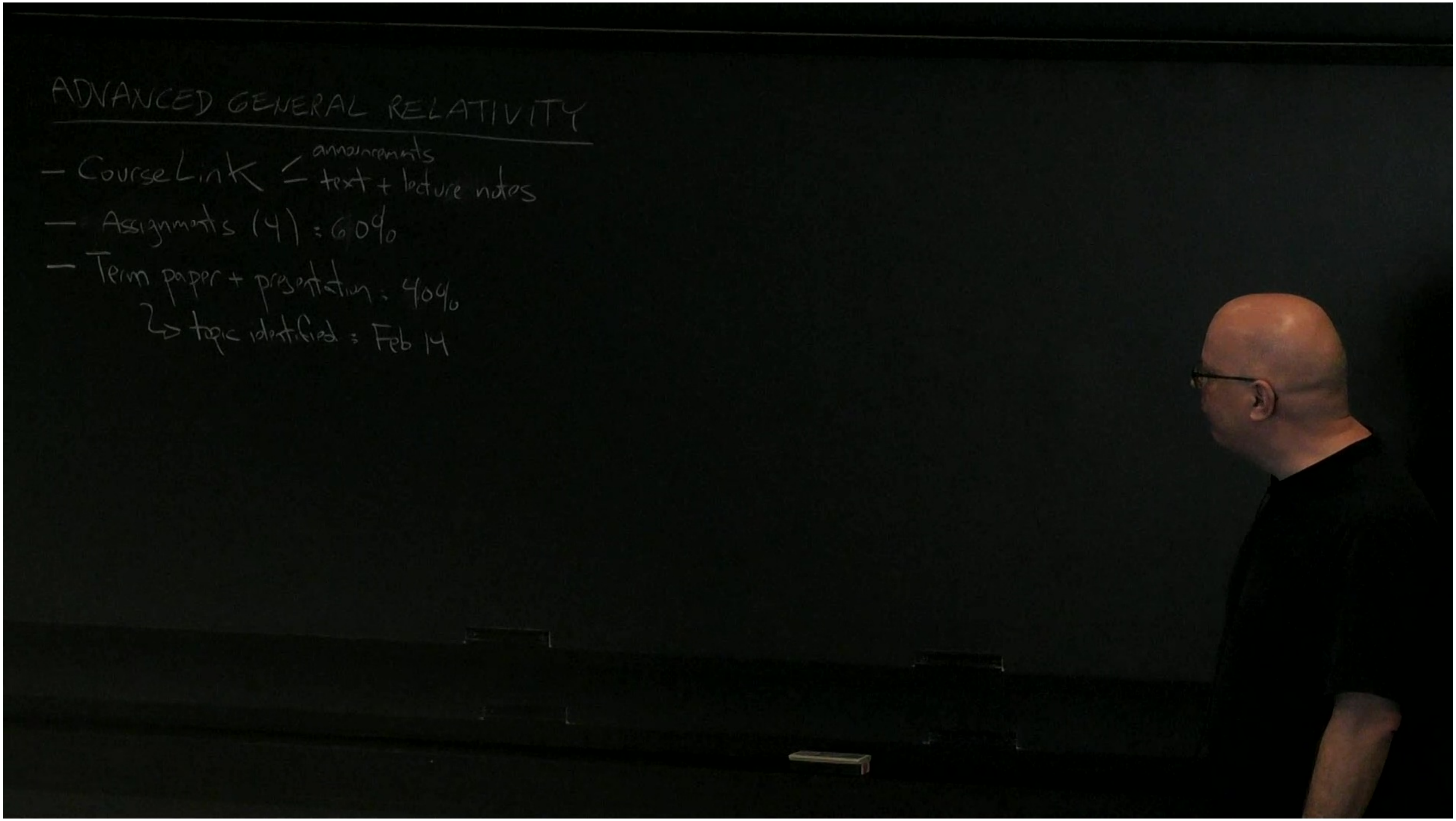
Title: Advanced General Relativity - 240110

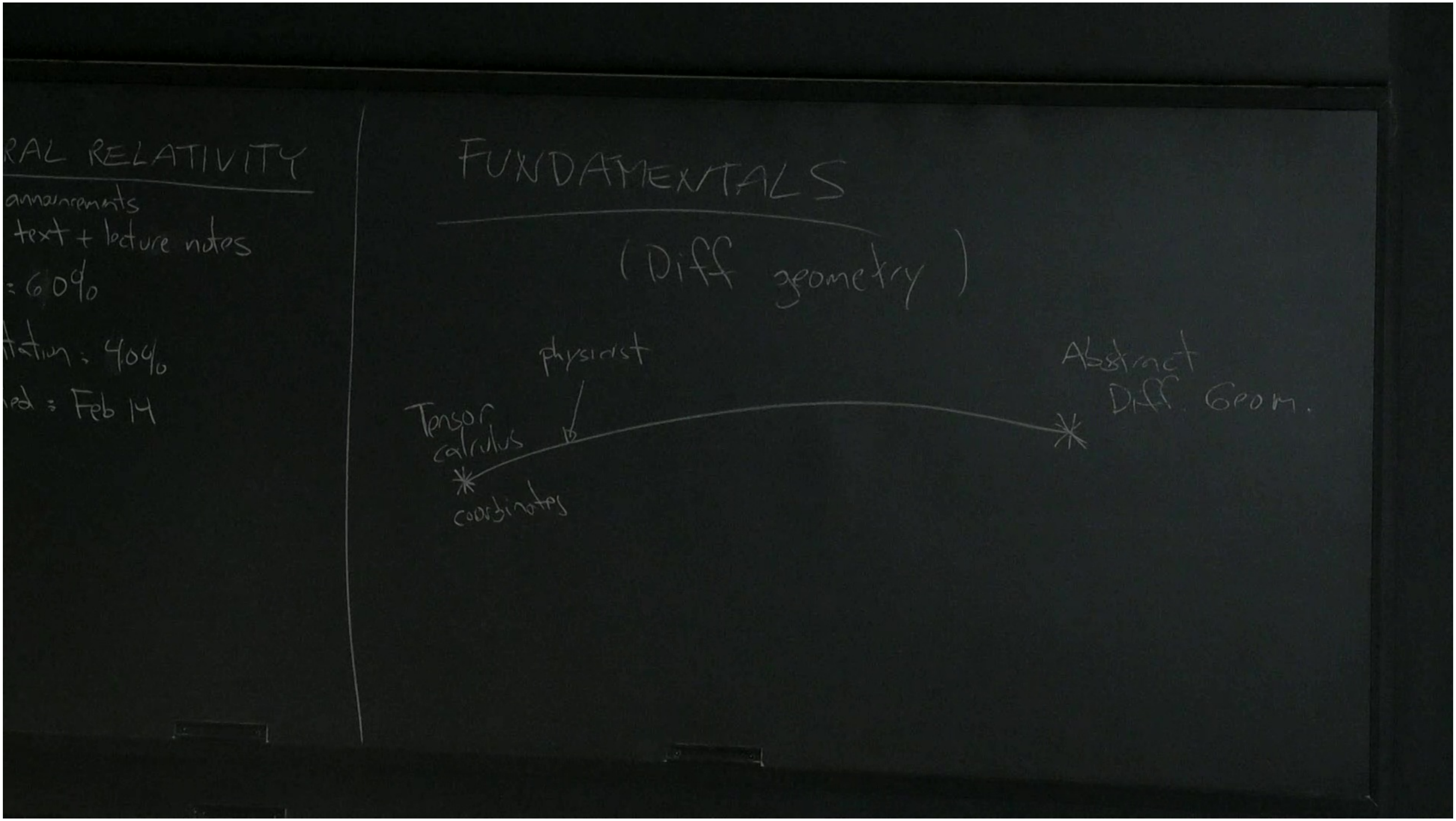
Speakers: Eric Poisson

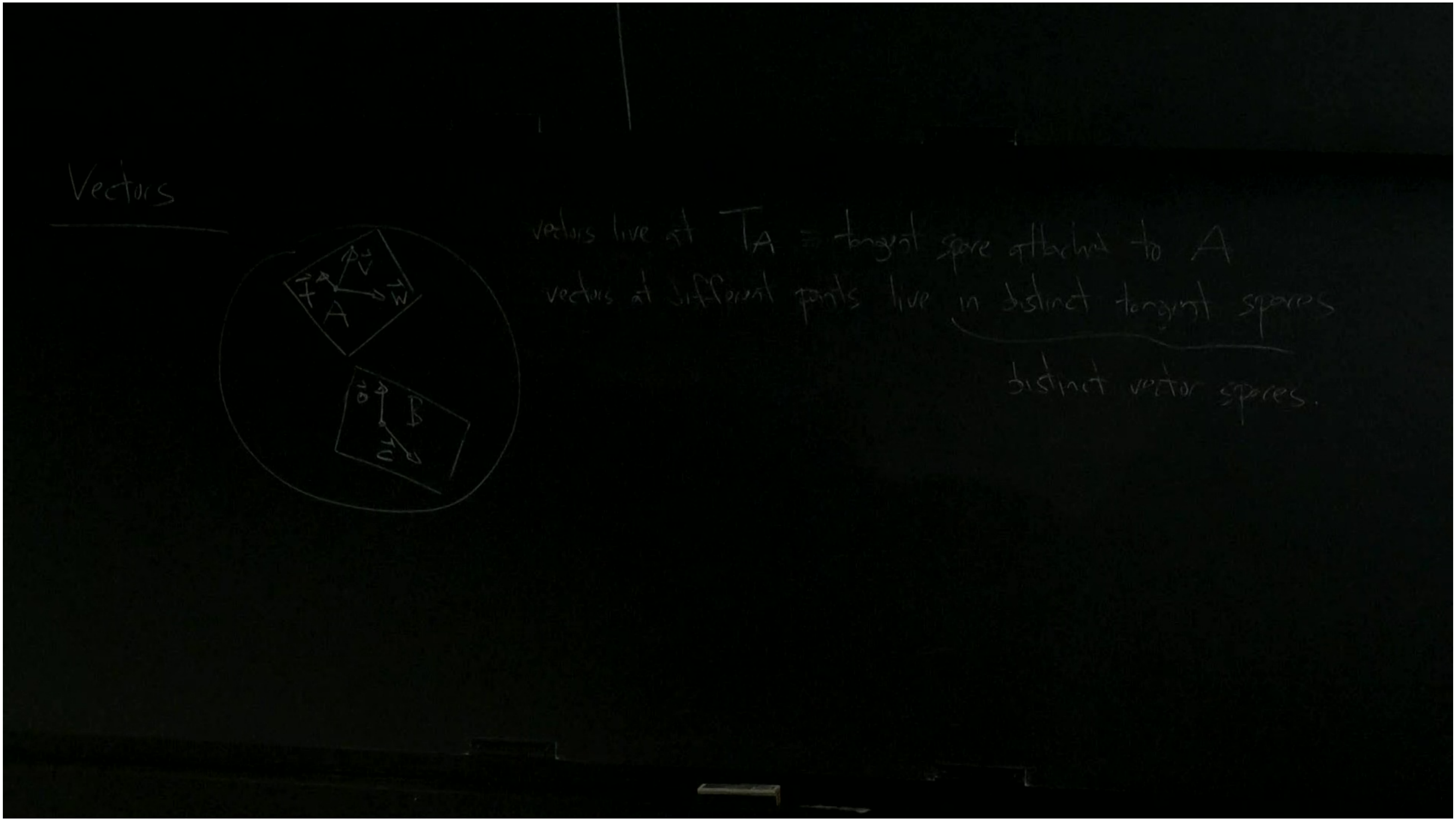
Collection: Advanced General Relativity (PHYS7840)

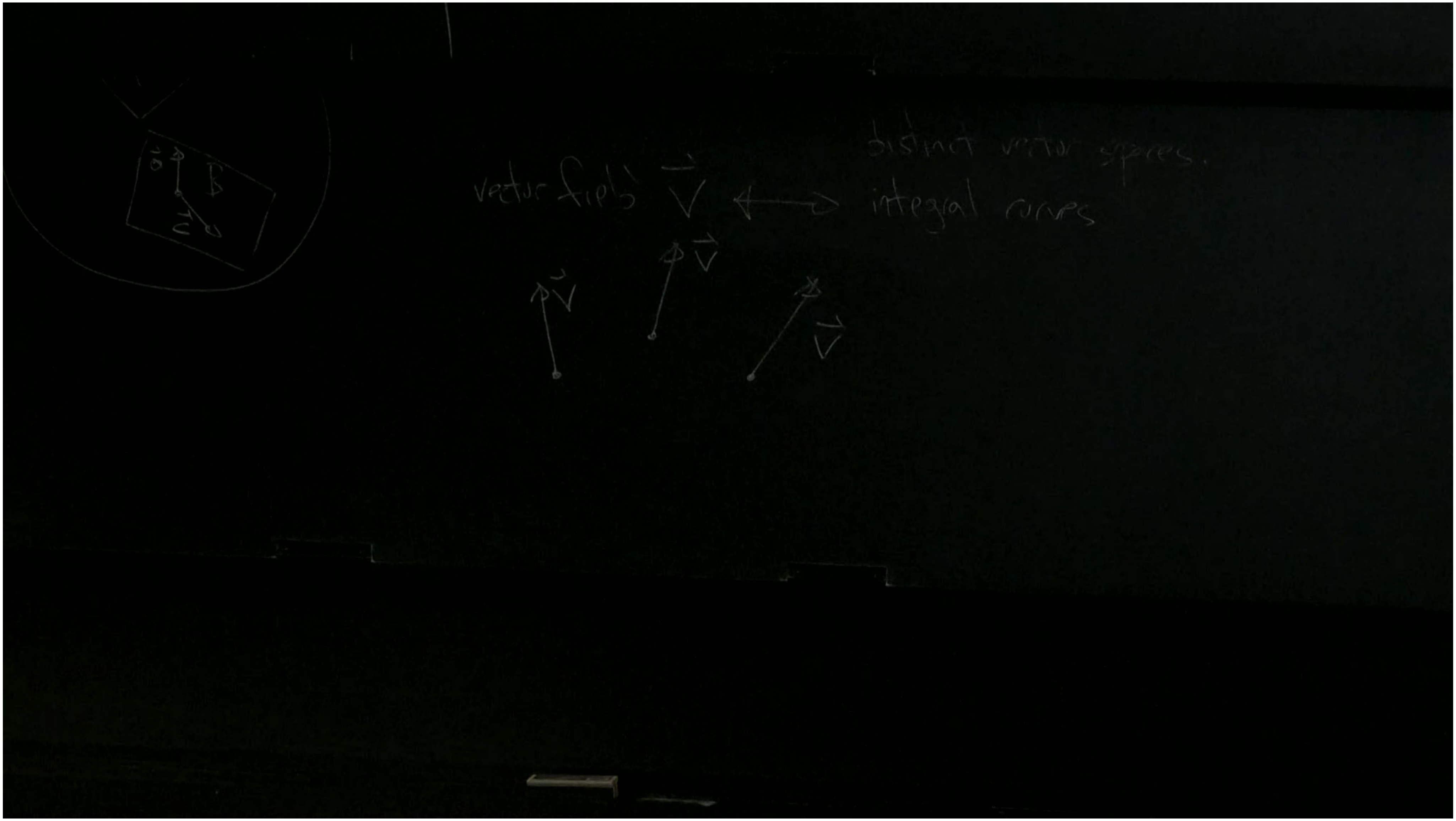
Date: January 10, 2024 - 10:30 AM

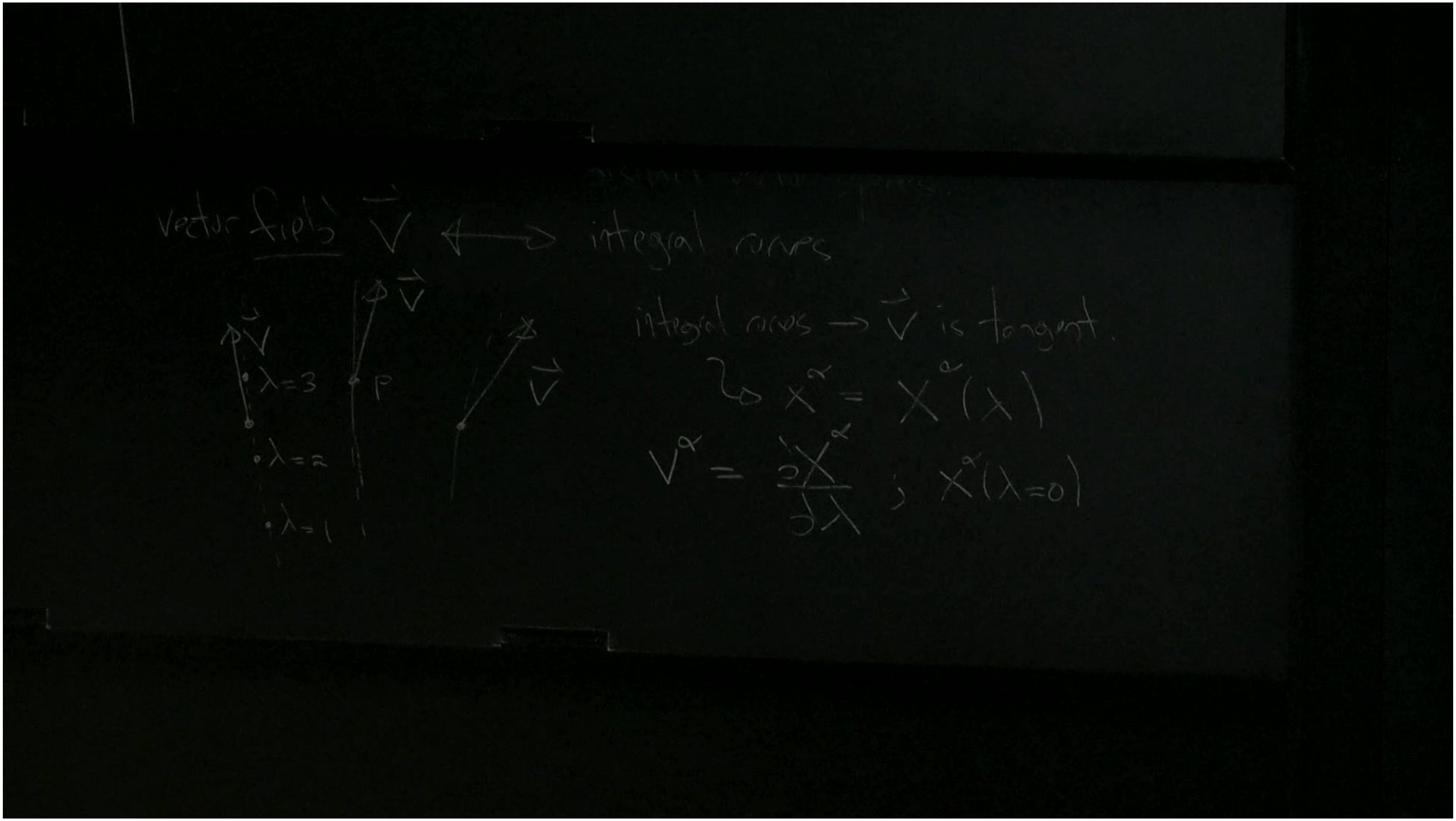
URL: https://pirsa.org/24010000

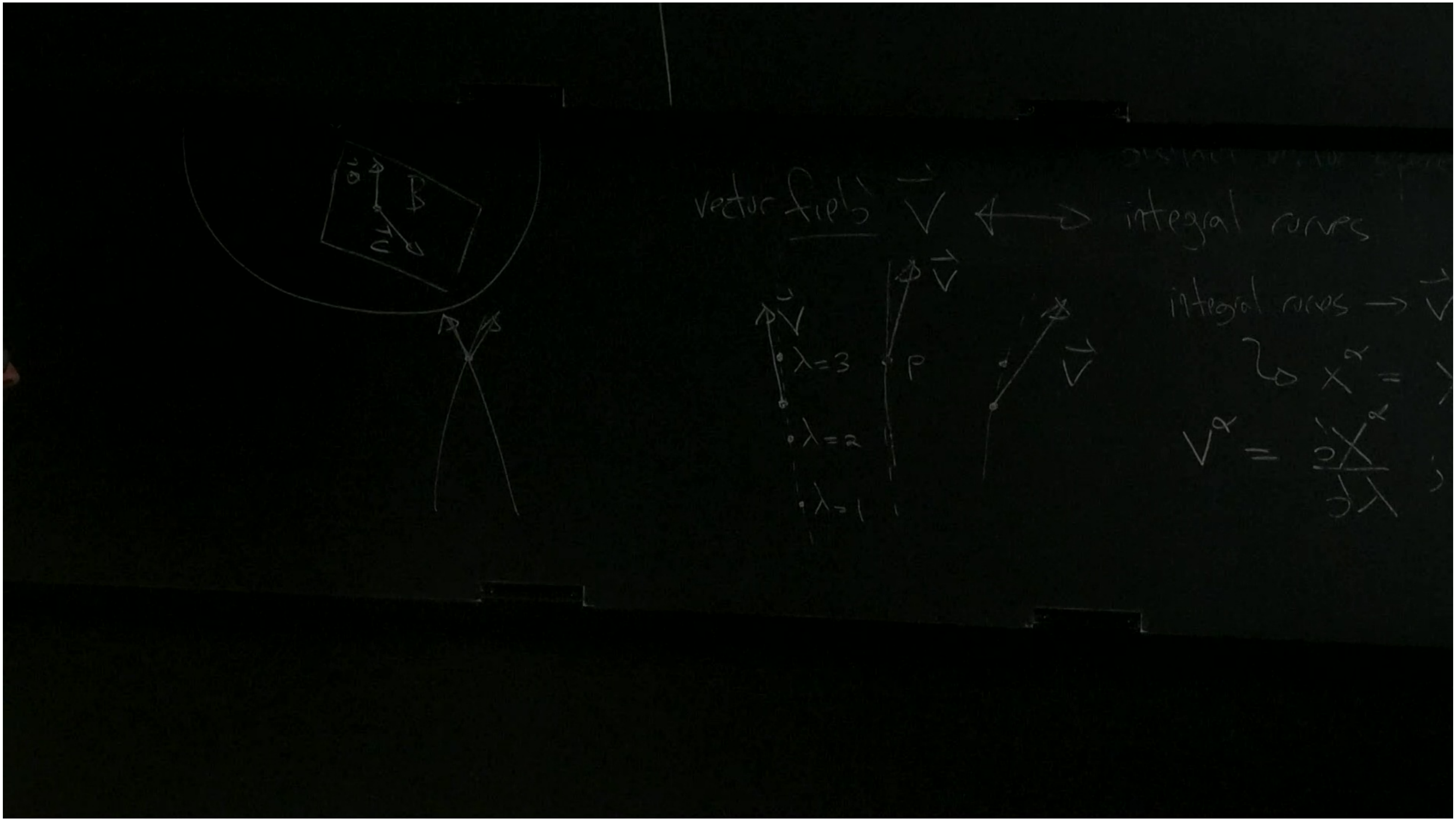

ADVANCED GENERAL RELATIVITY
– CourseLink
announcements
text + lecture notes
– Assignments (4): 60%
– Term paper + presentation: 40%
topic identified: Feb 14

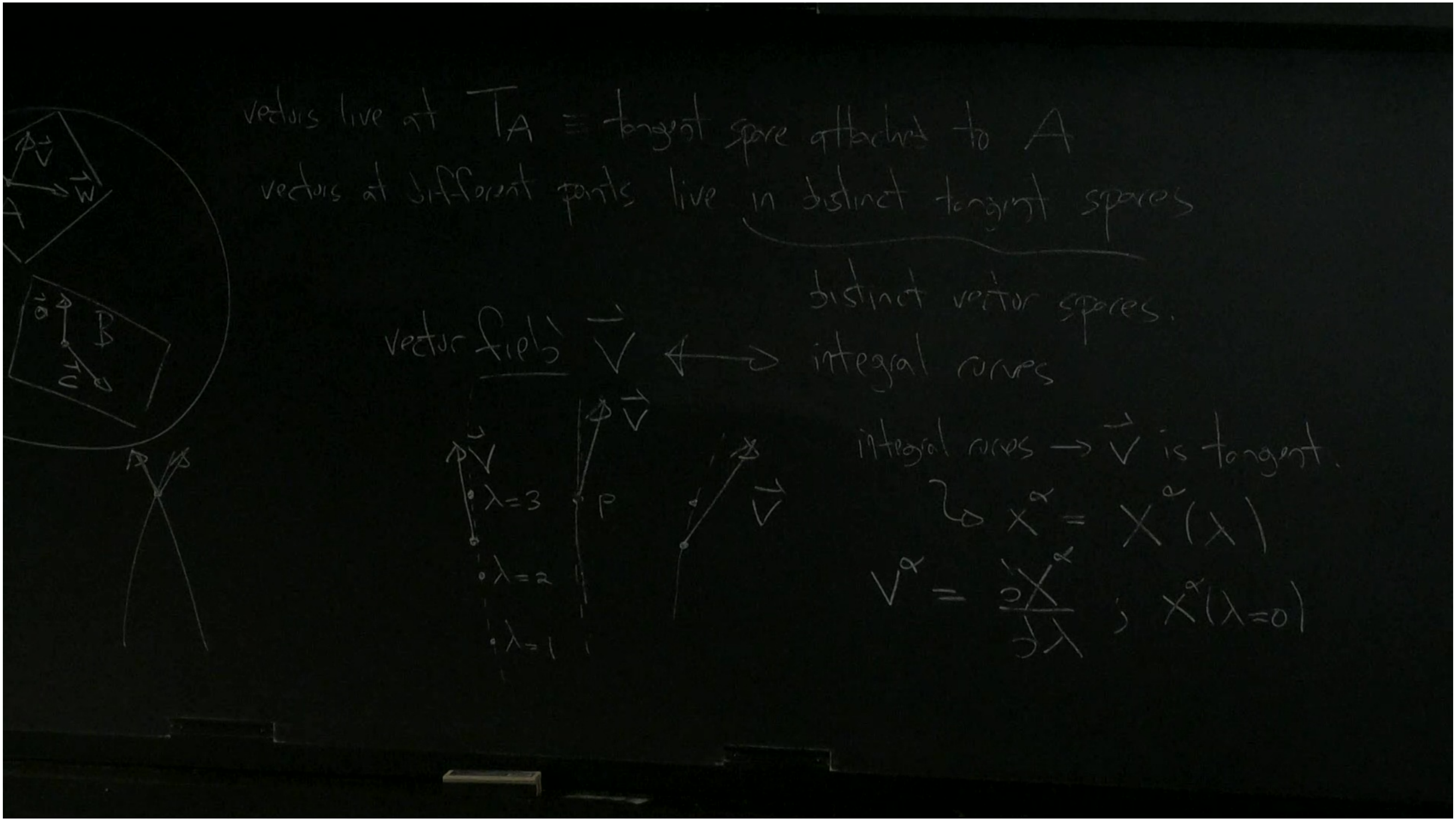

RAL RELATIVITY
announcements
text + lecture notes
: 60%
tation: 40%
ed: Feb 14
FUNDAMENTALS
(Diff geometry)
physicist
Abstract
Diff. Geom.
Tensor
calculus
coordinates

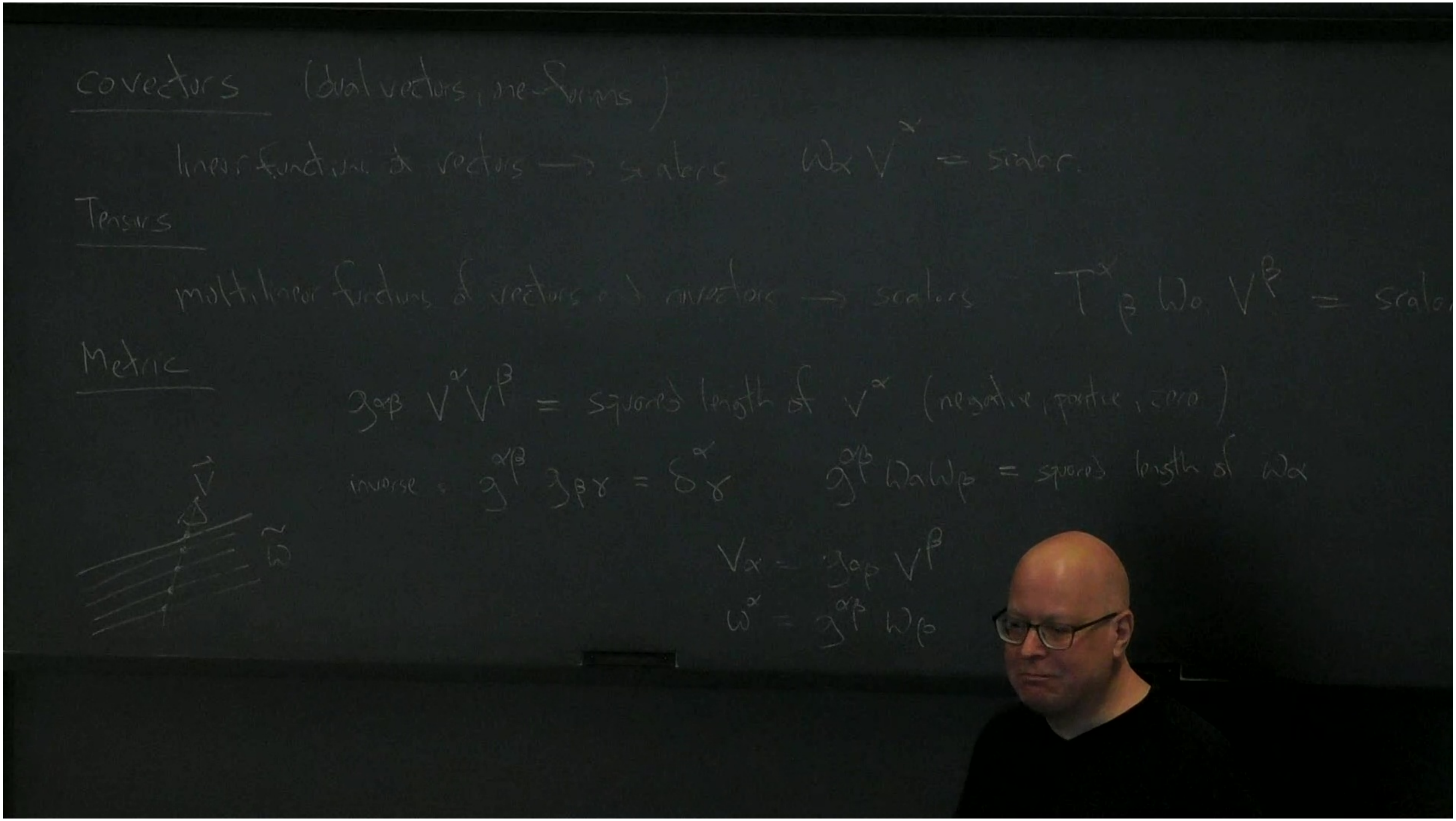

Vectors
vectors live at T_A = tangent space attached to A
vectors at different points live in distinct tangent spaces
distinct vector spaces.
A
B

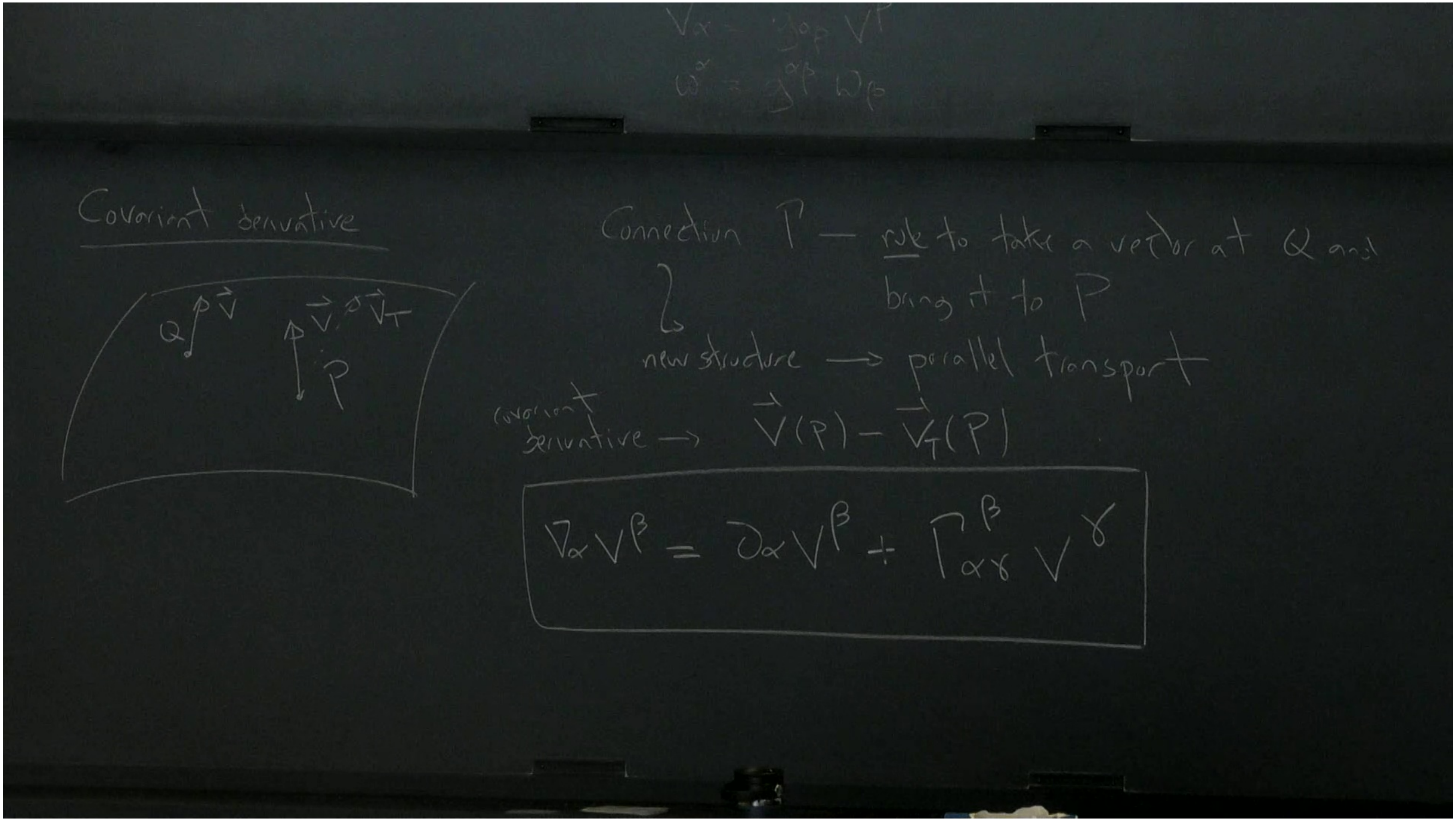

distinct vector spaces.
vector fields
integral curves


vector fields $\vec{V}$ ⟷ integral curves
integral curves → $\vec{V}$ is tangent.
$x^\alpha = X^\alpha(\lambda)$
$V^\alpha = \frac{dX^\alpha}{d\lambda}$; $X^\alpha(\lambda=0)$
$\lambda=3$
$\lambda=2$
$\lambda=1$
p


vector fields $\vec{V}$ ⟷ integral curves
integral curves → $\vec{V}$
$\lambda = 3$
$\lambda = 2$
$\lambda = 1$
P
B
$V^\alpha = \frac{\partial X^\alpha}{\partial \lambda}$


vectors live at T_A ≡ tangent space attached to A
vectors at different points live in distinct tangent spaces
distinct vector spaces.
vector field $\vec{V}$ ⟷ integral curves
integral curves → $\vec{V}$ is tangent.
$x^\alpha = X^\alpha(\lambda)$
$V^\alpha = \frac{dX^\alpha}{d\lambda}$; $X^\alpha(\lambda=0)$
λ=3
λ=2
λ=1
P
A
B


covectors (dual vectors, one-forms)
linear functions of vectors → scalars
Tensors
multilinear functions of vectors and covectors → scalars
Metric
squared length of
(negative, positive, zero)
inverse
squared length of


$V_\alpha = g_{\alpha\beta} V^\beta$
$w^\alpha = g^{\alpha\beta} w_\beta$
Covariant derivative
Q
$\vec{V}$
$\vec{V}$
$\vec{V}_T$
P
Connection Γ — rule to take a vector at Q and bring it to P
new structure → parallel transport
covariant derivative → $\vec{V}(P) - \vec{V}_T(P)$
$\nabla_\alpha V^\beta = \partial_\alpha V^\beta + \Gamma^\beta_{\alpha\gamma} V^\gamma$

Extend to other tensors $\longrightarrow$ Leibnitz

scalar: $\boxed{\nabla_\alpha f \equiv \partial_\alpha f}$

$$\nabla\left(A^{\cdots}_{\cdots}\, B^{\cdots}_{\cdots}\right) = \left(\nabla A^{\cdots}_{\cdots}\right) B^{\cdots}_{\cdots} + A^{\cdots}_{\cdots}\left(\nabla B^{\cdots}_{\cdots}\right)$$

covector: $\nabla_\alpha\left(\omega_\beta V^\beta\right) = \partial_\alpha\left(\omega_\beta V^\beta\right) = \left(\partial_\alpha \omega_\beta\right) V^\beta + \omega_\beta\left(\partial_\alpha V^\beta\right)$

$$\nabla_\alpha\left(\omega_\beta V^\beta\right) = \left(\nabla_\alpha \omega_\beta\right) V^\beta + \omega_\beta\left(\nabla_\alpha V^\beta\right)$$

$$\longrightarrow \left(\nabla_\alpha \omega_\beta\right) V^\beta = -\omega_\beta\left(\cancel{\partial_\alpha V^\beta} + \Gamma^\beta_{\alpha\gamma} V^\gamma\right) + \left(\partial_\alpha \omega_\beta\right) V^\beta + \omega_\beta\left(\cancel{\partial_\alpha V^\beta}\right)$$

$$= \left(\partial_\alpha \omega_\beta - \omega_\gamma \Gamma^\gamma_{\alpha\beta}\right) V^\beta$$

Extend to other tensors → Leibnitz

scalar: $\boxed{\nabla_\alpha f \equiv \partial_\alpha f}$

$$\nabla\left(A^{\cdots}_{\cdots} B^{\cdots}_{\cdots}\right) = \left(\nabla A^{\cdots}_{\cdots}\right) B^{\cdots}_{\cdots} + A^{\cdots}_{\cdots}\left(\nabla B^{\cdots}_{\cdots}\right)$$

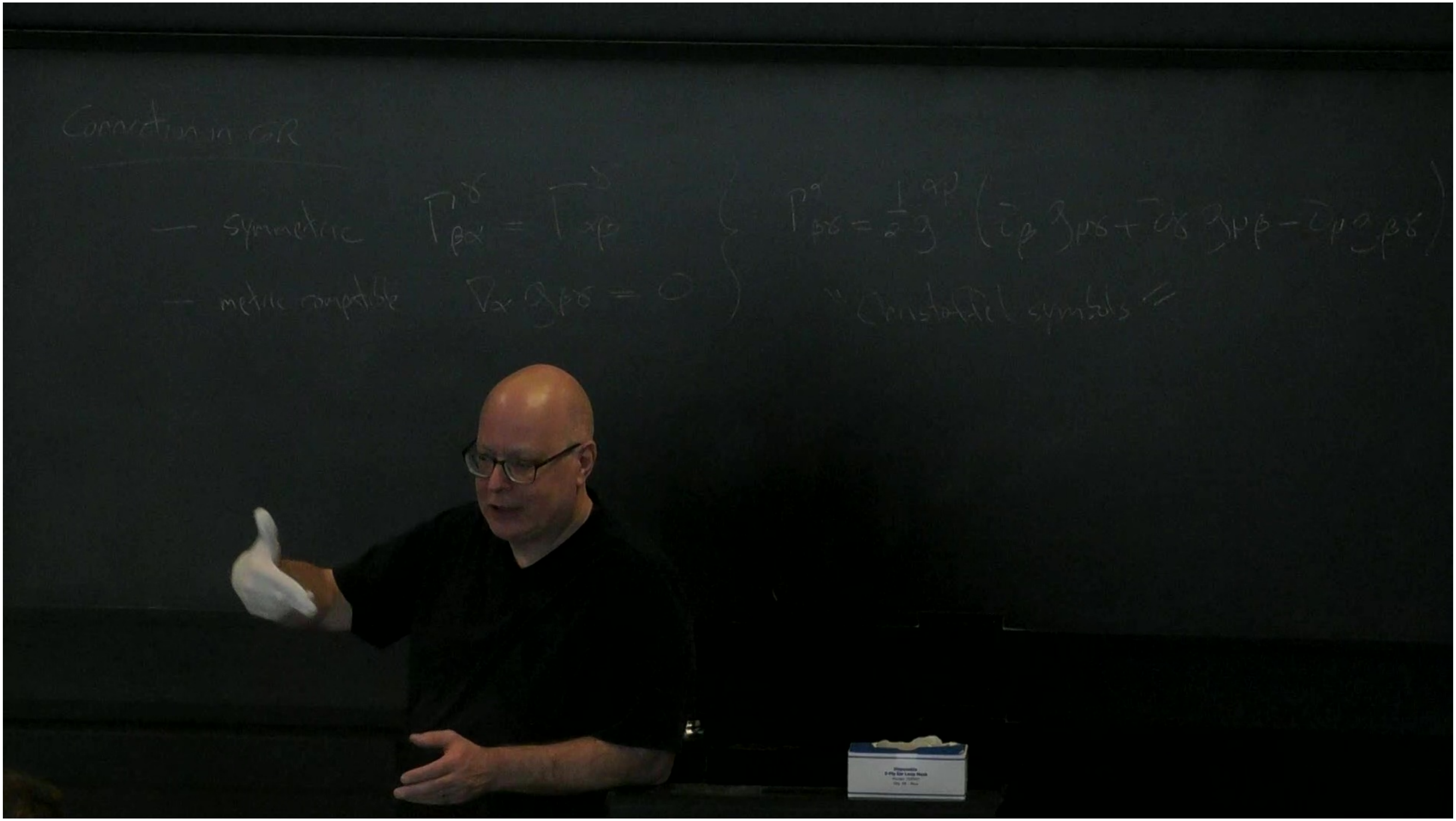
covector: $\nabla_\alpha\left(\omega_\beta V^\beta\right) = \partial_\alpha\left(\omega_\beta V^\beta\right) = \left(\partial_\alpha \omega_\beta\right) V^\beta + \omega_\beta\left(\partial_\alpha V^\beta\right)$

$$\nabla_\alpha\left(\omega_\beta V^\beta\right) = \left(\nabla_\alpha \omega_\beta\right) V^\beta + \omega_\beta\left(\nabla_\alpha V^\beta\right)$$

$$\rightarrow \left(\nabla_\alpha \omega_\beta\right) V^\beta = -\omega_\beta\left(\partial_\alpha V^\beta + \Gamma^\beta_{\alpha\gamma} V^\gamma\right) + \left(\partial_\alpha \omega_\beta\right) V^\beta + \omega_\beta\left(\partial_\alpha V^\beta\right)$$

$$= \left(\partial_\alpha \omega_\beta - \omega_\gamma \Gamma^\gamma_{\alpha\beta}\right) V^\beta$$

$$\boxed{\nabla_\alpha \omega_\beta = \partial_\alpha \omega_\beta - \Gamma^\gamma_{\alpha\beta}\,\omega_\gamma}$$


Connection in GR
— symmetric
— metric compatible
"Christoffel symbols"