Title: A Two-Part Exploration: A Foundational Topic + an Applicational Topic in the Context of Loop Quantum Gravity

Speakers: Sepideh Bakhoda

Series: Quantum Gravity

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Abstract: In this presentation, I will discuss two distinct topics, one relating to the foundational aspects of LQG and the other concerning its applicational implications.
Firstly, I will explore the U(1)^3 model of Euclidean Quantum Gravity, which serves as an interesting testing ground for the dynamics problem in LQG. With its analogous constraint structure to full gravity, the U(1)^3 model may hold the key to enhanced quantization techniques.

Secondly, I will delve into the asymptotic symmetries of General Relativity in the Ashtekar-Barbero formulation. New parity conditions for the Ashtekar-Barbero variables will be proposed, which do produce non-trivial supertranslation charges at spatial infinity. This development paves the way for investigating the quantum characteristics of supertranslation charges within the context of LQG.

Zoom link https://pitp.zoom.us/j/99532986538?pwd=cnU0VnpJbjU4TSt4MEEzVngxb2wvdz09

A Two-Part Exploration:
a Foundational Topic + an Applicational Topic
in the Context of Loop Quantum Gravity

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Content

- Foundational Aspect:

  The $U(1)^3$ Model of Euclidean Quantum Gravity

- Applicational Aspect:

  Boundary Conditions for Ashtekar-Barbero Variables with Supertranslations at i^0

Introduction and Motivation

- Phase space variables: $q_{ab}(x), \pi^{ab}(x), \quad \{q, \pi\} \sim \delta$
- Dynamics along the time flow generated by Hamiltonian $\boldsymbol{H}$.
- Useful to decompose time flow into components

$$\vec{t} = \underbrace{N}_{\text{Lapse}} \vec{n} + \underbrace{\vec{N}}_{\text{Shift}}$$

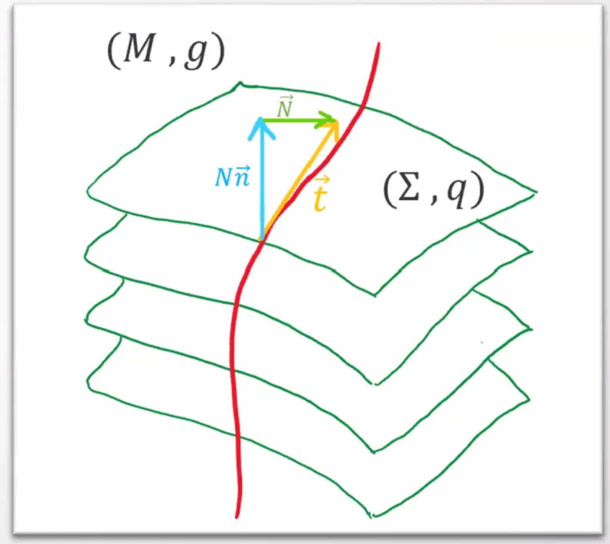
- Hamiltonian is combination of two generators

$$\boldsymbol{H} = \int_\Sigma d^3x \big(N \underbrace{H}_{\text{Hamiltonian constraint}} + N^a \underbrace{H_a}_{\text{Diffeomorphism constraint}}\big) + \cancel{\textbf{Boundary terms}}$$

$H_a[N^a]$ generates diffeos within spatial hypersurface Σ
$H[N]$ generates transversal displacements of Σ

- Classical evolution is governed by **the Hamiltonian constraint**.
  In quantum theory:
  the corresponding operator to $H[N]$ controls the dynamics.

Dirac '58; Arnowitt, Deser, Misner '62



ADM; Isham & Kuchar '85; Lee, Wald '90

Hypersurface deformation algebra:

$$\{H_a[N^a], H_b[M^b]\} = H_a[\mathcal{L}_{\vec{N}} M^a]$$
$$\{H_a[N^a], H[M]\} = H[\mathcal{L}_{\vec{N}} M]$$
$$\{H[N], H[M]\} = H_a[q^{ab}(N\partial_b M - M\partial_b N)]$$

Introduction and Motivation

- $H[N]$ is a very complicated function of q, π both for **Lorentzian** and **Euclidean** gravity.
- It has a simpler form in **Ashtekar-Barbero variables**

Sen, '82; Ashtekar '86; Barbero '95

E_i^a : densitized spatial triad ($i = 1,2,3$)

spatial metric information

A_a^i : conjugate connection ($i = 1,2,3$)

extrinsic curvature information

Immirzi parameter

Signature

$$H[N] = \int_\Sigma d^3x\ \tilde{N}\left[F_{ab}^i - (\beta^2 - s)\epsilon_{imn}K_a^m K_b^n\right]\epsilon_{ijk}E_j^a E_k^b \xrightarrow[\beta=1]{s=1} H[N] = \int_\Sigma d^3x\ \tilde{N}\ F_{ab}^i \epsilon_{ijk} E_j^a E_k^b$$

Euclidean Hamiltonian constraint

- Since spacetime is itself dynamical, LQG aims to construct this Hamiltonian constraint operator without relying on any fixed background spacetime. We need new ideas and techniques beyond those of QFT in fixed, flat spacetime.
- These were developed through early pioneering contributions

Jacobson, Smolin, Rovelli, Gambini and later by Blencowe, Pullin, Bruegmann, Borissov, Ashtekar, Lewandowski, Loll and etc.

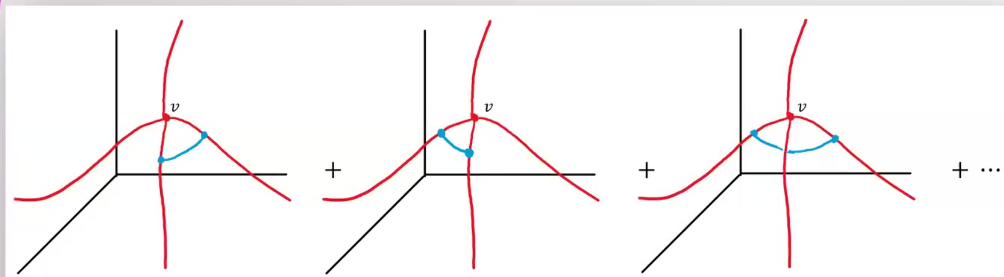
and Thiemann's construction of the **Euclidean** constraint operator in his **QSD papers**. Thiemann '98

- Thiemann showed how to construct the **Lorentzian** operator from the **Euclidean** one and the Volume operator. It is for this reason that we restrict attention to the **Euclidean** Hamiltonian constraint. Thiemann '98

Introduction and Motivation

Although the QSD construction of the Euclidean constraint operator is a great achievement, some open problems remain:

1. Many ambiguities in final operator action.
2. Constraint commutator $\left[\widehat{H}[N], \widehat{H}[M]\right]$ does not reproduce correct quantized structure functions.



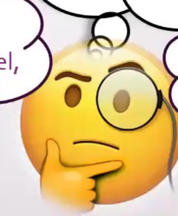
Algebraic Quantum Gravity
Thiemann, Giesel '07, '10

Master Constraint Programme
Thiemann '06
Thiemann, Dittrich '06

Spin Foam Models
Rovelli, Perez, Dittrich, Freidel, Vidotto, Many others

....

Toy Models



To improve the construction, working on simpler models sharing essential feature of GR can provide significant insight and primary directions for future progress.

1- Parametrized Field Theory Kuchar '89 Varadarajan '07; Laddha, Varadarajan '08, '10, '11; Thiemann '10; Thiemann '22

2- Husain-Kuchar Model Husain, Kuchar '90 Laddha, Varadarajan '11

Structure constants

3- **Smolin's $U(1)^3$ model** Smolin '92 Tomlin, Varadarajan '13; Varadarajan '13, '19; Handerson, Laddha, Tomlin '13; SB, Thiemann '21, '22; SB, Shojaie, Thiemann '21; Long, Ma '21; Thiemann '22

Structure functions

What is the $U(1)^3$ model? [Smolin; 92]

Constraints in Euclidean Gravity:

$$\mathcal{G}_i[\Lambda^i] = 2\int_\Sigma d^3x\, \Lambda^i\left(\partial_a E_i^a + \cancel{\epsilon_{ijk}A_a^j E_k^a}\right)$$

$$H_a[N^a] = -2\int_\Sigma d^3x\, N^a\left(F_{ab}^i E_i^b - A_a^i \mathcal{G}_i\right)$$

$$H[N] = \int_\Sigma d^3x \tilde{N}\; F_{ab}^i \epsilon_{ijk} E_j^a E_k^b$$

$$F_{ab}^i = 2\partial_{[a}A_{b]}^i + \cancel{\epsilon_{ijk}A_a^j A_b^k}$$

Constraints in Smolin's $\mathbf{U(1)^3}$ Model:

$$\boxed{\begin{gathered}\mathcal{G}_i[\Lambda^i] = 2\int_\Sigma d^3x\, \Lambda^i(\partial_a E_i^a) \\ H_a[N^a] = -2\int_\Sigma d^3x\, N^a\left(F_{ab}^i E_i^b - A_a^i \partial_b E_i^b\right) \\ H[N] = \int_\Sigma d^3x \tilde{N}\; F_{ab}^i \epsilon_{ijk} E_j^a E_k^b\end{gathered}}$$

$$\boxed{F_{ab}^i = 2\partial_{[a}A_{b]}^i}$$

Features:

- Gauge Group: $U(1)^3$
- Hypersurface deformation algebra between Ham. and diff. constraints is the same as that of GR
- It involves structure functions
- Constraints are at most linear in A

Towards Quantization

- ❑ Dirac quantization of this model has already been studied in great detail using Electric Shift method

Tomlin, Varadarajan '13
Varadarajan '13, '22
Ashtekar, Varadarajan '21

Main Idea:
Reason for the successfully implementation of the diffeomorphism constraint: intuition

$$H_a[N^a] = \int d^3x \, N^a (F^i_{ab} E^b_i - A^i_a \partial_b E^b_i) = \int d^3x \, E^a_i (\mathcal{L}_{\vec{N}} A^i_a)$$

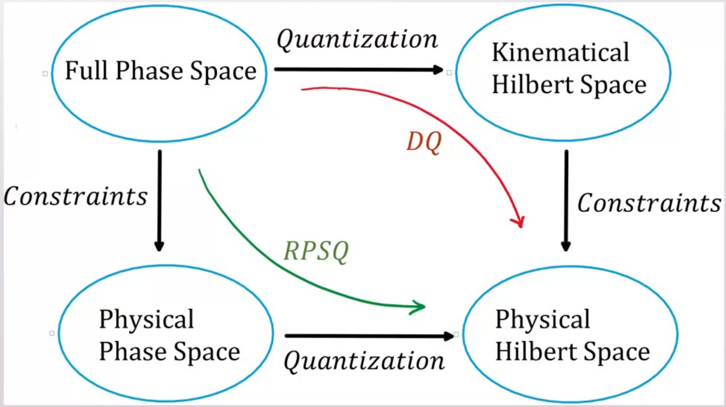
Is there such an intuition for the Hamiltonian constraint? Electric Shift $N^a_i \sim N E^a_i$

$$H[N] = \int d^3x \, \epsilon_{ijk} F^i_{ab} N^a_j E^b_k = \int d^3x \, \epsilon_{ijk} \, E^b_k (\mathcal{L}_{\vec{N_j}} A^i_b)$$

- ❑ We aim at moving forward its quantization through reduced phase space approach.

- ❑ RPSQ approach has the additional advantage that it frees us from the steps to compute

1. Kernal of constraint operators
2. Dirac observables

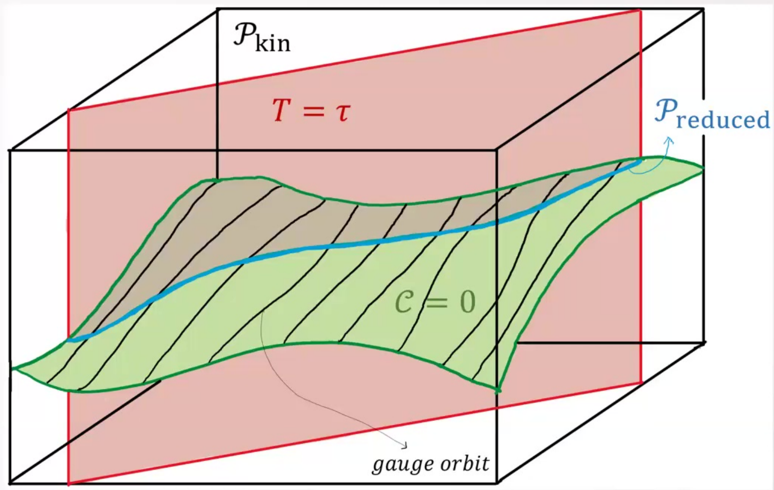


Reduced Phase Space Approach

Relational Formalism

Rovelli 90s; Dittrich '04, '05

- Take two gauge variant f, T and choose T as a clock
- Gauge invariant extension of f denoted by $F_{f,T}(\tau)$ in relation to values T takes
- $F_{f,T}(\tau)$: values of f when clock T takes values τ



Physical Hamiltonian:

1. Solve the constraints for as many momenta as there are constraints;

$$C(q^a, p_a) = 0 \rightarrow p_0 + h\big(q^a, p_{a(a\neq 0)}\big) = 0$$

1. Take their conjugate variables as clocks; $T = q^0$
2. Define gauge fixing conditions using the clocks; $G = T - \tau = 0$
3. Make sure that the gauge conditions are stable. To do this solve $\{C_I[\Lambda^I], G^J\} = 0$ for Λ^I.
4. Try to find a function h with this property (f is a function on the reduced phase space)

$$\dot{f} = \{\mathbf{H}_{\text{can}}, f\}_{C_I = G_I = 0, \Lambda^I = \Lambda^I_0} = \{h, f\}$$

One can add matter to the theory and use it as the clock variables.

Rovelli, Dittrich, Thiemann, Giesel, Husain, Kaminski, Lewandowski, Ashtekar, Marlof, Pullin, Gambini, Singh, Hohn, ...

Reduced Phase Space Approach

SB, Thiemann '21

Used for solving C_A Used for solving C_I

(A, E) description

Suppose (A^j_a, E^a_j) is divided to $w = (u^A, v_A)$, $z = (x^I, y_I)$, $r = (q^a, p_a)$ Physical degrees of freedom

Theorem: If the first class constraints of a system have the following form

Solved for u^A $C_A = C_A(u, x, q)$, $C_I = M_I{}^J(u, x, q)\, y_J + N_I{}^A(u, x, q)\, v_A + h_I(u, x, q, p)$

Contains differential operators

Solved for y_I

where the "matrices" $\sigma_{AB} := \{C_A, v_B\}$ and $M_I{}^J$ are non-singular.

Then, the physical Hamiltonian is

$$h = (\Lambda^I_0 h_I)_{C_A=0, G^I=0}$$

in which Λ^I_0 is the solutions of

$$\{\lambda^A C_A + \Lambda^I C_I, G^A\} = 0 = \{\lambda^A C_A + \Lambda^I C_I, G^J\}$$

Gauge fixings

1. $G_A := v_A = 0,$ Clocks related to C_A
2. $G^I := F^I(x) - \tau^I = 0$ Clocks related to C_I

$\mathcal{G}_i = \partial_a E^a_i,$ $H_a = F^i_{ab}E^b_i - A^i_a \partial_b E^b_i,$ $H = F^i_{ab}\epsilon_{ijk}E^a_j E^b_k$

$F^i_{ab} = 2\partial_{[a}A^i_{b]}$

- Using the theorem we investigated several gauge fixing conditions and obtained physical Hamiltonians.
- Thiemann in his recent work [Thiemann '22] introduced Exact quantization of the $U(1)^3$ model and in his work used the results of [S.B, Thiemann '21] to show that the exact quantization of this model matches with the reduced phase space quantization.

Covariant origin

SB, Thiemann '22

The $U(1)^3$ model was introduced in the Hamiltonian formulation.

Lagrangian?

The $U(1)^6$ Theory [S.B, T.Thiemann; CQG; 2022]

$$S = \frac{1}{2}\int d^4x \; F^{IJ}_{AB}\hat{\sigma}^{AB}_{IJ}$$

$$\hat{\sigma}^{AB}_{IJ} = \hat{\Sigma}^{AB}_{IJ} + \frac{1}{2}\gamma\epsilon_{IJ}{}^{KL}\hat{\Sigma}^{AB}_{KL}, \quad \hat{\Sigma}^{AB}_{IJ} = \hat{e}^A_{[I}\,\hat{e}^B_{J]}, \quad \hat{e}^A_I = \det(\{e^J_B\})^{1/2}\, e^A_I$$

- No local d.o.f.

Covariant origin

SB, Thiemann '22

The $U(1)^3$ model was introduced in the Hamiltonian formulation.

Lagrangian?

Twisted Self-Dual Model [SB, T.Thiemann; CQG; 2022]

$$S = \frac{1}{2} \int dt\, d^3x\, F^{IJ}_{AB}\, \hat{\Sigma}^{AB}_{IJ}$$

- $\hat{\Sigma}^{AB}_{IJ} = \hat{e}^A_{[I}\, \hat{e}^B_{J]}$
- $F^{0j} = F^j = \frac{1}{2\gamma}\epsilon_{jkl}\, F^{kl}$

Covariant origin

SB, Thiemann '22

The $U(1)^3$ model was introduced in the Hamiltonian formulation. **Lagrangian?**

Twisted Self-Dual Model [SB, T.Thiemann; CQG; 2022]

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- $\hat{\Sigma}^{AB}_{IJ} = \hat{e}^A_{[I}\, \hat{e}^B_{J]}$
- $F^{0j} = F^j = \frac{1}{2\gamma}\epsilon_{jkl}\, F^{kl}$

Features:

1. D.o.f: 2
2. Constraints: $G_j = \nabla_a \pi^a_j$,
   $C_a = F^j_{ab}\, \pi^b_j$,
   $C = F^j_{ab}\, \epsilon_{jkl} \pi^a_k\, \pi^b_l$

This leads to the Hamiltonian formulation of the $U(1)^3$, regardless of the value of $\gamma \neq 0$

Summary

Quantum Gravity

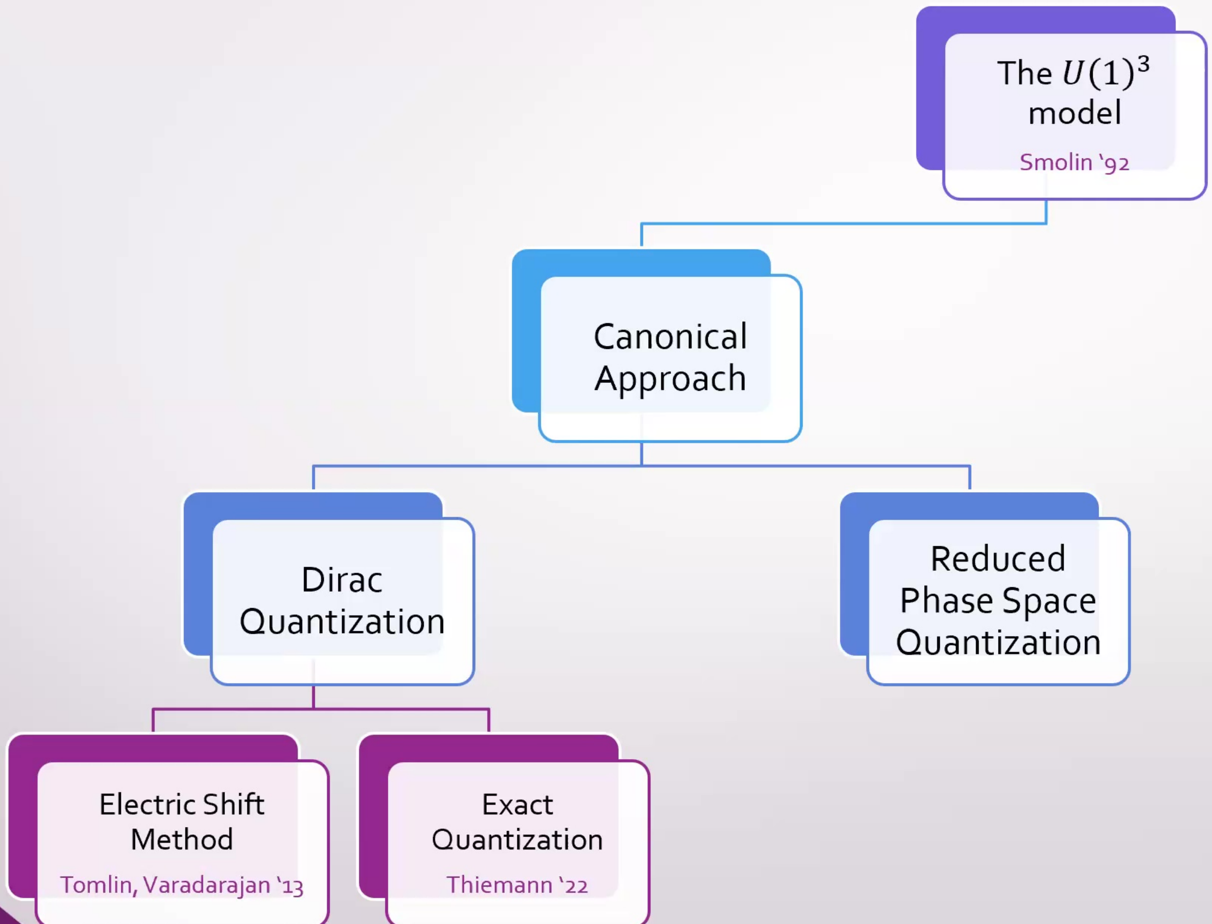
Loop Quantum Gravity

Problem of Dynamics

Toy models

$U(1)^3$ model

Summary


The $U(1)^3$ model
Smolin '92
Canonical Approach
Dirac Quantization
Reduced Phase Space Quantization
Electric Shift Method
Tomlin, Varadarajan '13
Exact Quantization
Thiemann '22

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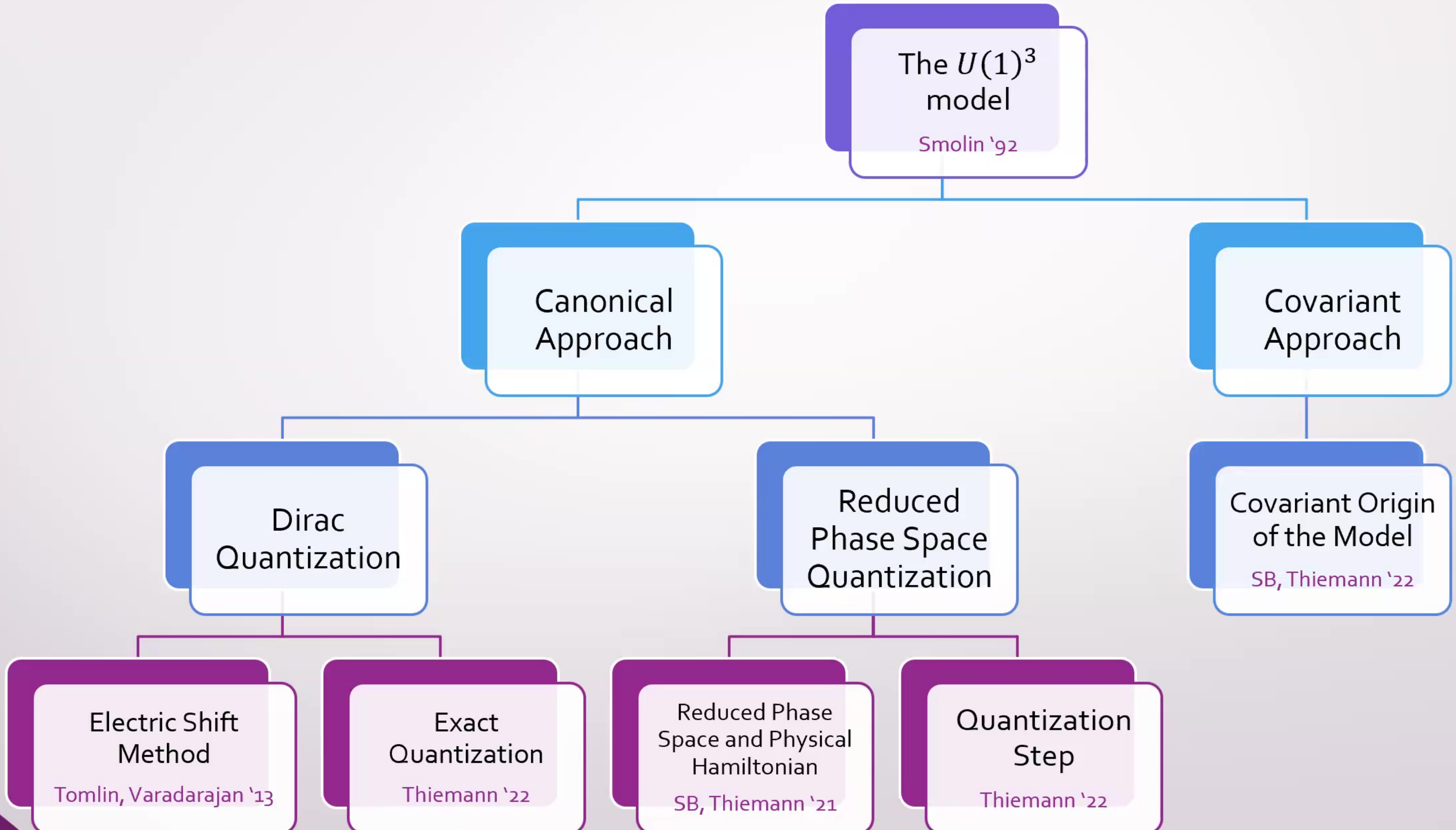

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Reduced Phase Space and Physical Hamiltonian
SB, Thiemann '21
Quantization Step
Thiemann '22

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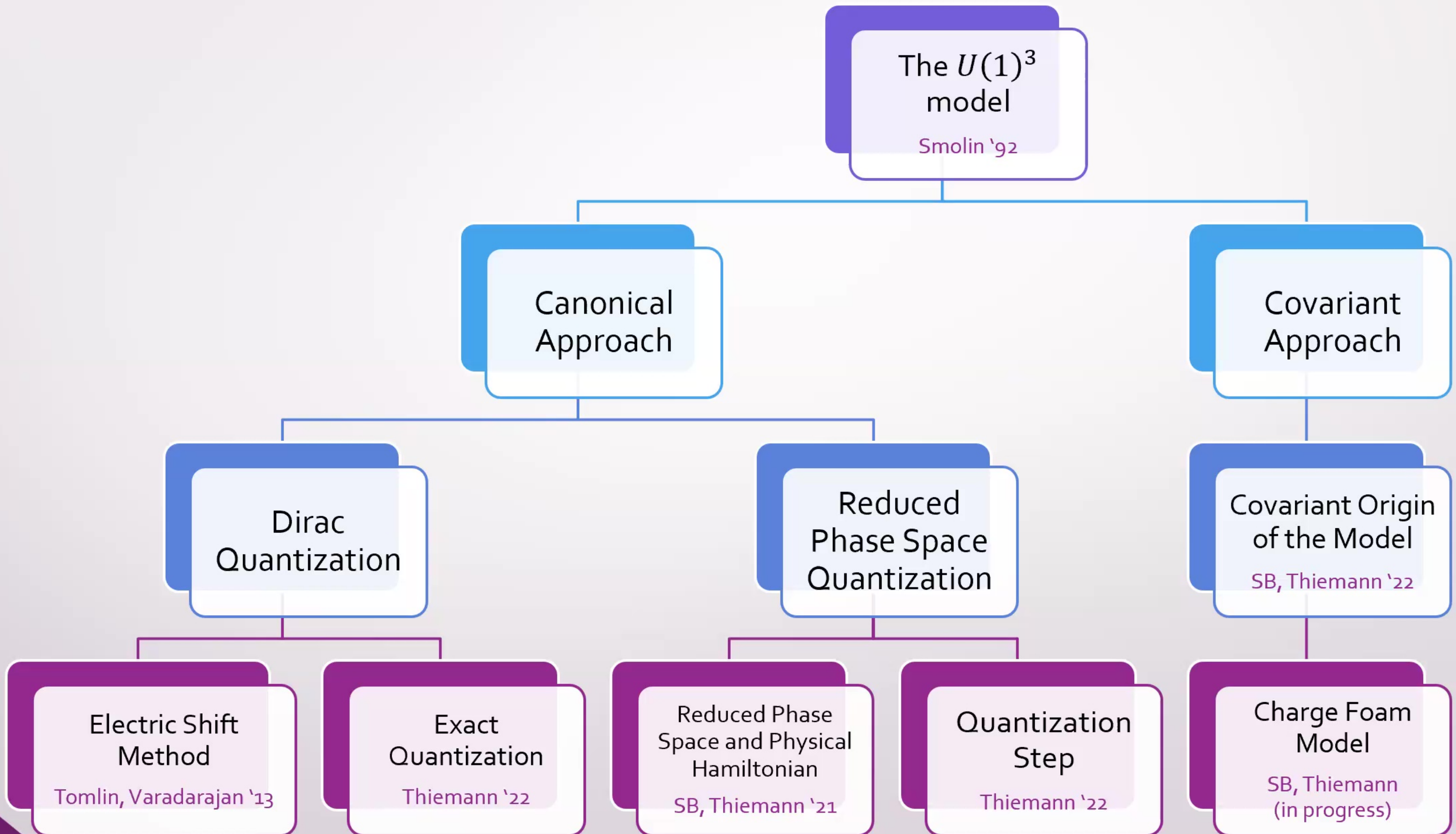
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Quantization Step
Thiemann '22
Charge Foam Model
SB, Thiemann (in progress)

Current and Future Projects

- Charge Foam Model (Thomas Thiemann)

It is anticipated that the spin foam model resulting from our study may be comparatively easier to manipulate from a technical standpoint compared to their non-Abelian counterparts. As such, it could serve as an intriguing experimental facility for exploring the spin foam approach to loop quantum gravity.

- Holonomy as Quantum Time Operator (Yongge Ma)

One of the disadvantageous of RPSQ is that time is not quantized in this approach. The goal of this work is to take advantageous of linearity of the Hamiltonian constraint operator in holonomy and try to interprent holonomy as quantum time operator.

Boundary Conditions for Ashtekar-Barbero Variables with Supertranslations at i^0

An Exploration into the Applicational Aspect of LQG

Based on [gr-qc/2311.01595]

Introduction and Motivation

- Phase space variables: $q_{ab}(x), \pi^{ab}(x)$, $\{q, \pi\} \sim \delta$
- Dynamics along the time flow generated by Hamiltonian $\boldsymbol{H}$.
  Useful to decompose time flow into components

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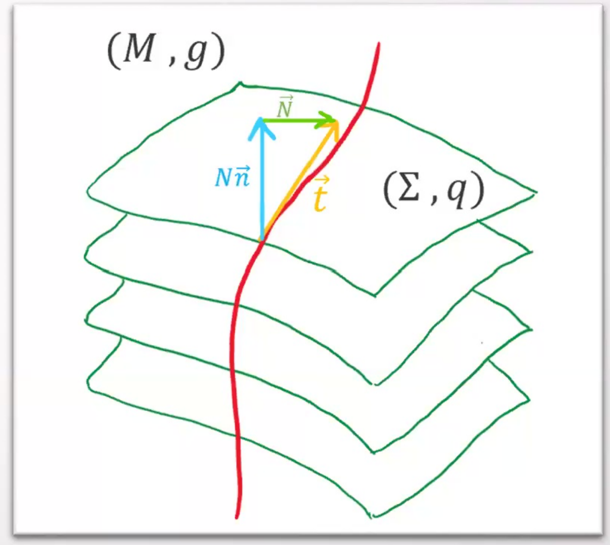

ADM; Isham & Kuchar '85; Lee, Wald '90

Hypersurface deformation algebra:

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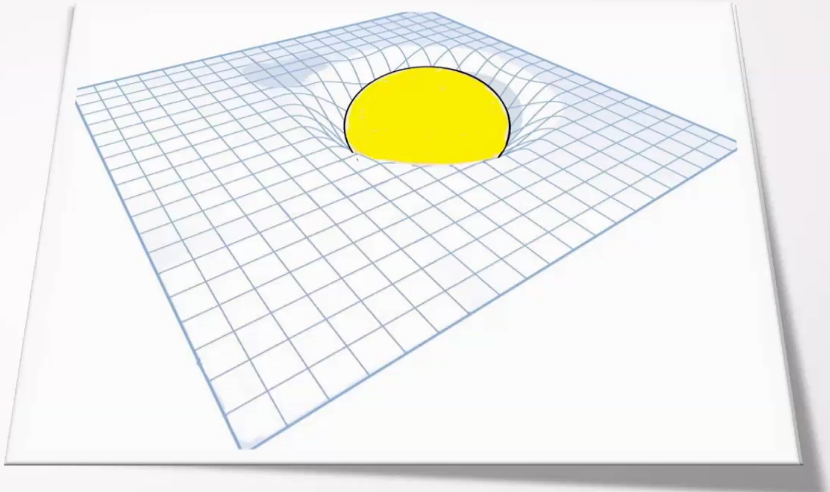
$$\{H_a[N^a], H[M]\} = H[\mathcal{L}_{\vec{N}}M]$$
$$\{H[N], H[M]\} = H_a[\underbrace{q^{ab}}_{\text{Structure functions}}(N\partial_b M - M\partial_b N)]$$

Introduction and Motivation

- Flat Spacetime

Symmetries: Poincare = Translation ⋉ Lorentz

Charges: Time translation → Energy
Spatial translation → Momentum
Rotation → Angular Momentum



Bondi, van der Burg, Mentyner '62
Sachs '62

- Asymptotically Flat Spacetimes

Symmetries: BMS = Supertranslation ⋉ Superrotation

Introduction and Motivation

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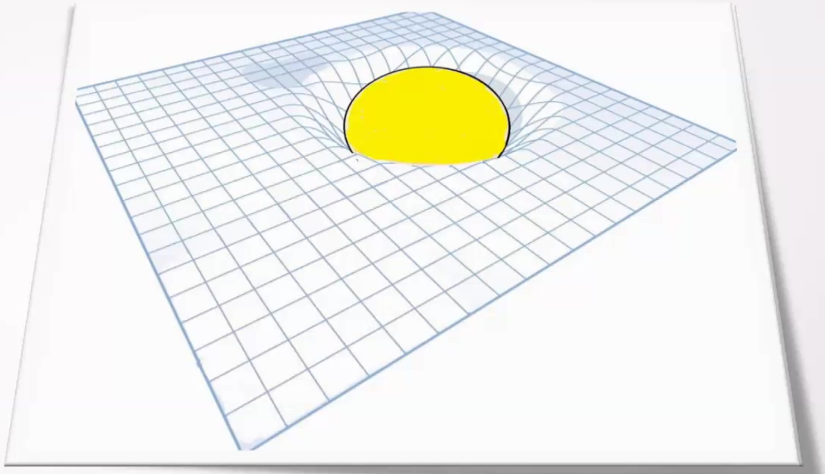

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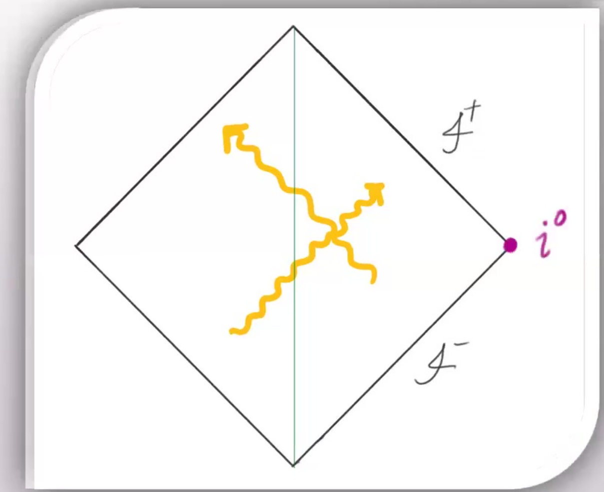
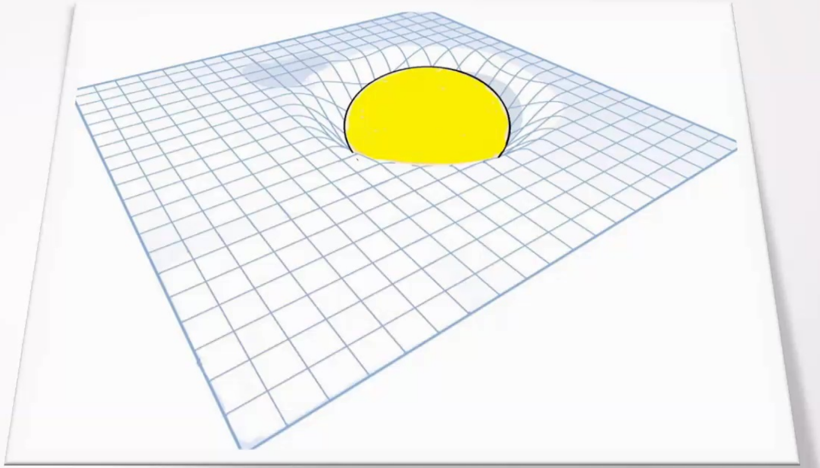


➢ First identified at Null infinity

➢ Can be identified at Spatial infinity?
Why is it important?

Introduction and Motivation

- Continuity of the boundary
- Quantum features of the charges



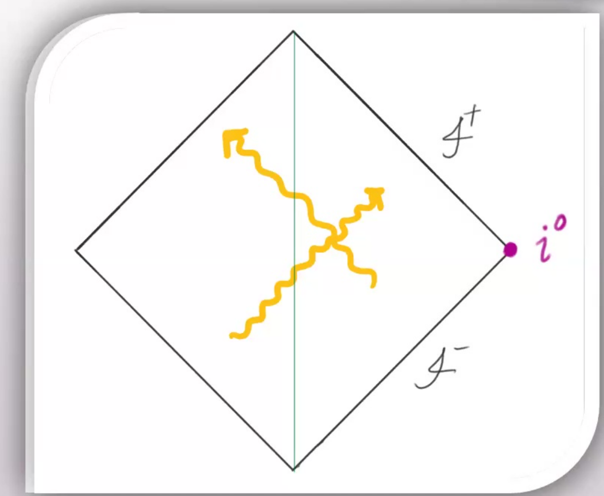
Which QG Theory ?

Canonical Loop Quantum Gravity

What is the language of LQG?

Ashtekar-Barbero variables

Goal:
To study the asymptotic structure of gravity in terms of Ashtekar-Barbero variables at i^0 in the asymptotically flat context.



Why?
In order to better understand the role of the BMS group in the quantum theory using LQG techniques.

Standard Boundary Conditions

- Fall-off conditions in Cartesian Coordinate: **[Thiemann; 95]**

$$E^I_i = \delta^I_i + \frac{\bar{f}^I_i}{r} + O(r^{-2})$$

$$A^i_I = \frac{\bar{g}^i_I}{r^2} + O(r^{-3})$$

- In Polar Coordinates: **[SB; 23]**

Asymptotic triads

$$E^r_i = r^2\sqrt{\bar{\gamma}}\,\bar{\gamma}^r_i + r\sqrt{\bar{\gamma}}\,\bar{F}^r_a\bar{\gamma}^a_i + O(1)$$

$$E^A_i = r\sqrt{\bar{\gamma}}\,\bar{\gamma}^A_i + \sqrt{\bar{\gamma}}\,\bar{F}^A_a\bar{\gamma}^a_i + O(r^{-1})$$

$$A^i_r = \frac{1}{r^2}\bar{G}^a_r\bar{\gamma}^i_a + O(r^{-3})$$

$$A^i_A = \frac{1}{r}\bar{G}^a_A\bar{\gamma}^i_a + O(r^{-2})$$

$$\bar{\gamma}^a_i\bar{\gamma}^b_i = \bar{\gamma}^{ab} = \mathrm{diag}(1, \bar{\gamma}^{AB}),\ a, b \in \{r, \theta, \varphi\},\ A, B \in \{\theta, \varphi\}$$

$\bar{\gamma}_{AB}$ is the unit metric on the sphere

Standard Boundary Conditions

- Fall-off conditions in Cartesian Coordinate: **[Thiemann; 95]**

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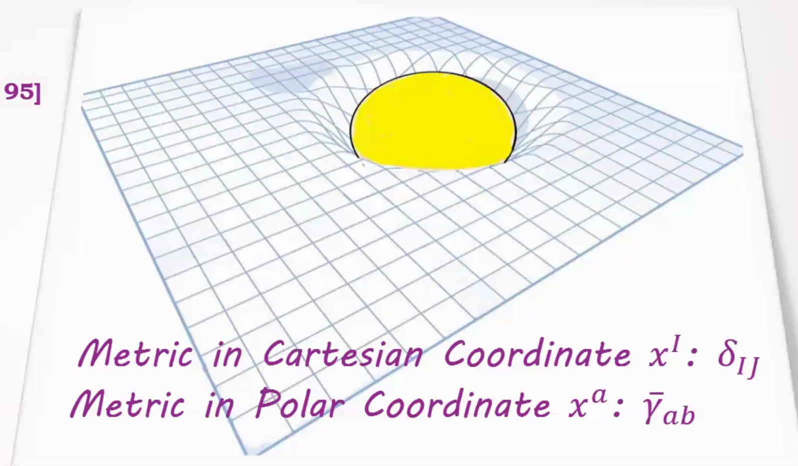

- In Polar Coordinates: **[SB; 23]**

Asymptotic triads

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$$E^A_i = r\sqrt{\bar{\gamma}}\,\bar{\gamma}^A_i + \sqrt{\bar{\gamma}}\,\bar{F}^A_a\bar{\gamma}^a_i + O(r^{-1}) \qquad A^i_A = \frac{1}{r}\bar{G}^a_A\bar{\gamma}^i_a + O(r^{-2})$$

Divergent part of the symplectic form: — even

$$\Omega = \int_\Sigma d^3x\, \delta A^i_a \wedge \delta E^a_i$$
$$= \int \frac{dr}{r}\int_{S^2} d\theta d\varphi\,(\delta\bar{G}^r_r \wedge \delta\bar{F}^r_r + \delta\bar{G}^A_r \wedge \delta\bar{F}^r_A + \delta\bar{G}^r_A \wedge \delta\bar{F}^A_r + \delta\bar{G}^A_B \wedge \delta\bar{F}^B_A) + \text{Finite}$$

Standard Boundary Conditions

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- In Polar Coordinates: **[SB; 23]**

Asymptotic triads

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$$A^i_A = \frac{1}{r}\bar{G}^a_A\bar{\gamma}^i_a + O(r^{-2})$$

Divergent part of the symplectic form:

—— even
—— odd

Standard Parity Conditions
[Regge, Teitelboim; 74]
[Thiemann; 95] [Campiglia; 15]

$$\Omega = \int_\Sigma d^3x\, \delta A^i_a \wedge \delta E^a_i$$

$$= \int \frac{dr}{r} \int_{S^2} d\theta d\varphi\, (\delta\bar{G}^r_r \wedge \delta\bar{F}^r_r + \delta\bar{G}^A_r \wedge \delta\bar{F}^r_A + \delta\bar{G}^r_A \wedge \delta\bar{F}^A_r + \delta\bar{G}^A_B \wedge \delta\bar{F}^B_A) + \text{Finite}$$

Standard Boundary Conditions

- Consistency requirement:
  The fall-off and parity conditions must be preserved under hypersurface deformations

- Yet another requirement
  Generators of the asymptotic symmetries must be well-defined
  1) They should be finite
  2) They should be functionally differentiable

$$\mathcal{G}_i[\Lambda^i] = \frac{2}{\beta}\int_\Sigma d^3x\, \Lambda^i \left(\partial_a E^a_i + \epsilon_{ijk} A^j_a E^a_k\right)$$

$$H_a[N^a] = \frac{-2s}{\beta}\int_\Sigma d^3x\, N^a \left(F^i_{ab} E^b_i - A^i_a \mathcal{G}_i\right)$$

$$H[N] = \int_\Sigma d^3x\, \tilde{N}\left[F^i_{ab} - (\beta^2 - s)\epsilon_{imn} K^m_a K^n_b\right] \epsilon_{ijk} E^a_j E^b_k$$

boost

Temporal translation

Temporal Supertranslation

Radial translation

Rotation

$$N = rb + f_0 + S_T + O(r^{-1}),$$

$$N^r = W_R + S_R + O(r^{-1}),$$

$$N^A = Y^A + \frac{1}{r} I^A + O(r^{-2})$$

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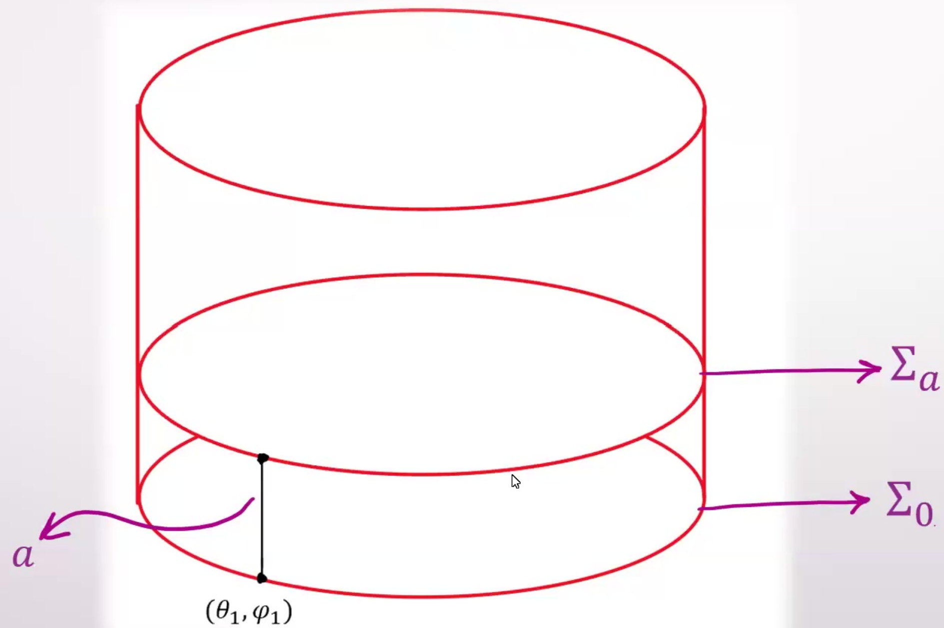
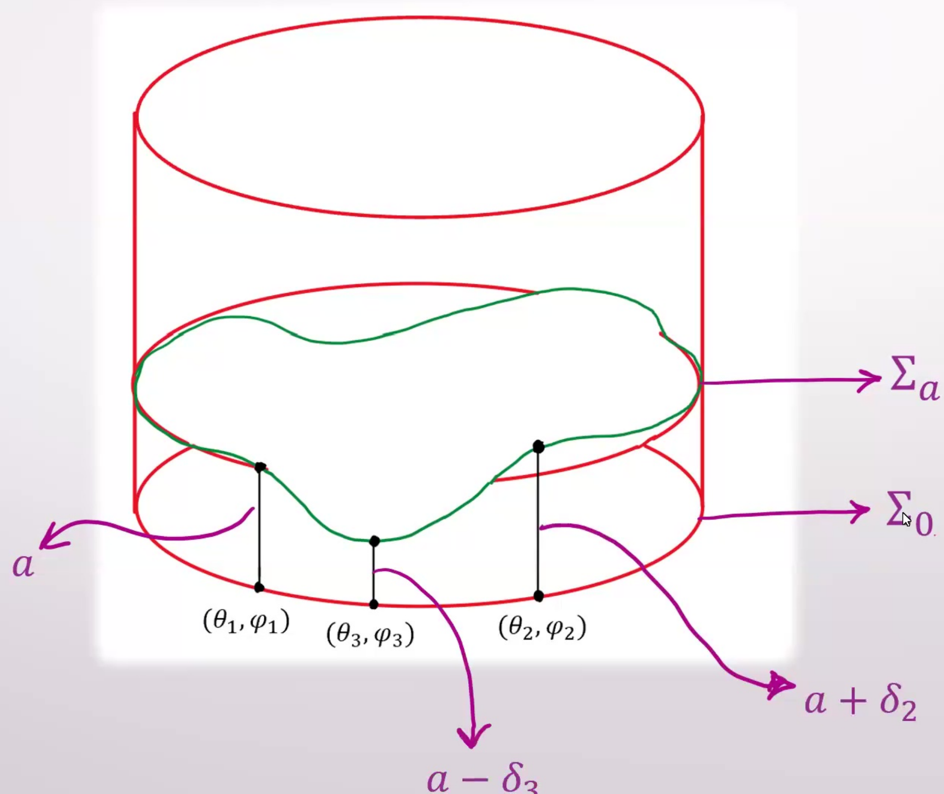
Intuition



Intuition

Ordinary time translation: $t \to t + a$
Conserved charge: Energy



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In **[Thiemann; 95] [Campiglia; 15]** the well-defined generators were derived and Poincare symmetry was recovered but

$$S_T \sim \text{odd}, \quad S_R \sim \text{even}$$

Supertranslation Charge: $\int_{S^2} d\Omega\, \left(S_T \bar{F}^r_r + S_R \bar{G}^r_r\right) = 0$

New Boundary Conditions

- The following boundary conditions are the desired ones **[SB; 23]**

$$\bar{F}^\theta_r \sim \bar{F}^\theta_\varphi \sim \bar{F}^\varphi_\theta = even$$
$$\bar{F}^\varphi_r \sim \bar{F}^\theta_\theta \sim \bar{F}^\varphi_\varphi \sim \bar{G}^r_r = odd$$
$$(\bar{F}^r_r - \bar{F}^A_A) \sim (\bar{G}^\theta_\theta + \bar{G}^r_r) \sim (\bar{G}^\varphi_\varphi + \bar{G}^r_r) = even$$
$$\bar{G}^\theta_r + \frac{1}{2\sqrt{\bar{\gamma}}}\bar{D}_\varphi(\bar{F}^r_r - \bar{F}^A_A) = odd$$
$$\bar{G}^r_\theta + \frac{1}{2\sqrt{\bar{\gamma}}}\bar{D}_\varphi(\bar{F}^r_r - \bar{F}^A_A) = odd$$
$$\bar{G}^\varphi_r - \frac{1}{2\sqrt{\bar{\gamma}}}\bar{D}_\theta(\bar{F}^r_r - \bar{F}^A_A) = even$$
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$$\bar{G}^\varphi_\theta - \frac{1}{2\sqrt{\bar{\gamma}}}\left(\bar{F}^r_r - \bar{F}^A_A\right) = odd$$

$$\bar{\gamma}_{AB}\bar{F}^A_r + \bar{F}^r_B = 0$$
$$\bar{\gamma}_{B[E}\delta\bar{F}^B_{D]} = 0$$

$$N = rb + f_0 + S_T + O(r^{-1}),$$
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\begin{aligned}
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&\bar{G}^{\varphi}_{\theta} - \frac{1}{2\sqrt{\bar{\gamma}}}\left(\bar{F}^{r}_{r} - \bar{F}^{A}_{A}\right) = odd
\end{aligned}
$$

$$
\begin{aligned}
&I_D = \partial_D W - \beta\, b\left[\frac{s}{\beta}(\bar{k}^{r}_{D} + \bar{\gamma}_{BD}\bar{k}^{B}_{r}) - \frac{\epsilon^{AC}}{\sqrt{\bar{\gamma}}}\bar{\gamma}_{AD}\bar{\gamma}_{BC}\bar{F}^{B}_{r}\right]\\
&2\bar{\Lambda}^{r} = -s\,\frac{\epsilon^{DA}}{\sqrt{\bar{\gamma}}}\bar{D}_{A}\left(b\,[\bar{k}^{r}_{D} + \bar{\gamma}_{BD}\bar{k}^{B}_{r}]\right) - \beta\left[\bar{F}^{A}_{r}(\bar{D}_{A}b) - 2b\bar{D}_{A}\bar{F}^{A}_{r} + b\,\frac{\epsilon^{AB}}{\sqrt{\bar{\gamma}}}\bar{\gamma}_{BC}\bar{G}^{C}_{A}\right]\\
&\bar{\Lambda}_{\theta} + \beta\frac{b}{2}\bar{D}_{\theta}(\bar{F}^{r}_{r} - \bar{F}^{A}_{A}) = odd\\
&\bar{\Lambda}_{\varphi} + \beta\frac{b}{2}\bar{D}_{\varphi}(\bar{F}^{r}_{r} - \bar{F}^{A}_{A}) = even
\end{aligned}
$$

$$
\begin{aligned}
&\bar{\gamma}_{AB}\bar{F}^{A}_{r} + \bar{F}^{r}_{B} = 0\\
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\begin{aligned}
N &= rb + f_0 + S_T + O(r^{-1}),\\
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\end{aligned}
$$

- The transformations that preserve the standard boundary conditions are [SB; 23]

$S_T \sim$ even, $S_R \sim$ odd

$$
Q_{\text{Supertranslation}} = -2\int_{S^2} d\Omega\,\sqrt{\bar{\gamma}}\left[\underbrace{S_T}_{\text{even}}\underbrace{(\bar{F}^{r}_{r} - \bar{F}^{A}_{A})}_{\text{even}} - \frac{2}{\beta}\underbrace{S_R}_{\text{odd}}\bar{G}^{r}_{r}\right]
$$

New Boundary Conditions

- The charges associated with f_0, W_R and S_T, S_R are all finite and non-vanishing **[SB; 23]**
- But it turns out that the charges associated with b and Y^A are not finite
- In terms of ADM variables, if one sets the leading term of the constraints equal to zero the divergences associated with boost and rotation charge is removed. **[Henneaux, Troessaert; 18]**
- Surprisingly this method does not work in Ashtekar-Barbero formulation. **[SB; 23]**

- By examining the process of obtaining Ashtekar-Barbero variables from ADM variables, it becomes evident that in order to establish A_a^i and E_i^a as conjugate variables, particularly to demonstrate that the Poisson bracket between two A_a^i is zero, it is necessary to prove

$$\frac{\delta \Gamma_a^j(x)}{\delta E_k^b(y)} - \frac{\delta \Gamma_b^k(y)}{\delta E_j^a(x)} = 0$$

which represents the integrability condition for Γ_a^j a to possess a generating function F.

✓ For a manifold without boundary

✓ For the asymptotically flat manifold with the standard parity conditions

The function F is not well-defined for the new boundary conditions.

If one wants to keep the connection with ADM formulation, boundary conditions should make F well-defined.

Outlook

- Find appropriate parity conditions for Ashtekar-Barbero variables which produce the full Poincare + Supertranslation charges at spatial infinity.
- Trying to figure out how to impose the boundary conditions in quantum level properly and use LQG techniques to quantize the supertranslation charges.
- Once we have successfully accomplished the previous goal, our next aim is to determine boundary conditions that not only yield supertranslations at spatial infinity, but also incorporate superrotations

Thank you for your attention!