Title: Advanced General Relativity - 240124

Speakers: Eric Poisson

Collection: Advanced General Relativity (PHYS7840)

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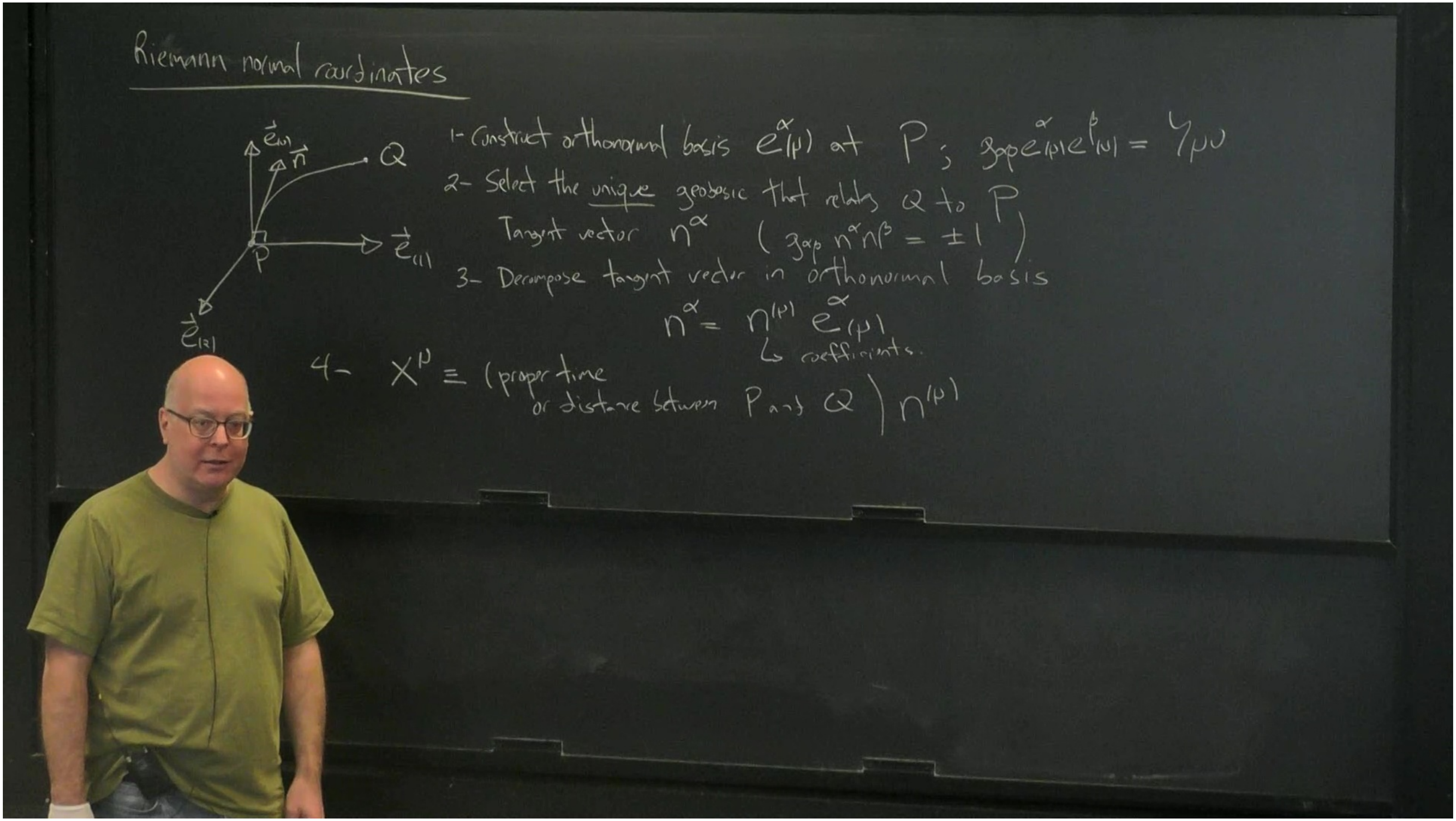

Riemann normal coordinates
$\vec{e}_{(0)}$
$\vec{n}$
Q
P
$\vec{e}_{(1)}$
$\vec{e}_{(2)}$
1- Construct orthonormal basis $e^{\alpha}_{(\mu)}$ at P ; $g_{\alpha\beta} e^{\alpha}_{(\mu)} e^{\beta}_{(\nu)} = \eta_{\mu\nu}$
2- Select the unique geodesic that relates Q to P
Tangent vector n^{α} ($g_{\alpha\beta} n^{\alpha} n^{\beta} = \pm 1$)
3- Decompose tangent vector in orthonormal basis
$n^{\alpha} = n^{(\mu)} e^{\alpha}_{(\mu)}$
↳ coefficients.
4- $X^{\mu} \equiv$ (proper time or distance between P and Q) $n^{(\mu)}$

$\vec{e}_{(2)}$

3- Decompose tangent vector in orthonormal basis

$$n^{\alpha} = n^{(\mu)} e^{\alpha}_{(\mu)}$$

↳ coefficients.

4- $X^{\mu}_{Q} \equiv \left(\text{proper time or distance between } P \text{ and } Q\right) n^{(\mu)}$

$$\longrightarrow \quad g_{\mu\nu} = \eta_{\mu\nu} - \frac{1}{3} \underbrace{R_{\mu\lambda\nu\rho}(P)}_{1/R^2} \underbrace{X^{\lambda} X^{\rho}}_{\sim s^2} + O(s^3/R^3)$$

$$\begin{cases} g_{\mu\nu}(P) = \eta_{\mu\nu} \\ \partial_{\lambda} g_{\mu\nu}(P) = 0 \longrightarrow \Gamma^{\lambda}_{\mu\nu}(P) = 0 \end{cases}$$

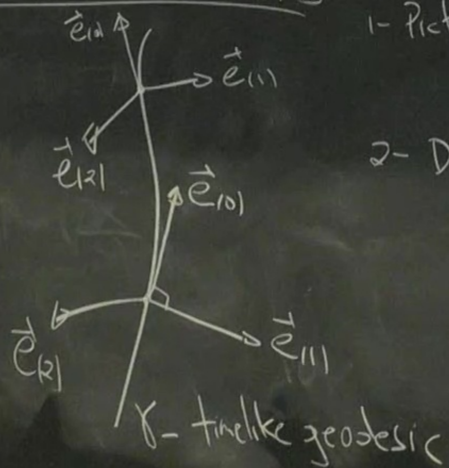
3- Decompose tangent vector in orthonormal basis

$$n^\alpha = n^{(\mu)} e^\alpha_{(\mu)}$$

↳ coefficients.

4- $x^\mu_Q \equiv$ (proper time or distance between P and Q) $n^{(\mu)}$

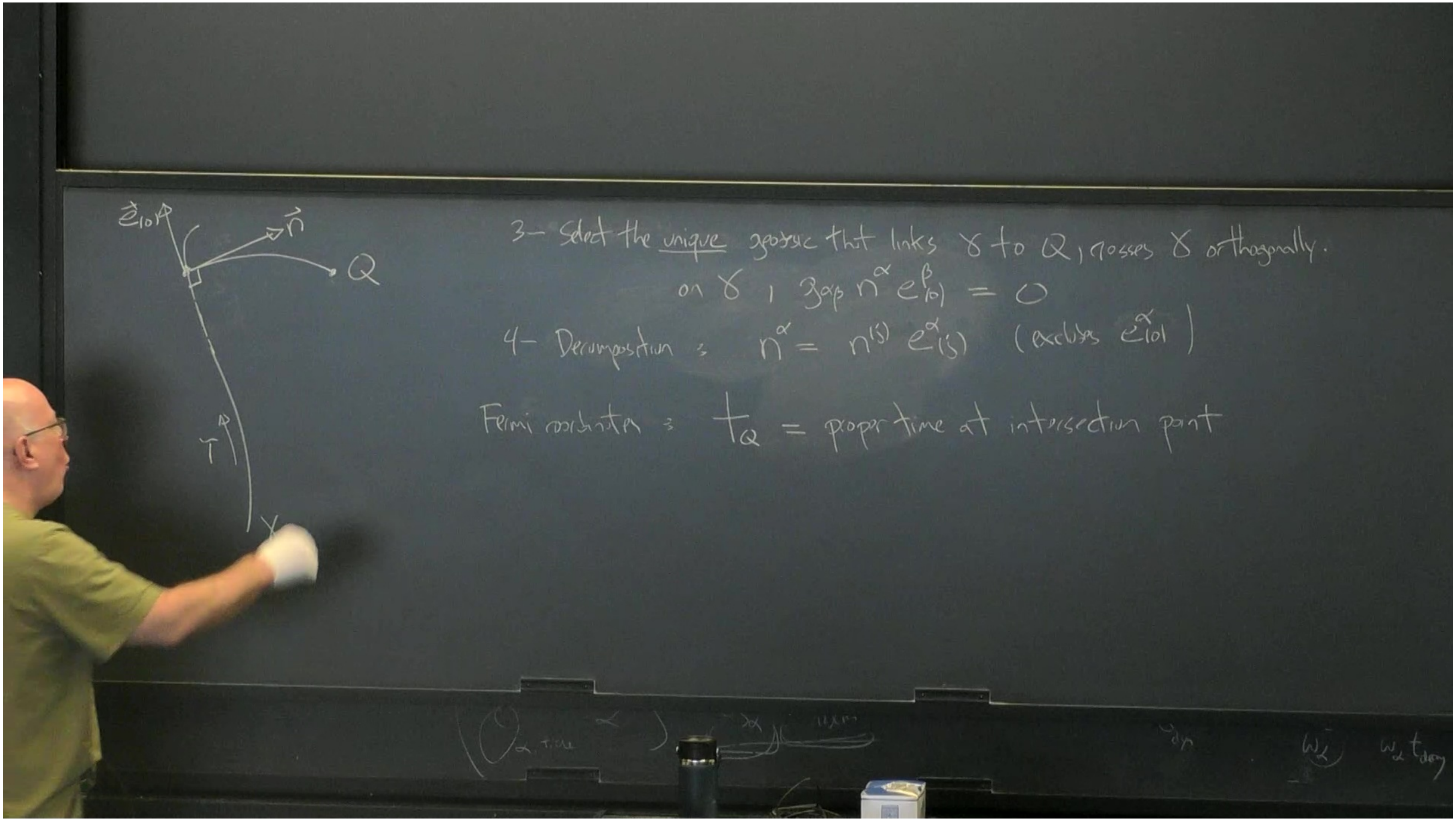
Fermi normal coordinates



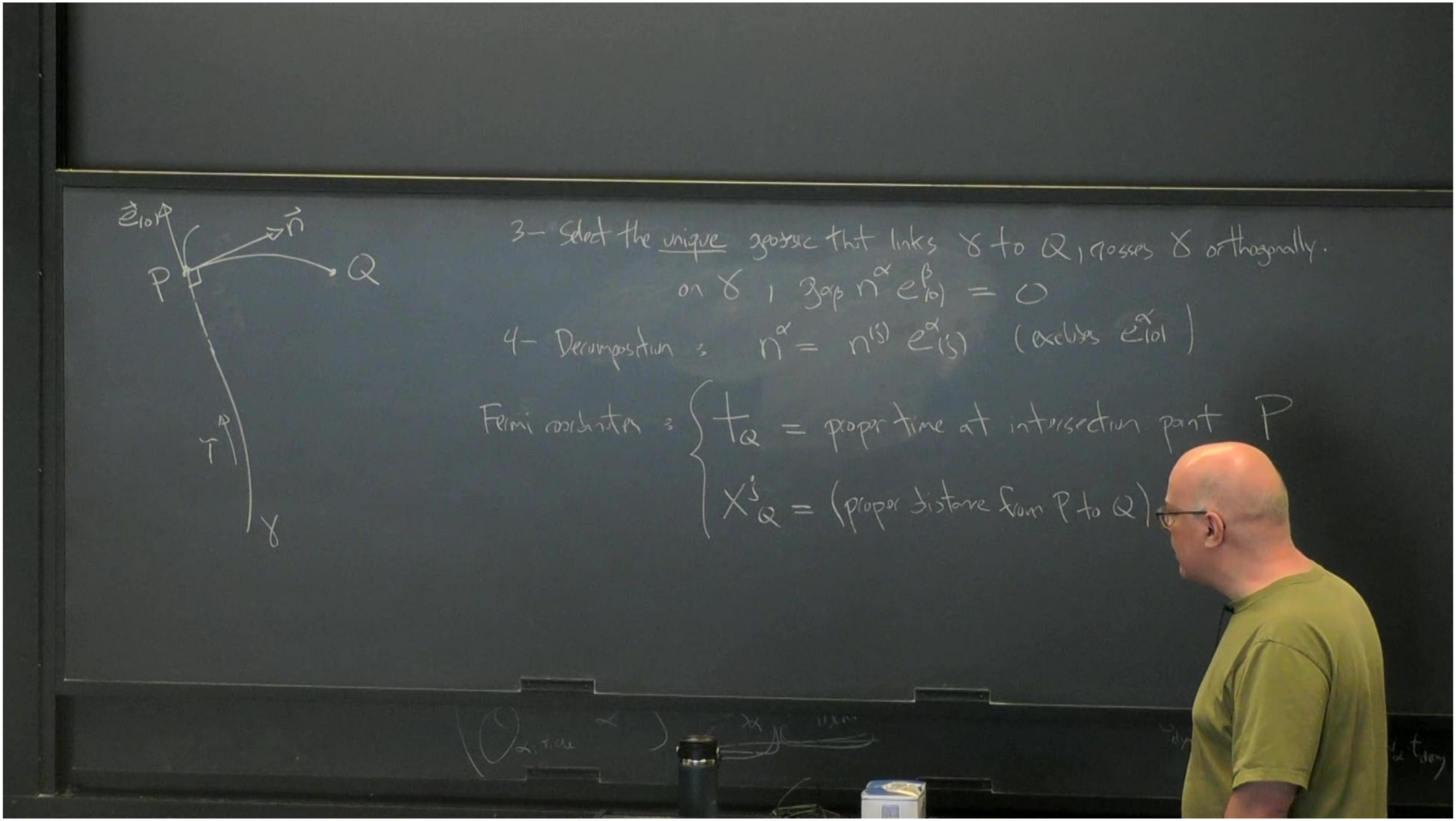
1- Pick a point on γ, construct orthonormal basis, with $e^\alpha_{(0)} \equiv u^\alpha$

$$g_{\alpha\beta} e^\alpha_{(\mu)} e^\beta_{(\nu)} = \eta_{\mu\nu}$$

2- Define $e^\alpha_{(\mu)}$ at any point on γ by parallel transport



$\vec{e}_{(0)}$
$\vec{n}$
Q
τ
γ
3— Select the unique geodesic that links γ to Q, crosses γ orthogonally.
on γ, $g_{\alpha\beta}\, n^\alpha e^\beta_{(0)} = 0$
4— Decomposition: $n^\alpha = n^{(s)} e^\alpha_{(s)}$ (excludes $e^\alpha_{(0)}$)
Fermi coordinates: t_Q = proper time at intersection point



$\vec{e}_{(0)}$
$\vec{n}$
P
Q
τ
γ
3– Select the unique geodesic that links γ to Q, crosses γ orthogonally.
on γ, $g_{\alpha\beta} n^\alpha e^\beta_{(0)} = 0$
4– Decomposition : $n^\alpha = n^{(j)} e^\alpha_{(j)}$ (excludes $e^\alpha_{(0)}$)
Fermi coordinates :
t_Q = proper time at intersection point P
x^j_Q = (proper distance from P to Q)

$$X^j_Q = (\text{proper distance from } P \text{ to } Q)\, n^{(j)}$$

$$\boxed{\begin{aligned} g_{tt} &= -1 - R_{tptq}(t)\, x^p x^q + O(s^3/\mathcal{R}^3) \\ g_{tj} &= \tfrac{2}{3} R_{jpqt}(t)\, x^p x^q + O(s^3/\mathcal{R}^3) \\ g_{jk} &= \delta_{jk} - \tfrac{1}{3} R_{jpkq}\, x^p x^q + O(s^3/\mathcal{R}^3) \end{aligned}}$$

$$\boxed{\begin{aligned} g_{\mu\nu}(\gamma) &= \eta_{\mu\nu} \\ \Gamma^\lambda_{\mu\nu}(\gamma) &= 0 \end{aligned}}$$

γ

$X^j_Q = (\text{proper distance from } P \text{ to } Q)\, n^{(j)}$

$$\boxed{\begin{aligned} g_{tt} &= -1 - R_{tptq}(t)\, x^p x^q + O(s^3/\mathcal{R}^3) \\ g_{tj} &= \tfrac{2}{3} R_{jpqt}(t)\, x^p x^q + O(s^3/\mathcal{R}^3) \\ g_{jk} &= \delta_{jk} - \tfrac{1}{3} R_{jpkq}\, x^p x^q + O(s^3/\mathcal{R}^3) \end{aligned}}$$

$$\boxed{\begin{aligned} g_{\mu\nu}(\gamma) &= \eta_{\mu\nu} \\ \Gamma^\lambda_{\mu\nu}(\gamma) &= 0 \end{aligned}}$$

TIMELIKE

U^α

$\nabla_\beta U_\alpha$ 16

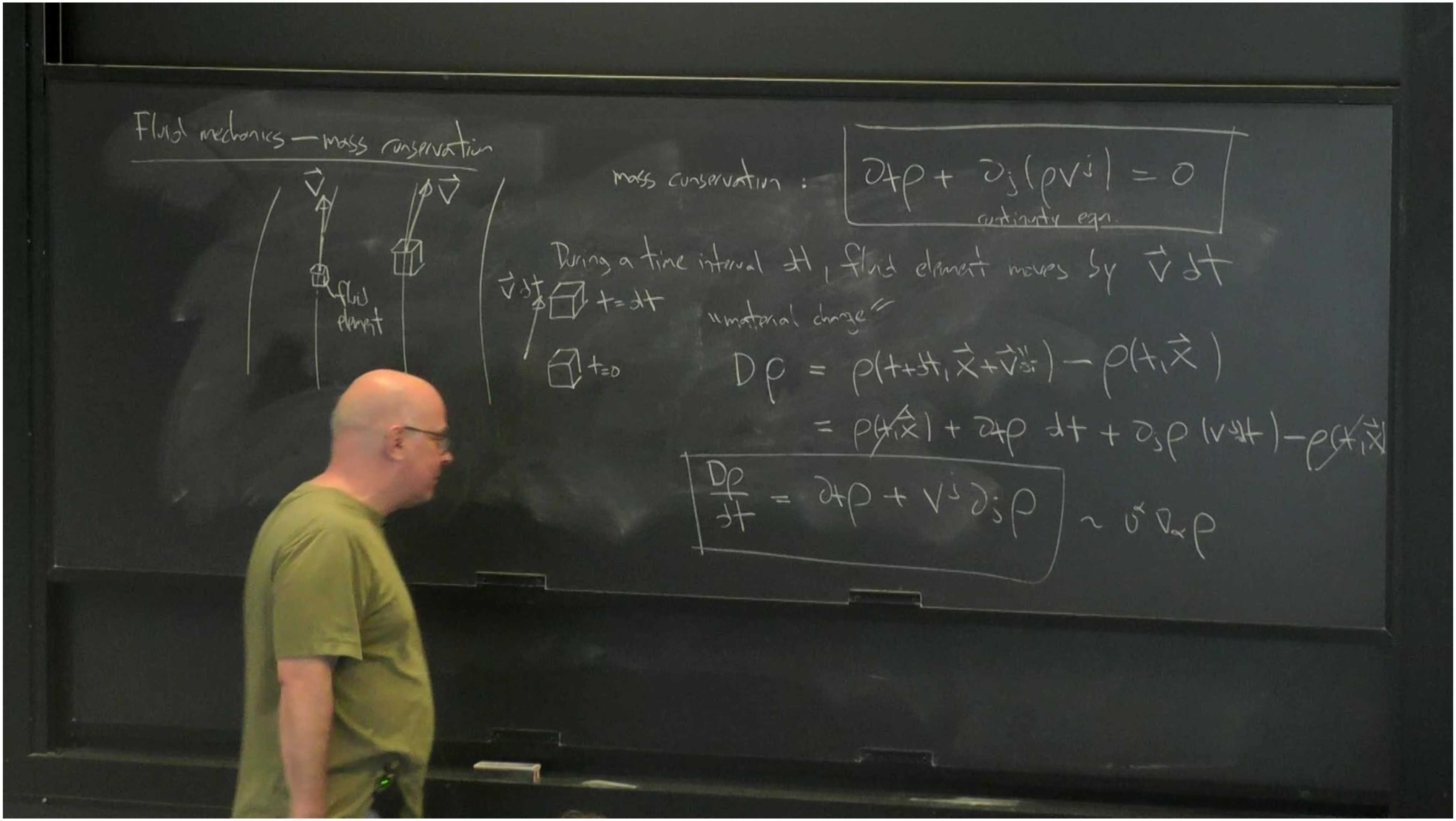
NEWTONIAN

$\underset{3}{\partial_j} V_k^{\sim 3} \sim 9$

k = tangent to null geodesics

$k_\alpha k^\alpha = 0$

relative behaviour $\underset{4}{\nabla_\beta} k_\alpha^{\;4} \sim 16$


Fluid mechanics — mass conservation
$\vec v$
$\vec v$
fluid element
mass conservation : $\boxed{\partial_t \rho + \partial_j(\rho v^j) = 0}$
continuity eqn.
During a time interval dt, fluid element moves by $\vec v\, dt$
$\vec v\, dt$
$t = dt$
$t = 0$
"material change"
$D\rho = \rho(t+dt, \vec x + \vec v\, dt) - \rho(t, \vec x)$
$= \cancel{\rho(t, \vec x)} + \partial_t \rho\, dt + \partial_j \rho\, (v^j dt) - \cancel{\rho(t, \vec x)}$
$\boxed{\frac{D\rho}{dt} = \partial_t \rho + v^j \partial_j \rho}$ ~ $v^\alpha \nabla_\alpha \rho$

$t=0$

$$D\rho = \rho(t+\delta t, \vec{x}+\vec{v}\,\delta t) - \rho(t, x)$$

$$= \cancel{\rho(t,\vec{x})} + \partial_t\rho\, dt + \partial_j\rho\,(v^j dt) - \cancel{\rho(t,\vec{x})}$$

$$\boxed{\frac{D\rho}{dt} = \partial_t\rho + v^j\partial_j\rho} \sim v^\alpha \nabla_\alpha \rho$$

$$0 = \partial_t\rho + v^j\partial_j\rho + \rho\,\partial_j v^j = \frac{D\rho}{dt} + \rho\,\vec{\nabla}\cdot\vec{v} = 0$$

$$\boxed{-\frac{1}{\rho}\frac{D\rho}{dt} = \vec{\nabla}\cdot\vec{v}} \to \text{fractional rate of change of density}$$

$$\rho = \frac{m}{V}$$

$$\frac{D\rho}{dt} = -\frac{m}{V^2}\frac{dV}{dt}$$

$$= -\frac{\rho}{V}\frac{dV}{dt} \longrightarrow \boxed{\frac{1}{V}\frac{DV}{dt} = \vec{\nabla}\cdot\vec{v}} \to \text{fractional rate of change of volumes}$$