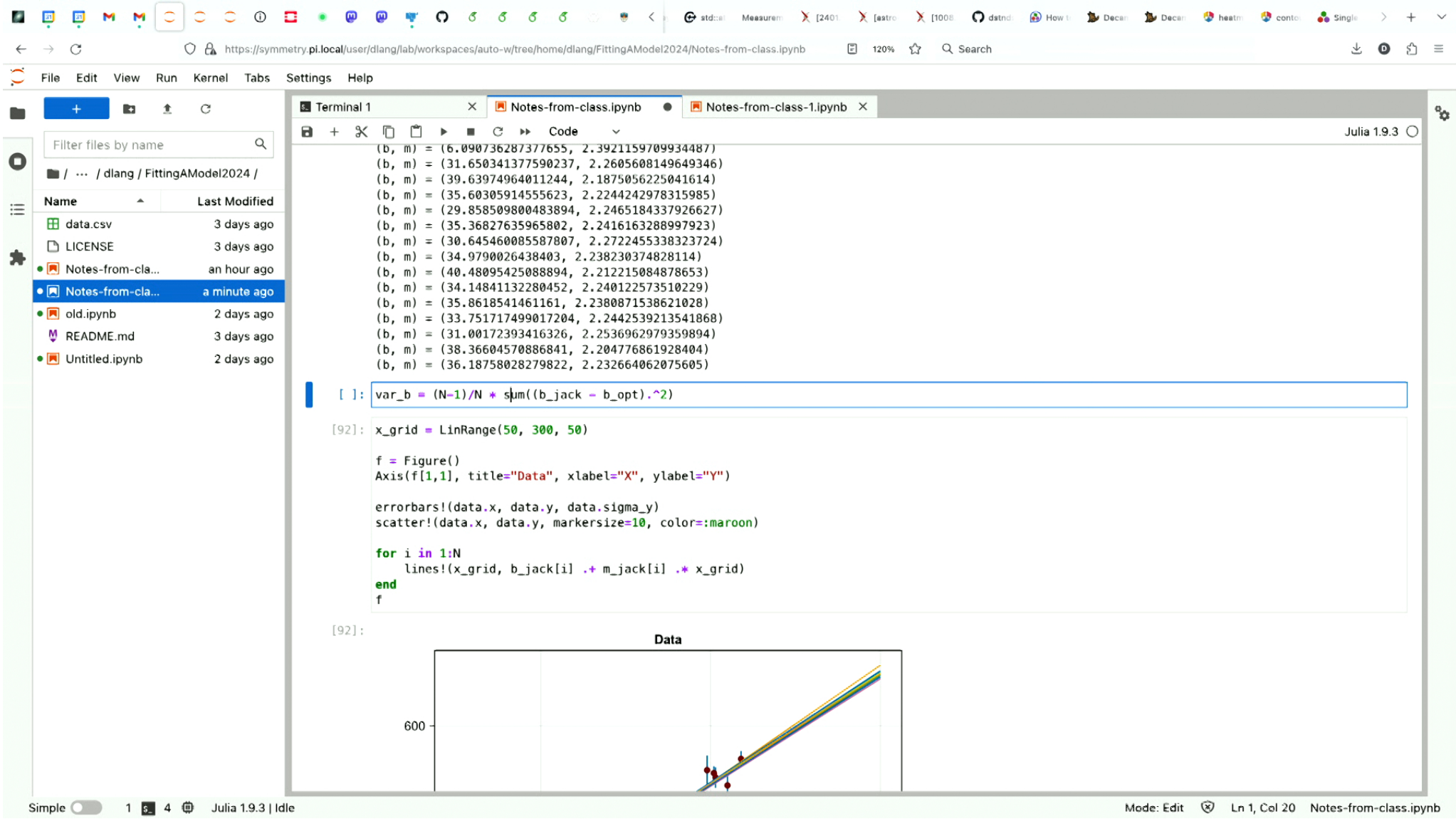
[ ]:
```
var_b = 1/N
```

[92]:
```
x_grid = LinRange(50, 300, 50)

f = Figure()
Axis(f[1,1], title="Data", xlabel="X", ylabel="Y")

errorbars!(data.x, data.y, data.sigma_y)
scatter!(data.x, data.y, markersize=10, color=:maroon)

for i in 1:N
    lines!(x_grid, b_jack[i] .+ m_jack[i] .* x_grid)
end
f
```

[92]:



Simple 1 4 Julia 1.9.3 | Idle Mode: Edit Ln 1, Col 12 Notes-from-class.ipynb


https://symmetry.pi.local/user/dlang/lab/workspaces/auto-w/tree/home/dlang/FittingAModel2024/Notes-from-class.ipynb
File Edit View Run Kernel Tabs Settings Help
Terminal 1
Notes-from-class.ipynb
Notes-from-class-1.ipynb
Code
Julia 1.9.3
Filter files by name
/ ··· / dlang / FittingAModel2024 /
Name
Last Modified
data.csv 3 days ago
LICENSE 3 days ago
Notes-from-cla... an hour ago
Notes-from-cla... a minute ago
old.ipynb 2 days ago
README.md 3 days ago
Untitled.ipynb 2 days ago
(b, m) = (6.090736287377655, 2.3921159709934487)
(b, m) = (31.650341377590237, 2.2605608149649346)
(b, m) = (39.63974964011244, 2.1875056225041614)
(b, m) = (35.60305914555623, 2.2244242978315985)
(b, m) = (29.858509800483894, 2.2465184337926627)
(b, m) = (35.36827635965802, 2.2416163288997923)
(b, m) = (30.645460085587807, 2.2722455338323724)
(b, m) = (34.9790026438403, 2.238230374828114)
(b, m) = (40.48095425088894, 2.212215084878653)
(b, m) = (34.14841132280452, 2.240122573510229)
(b, m) = (35.8618541461161, 2.2380871538621028)
(b, m) = (33.751717499017204, 2.2442539213541868)
(b, m) = (31.00172393416326, 2.2536962979359894)
(b, m) = (38.36604570886841, 2.204776861928404)
(b, m) = (36.18758028279822, 2.232664062075605)
[ ]: var_b = (N-1)/N * sum((b_jack - b_opt).^2)
[92]: x_grid = LinRange(50, 300, 50)
f = Figure()
Axis(f[1,1], title="Data", xlabel="X", ylabel="Y")
errorbars!(data.x, data.y, data.sigma_y)
scatter!(data.x, data.y, markersize=10, color=:maroon)
for i in 1:N
lines!(x_grid, b_jack[i] .+ m_jack[i] .* x_grid)
end
f
[92]:
Data
600
Simple 1 4 Julia 1.9.3 | Idle
Mode: Edit Ln 1, Col 20 Notes-from-class.ipynb

https://symmetry.pi.local/user/dlang/lab/workspaces/auto-w/tree/home/dlang/FittingAModel2024/Notes-from-class.ipynb

File Edit View Run Kernel Tabs Settings Help

Terminal 1 | Notes-from-class.ipynb | Notes-from-class-1.ipynb

Code | Julia 1.9.3

Filter files by name

/ ··· / dlang / FittingAModel2024 /

Name	Last Modified
data.csv	3 days ago
LICENSE	3 days ago
Notes-from-cla...	an hour ago
Notes-from-cla...	in a few seconds
old.ipynb	2 days ago
README.md	3 days ago
Untitled.ipynb	2 days ago

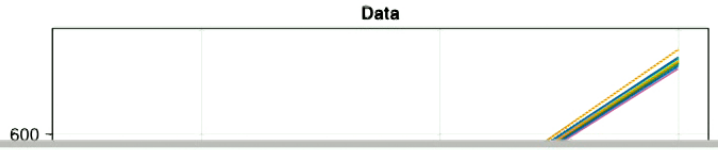
```
(b, m) = (6.090736287377655, 2.3921159709934487)
(b, m) = (31.650341377590237, 2.2605608149649346)
(b, m) = (39.63974964011244, 2.1875056225041614)
(b, m) = (35.60305914555623, 2.2244242978315985)
(b, m) = (29.858509800483894, 2.2465184337926627)
(b, m) = (35.36827635965802, 2.2416163288997923)
(b, m) = (30.645460085587807, 2.2722455338323724)
(b, m) = (34.9790026438403, 2.238230374828114)
(b, m) = (40.48095425088894, 2.212215084878653)
(b, m) = (34.14841132280452, 2.240122573510229)
(b, m) = (35.8618541461161, 2.2380871538621028)
(b, m) = (33.751717499017204, 2.2442539213541868)
(b, m) = (31.00172393416326, 2.2536962979359894)
(b, m) = (38.36604570886841, 2.204776861928404)
(b, m) = (36.18758028279822, 2.232664062075605)
```

[93]:
```
var_b = (N-1)/N * sum((b_jack .- b_opt).^2)
var_m = (N-1)/N * sum((b_jack .- b_opt).^2)
cov_b = (N-1)/N * sum((b_jack .- b_opt).^2)
```

[93]: 872.5127292593073

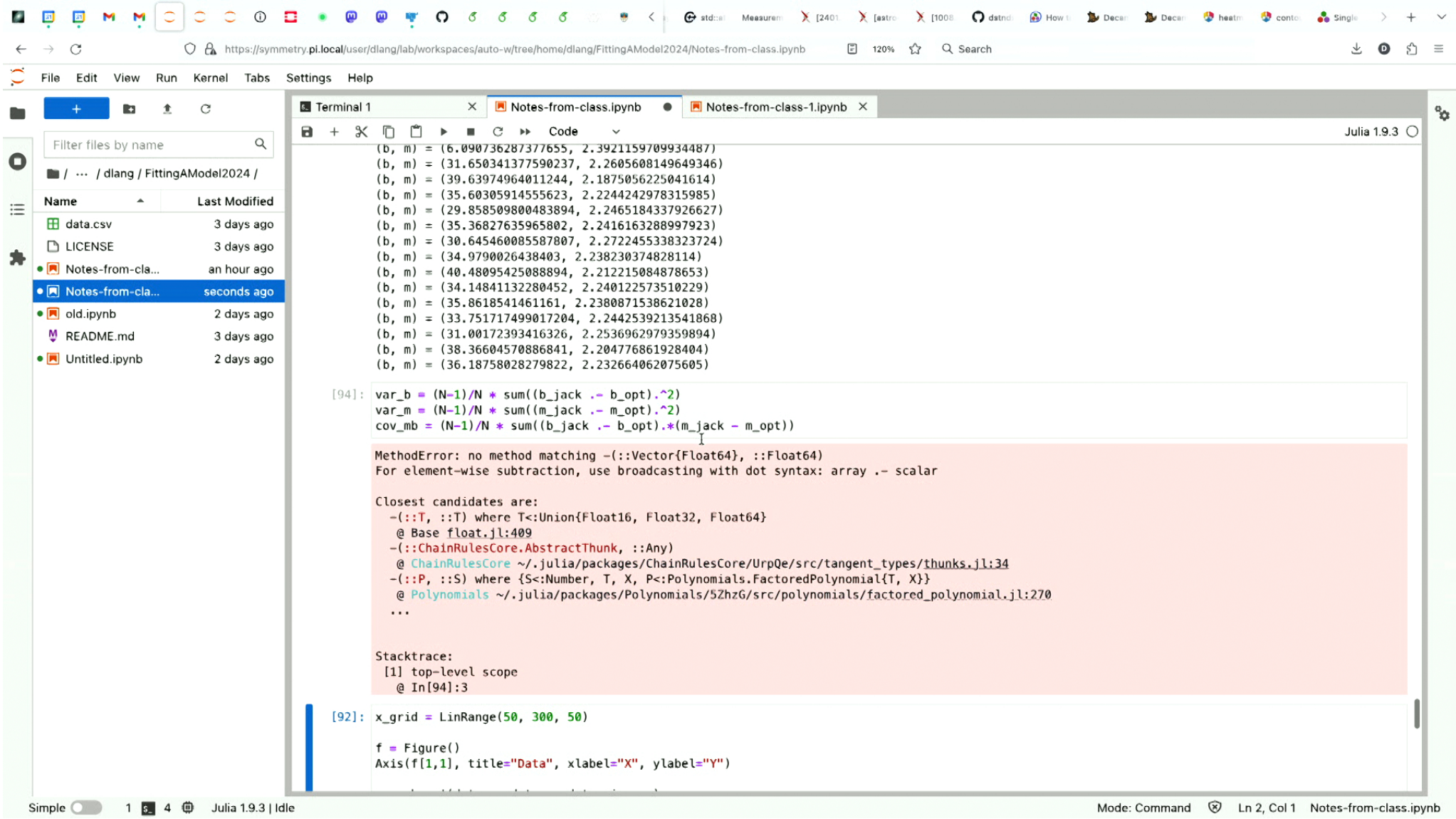
[92]:
```
x_grid = LinRange(50, 300, 50)

f = Figure()
Axis(f[1,1], title="Data", xlabel="X", ylabel="Y")

errorbars!(data.x, data.y, data.sigma_y)
scatter!(data.x, data.y, markersize=10, color=:maroon)

for i in 1:N
    lines!(x_grid, b_jack[i] .+ m_jack[i] .* x_grid)
end
f
```

[92]:



Simple | 1 | 4 | Julia 1.9.3 | Idle

Mode: Edit | Ln 3, Col 4 | Notes-from-class.ipynb


https://symmetry.pi.local/user/dlang/lab/workspaces/auto-w/tree/home/dlang/FittingAModel2024/Notes-from-class.ipynb
File Edit View Run Kernel Tabs Settings Help
Terminal 1
Notes-from-class.ipynb
Notes-from-class-1.ipynb
Code
Julia 1.9.3
Filter files by name
/ ··· / dlang / FittingAModel2024 /
Name
Last Modified
data.csv 3 days ago
LICENSE 3 days ago
Notes-from-cla... an hour ago
Notes-from-cla... seconds ago
old.ipynb 2 days ago
README.md 3 days ago
Untitled.ipynb 2 days ago
(b, m) = (6.090736287377655, 2.3921159709934487)
(b, m) = (31.650341377590237, 2.2605608149649346)
(b, m) = (39.63974964011244, 2.1875056225041614)
(b, m) = (35.60305914555623, 2.2244242978315985)
(b, m) = (29.858509800483894, 2.2465184337926627)
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(b, m) = (30.645460085587807, 2.2722455338323724)
(b, m) = (34.9790026438403, 2.238230374828114)
(b, m) = (40.48095425088894, 2.212215084878653)
(b, m) = (34.14841132280452, 2.240122573510229)
(b, m) = (35.8618541461161, 2.2380871538621028)
(b, m) = (33.751717499017204, 2.2442539213541868)
(b, m) = (31.00172393416326, 2.2536962979359894)
(b, m) = (38.36604570886841, 2.204776861928404)
(b, m) = (36.18758028279822, 2.232664062075605)
[94]:
var_b = (N-1)/N * sum((b_jack .- b_opt).^2)
var_m = (N-1)/N * sum((m_jack .- m_opt).^2)
cov_mb = (N-1)/N * sum((b_jack .- b_opt).*(m_jack - m_opt))
MethodError: no method matching -(::Vector{Float64}, ::Float64)
For element-wise subtraction, use broadcasting with dot syntax: array .- scalar
Closest candidates are:
-(::T, ::T) where T<:Union{Float16, Float32, Float64}
@ Base float.jl:409
-(::ChainRulesCore.AbstractThunk, ::Any)
@ ChainRulesCore ~/.julia/packages/ChainRulesCore/UrpQe/src/tangent_types/thunks.jl:34
-(::P, ::S) where {S<:Number, T, X, P<:Polynomials.FactoredPolynomial{T, X}}
@ Polynomials ~/.julia/packages/Polynomials/5ZhzG/src/polynomials/factored_polynomial.jl:270
...
Stacktrace:
[1] top-level scope
@ In[94]:3
[92]:
x_grid = LinRange(50, 300, 50)
f = Figure()
Axis(f[1,1], title="Data", xlabel="X", ylabel="Y")
Simple 1 4 Julia 1.9.3 | Idle
Mode: Command Ln 2, Col 1 Notes-from-class.ipynb

https://symmetry.pi.local/user/dlang/lab/workspaces/auto-w/tree/home/dlang/FittingAModel2024/Notes-from-class.ipynb

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Terminal 1 | Notes-from-class.ipynb | Notes-from-class-1.ipynb

Code | Julia 1.9.3

Filter files by name

/ ··· / dlang / FittingAModel2024 /

Name	Last Modified
data.csv	3 days ago
LICENSE	3 days ago
Notes-from-cla...	an hour ago
Notes-from-cla...	seconds ago
old.ipynb	2 days ago
README.md	3 days ago
Untitled.ipynb	2 days ago

```
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(b, m) = (31.650341377590237, 2.2605608149649346)
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(b, m) = (34.9790026438403, 2.238230374828114)
(b, m) = (40.48095425088894, 2.212215084878653)
(b, m) = (34.14841132280452, 2.240122573510229)
(b, m) = (35.8618541461161, 2.2380871538621028)
(b, m) = (33.751717499017204, 2.2442539213541868)
(b, m) = (31.00172393416326, 2.2536962979359894)
(b, m) = (38.36604570886841, 2.204776861928404)
(b, m) = (36.18758028279822, 2.232664062075605)
```

[95]:
```
var_b = (N-1)/N * sum((b_jack .- b_opt).^2)
var_m = (N-1)/N * sum((m_jack .- m_opt).^2)
cov_mb = (N-1)/N * sum((b_jack .- b_opt) .* (m_jack .- m_opt))
```

[95]:
```
-4.834212105071833
```

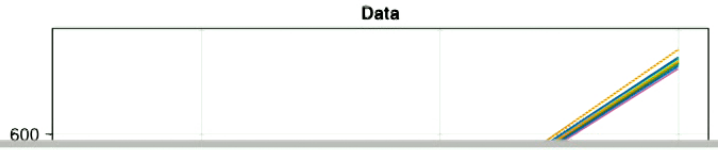
[92]:
```
x_grid = LinRange(50, 300, 50)

f = Figure()
Axis(f[1,1], title="Data", xlabel="X", ylabel="Y")

errorbars!(data.x, data.y, data.sigma_y)
scatter!(data.x, data.y, markersize=10, color=:maroon)

for i in 1:N
    lines!(x_grid, b_jack[i] .+ m_jack[i] .* x_grid)
end
f
```

[92]:



Simple 1 4 Julia 1.9.3 | Idle Mode: Command Ln 2, Col 1 Notes-from-class.ipynb