Title: Gauging spacetime inversions - VIRTUAL

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Series: Quantum Fields and Strings

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Abstract: Spacetime inversion symmetries such as parity and time reversal play a central role in physics, but they are usually treated as global symmetries. In quantum gravity there are no global symmetries, so any spacetime inversion symmetries must be gauge symmetries. In particular this includes CRT symmetry (in even
dimensions usually combined with a rotation to become CPT), which in quantum field theory is always a symmetry and seems likely to be a symmetry of quantum gravity as well. I'll discuss what it means to gauge a spacetime inversion symmetry, and explain some of the more unusual consequences of doing this. In particular I'll argue that the gauging of CRT is automatically implemented by the sum over topologies in the Euclidean gravity path integral, that in a closed universe the Hilbert space of quantum gravity must be a real vector space, and that in Lorentzian signature manifolds which are not time-orientable must be included as valid configurations of the theory.

Zoom link https://pitp.zoom.us/j/98089579253?pwd=NDBhUjBUTTEwT0R5YmhYVElZenJtZz09

Gauging spacetime inversions

Daniel Harlow

MIT

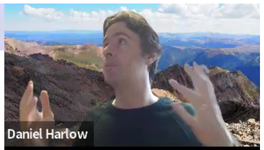
December 5, 2023

Introduction

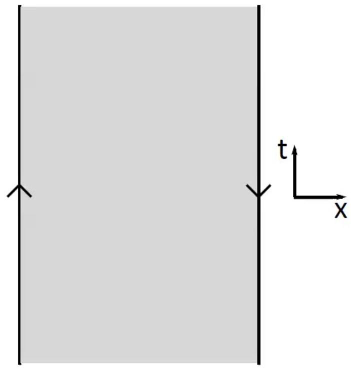
I'll begin with an apparent tension between two plausible statements about quantum gravity:

- **$\mathcal{CRT}$ is a symmetry in quantum gravity**
  - True in AdS/CFT
  - True in string perturbation theory
- **In quantum gravity there are no global symmetries**
  - Heuristic black hole argument in the continuous case Banks/Seiberg 2010
  - True in AdS/CFT Harlow/Ooguri 2018
  - Necessary for "island" picture of unitary black hole evaporation Harlow/Shaghoulian 2020

The only way out of this tension is that $\mathcal{CRT}$ *must be a gauge symmetry*. More generally any spacetime inversion symmetry which is not broken must be gauged.



In Euclidean signature there is a long history of gauging parity going back to the unoriented string worldsheet, but in Lorentzian signature the gauging of any symmetry involving $\mathcal{T}$ leads to potentially alarming consequences.



For example if $\mathcal{T}$ is gauged we should allow time-unorientable spacetimes such as the *Lorentzian Möbius strip*, where traversing a spatial circle reverses the direction of time.
Why doesn't this lead to causal pathologies?
Does quantum field theory even make sense in such spacetimes?

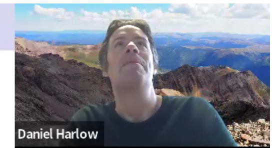
Another puzzle arises when we consider the Hilbert space of quantum gravity in a closed universe (such as $\mathbb{S}^3$):

- Any symmetry which reverses time is represented on Hilbert space by an antiunitary operator Θ.
- In a closed universe all physical states should be gauge-invariant.
- If $|\psi\rangle$ is invariant under Θ then $i|\psi\rangle$ is *not* invariant:

$$\Theta i|\psi\rangle = -i\Theta|\psi\rangle = -i|\psi\rangle.$$

- More generally the set of Θ-invariant states form a *real* vector space.

Since $\mathcal{CRT}$ reverses time and (by our assumptions) is always gauged, we see that the Hilbert space of quantum gravity in a closed universe is real! How is this consistent with quantum mechanics?



In this talk I will explain in more detail what it means to gauge spacetime inversion symmetries, focusing on the Lorentzian interpretation of the cases that involve time-reversal.

The plan:

- Review the gauging of discrete internal symmetries
- Background gauge fields for spacetime inversions in quantum field theory
- An example from AdS/CFT where time-unorientable geometries must be included
- Quantum mechanics in a closed universe

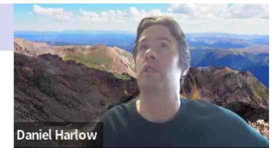
Based on 23111.09978 with Tokiro Numasawa.

Gauging discrete internal symmetries

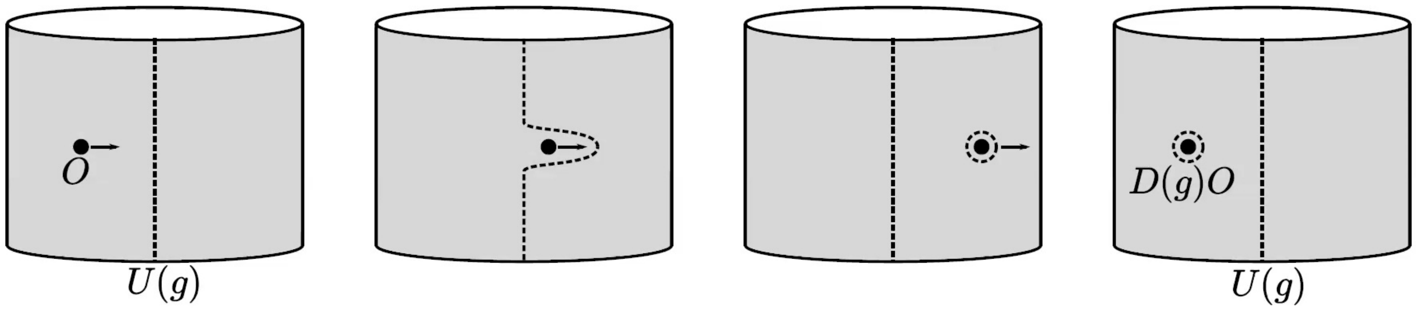
- In general a background gauge field on a spacetime M for an internal symmetry with symmetry group G is a connection on principal G-bundle over M.
- When G is discrete this reduces to a rule for assigning a G-holonomy to each loop in M.
- More formally a discrete background gauge field is a homomorphism

$$w : \pi_1(M) \to G,$$

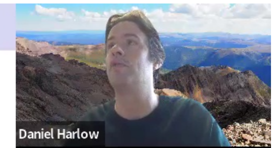
with an equivalence relation $w \sim gwg^{-1}$ for all $g \in G$.



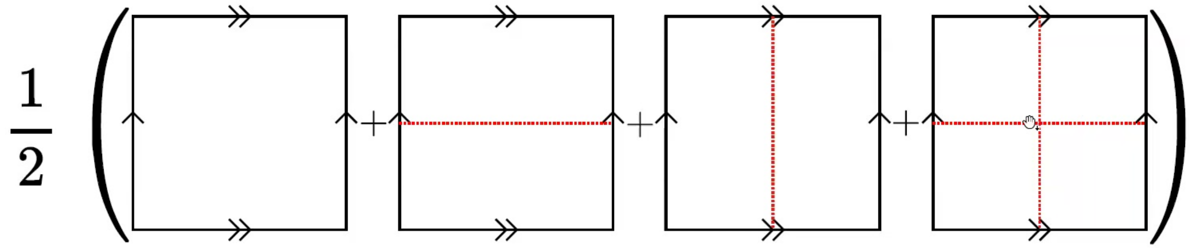
In practice we can implement a background gauge field by wrapping the codimension-one symmetry operator $U(g)$ on $(d-1)$-cycles that are dual to the generators of $\pi_1(M)$:



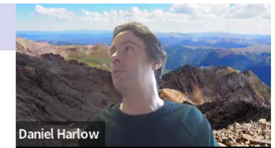
I emphasize for future reference that charged operators can detect the location of $U(g)$, although this will stop being the case in a moment when we gauge the symmetry.



To gauge the symmetry, we should now sum over background gauge fields and divide by the size of the group.



- For example if $M = T^2$ and $G = \mathbb{Z}_2$ there are four terms: the first two project onto singlet states in the ungauged theory, while the second two project onto singlets in a new "twisted" sector.
- Note the intersection of surface operators in the fourth term: if this cannot be consistently defined then the symmetry has an anomaly and cannot be gauged. Lin/Shao 2019



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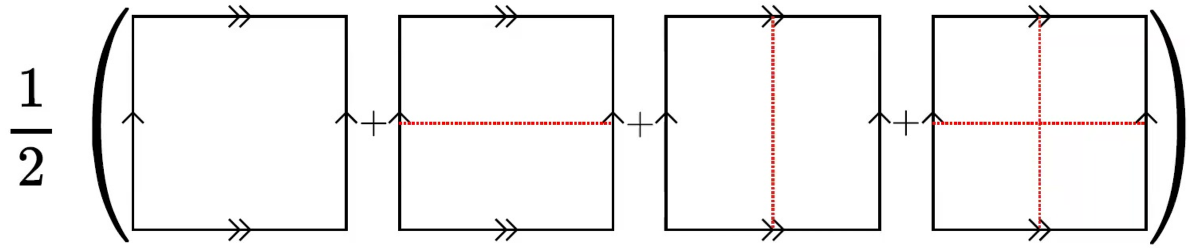

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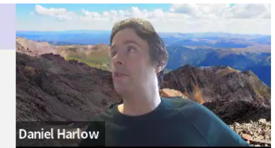
- Examples: orbifold of the compact scalar, fermion parity in the Ising model.

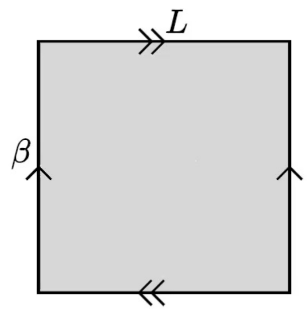


Background gauge fields for $\mathcal{R}$

We now turn to constructing backgrounds for $\mathcal{R}$. We'll discuss the general rules later, and first proceed by analogy.

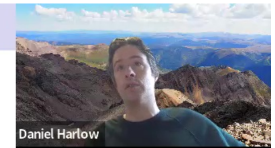
- For concreteness we will consider a free scalar field in $1+1$ dimensions on a spatial circle of circumference L. $\mathcal{R}$ symmetry is implemented by a unitary operator $U_\mathcal{R}$ which acts as

$$U_\mathcal{R}^\dagger \phi(t,x) U_\mathcal{R} = \phi(t,-x).$$



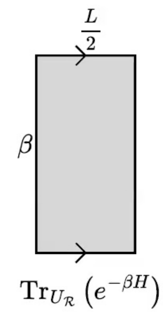
$\mathrm{Tr}\left(U_\mathcal{R} e^{-\beta H}\right)$

- Inserting $U_\mathcal{R}$ into the thermal trace gives the partition function on the Klein bottle, and the sum of the torus and the Klein bottle gives a projection onto $\mathcal{R}$-invariant states.



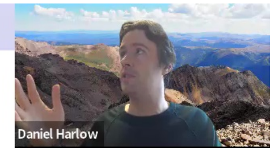
- Perhaps more surprising is what happens if we take $U_{\mathcal{R}}$ to be extended in the time direction, so that there is a spatial holonomy for $\mathcal{R}$.
- In other words we want to construct a twisted sector of states obeying the boundary conditions

$$\phi(t, x+L) = \phi(t, -x).$$



- What this does is remove half of the spacetime (which we can take to be the region $x \in (L/2, L)$) and introduce boundaries at the fixed points $x = 0$ and $x = L/2$.
- The boundaries are Neumann boundaries since

$$\partial_x \phi(t, 0) = \lim_{\epsilon \to 0} \frac{\phi(t, \epsilon) - \phi(t, -\epsilon)}{2\epsilon} = 0.$$



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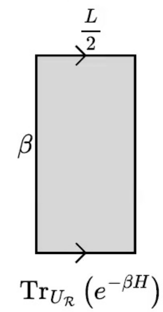

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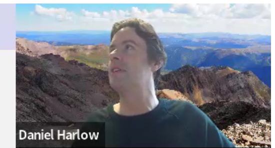
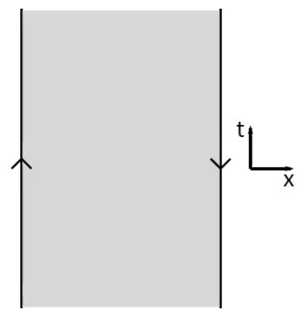
- In string theory language, the twisted states of gauging $\mathcal{R}$ are open strings! Sagnotti, Horava



Holonomy for $\mathcal{T}$

Where things really start getting interesting is when we try to turn on a spatial holonomy for $\mathcal{T}$:

$$\phi(t, x + L) = \phi(-t, x).$$



Classically this has a perfectly reasonable-looking initial value formulation: we need

$$\phi(0, x + L) = \phi(0, x)$$
$$\dot{\phi}(0, x + L) = -\dot{\phi}(0, x).$$

We can easily write down a reasonable set of solutions that can accommodate this initial data:

$$\phi(t,x) = A_0 + \sum_{n=1}^{\infty}\Big(A_n\cos(\omega_n t) + B_n\sin(\omega_n t)\Big)\cos(\omega_n t) + \sum_{n=1}^{\infty}\Big(C_n\cos(\hat{\omega}_n t) + D_n\sin(\hat{\omega}_n t)\Big)\sin(\hat{\omega}_n t),$$

with $\omega_n = \frac{2\pi n}{L}$ and $\hat{\omega}_n = \frac{2\pi(n+1/2)}{L}$.

To construct the phase space we also need a symplectic form, and here there is a puzzle: in quantum language we would like to have

$$[\phi(0,x),\dot{\phi}(0,y)] = i\delta(x-y),$$

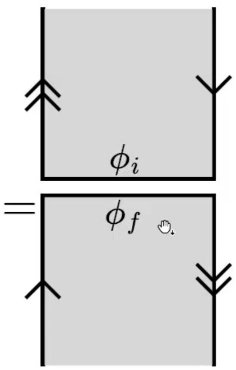
but which direction should we take the time derivative in? As we go around the circle there must be a discontinuity! Kay 1992, Friedman/Higuchi 1995

Our approach is to accept the existence of this discontinuity as the location of the symmetry brane: as in the internal case, we only need it to become invisible once we gauge the symmetry.

A natural state?

- We thus can construct a Hilbert space for this theory out of wave functionals of $\phi(x)$ on which $\frac{\delta}{\delta\phi(x)}$ flips sign at $x = 0 = L$.
- This Hilbert space however does not have any particularly nice states in it: there is no time-translation symmetry, so it does not have a ground state.

$$\langle\phi_f|\hat{\rho}|\phi_i\rangle =$$

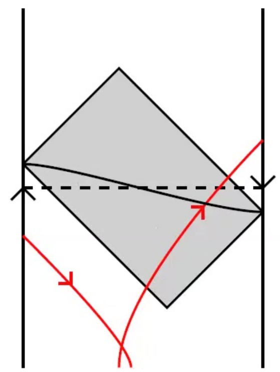


- We can try using the Euclidean path integral to prepare a state. This state however is not pure since the boundary conditions connect the bra and the ket, and in fact it is not even positive (as we will see in a moment). We'll call $\hat{\rho}$ the Möbius pseudostate.

- In the past this lack of a natural state was used to argue that quantum field theory in time-unorientable backgrounds does not make sense. Kay 1992, Friedman/Higuchi 1995
- Our attitude will instead be that this just means we need to think harder about what to compute: the Hilbert space of the theory is perfectly conventional.
- The nicest things to compute seem to be correlation functions in the Möbius pseudostate, which we will view as "computables" with which to characterize the theory rather than being directly observable.
- I'll spare you the details of the calculation of the correlators, but here are some formulas:

$$
\begin{aligned}
\partial_{x_1^-}\partial_{x_2^-}G &= -\frac{\pi}{16L^2\sin^2\left(\frac{\pi}{2L}\left(x_2^- - x_1^- - i\epsilon\right)\right)}\\
\partial_{x_1^+}\partial_{x_2^+}G &= -\frac{\pi}{16L^2\sin^2\left(\frac{\pi}{2L}\left(x_2^+ - x_1^+ - i\epsilon\right)\right)}\\
\partial_{x_1^-}\partial_{x_2^+}G &= \frac{\pi}{16L^2\cos^2\left(\frac{\pi}{2L}\left(x_2^+ + x_1^- - i\epsilon\right)\right)}\\
\partial_{x_1^+}\partial_{x_2^-}G &= \frac{\pi}{16L^2\cos^2\left(\frac{\pi}{2L}\left(x_2^- + x_1^+ - i\epsilon\right)\right)}.
\end{aligned}
$$

The main lesson of these correlators is that they have conventional causal properties in a "diamond" sitting on any spatial slice with the discontinuous point removed:



In particular they are compatible with the canonical commutation relations at $t = 0$.
Outside of this region however the algebra is more complicated, since outside of this region there can be self-intersections of time-like curves.

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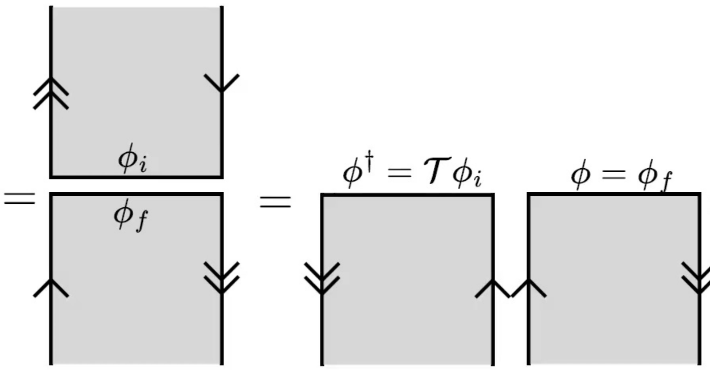
$$\partial_{x_1^-}\partial_{x_2^+}G = \frac{\pi}{16L^2\cos^2\left(\frac{\pi}{2L}\left(x_2^+ + x_1^- - i\epsilon\right)\right)}$$

$$\partial_{x_1^+}\partial_{x_2^-}G = \frac{\pi}{16L^2\cos^2\left(\frac{\pi}{2L}\left(x_2^- + x_1^+ - i\epsilon\right)\right)}.$$

In 2D CFT we can give a rather explicit characterization of the Möbius pseudostate:

$$\langle \widetilde{\phi}_f | \hat{\rho} | \phi_i \rangle =$$

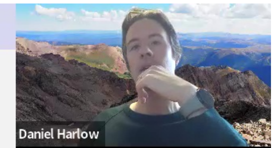


$$= \langle \mathcal{T}\phi_i, \widetilde{\phi}_f | \Omega \rangle$$

This leads to

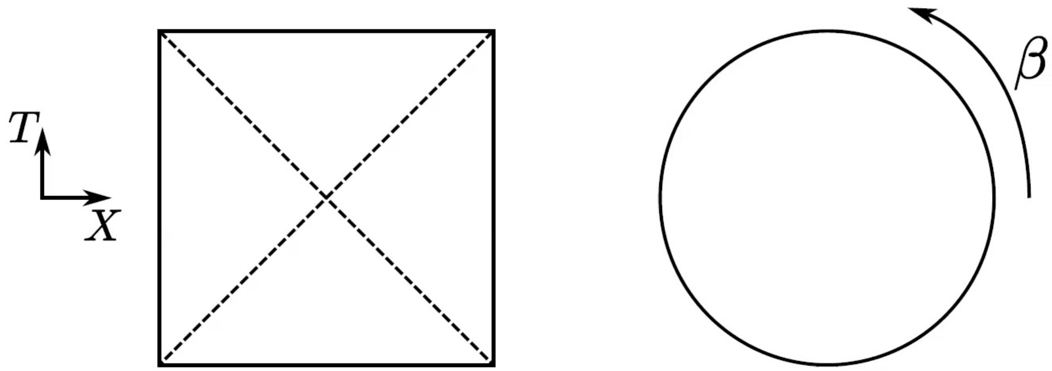
$$\hat{\rho} = e^{-\pi K_{cyl}} \Theta_{\mathcal{CRT}} \Theta_{\mathcal{T}},$$

so instead of a positive operator we have a positive operator times a unitary operator.



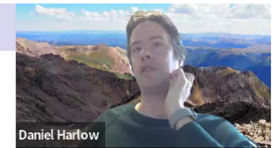
An example from AdS/CFT

You may be thinking I'm crazy, but I'll now explain how these results can arise quite naturally in the context of AdS/CFT.

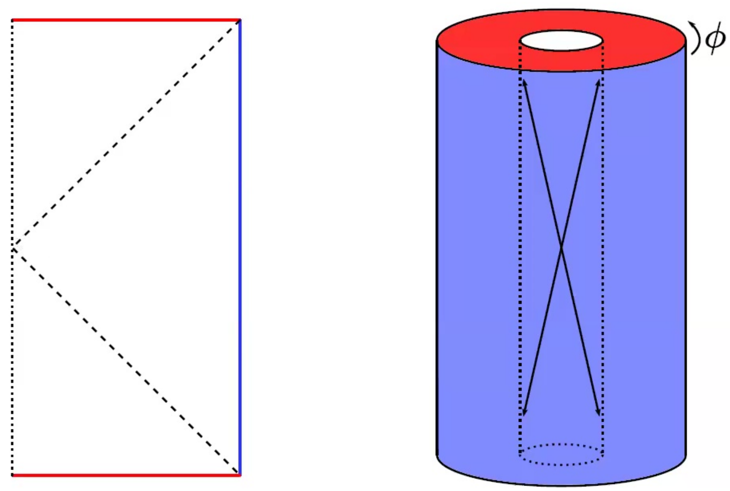


Let's first recall the BTZ geometry in Lorentzian and Euclidean signature. We usually study the former in the Hartle-Hawking state, while we use the latter to compute the thermal partition function

$$Z_{BTZ}(\beta) \sim e^{\frac{\pi^2}{2G\beta}} = e^{\frac{\pi^2 c}{3\beta}}.$$



We now construct a new geometry, which we call the $\mathcal{CRT}$-twisted black hole, by a $\mathbb{Z}_2$ quotient:



$$(T, X, \phi) \sim (-T, -X, \phi + \pi).$$

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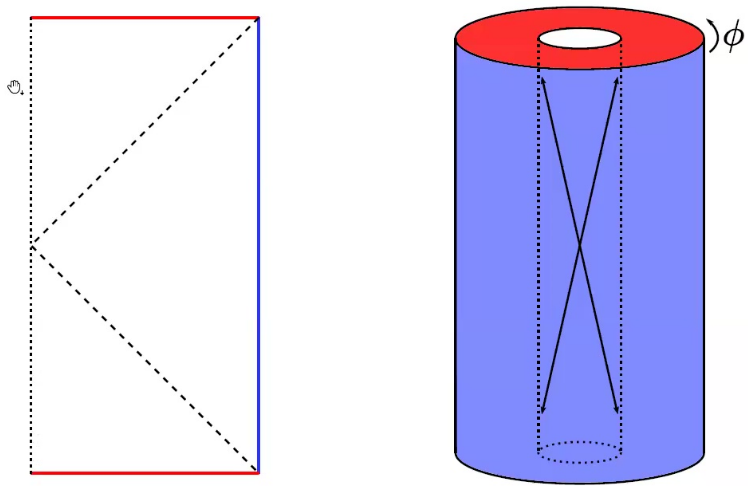
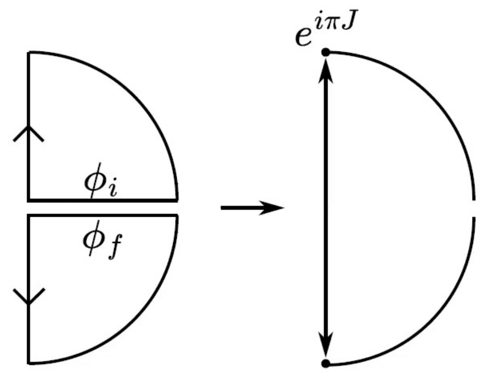


$$(T, X, \phi) \sim (-T, -X, \phi + \pi).$$

This is a smooth asymptotically-AdS Lorentzian manifold, whose asymptotic boundary is the usual Lorentzian cylinder. It however is not time-orientable, and has self-intersecting timelike curves.

Choice of pseudostate

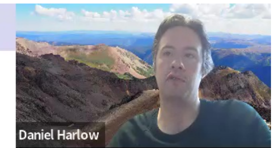
As in the Lorentzian Möbius strip there is not a natural state for this geometry, but there is a natural Euclidean pseudostate:



Its boundary interpretation is that

$$\hat{\rho} = e^{-\frac{\beta}{4}H} e^{i\pi J} e^{-\frac{\beta}{4}H} = e^{-\frac{\beta}{2}H} e^{i\pi J},$$

which again is a positive operator times a unitary.



We can actually check this proposal in two different ways. Let's first compute the trace of the psuedostate using the bulk saddle point:

$$Z_{bulk}(\beta) = \sqrt{Z_{BTZ}(\beta)} \sim e^{\frac{\pi^2 c}{6\beta}}.$$

We can do the CFT calculation using Cardyology: the quantity

$$Z_{CFT} = \mathrm{tr}\left(e^{-\frac{\beta}{2}H} e^{i\pi J}\right)$$

is just the torus partition function with

$$\tau = \frac{1}{2} + i\frac{\beta}{4\pi} \equiv \frac{1}{2} + i\epsilon.$$

Therefore we have

$$Z_{CFT}\left(\frac{1}{2} + i\epsilon\right) = Z\left(-\frac{1}{\frac{1}{2} + i\epsilon}\right) \approx Z(-2 + 4i\epsilon) = Z(4i\epsilon) = Z\left(\frac{i}{4\epsilon}\right) \sim e^{\frac{\pi^2 c}{6\beta}}.$$

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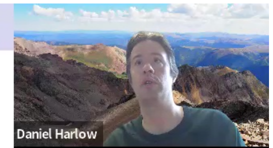
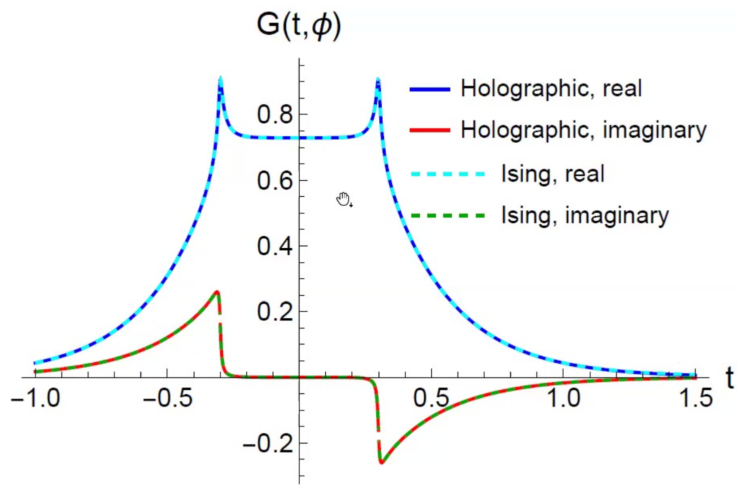
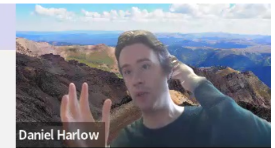


We can also look at the Wightman two-point function directly in Lorentzian signature:

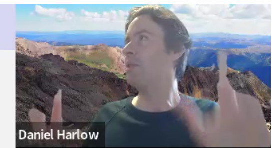


Here we are comparing to the high-temperature torus correlator in the Ising model with $\tau = \frac{1}{2} + i\frac{\beta}{4\pi}$, as this should be universal. (Could also probably be done using the torus Virasoro identy block).



The general story

- So far we have considered a number of examples, in this slide I'll briefly say what the general rules are for gauging $\mathcal{R}$ and $\mathcal{T}$ (I'll ignore fermions, see the paper for how to handle them). This question only really makes sense in gravitational theories, since trying to gauge them in QFT would break Lorentz symmetry.
- The organizing principle is the *structure group of the tangent bundle*. In any Lorentzian manifold we can reduce the structure group from $GL(d-1,1)$ to $O(d-1,1)$. The latter has four connected components, so it is natural to ask if we can reduce it further. There are five possibilities: $\mathcal{R}$ and $\mathcal{CT}$ both gauged, $\mathcal{R}$ and $\mathcal{CT}$ both not gauged, $\mathcal{R}$ gauged but not $\mathcal{CT}$, $\mathcal{CT}$ gauged but not $\mathcal{R}$, and $\mathcal{CT}$ gauged but not $\mathcal{R}$ or $\mathcal{CT}$. All possibilities make sense, and constrain which kinds of geometries we include.
- In Euclidean signature we instead reduce to $O(d)$, which has only two connected components. $\mathcal{CRT}$ is automatically gauged since it is in the identity component, so the only choice is whether $\mathcal{R}$ and $\mathcal{CT}$ are both gauged or neither gauged. In more conventional language, do we include unoriented manifolds or not? As usual Euclidean gravity knows something that Lorentzian gravity doesn't: that $\mathcal{CRT}$ must be gauged.



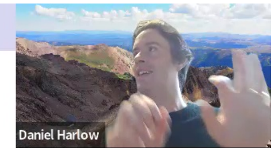
Some remarks on quantum cosmology

- In a closed universe all physical states must be gauge-invariant. In electromagnetism this is a consequence of integrating Gauss's law, and the same is true for discrete symmetries.
- For antiunitary symmetries such as $\mathcal{CRT}$ this has a surprising consequence: if $|\psi\rangle$ and $|\phi$ are linearly-independent $\mathcal{CRT}$-invariant states then we have

$$\Theta(a|\psi\rangle + b|\phi\rangle) = a^*|\psi + b^*|\phi\rangle,$$

  which will only be equal to $a|\psi\rangle + b|\phi\rangle$ if a and b are real.
- Thus the Hilbert space of quantum gravity in a closed universe is real!

This seems to be a powerful constraint on any putative holographic dual of cosmology in a closed universe.



You can immediately ask why this doesn't destroy quantum mechanics. After all in most presentations of quantum mechanics the phases seem rather important, e.g. in the double slit experiment or Shor's algorithm.

- The reason it doesn't goes back to an old issue in quantum cosmology: you need to include a clock!
- Indeed if we define

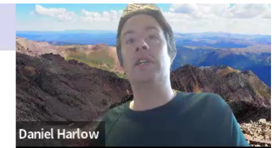
$$|\widetilde{\psi}\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle_C|\psi\rangle_S + |-\rangle_C\Theta_S|\psi\rangle_S\right)$$

$$\widetilde{P} = |+\rangle\langle+|_C \otimes |\chi\rangle\langle\chi|_S + |-\rangle\langle-| \otimes \Theta_S|\chi\rangle\langle\chi|\Theta_S^\dagger,$$

  then we simply have

$$\langle\widetilde{\psi}|\widetilde{P}|\widetilde{\psi}\rangle = |\langle\chi|\psi\rangle|^2$$

  for any states $|\psi\rangle$ and $|\chi\rangle$ of the system S.
- This may seem like bookkeeping, but it isn't: the clock needs to be part of the system, so this calculation is only exact in the limit of an infinitely big universe. When it is finite, there are corrections to quantum mechanics!



Another interesting consequence of this is that in quantum cosmology there are the "Hartle-Hawking" and "Vilenkin" proposals for the wave function of the universe:

- HH: sum over expanding and contracting branches
- V: expanding only

The arguments on both sides are not particularly convincing, but the gauging of $\mathcal{CRT}$ comes down strongly on the side of Hartle-Hawking!

Conclusions

Thus we've learned (I hope) several interesting things:

- Gauging spatial reflections can introduce boundaries of spacetime, which in string theory are open strings.
- Quantum field theory makes sense on backgrounds which are not time-orientable, but doesn't have a preferred state.
- In AdS/CFT it is necessary to include time-unorientable geometries to match boundary CFT calculations.
- In a closed universe the Hilbert space of quantum gravity is real.

Thanks!