Title: Effective metrics from group field theory

Speakers: Lisa Mickel

Series: Quantum Gravity

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Abstract: Group field theory (GFT) is a background independent approach to quantum gravity in which the quanta represent "building blocks of space". It has been successfully applied to the cosmological setting and shown to have the potential for rich phenomenology. In this talk we will explore a novel proposal to construct operators in GFT based on symmetries of the action, which can then be identified with respective classically conserved currents. This construction relies on a relational coordinate system spanned by four massless scalar fields. As the classical currents are related to the components of the metric, the expectation values of the new GFT operators can be interpreted as giving rise to an effective metric originating directly from the quantum theory. We proceed to investigate the implications of this proposition for a flat homogeneous universe and its perturbations.

Zoom link https://pitp.zoom.us/j/98843634192?pwd=OWs1SEdiWnIyYmlSbTh3bU9MaDBDdz09

EFFECTIVE METRICS FROM GROUP FIELD THEORY

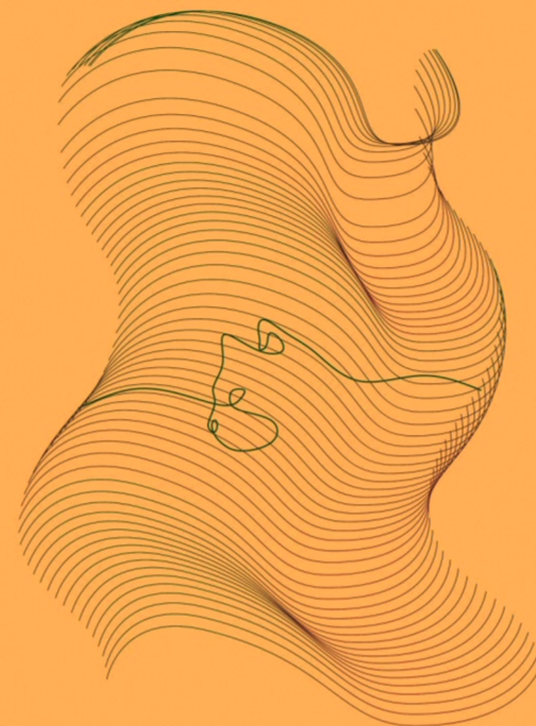
arXiv:2312.10016 and work in progress

Lisa Mickel, University of Sheffield
Supervisor: Steffen Gielen

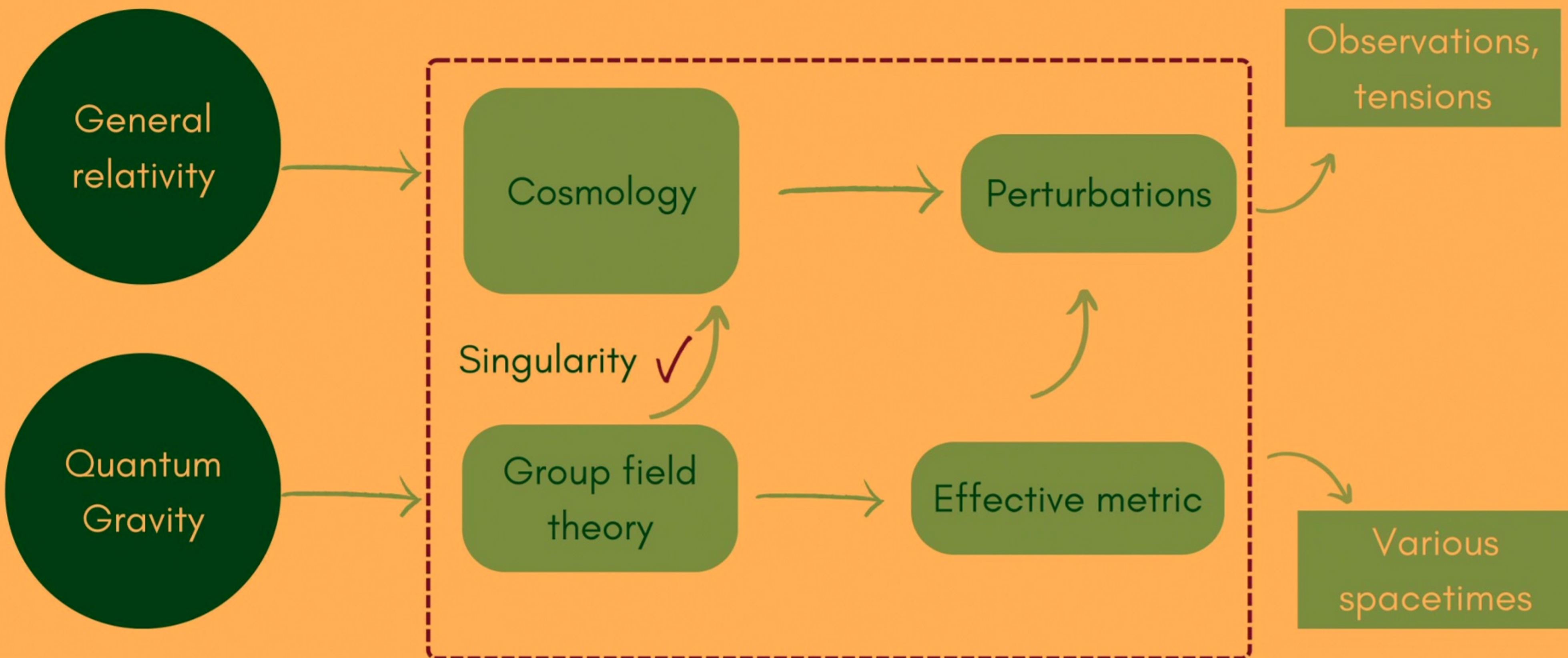
University of Sheffield

Quantum Gravity Seminar
Perimeter Institute
21st Dec 2023

PERIMETER INSTITUTE



OVERVIEW


General relativity
Quantum Gravity
Cosmology
Perturbations
Observations, tensions
Singularity ✓
Group field theory
Effective metric
Various spacetimes

CONTENT

- Group field theory and its application to cosmology
- Relational coordinate system: GFT operators and the effective metric
- Application to cosmology
  - Background
  - Perturbations

GROUP FIELD THEORY

- Field theory on a group manifold
  - bosonic group field $\varphi(g_i, \chi^A)$ real valued functions, group elements — e.g. $\varphi : \mathrm{SU}(2)^4 \times \mathbb{R}^4 \to \mathbb{R}$
- Field excitations: building blocks of spacetime
  - tetrahedron or spin network vertex

- Action
  - Laplacian on $\mathbb{R}^4$

$$S[\varphi] = \int \mathrm{d}^4\chi \left(\frac{1}{2} \sum_J \sum_{n=0}^{\infty} \mathcal{K}_J^{(2n)} \varphi_J(\chi^A) \Delta^n \varphi_J(\chi^A) - V(\varphi) \right)$$

- Macroscopic geometry: several building blocks

GROUP FIELD THEORY COSMOLOGY

- Relational dynamics: single massless scalar field χ^0 as a clock
- Effective scale factor from volume operator of loop quantum gravity $\langle \hat{V} \rangle = a^3$

- Dynamics
  - Mean field [Oriti, Sindoni, Wilson-Ewing ('16), ('17)]
    - Schwinger Dyson equations
  - Hamiltonian [Wilson-Ewing ('18)], [Gielen, Polaczek, Wilson-Ewing ('19)]
    - Deparametrisation w.r.t. clock field

- Effective Friedmann equation
  - Bounce
  - Recover GR at late times
- State dependence differs in approaches [Gielen, Calcinari ('22)]

EXTENDED GFT COSMOLOGY

- Other phenomenology:
  - Cyclic universe with interactions [de Cesare, Pithis, Sakellariadou, ('16)]
  - Dark energy with multiple field modes [Oriti, Pang, ('21)]
  - Anisotropic universe [Gielen, Calcinari, ('23)]
  - Perturbations [Marchetti, Oriti ('21) ('22)], [Jercher, Marchetti, Pithis ('23)]

RELATIONAL COORDINATE SYSTEM

- Couple four massless scalar fields $\varphi : \mathrm{SU}(2)^4 \times \mathbb{R}^4 \to \mathbb{R}$
  - Relate to coordinate system in which $\partial_\mu \chi^A = \delta^A_\mu$
  - One clock field χ^0, three 'spatial' fields $\chi^1 \; \chi^2 \; \chi^3$

- Studies in mean field approach [Marchetti, Oriti ('21) ('22)] [Jercher, Marchetti, Pithis ('23)]
  - Dynamics of the perturbed volume operator
  - Two-sector GFT: spacelike and timelike tetrahedra

Here:
Hamiltonian approach and additional operators

CONSERVATION LAWS FROM SYMMETRY

- Translational invariance $\chi^A \mapsto \chi^A + \epsilon^A$

- Classically conserved current from free scalar field action

$$(j^\mu)^A = -\sqrt{-g} g^{\mu\nu} \partial_\nu \chi^A = -\sqrt{-g} g^{\mu A}$$

- Klein Gordon equation

$$\partial_\mu (j^\mu)^A = 0$$

- GFT stress energy tensor

$$T^{AB} := -\frac{\partial \mathcal{L}}{\partial(\partial_A \varphi)} \partial_B \varphi + \delta^{AB} \mathcal{L}$$

- Simplifications: finite sum, no interactions

$$\mathcal{L} = \sum_J \left(\frac{1}{2} \mathcal{K}_J^{(0)} \varphi_J - \frac{1}{2} \mathcal{K}_J^{(2)} (\partial_A \varphi_J)^2 \right)$$

- Conservation law $\partial_A T^{AB} = 0$

- Identify in relational coordinate system $j^{AB} = \langle \mathcal{T}^{AB} \rangle$

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QUANTIZATION PROCEDURE

- Fourier decomposition w.r.t. spatial fields
- Hamiltonian approach [Gielen, Polaczek (`21)]
  - Promote field and its conjugate momentum to operators

$$[\varphi_{J,k}(\chi^0), \pi_{J',k'}(\chi^0)] = \mathrm{i}\,\delta_{JJ'}(2\pi)^3\delta(\vec{k}+\vec{k'})$$

$$\boxed{H = \int \frac{\mathrm{d}^3k}{(2\pi)^3} \sum_J \frac{\mathcal{K}_J^{(2)}}{2}\left(-\frac{1}{|\mathcal{K}_J^{(2)}|^2}\pi_{J,-k}(\chi^0)\pi_{J,k}(\chi^0) + \omega_{J,k}^2\,\varphi_{J,-k}(\chi^0)\varphi_{J,k}(\chi^0)\right)}$$

$$\boxed{\begin{aligned}\varphi_{J,k}(\chi^0) &= \frac{1}{2\alpha_{J,k}}(A_{J,k} + A_{J,-k}^\dagger) \\ \pi_{J,k}(\chi^0) &= -\mathrm{i}\alpha_{J,k}(A_{J,k} - A_{J,-k}^\dagger)\end{aligned}}$$

$$\alpha_{J,k} = \sqrt{\frac{|\omega_{J,k}||\mathcal{K}^{(2)}|}{2}}$$

$$\omega_{J,k}^2 = m_J^2 + \vec{k}^2$$

$$m_J^2 = -\frac{\mathcal{K}^{(0)}}{\mathcal{K}^{(2)}}$$

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- Two types of Hamiltonian:
  - squeezing $\omega_{J,k}^2 > 0$: $H_{J,k} = \mathrm{sgn}(\mathcal{K}_J^{(2)})\frac{|\omega_{J,k}|}{2}\left(a_{J,k}a_{J,-k} + a_{J,k}^\dagger a_{J,-k}^\dagger\right)$
  - oscillating $\omega_{J,k}^2 < 0$: $H_{J,k} = -\mathrm{sgn}(\mathcal{K}_J^{(2)})\frac{|\omega_{J,k}|}{2}\left(a_{J,-k}a_{J,-k}^\dagger + a_{J,k}^\dagger a_{J,k}\right)$

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QUANTIZE GFT ENERGY MOMENTUM TENSOR

- Focus on single J mode - generalisation possible
- Replace $\varphi_k,\, \pi_k$ with operators in classical Fourier decomposition $T^{AB} \to \mathcal{T}^{AB}$
- Impose normal ordering for $a_k,\, a_k^\dagger$
- Conservation law $\partial_0 : \mathcal{T}_k^{0B} : + \mathrm{i} \sum_a k_a : \mathcal{T}_k^{aB} : = 0$ holds at operator level also after normal ordering
  - Independent of state choice
  - Klein-Gordon equation holds exactly in GFT

- Can now construct effective metric for any sufficiently semiclassical state
  - State determines physical scenario

EFFECTIVE FLRW METRIC

- Apply this construction to cosmology: flat FLRW metric

$$\mathrm{d}s^2 = -N^2(t)\mathrm{d}t^2 + a^2(t)\mathrm{d}x^i\mathrm{d}x^j$$

- State determines physical scenario
  - Coherent state is sufficiently semiclassical

[Gielen, Polaczek (`20)]

$$a_k|\sigma\rangle = \sigma(\vec{k})|\sigma\rangle$$

  - Gaussian peaking function: almost homogeneous

$$\sigma(\vec{k}) = \frac{\mathcal{A} + \mathrm{i}\mathcal{B}}{c_\sigma}\, e^{-\frac{(\vec{k}-\vec{k}_0)^2}{2s^2}}$$

- Cosmology: Peak around homogeneous mode $\vec{k}_0 = 0$

Contrast to standard cosmology: No clear split into background and perturbations

EFFECTIVE FLRW METRIC

- Classical currents

clock field conjugate momentum

$$j^{AB} = \begin{pmatrix} |\pi_0| & 0 \\ 0 & -\frac{a^4}{|\pi_0|}\delta^{ab} \end{pmatrix}$$

- Identification with operator expectation values

$$\boxed{\begin{gathered} |\pi_0| = \langle \mathcal{T}_0^{00} \rangle = |m|(\mathcal{B}^2 - \mathcal{A}^2)\,, \qquad \langle \mathcal{T}_0^{0b} \rangle = 0, \qquad \langle \mathcal{T}_0^{ab} \rangle = 0 \\ a^4 = -|\pi_0| \langle \mathcal{T}_0^{aa} \rangle = m^2(\mathcal{B}^2 - \mathcal{A}^2)\left((\mathcal{A}^2 + \mathcal{B}^2)\cosh(2|m|\chi^0) - 2\mathcal{A}\mathcal{B}\sinh(2|m|\chi^0)\right) \end{gathered}}$$

  - Spatially flat metric
  - Lorentzian signature dependent on initial conditions

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  - Spatially flat metric
  - Lorentzian signature dependent on initial conditions

COMPARISON

- GFT Friedmann equation: $$H^2 = \frac{1}{4}m^2\left(1-\frac{|\pi_0|^4}{a^8}\right) \xrightarrow[\text{late times}]{} \frac{1}{4}m^2$$

- Classical Friedmann equation with a single scalar field $$H^2 = \frac{\kappa}{6}$$ $\kappa = 8\pi G$

- GFT with a clock field: $$H^2 = \frac{\kappa}{6}\left(1+\frac{v_0}{a^3}+\frac{\mathcal{Y}}{a^6}\right)$$ negative constant (initial condition)

  [Gielen, Polaczek (`20)]

- Effective Friedmann equation from volume operator $$a^3 = \langle\sigma|V|\sigma\rangle \propto \langle\sigma|N|\sigma\rangle$$

- Here: $$a^4 \propto \langle\sigma|N|\sigma\rangle$$

GR IN RELATIONAL FRAMEWORK

- GFT Friedmann equation: $$H^2 = \frac{1}{4}m^2\left(1 - \frac{|\pi_0|^4}{a^8}\right) \xrightarrow[\text{late times}]{} \frac{1}{4}m^2$$

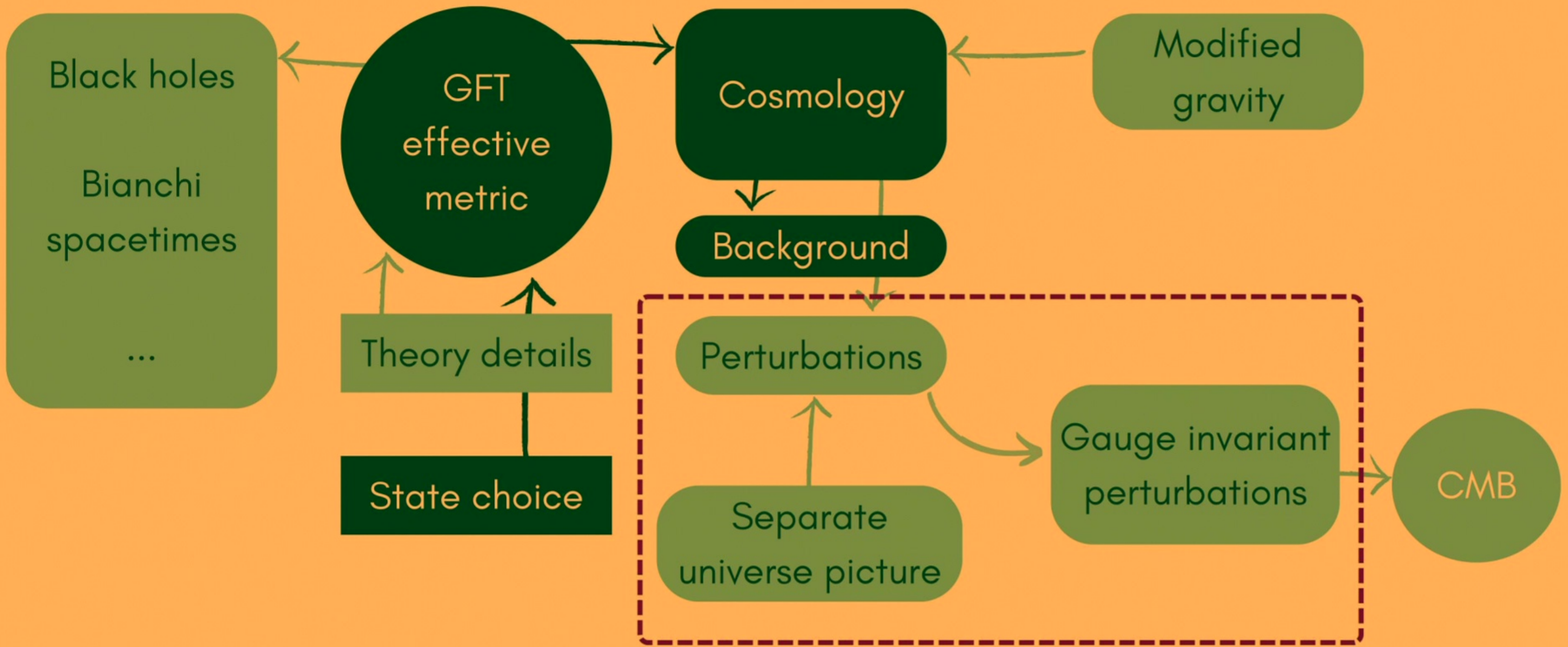
- Classically, the gradients of spatial fields change background Friedmann equation

$$H^2 = \left(\frac{a'}{a}\right)^2 = \frac{\kappa}{6}\left(1 + 3\frac{a^4}{\pi_0^2}\right), \qquad \frac{a''}{a} = \frac{\kappa}{6}\left(1 + 9\frac{a^4}{\pi_0^2}\right)$$

- GFT bounce at $a^4 = \pi_0^2$
  - Can only reproduce GR with single massless scalar field at late times
- Gradient terms absent also in other GFT approaches
  - Consistency with GFT literature

$\rightarrow$ Conflict between homogeneity on GFT side and relational coordinate system

BIGGER PICTURE



PERTURBATIONS FROM OPERATORS

- General relativity: Add small inhomogeneous perturbations on homogeneous background
  - Perturbed line element

$$\mathrm{d}s^2 = -N^2(1+2\tilde{\Phi})\mathrm{d}t^2 + 2Na\,\partial_i B\,\mathrm{d}t\mathrm{d}x^i + a^2\left((1-2\psi)\delta_{ij} + 2\partial_i\partial_j E\right)\mathrm{d}x^i\mathrm{d}x^j$$

- Effective metric identification gives ($k \neq 0$)

$$T_k^{00} = |\pi_0|(\tilde{\Phi} + 3\psi + k^2 E) \qquad T_k^{A\neq B} = 2\frac{a^4}{|\pi_0|}k_A k_B E$$

$$T_k^{AA} = \frac{a^4}{|\pi_0|}(\tilde{\Phi} - \psi - k^2 E + 2k_A^2 E) \qquad T_k^{0A} = \mathrm{i}\,a^2 k_A B$$

Access to all perturbative quantities

PERTURBATIONS AROUND THE BOUNCE

- Gaussian state: peaked on homogeneous mode
  - Study low k-modes

- Compare to

General relativity

Separate universe picture

CLASSICAL PERTURBATIONS

- Perturbed stress energy tensor

$$
{}^{(x)}\delta T^0_0 = \frac{\pi_0^2}{a^6}\tilde{\Phi} + \boxed{\frac{1}{a^2}\left(-3\psi + \nabla^2 E\right)}
$$

$$
{}^{(x)}\delta T^i_i = -\frac{\pi_0^2}{a^6}\tilde{\Phi} + \boxed{\frac{1}{a^2}\left(-\psi + (\nabla^2 - 2\partial_i^2)E\right)}
$$

Needs to vanish for adiabatic perturbations: $\frac{\delta P}{\delta \rho} = \frac{P'}{\rho'}$

- Non-adiabatic perturbations in GR with relational coordinate system
  - Additional terms not negligible, as bounce happens at $a^4 = \pi_0^2$

COMPARING DYNAMICS

- Consider e.g. $$T_k^{A\neq B} = 2\frac{a^4}{|\pi_0|}k_A k_B E$$

Classical	Quantum
$k_A k_B E'' = -k_A k_B \frac{a^4}{\pi_0^2}\left(2\kappa E + k^2 E\right)$	$k_A k_B E'' = k_A k_B \left(E\left(k^2 - 7H^2\right) - 8HE' - \frac{C_k\lvert\pi_0\rvert}{4a^4}\right)$

- Different small wavelength limit (sign of k^2): Euclidean signature
- Different overall factor a^4

PERTURBATIONS AND GAUGE

[Gielen, LM, ('23)]

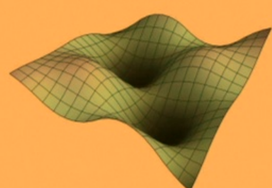
- Perturb Friedmann equation at linear order: $H^2 = \frac{\kappa}{3}N^2\rho\mathcal{F}$

$$\downarrow$$

$$H\psi' = -H^2\left(\tilde{\Phi} + \frac{\delta\rho}{2\rho} + \frac{\delta\mathcal{F}}{2\mathcal{F}}\right)$$

Lapse perturbation

- Gauge freedom: choice of perturbed coordinate system



- Gauge invariant variables
  - Comoving curvature perturbation

$$\boxed{\mathcal{R} = \psi + \frac{H}{\phi'}\delta\phi}$$

  - Curvature perturbation on equal density hypersurfaces

$$\boxed{-\zeta = \psi + \frac{H}{\rho'}\delta\rho}$$

COMPARING TO SU RESULTS

- Statements about long wavelength perturbations from Friedmann equation only
  - Consistency check
    - E.g. conservation law for adiabatic perturbations for ζ

$$\zeta = \psi + \frac{\tilde{\Phi}}{3} \approx \frac{T^{00}_{\mathrm{SU}}}{3\pi_0} \approx \text{const.}$$

- Connection to other theories with modified Friedmann equations?

CONCLUSION

- GFT is a background independent approach to quantum gravity and has been applied to cosmology in various ways
- By introducing new operators that allow to reconstruct an effective metric we establish a new connection to GR

- Application to cosmology: recover Friedmann equation for single scalar field in GR
- Cosmological perturbations

SIMPLIFICATIONS

- Used various simplifications to make progress
  - Single mode
    - Phenomenology, particularly in early universe?
  - Interactions
    - Dominate at late times – effect on Friedmann equation? [Gielen, Polaczek (`20)]

OUTLOOK

- Fifth dominating matter field [Marchetti, Oriti ('21) ('22)], [Jercher, Marchetti, Pithis ('23)]
    - Intermediate regime with agreeing Friedmann equations?
- State choices
    - Peaked around different value
    - Other spacetimes
- Connection to observables: study gauge invariant variables to connect to the CMB


THANK YOU!
Questions...?