Title: Boundary and plane defect criticality in the 3d O(N) model

Speakers: Max Metlitski

Series: Quantum Matter

Date: December 12, 2023 - 11:00 AM

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Abstract: It is known that the classical O(N) model in dimension d > 3 at its bulk critical point admits three boundary universality classes: the ordinary, the extraordinary and the special. The extraordinary fixed point corresponds to the bulk transition occurring in the presence of an ordered boundary, while the special fixed point corresponds to a boundary phase transition between the ordinary and the extra-ordinary classes. While the ordinary fixed point survives in d = 3, it is less clear what happens to the extraordinary and special fixed points when d = 3 and N is greater or equal to 2. I'll show that formally treating N as a continuous parameter, there exists a critical value Nc > 2 separating two distinct regimes. For N < Nc the extra-ordinary fixed point survives in d = 3, albeit in a modified form: the long-range boundary order is lost, instead, the order parameter correlation function decays as a power of log r. For N > Nc there is no fixed point with order parameter correlations decaying slower than power law. I'll discuss how these findings compare to recent Monte-Carlo studies of classical and quantum spin models.

Zoom link https://pitp.zoom.us/j/97209122334?pwd=UHQ2OXR4bnVZREV0SlJOYXphWjh0QT09

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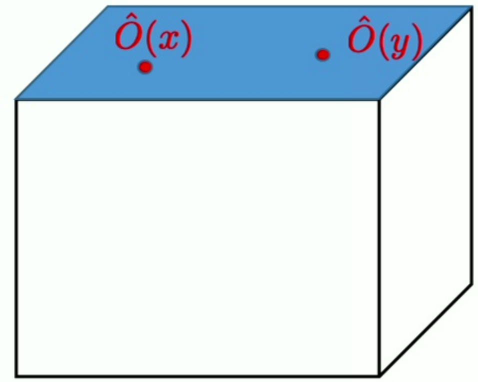
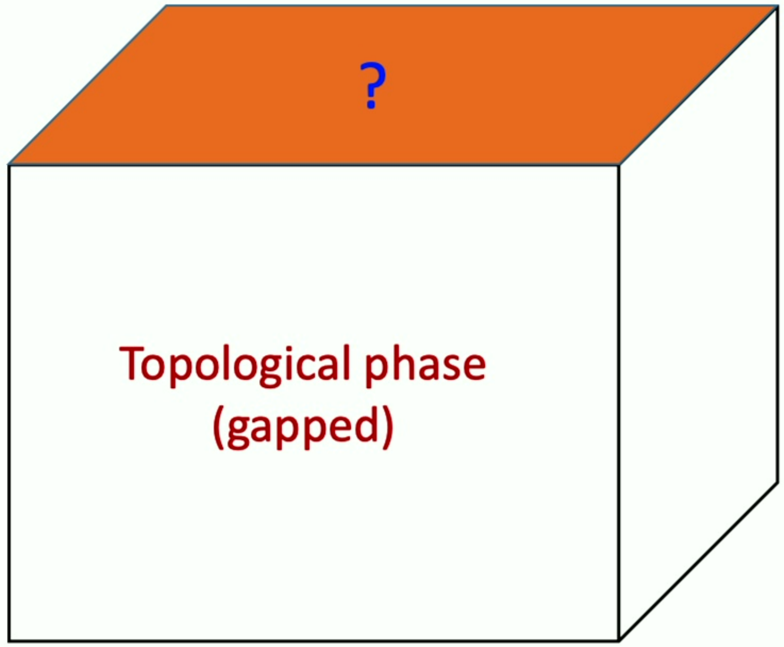
Boundary and plane defect criticality in the 3d O(N) model

Max Metlitski
MIT



Perimeter Institute
December 12, 2023

Boundaries of TQFTs



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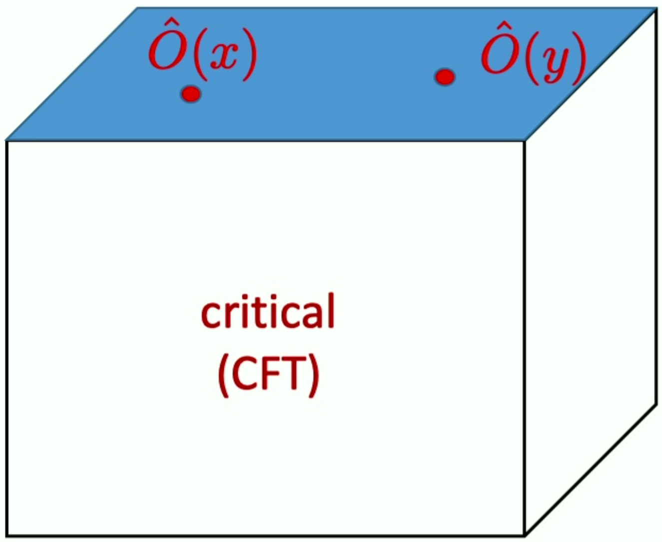
Boundary criticality



$$\langle \hat{O}(x)\hat{O}(y)\rangle \sim \frac{1}{|x-y|^{2\hat{\Delta}}}$$

- BCFT - not unique

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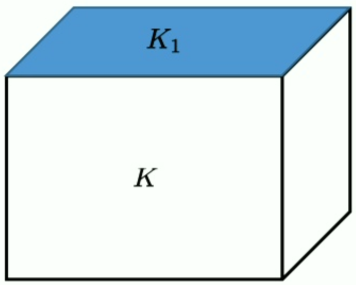
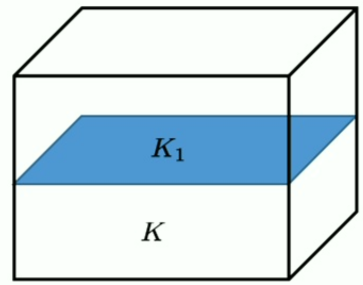
Classical O(N) model, d = 3

$$H = -\sum_{\langle ij \rangle} K_{ij} \vec{S}_i \cdot \vec{S}_j$$



Boundary



Plane defect

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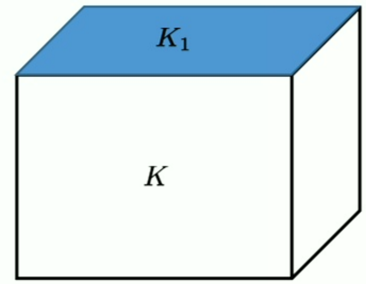
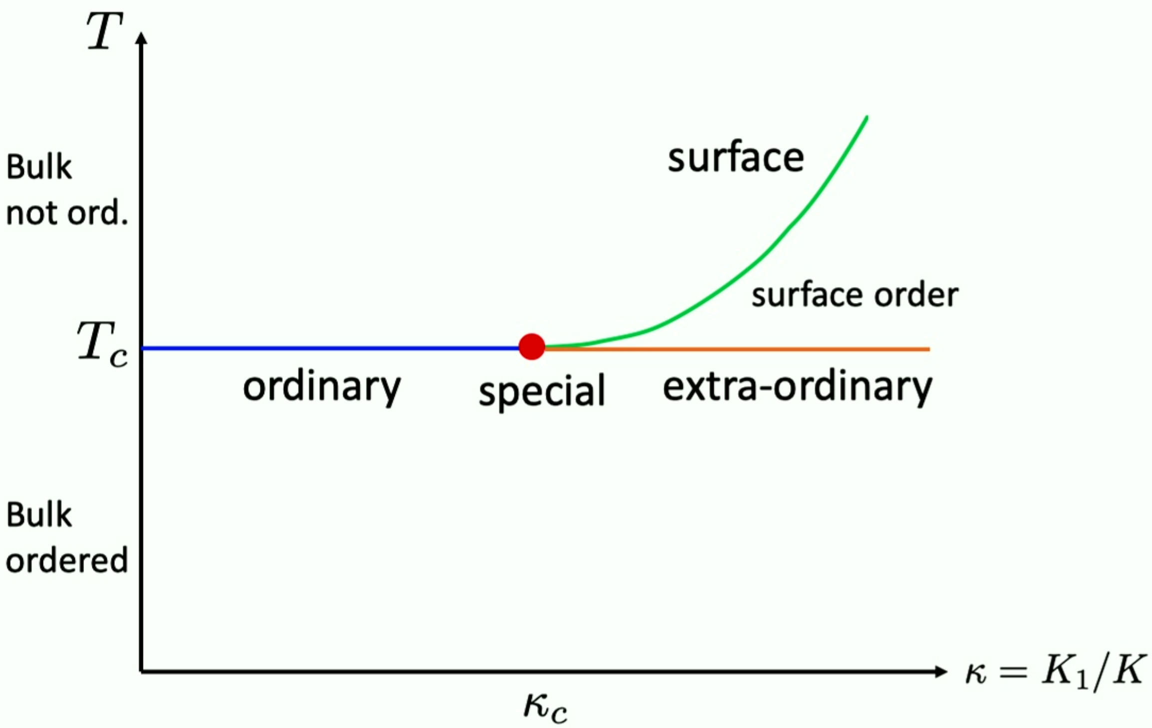
Boundary criticality: N = 1

$$H = -\sum_{\langle ij \rangle} K_{ij} \vec{S}_i \cdot \vec{S}_j$$





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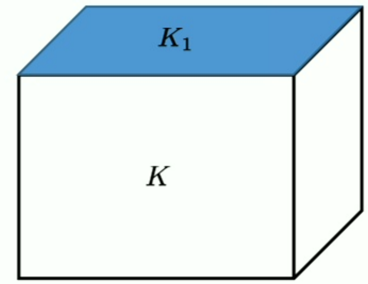
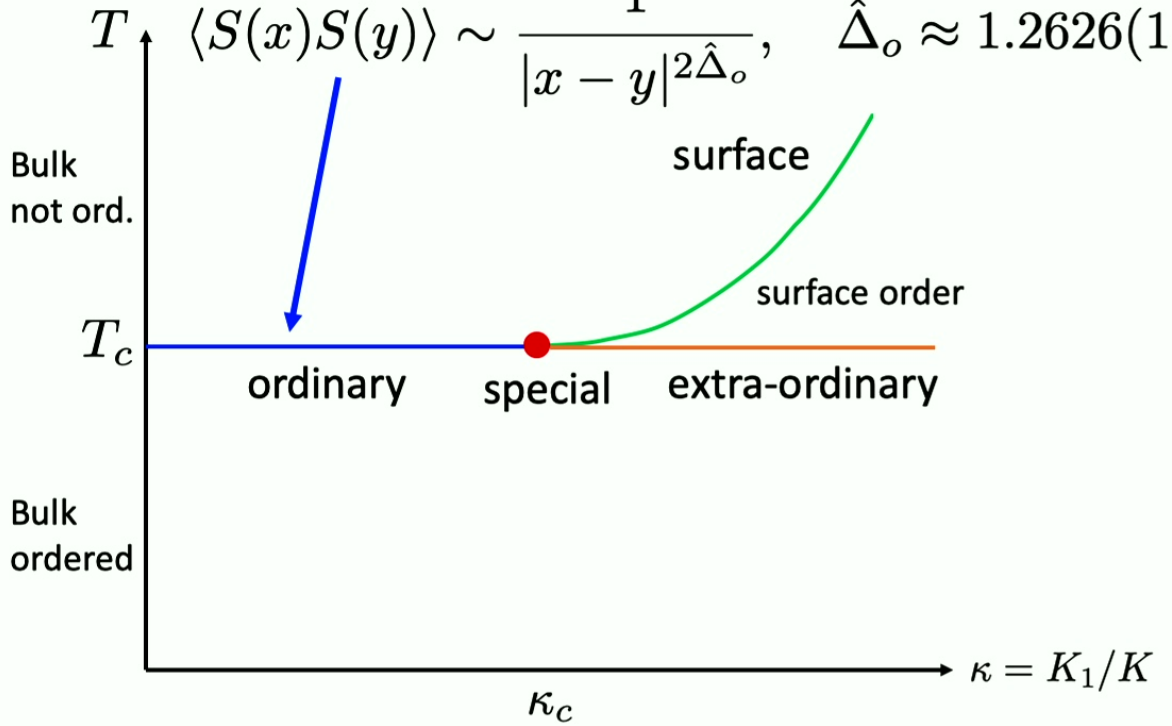
Boundary criticality: N = 1

$$H = -\sum_{\langle ij \rangle} K_{ij} \vec{S}_i \cdot \vec{S}_j$$



$$\langle S(x)S(y)\rangle \sim \frac{1}{|x-y|^{2\hat{\Delta}_o}}, \quad \hat{\Delta}_o \approx 1.2626(15)$$



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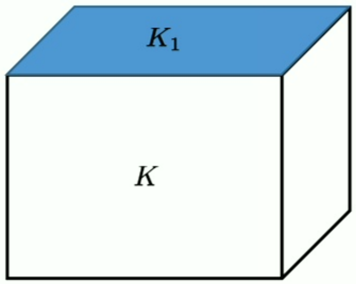
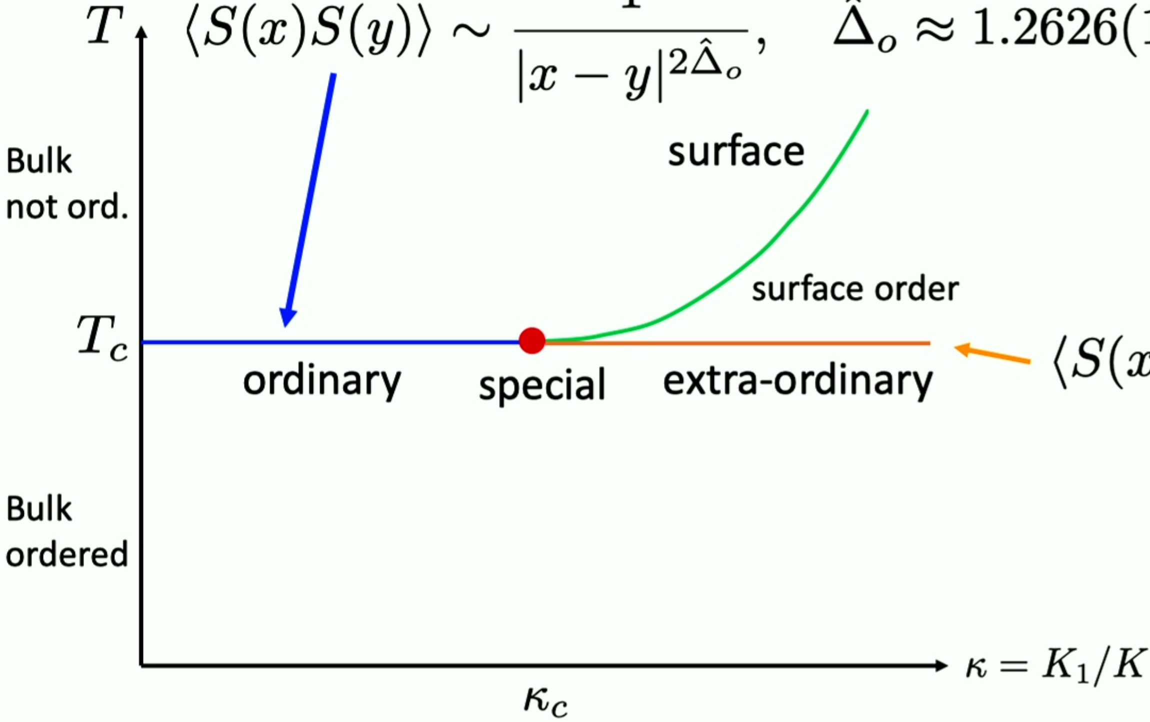
Boundary criticality: N = 1

$$H = -\sum_{\langle ij \rangle} K_{ij} \vec{S}_i \cdot \vec{S}_j$$



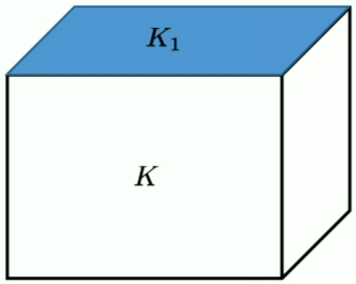
$$\langle S(x) S(y) \rangle \sim \frac{1}{|x-y|^{2\hat{\Delta}_o}}, \quad \hat{\Delta}_o \approx 1.2626(15)$$

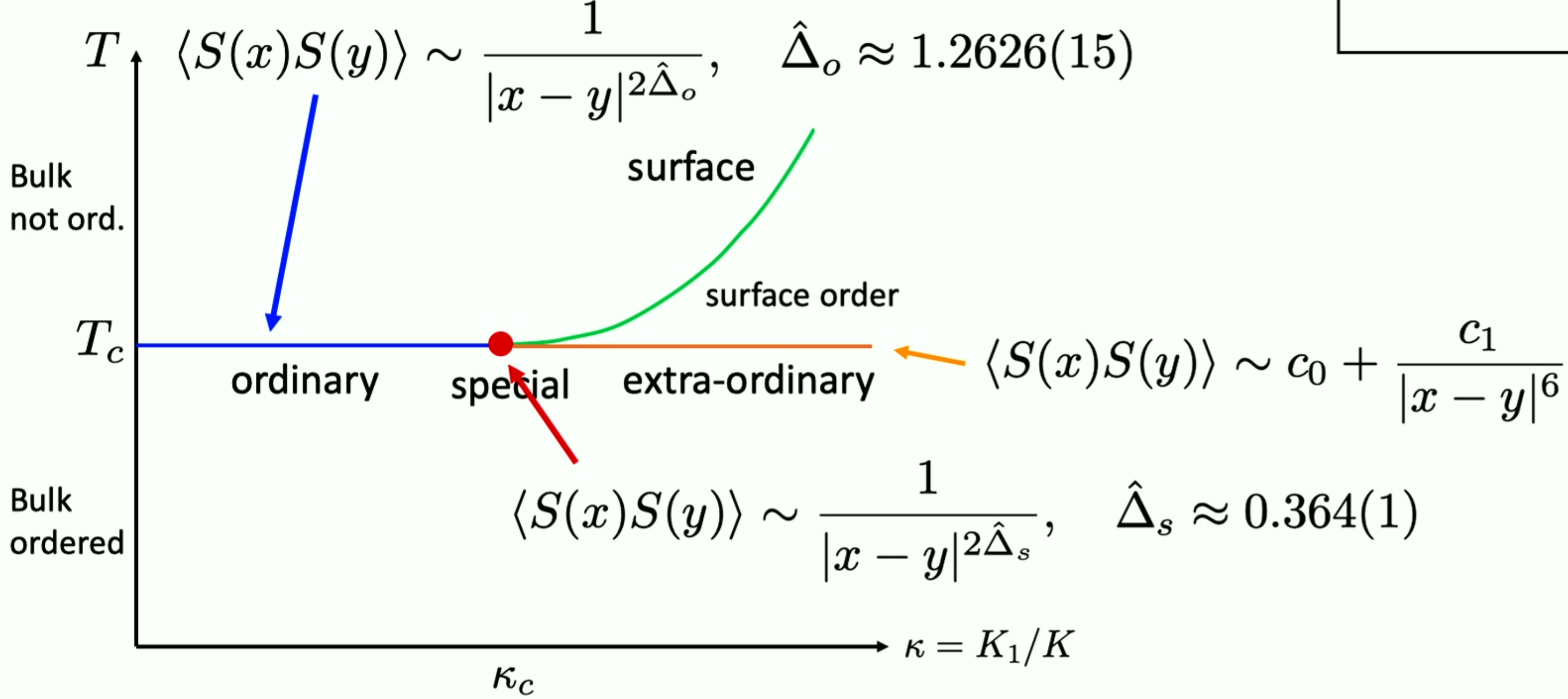


$$\langle S(x) S(y) \rangle \sim c_0 + \frac{c_1}{|x-y|^6}$$

Boundary criticality: N = 1

$$H = -\sum_{\langle ij \rangle} K_{ij} \vec{S}_i \cdot \vec{S}_j$$





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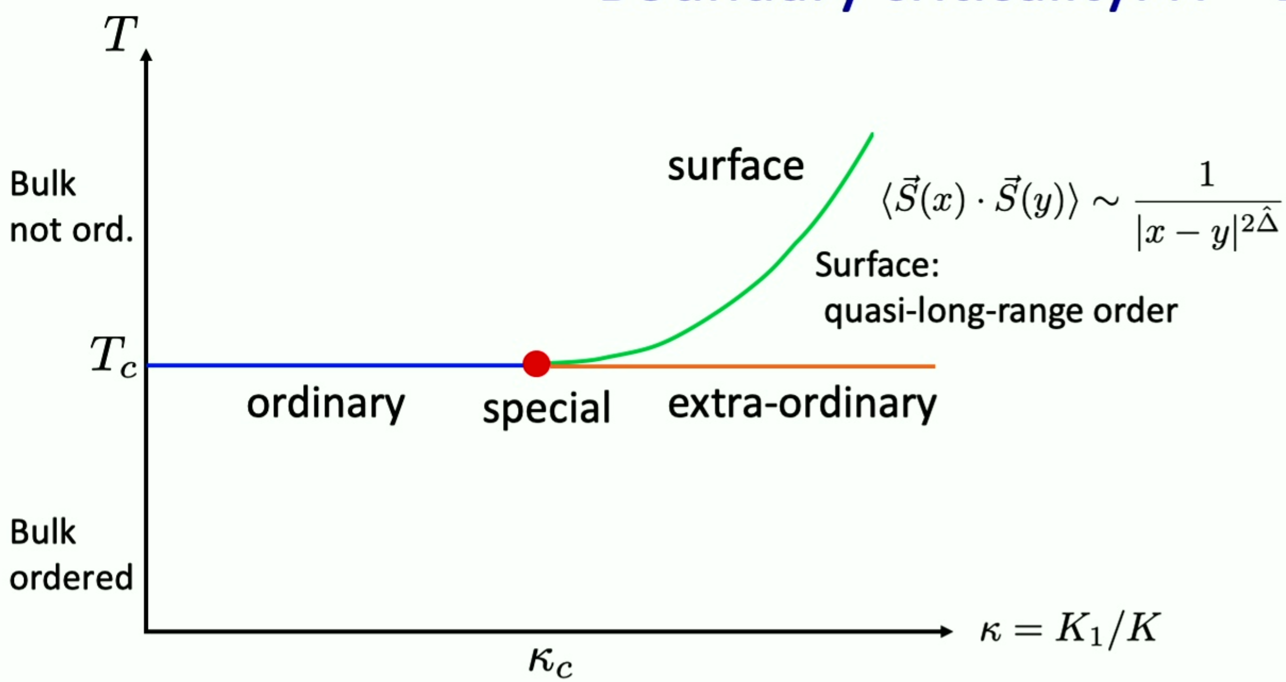
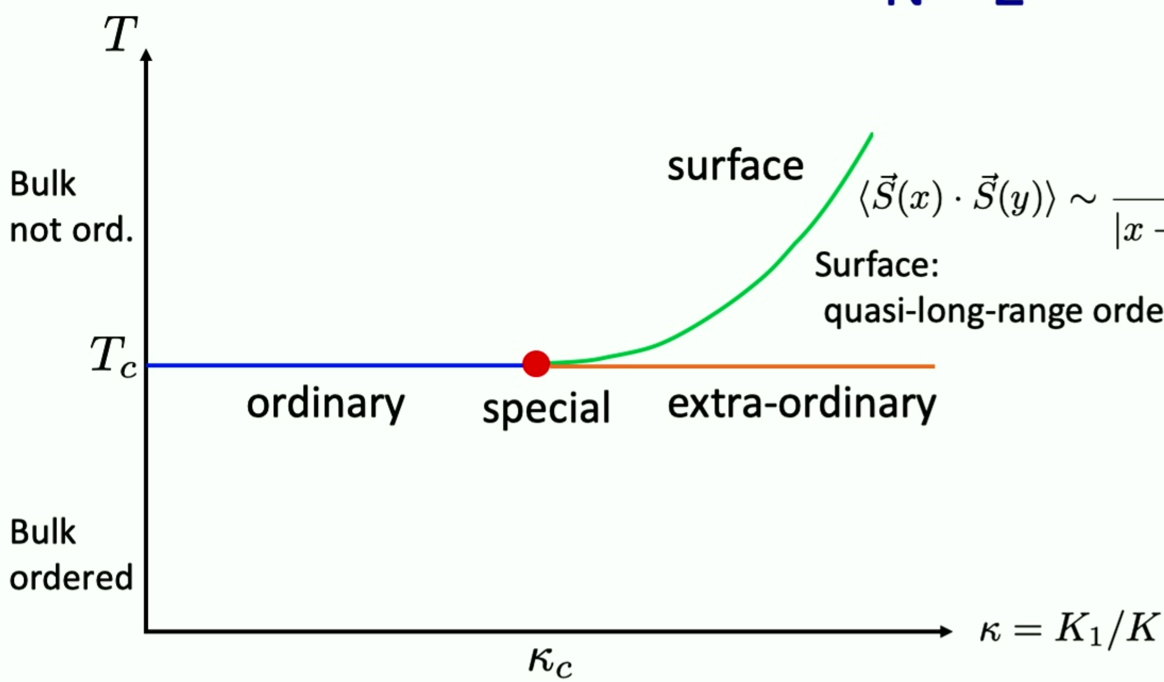
Boundary criticality: N = 2



- Mermin-Wagner-Hohenberg theorem: 2d + local interactions

$$\langle \vec{S}(x) \cdot \vec{S}(y) \rangle \to 0, \qquad |x-y| \to \infty$$

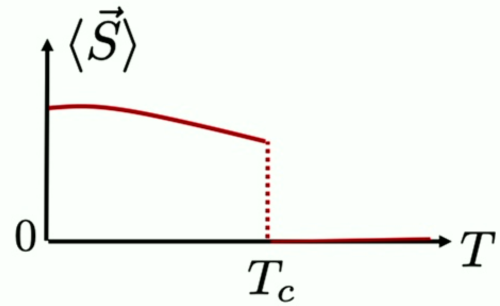
N = 2



- Extra-ordinary??

- Long range order at T_c ?

Deng, Blote, Nightingale, 2005



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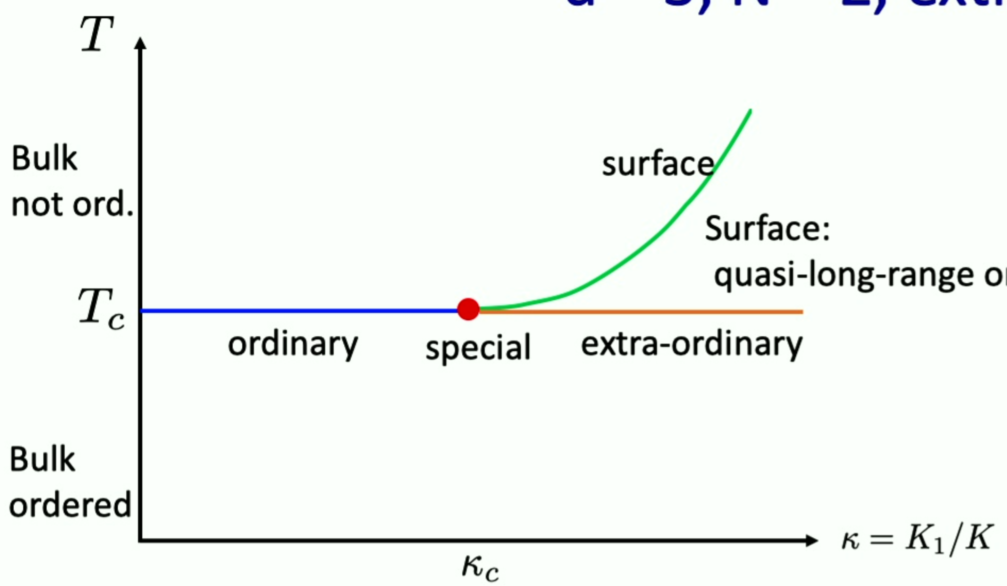
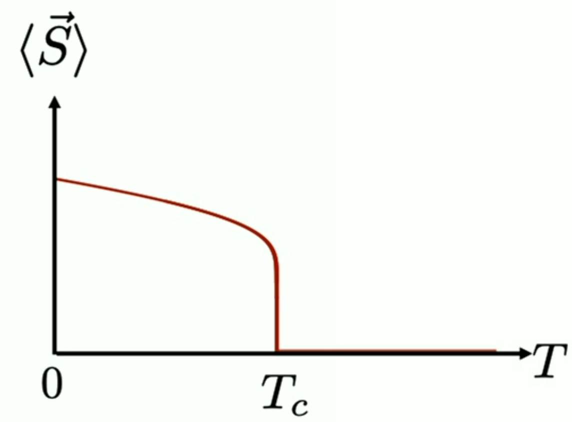
d = 3, N = 2, extra-ordinary



- $\langle \vec{S}(x) \cdot \vec{S}(0) \rangle \sim \frac{1}{(\log x)^q}$ "extra-ordinary-log" MM, 2020.

$q \approx 0.531(9)$

M. Hu, Y. Deng, J.-P. Lv, 2021
Francesco Parisen Toldin, MM 2021



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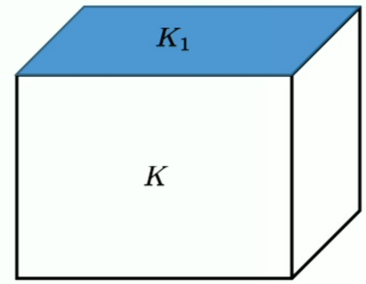
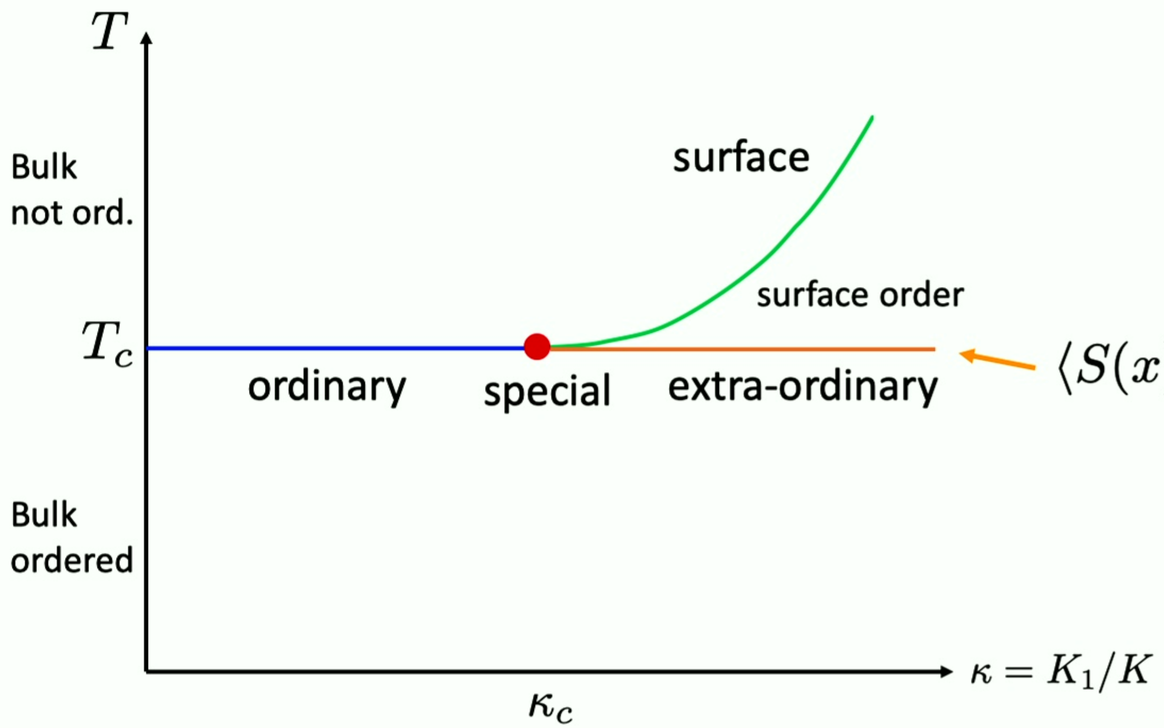
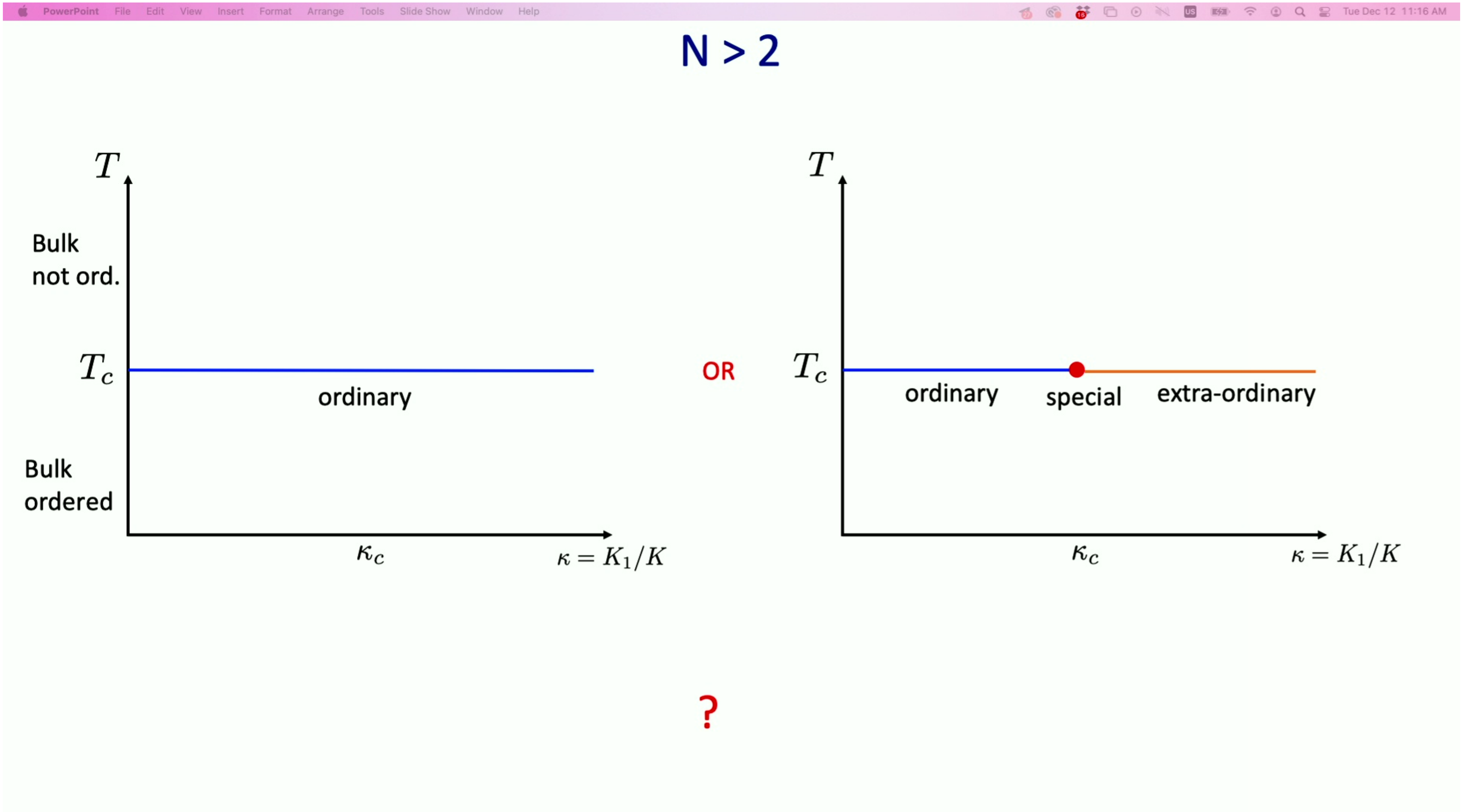
Boundary criticality: N = 1

$$H = -\sum_{\langle ij \rangle} K_{ij} \vec{S}_i \cdot \vec{S}_j$$





$$\langle S(x)S(y)\rangle \sim c_0 + \frac{c_1}{|x-y|^6}$$



N > 2
T
Bulk
not ord.
T_c
ordinary
Bulk
ordered
κ_c
$\kappa = K_1/K$
OR
T
T_c
ordinary
special
extra-ordinary
κ_c
$\kappa = K_1/K$
?

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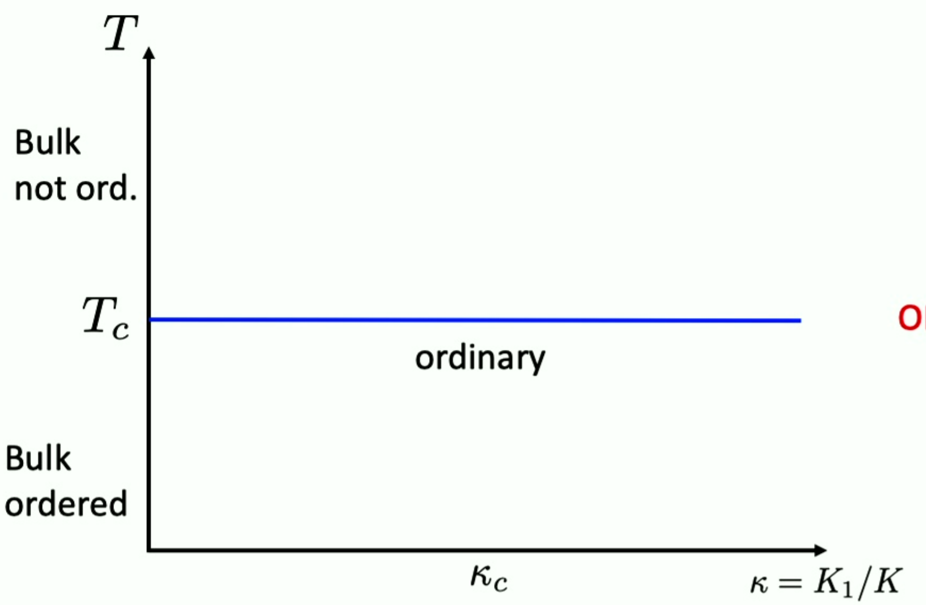
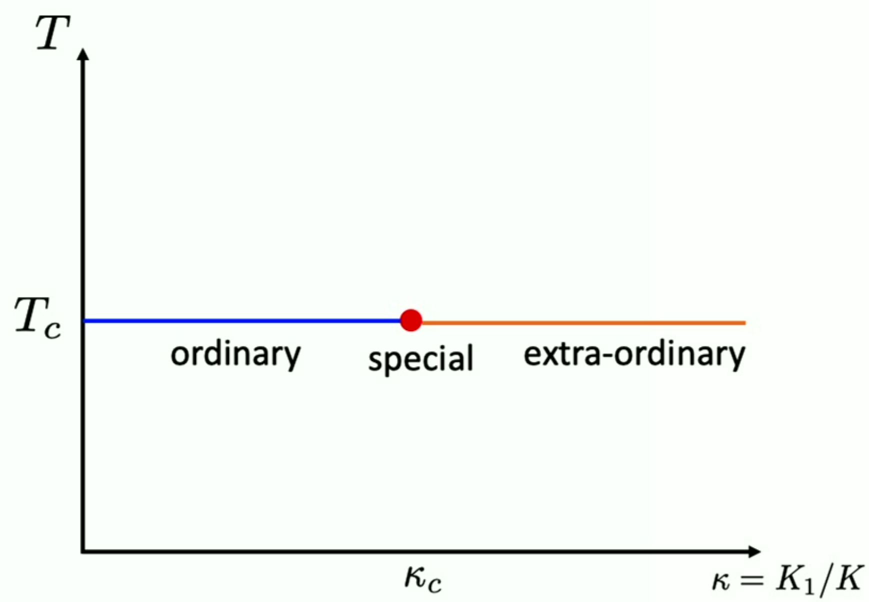
N > 2



OR



Large finite N

$$N \to 2^+$$

$$\langle \vec{S}(x) \cdot \vec{S}(0) \rangle \sim \frac{1}{(\log x)^q}$$

MM, 2020

RG

- Surface: $$S = \frac{1}{2g}\int d^2\mathrm{x}\,(\partial_\mu \vec{n})^2, \qquad \vec{n}^2 = 1$$

Polyakov:

$$\mathrm{x} \to e^{\ell}\mathrm{x} : \qquad \frac{dg}{d\ell} \approx \frac{N-2}{2\pi} g^2, \quad \vec{n} \to \left(1 - \frac{\eta_n(g)}{2} d\ell\right)\vec{n}, \qquad \eta_n \approx \frac{N-1}{2\pi} g.$$

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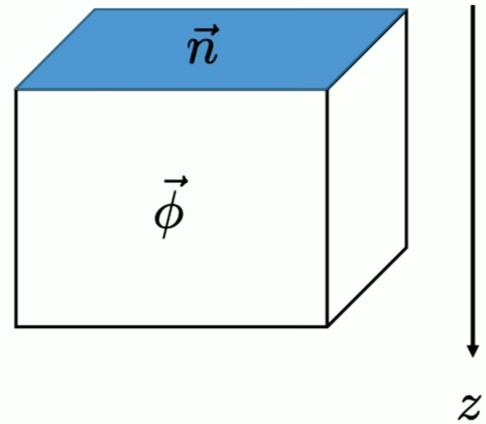
RG: adding the bulk

$$S_n = \frac{1}{2g} \int d^2\mathrm{x} \, (\partial_\mu \vec{n})^2$$

$$S = S_{ord}[\vec{\phi}] + S_n - \tilde{s} \int d^2\mathrm{x} \, \vec{n} \cdot \vec{\phi}(\vec{\mathrm{x}}, z = 0)$$



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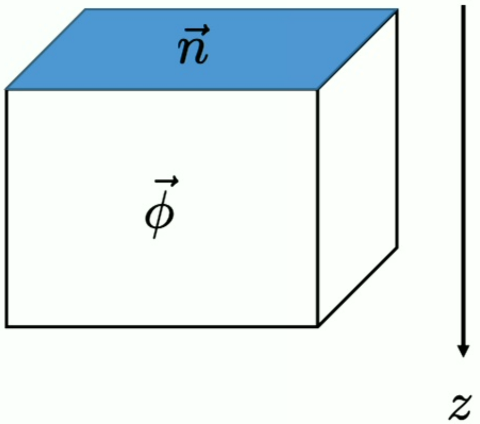
RG: adding the bulk

$$S_n = \frac{1}{2g}\int d^2\mathrm{x}\ (\partial_\mu \vec{n})^2$$

$$S = S_{ord}[\vec{\phi}] + S_n - \tilde{s}\int d^2\mathrm{x}\ \vec{n}\cdot\vec{\phi}(\vec{\mathrm{x}}, z=0)$$

$$\vec{n} = (\vec{\pi}, \sqrt{1-\vec{\pi}^2})$$



- $g = 0\,,\ \ \vec{n} = (\vec{0}, 1)$ - flows to normal universality class

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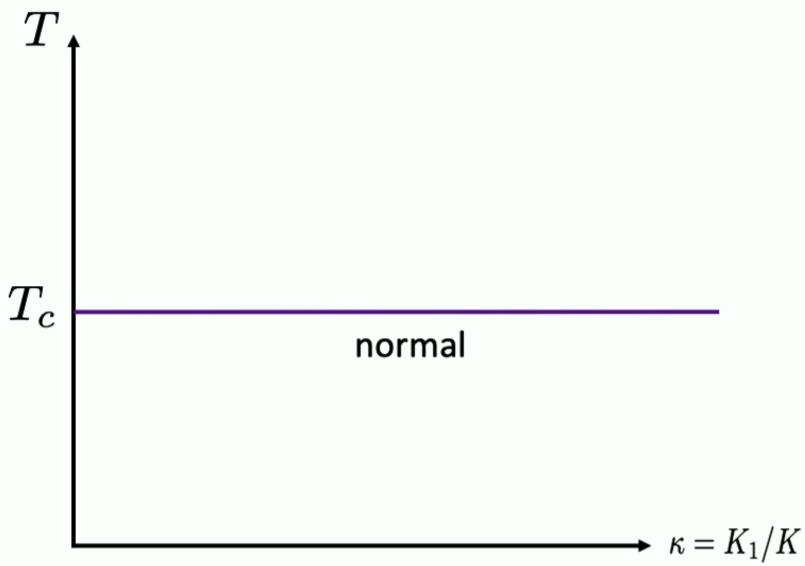
"Normal" universality class

$$H = -\sum_{\langle ij \rangle} K_{ij} \vec{S}_i \cdot \vec{S}_j - \sum_{i \in surf} \vec{h}_1 \cdot \vec{S}_i$$

$$O(N) \to O(N-1)$$



$$h_1 \neq 0$$

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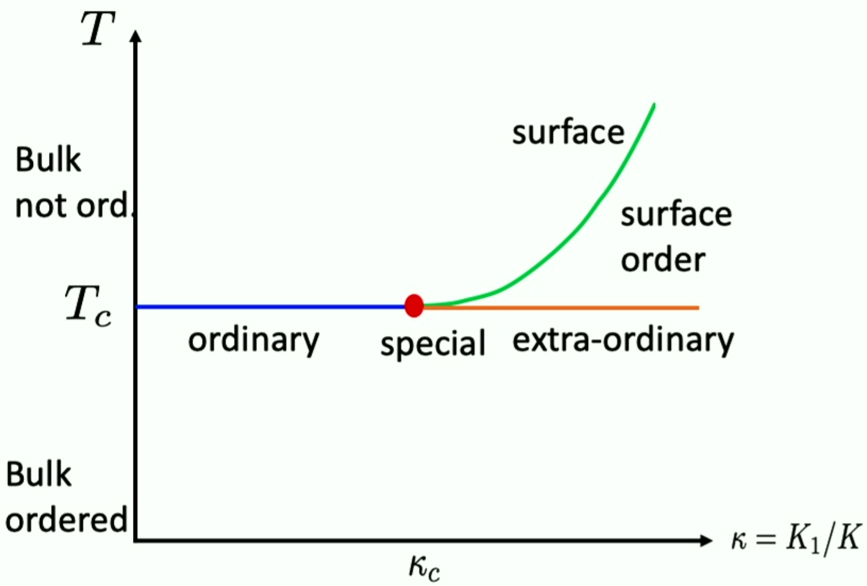
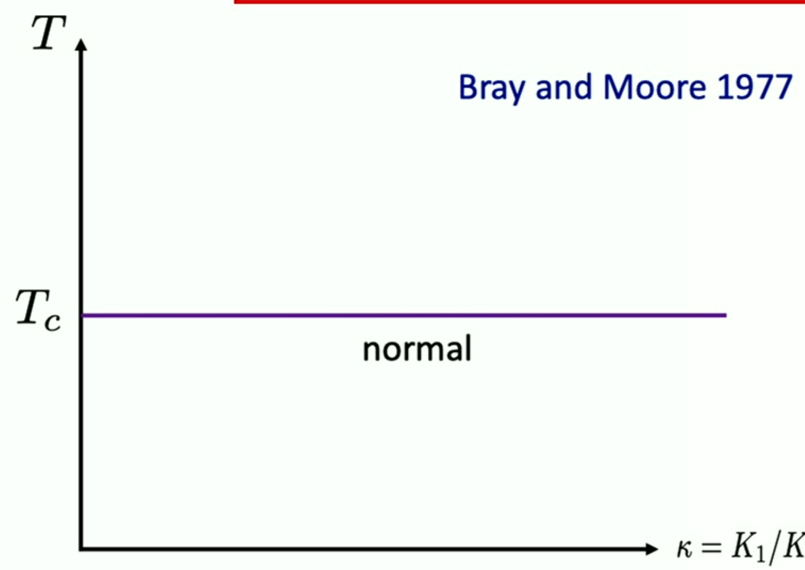
N = 1

$$H = -\sum_{\langle ij \rangle} K_{ij} \vec{S}_i \cdot \vec{S}_j - \sum_{i \in surf} \vec{h}_1 \cdot \vec{S}_i$$

N = 1:
Extra-ordinary = Normal



$h_1 = 0$



$h_1 \neq 0$

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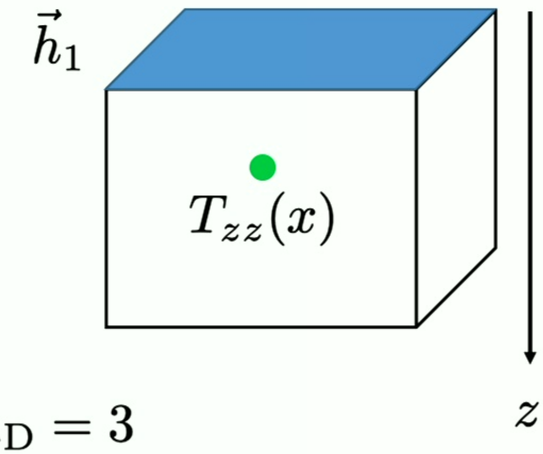
Normal universality class

$$O(N) \to O(N-1)$$



- **Protected boundary operators:**
  - Displacement: $$T_{zz}(\mathbf{x}, z \to 0) = \sqrt{C_D}\,\mathrm{D}(\mathbf{x}), \qquad \Delta_{\mathrm{D}} = 3$$
  - Tilt: $$j_z^{Ni}(\mathbf{x}, z \to 0) = s\,\mathrm{t}_i(\mathbf{x}), \qquad i = 1 \ldots N-1, \quad \Delta_{\mathrm{t}} = 2$$

Bray and Moore, 1977;
Burkhardt and Cardy, 1987.

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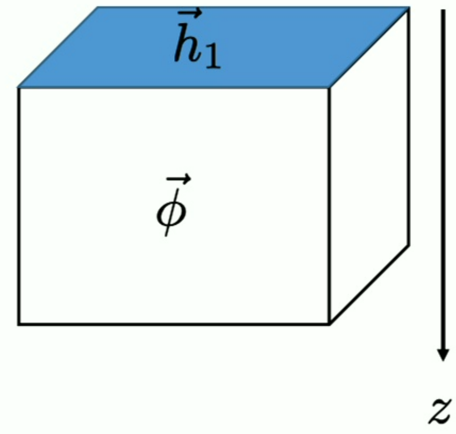
Ward identity

$$j_z^{Ni}(\mathbf{x}, z \to 0) = s\,\mathrm{t}_i(\mathbf{x}) \qquad \langle \mathrm{t}_i(\mathrm{x})\mathrm{t}_j(\mathrm{x}')\rangle = \frac{\delta_{ij}}{|\mathrm{x}-\mathrm{x}'|^4}$$



$$\phi_N(\mathbf{x}, z) = \frac{a_\sigma}{(2z)^{\Delta_\phi}} + b_{\mathrm{D}}(2z)^{3-\Delta_\phi}\mathrm{D}(\mathbf{x}) + \ldots, \quad z \to 0$$

$$\phi_i(\mathbf{x}, z) = b_{\mathrm{t}}(2z)^{2-\Delta_\phi}\mathrm{t}_i(\mathbf{x}) + \ldots, \quad i = 1 .. N-1, \quad z \to 0$$

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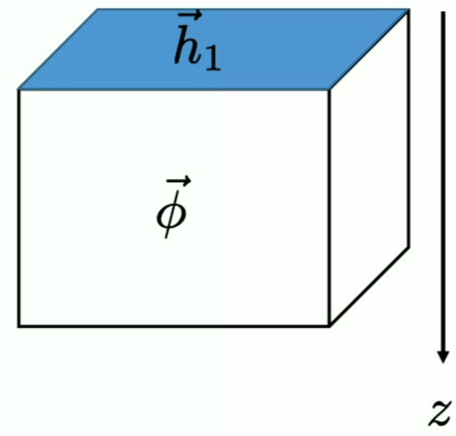
Ward identity

$$j_z^{Ni}(\mathbf{x}, z \to 0) = s\, \mathrm{t}_i(\mathbf{x}) \qquad \langle \mathrm{t}_i(\mathrm{x})\mathrm{t}_j(\mathrm{x}')\rangle = \frac{\delta_{ij}}{|\mathrm{x}-\mathrm{x}'|^4}$$



$$\phi_N(\mathbf{x}, z) = \frac{a_\sigma}{(2z)^{\Delta_\phi}} + b_{\mathrm{D}}(2z)^{3-\Delta_\phi}\mathrm{D}(\mathbf{x}) + \ldots, \quad z \to 0$$

$$\phi_i(\mathbf{x}, z) = b_{\mathrm{t}}(2z)^{2-\Delta_\phi}\mathrm{t}_i(\mathbf{x}) + \ldots, \quad i = 1\,..\,N-1, \quad z \to 0$$

$$\boxed{s = \frac{a_\sigma}{4\pi b_{\mathrm{t}}}}$$

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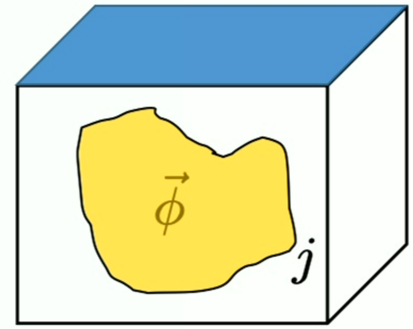
$$\phi_N(\mathbf{x}, z) = \frac{a_\sigma}{(2z)^{\Delta_\phi}} + b_{\mathrm{D}}(2z)^{3-\Delta_\phi}\mathrm{D}(\mathbf{x}) + \dots, \quad z \to 0$$



$$\phi_i(\mathbf{x}, z) = b_{\mathrm{t}}(2z)^{2-\Delta_\phi}\mathrm{t}_i(\mathbf{x}) + \dots, \quad i = 1..N-1, \quad z \to 0$$

$$\delta_\omega\langle\phi^a(x)\rangle = \omega^{ab}\langle\phi^b(x)\rangle = \frac{1}{2}\int_{y\in S} dS_\mu\,\omega^{bc}\langle\phi^a(x) j^{bc}_\mu(y)\rangle$$

$$j^{Ni}_z(\mathbf{x}, z \to 0) = s\,\mathrm{t}_i(\mathbf{x})$$

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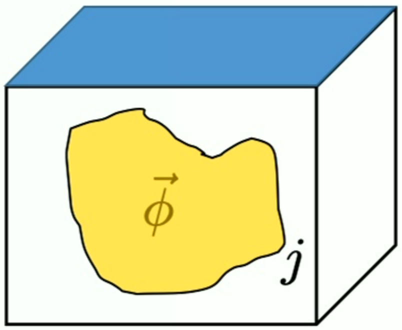
$$\phi_N(\mathbf{x},z) = \frac{a_\sigma}{(2z)^{\Delta_\phi}} + b_{\mathrm{D}}(2z)^{3-\Delta_\phi}\mathrm{D}(\mathbf{x}) + \ldots, \quad z\to 0$$



$$\phi_i(\mathbf{x},z) = b_{\mathrm{t}}(2z)^{2-\Delta_\phi}\mathrm{t}_i(\mathbf{x}) + \ldots, \quad i = 1..N-1, \quad z\to 0$$

$$\delta_\omega\langle\phi^a(x)\rangle = \omega^{ab}\langle\phi^b(x)\rangle = \frac{1}{2}\int_{y\in S} dS_\mu\,\omega^{bc}\langle\phi^a(x)j^{bc}_\mu(y)\rangle$$

$$j_z^{Ni}(\mathbf{x}, z\to 0) = s\,\mathrm{t}_i(\mathbf{x})$$

$$\langle\phi^i(\mathbf{x},z)\mathrm{t}^j(\mathbf{y})\rangle = b_{\mathrm{t}}\delta_{ij}\frac{(2z)^{2-\Delta_\phi}}{(|\mathbf{x}-\mathbf{y}|^2+z^2)^2}$$

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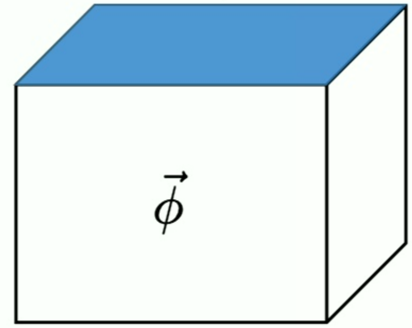
Adding fluctuations

$$\vec{n} = (\vec{\pi}, \sqrt{1 - \vec{\pi}^2})$$



$$S = S_{ord}[\vec{\phi}] + S_n - \tilde{s} \int d^2\mathrm{x}\; \vec{n} \cdot \vec{\phi}(\vec{\mathrm{x}}, z = 0)$$

$$\to S_{norm}[\vec{\phi}] + S_n - s \int d^2\mathrm{x}\, \pi_i \mathrm{t}_i$$

$$j_z^{N\,i}(\vec{\mathrm{x}}, z \to 0) = s\, \mathrm{t}^i(\vec{\mathrm{x}}), \qquad \langle \mathrm{t}^i(\vec{\mathrm{x}}) \mathrm{t}^j(0) \rangle = \frac{\delta^{ij}}{|\vec{\mathrm{x}}|^4}$$

MM, 2020. Jay Padayasi, Abijith Krishnan, MM, Ilya Gruzberg and Marco Meineri, 2021.

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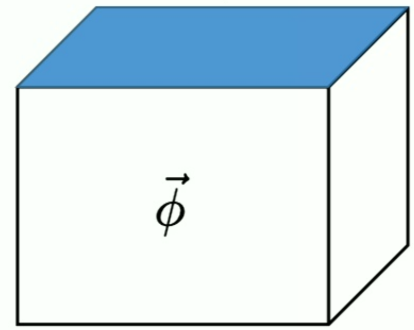
Adding fluctuations

$$\vec{n} = (\vec{\pi}, \sqrt{1-\vec{\pi}^2})$$



$$S = S_{ord}[\vec{\phi}] + S_n - \tilde{s}\int d^2\mathrm{x}\ \vec{n}\cdot\vec{\phi}(\vec{\mathrm{x}}, z=0)$$

$$\to S_{norm}[\vec{\phi}] + S_n - s\int d^2\mathrm{x}\,\pi_i \mathrm{t}_i$$

$$j_z^{N\,i}(\vec{\mathrm{x}}, z\to 0) = s\,\mathrm{t}^i(\vec{\mathrm{x}}), \qquad \langle \mathrm{t}^i(\vec{\mathrm{x}})\mathrm{t}^j(0)\rangle = \frac{\delta^{ij}}{|\vec{\mathrm{x}}|^4}$$

O(N) rotation: $\pi^i \to \pi^i + \omega^i, \qquad S_{norm} \to S_{norm} + \int d^2\mathrm{x}\,\omega^i j_z^{N\,i}(\vec{\mathrm{x}})$

MM, 2020. Jay Padayasi, Abijith Krishnan, MM, Ilya Gruzberg and Marco Meineri, 2021.

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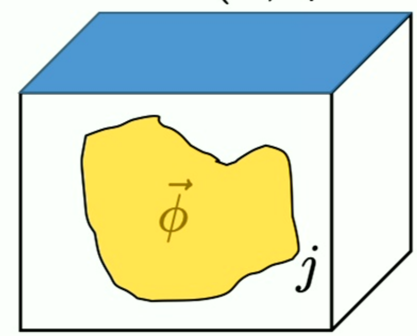
Adding fluctuations

$$\vec{n} = (\vec{\pi}, \sqrt{1-\vec{\pi}^2})$$



$$S = S_{ord}[\vec{\phi}] + S_n - \tilde{s}\int d^2\mathrm{x}\; \vec{n}\cdot\vec{\phi}(\vec{\mathrm{x}}, z=0)$$

$$\to S_{norm}[\vec{\phi}] + S_n - s\int d^2\mathrm{x}\,\pi_i \mathrm{t}_i$$

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O(N) rotation: $\pi^i \to \pi^i + \omega^i, \qquad S_{norm} \to S_{norm} + \int d^2\mathrm{x}\,\omega^i j_z^{N\,i}(\vec{\mathrm{x}})$

MM, 2020. Jay Padayasi, Abijith Krishnan, MM, Ilya Gruzberg and Marco Meineri, 2021.

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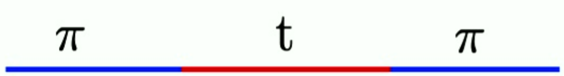
RG

$$S = S_{norm}[\vec{\phi}] + S_n - s \int d^2\mathrm{x}\, \pi_i \mathrm{t}_i$$

$$S_n = \frac{1}{2g} \int d^2x \left((\partial_\mu \vec{\pi})^2 + \frac{1}{1-\vec{\pi}^2} (\vec{\pi} \cdot \partial_\mu \vec{\pi})^2 \right)$$



$$\langle \mathrm{t}_i(\mathrm{x}) \mathrm{t}_j(\mathrm{x}') \rangle = \frac{\delta_{ij}}{|\mathrm{x}-\mathrm{x}'|^4}$$

$$\frac{dg}{d\ell} = -\alpha g^2 \qquad \alpha = \frac{\pi s^2}{2} - \frac{N-2}{2\pi} \qquad \eta_n = \frac{N-1}{2\pi} g$$

RG results

$$\frac{dg}{d\ell} = -\alpha g^2 \qquad \alpha = \frac{\pi s^2}{2} - \frac{N-2}{2\pi} \qquad \eta_n = \frac{N-1}{2\pi} g$$

- $\alpha > 0, \qquad g \to 0$

  - Extra-ordinary-log fixed point

$$\langle \vec{n}(x) \cdot \vec{n}(0) \rangle \sim \frac{1}{(\log x)^q}$$

$$q = \frac{N-1}{2\pi\alpha}$$

- $\alpha < 0, \qquad g = 0$ - unstable

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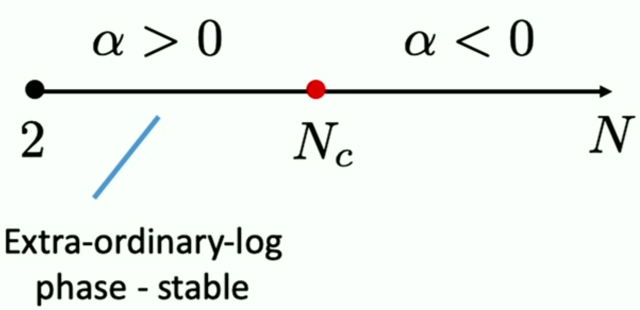
N_c

$$\frac{dg}{d\ell} = -\alpha g^2, \qquad \alpha = \frac{\pi s^2}{2} - \frac{N-2}{2\pi} \qquad s = \frac{a_\sigma}{4\pi b_t}$$

$$\alpha(N=2) = \frac{\pi s^2}{2} > 0$$

$$\alpha(N\to\infty) \approx -\frac{N-4}{4\pi} < 0$$



- Monte-Carlo: $N_c > 3$
- Bootstrap: $N_c \sim 5$

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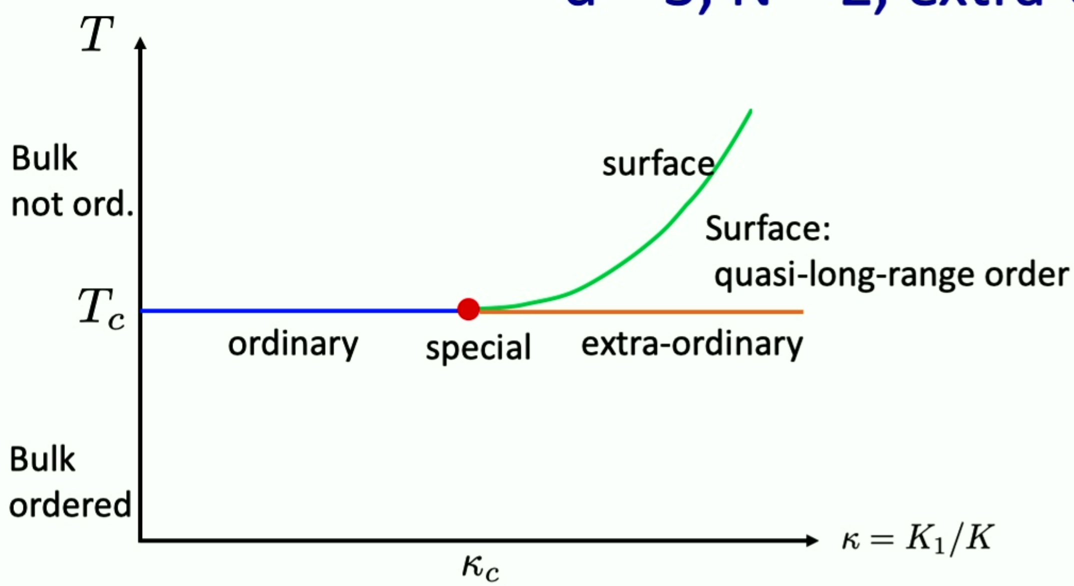
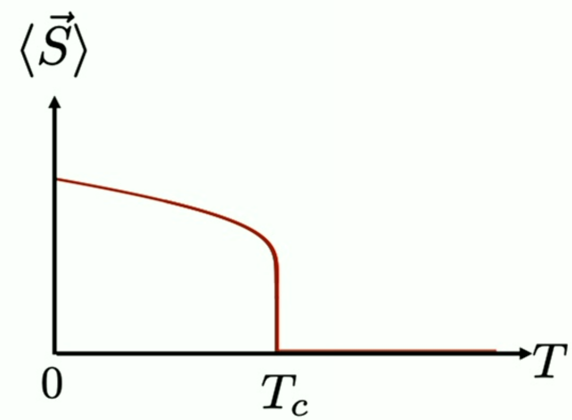
d = 3, N = 2, extra-ordinary



- $$\langle \vec{S}(x) \cdot \vec{S}(0) \rangle \sim \frac{1}{(\log x)^q}$$ "extra-ordinary-log"
MM, 2020.

$$q \approx 0.531(9)$$

M. Hu, Y. Deng, J.-P. Lv, 2021
Francesco Parisen Toldin, MM 2021



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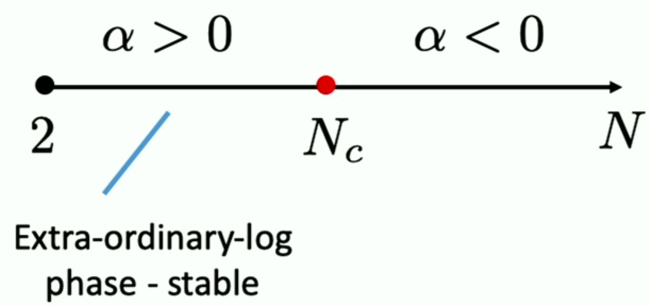
N_c

$$\frac{dg}{d\ell} = -\alpha g^2, \qquad \alpha = \frac{\pi s^2}{2} - \frac{N-2}{2\pi} \qquad s = \frac{a_\sigma}{4\pi b_t}$$

$$\alpha(N=2) = \frac{\pi s^2}{2} > 0$$

$$\alpha(N \to \infty) \approx -\frac{N-4}{4\pi} < 0$$



- Monte-Carlo: $N_c > 3$
- Bootstrap: $N_c \sim 5$

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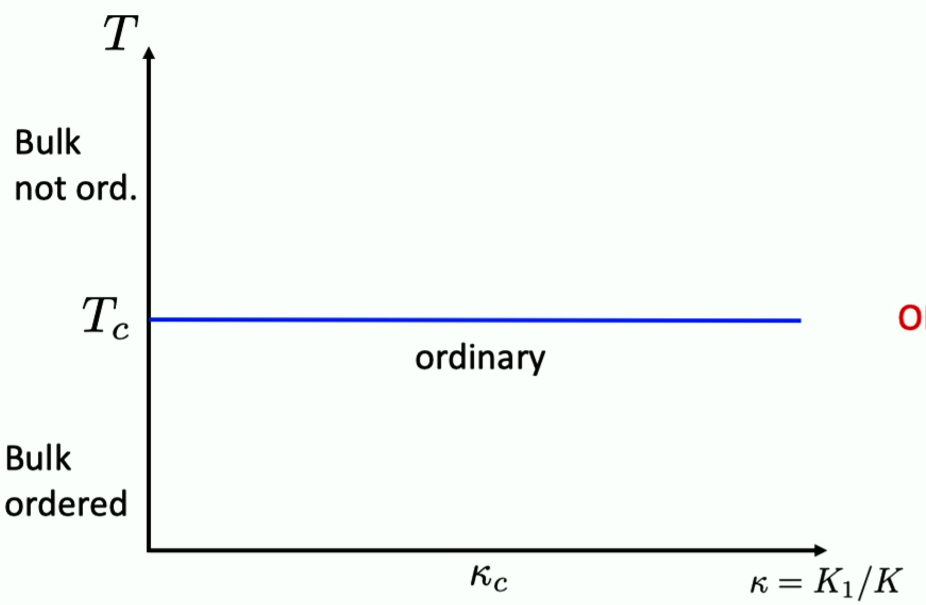
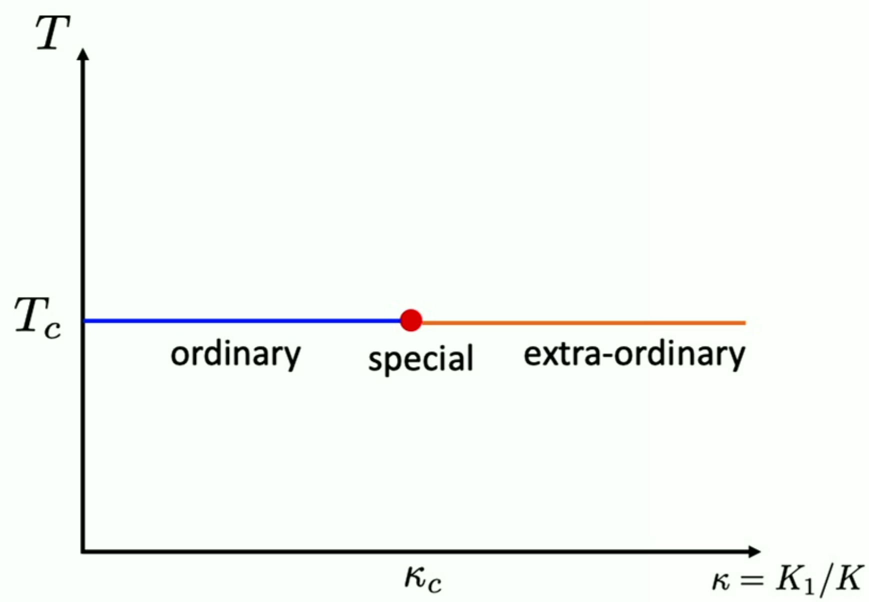
N > 2



OR



Large finite N

$$N \to 2^+$$

$$\langle \vec{S}(x) \cdot \vec{S}(0) \rangle \sim \frac{1}{(\log x)^q}$$

MM, 2020

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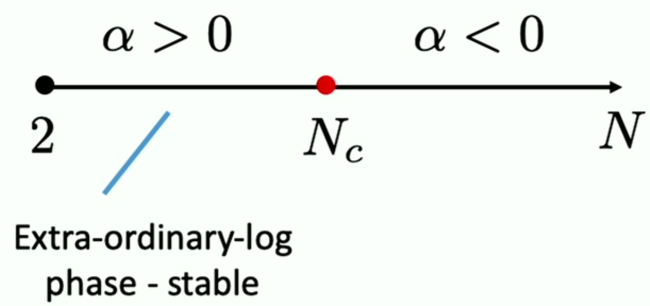
N_c

$$\frac{dg}{d\ell} = -\alpha g^2, \qquad \alpha = \frac{\pi s^2}{2} - \frac{N-2}{2\pi} \qquad s = \frac{a_\sigma}{4\pi b_t}$$

$$\alpha(N=2) = \frac{\pi s^2}{2} > 0$$

$$\alpha(N \to \infty) \approx -\frac{N-4}{4\pi} < 0$$



- Monte-Carlo: $N_c > 3$
- Bootstrap: $N_c \sim 5$

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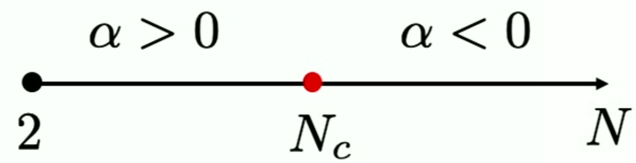
Near N_c

$$\frac{dg}{d\ell} \approx a(N - N_c)g^2 + bg^3, \qquad a > 0$$



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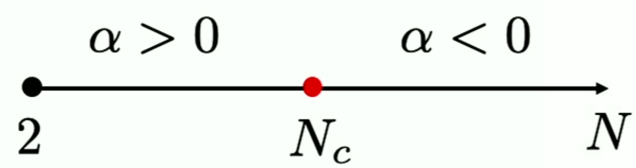
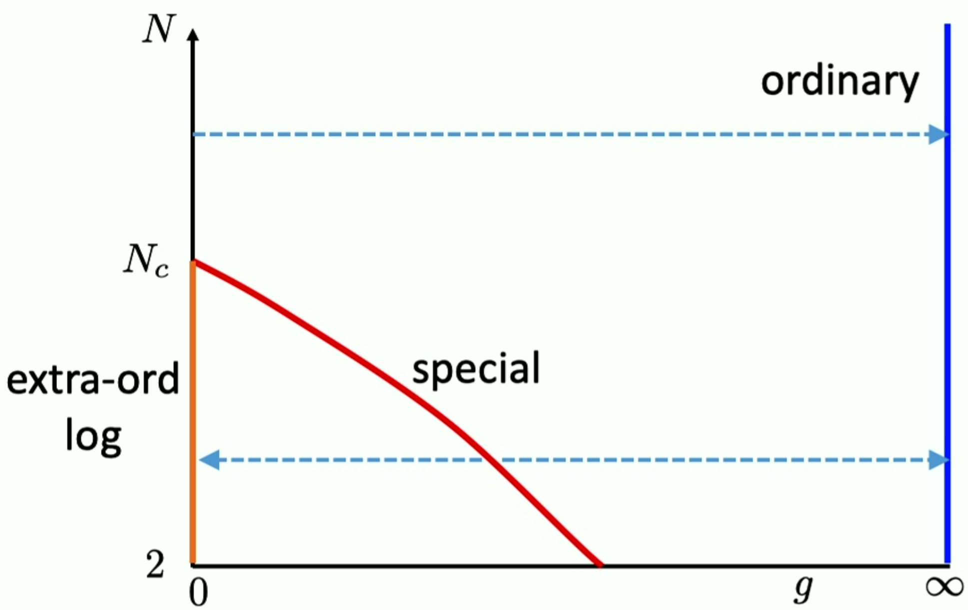
Near N_c

$$\frac{dg}{d\ell} \approx a(N - N_c)g^2 + bg^3, \qquad a > 0$$





Scenario I: $b > 0$

$$N \to N_c^-, \qquad g_*^{spec} \sim \frac{a(N_c - N)}{b}$$

$$\Delta_{\vec{n}} \approx \frac{N-1}{4\pi} g_*$$

$$\nu^{-1} = \frac{a^2(N_c - N)^2}{b}$$

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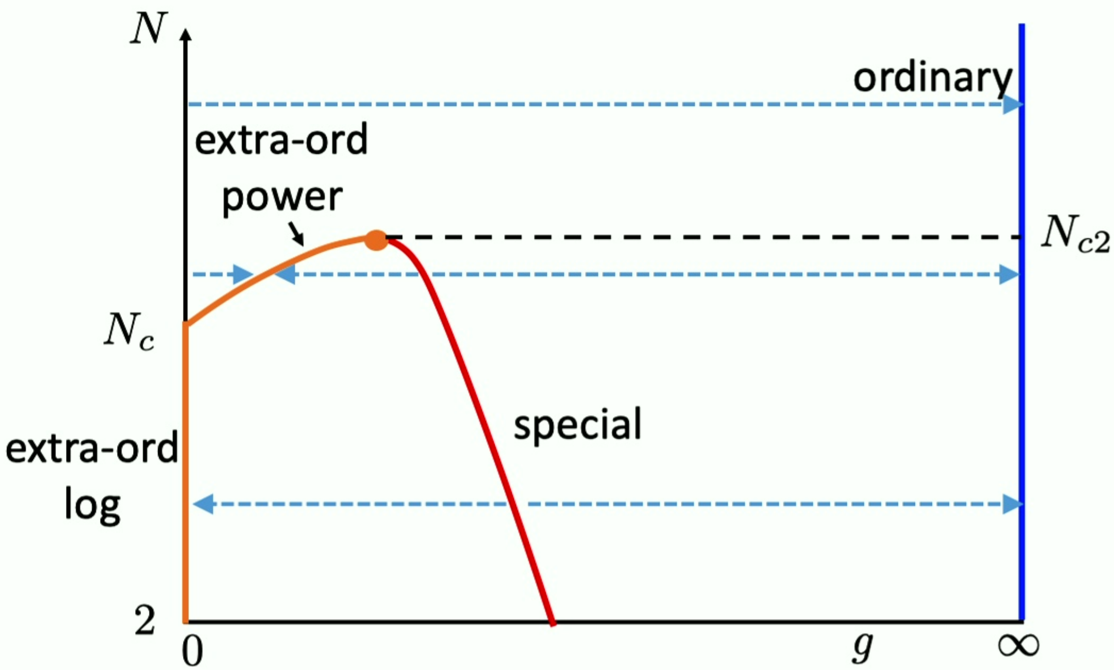
Near N_c

$$\frac{dg}{d\ell} \approx a(N - N_c)g^2 + bg^3, \qquad a > 0$$



Scenario II: $b < 0$

$$N \to N_c^+, \qquad g_* = \frac{a(N - N_c)}{|b|}$$

$$\Delta_{\vec{n}} \approx \frac{N-1}{4\pi} g_*$$

"Extra-ordinary-power" class.

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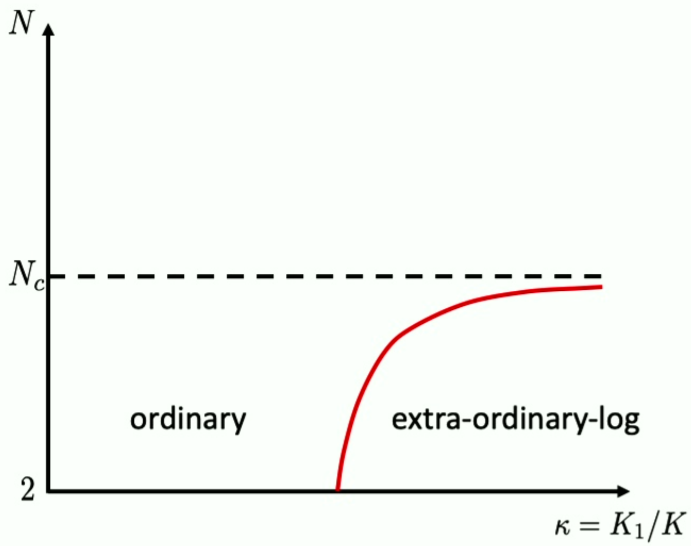
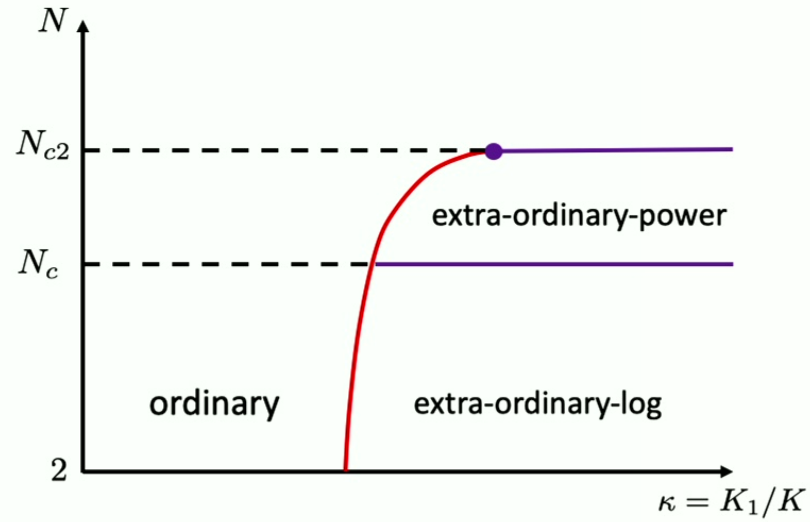


Scenario I



Scenario II

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Sign of b?

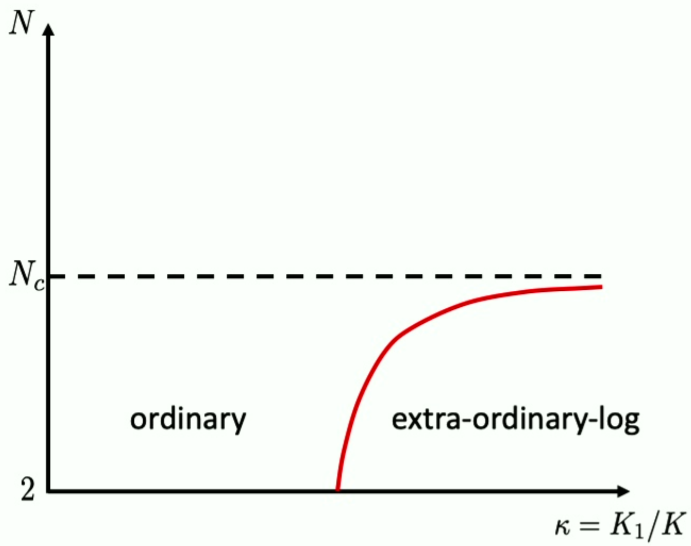
$$\frac{dg}{d\ell} = -\alpha g^2 + b g^3$$

$$(2\pi)^2 b \approx \left(\frac{4}{3} \pm 2.5 \cdot 10^{-3}\right) N + O(N^0)$$

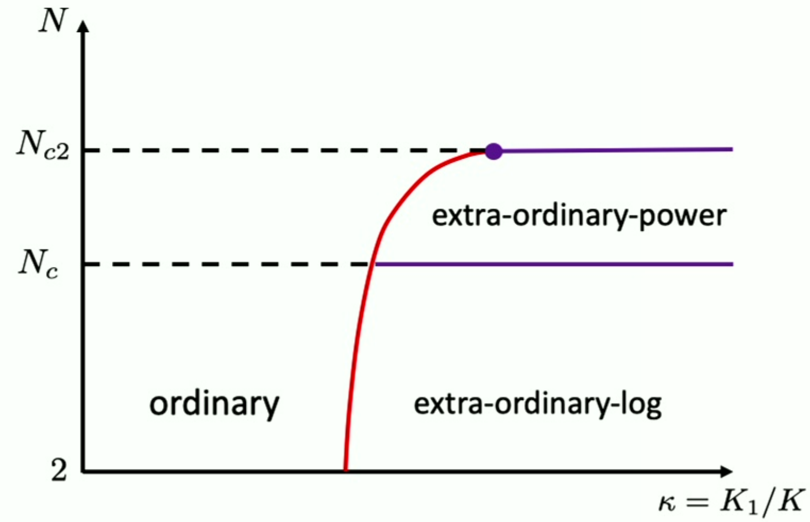
A. Krishnan, MM, 2023.

$$(2\pi)^2 b = (N-2)$$

- pure 2d nlσm



Scenario I



Scenario II

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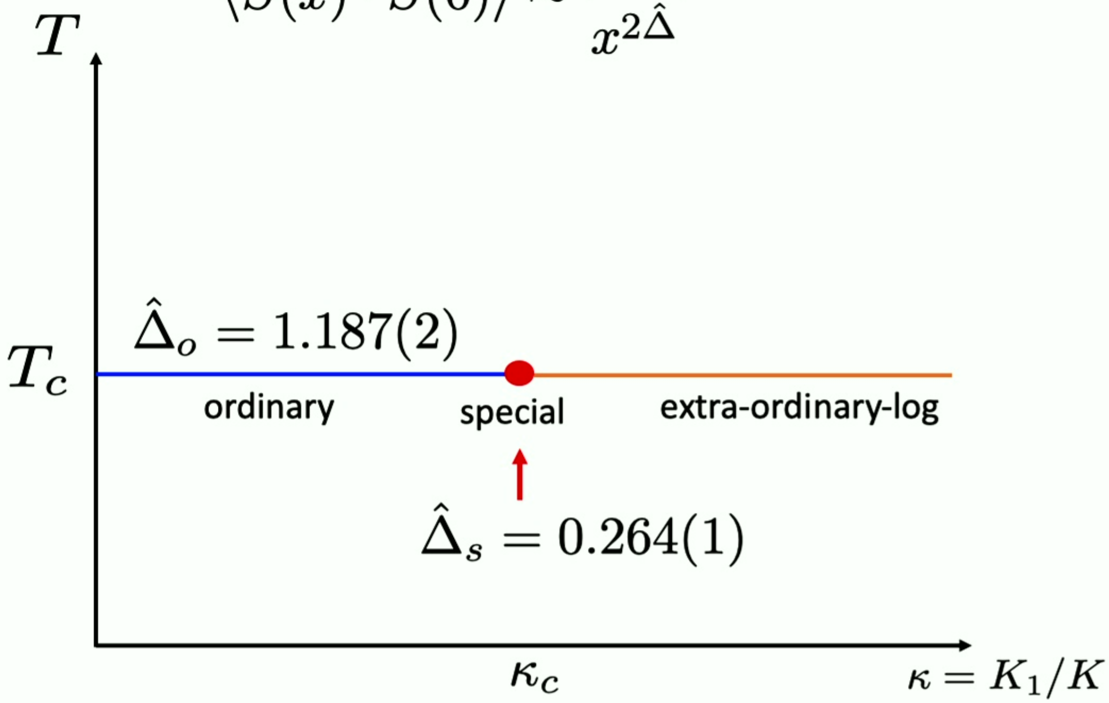
Monte Carlo: N = 3

$$\langle \vec{S}(x) \cdot \vec{S}(0) \rangle \sim \frac{1}{x^{2\hat{\Delta}}}$$



F. Parisen Toldin, Aachen, 2020

Deng et al, 2005

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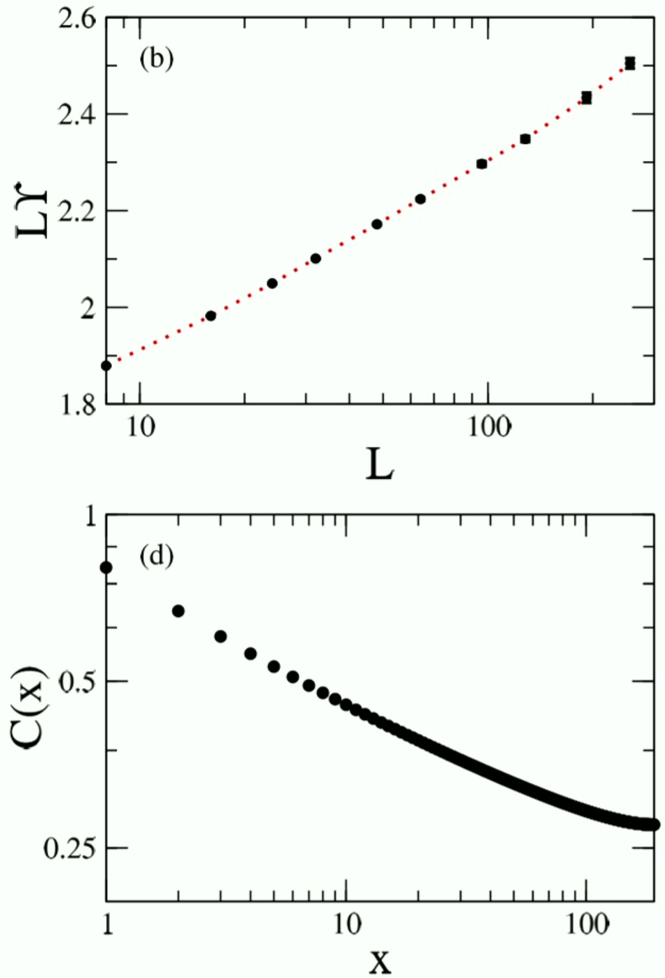
N = 3 – extra-ordinary log phase?



$$L\Upsilon = \frac{2}{Ng(L)} = \frac{2}{N}\left(\frac{1}{g} + \alpha \log L\right)$$

F. Parisen Toldin, 2020

$$C(x) = \langle \vec{S}(x) \cdot \vec{S}(0) \rangle \sim \frac{1}{(\log x)^q}$$

$$\alpha = 0.15(2)$$

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Monte Carlo

$$\alpha = \frac{\pi s^2}{2} - \frac{N-2}{2\pi} \qquad s = \frac{a_\sigma}{4\pi b_t}$$

N	α_{eo}^{MC}	α_{norm}^{MC}	α_{norm}^{boot}
2	0.27(1)	0.300(5)	0.36
3	0.15(2)	0.190(4)	0.22
4			0.13
5			0.02
10			-0.45

Francesco Parisen Toldin, 2021.

M. Hu, Y. Deng and J.-P. Lv, 2021.

Francesco Parisen Toldin, MM, 2021.

Jay Padayasi, Abijith Krishnan, MM, Ilya Gruzberg and Marco Meineri, 2021.

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Bootstrapping the normal universality class

Jay Padayasi
(Ohio State)

Marco Meineri
(U. Turin)

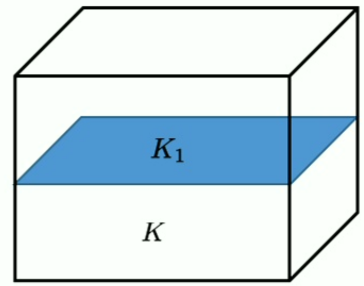
Abijith Krishnan
(MIT)

Ilya Gruzberg
(Ohio State)

Jay Padayasi, Abijith Krishnan, MM, Ilya Gruzberg and Marco Meineri, 2111.03071.

Plane defect, d = 3

$$H = -\sum_{\langle ij \rangle} K_{ij} \vec{S}_i \cdot \vec{S}_j$$



$$S = S_{\text{inf}} + c \int d^2 \mathrm{x}\, \epsilon(\mathrm{x}, z = 0), \quad c \sim K_c - K_1$$

$c = 0$ - special fixed point, exists for any N.

A.J. Bray, M.A. Moore, 1977

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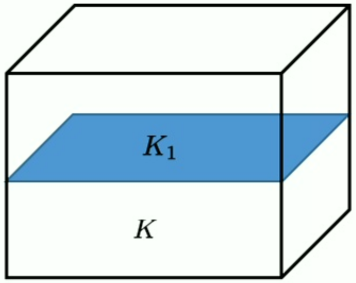
Plane defect in O(N) model



Abijith Krishnan
(MIT)

A. Krishnan, MM, 2023.

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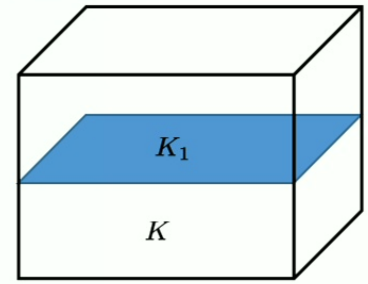
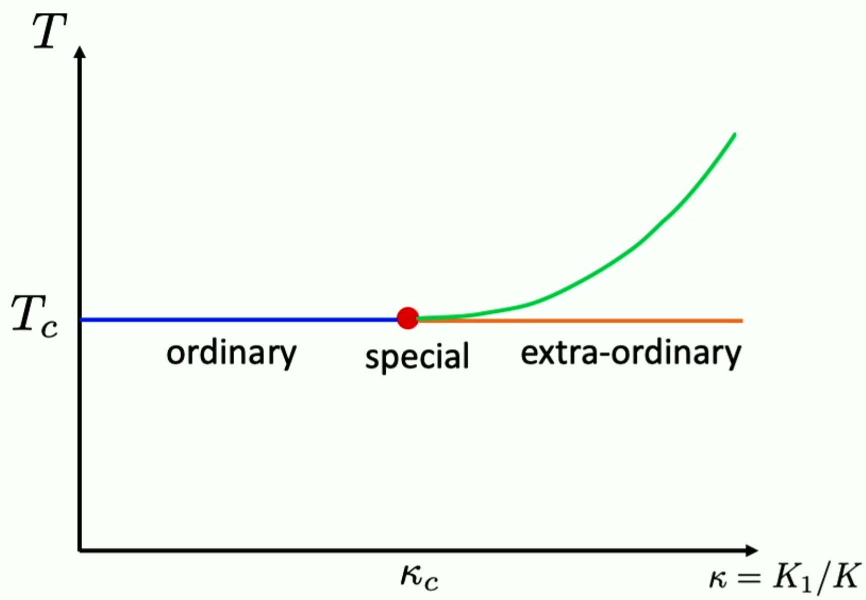
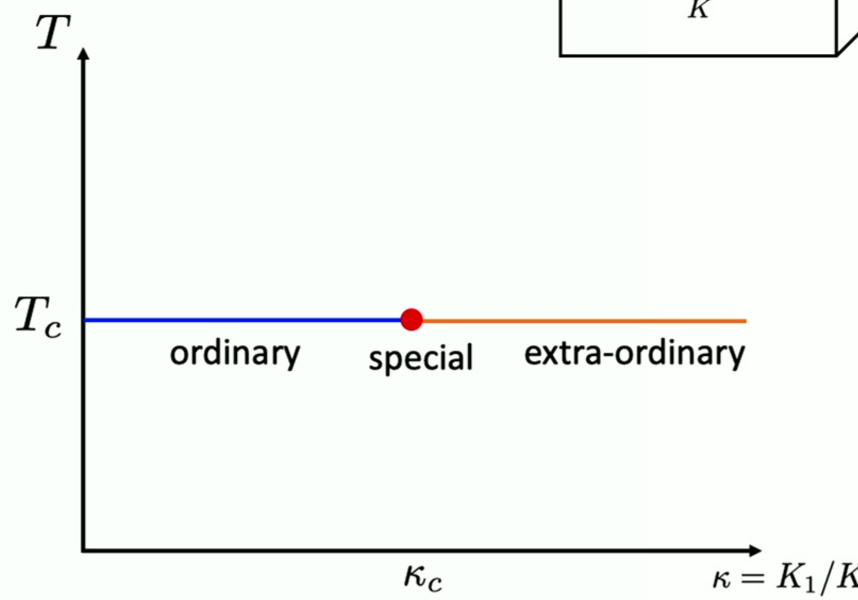
Plane defect, d = 3





$N = 1, 2$



$N > 2$

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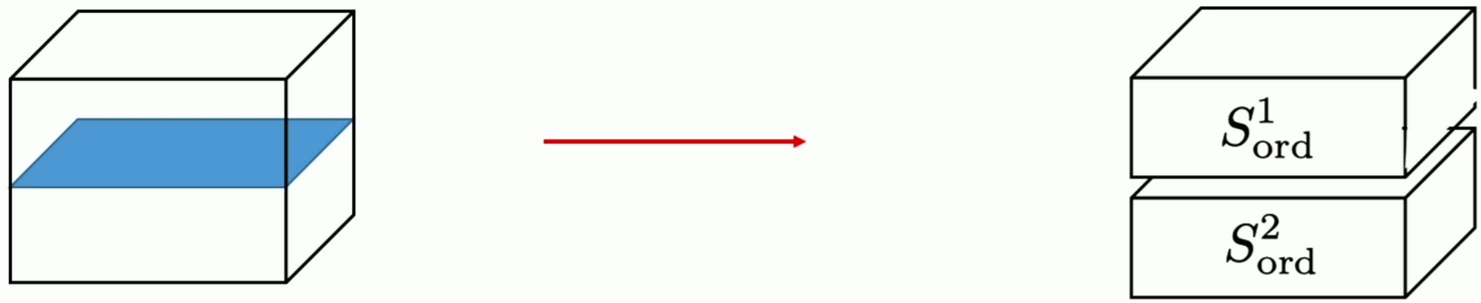
Plane defect, ordinary



$$S = S^1_{\mathrm{ord}} + S^2_{\mathrm{ord}} + u \int d^2\mathrm{x}\, \hat{\phi}^1_a \hat{\phi}^2_a$$

$$\Delta^{\mathrm{ord}}_{\hat{\phi}} = 1 + \frac{2}{3N} + O(N^{-2}) > 1$$

A.J. Bray, M.A. Moore, 1977

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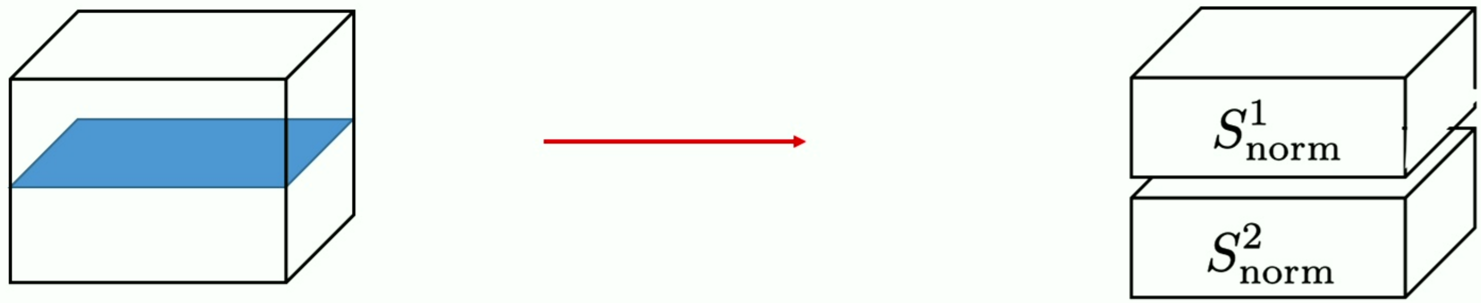
Plane defect, normal



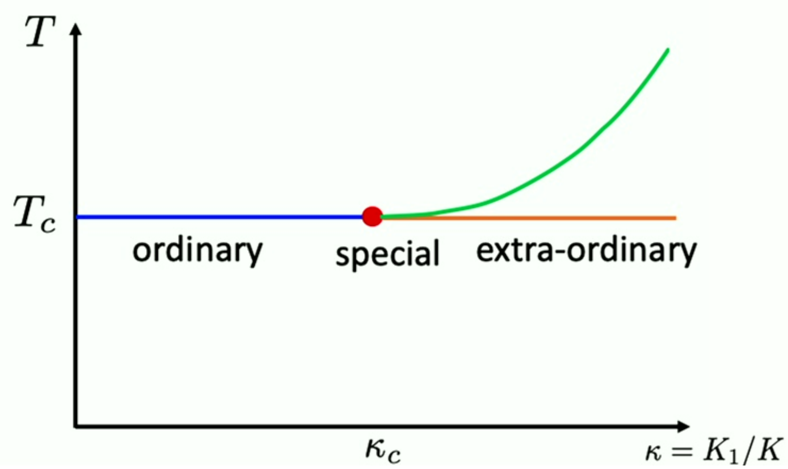
$$S = S^1_{\text{norm}} + S^2_{\text{norm}}$$

$$\delta S = u \int d^2x \, D_1 \, D_2$$

$$\delta S = v \int d^2x \, t^1_i \, t^2_i$$

A.J. Bray, M.A. Moore, 1977

- N = 1: extra-ordinary = normal



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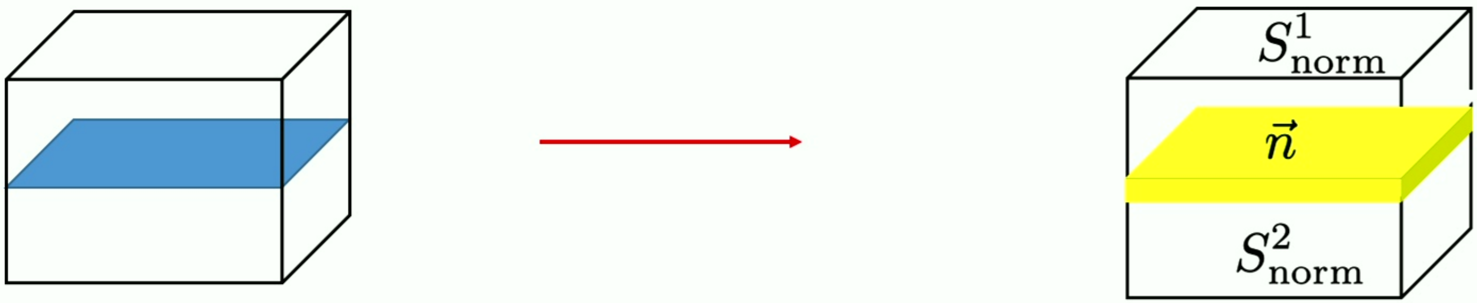
Extra-ordinary, N≥2



$$S = S^1_{\mathrm{norm}} + S^2_{\mathrm{norm}} + \frac{1}{2g}\int d^2x\,(\partial_\mu \vec{n})^2 - s\int d^2x\,\pi_i(\mathrm{t}^1_i + \mathrm{t}^2_i) \qquad s = \frac{a_\sigma}{4\pi b_{\mathrm{t}}}$$

$$\frac{dg}{d\ell} \approx -\alpha g^2, \qquad \alpha = 2\cdot\frac{\pi s^2}{2} - \frac{N-2}{2\pi}, \qquad \eta_n = \frac{(N-1)g}{2\pi}$$

$$\alpha(N) \to \frac{1}{\pi}, \quad N\to\infty$$

A. Krishnan, MM, 2023

N=2,3 Monte Carlo: Y. Sun, M. Hu, J.-P. Lv, 2023

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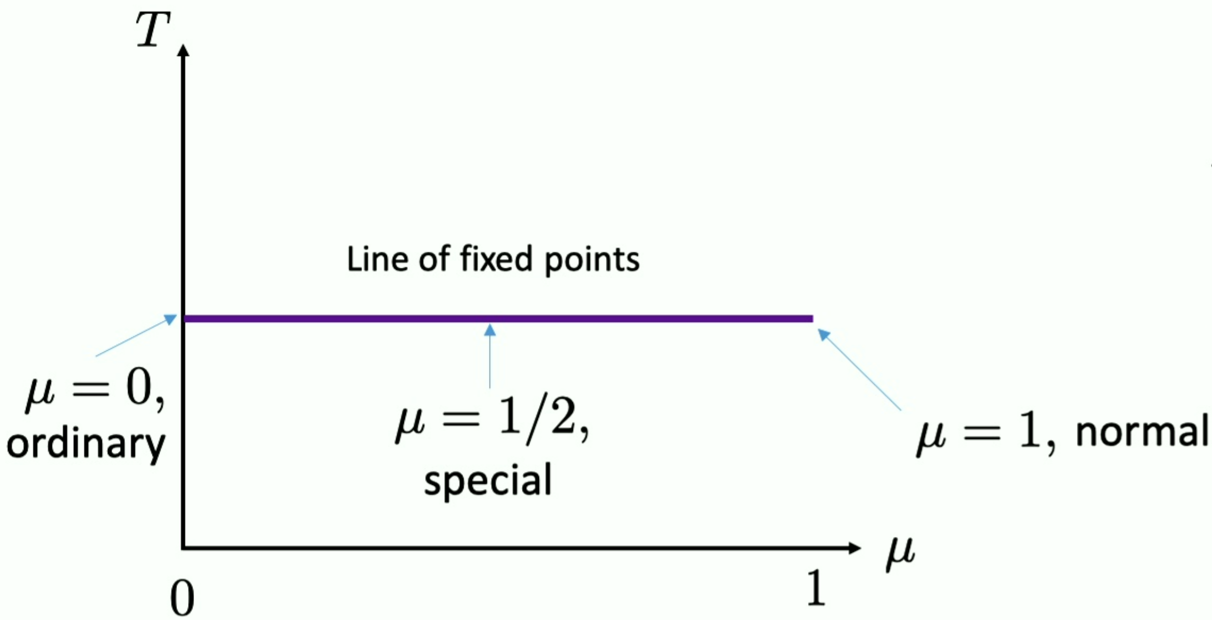
N = ∞



$$\hat{\Delta}_{\phi,S} = 1 - \mu$$

$$\hat{\Delta}_{\phi,A} = 1 + \mu$$

E. Eisenriegler, T.W. Burkhardt, 1982

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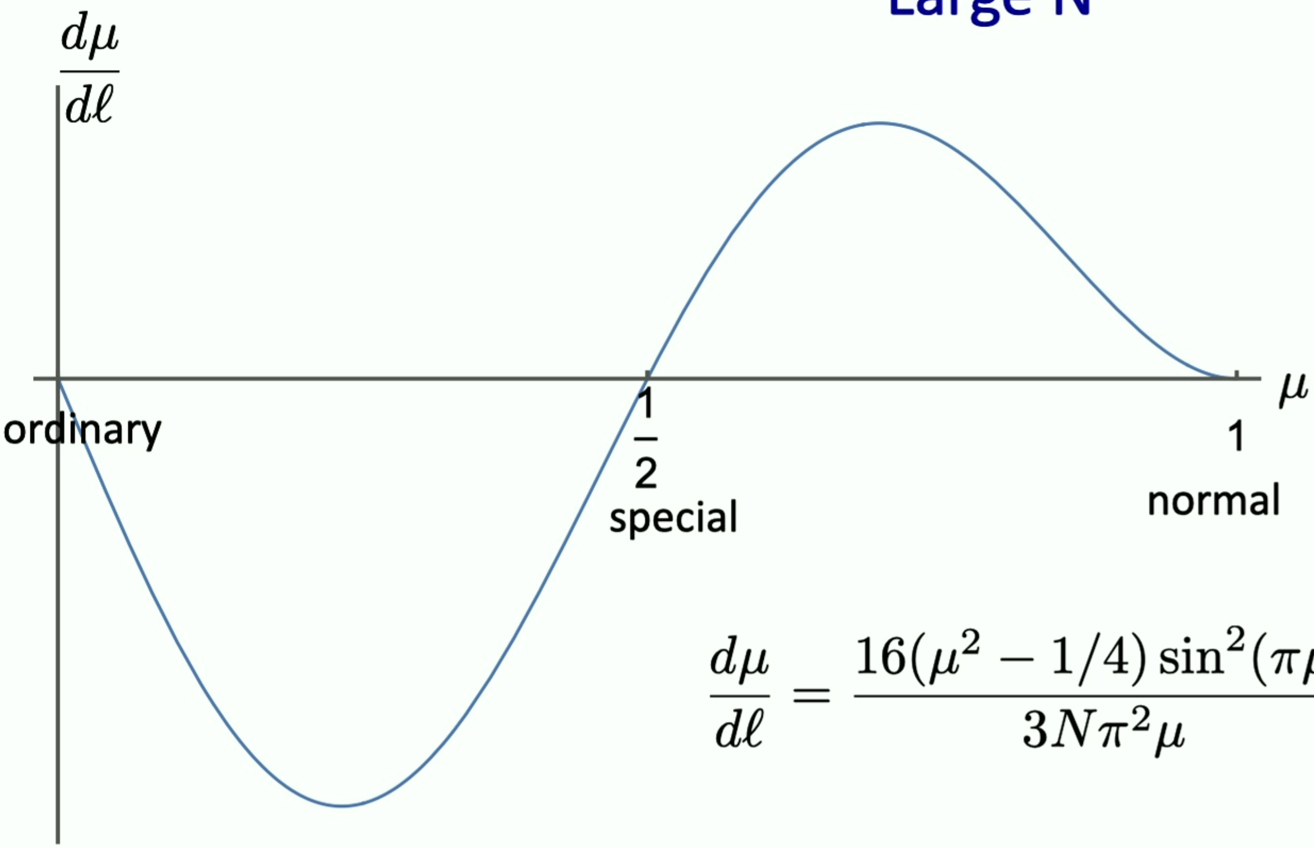
Large N



$$\hat{\Delta}_{\phi,S} = 1 - \mu$$

$$\hat{\Delta}_{\phi,A} = 1 + \mu$$

$$\frac{d\mu}{d\ell} = \frac{16(\mu^2 - 1/4)\sin^2(\pi\mu)}{3N\pi^2\mu}$$

$$\frac{dg}{d\ell} = -\alpha g^2, \quad \eta_n = \frac{(N-1)g}{2\pi}, \quad \alpha \to \frac{1}{\pi}, \quad N \to \infty$$

A. Krishnan, MM, 2023

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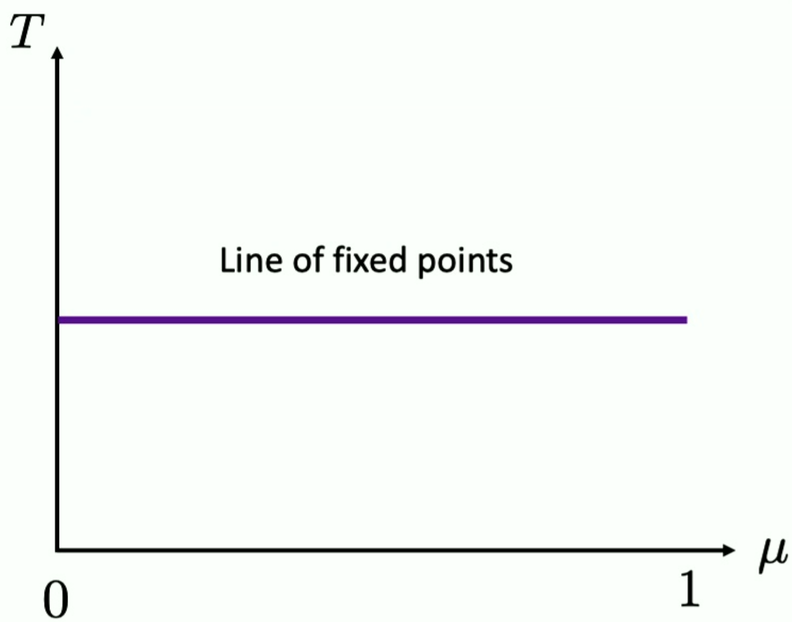
Boundary entropy

$$T^{\mu}_{\mu} = \frac{1}{24\pi}\delta(x_{\perp})(a_{3d}\hat{R} + b\,\mathrm{tr}\hat{K}^2)$$

K. Jensen, A. O'Bannon (2015)

$$N \to \infty : \ a^{\mathrm{plane}}_{3d}(\mu) = 0$$

A. Krishnan, MM, 2023; S. Giombi and H. Khanchandani (2021)



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Boundary entropy, finite N

$$T^{\mu}_{\mu} = \frac{1}{24\pi}\delta(x_{\perp})(a_{3d}\hat{R} + b\,\mathrm{tr}\hat{K}^2)$$ K. Jensen, A. O'Bannon (2015)

$$a^{sp}_{int} = 0$$

$$a^{O}_{bound} = -\frac{1}{16} + O(N^{-1}), \qquad a^{O}_{int} = -\frac{1}{8} + O(N^{-1})$$

$$a^{N}_{bound} = -\frac{N}{2} - \frac{1}{16} + O(N^{-1}), \qquad a^{eo}_{int} = 2a^{N}_{bound} + N - 1 = -\frac{9}{8} + O(N^{-1})$$

N-1 π fields

A. Krishnan, MM, 2023

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Boundary entropy, finite N

$$T^{\mu}_{\mu} = \frac{1}{24\pi}\delta(x_{\perp})(a_{3d}\hat{R} + b\,\mathrm{tr}\hat{K}^2)$$ K. Jensen, A. O'Bannon (2015)

$$a_{\mathrm{UV}} - a_{\mathrm{IR}} = 3\pi \int d^2\mathrm{x}\,\mathrm{x}^2 \langle \mathcal{T}(\mathrm{x})\mathcal{T}(0)\rangle_{\mathrm{c}}$$

$$a^{sp}_{int} = 0$$

$$a^{O}_{bound} = -\frac{1}{16} + O(N^{-1}), \qquad a^{O}_{int} = -\frac{1}{8} + O(N^{-1})$$

$$a^{N}_{bound} = -\frac{N}{2} - \frac{1}{16} + O(N^{-1}), \qquad a^{eo}_{int} = 2a^{N}_{bound} + N - 1 = -\frac{9}{8} + O(N^{-1})$$

A. Krishnan, MM, 2023

N-1 π fields

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Recent developments

- Can a continuous symmetry be spontaneously broken on a 2d defect in a (unitary) CFT?

  - Generically, "No"!
  - Generically, extra-ordinary-log is as close as you can get.

31 May 2023

Spontaneous symmetry breaking on surface defects

Gabriel Cuomo,[a,b] **Shuyu Zhang**[c]

[a] *Simons Center for Geometry and Physics, SUNY, Stony Brook, NY 11794, USA*
[b] *C. N. Yang Institute for Theoretical Physics, Stony Brook University, Stony Brook, NY 11794, USA*
[c] *Department of Physics and Astronomy, Stony Brook University, Stony Brook, NY 11794, USA*

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Recent developments

- Study of 2d defects in O(N) model in general d.

Avia Raviv-Moshe and Siwei Zhong, 2305.11370
Maxime Trepanier, 2305.10486
Simone Giombi and Bowei Liu, 2305.11402

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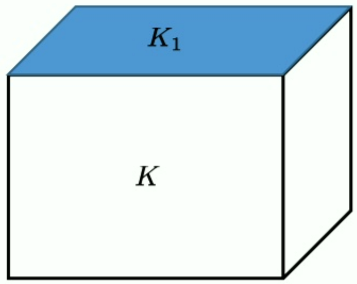
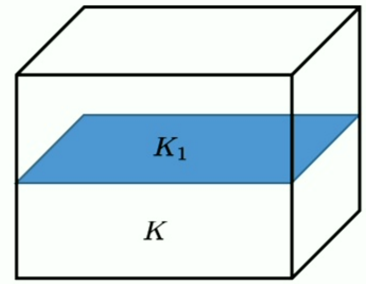
Conclusion



Boundary



Plane defect

- Monte-Carlo
- Conformal bootstrap
- Quantum models

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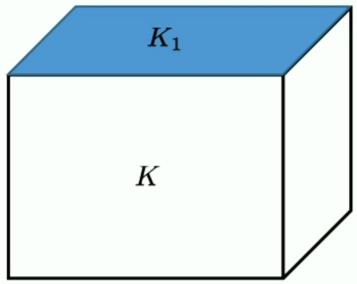
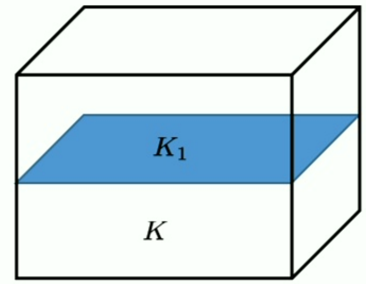
Conclusion



Boundary



Plane defect

- Monte-Carlo
- Conformal bootstrap
- Quantum models

Thank you!