Title: Numerical Methods Lecture

Speakers: Dustin Lang

Collection: Numerical Methods 2023/24

Date: January 11, 2024 - 10:15 AM

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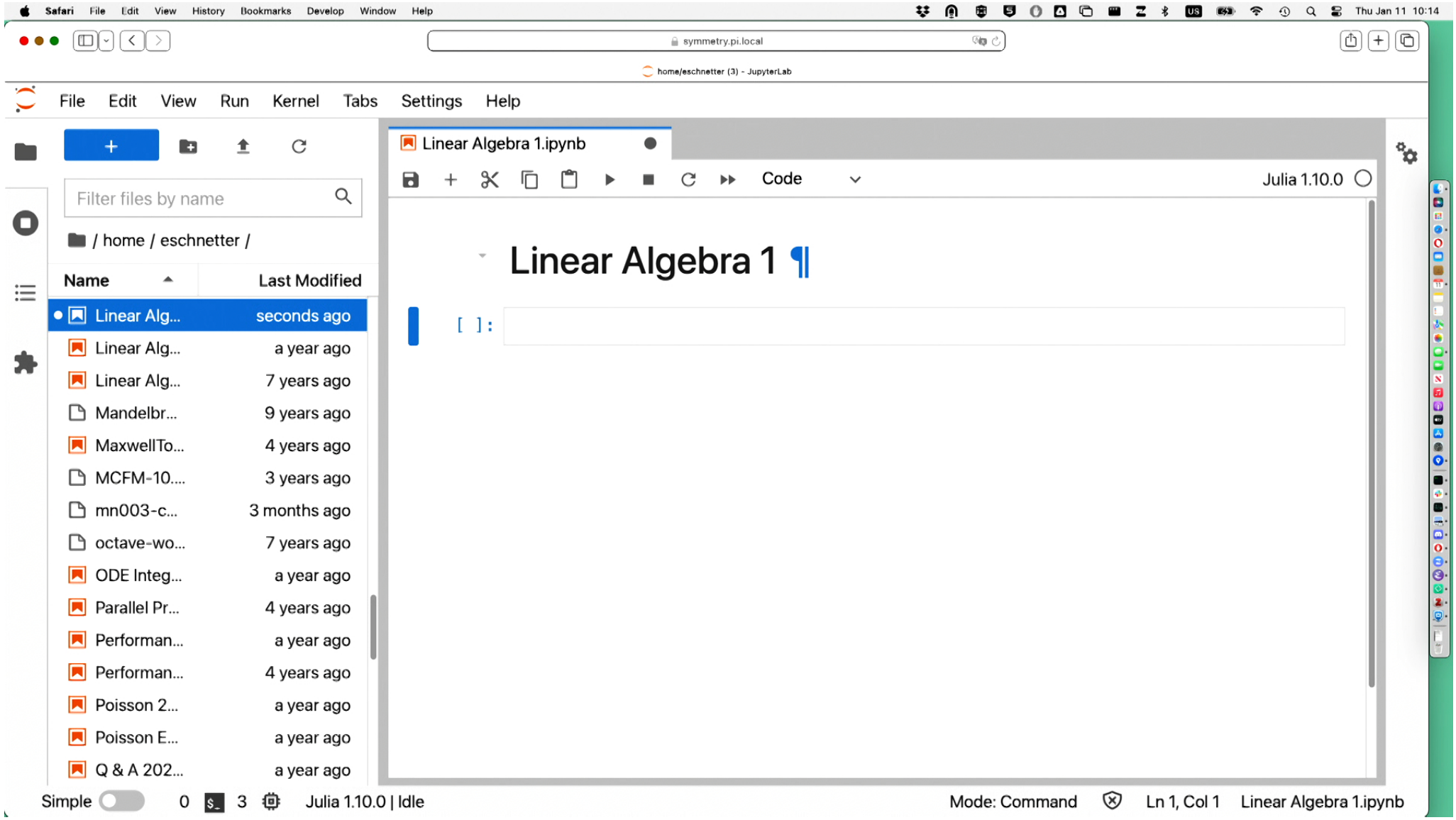
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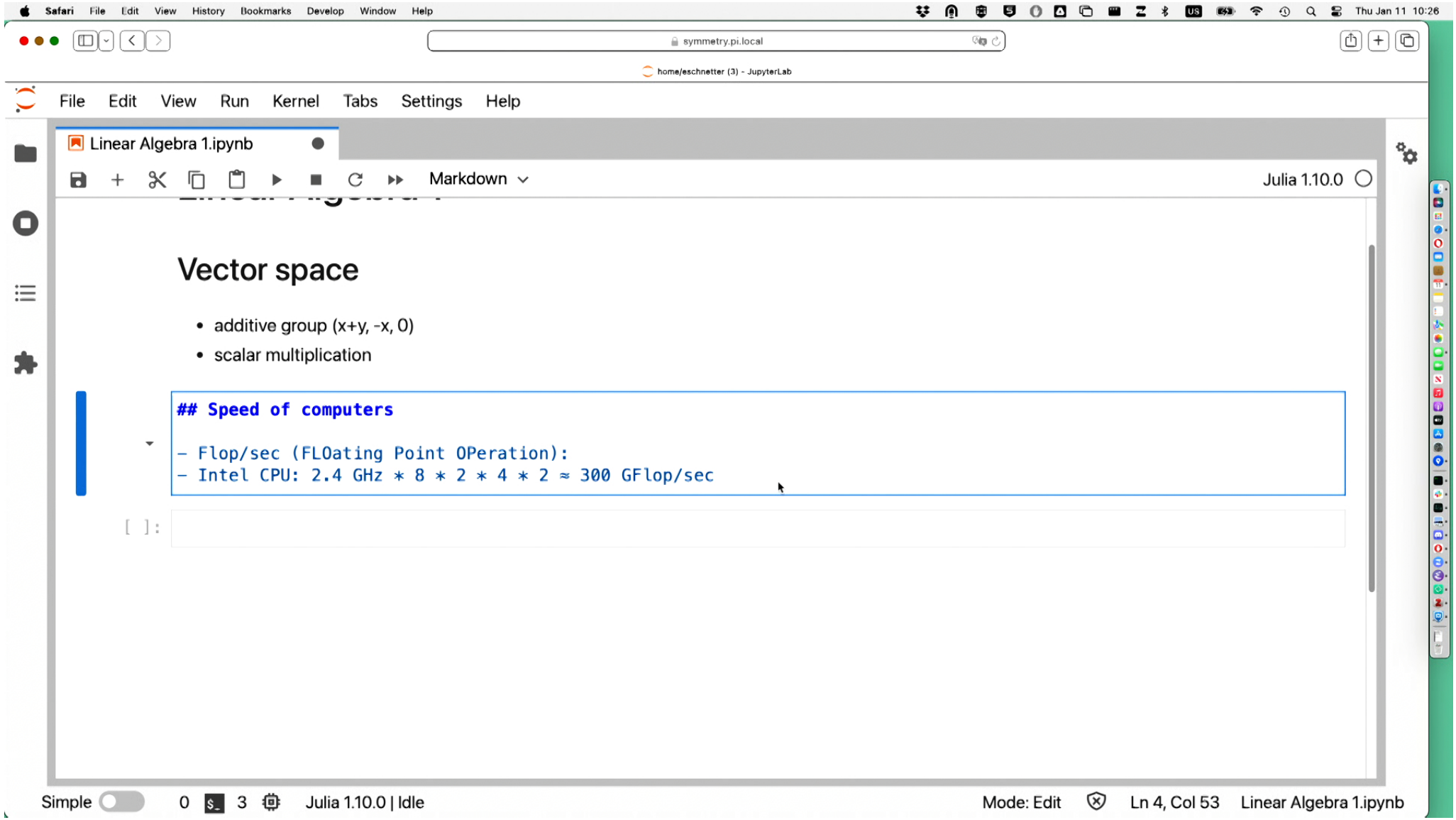
Name
Last Modified
Linear Alg... seconds ago
Linear Alg... a year ago
Linear Alg... 7 years ago
Mandelbr... 9 years ago
MaxwellTo... 4 years ago
MCFM-10.... 3 years ago
mn003-c... 3 months ago
octave-wo... 7 years ago
ODE Integ... a year ago
Parallel Pr... 4 years ago
Performan... a year ago
Performan... 4 years ago
Poisson 2... a year ago
Poisson E... a year ago
Q & A 202... a year ago
Linear Algebra 1.ipynb
Code
Julia 1.10.0
Linear Algebra 1 ¶
[ ]:
Simple
0
3
Julia 1.10.0 | Idle
Mode: Command
Ln 1, Col 1
Linear Algebra 1.ipynb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Thu Jan 11 10:26
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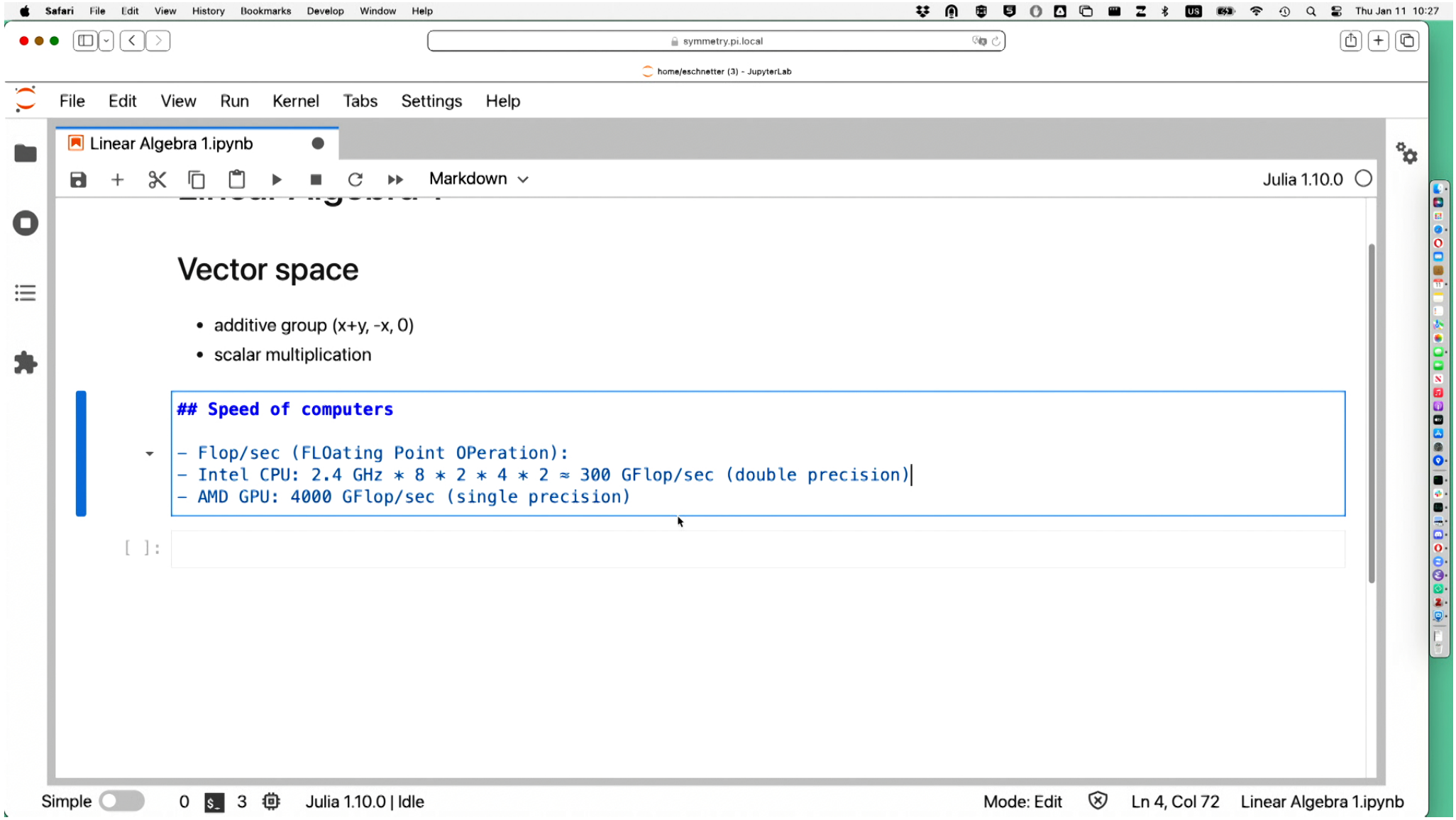
File Edit View Run Kernel Tabs Settings Help
Linear Algebra 1.ipynb
Markdown
Julia 1.10.0
Vector space
additive group (x+y, -x, 0)
scalar multiplication
Speed of computers
- Flop/sec (FLOating Point OPeration):
- Intel CPU: 2.4 GHz * 8 * 2 * 4 * 2 ≈ 300 GFlop/sec
[ ]:
Simple
0
3
Julia 1.10.0 | Idle
Mode: Edit
Ln 4, Col 53
Linear Algebra 1.ipynb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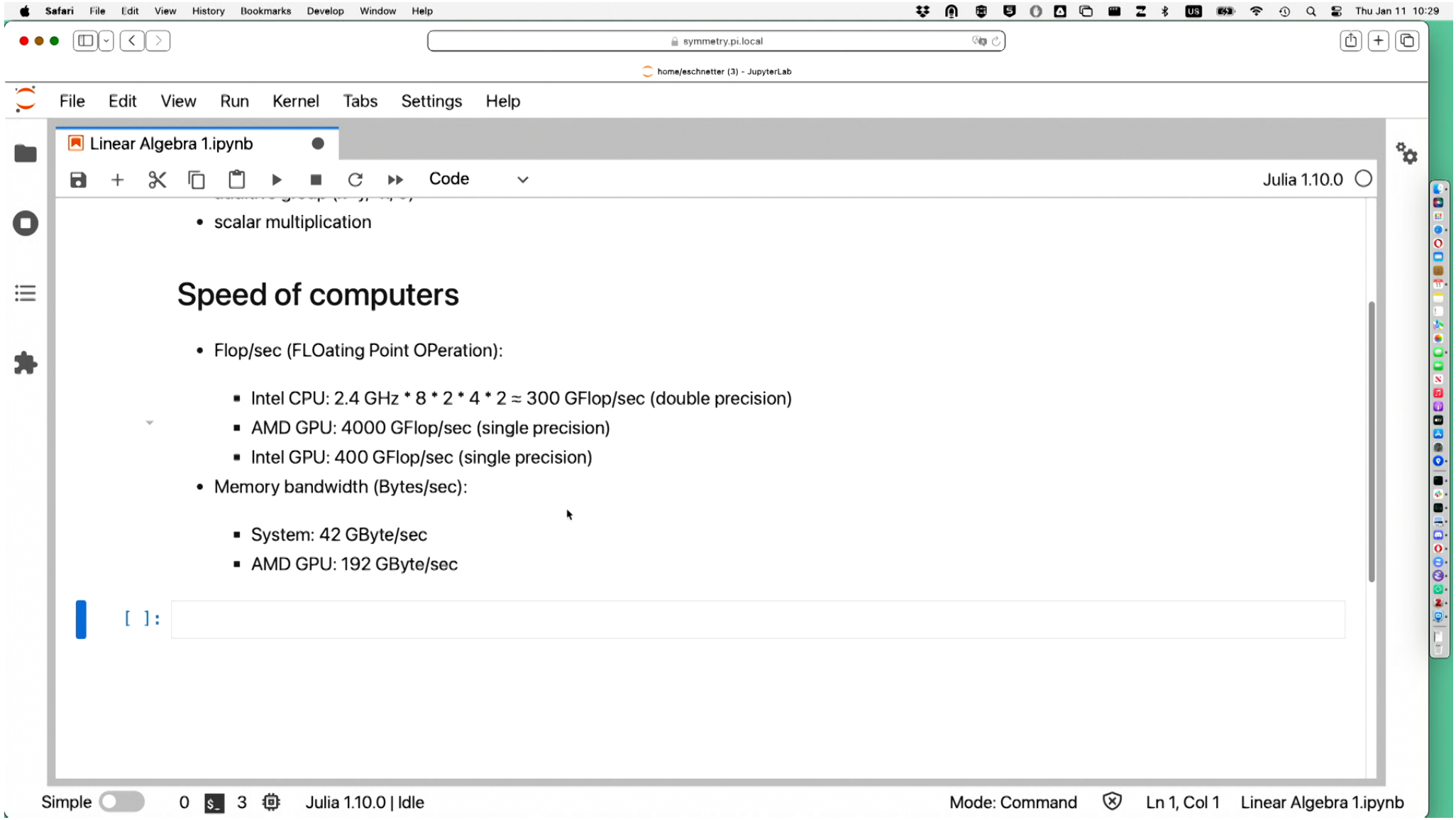
home/eschnetter (3) - JupyterLab
File Edit View Run Kernel Tabs Settings Help
Linear Algebra 1.ipynb
Markdown
Julia 1.10.0
Vector space
additive group (x+y, -x, 0)
scalar multiplication
Speed of computers
- Flop/sec (FLOating Point OPeration):
- Intel CPU: 2.4 GHz * 8 * 2 * 4 * 2 ≈ 300 GFlop/sec (double precision)
- AMD GPU: 4000 GFlop/sec (single precision)
[ ]:
Simple
0
3
Julia 1.10.0 | Idle
Mode: Edit
Ln 4, Col 72
Linear Algebra 1.ipynb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Thu Jan 11 10:29
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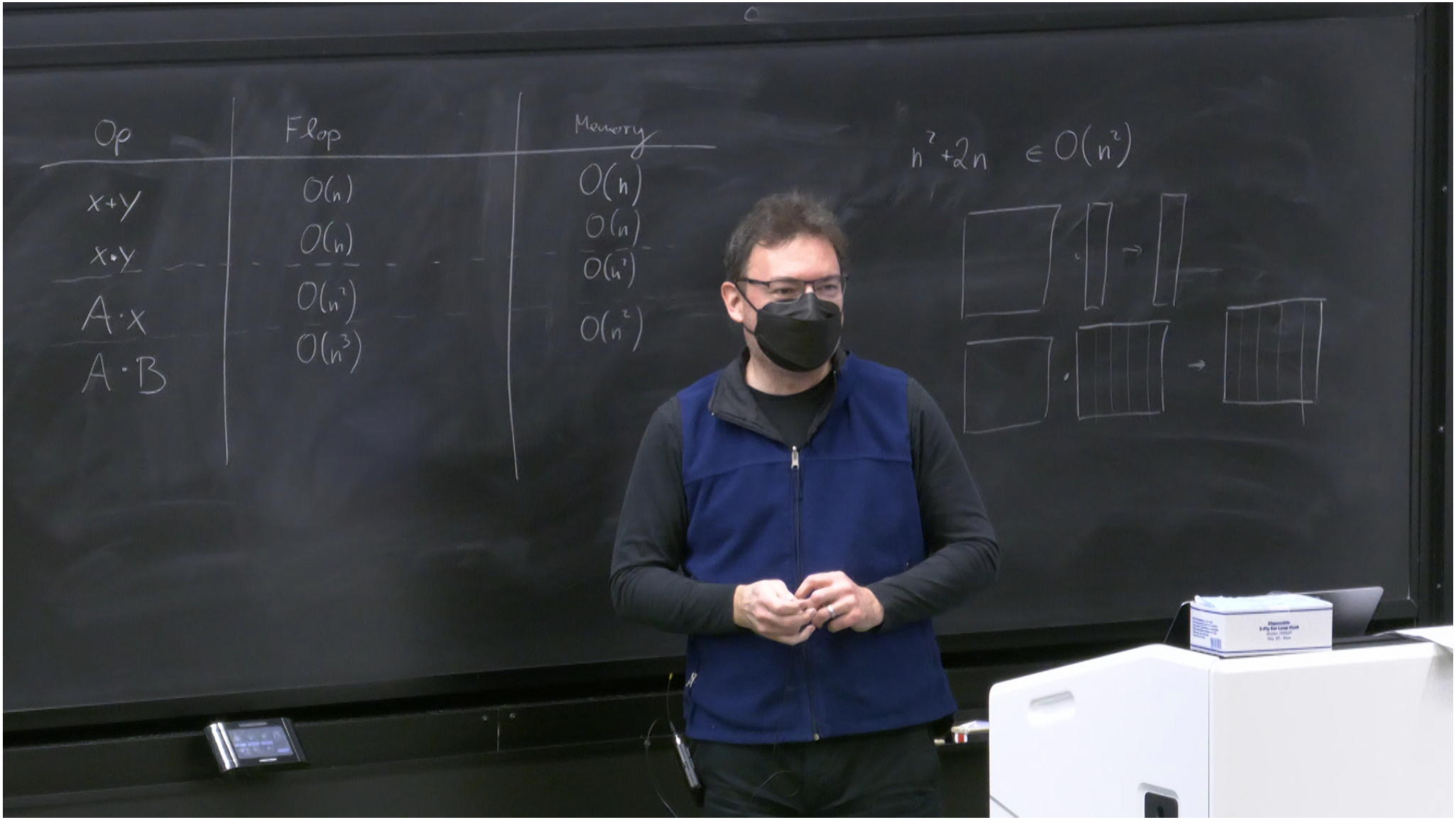
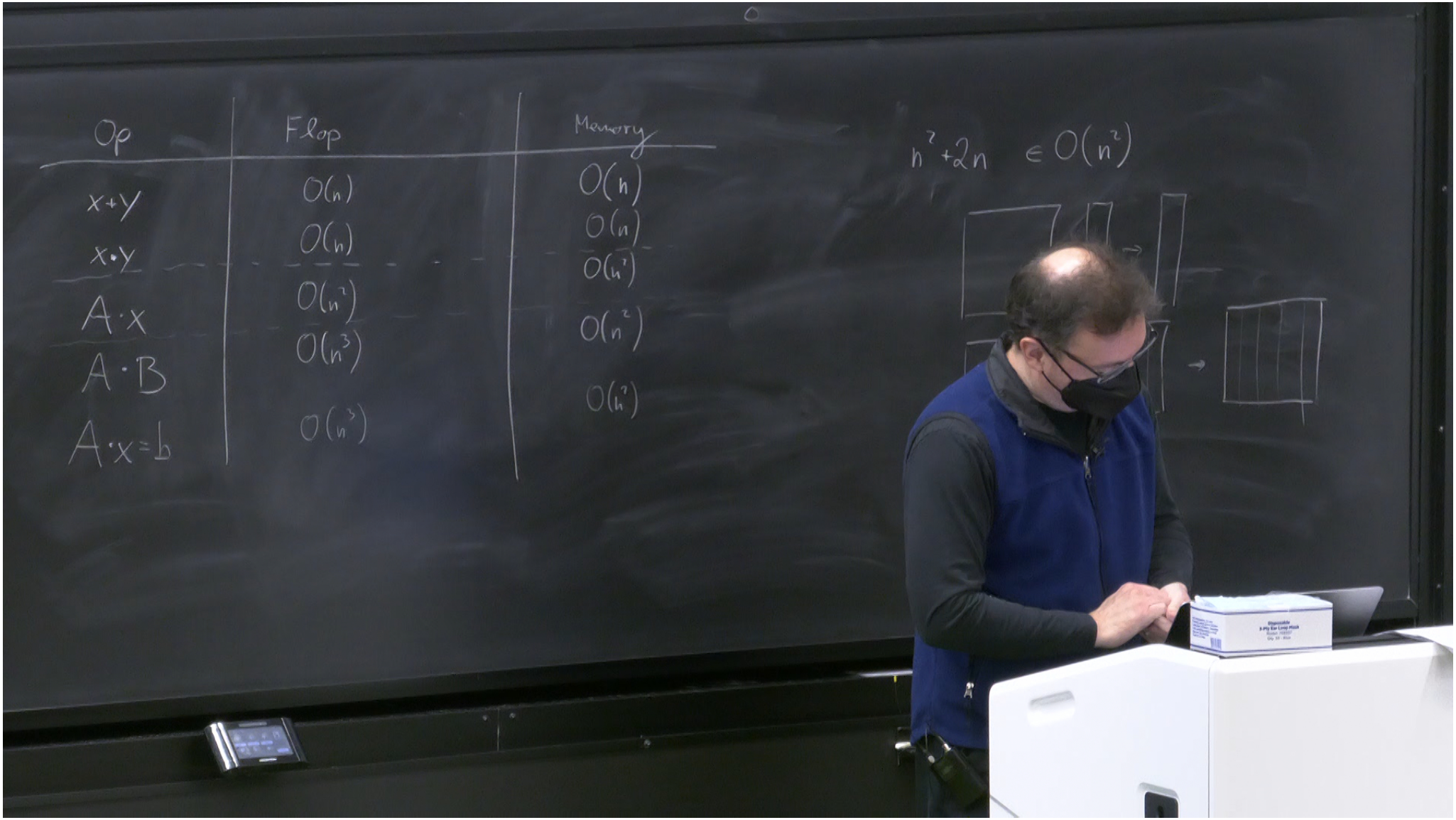
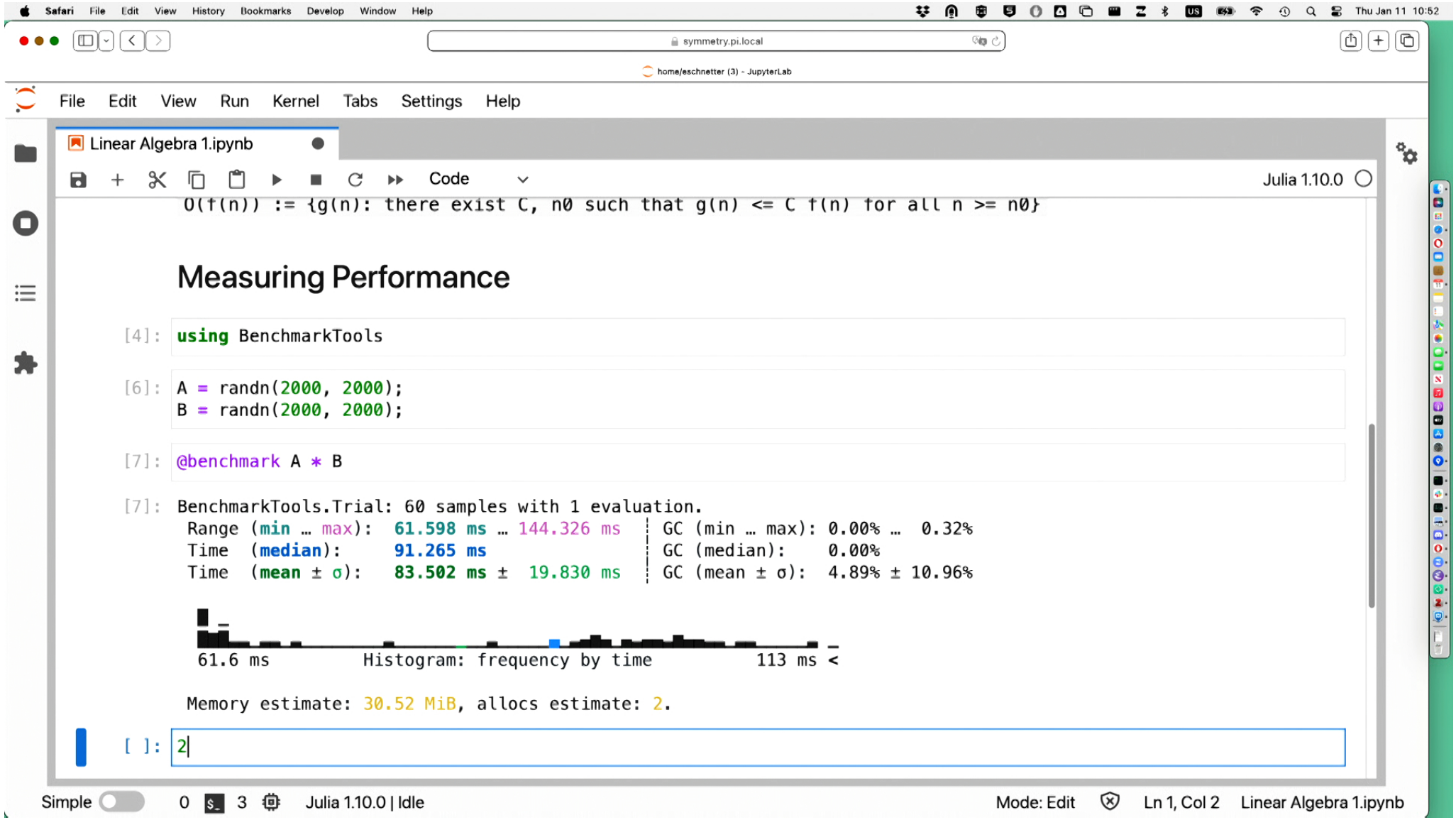
home/eschnetter (3) - JupyterLab
File Edit View Run Kernel Tabs Settings Help
Linear Algebra 1.ipynb
Code
Julia 1.10.0
scalar multiplication
Speed of computers
Flop/sec (FLOating Point OPeration):
Intel CPU: 2.4 GHz * 8 * 2 * 4 * 2 ≈ 300 GFlop/sec (double precision)
AMD GPU: 4000 GFlop/sec (single precision)
Intel GPU: 400 GFlop/sec (single precision)
Memory bandwidth (Bytes/sec):
System: 42 GByte/sec
AMD GPU: 192 GByte/sec
[ ]:
Simple
0
3
Julia 1.10.0 | Idle
Mode: Command
Ln 1, Col 1
Linear Algebra 1.ipynb


Op
Flop
Memory
x+y
O(n)
O(n)
x·y
O(n)
O(n)
A·x
O(n²)
O(n²)
A·B
O(n³)
n²+2n ∈ O(n²)


Op
Flop
Memory
x+y
O(n)
O(n)
x·y
O(n)
O(n)
A·x
O(n²)
O(n²)
A·B
O(n³)
O(n²)
A·x=b
O(n³)
O(n²)
n²+2n ∈ O(n²)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symmetry.pi.local
home/eschnetter (3) - JupyterLab
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Linear Algebra 1.ipynb
Code
Julia 1.10.0
O(f(n)) := {g(n): there exist C, n0 such that g(n) <= C f(n) for all n >= n0}
Measuring Performance
[4]: using BenchmarkTools
[6]: A = randn(2000, 2000);
B = randn(2000, 2000);
[7]: @benchmark A * B
[7]: BenchmarkTools.Trial: 60 samples with 1 evaluation.
Range (min … max): 61.598 ms … 144.326 ms | GC (min … max): 0.00% … 0.32%
Time (median): 91.265 ms | GC (median): 0.00%
Time (mean ± σ): 83.502 ms ± 19.830 ms | GC (mean ± σ): 4.89% ± 10.96%
61.6 ms Histogram: frequency by time 113 ms <
Memory estimate: 30.52 MiB, allocs estimate: 2.
[ ]: 2
Simple
0 3
Julia 1.10.0 | Idle
Mode: Edit
Ln 1, Col 2
Linear Algebra 1.ipynb

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Thu Jan 11 10:54

symmetry.pi.local

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Linear Algebra 1.ipynb

Code

Julia 1.10.0

[4]:
```
using BenchmarkTools
```

[6]:
```
A = randn(2000, 2000);
B = randn(2000, 2000);
```

[7]:
```
@benchmark A * B
```

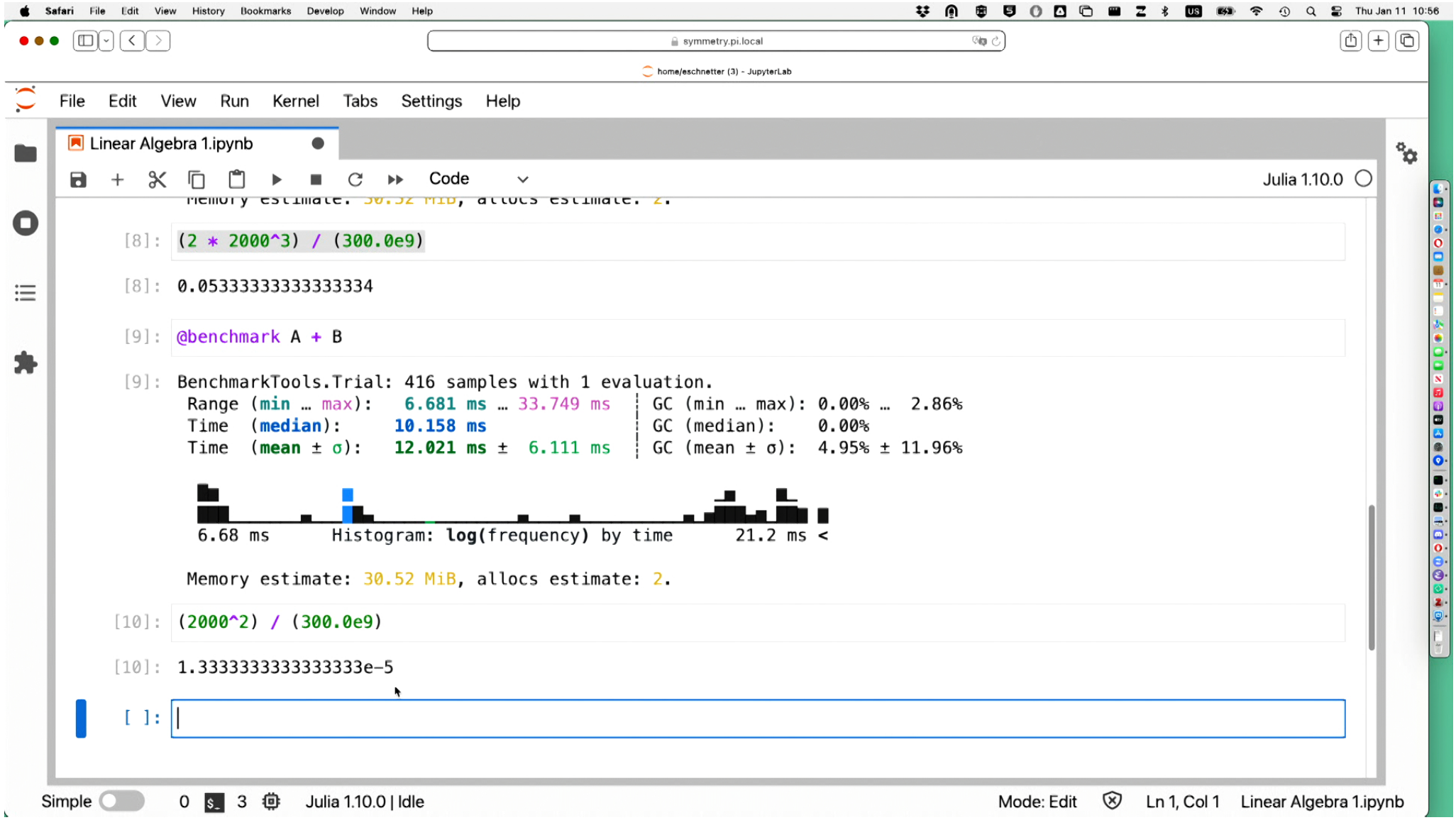
[7]:
```
BenchmarkTools.Trial: 60 samples with 1 evaluation.
 Range (min … max):  61.598 ms … 144.326 ms  ┊ GC (min … max): 0.00% …  0.32%
 Time  (median):     91.265 ms               ┊ GC (median):    0.00%
 Time  (mean ± σ):   83.502 ms ±  19.830 ms  ┊ GC (mean ± σ):  4.89% ± 10.96%

 ▁                                                                  ▁
 61.6 ms         Histogram: frequency by time          113 ms <

 Memory estimate: 30.52 MiB, allocs estimate: 2.
```

[8]:
```
(2 * 2000^3) / (300.0e9)
```

[8]:
```
0.05333333333333334
```

[ ]:

Simple 0 3 Julia 1.10.0 | Idle Mode: Edit Ln 1, Col 1 Linear Algebra 1.ipynb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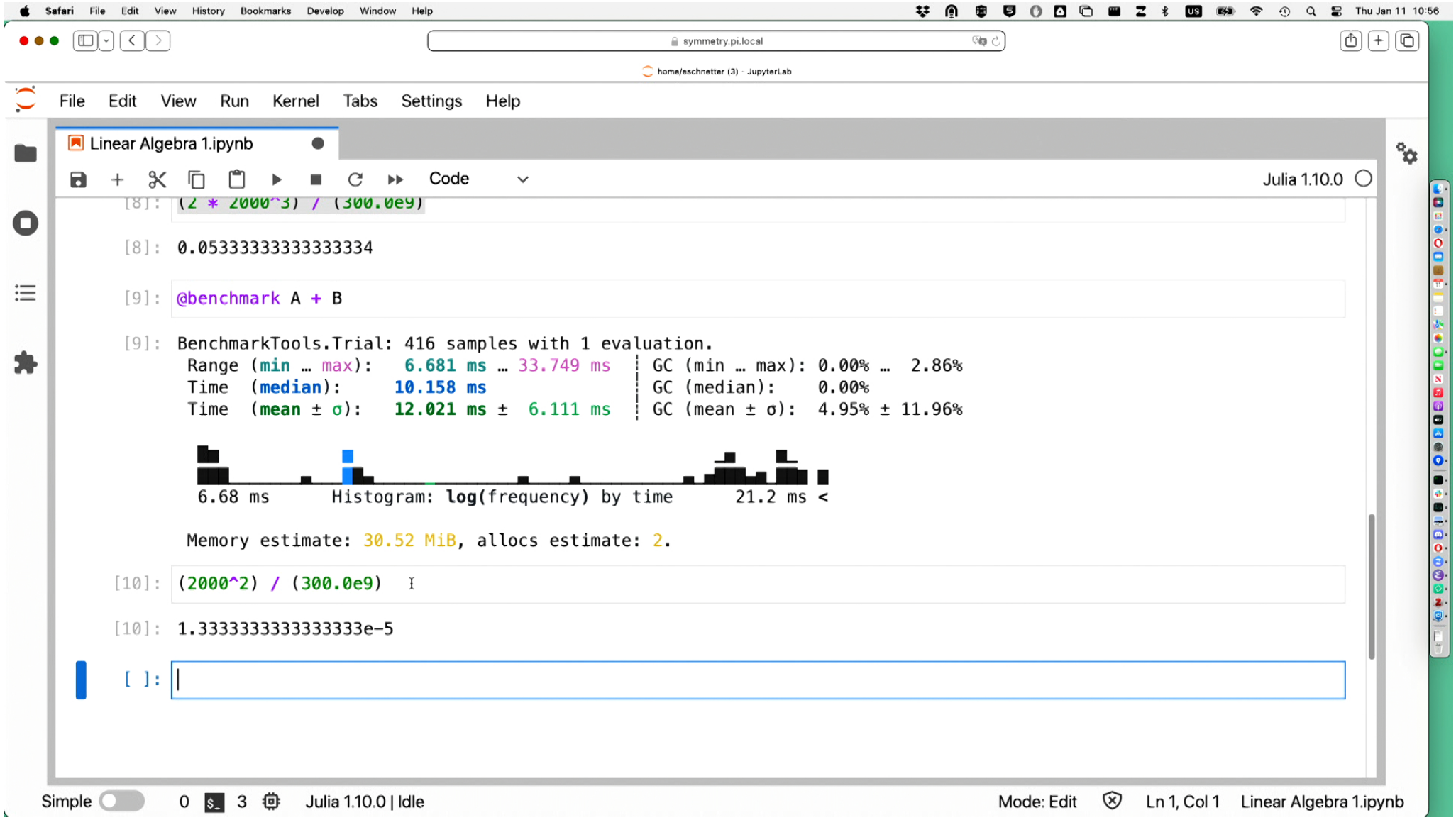
File Edit View Run Kernel Tabs Settings Help
Linear Algebra 1.ipynb
Code
Julia 1.10.0
Memory estimate: 30.52 MiB, allocs estimate: 2.
[8]: (2 * 2000^3) / (300.0e9)
[8]: 0.05333333333333334
[9]: @benchmark A + B
[9]: BenchmarkTools.Trial: 416 samples with 1 evaluation.
Range (min … max): 6.681 ms … 33.749 ms | GC (min … max): 0.00% … 2.86%
Time (median): 10.158 ms | GC (median): 0.00%
Time (mean ± σ): 12.021 ms ± 6.111 ms | GC (mean ± σ): 4.95% ± 11.96%
6.68 ms Histogram: log(frequency) by time 21.2 ms <
Memory estimate: 30.52 MiB, allocs estimate: 2.
[10]: (2000^2) / (300.0e9)
[10]: 1.3333333333333333e-5
[ ]:
Simple
0
3
Julia 1.10.0 | Idle
Mode: Edit
Ln 1, Col 1
Linear Algebra 1.ipynb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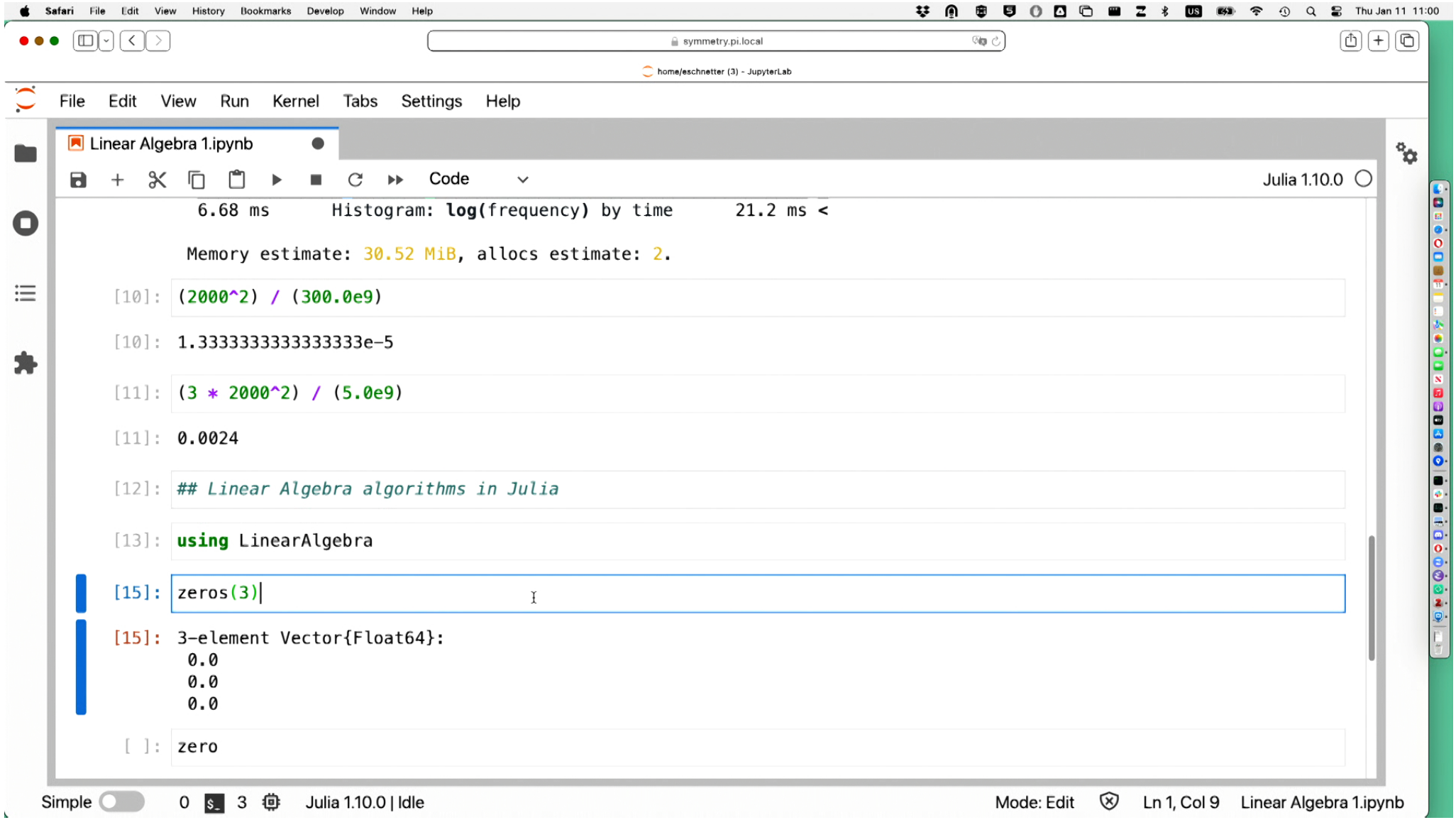
symmetry.pi.local
home/eschnetter (3) - JupyterLab
File Edit View Run Kernel Tabs Settings Help
Linear Algebra 1.ipynb
Code
Julia 1.10.0
[8]: (2 * 2000^3) / (300.0e9)
[8]: 0.05333333333333334
[9]: @benchmark A + B
[9]: BenchmarkTools.Trial: 416 samples with 1 evaluation.
Range (min … max): 6.681 ms … 33.749 ms | GC (min … max): 0.00% … 2.86%
Time (median): 10.158 ms | GC (median): 0.00%
Time (mean ± σ): 12.021 ms ± 6.111 ms | GC (mean ± σ): 4.95% ± 11.96%
6.68 ms Histogram: log(frequency) by time 21.2 ms <
Memory estimate: 30.52 MiB, allocs estimate: 2.
[10]: (2000^2) / (300.0e9)
[10]: 1.3333333333333333e-5
[ ]:
Simple
0
3
Julia 1.10.0 | Idle
Mode: Edit
Ln 1, Col 1
Linear Algebra 1.ipynb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symmetry.pi.local
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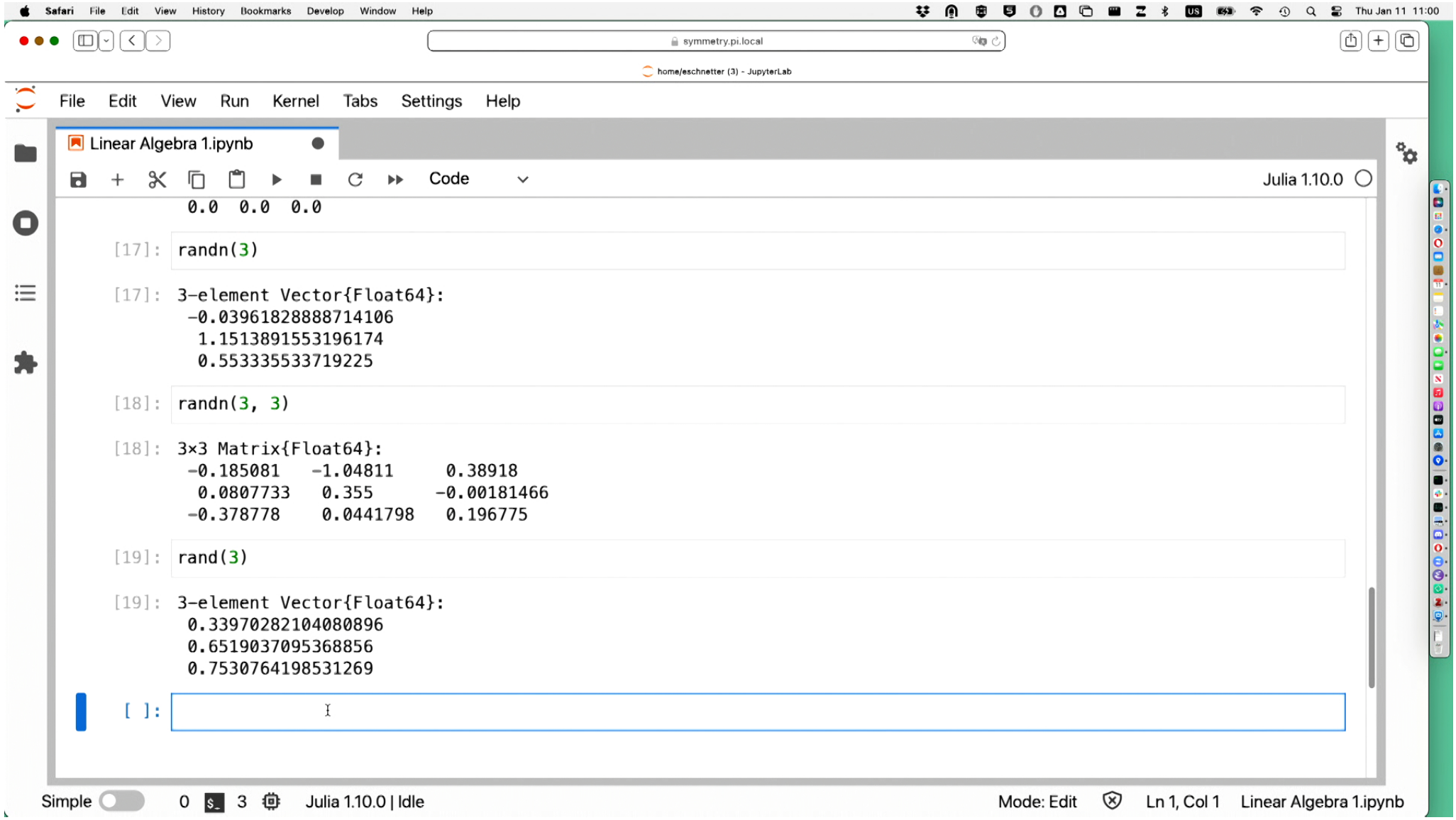
File Edit View Run Kernel Tabs Settings Help
Linear Algebra 1.ipynb
Code
Julia 1.10.0
6.68 ms Histogram: log(frequency) by time 21.2 ms <
Memory estimate: 30.52 MiB, allocs estimate: 2.
[10]: (2000^2) / (300.0e9)
[10]: 1.3333333333333333e-5
[11]: (3 * 2000^2) / (5.0e9)
[11]: 0.0024
[12]: ## Linear Algebra algorithms in Julia
[13]: using LinearAlgebra
[15]: zeros(3)
[15]: 3-element Vector{Float64}:
0.0
0.0
0.0
[ ]: zero
Simple 0 3 Julia 1.10.0 | Idle
Mode: Edit Ln 1, Col 9 Linear Algebra 1.ipynb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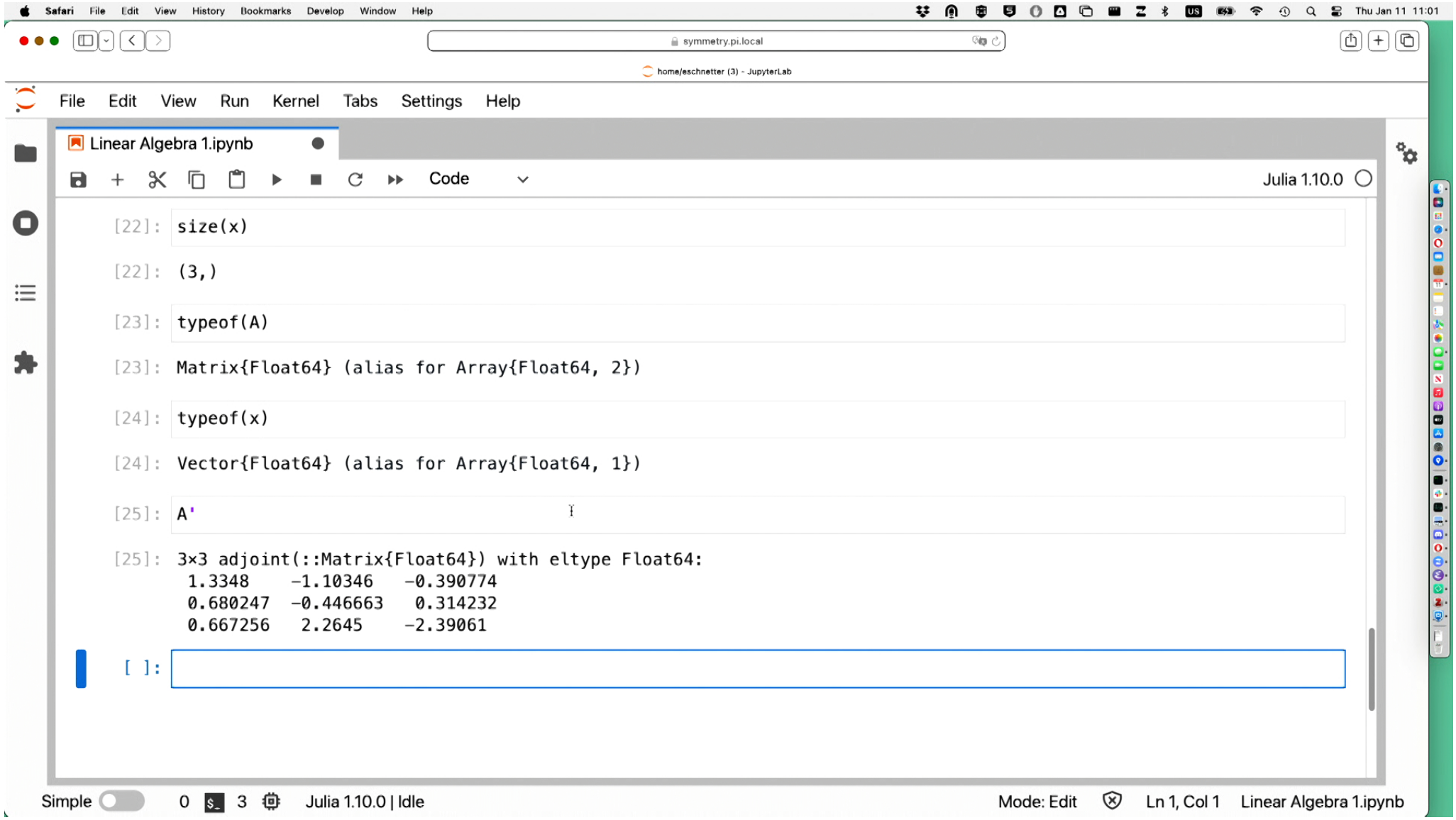
Linear Algebra 1.ipynb
Code
Julia 1.10.0
0.0 0.0 0.0
[17]: randn(3)
[17]: 3-element Vector{Float64}:
-0.03961828888714106
1.1513891553196174
0.553335533719225
[18]: randn(3, 3)
[18]: 3×3 Matrix{Float64}:
-0.185081 -1.04811 0.38918
0.0807733 0.355 -0.00181466
-0.378778 0.0441798 0.196775
[19]: rand(3)
[19]: 3-element Vector{Float64}:
0.33970282104080896
0.6519037095368856
0.7530764198531269
[ ]:
Simple 0 3 Julia 1.10.0 | Idle
Mode: Edit Ln 1, Col 1 Linear Algebra 1.ipynb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Thu Jan 11 11:01
symmetry.pi.local
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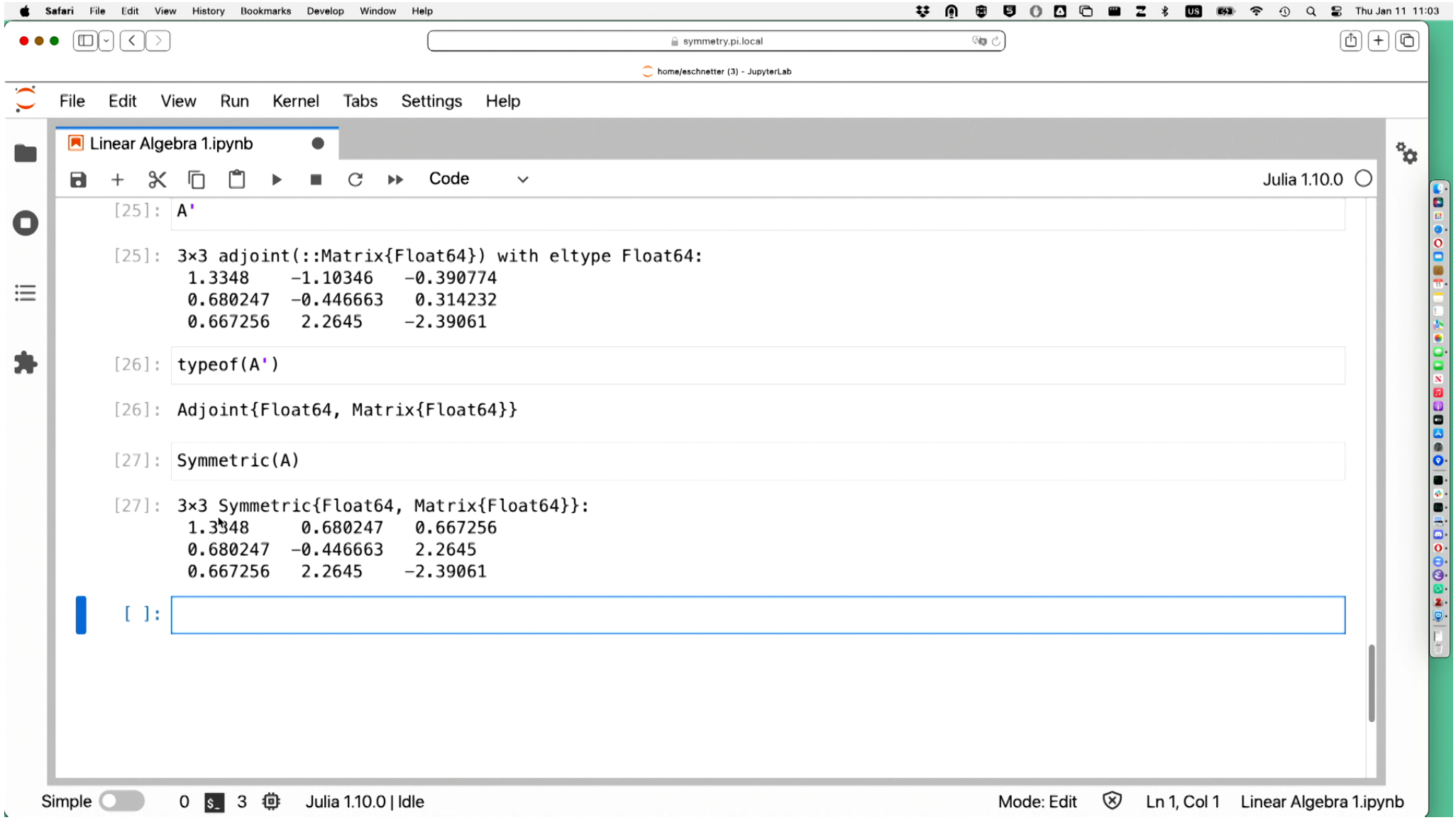
File Edit View Run Kernel Tabs Settings Help
Linear Algebra 1.ipynb
Code
Julia 1.10.0
[22]: size(x)
[22]: (3,)
[23]: typeof(A)
[23]: Matrix{Float64} (alias for Array{Float64, 2})
[24]: typeof(x)
[24]: Vector{Float64} (alias for Array{Float64, 1})
[25]: A'
[25]: 3×3 adjoint(::Matrix{Float64}) with eltype Float64:
1.3348 -1.10346 -0.390774
0.680247 -0.446663 0.314232
0.667256 2.2645 -2.39061
[ ]:
Simple
0
3
Julia 1.10.0 | Idle
Mode: Edit
Ln 1, Col 1
Linear Algebra 1.ipynb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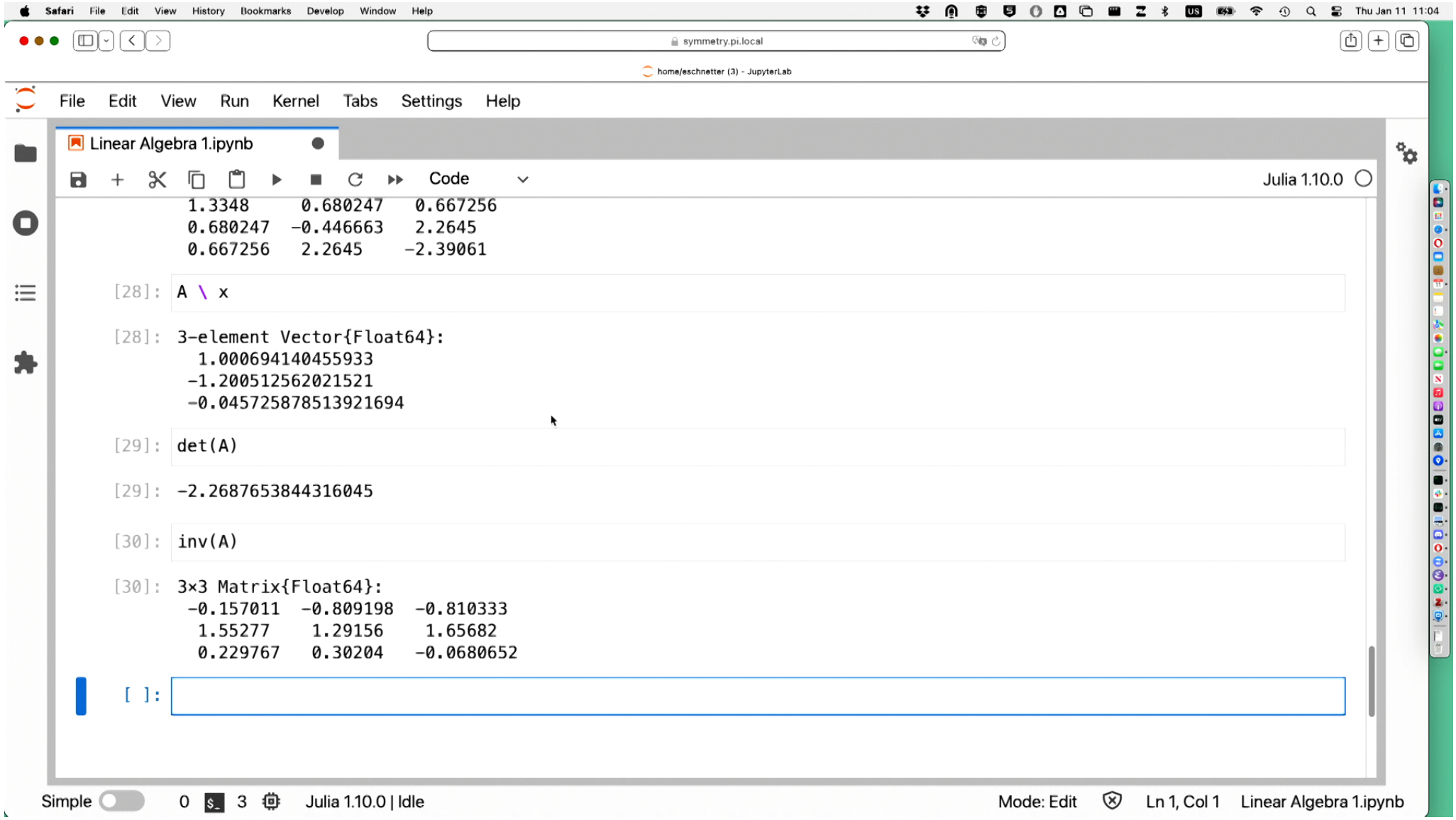
Thu Jan 11 11:03
symmetry.pi.local
home/eschnetter (3) - JupyterLab
File Edit View Run Kernel Tabs Settings Help
Linear Algebra 1.ipynb
Code
Julia 1.10.0
[25]: A'
[25]: 3×3 adjoint(::Matrix{Float64}) with eltype Float64:
1.3348 -1.10346 -0.390774
0.680247 -0.446663 0.314232
0.667256 2.2645 -2.39061
[26]: typeof(A')
[26]: Adjoint{Float64, Matrix{Float64}}
[27]: Symmetric(A)
[27]: 3×3 Symmetric{Float64, Matrix{Float64}}:
1.3348 0.680247 0.667256
0.680247 -0.446663 2.2645
0.667256 2.2645 -2.39061
[ ]:
Simple
0
3
Julia 1.10.0 | Idle
Mode: Edit
Ln 1, Col 1
Linear Algebra 1.ipynb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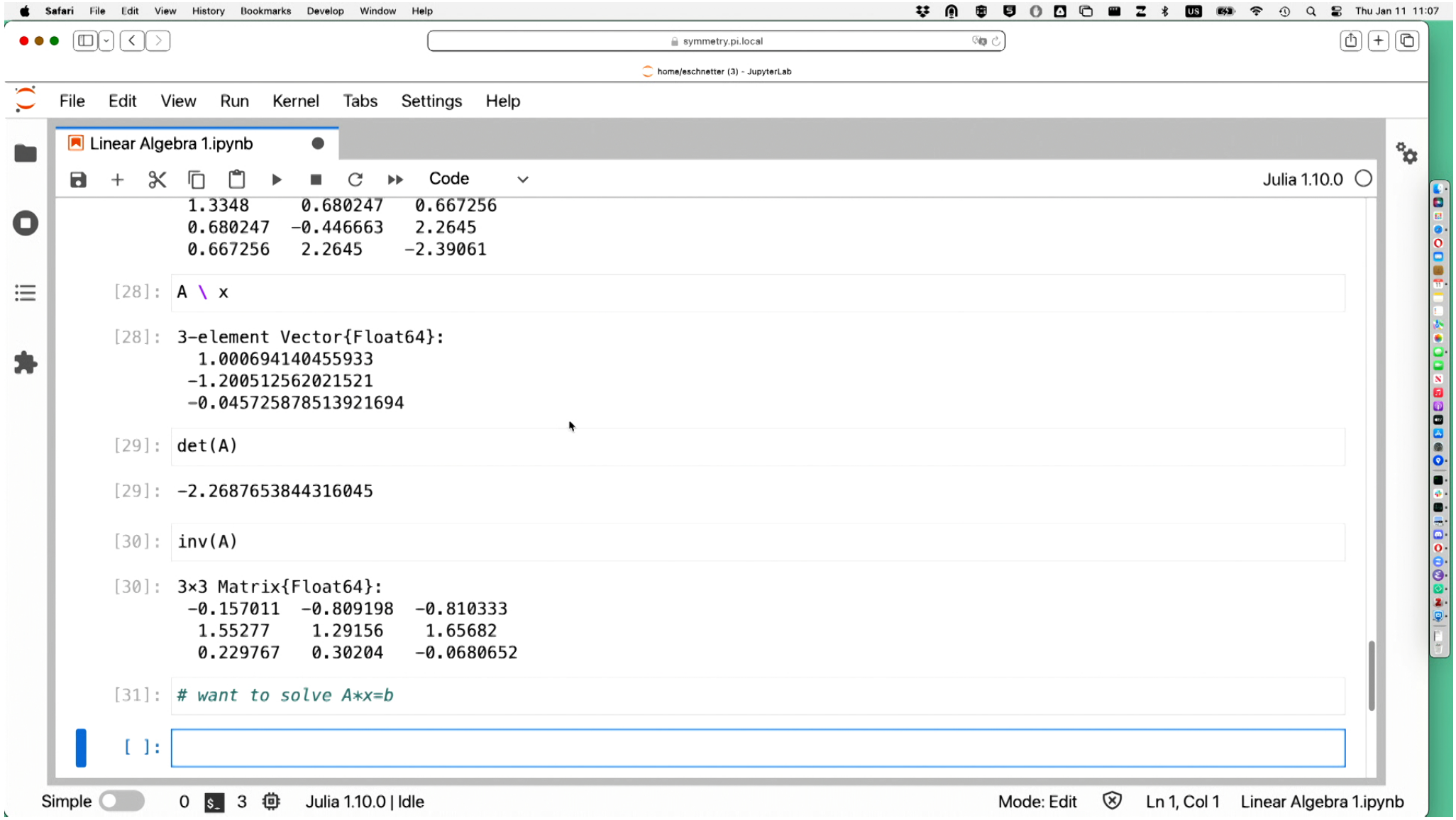
Linear Algebra 1.ipynb
Code
Julia 1.10.0
1.3348 0.680247 0.667256
0.680247 -0.446663 2.2645
0.667256 2.2645 -2.39061
[28]: A \ x
[28]: 3-element Vector{Float64}:
1.000694140455933
-1.200512562021521
-0.045725878513921694
[29]: det(A)
[29]: -2.2687653844316045
[30]: inv(A)
[30]: 3×3 Matrix{Float64}:
-0.157011 -0.809198 -0.810333
1.55277 1.29156 1.65682
0.229767 0.30204 -0.0680652
[ ]:
Simple
0
3
Julia 1.10.0 | Idle
Mode: Edit
Ln 1, Col 1
Linear Algebra 1.ipynb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Thu Jan 11 11:07
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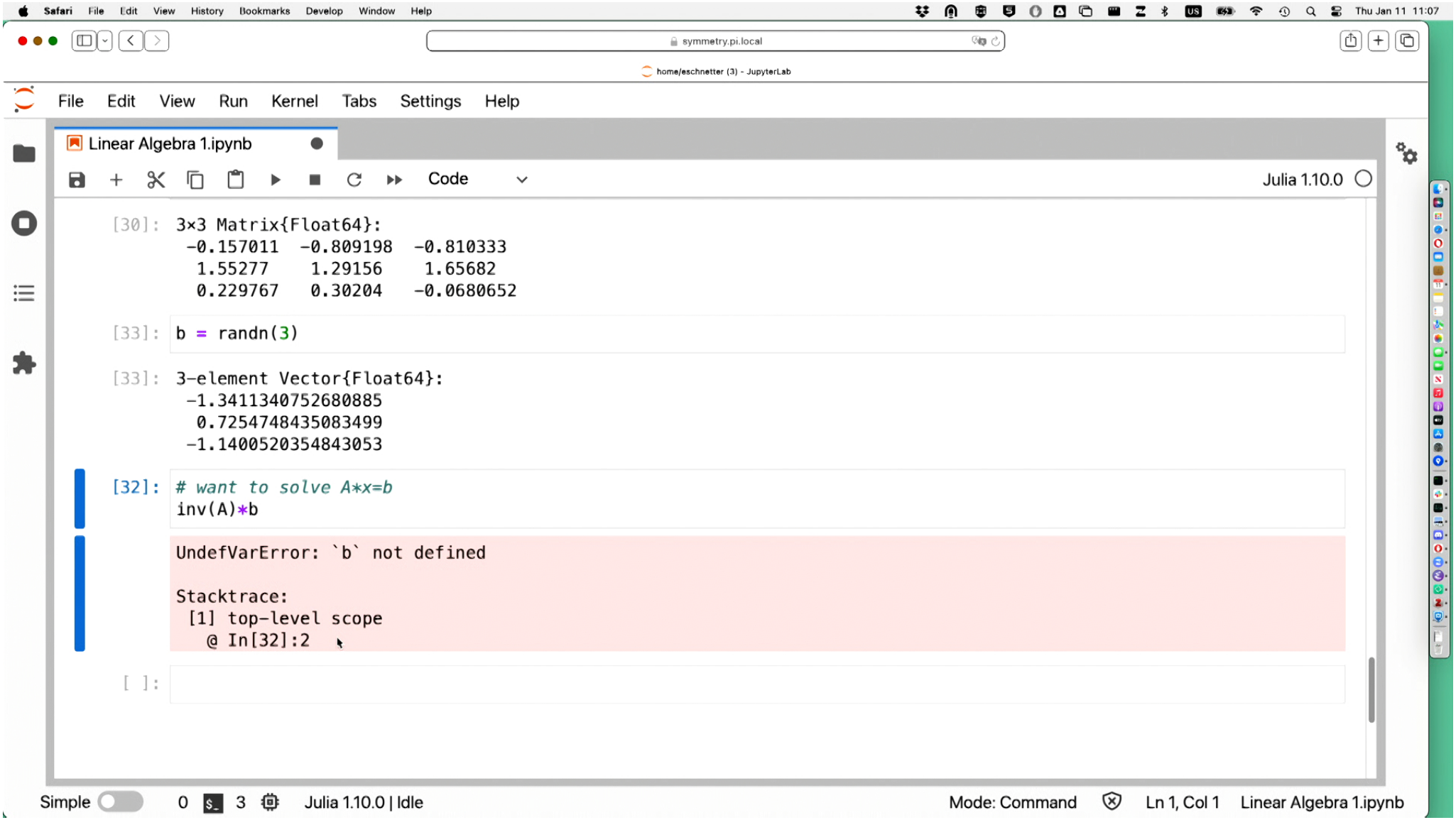
Linear Algebra 1.ipynb
Code
Julia 1.10.0
1.3348 0.680247 0.667256
0.680247 -0.446663 2.2645
0.667256 2.2645 -2.39061
[28]: A \ x
[28]: 3-element Vector{Float64}:
1.000694140455933
-1.200512562021521
-0.045725878513921694
[29]: det(A)
[29]: -2.2687653844316045
[30]: inv(A)
[30]: 3×3 Matrix{Float64}:
-0.157011 -0.809198 -0.810333
1.55277 1.29156 1.65682
0.229767 0.30204 -0.0680652
[31]: # want to solve A*x=b
[ ]:
Simple
0
3
Julia 1.10.0 | Idle
Mode: Edit
Ln 1, Col 1
Linear Algebra 1.ipynb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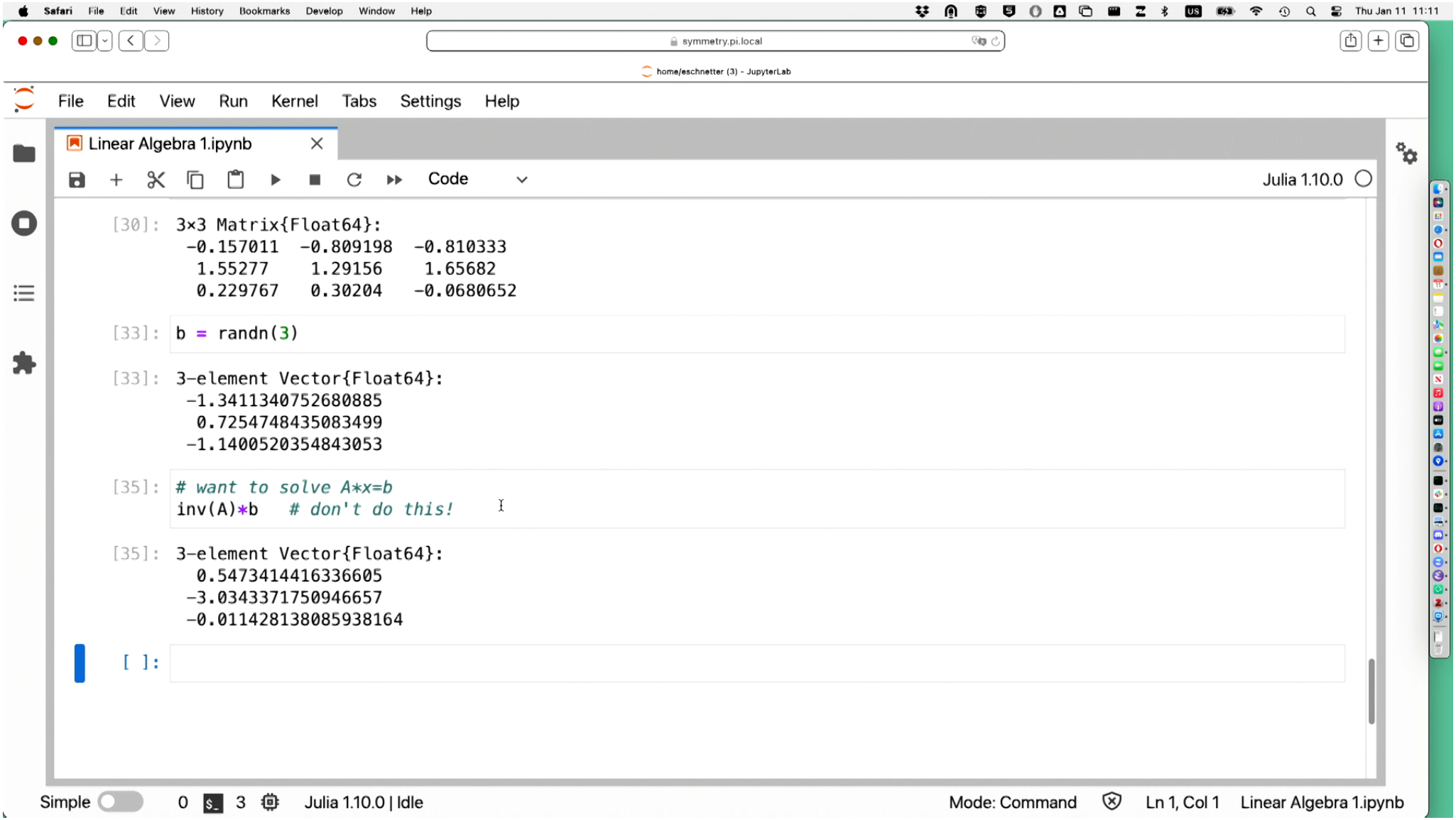
Thu Jan 11 11:07
symmetry.pi.local
home/eschnetter (3) - JupyterLab
File Edit View Run Kernel Tabs Settings Help
Linear Algebra 1.ipynb
Code
Julia 1.10.0
[30]: 3×3 Matrix{Float64}:
-0.157011 -0.809198 -0.810333
1.55277 1.29156 1.65682
0.229767 0.30204 -0.0680652
[33]: b = randn(3)
[33]: 3-element Vector{Float64}:
-1.3411340752680885
0.7254748435083499
-1.1400520354843053
[32]: # want to solve A*x=b
inv(A)*b
UndefVarError: `b` not defined
Stacktrace:
[1] top-level scope
@ In[32]:2
[ ]:
Simple
0
3
Julia 1.10.0 | Idle
Mode: Command
Ln 1, Col 1
Linear Algebra 1.ipynb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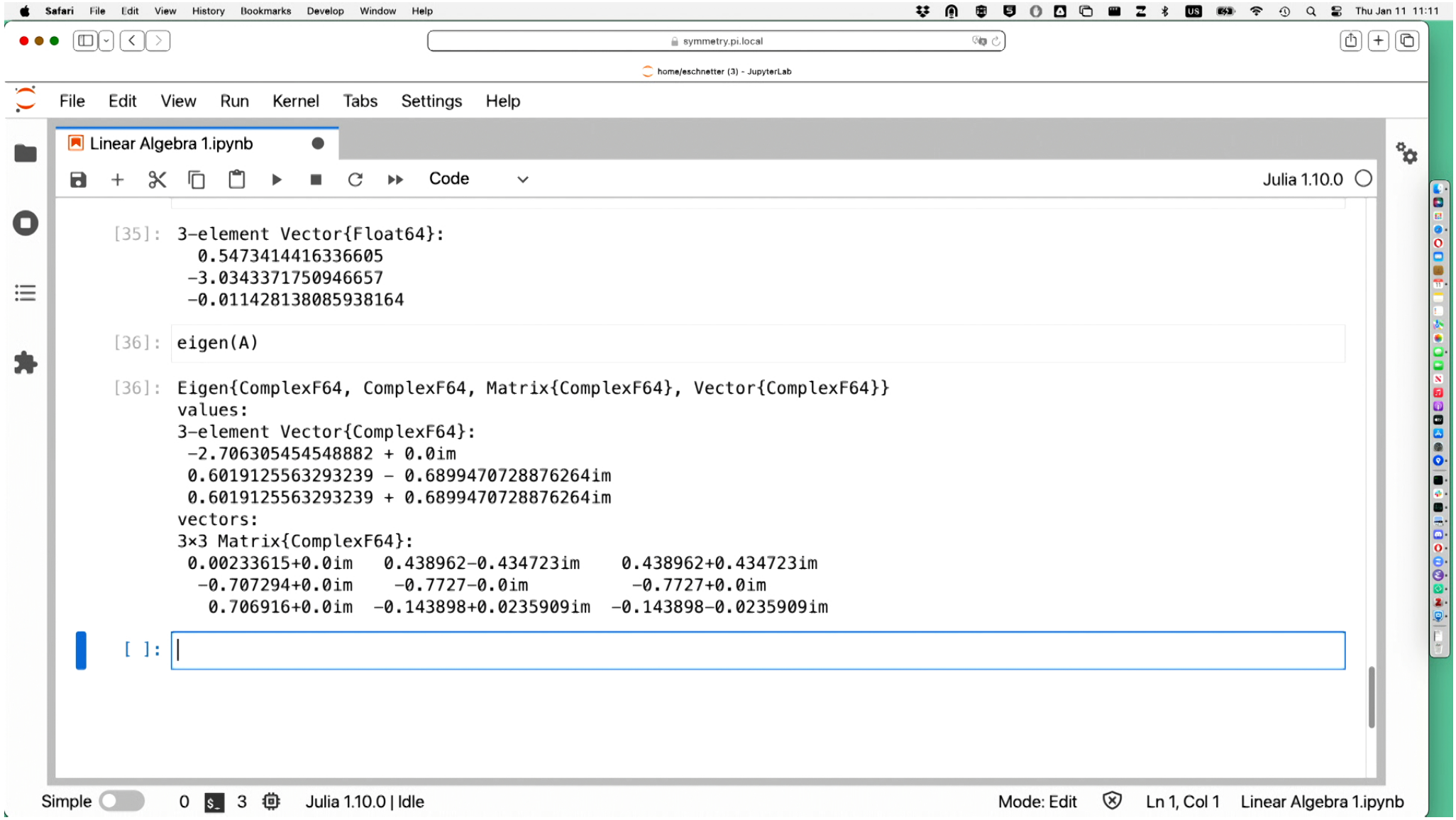
Thu Jan 11 11:11
symmetry.pi.local
home/eschnetter (3) - JupyterLab
File Edit View Run Kernel Tabs Settings Help
Linear Algebra 1.ipynb
Code
Julia 1.10.0
[30]: 3×3 Matrix{Float64}:
-0.157011 -0.809198 -0.810333
1.55277 1.29156 1.65682
0.229767 0.30204 -0.0680652
[33]: b = randn(3)
[33]: 3-element Vector{Float64}:
-1.3411340752680885
0.7254748435083499
-1.1400520354843053
[35]: # want to solve A*x=b
inv(A)*b # don't do this!
[35]: 3-element Vector{Float64}:
0.5473414416336605
-3.0343371750946657
-0.011428138085938164
[ ]:
Simple
0
3
Julia 1.10.0 | Idle
Mode: Command
Ln 1, Col 1
Linear Algebra 1.ipynb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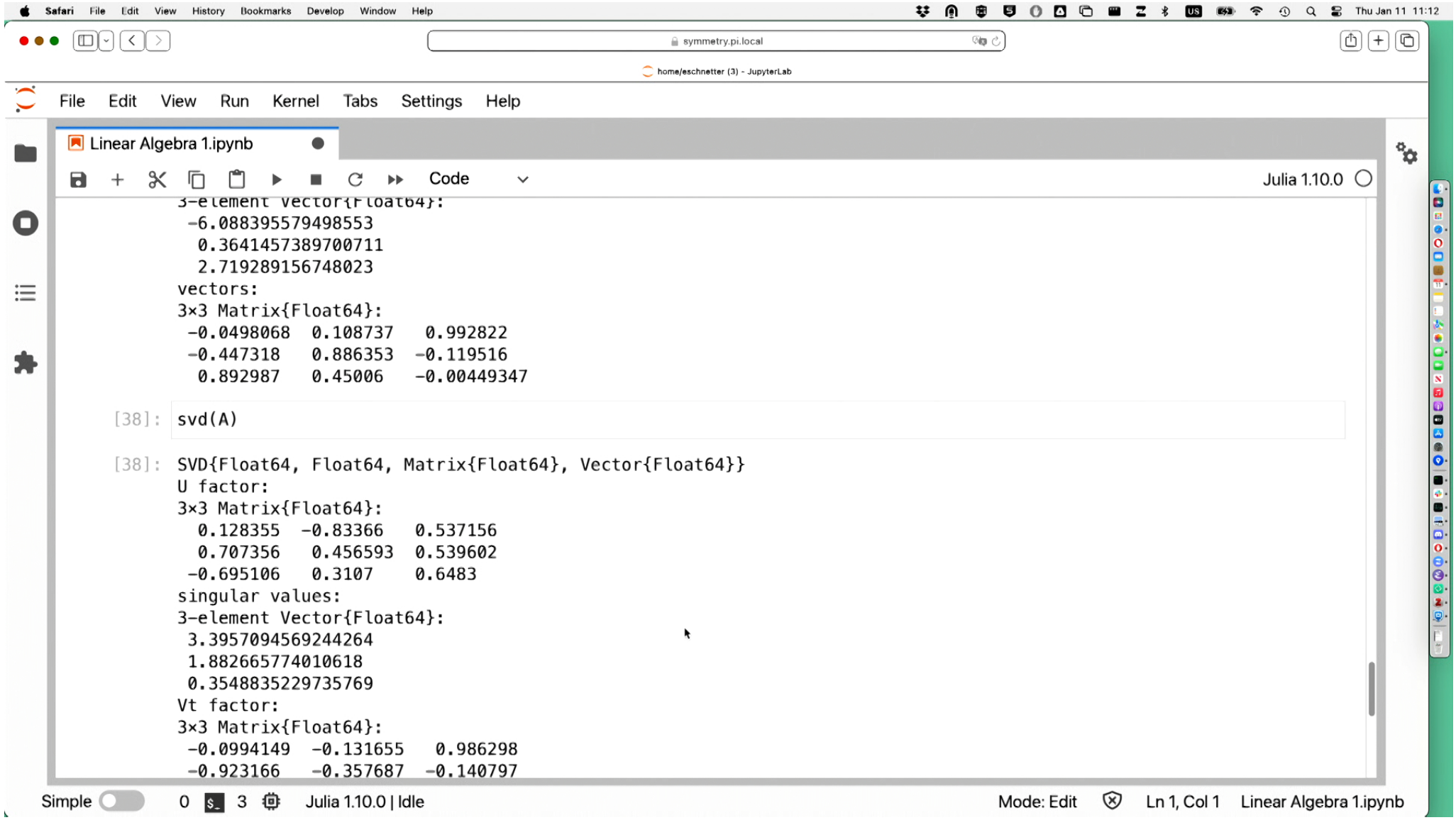
Thu Jan 11 11:11
symmetry.pi.local
home/eschnetter (3) - JupyterLab
File Edit View Run Kernel Tabs Settings Help
Linear Algebra 1.ipynb
Code
Julia 1.10.0
[35]: 3-element Vector{Float64}:
0.5473414416336605
-3.0343371750946657
-0.011428138085938164
[36]: eigen(A)
[36]: Eigen{ComplexF64, ComplexF64, Matrix{ComplexF64}, Vector{ComplexF64}}
values:
3-element Vector{ComplexF64}:
-2.706305454548882 + 0.0im
0.6019125563293239 - 0.6899470728876264im
0.6019125563293239 + 0.6899470728876264im
vectors:
3×3 Matrix{ComplexF64}:
0.00233615+0.0im 0.438962-0.434723im 0.438962+0.434723im
-0.707294+0.0im -0.7727-0.0im -0.7727+0.0im
0.706916+0.0im -0.143898+0.0235909im -0.143898-0.0235909im
[ ]:
Simple
0
3
Julia 1.10.0 | Idle
Mode: Edit
Ln 1, Col 1
Linear Algebra 1.ipynb


Safari File Edit View History Bookmarks Develop Window Help
Thu Jan 11 11:12
symmetry.pi.local
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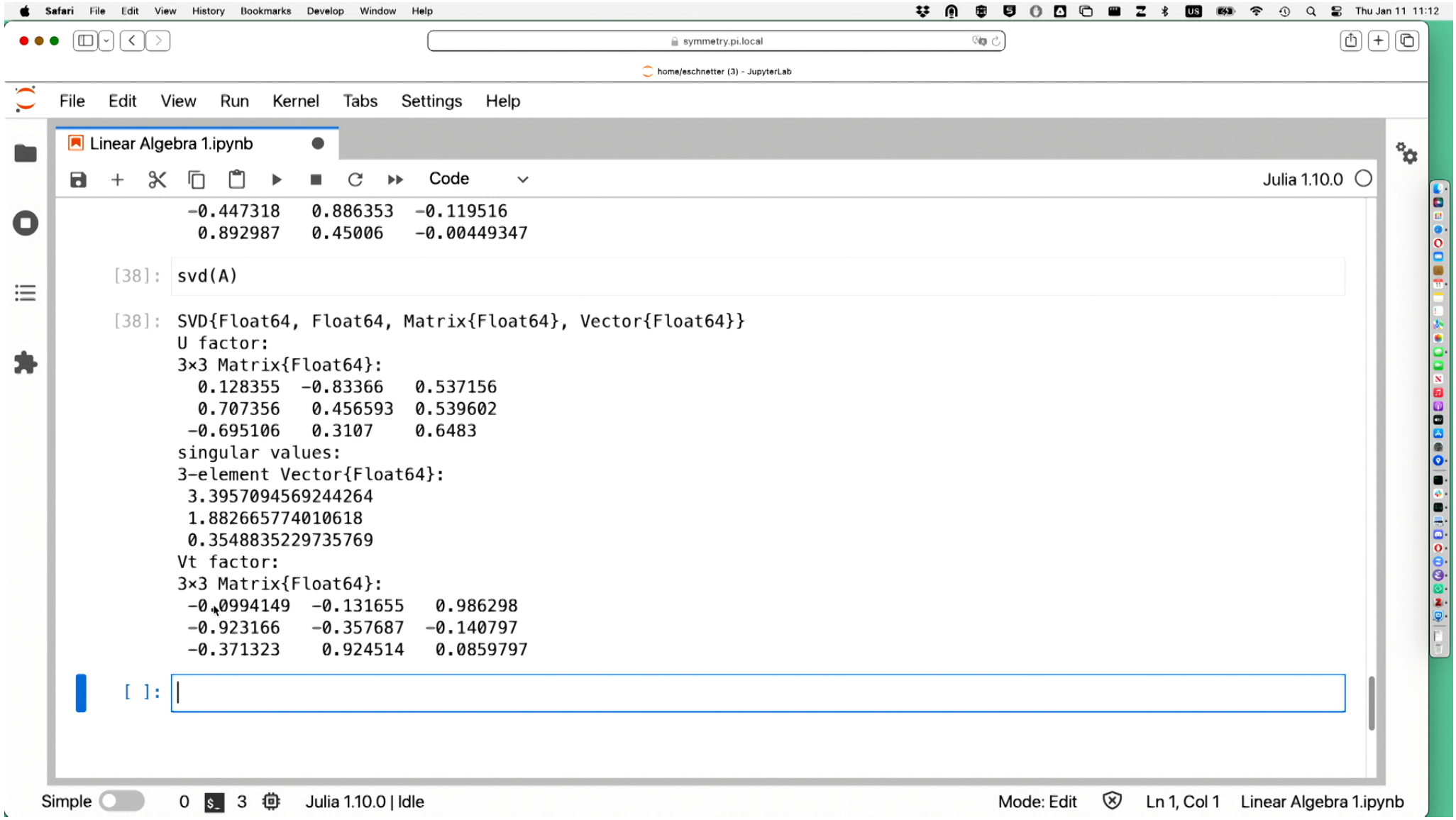
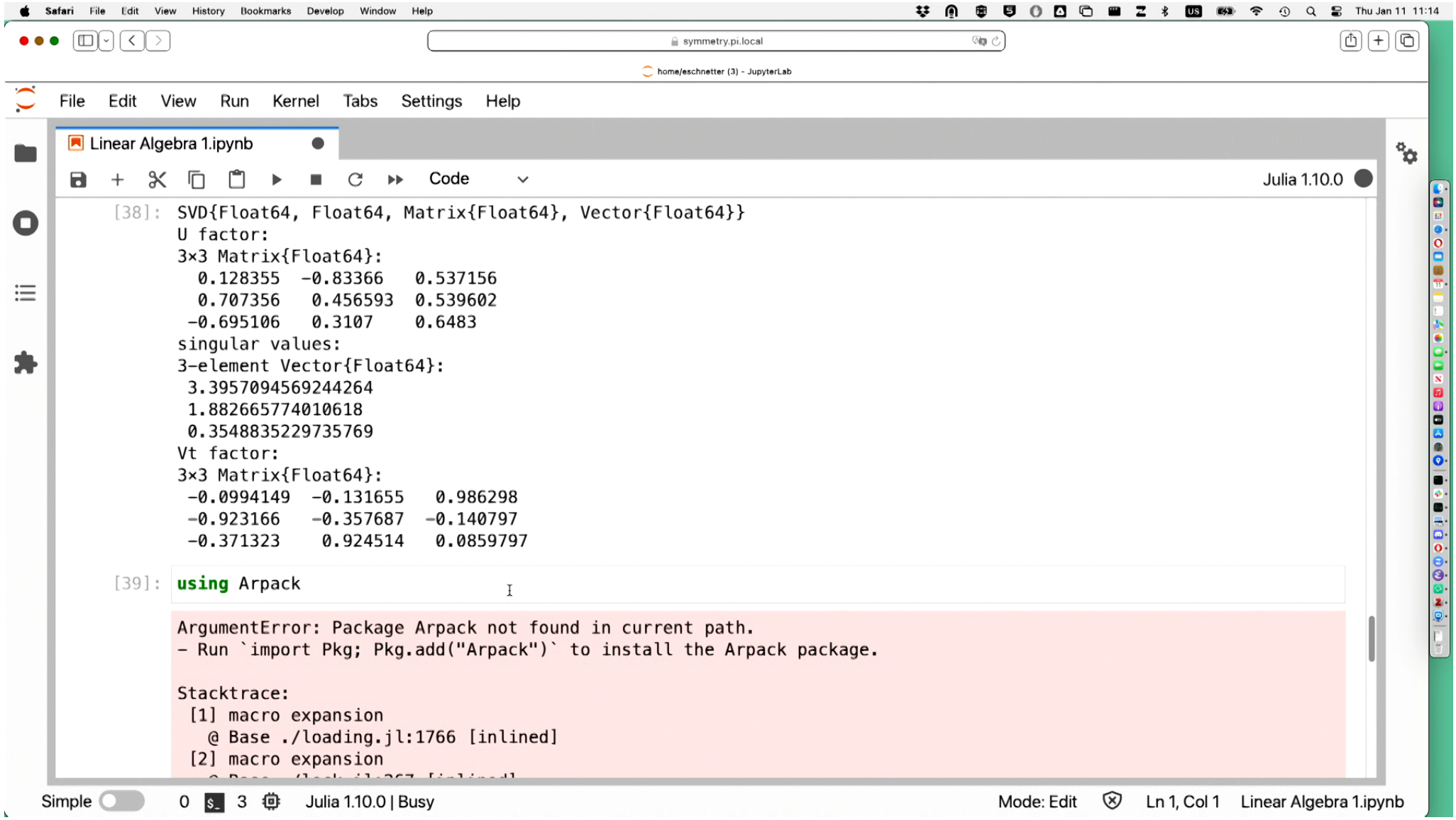
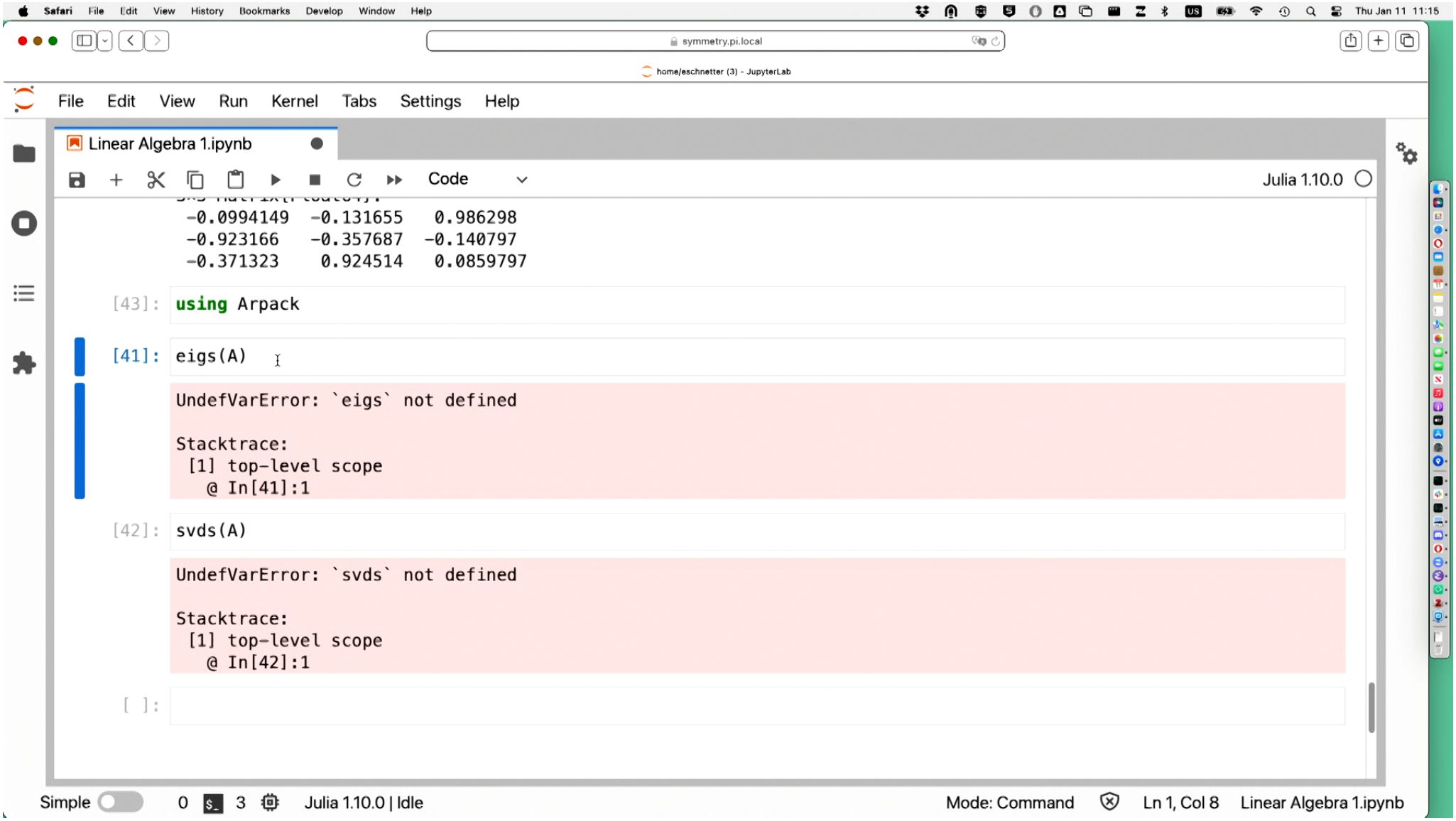
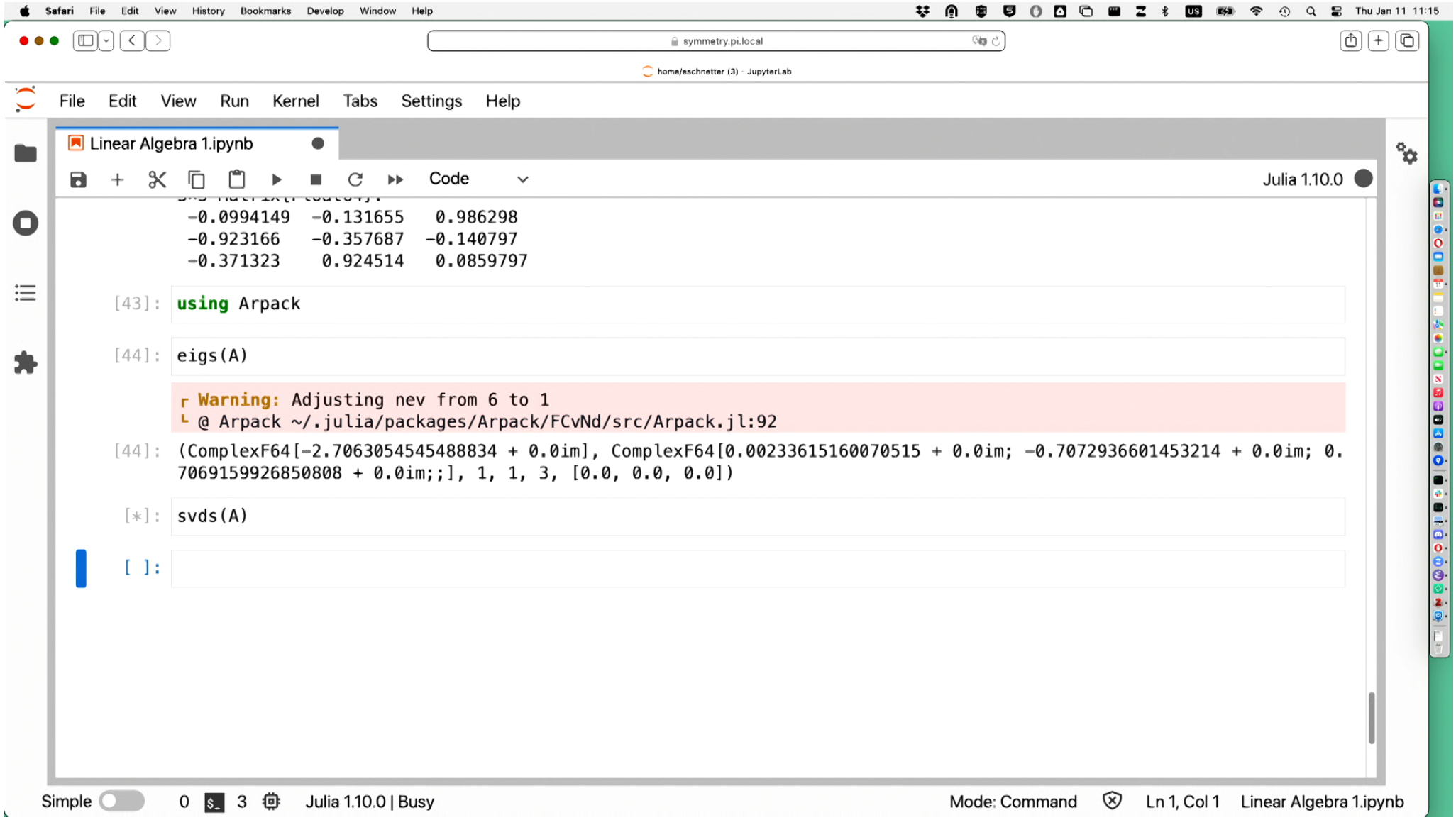
Linear Algebra 1.ipynb
Code
Julia 1.10.0
3-element Vector{Float64}:
-6.088395579498553
0.3641457389700711
2.719289156748023
vectors:
3×3 Matrix{Float64}:
-0.0498068 0.108737 0.992822
-0.447318 0.886353 -0.119516
0.892987 0.45006 -0.00449347
[38]: svd(A)
[38]: SVD{Float64, Float64, Matrix{Float64}, Vector{Float64}}
U factor:
3×3 Matrix{Float64}:
0.128355 -0.83366 0.537156
0.707356 0.456593 0.539602
-0.695106 0.3107 0.6483
singular values:
3-element Vector{Float64}:
3.3957094569244264
1.882665774010618
0.3548835229735769
Vt factor:
3×3 Matrix{Float64}:
-0.0994149 -0.131655 0.986298
-0.923166 -0.357687 -0.140797
Simple
0
3
Julia 1.10.0 | Idle
Mode: Edit
Ln 1, Col 1
Linear Algebra 1.ipynb


Safari File Edit View History Bookmarks Develop Window Help
Thu Jan 11 11:12
symmetry.pi.local
home/eschnetter (3) - JupyterLab
File Edit View Run Kernel Tabs Settings Help
Linear Algebra 1.ipynb
Code
Julia 1.10.0
-0.447318 0.886353 -0.119516
0.892987 0.45006 -0.00449347
[38]: svd(A)
[38]: SVD{Float64, Float64, Matrix{Float64}, Vector{Float64}}
U factor:
3×3 Matrix{Float64}:
0.128355 -0.83366 0.537156
0.707356 0.456593 0.539602
-0.695106 0.3107 0.6483
singular values:
3-element Vector{Float64}:
3.3957094569244264
1.882665774010618
0.3548835229735769
Vt factor:
3×3 Matrix{Float64}:
-0.0994149 -0.131655 0.986298
-0.923166 -0.357687 -0.140797
-0.371323 0.924514 0.0859797
[ ]:
Simple
0
3
Julia 1.10.0 | Idle
Mode: Edit
Ln 1, Col 1
Linear Algebra 1.ipynb


Safari File Edit View History Bookmarks Develop Window Help
Thu Jan 11 11:14
symmetry.pi.local
home/eschnetter (3) - JupyterLab
File Edit View Run Kernel Tabs Settings Help
Linear Algebra 1.ipynb
Code
Julia 1.10.0
[38]:
SVD{Float64, Float64, Matrix{Float64}, Vector{Float64}}
U factor:
3×3 Matrix{Float64}:
0.128355 -0.83366 0.537156
0.707356 0.456593 0.539602
-0.695106 0.3107 0.6483
singular values:
3-element Vector{Float64}:
3.3957094569244264
1.882665774010618
0.3548835229735769
Vt factor:
3×3 Matrix{Float64}:
-0.0994149 -0.131655 0.986298
-0.923166 -0.357687 -0.140797
-0.371323 0.924514 0.0859797
[39]:
using Arpack
ArgumentError: Package Arpack not found in current path.
- Run `import Pkg; Pkg.add("Arpack")` to install the Arpack package.
Stacktrace:
[1] macro expansion
@ Base ./loading.jl:1766 [inlined]
[2] macro expansion
Simple
0
3
Julia 1.10.0 | Busy
Mode: Edit
Ln 1, Col 1
Linear Algebra 1.ipynb


Safari File Edit View History Bookmarks Develop Window Help
Thu Jan 11 11:15
symmetry.pi.local
home/eschnetter (3) - JupyterLab
File Edit View Run Kernel Tabs Settings Help
Linear Algebra 1.ipynb
Code
Julia 1.10.0
-0.0994149 -0.131655 0.986298
-0.923166 -0.357687 -0.140797
-0.371323 0.924514 0.0859797
[43]: using Arpack
[41]: eigs(A)
UndefVarError: `eigs` not defined
Stacktrace:
[1] top-level scope
@ In[41]:1
[42]: svds(A)
UndefVarError: `svds` not defined
Stacktrace:
[1] top-level scope
@ In[42]:1
[ ]:
Simple
0
3
Julia 1.10.0 | Idle
Mode: Command
Ln 1, Col 8
Linear Algebra 1.ipynb


Safari File Edit View History Bookmarks Develop Window Help
Thu Jan 11 11:15
symmetry.pi.local
home/eschnetter (3) - JupyterLab
File Edit View Run Kernel Tabs Settings Help
Linear Algebra 1.ipynb
Code
Julia 1.10.0
-0.0994149 -0.131655 0.986298
-0.923166 -0.357687 -0.140797
-0.371323 0.924514 0.0859797
[43]: using Arpack
[44]: eigs(A)
Warning: Adjusting nev from 6 to 1
@ Arpack ~/.julia/packages/Arpack/FCvNd/src/Arpack.jl:92
[44]: (ComplexF64[-2.7063054545488834 + 0.0im], ComplexF64[0.00233615160070515 + 0.0im; -0.7072936601453214 + 0.0im; 0.7069159926850808 + 0.0im;;], 1, 1, 3, [0.0, 0.0, 0.0])
[*]: svds(A)
[ ]:
Simple
0
3
Julia 1.10.0 | Busy
Mode: Command
Ln 1, Col 1
Linear Algebra 1.ipynb