Title: Quantum scars in quantum field theory

Speakers: Annie Wei

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Abstract: We develop the theory of quantum scars for quantum fields. By generalizing the formalisms of Heller and Bogomolny from few-body quantum mechanics to quantum fields, we find that unstable periodic classical solutions of the field equations imprint themselves in a precise manner on bands of energy eigenfunctions. This indicates a breakdown of thermalization at certain energy scales, in a manner that can be characterized via semiclassics. As an explicit example, we consider time-periodic non-topological solitons in complex scalar field theories. We find that an unstable variant of Q-balls, called Q-clouds, induce quantum scars. Some technical contributions of our work include methods for characterizing moduli spaces of periodic orbits in field theories, which are essential for formulating our quantum scar formula. We further discuss potential connections with quantum many-body scars in Rydberg atom arrays. Based on work in arXiv:2212.01637 with Jordan Cotler.

Zoom link https://pitp.zoom.us/j/91572728134?pwd=Q0Jzb0lwQW5VU0ptRnRWL2tOTTdLdz09

Quantum Scars in Quantum Field Theory

Annie Wei

Joint work with Jordan Cotler
arXiv:2212.01637

December 4, 2023

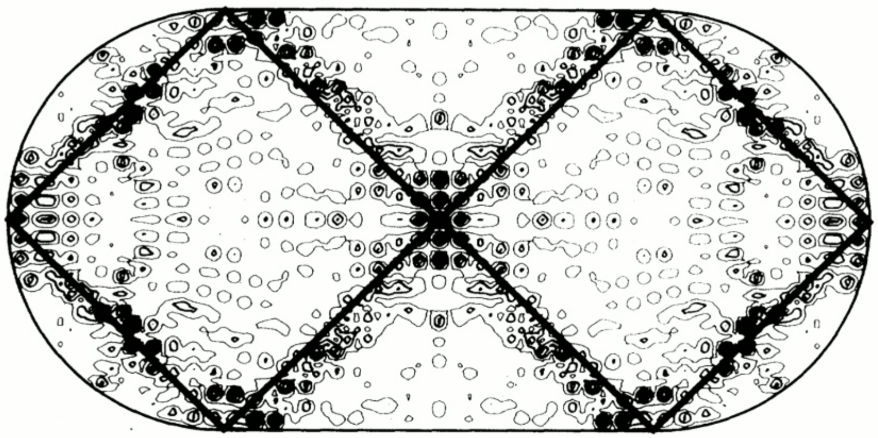
Outline

Motivation

- In general: how do we describe chaotic quantum systems?
- Quantum scars: a non-perturbative phenomenon occurring when we might otherwise expect thermalization
- We generalize the formulation of quantum scars from quantum mechanics to quantum field theory
- Potential applications to
  - Rydberg quantum scars
  - Quantum gravity

Quantum Scars

- Classically, we would expect chaotic systems to be ergodic
- Quantum scars: eigenstates of chaotic quantum system have enhanced probability distribution around unstable periodic orbits (Heller '84)
- Oscillatory fringes



- Quantum scars provide a correction to the microcanonical ensemble (Heller '84; Bogomolny '87, Berry '89)
- Consider eigenstates $\Psi_n(\mathbf{q})$, and average $|\Psi_n(\mathbf{q})|^2$ over eigenstates with energy in $[E - \epsilon/2, E + \epsilon/2]$
- In an ergodic system, we would expect this quantity to obey the microcanonical ensemble:

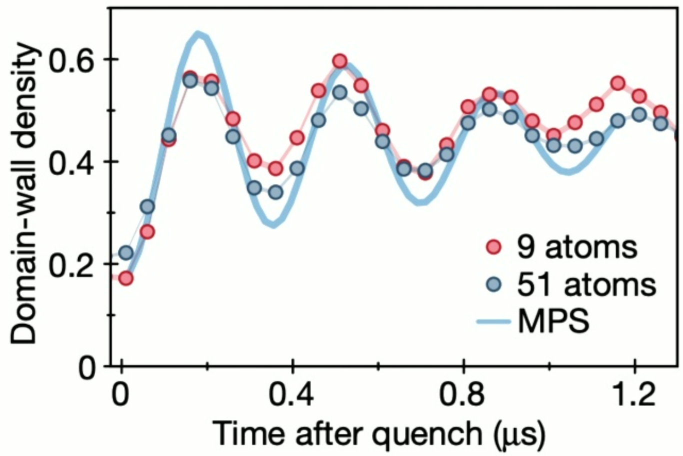
$$P_{micro}(\mathbf{q}) \propto \int d^d\mathbf{p}\, \delta_\epsilon(E - H(\mathbf{p}, \mathbf{q}))$$

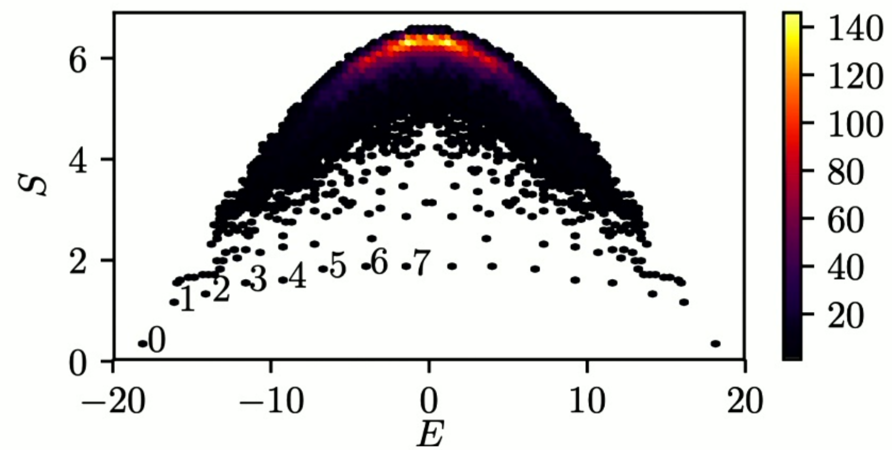
- Due to quantum scars, we have

$$P(\mathbf{q}) \approx P_{micro}(\mathbf{q}) + \delta P_{scar}(\mathbf{q})$$

Motivating Applications

- Rydberg many-body scars: experimental observation of periodic revivals after quantum quench (Bernien '17, Turner '18)





Are these quantum scars in the original sense of Heller?

- Quantum gravity: Do unstable orbits around black holes correspond to scars in the dual CFT? (Dodelson, Zhiboedov '22, Milekhin, Sukhov '23)

Objectives

- We will clarify and re-derive results from few-body semiclassical quantum chaos so that they can be generalized to QFT, based on Bogomolny '87
- We will generalize these results to QFT
- We will give examples of QFTs containing scar states

QM Scar Formula: Derivation Bogomolny '87, Cotler, Wei '22

- We would like to compute $\langle|\Psi(\mathbf{q})|^2\rangle_{E,\Delta}$
- Position smearing:

$$\langle f(\mathbf{q})\rangle_\Delta = \frac{1}{(2\pi\Delta^2)^{d/2}}\int d^d\mathbf{z}\, e^{-\frac{1}{2\Delta^2}(\mathbf{q}-\mathbf{z})^2} f(\mathbf{z})$$

- Energy smearing: For eigenstates $\Psi_n(\mathbf{q})$ in the energy window $[E-\epsilon/2, E+\epsilon/2]$,

$$\langle|\Psi(\mathbf{q})|^2\rangle_E = \frac{\sum_n |\Psi_n(\mathbf{q})|^2 \delta_\epsilon(E-E_n)}{\sum_n \delta_\epsilon(E-E_n)}$$

- We will find that

$$\langle|\Psi(\mathbf{q})|^2\rangle_{E,\Delta} \approx P_{micro}(\mathbf{q}) + \delta P_{scar}(\mathbf{q})$$

To evaluate the expression

$$\langle|\Psi(\mathbf{q})|^2\rangle_{E,\Delta} = \frac{\sum_n \langle|\Psi_n(\mathbf{q})|^2\rangle_\Delta \delta_\epsilon(E-E_n)}{\sum_n \delta_\epsilon(E-E_n)}$$

we will use the fact that

$$-\frac{1}{\pi}\text{Im}\, G(\mathbf{q},\mathbf{q},E+i\epsilon) = \sum_n |\psi_n(\mathbf{q})|^2 \delta_\epsilon(E-E_n)$$

$$-\frac{1}{\pi}\text{Im}\int d^d\mathbf{q}\, G(\mathbf{q},\mathbf{q},E+i\epsilon) = \sum_n \delta_\epsilon(E-E_n)$$

Van Vleck Propagator

- Start with semiclassical Green's function:

$$G(\mathbf{q}^A, \mathbf{q}^B, t) \approx \sum_{\substack{\text{classical} \\ \text{paths } c}} \frac{1}{\sqrt{(2\pi i\hbar)^d}} \left| \det\left(\frac{\partial^2 S_c(\mathbf{z}^a, \mathbf{z}^B, t)}{\partial \mathbf{z}^A \partial \mathbf{z}^B} \right) \right|^{1/2}_{\substack{\mathbf{z}^A = \mathbf{q}^A \\ \mathbf{z}^B = \mathbf{q}^B}}$$

$$\exp\left[\frac{i}{\hbar} S_c(\mathbf{q}^A, \mathbf{q}^B, t) - i\nu_c \frac{\pi}{2} \right]$$

- Transform from time to energy variables

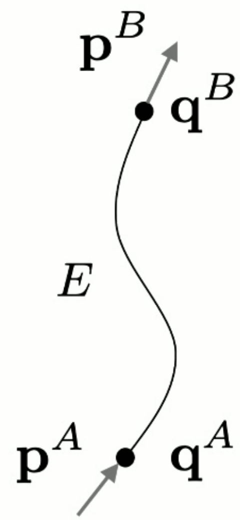
$$(\mathbf{q}^A, \mathbf{q}^B, t) \Rightarrow (\mathbf{q}^A, \mathbf{q}^B, E)$$

- Abbreviated action:

$$S(\mathbf{q}^A, \mathbf{q}^B, E) = \int_{\mathbf{q}^A}^{\mathbf{q}^B} \mathbf{p} \cdot d\mathbf{q}$$

- At the endpoints,

$$\frac{\partial S(\mathbf{q}^A, \mathbf{q}^B, E)}{\partial \mathbf{q}^A} = -\mathbf{p}^A$$
$$\frac{\partial S(\mathbf{q}^A, \mathbf{q}^B, E)}{\partial \mathbf{q}^B} = \mathbf{p}^B$$



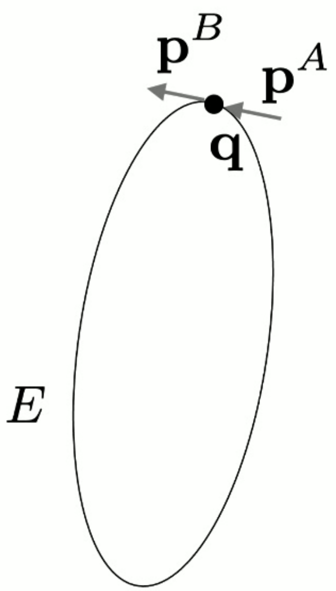
Periodic Orbits

- For periodic orbits, set $\mathbf{q}^A = \mathbf{q}^B = \mathbf{q}$
- Call the period T
- Also have

$$\left(\frac{\partial S(\mathbf{q}^A, \mathbf{q}^B, E)}{\partial \mathbf{q}^A} + \frac{\partial S(\mathbf{q}^A, \mathbf{q}^B, E)}{\partial \mathbf{q}^B}\right)_{\mathbf{q}^A=\mathbf{q}^B=\mathbf{q}} = -\mathbf{p}^A + \mathbf{p}^B = 0$$

- We will abbreviate

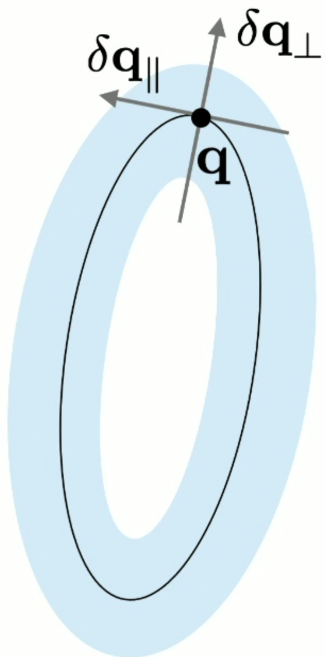
$$A_{ij}(\mathbf{q}) = \left(\frac{\partial^2 S(\mathbf{z}^A, \mathbf{z}^B, E)}{\partial z_i^A \partial z_j^A} + 2\frac{\partial^2 S(\mathbf{z}^A, \mathbf{z}^B, E)}{\partial z_i^A \partial z_j^B} + \frac{\partial^2 S(\mathbf{z}^A, \mathbf{z}^B, E)}{\partial z_i^B \partial z_j^B}\right)_{\mathbf{z}^A=\mathbf{z}^B=\mathbf{q}}$$



We will consider nearly periodic orbits: $\mathbf{q}^A = \mathbf{q}^B = \mathbf{q}$, and

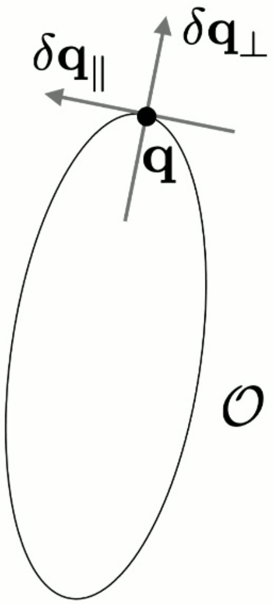
$$\left(\frac{\partial S(\mathbf{q}^A, \mathbf{q}^B, E)}{\partial \mathbf{q}^A} + \frac{\partial S(\mathbf{q}^A, \mathbf{q}^B, E)}{\partial \mathbf{q}^B} \right)_{\mathbf{q}^A = \mathbf{q}^B = \mathbf{q}}$$

$$= -\mathbf{p}^A + \mathbf{p}^B \approx 0$$



Letting $\mathcal{O}$ be the space of the images of the periodic orbits, we can split into directions parallel to and perpendicular to a periodic orbit:

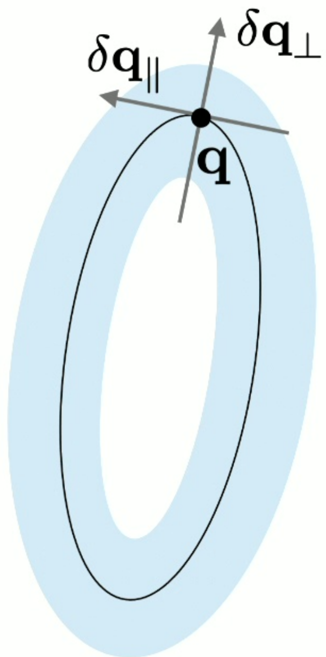
$$\delta\mathbf{q}_{\|} \in T_{\mathbf{q}}\mathcal{O}$$
$$\delta\mathbf{q}_{\perp} \in N_{\mathbf{q}}\mathcal{O}$$



We will consider nearly periodic orbits: $\mathbf{q}^A = \mathbf{q}^B = \mathbf{q}$, and

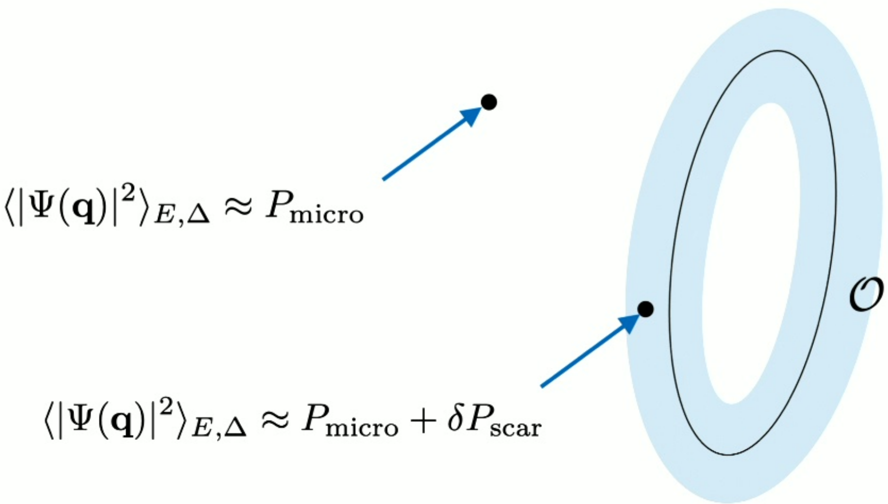
$$\left(\frac{\partial S(\mathbf{q}^A, \mathbf{q}^B, E)}{\partial \mathbf{q}^A} + \frac{\partial S(\mathbf{q}^A, \mathbf{q}^B, E)}{\partial \mathbf{q}^B}\right)_{\mathbf{q}^A = \mathbf{q}^B = \mathbf{q}}$$

$$= -\mathbf{p}^A + \mathbf{p}^B \approx 0$$



QM Scar Formula Bogomolny '87, Cotler, Wei '22

$$\langle|\Psi(\mathbf{q})|^2\rangle_{E,\Delta} \approx \begin{cases} P_{\text{micro}}[\mathbf{q}] + \delta P_{\text{scar}}[\mathbf{q}_c, \delta\mathbf{q}_\perp] & \text{if } \mathbf{q} = \mathbf{q}_c + \delta\mathbf{q}_\perp \text{ where } \mathbf{q}_c \in \mathcal{O},\ \delta\mathbf{q}_\perp \in \mathrm{N}_{\mathbf{q}_c}\mathcal{O}, \\ & \quad \|\delta\mathbf{q}_\perp\|_2 \lesssim \frac{\hbar}{\Delta} \\ P_{\text{micro}}[\mathbf{q}] & \text{if } \left\|\mathbf{p}^A - \mathbf{p}^B\right\|_2 \gg \frac{\hbar}{\Delta} \end{cases}$$



Here

$$\Delta = \sqrt{\frac{\hbar T_{max}}{m}} \left(\frac{\hbar}{E T_{max}}\right)^{\gamma}, \ \frac{1}{4} < \gamma < \frac{1}{2}$$

and

$$P_{\text{micro}}(\mathbf{q}) = \frac{\int \frac{d^d\mathbf{p}}{(2\pi\hbar)^d}\, \delta_\varepsilon(E - H(\mathbf{q},\mathbf{p}))}{\int d^d\mathbf{z}\, \frac{d^d\mathbf{p}}{(2\pi\hbar)^d}\, \delta_\varepsilon(E - H(\mathbf{z},\mathbf{p}))}$$

$$\delta P_{\text{scar}}(\mathbf{q}_c, \delta\mathbf{q}_\perp) = -\frac{2}{\pi\hbar \int d^d\mathbf{z}\, \frac{d^d\mathbf{p}}{(2\pi\hbar)^d}\, \delta_\varepsilon(E - H(\mathbf{z},\mathbf{p}))} \,\text{Im}\Bigg\{ \frac{1}{i} \frac{1}{\sqrt{(2\pi i\hbar)^{d-1}}} \frac{1}{|\dot{\mathbf{q}}|}$$

$$\left|\det\left(\frac{\partial^2 S(\mathbf{q}^A, \mathbf{q}^B, E)}{\partial \mathbf{q}_\perp^A \partial \mathbf{q}_\perp^B}\right)\right|^{1/2}_{\mathbf{q}^A=\mathbf{q}^B=\mathbf{q}} \times \exp\left[-\frac{\varepsilon}{\hbar}\, T(\mathbf{q}_c,\mathbf{q}_c,E) - i\nu(\mathbf{q}_c,\mathbf{q}_c,E)\frac{\pi}{2}\right.$$

$$\left. + \frac{i}{\hbar}\left(S(\mathbf{q}_c,\mathbf{q}_c,E) + \frac{1}{2}\,\delta\mathbf{q}_\perp \cdot \mathbf{A}(\mathbf{q}_c) \cdot \delta\mathbf{q}_\perp\right)\right] \Bigg\},$$

- Note that scars can be seen an oscillatory fringes Heller '84, Berry '89
- Energy smearing: $\delta P_{scar} \sim \exp(-\epsilon T/\hbar)$, so long orbits are suppressed. In a chaotic system we can't resolve individual eigenstates, so we need to smear over an energy window. Bogomolny '87, Berry '89
- Position smearing: picks out nearly periodic orbits: Bogomolny '87

$$\delta P_{scar} \propto \exp\left(-\frac{1}{2}\frac{\Delta^2}{\hbar}(\mathbf{p}^A - \mathbf{p}^B)^2\right)$$

means that contributions come from orbits with

$$\left\|\mathbf{p}^A - \mathbf{p}^B\right\|_2 \lesssim \frac{\hbar}{\Delta}$$

- Formula applies to both stable and unstable orbits

- Note that previously in the literature, the semiclassical limit is taken to be $\hbar \to 0$
- This doesn't make sense since $\hbar$ is dimensionful!
- Instead, we take our small quantity to be $\frac{\hbar}{ET_{max}}$

Notation

- We will work with a complex scalar field $\phi(\mathbf{x}) = (\phi_1(\mathbf{x}), \phi_2(\mathbf{x}))$
- We will define the following notation:

$$\langle \mathbf{f}, \mathbf{g} \rangle_{L^2} := \int d^d\mathbf{x} \, \left(f_1(\mathbf{x}) g_1(\mathbf{x}) + f_2(\mathbf{x}) g_2(\mathbf{x}) \right)$$

$$\mathbf{f} \cdot \mathbf{M} \cdot \mathbf{g} := \int d^d\mathbf{x} \, d^d\mathbf{y} \sum_{i,j=1}^{2} f_i(\mathbf{x}) M_{ij}(\mathbf{x}, \mathbf{y}) g_j(\mathbf{y})$$

$$\frac{\delta \mathcal{F}}{\delta \mathbf{f}} \cdot \frac{\delta \mathcal{G}}{\delta \mathbf{g}} := \int d^d\mathbf{x} \left(\frac{\delta \mathcal{F}}{\delta f_1(\mathbf{x})} \frac{\delta \mathcal{G}}{\delta g_1(\mathbf{x})} + \frac{\delta \mathcal{F}}{\delta f_2(\mathbf{x})} \frac{\delta \mathcal{G}}{\delta g_2(\mathbf{x})} \right)$$

QFT Scar Formula: Derivation Cotler, Wei '22

- With eigenstates $\Psi_n[\phi]$ and energy window $[E - \epsilon/2, E + \epsilon/2]$, we want to compute

$$\langle |\Psi[\phi]|^2 \rangle_{E,\Delta} = \frac{\sum_n \langle |\Psi_n[\phi]|^2 \rangle_\Delta \delta_\epsilon(E - E_n)}{\sum_n \delta_\epsilon(E - E_n)}$$

- Again, we perform position smearing,

$$\langle f[\phi] \rangle_\Delta \propto \int [d\chi] e^{-\frac{1}{2\Delta^2} \|\chi - \phi\|^2_{L^2}} f[\chi]$$

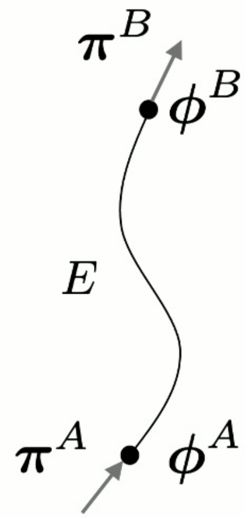
- First we derive the Van Vleck propagator $G(\phi^A, \phi^B, t)$ in field theory
- Then we transform from time to energy variables: $(\phi^A, \phi^B, t) \Rightarrow (\phi^A, \phi^B, E)$
- Abbreviated action:

$$S(\phi^A, \phi^B, E) = \int_{t^A}^{t^B} dt \, \langle \pi, \dot{\phi} \rangle_{L^2}$$

- At the endpoints:

$$\frac{\delta S(\phi^A, \phi^B, E)}{\delta \phi^A} = -\pi^A$$

$$\frac{\delta S(\phi^A, \phi^B, E)}{\delta \phi^B} = \pi^B$$



Periodic Orbits

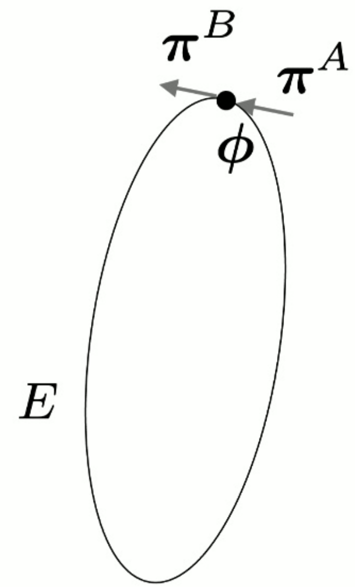
- Obtain periodic orbit by setting $\phi^A = \phi^B = \phi$
- Also have

$$\left(\frac{\delta S(\phi^A, \phi^B, E)}{\delta \phi^A} + \frac{\delta S(\phi^A, \phi^B, E)}{\delta \phi^B}\right)_{\phi^A=\phi^B=\phi}$$

$$= -\pi^A + \pi^B = 0$$

- We will abbreviate

$$A_{ij}[\phi](x, y) = \left(\frac{\delta^2 S(\chi^A, \chi^B, E)}{\delta \chi_i^A \delta \chi_j^A} + 2\frac{\delta^2 S(\phi^A, \phi^B, E)}{\delta \chi_i^A \delta \chi_j^B} + \frac{\delta^2 S(\chi^A, \chi^B, E)}{\delta \chi_i^B \delta \chi_j^B}\right)_{\chi^A=\chi^B=\phi}$$

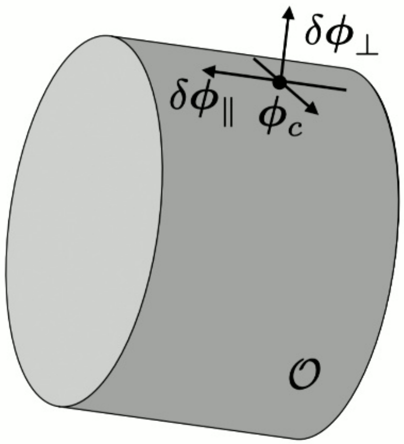


- Space of images of the periodic orbits $\mathcal{O}$ can have interesting structure in QFT: there is translation symmetry and potentially additional symmetry, so that

$$\mathcal{O} \sim \mathbb{S}^1 \times \mathbb{R}^d \times \mathcal{M}^k$$

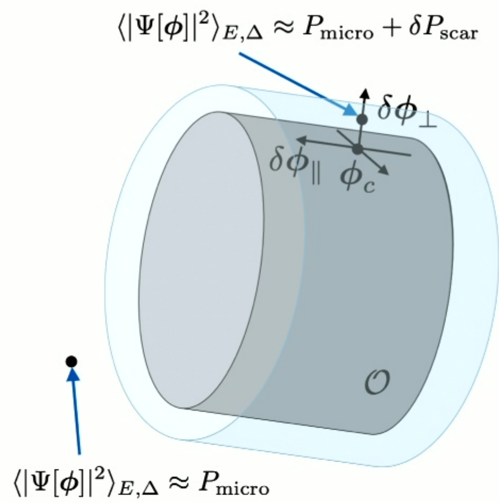
- Can split into directions parallel to and perpendicular to periodic orbit:

$$\delta\phi_{\parallel} \in T_\phi\mathcal{O}$$
$$\delta\phi_{\perp} \in N_\phi\mathcal{O}$$



QFT Scar Formula Cotler, Wei '22

$$\langle|\Psi[\phi]|^2\rangle_{E,\Delta} \approx \begin{cases} P_{\text{micro}}[\phi] + \delta P_{\text{scar}}[\phi_c, \delta\phi_\perp] & \text{if } \phi = \phi_c + \delta\phi_\perp \text{ where } \phi_c \in \mathcal{O},\ \delta\phi_\perp \in \mathrm{N}_{\phi_c}\mathcal{O}, \\ & \quad \|\delta\phi_\perp\|_{L^2} \lesssim \frac{\hbar}{\Delta} \\ P_{\text{micro}}[\phi] & \text{if } \left\|\pi^A - \pi^B\right\|_{L^2} \gg \frac{\hbar}{\Delta} \end{cases}$$



Here

$$\Delta = \sqrt{\hbar T_{max}} \left(\frac{\hbar}{E T_{max}} \right)^{\gamma}, \; \frac{1}{4} < \gamma < \frac{1}{2}$$

and

$$P_{\text{micro}}[\phi] = \frac{\int [d\pi] \, \delta_{\varepsilon}(E - H(\phi, \pi))}{\int [d\phi] \, [d\pi] \, \delta_{\varepsilon}(E - H(\phi, \pi))}$$

$$\delta P_{\text{scar}}[\phi_c, \delta\phi_{\perp}] = -\frac{2}{\pi\hbar \int [d\chi] \left[\frac{d\pi}{2\pi\hbar}\right] \delta_{\varepsilon}(E - H(\chi, \pi))} \, \text{Im} \left\{ \frac{1}{i} \frac{1}{\|\dot{\phi}\|_{L^2}} \right.$$
$$\left| \det \left(\frac{1}{2\pi i\hbar} \frac{\delta^2 S(\phi^A, \phi^B, E)}{\delta\phi_{\perp}^A \delta\phi_{\perp}^B} \right) \right|^{1/2}_{\phi^A = \phi^B = \phi} \times \exp \left[-\frac{\varepsilon}{\hbar} T(\boldsymbol{\phi}_c, \boldsymbol{\phi}_c, E) - i\nu(\boldsymbol{\phi}_c, \boldsymbol{\phi}_c, E) \frac{\pi}{2} \right.$$
$$\left. \left. + \frac{i}{\hbar} \left(S(\boldsymbol{\phi}_c, \boldsymbol{\phi}_c, E) + \frac{1}{2} \delta\boldsymbol{\phi}_{\perp} \cdot \mathbf{A}[\boldsymbol{\phi}_c] \cdot \delta\boldsymbol{\phi}_{\perp} \right) \right] \right\}.$$

- The scars can be seen an oscillatory fringes
- Due to energy smearing, $\delta P_{scar} \sim \exp(-\epsilon T/\hbar)$, so only orbits with $T \lesssim \hbar/\epsilon$ contribute
- Position smearing picks out nearly periodic orbits
- Formula applies to both stable and unstable orbits

Example: Q-balls and Q-clouds Coleman '85, Alford '88

- Consider a soliton stabilized by a $U(1)$ charge, with Lagrangian

$$\mathcal{L} = \partial_\mu \Phi^* \partial^\mu \Phi - U(|\Phi|^2)$$

and potential

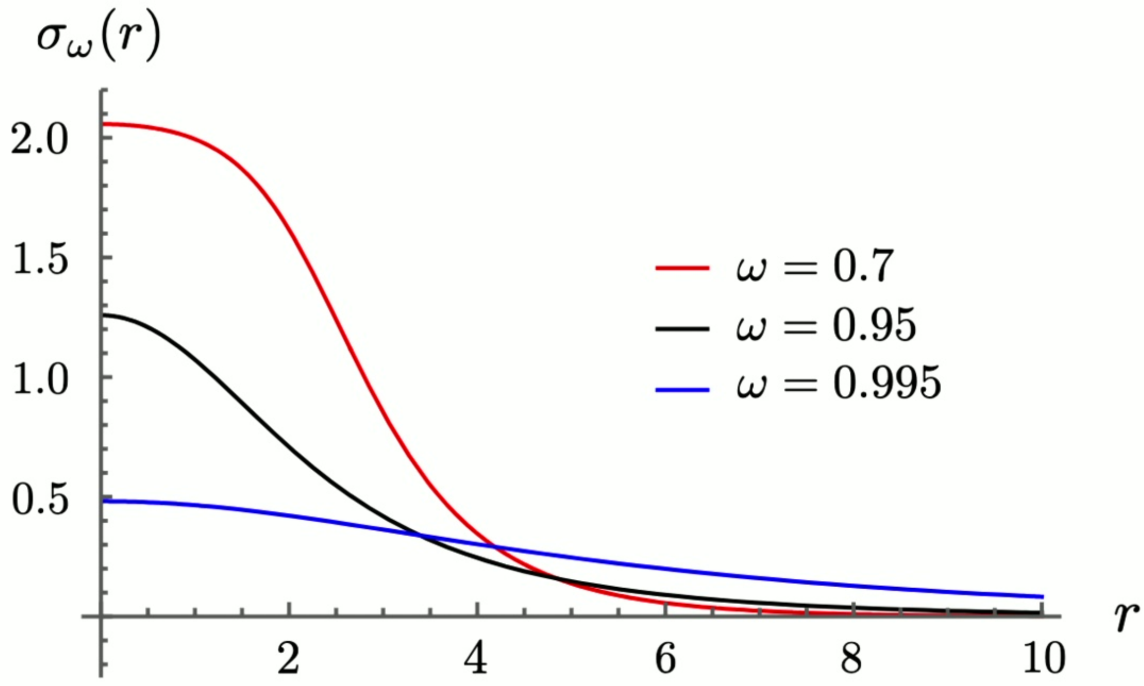
$$U(|\Phi|^2) = m^2|\Phi|^2 - \frac{f}{2}|\Phi|^4 + O(|\Phi|^6)$$

where $f > 0$

- Look for solutions that minimize energy at fixed charge
- This potential has solutions that are oscillating lumps,

$$\phi(\mathbf{x}, t) = e^{i\omega t}\sigma_\omega(|\mathbf{x}|)$$

- Stable solutions are known as Q-balls, unstable solutions as Q-clouds
- $\omega^2 \to m^2$ from below is the Q-cloud limit



Space of Q-cloud solutions Cotler, Wei '22

- Work in 3+1 dimensions, in energy window $[E - \epsilon/2, E + \epsilon/2]$
- Consider deformation of existing solution, and enforce that it satisfies EOM $\Rightarrow$ obtain system of ODEs that can either be characterized analytically or solved numerically

- We find that the space $\mathcal{O}$ of Q-cloud solutions is 5-dimensional:
  - time translations
  - spatial translations
  - energy deformations
- That is,

$$\mathcal{O} \sim \mathbb{S}^1 \times \mathbb{R}^3 \times [E - \epsilon/2, E + \epsilon/2]$$

- It also satisfies the conditions for the QFT scar formula

Outline

Takeaways

- Tools from semiclassical chaos can be adapted to QFT
- In semiclassical quantum chaos we often need to consider an ensemble of eigenstates rather than individual eigenstates
- Need to smear to get a saddle

Future Directions

- Find other examples of quantum scars in QFT, like oscillons (near-periodic solitons)
- Application to Rydberg many-body scars: can we write down an EFT of the PXP model?
- Application to AdS/CFT: can we use our tools to get a bulk path integral understanding of holographic scars? (Dodelson, Zhiboedov '22, Milekhin, Sukhov '23)
- Can we adapt other tools from semiclassical quantum chaos to QFT?

Thank You!