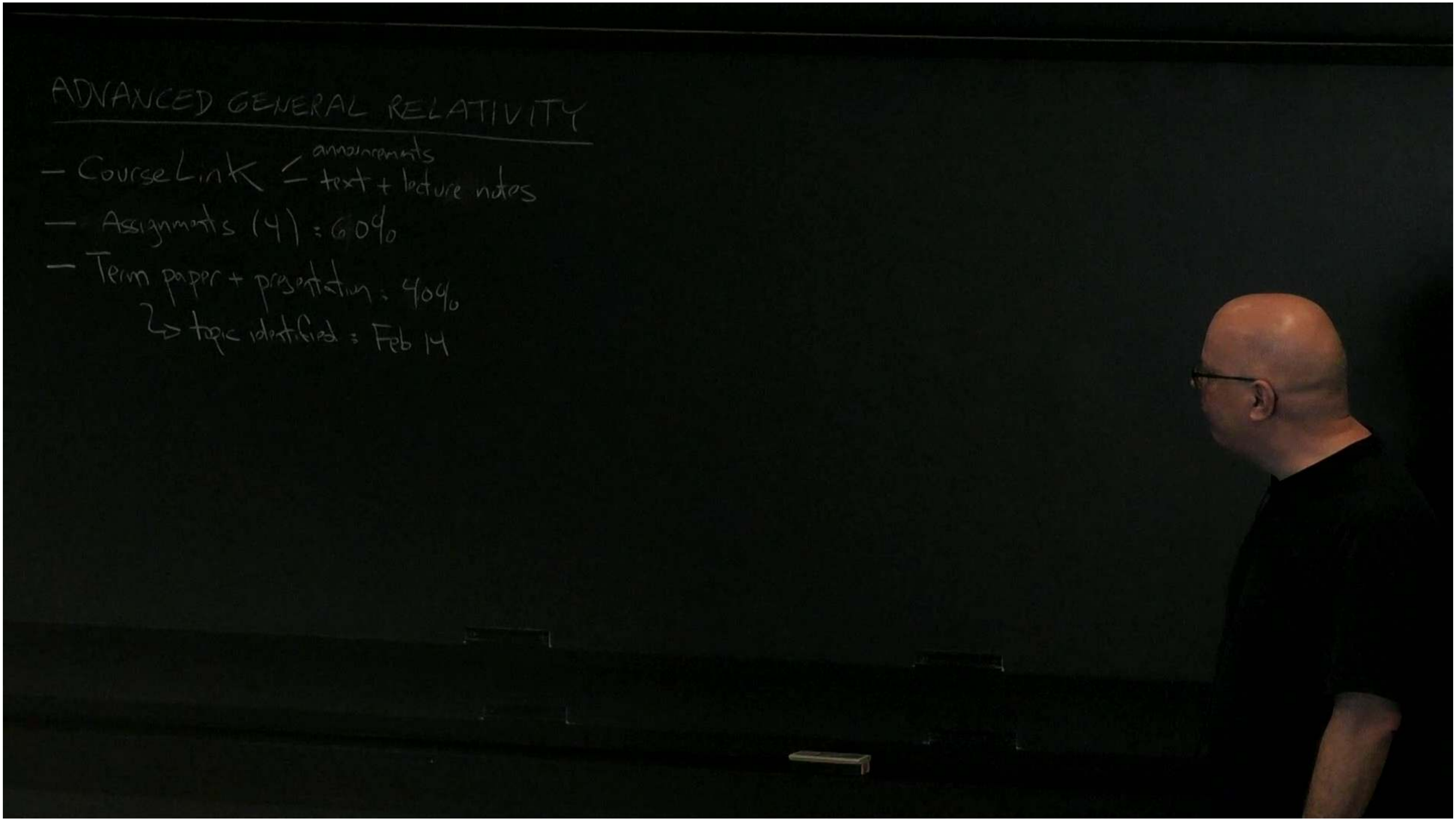
Title: Advanced General Relativity - 240110

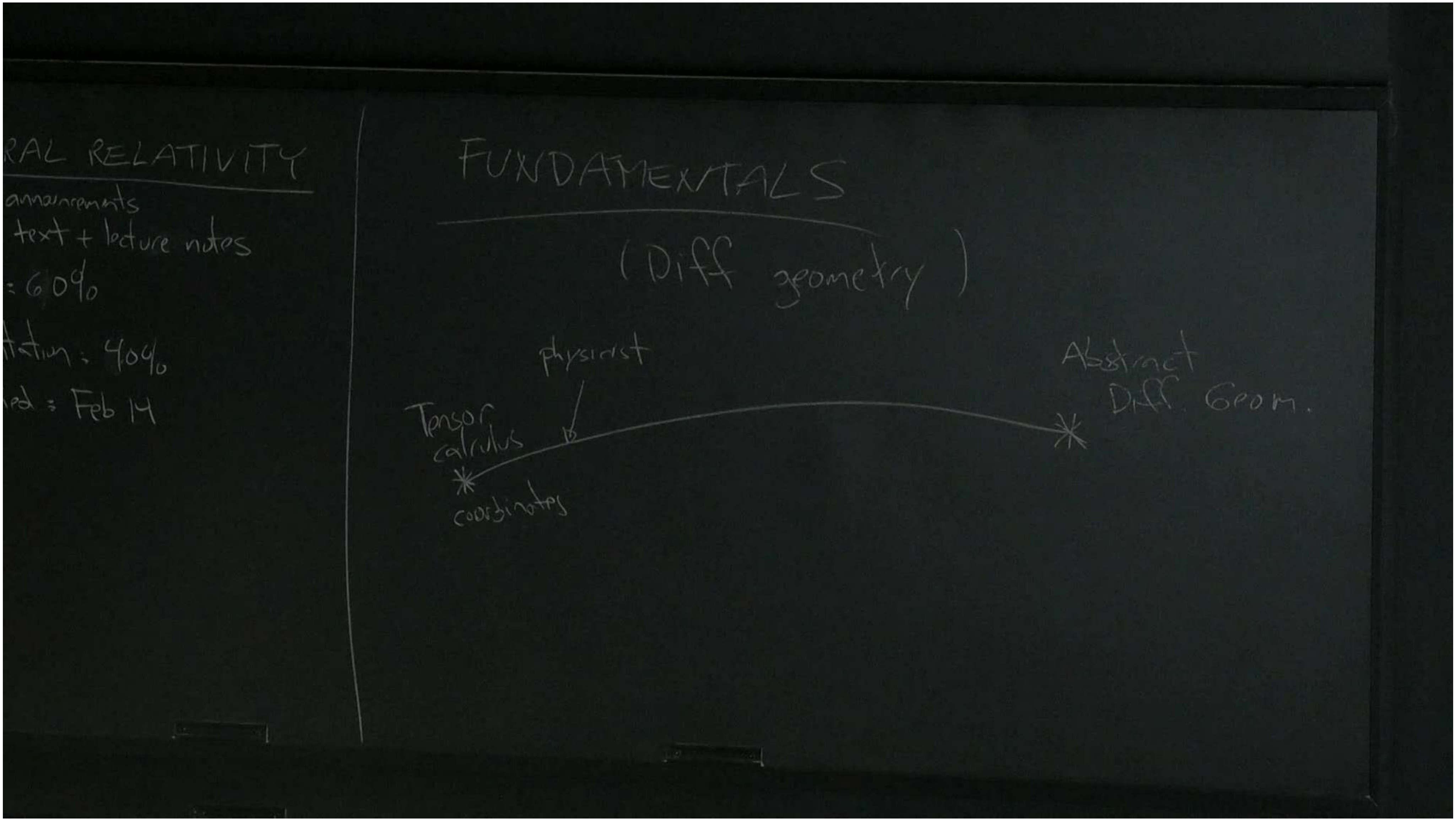
Speakers: Eric Poisson

Collection: Advanced General Relativity (PHYS7840)

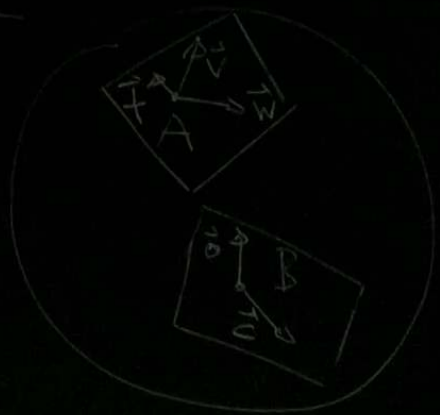
Date: January 10, 2024 - 10:30 AM

URL: https://pirsa.org/24010000


ADVANCED GENERAL RELATIVITY
- CourseLink < announcements
text + lecture notes
- Assignments (4) : 60%
- Term paper + presentation : 40%
↳ topic identified : Feb 14

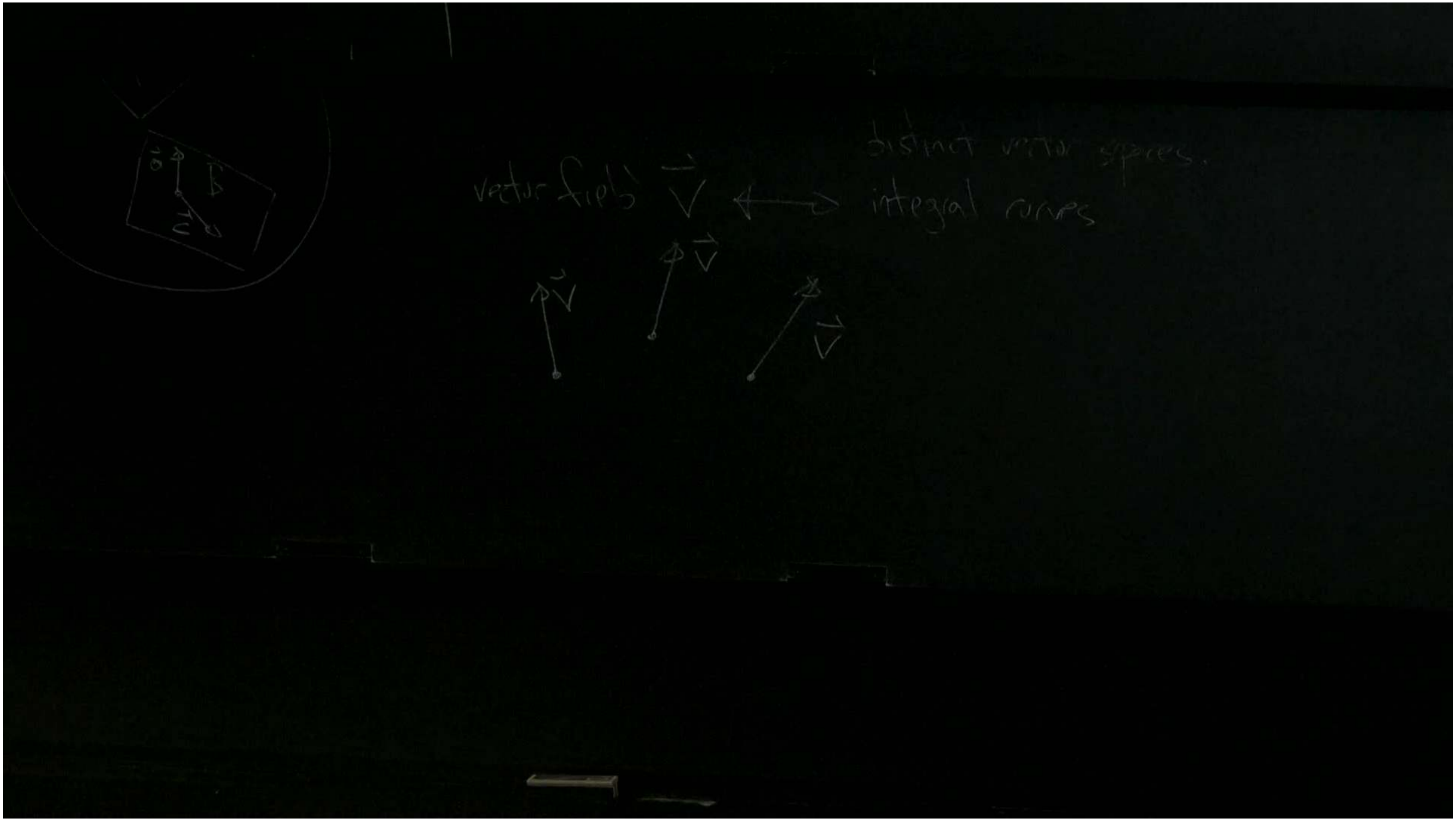

RAL RELATIVITY
annaincements
text + lecture notes
= 60%
tation : 40%
ed : Feb 14
FUNDAMENTALS
(Diff geometry)
physicist
Tensor calculus
coordinates
Abstract Diff. Geom.

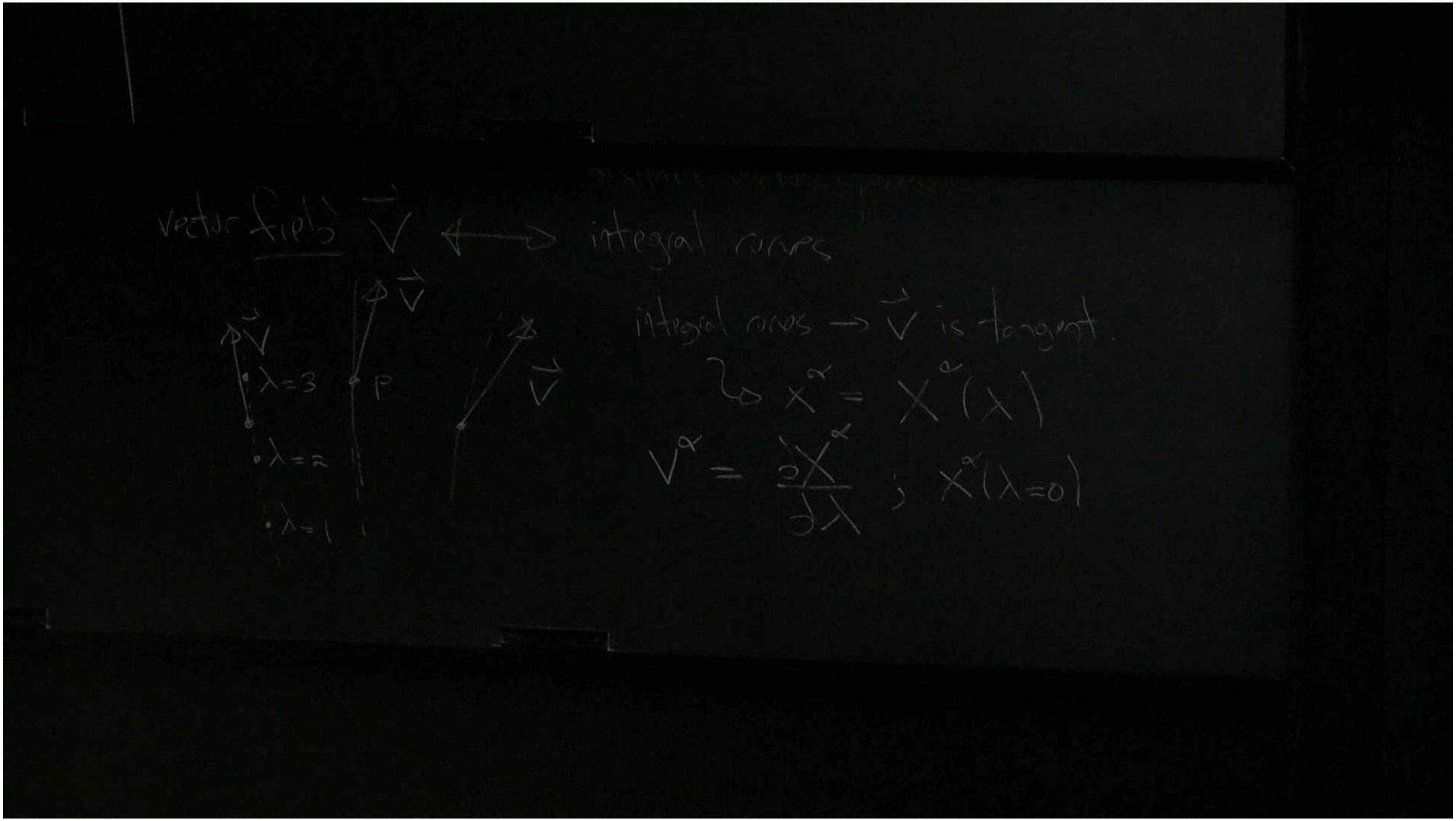
Vectors

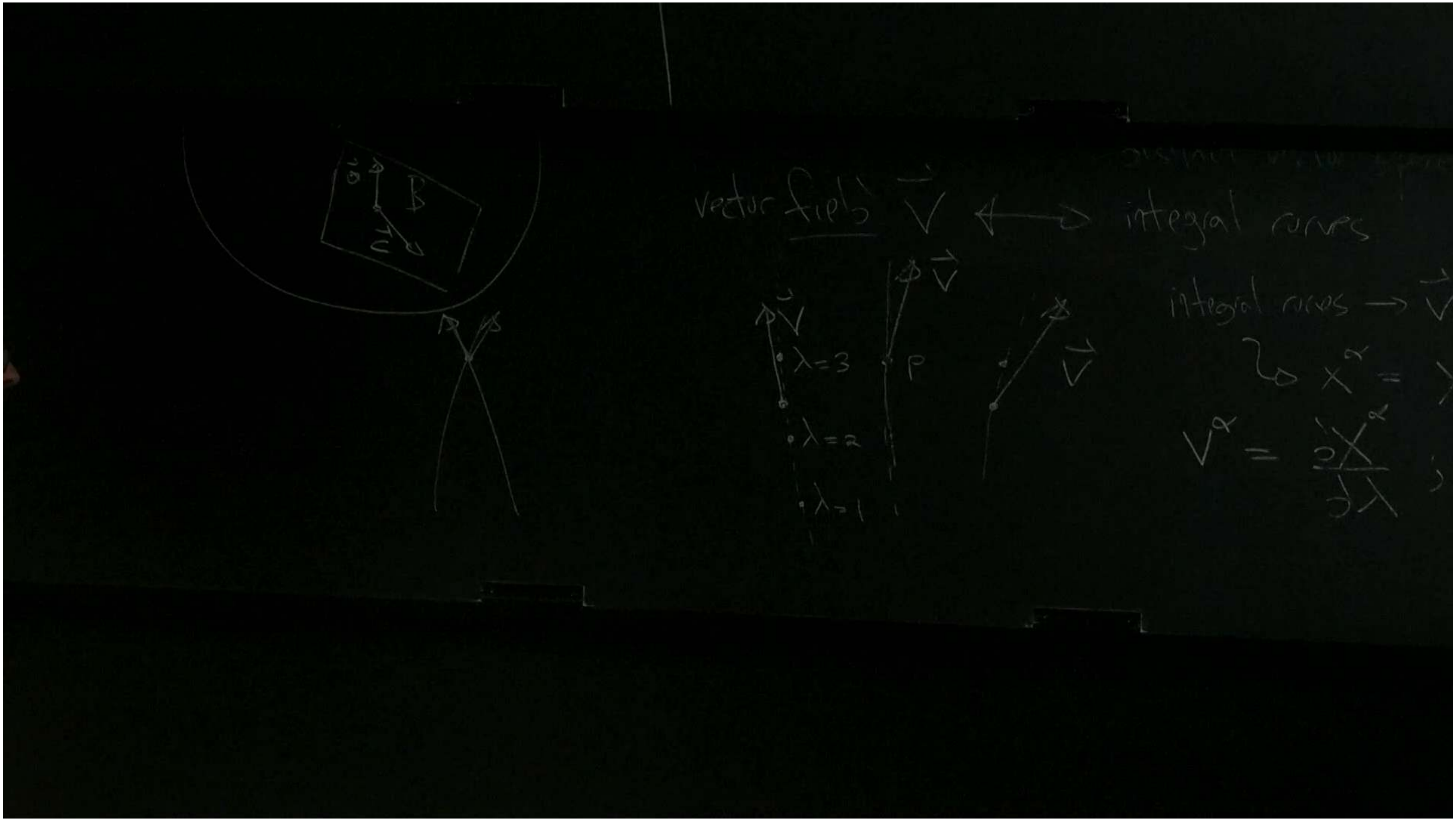


vectors live at T_A = tangent space attached to A

vectors at different points live in $\underbrace{\text{distinct tangent spaces}}_{\text{distinct vector spaces.}}$


distinct vector spaces.
vector fields
integral curves

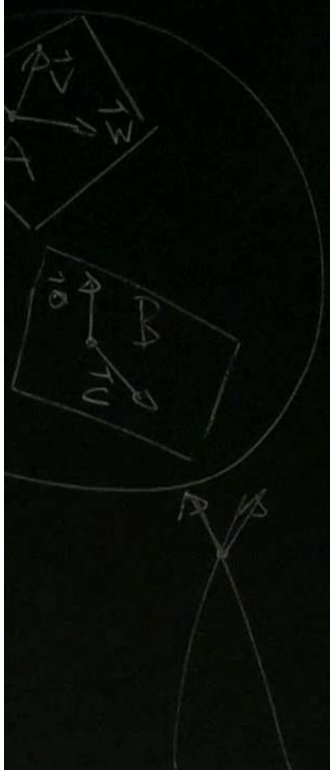

vector fields $\vec{V} \longleftrightarrow$ integral curves
integral curves $\rightarrow$ $\vec{V}$ is tangent.
$x^\alpha = X^\alpha(\lambda)$
$V^\alpha = \frac{\partial X^\alpha}{\partial \lambda}$; $X^\alpha(\lambda=0)$
$\vec{V}$
$\lambda=3$
$\lambda=2$
$\lambda=1$
P


vector field V ⟷ integral curves
integral curves → V
V^α = ∂X^α/∂λ
λ=3
λ=2
λ=1
p
B

vectors live at $T_A \equiv$ tangent space attached to A

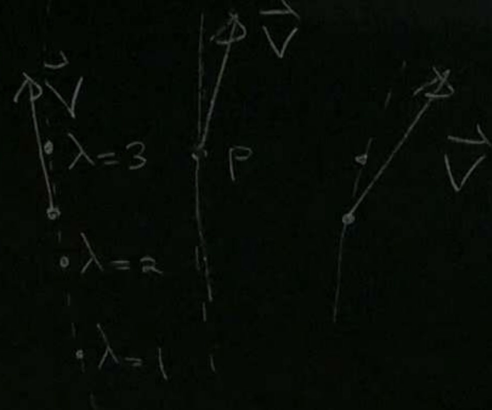
vectors at different points live in distinct tangent spaces

distinct vector spaces.



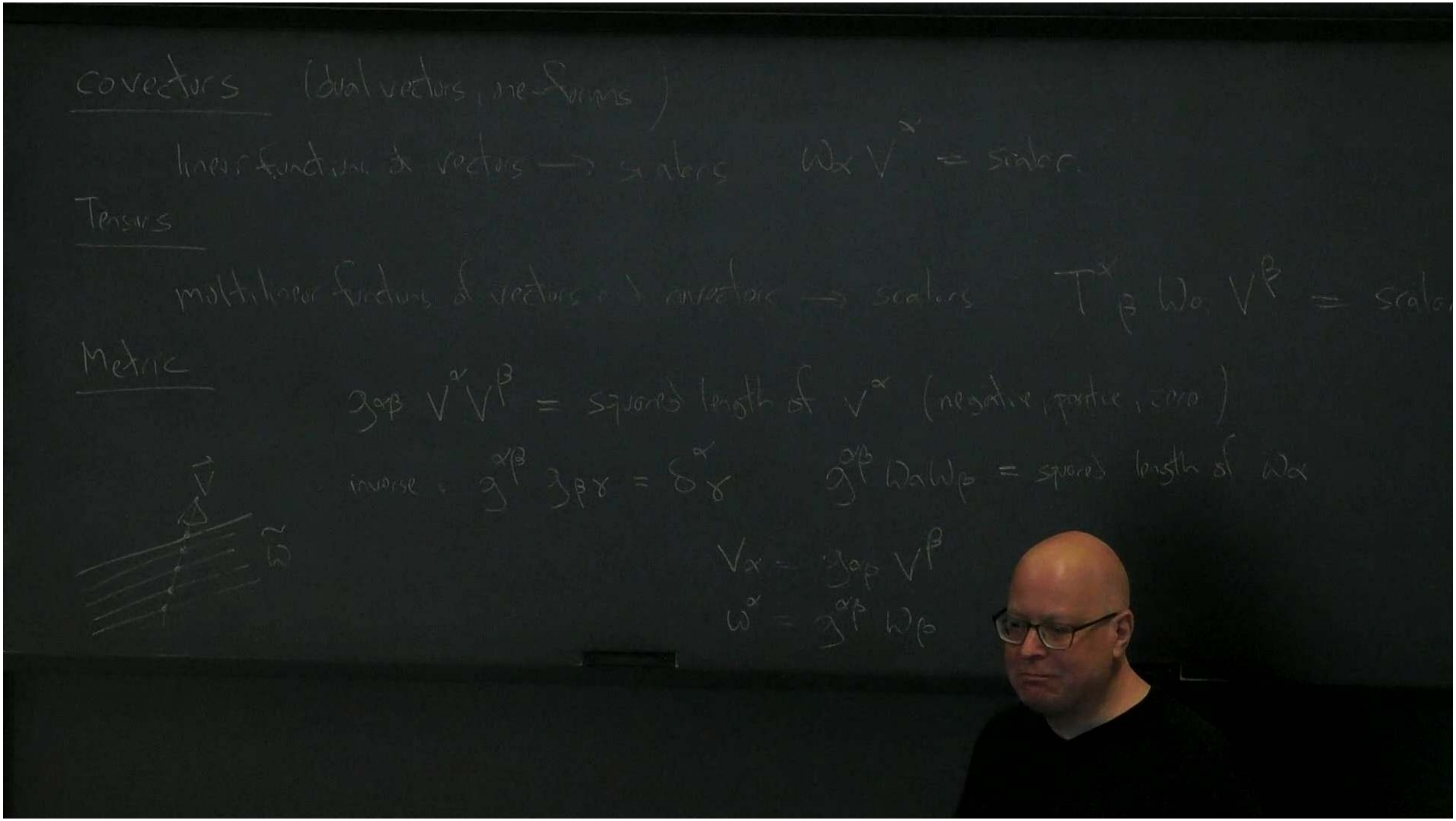
vector fields $\vec{V} \longleftrightarrow$ integral curves

integral curves $\rightarrow \vec{V}$ is tangent.



$$x^\alpha = X^\alpha(\lambda)$$

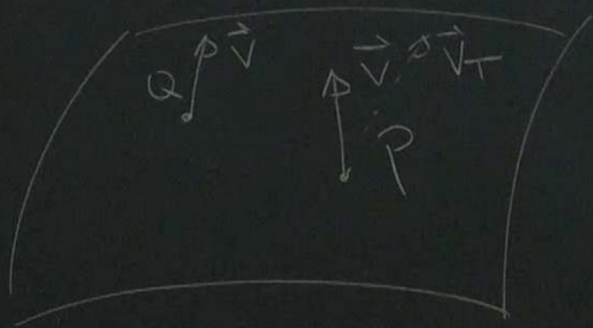
$$V^\alpha = \frac{\partial X^\alpha}{\partial \lambda} \; ; \; X^\alpha(\lambda=0)$$


covectors (dual vectors, one-forms)
linear functions of vectors → scalars $\omega_\alpha V^\alpha$ = scalar.
Tensors
multilinear functions of vectors and covectors → scalars $T^\alpha{}_\beta\, \omega_\alpha V^\beta$ = scalar
Metric
$g_{\alpha\beta} V^\alpha V^\beta$ = squared length of V^α (negative, positive, zero)
inverse: $g^{\alpha\beta} g_{\beta\gamma} = \delta^\alpha{}_\gamma$
$g^{\alpha\beta}\omega_\alpha\omega_\beta$ = squared length of ω_α
$V_\alpha = g_{\alpha\beta} V^\beta$
$\omega^\alpha = g^{\alpha\beta}\omega_\beta$
$\vec{V}$
$\tilde{\omega}$

$V_\alpha = g_{\alpha\beta} V^\beta$

$\omega^\alpha = g^{\alpha\beta} \omega_\beta$

Covariant derivative



Connection Γ — rule to take a vector at Q and bring it to P

↳ new structure → parallel transport

covariant derivative → $\vec{V}(P) - \vec{V}_T(P)$

$$\boxed{\nabla_\alpha V^\beta = \partial_\alpha V^\beta + \Gamma^\beta_{\alpha\gamma} V^\gamma}$$

Extend to other tensors $\longrightarrow$ Leibnitz

scalar: $\boxed{\nabla_\alpha f \equiv \partial_\alpha f}$

$$\nabla\left(A^{\cdots}_{\cdots}\, B^{\cdots}_{\cdots}\right) = \left(\nabla A^{\cdots}_{\cdots}\right) B^{\cdots}_{\cdots} + A^{\cdots}_{\cdots}\left(\nabla B^{\cdots}_{\cdots}\right)$$

covector: $\nabla_\alpha(\omega_\beta V^\beta) = \partial_\alpha(\omega_\beta V^\beta) = (\partial_\alpha \omega_\beta) V^\beta + \omega_\beta (\partial_\alpha V^\beta)$

$$\nabla_\alpha(\omega_\beta V^\beta) = (\nabla_\alpha \omega_\beta) V^\beta + \omega_\beta(\nabla_\alpha V^\beta)$$

$$\rightarrow (\nabla_\alpha \omega_\beta) V^\beta = -\omega_\beta\left(\cancel{\partial_\alpha V^\beta} + \Gamma^\beta_{\alpha\gamma} V^\gamma\right) + (\partial_\alpha \omega_\beta) V^\beta + \cancel{\omega_\beta(\partial_\alpha V^\beta)}$$

$$= \left(\partial_\alpha \omega_\beta - \omega_\gamma \Gamma^\gamma_{\alpha\beta}\right) V^\beta$$

Extend to other tensors → Leibnitz

scalar: $\boxed{\nabla_\alpha f \equiv \partial_\alpha f}$

$$\nabla(A^{\cdots}_{\cdots}\, B^{\cdots}_{\cdots}) = (\nabla A^{\cdots}_{\cdots})\, B^{\cdots}_{\cdots} + A^{\cdots}_{\cdots}\, (\nabla B^{\cdots}_{\cdots})$$

covector: $\nabla_\alpha(\omega_\beta V^\beta) = \partial_\alpha(\omega_\beta V^\beta) = (\partial_\alpha \omega_\beta) V^\beta + \omega_\beta(\partial_\alpha V^\beta)$

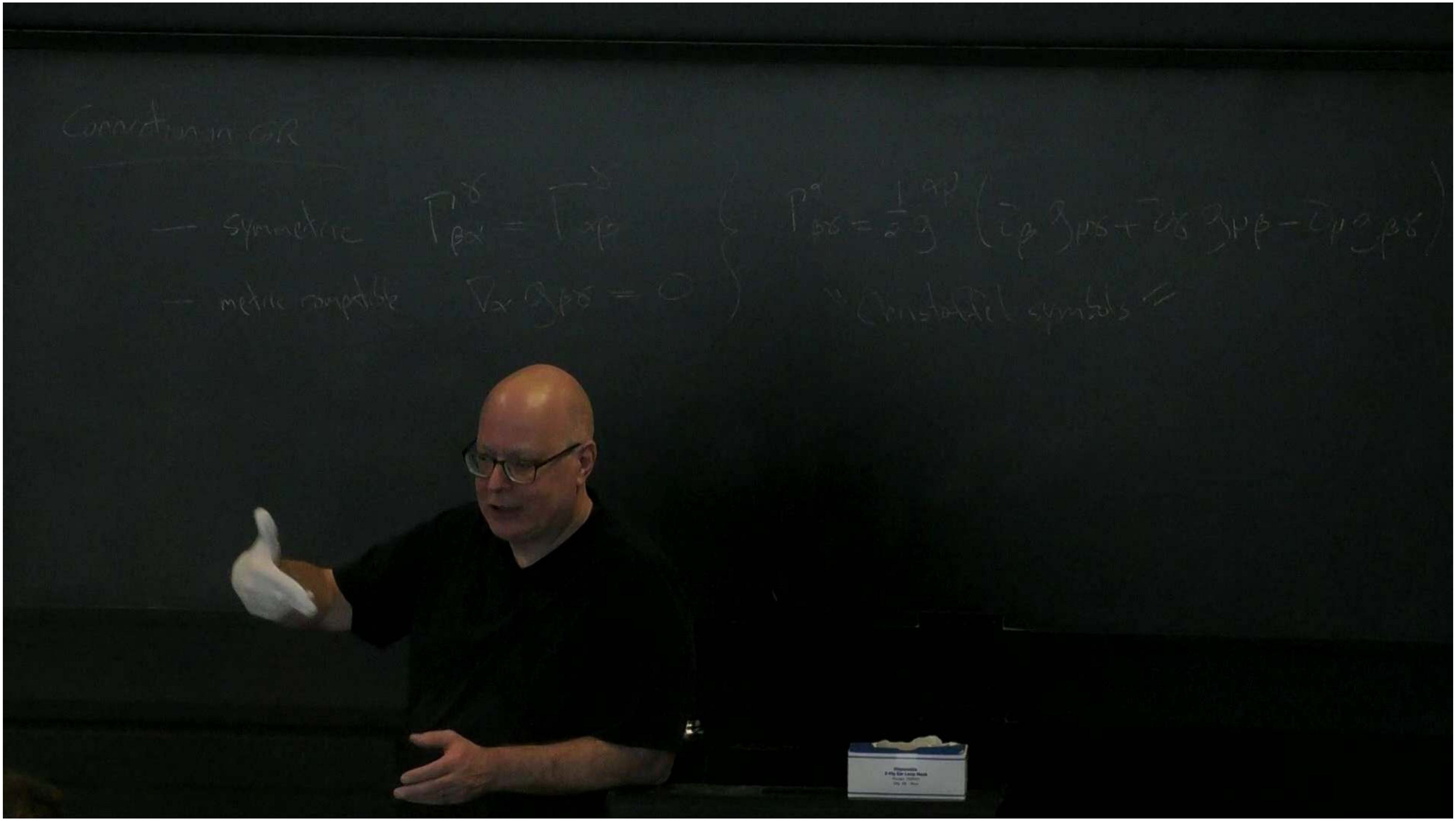
$$\nabla_\alpha(\omega_\beta V^\beta) = (\nabla_\alpha \omega_\beta) V^\beta + \omega_\beta(\nabla_\alpha V^\beta)$$

$$\rightarrow \underline{(\nabla_\alpha \omega_\beta)} V^\beta = -\omega_\beta(\cancel{\partial_\alpha V^\beta} + \Gamma^\beta_{\alpha\gamma} V^\gamma) + (\partial_\alpha \omega_\beta) V^\beta + \cancel{\omega_\beta(\partial_\alpha V^\beta)}$$

$$= \underline{(\partial_\alpha \omega_\beta - \omega_\gamma \Gamma^\gamma_{\alpha\beta})} V^\beta$$

$$\boxed{\nabla_\alpha \omega_\beta = \partial_\alpha \omega_\beta - \Gamma^\gamma_{\alpha\beta}\, \omega_\gamma}$$

integral curves → $\vec{V}$ is


Connection in GR
— symmetric $\Gamma^{\gamma}_{\beta\alpha} = \Gamma^{\gamma}_{\alpha\beta}$
— metric compatible $\nabla_\alpha g_{\beta\gamma} = 0$
$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\lambda} \left(\partial_\beta g_{\lambda\gamma} + \partial_\gamma g_{\lambda\beta} - \partial_\lambda g_{\beta\gamma} \right)$
"Christoffel symbols"