Title: Numerical Methods Lecture

Speakers: Erik Schnetter

Collection: Numerical Methods 2023/24

Date: January 15, 2024 - 2:00 PM

URL: https://pirsa.org/24010016

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Linear Algebra 2.ipynb | Gradient Descent.ipynb | Congugate Gradients.ipynb | Eigenvectors.ipynb

Code — Julia 1.10.0

Linear Algebra 2

```
[1]: xs = 0:0.01:pi
```

```
[1]: 0.0:0.01:3.14
```

```
[6]: @time ys = sin.(xs)
```

```
  0.097673 seconds (172.87 k allocations: 10.993 MiB, 99.92% compilation time)
[6]: 315-element Vector{Float64}:
 0.0
 0.009999833334166664
 0.01999866669333308
 0.02999550020249566
 0.03998933418663416
 0.04997916927067833
 0.059964006479444595
 0.06994284733753277
 0.0799146939691727
 0.08987854919801104
 0.09983341664682815
```

Simple 0 $_ 4 Julia 1.10.0 | Idle Saving completed Mode: Command Ln 1, Col 21 Linear Algebra 2.ipynb

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Linear Algebra 2.ipynb | Gradient Descent.ipynb | Congugate Gradients.ipynb | Eigenvectors.ipynb

Code — Julia 1.10.0

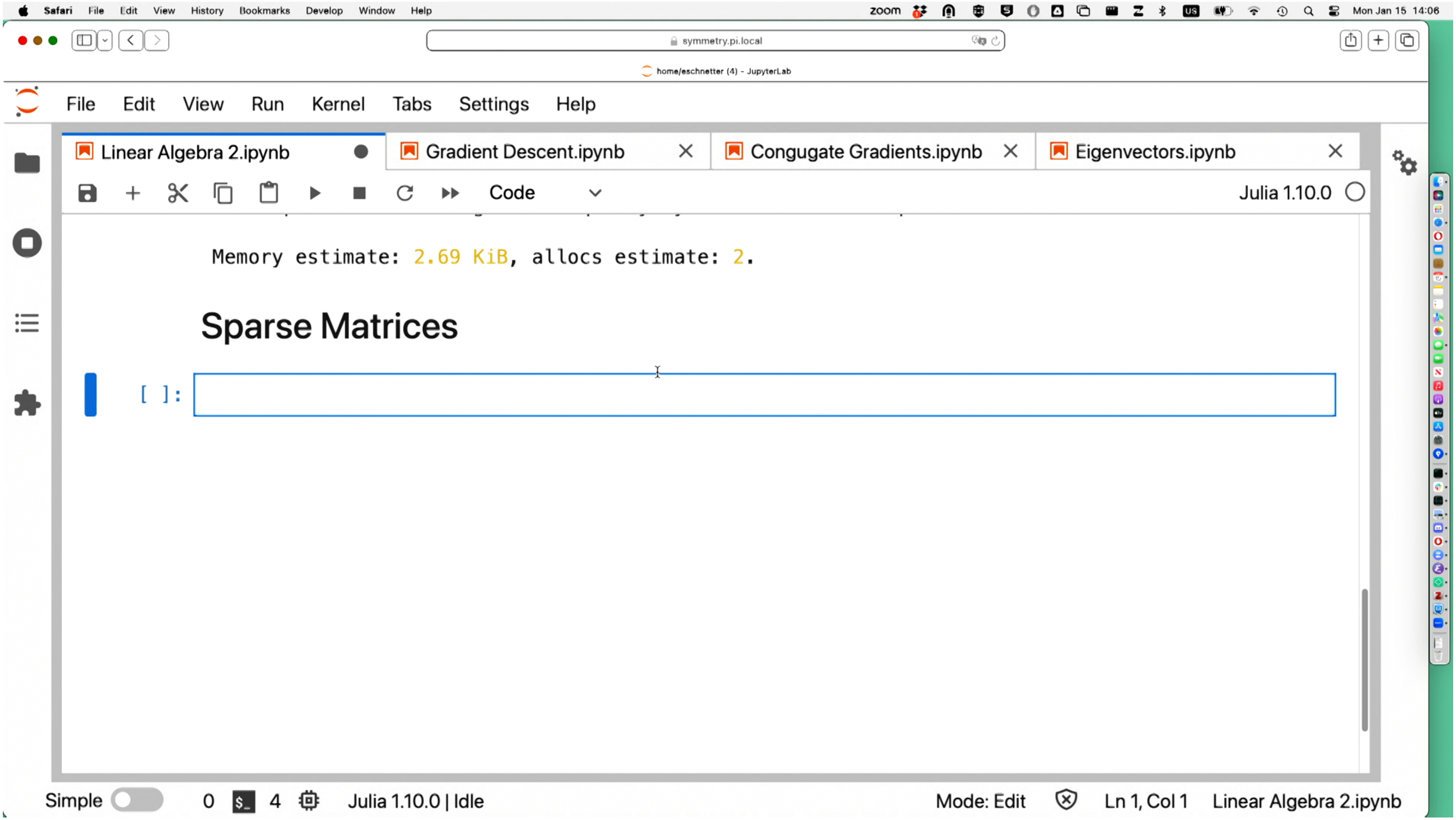
```
0.04158066243329049
0.031587398436453896
0.02159097572609596
0.011592393936158275
0.0015926529164868282
```

[4]:
```julia
using BenchmarkTools
```

[*]:
```julia
@benchmark ys = sin.(xs)
```

[ ]:

Simple 0 $_ 4 Julia 1.10.0 | Busy Mode: Command Ln 1, Col 1 Linear Algebra 2.ipynb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File Edit View Run Kernel Tabs Settings Help
Linear Algebra 2.ipynb
Gradient Descent.ipynb
Congugate Gradients.ipynb
Eigenvectors.ipynb
Code
Julia 1.10.0
Memory estimate: 2.69 KiB, allocs estimate: 2.
Sparse Matrices
[ ]:
Simple
0
4
Julia 1.10.0 | Idle
Mode: Edit
Ln 1, Col 1
Linear Algebra 2.ipynb

File Edit View Run Kernel Tabs Settings Help

Linear Algebra 2.ipynb | Gradient Descent.ipynb | Congugate Gradients.ipynb | Eigenvectors.ipynb

Code — Julia 1.10.0

[10]:
```julia
using LinearAlgebra
using SparseArrays
```

[11]:
```julia
sprand(10, 10, 0.2)
```

[11]:
```
10×10 SparseMatrixCSC{Float64, Int64} with 16 stored entries:
  ⋅          ⋅          ⋅          ⋅         …   ⋅           ⋅          ⋅
  ⋅          ⋅          ⋅         0.922262       0.0461883   ⋅         0.17296
 0.710916    ⋅          ⋅         0.0222049       ⋅           ⋅          ⋅
  ⋅         0.823887   0.530032    ⋅              ⋅           ⋅          ⋅
 0.675312    ⋅          ⋅          ⋅              ⋅           ⋅          ⋅
  ⋅          ⋅          ⋅          ⋅         …   ⋅           ⋅          ⋅
  ⋅          ⋅          ⋅          ⋅             0.233278     ⋅          ⋅
  ⋅          ⋅          ⋅          ⋅             0.781869    0.943912    ⋅
  ⋅          ⋅          ⋅         0.193249        ⋅           ⋅         0.665795
 0.519597    ⋅          ⋅          ⋅              ⋅           ⋅          ⋅
```

[ ]:

Simple 0 4 Julia 1.10.0 | Idle Mode: Edit Ln 1, Col 1 Linear Algebra 2.ipynb

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Linear Algebra 2.ipynb | Gradient Descent.ipynb | Congugate Gradients.ipynb | Eigenvectors.ipynb

Code — Julia 1.10.0

```
 ⋅          ⋅          ⋅  0.00586887  …  0.937074  ⋅  ⋅         0.820368
 0.0285838  ⋅          ⋅   ⋅                ⋅      ⋅  0.382854  0.504618
 0.0176372  0.520039   ⋅   ⋅                ⋅      ⋅   ⋅         ⋅
 ⋅          ⋅          ⋅   ⋅                ⋅      ⋅  0.251513   ⋅
 0.488654   ⋅          ⋅  0.556611          ⋅      ⋅   ⋅         ⋅
 ⋅          ⋅          ⋅   ⋅             …  ⋅      ⋅   ⋅         ⋅
 ⋅          ⋅          ⋅   ⋅                ⋅      ⋅   ⋅         ⋅
 ⋅          ⋅          ⋅   ⋅                ⋅      ⋅   ⋅        0.962364
 ⋅          ⋅          ⋅   ⋅                ⋅      ⋅   ⋅         ⋅
 0.680894   ⋅          ⋅   ⋅                ⋅      ⋅   ⋅         ⋅
```

[14]:

```
inv(A)
```

```
The inverse of a sparse matrix can often be dense and can cause the computer to run out of memor
y. If you are sure you have enough memory, please either convert your matrix to a dense matrix,
e.g. by calling `Matrix` or if `A` can be factorized, use `\` on the dense identity matrix, e.g.
`A \ Matrix{eltype(A)}(I, size(A)...)` restrictions of `\` on sparse lhs applies. Alternatively,
`A\b` is generally preferable to `inv(A)*b`

Stacktrace:
 [1] error(s::String)
   @ Base ./error.jl:35
```

Simple 0 4 Julia 1.10.0 | Idle — Mode: Edit — Ln 1, Col 1 — Linear Algebra 2.ipynb

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Linear Algebra 2.ipynb | Gradient Descent.ipynb | Congugate Gradients.ipynb | Eigenvectors.ipynb

Code — Julia 1.10.0

```
 0.0176372  0.320039   ·          ·                    ·          ·         ·          ·
  ·          ·          ·          ·                    ·          ·        0.251513   ·
 0.488654    ·          ·         0.556611              ·          ·         ·          ·
  ·          ·          ·          ·         …          ·          ·         ·          ·
  ·          ·          ·          ·                    ·          ·         ·          ·
  ·          ·          ·          ·                    ·          ·         ·         0.962364
  ·          ·          ·          ·                    ·          ·         ·          ·
 0.680894    ·          ·          ·                    ·          ·         ·          ·
```

[16]:
```
pinv(Matrix(A))
```

[16]:
```
10×10 Matrix{Float64}:
  0.0577932      0.221219       0.164213     …  -0.231032      0.0   0.862024
 -0.0216246      1.07853        1.86149         -0.522485      0.0   0.128425
  1.62602e-16   -4.06114e-16   -1.34701e-16     -1.10676e-16   0.0   2.56886e-17
 -0.0507372     -0.19421       -0.144164         0.202825      0.0  -0.75678
 -0.0829747     -0.317608      -0.235763         0.331697      0.0   0.870955
  0.016606      -0.917114       0.0471841    …   0.447837      0.0  -0.133139
  1.05344       -0.789497      -0.038966        -0.468424      0.0   0.182963
  0.0            0.0            0.0              0.0           0.0   0.0
 -0.0254399      1.40499       -0.0722846       -0.686072      0.0   0.203964
  0.0160276      0.9032         0.0455406        0.53361       0.0  -0.203577
```

Simple 0 4 Julia 1.10.0 | Idle — Mode: Command — Ln 1, Col 1 — Linear Algebra 2.ipynb

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Linear Algebra 2.ipynb | Gradient Descent.ipynb | Congugate Gradients.ipynb | Eigenvectors.ipynb

Code   Julia 1.10.0

[16]:
```
pinv(Matrix(A))
```

[16]:
```
10×10 Matrix{Float64}:
  0.0577932    0.221219     0.164213    …  -0.231032    0.0   0.862024
 -0.0216246    1.07853      1.86149        -0.522485    0.0   0.128425
  1.62602e-16 -4.06114e-16 -1.34701e-16    -1.10676e-16 0.0   2.56886e-17
 -0.0507372   -0.19421     -0.144164        0.202825    0.0  -0.75678
 -0.0829747   -0.317608    -0.235763        0.331697    0.0   0.870955
  0.016606    -0.917114     0.0471841   …   0.447837    0.0  -0.133139
  1.05344     -0.789497    -0.038966       -0.468424    0.0   0.182963
  0.0          0.0          0.0             0.0         0.0   0.0
 -0.0254399    1.40499     -0.0722846      -0.686072    0.0   0.203964
  0.0160276    0.9032       0.0455406       0.53361     0.0  -0.203577
```

[17]:
```
spzeros(10, 10)
```

[17]:
```
10×10 SparseMatrixCSC{Float64, Int64} with 0 stored entries:
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅
```

Simple 0 $_ 4   Julia 1.10.0 | Idle   Mode: Edit   Ln 1, Col 1   Linear Algebra 2.ipynb

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Linear Algebra 2.ipynb | Gradient Descent.ipynb | Congugate Gradients.ipynb | Eigenvectors.ipynb

Code — Julia 1.10.0

```
 -0.0254399    1.40499    -0.0722846    -0.686072    0.0   0.203964
  0.0160276    0.9032      0.0455406     0.53361     0.0  -0.203577
```

[17]:
```
spzeros(10, 10)
```

[17]:
```
10×10 SparseMatrixCSC{Float64, Int64} with 0 stored entries:
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅
```

[ ]:

Simple 0 4 Julia 1.10.0 | Idle Mode: Edit Ln 1, Col 1 Linear Algebra 2.ipynb

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Linear Algebra 2.ipynb | Gradient Descent.ipynb | Congugate Gradients.ipynb | Eigenvectors.ipynb

Code — Julia 1.10.0

```
 .  .  .  .  .  .  .  .  .  .
 .  .  .  .  .  .  .  .  .  .
 .  .  .  .  .  .  .  .  .  .
 .  .  .  .  .  .  .  .  .  .
```

[19]:
```julia
sparse(Diagonal(1:10))
```

[19]:
```
10×10 SparseMatrixCSC{Int64, Int64} with 10 stored entries:
 1  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅   ⋅
 ⋅  2  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅   ⋅
 ⋅  ⋅  3  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅   ⋅
 ⋅  ⋅  ⋅  4  ⋅  ⋅  ⋅  ⋅  ⋅   ⋅
 ⋅  ⋅  ⋅  ⋅  5  ⋅  ⋅  ⋅  ⋅   ⋅
 ⋅  ⋅  ⋅  ⋅  ⋅  6  ⋅  ⋅  ⋅   ⋅
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  7  ⋅  ⋅   ⋅
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  8  ⋅   ⋅
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  9   ⋅
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  10
```

[ ]:

Simple 0 4 Julia 1.10.0 | Idle Mode: Command Ln 1, Col 1 Linear Algebra 2.ipynb

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Linear Algebra 2.ipynb | Gradient Descent.ipynb | Congugate Gradients.ipynb | Eigenvectors.ipynb

Code — Julia 1.10.0

```
[19]: 10×10 SparseMatrixCSC{Int64, Int64} with 10 stored entries:
 1  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅   ⋅
 ⋅  2  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅   ⋅
 ⋅  ⋅  3  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅   ⋅
 ⋅  ⋅  ⋅  4  ⋅  ⋅  ⋅  ⋅  ⋅   ⋅
 ⋅  ⋅  ⋅  ⋅  5  ⋅  ⋅  ⋅  ⋅   ⋅
 ⋅  ⋅  ⋅  ⋅  ⋅  6  ⋅  ⋅  ⋅   ⋅
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  7  ⋅  ⋅   ⋅
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  8  ⋅   ⋅
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  9   ⋅
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  10
```

```
[20]: Bidiagonal(1:10, 1:10)
```

```
MethodError: no method matching Bidiagonal(::UnitRange{Int64}, ::UnitRange{Int64})

Closest candidates are:
  Bidiagonal(::V, ::V, ::Symbol) where {T, V<:AbstractVector{T}}
   @ LinearAlgebra ~/.julia/juliaup/julia-1.10.0+0.x64.linux.gnu/share/julia/stdlib/v1.10/Linear
Algebra/src/bidiag.jl:68
  Bidiagonal(::V, ::V, ::AbstractChar) where {T, V<:AbstractVector{T}}
   @ LinearAlgebra ~/.julia/juliaup/julia-1.10.0+0.x64.linux.gnu/share/julia/stdlib/v1.10/Linear
```

Simple 0 $_ 4 Julia 1.10.0 | Idle — Mode: Edit — Ln 1, Col 20 — Linear Algebra 2.ipynb

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Linear Algebra 2.ipynb | Gradient Descent.ipynb | Congugate Gradients.ipynb | Eigenvectors.ipynb

Code — Julia 1.10.0

```
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  8  ⋅   ⋅
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  9   ⋅
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  10
```

[24]:

```julia
Bidiagonal(1:10, 1:9, :U)
```

[24]:

```
10×10 Bidiagonal{Int64, UnitRange{Int64}}:
 1  1  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅   ⋅
 ⋅  2  2  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅   ⋅
 ⋅  ⋅  3  3  ⋅  ⋅  ⋅  ⋅  ⋅   ⋅
 ⋅  ⋅  ⋅  4  4  ⋅  ⋅  ⋅  ⋅   ⋅
 ⋅  ⋅  ⋅  ⋅  5  5  ⋅  ⋅  ⋅   ⋅
 ⋅  ⋅  ⋅  ⋅  ⋅  6  6  ⋅  ⋅   ⋅
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  7  7  ⋅   ⋅
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  8  8   ⋅
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  9   9
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  10
```

[ ]:

Simple 0 4 Julia 1.10.0 | Idle Mode: Command Ln 1, Col 1 Linear Algebra 2.ipynb

```
 .  .  .  .  .  .  7  7  .     .
 .  .  .  .  .  .  .  8  8     .
 .  .  .  .  .  .  .  .  9     9
 .  .  .  .  .  .  .  .  .    10
```

[25]:

```
Tridiagonal(1:10, 2:9, 3:10)
```

```
ArgumentError: cannot construct Tridiagonal from incompatible lengths of subdiagonal, diagonal and superdiagonal: (10, 8, 8)

Stacktrace:
 [1] Tridiagonal
   @ LinearAlgebra ~/.julia/juliaup/julia-1.10.0+0.x64.linux.gnu/share/julia/stdlib/v1.10/LinearAlgebra/src/tridiag.jl:463 [inlined]
 [2] Tridiagonal(dl::UnitRange{Int64}, d::UnitRange{Int64}, du::UnitRange{Int64})
   @ LinearAlgebra ~/.julia/juliaup/julia-1.10.0+0.x64.linux.gnu/share/julia/stdlib/v1.10/LinearAlgebra/src/tridiag.jl:502
 [3] top-level scope
   @ In[25]:1
```

[ ]:

Simple 0 4 Julia 1.10.0 | Idle Mode: Edit Ln 1, Col 27 Linear Algebra 2.ipynb

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Linear Algebra 2.ipynb | Gradient Descent.ipynb | Congugate Gradients.ipynb | Eigenvectors.ipynb

Code — Julia 1.10.0

```
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  9   9
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  10
```

[29]:

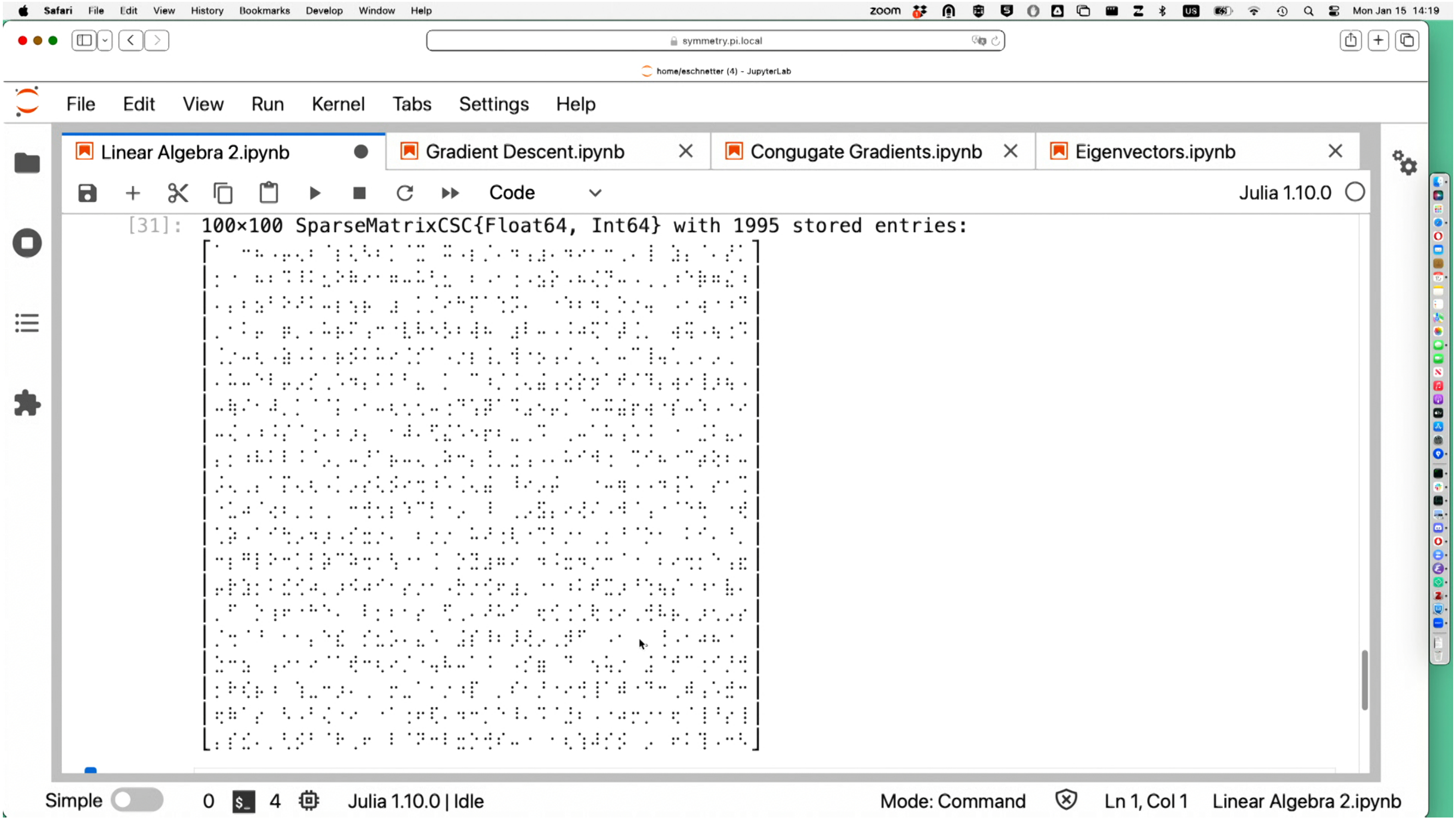
```julia
Tridiagonal(2:10, 1:10, 3:11)
```

[29]:

```
10×10 Tridiagonal{Int64, UnitRange{Int64}}:
 1  3  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅   ⋅   ⋅
 2  2  4  ⋅  ⋅  ⋅  ⋅  ⋅   ⋅   ⋅
 ⋅  3  3  5  ⋅  ⋅  ⋅  ⋅   ⋅   ⋅
 ⋅  ⋅  4  4  6  ⋅  ⋅  ⋅   ⋅   ⋅
 ⋅  ⋅  ⋅  5  5  7  ⋅  ⋅   ⋅   ⋅
 ⋅  ⋅  ⋅  ⋅  6  6  8  ⋅   ⋅   ⋅
 ⋅  ⋅  ⋅  ⋅  ⋅  7  7  9   ⋅   ⋅
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  8  8  10   ⋅
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  9   9  11
 ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  ⋅  10  10
```

[ ]:

Simple 0 $_ 4 Julia 1.10.0 | Idle — Mode: Command — Ln 1, Col 1 — Linear Algebra 2.ipynb


symmetry.pi.local
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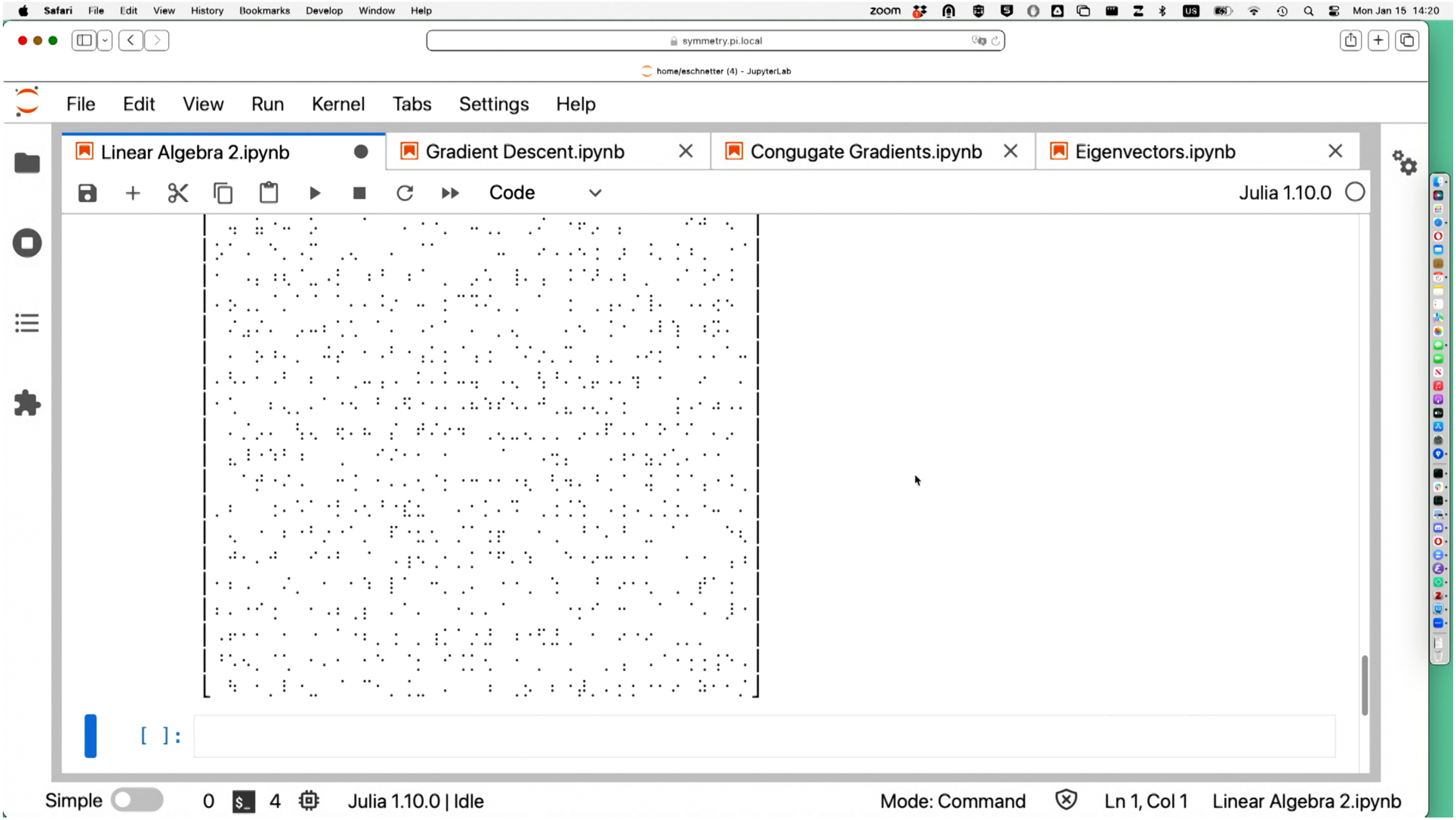
File Edit View Run Kernel Tabs Settings Help
Linear Algebra 2.ipynb
Gradient Descent.ipynb
Congugate Gradients.ipynb
Eigenvectors.ipynb
Code
Julia 1.10.0
[31]: 100×100 SparseMatrixCSC{Float64, Int64} with 1995 stored entries:
Simple 0 4 Julia 1.10.0 | Idle
Mode: Command Ln 1, Col 1 Linear Algebra 2.ipynb


Safari File Edit View History Bookmarks Develop Window Help
Mon Jan 15 14:20
symmetry.pi.local
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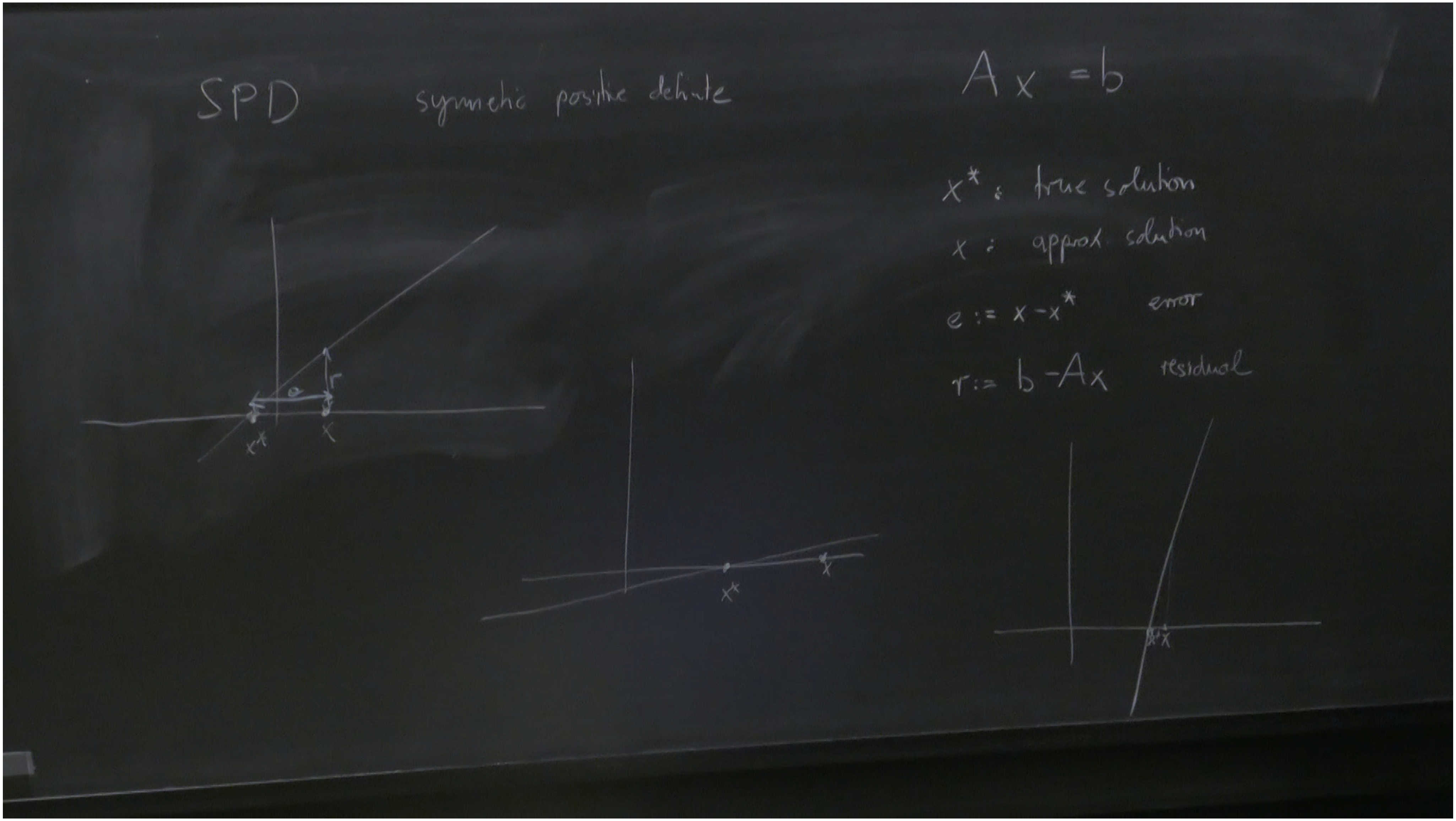
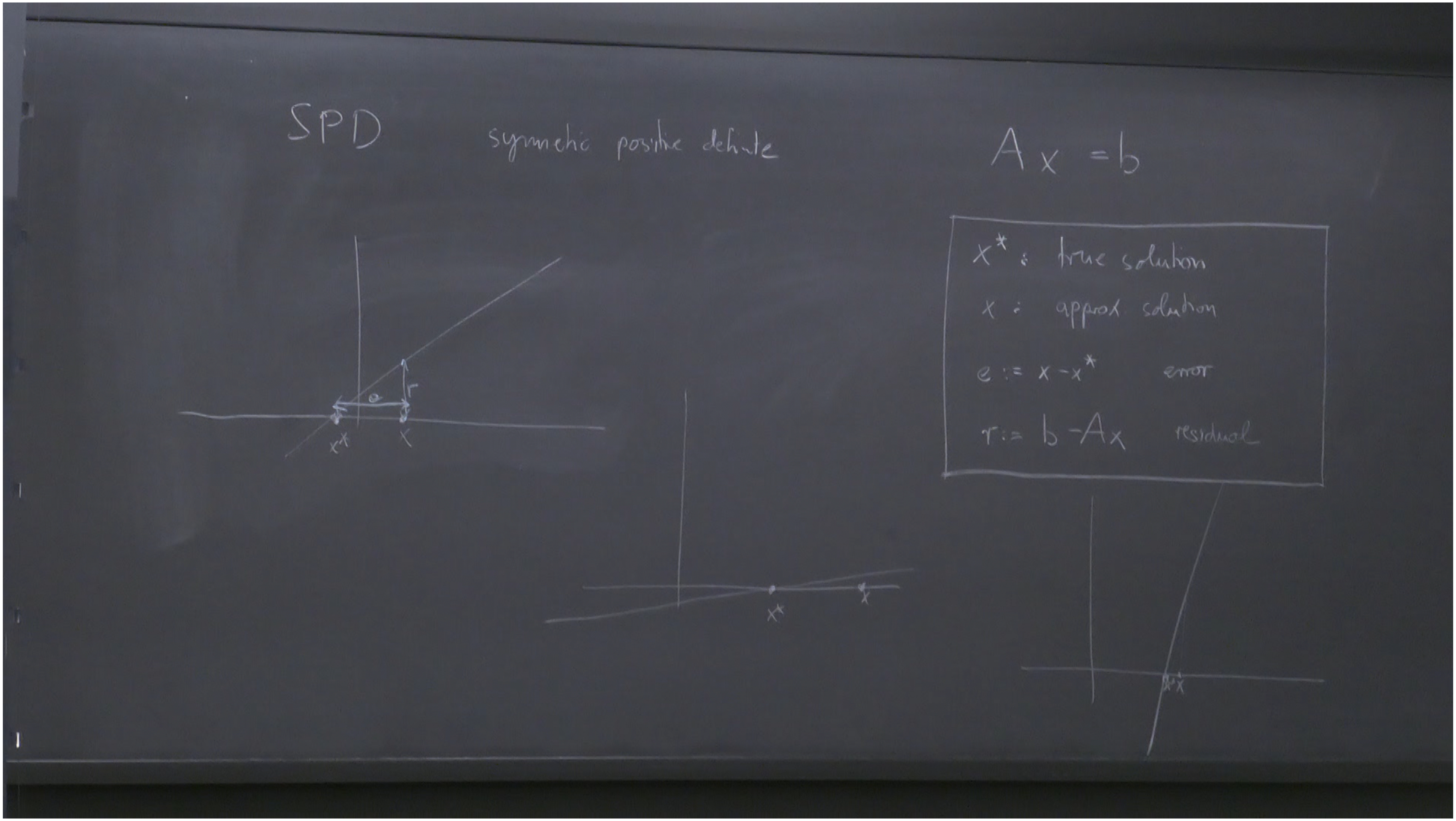
File Edit View Run Kernel Tabs Settings Help
Linear Algebra 2.ipynb
Gradient Descent.ipynb
Congugate Gradients.ipynb
Eigenvectors.ipynb
Code
Julia 1.10.0
[ ]:
Simple
0
4
Julia 1.10.0 | Idle
Mode: Command
Ln 1, Col 1
Linear Algebra 2.ipynb

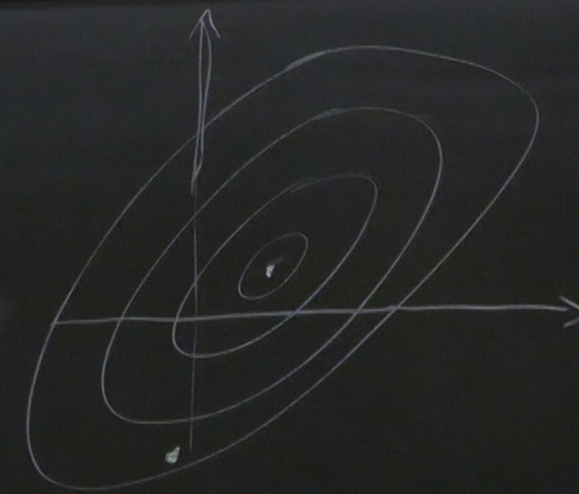

SPD symmetric positive definite
Ax = b
x* : true solution
x : approx. solution
e := x - x* error
r := b - Ax residual
e
r
x*
x
x*
x
x*
x

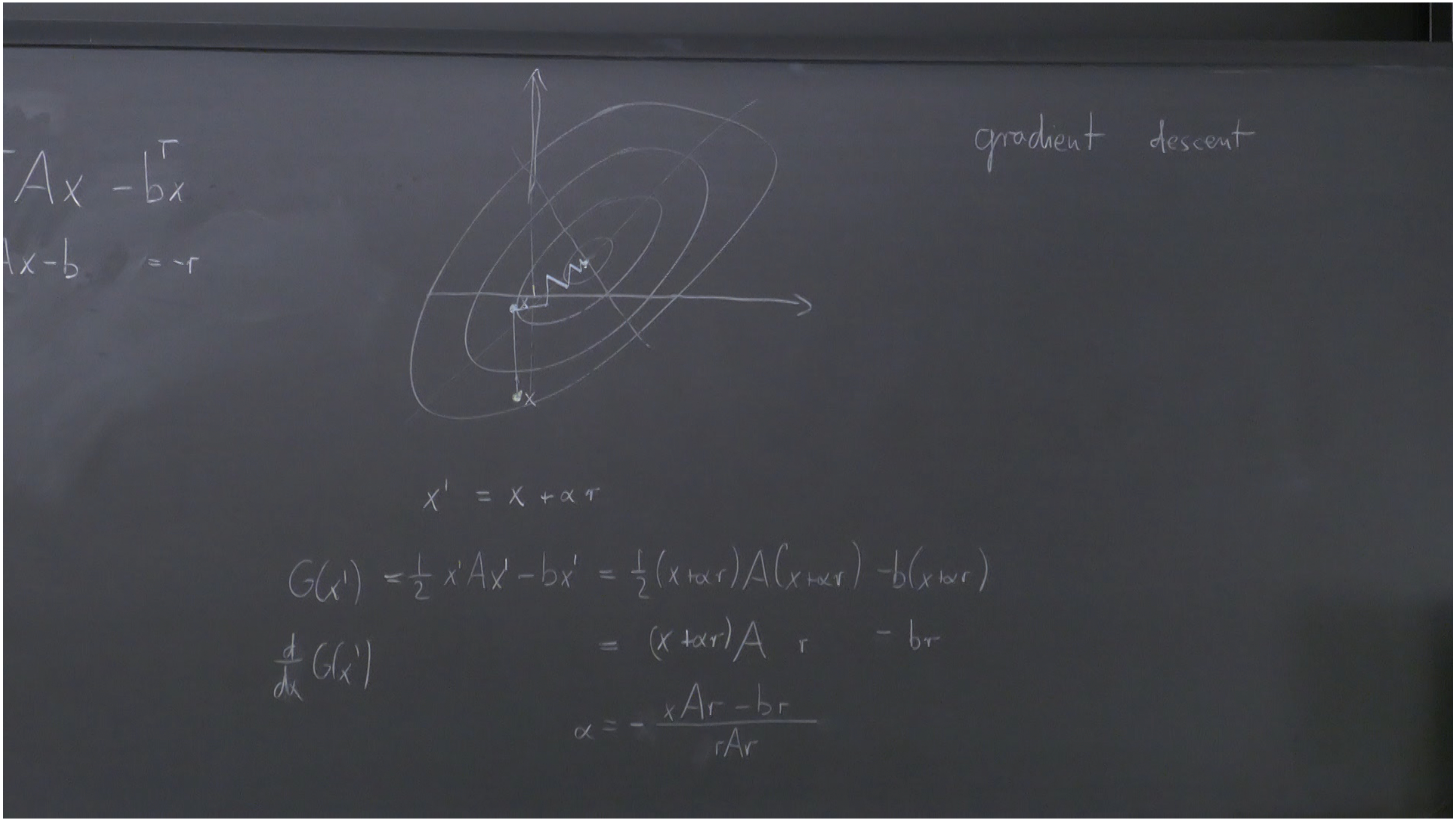

SPD
symmetric positive definite
Ax = b
x* : true solution
x : approx solution
e := x - x* error
r := b - Ax residual
e
r
x*
x
x*
x
x*x

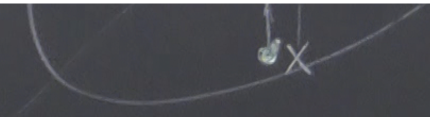
$Ax = b$

$$G := \frac{1}{2}x^T A x - b^T x$$

$$\nabla G = Ax - b = -r$$




gradient descent
Ax - bx
Ax-b = -r
x' = x + αr
G(x') = ½ x'Ax' - bx' = ½ (x+αr) A (x+αr) - b(x+αr)
= (x+αr) A r - br
d/dα G(x')
α = - (xAr - br)/(rAr)

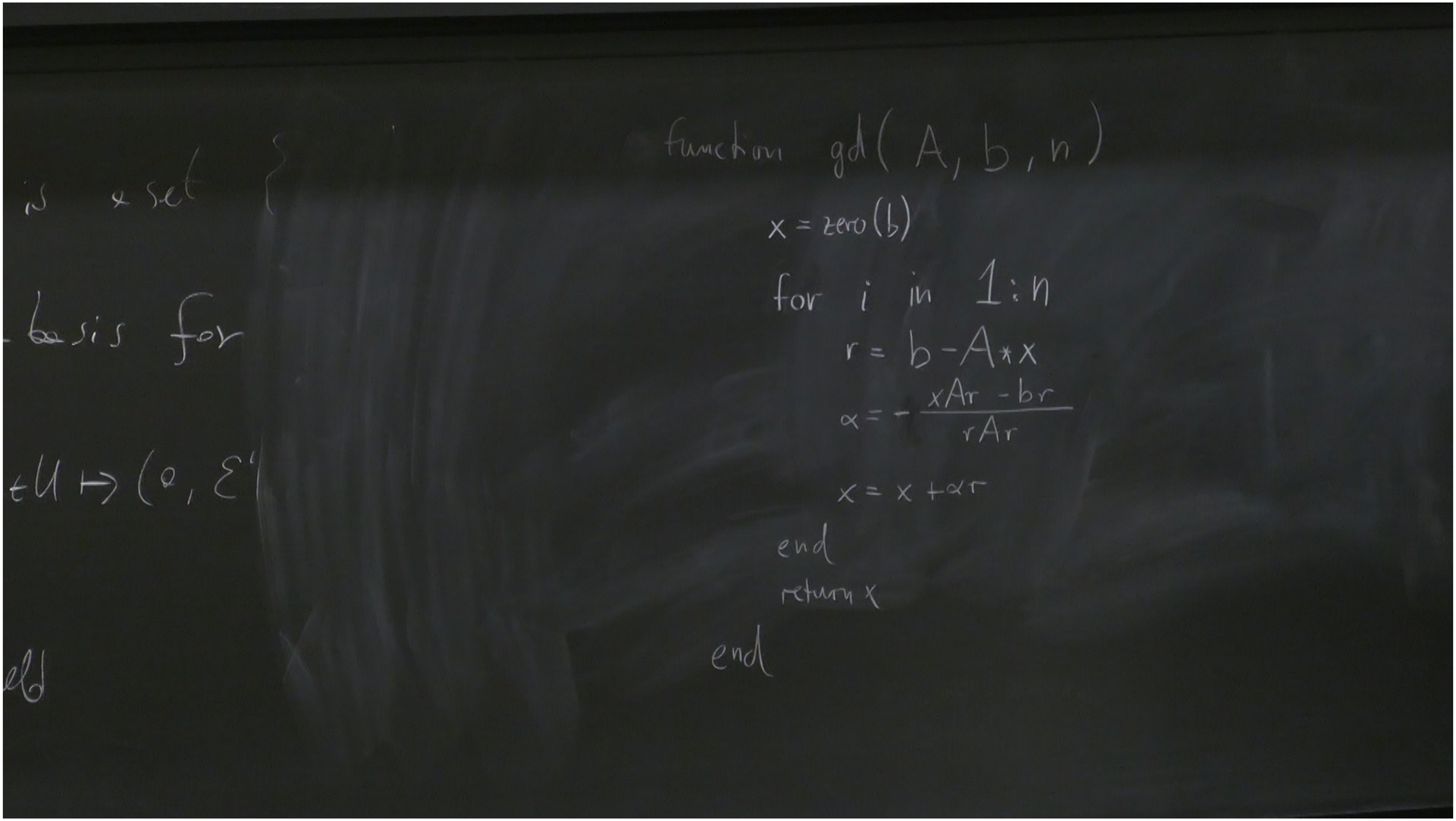


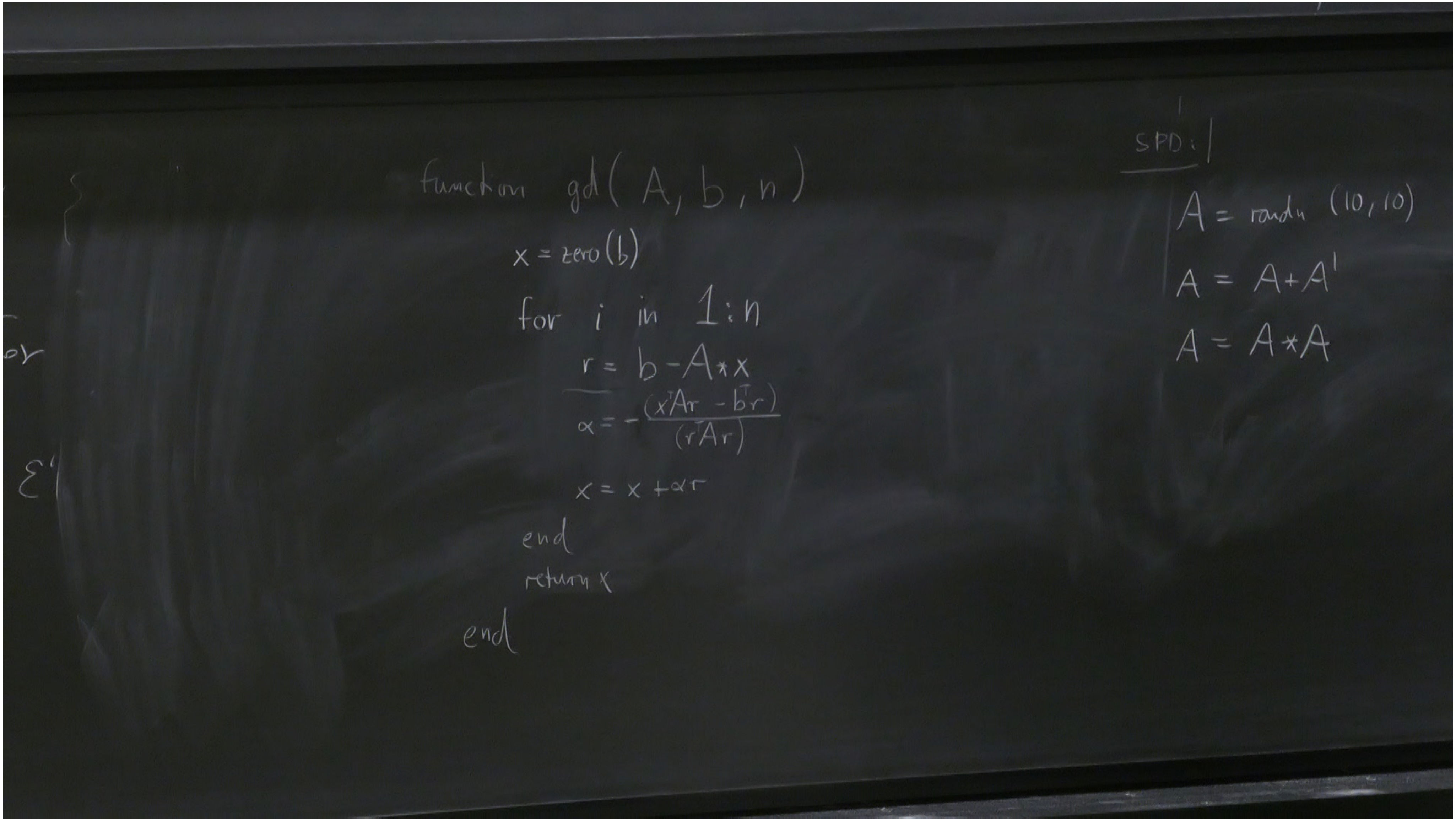
$$x' = x + \alpha r$$

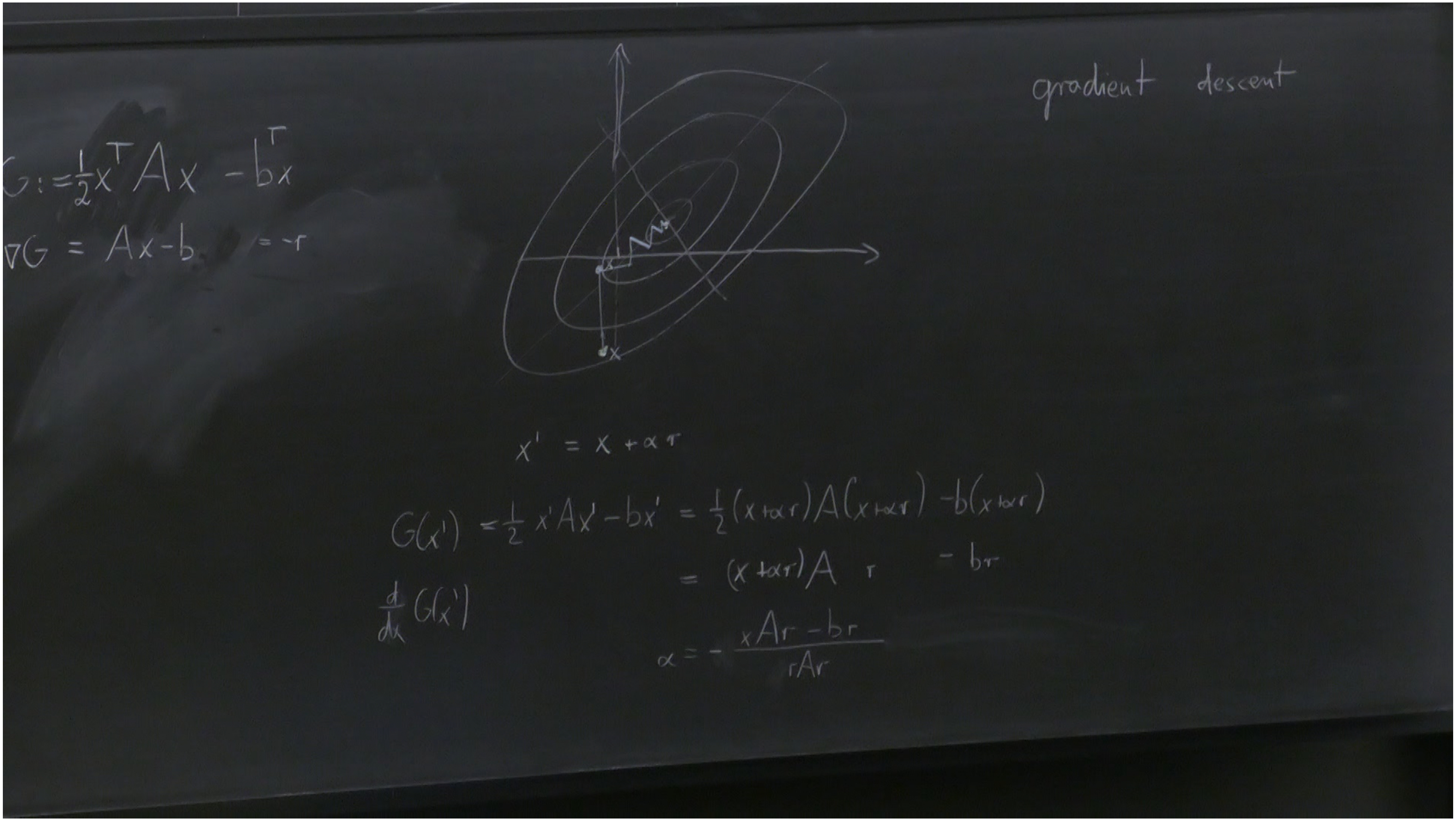
$$G(x') = \frac{1}{2} x' A x' - b x' = \frac{1}{2}(x + \alpha r) A (x + \alpha r) - b(x + \alpha r)$$

$$\frac{d}{dx} G(x') = (x + \alpha r) A \; r - br$$

$$\alpha = - \frac{xAr - br}{rAr}$$


is a set
basis for
end
function gd(A, b, n)
x = zero(b)
for i in 1:n
r = b - A*x
α = - (xAr - br) / (rAr)
x = x + αr
end
return x
end


function gd(A, b, n)
x = zero(b)
for i in 1:n
r = b - A*x
α = - (xᵀAr - bᵀr) / (rᵀAr)
x = x + αr
end
return x
end
SPD:
A = randn (10,10)
A = A+A'
A = A*A


gradient descent
G := 1/2 x^T A x - b^T x
∇G = Ax - b = -r
x' = x + α r
G(x') = 1/2 x'Ax' - bx' = 1/2 (x+αr) A (x+αr) - b(x+αr)
= (x+αr) A r - br
d/dα G(x')
α = - (xAr - br) / rAr