Title: Quantum Field Theory for Cosmology - Lecture 20240109

Speakers: Achim Kempf

Collection: Quantum Field Theory for Cosmology (PHYS785/AMATH872)

Date: January 09, 2024 - 4:00 PM

URL: https://pirsa.org/24010008

Mail - Achim Kempf - Outlook | [2312.10794] A mathematical p | Quantum Field Theory for Cosm | Sharing Desktop | Stop Sharing

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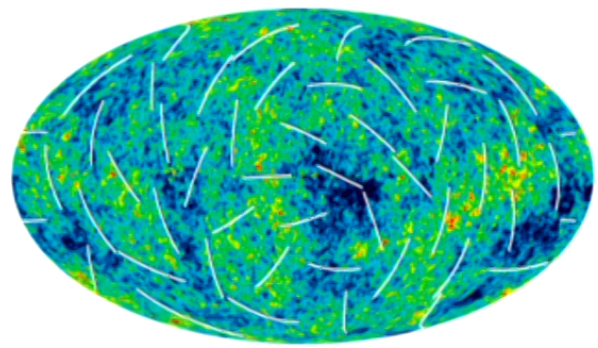
Quantum Field Theory for Cosmology, AMATH872/PHYS785, in W2024



- **Term:** Winter 2024.
- **Course codes:** AMATH872 / PHYS785
- **Instructor:** Achim Kempf
- **Prerequisite:** AMATH673 or PHYS702 or consent of instructor. Some knowledge of general relativity.
- **Time/venue:** Tuesdays and Thursdays 4-5:20pm in the **Alice room** at Perimeter Institute.
- **Also accessible on Zoom:** here
- **Reading week:** no lectures from Sat, Feb 17, 2024 to Sun Feb 25, 2024
- **Discussions/tutorial with prof:** after class
- **Office hours:** by arrangement.

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- The grades are currently planned to be based on a project, and essay and a presentation. Details TBA.

Health precautions (permanent)

- *If you have cold/flu/covid symptoms, do not come to class. Do the right thing, which is taking care of yourself and getting healthy first.*
    - For Lecture Videos (of my previous teaching), scroll down to the bottom of this page.
    - All important updates will always be posted here.

Lectures, lecture notes and videos:

In the list below, the videos are from my pre-pandemic teaching of this course. New recordings are planned for Winter 2024.

Jan 9 (Tue), Lecture 1: Notes, Old Video.

Historical introduction. The role of QFT in the standard models of particle physics and cosmology

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QFT for Cosmology, Achim Kempf, Lecture 1

Historical background:

≈ 1900:

Classical mechanics became experimentally untenable:

- Black body radiation ("Ultraviolet catastrophe")
- Photoelectric effect (Ionization depends on color, not intensity)
- Stability of matter ($\Delta x \Delta p \geq \frac{\hbar}{2}$ implies that e^- do not spiral into the nuclei)

Finally, in 1925:

Heisenberg discovers nonrelativistic quantum mechanics (QM)

In essence:

- Equations of motion stay the same, e.g.:

$$m\ddot{\hat{x}} = -k\hat{x} \quad \text{(harm. oscillator)}$$

- but we have noncommutativity:

$$[\hat{x}, \underbrace{m\dot{\hat{x}}}_{=\hat{p}}] = i\hbar \quad \text{"canonical commutation relation"}$$

o A few months later:

Schrödinger discovered his equation

Quantization implied fundamental changes:

Math: $[\hat{x}(t), \hat{p}(t)] = i\hbar 1 \neq 0 \Rightarrow \hat{x}(t), \hat{p}(t)$ not number-valued.

Q: Could $\hat{x}(t), \hat{p}(t)$ take values in finite dimensional matrices?

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Schrödinger discovered his equation

$$i\hbar \frac{d}{dt}\Psi(x,t) = -\frac{\hbar^2}{2m}\Delta\Psi(x,t) + V(x,t)\Psi(x,t)$$

o A few more months later:

Dirac showed equivalence to Heisenberg's.

$m\ddot{\hat{x}} = -k\hat{x}$ (harm. oscillator)

- but we have noncommutativity:

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Q: Could $\hat{x}(t), \hat{p}(t)$ take values in finite dimensional matrices?

A: No: If $\hat{x}(t), \hat{p}(t)$ were $N \times N$ matrices, then:

$$Tr([\hat{x},\hat{p}]) = Tr(i\hbar 1) \Rightarrow 0 = i\hbar N \quad ↯$$

$\Rightarrow$ $\hat{x}(t), \hat{p}(t)$ must not have well-defined trace, i.e., must act on ∞ dim. Hilbert space, i.e., must be operator-valued.

Physics: $\Delta x_i \Delta p_j \geq \frac{\hbar}{2}\delta_{ij}$

⇝ Uncertainty, i.e. "quantum fluctuations", are seen as being part of nature

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→ Uncertainty, i.e. "quantum fluctuations", are seen as being part of nature.

- But: Nonrelativistic quantum mechanics, i.e.,

$$[\hat{x}_i, \hat{p}_j] = i\hbar \delta_{ij} \quad \text{and} \quad i\hbar \frac{d}{dt} \hat{f}(\hat{x},\hat{p}) = [\hat{f}(\hat{x},\hat{p}), \hat{H}]$$

soon became unsatisfactory.

- Why? QM is not consistent with special relativity:

E.g. typical momentum of e^- in ground state of H-atom corresponds to $\approx 1\%$ of speed of light.

⇒ The effects of special relativity were soon spectroscopically measurable.

⇒ measurable contradiction to QM!

- Attempts to find a covariant generalization of the Schrödinger equation led to:
  - "Dirac Equation"
  - "Klein Gordon Equation" (see later)

- They had some success, but suffer serious problems too:
  - Energy not bounded from below ⇒ "instability"
  - Unitarity of time evolution unclear
  - Also: It remained unclear how particle creation and annihilation processes could be calculated.

- Thus, a new idea was needed!

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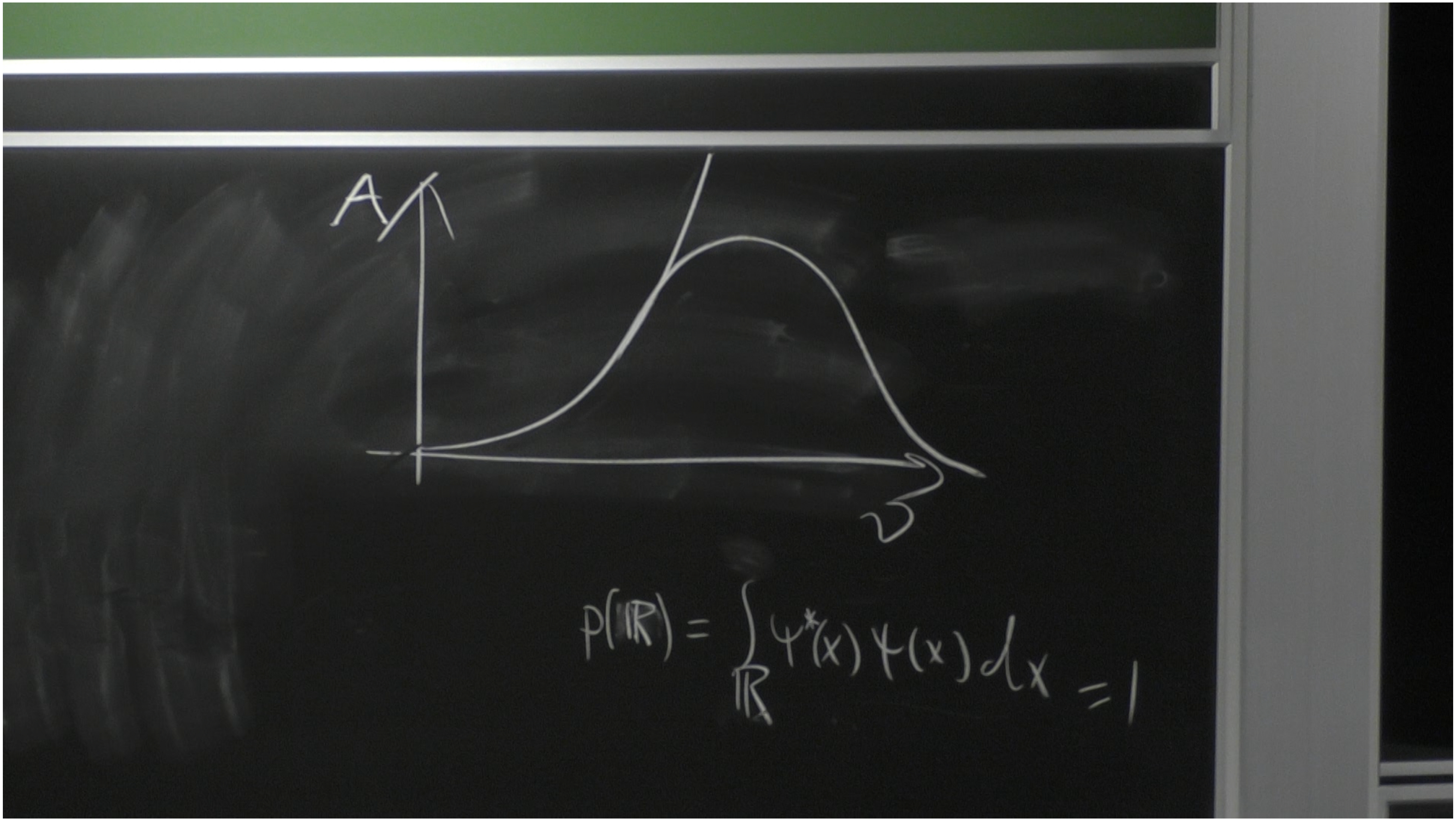
A
p(ℝ) = ∫ℝ ψ*(x) ψ(x) dx = 1

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□ **But:** Nonrelativistic quantum mechanics, i.e.,

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The idea of 2nd quantization: (Heisenberg and others, 1930s)

□ **Observation:**

In QM, all is subject to quantum fluctuations

□ **Idea:**

In 2nd quantization, quantize ψ!

□ **Program:**

Similar to $\hat{p} = \dot{\hat{x}}$ (in suitable units)

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As in classical theories, if the wave function's initial conditions are known, then the equation of motion (say the Schrödinger, Klein Gordon or Dirac equation) determines the evolution of $\Psi(x,t)$ without any uncertainty.

▫ Consequences:

Math:

→ $\hat{\Psi}(x,t)$ and $\hat{\pi}(x,t)$ can no longer be number-valued.

→ For each x and t the "value"

$$\hat{\Psi}(x,t)$$

is an operator on a Hilbert space!

Notice: (Recall: The eqns of motion stay the same also in 1st quantization)

The equations of motion (Schrödinger, Klein Gordon or Dirac equation) stay the same only now with $\hat{\Psi}, \hat{\pi}$ noncommutative.

Then, similar to $[\hat{x}_i, \hat{p}_j] = i\hbar\delta_{ij}$, require:

$$[\hat{\Psi}(x,t), \hat{\pi}(x',t)] = i\hbar\,\delta(x-x')$$

▫ Success!

Problems with energy positivity, unitarity etc can be solved.

Physics:

$$\Delta\Psi(x,t)\,\Delta\pi(x',t) \geq \frac{\hbar}{2}\,\delta^3(x-x')$$

(we'll need to discuss that) ($x = (x_1, x_2, x_3)$)

⇒ The "wave function" is now subject to quantum fluctuations and uncertainty!

⇒ New phenoma now predicted and described:

1.) Regarding particles:

Particle creation/annihilation ← (E.g. norm of wave fctn i.e. particle number no longer fixed)

Existence of antiparticles ← (the negative energy (or mass) states can be interpreted as particles propagating backwards in time, thus to us appearing to have positive energy (or mass).)

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1.) **Regarding particles:**

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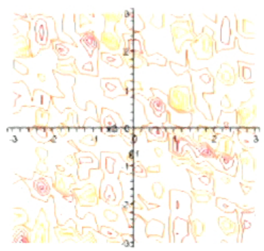
2.) **Regarding fields:**

Even in the lowest energy state (i.e. no particles, i.e., in the Vacuum, the statement

$$\overline{\varphi}(x,t) = \langle vacuum | \hat{\varphi}(x,t) | vacuum \rangle = 0$$

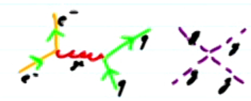
allows for the values of $\hat{\varphi}(x,t)$ when measured, to fluctuate:



⇝ 2 main uses of quantum field theory:

1) The Standard Model of Particle Physics

* EM, weak and strong forces



* Screening, anti-screening and renormalization

* How fundamentally massless particles can effectively acquire a mass: "Spontaneous symmetry breaking"
  Namely: Groundstate has less symmetry than the action: "Higgs" particle.

* Anomalies: Quantum fluctuations reduce symmetry of the action itself.

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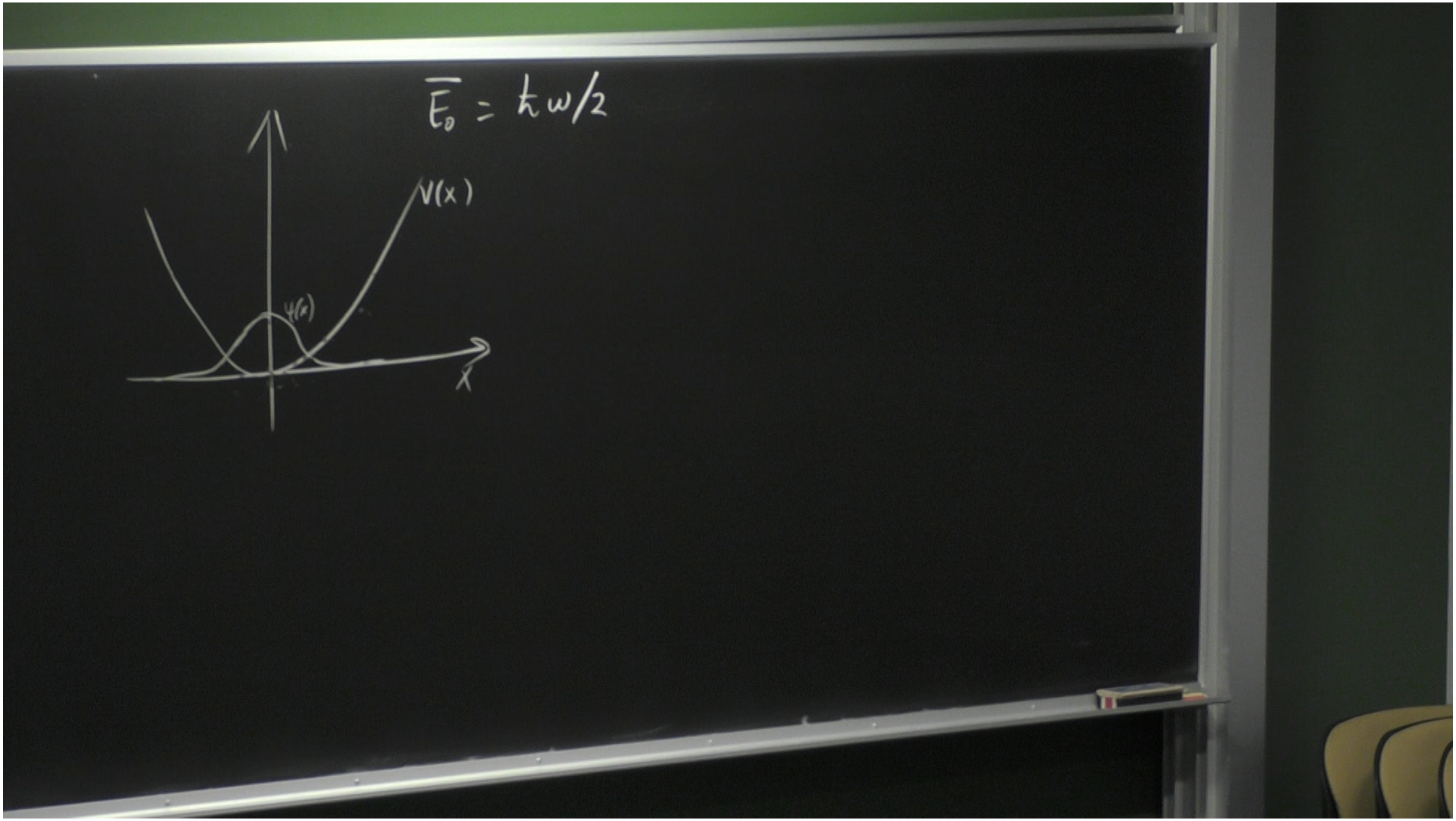
$\overline{E_0} = \hbar\omega/2$
V(x)
ψ(x)
x

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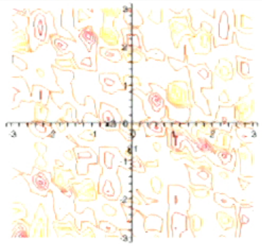

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  Namely: Ground state has less symmetry than the action:
  "Higgs" particle.

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2) The Standard Model of Cosmology

(the aim of this course)

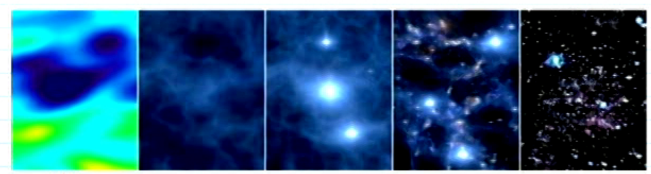
Classical General Relativity + QFT
  ↳ Mostly

i.e.: Accelerations, curvature, horizons + QFT

* Unruh Effect: What is a "particle"?
* Hawking Effect: Can nature destroy information?
* Cosmic Inflation: Where did it all come from?

Cosmic Inflation:

- A local quantum fluctuation of high potential $V(\phi)$ may occur.
- Acting as temporary cosm. constant, may spawn a rapidly-expanding daughter universe.
- Finally, $V(\phi) \to 0$, energy goes into particle production: plasma
- Rapid expansion amplified quantum field fluctuations.
- These fluctuations imprinted on primordial plasma, seeding galaxy formation:



↑ Our target

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