Title: Do Euclidean Wormhole Saddles Contribute to the Factorization Problem?

Speakers: Molly Kaplan

Series: Quantum Fields and Strings

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Abstract: We investigate the nature of the Giddings-Strominger wormhole in axion gravity, whose stability remains contested despite previous work in this direction. Unlike what is done in these works, we follow a Lorentzian approach which directly addresses the factorization problem. To probe the thermodynamic stability of wormholes in the gravitational path integral, we look for fixed-area saddle points. However, taking care to choose the appropriate boundary conditions, we find that there are no fixed-area axion wormholes in Lorentzian signature. We then discuss how we could go beyond this analysis, considering off-shell axion wormhole configurations with fixed-length.

Zoom link https://pitp.zoom.us/j/98119789932?pwd=N1RCanJCdk56eWJ3RHJCRG5KU21lQT09

Do Axion Wormhole Saddles Contribute to the Factorization Problem?

Based on work to appear with Jesse Held and Don Marolf

Molly Kaplan (UCSB)

December 5, 2023

Perimeter Institute Quantum Fields & Strings Seminar

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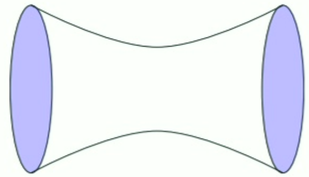
Spacetime wormholes



- We define a spacetime wormhole as a geometry (real or complex) with boundaries that have two or more disconnected components
- Most constructions considered have been in Euclidean signature
- Ubiquitous in string theory
- Supported by "bouncing" energy in FLRW: $ds^2 = d\tau^2 + a^2(\tau)d\Sigma_d^2$ (e.g., **negative kinetic energy**, gradient energy, negative boundary curvature)
- A renewed importance:
  - Replica wormholes producing the Page curve [Almheiri et. al. '20] [Penington et. al. '19] (though these are distinct from spacetime wormholes)

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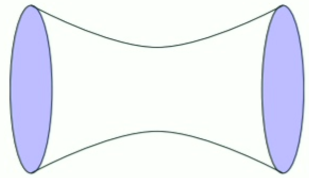

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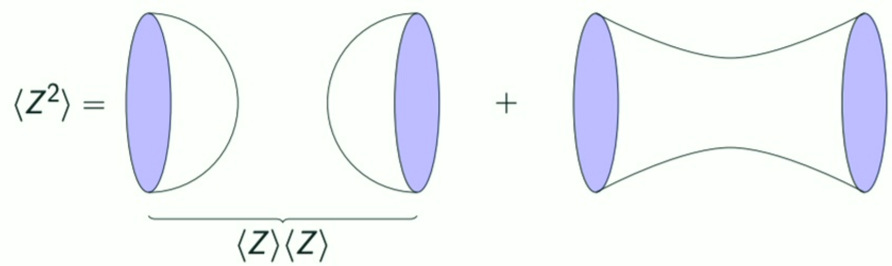
  - Replica wormholes relate to physical wormholes [Penington et. al. '19] [Marolf, Maxfield '20]

Factorization problem

- Wormholes are thus an important part of the quantum gravity story
- But including them in the gravitational path integral (GPI), i.e.,

$$Z_{grav} = \int \mathcal{D}g\mathcal{D}\phi e^{-S_E[g,\phi]} \tag{1}$$

leads to the "factorization puzzle"



- The gravitational theory seems to be dual to an ensemble of CFTs, contradicting our established intuition of UV-complete bulk gravity theories [Maldacena '97], etc.

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Potential resolutions to the factorization problem

- Ensembles are right (loophole for limits on SUSY CFTs in high-dim? [Seiberg '10])
- Precise cancellations of wormholes in UV-complete theories, like in SYK [Saad et. al. '21]
- Euclidean wormholes unstable due to brane nucleation (e.g. wormholes with negative-curvature boundaries [Witten, Yau '99], or Euclidean wormholes from 10- and 11-dim SUGRA [Marolf, Santos '21])
- Euclidean wormholes unstable due to field-theoretic negative modes
- **Our perspective: we should really consider *Lorentzian* wormholes** - whose contribution could could be forbidden due to brane nucleation or negative modes, or for some other reason

Why Lorentzian?

- More fundamental description
- Avoids the conformal factor problem in Euclidean signature
  - Over real contours, the Euclidean action is unbounded below
  - Can avoid this by choosing a complex contour, but in many cases there is no "natural" choice for this contour
- For the axion wormhole, extra subtlety in Euclidean due to the sign of the kinetic term

However still some subtleties in Lorentzian signature, especially with the domain of integration $\Longrightarrow$ we will include geometries with mild singularities (codimension-2 conical signularities)

Axion wormholes: definition

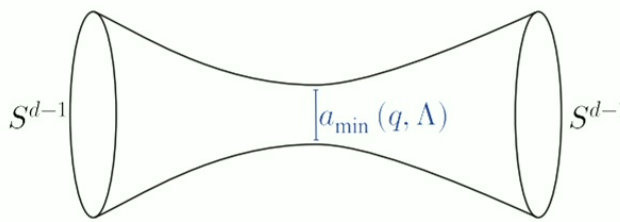
Originally formulated in Euclidean signature, where the kinetic term has the "wrong" sign [Giddings, Strominger '88]

$$S_E = -\frac{1}{16\pi G}\int_{\mathcal{M}} \sqrt{g}\left(R - \Lambda + \partial_\mu \chi \partial^\mu \chi\right) + S_{\text{bndy.}} \qquad (2)$$

The solution is an FLRW metric

$$ds^2 = dr^2 + a^2(r) d\Omega^2_{d-1} \qquad (3)$$

where $a(r)$ depends on Λ and the conserved axion flux $q \equiv \int_{r=\text{constant}} d\Omega \sqrt{h} \partial_r \chi$



Axion wormholes: previous studies

Its stability under negative modes has been studied, but with conflicting results:

- Hertog et. al. '19
  - Euclidean analysis
  - Asymptotically AdS
  - Neumann boundary conditions for the axion
  - Unstable: infinitely many negative modes
  - Unstable perhaps due to the choice of boundary conditions? [Andade, Marolf '11]
- Loges et. al. '22:
  - Lorentzian analysis, but does not address factorization problem - studies transition amplitudes
  - Asymptotically flat
  - Stable: no negative modes

Our research question

We would like to study the axion wormhole contribution to the Lorentzian path integral, i.e.

- Given boundary conditions on two disconnected spacetime boundaries, does there exist a connected solution to (most of) the equations of motion?
- If not, can the fields in the theory be analytically continued to make one?

To study this in a way that addresses the factorization problem, we first need to introduce the approach of [Marolf '22] for constructing Lorentzian partition functions

Gravitational Thermodynamics of the Lorentzian GPI

We define the partition function

$$Z(\beta) \equiv \text{Tr} e^{-\beta H} = \int dT f_\beta(T) Z_L(T) \tag{4}$$

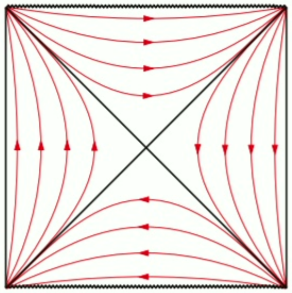
In AdS/CFT, $Z_L(T) = \text{Tr} e^{-iHT}$.
Just as $Z_E(\beta)$ is defined with boundaries $S^1 \times Y$ where S^1 has proper length β, $Z_L(\beta)$ is defined with boundaries $S^1 \times Y$ where T is the proper time around S^1. In practice, this means...

- Start with a boundary $\partial\mathcal{M} = Y \times \mathbb{R}$ and a killing vector ξ_∂ which generates time translations along $\mathbb{R}$
- Quotient the boundary by the isometry $e^{\xi_\partial T} \implies$ get boundary $Y \times S^1$ with T the proper time around S^1

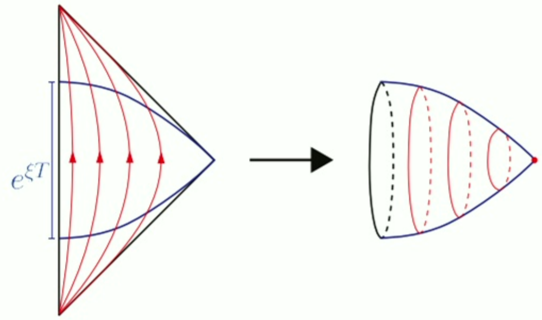
$Z_L(T)$ is a GPI (integral over e^{iS}) for geometries with these boundary conditions.

The Lorentzian GPI: AdS-Schwarzschild example

For example, we take the AdS-Schwarzschild black hole



- ξ_∂ can be extended into the bulk but has a non-trivial killing horizon
- $\implies$ when we periodically identify, get a codimension-2 conical singularity:



The Lorentzian GPI

For AdS-Schwarzschild and similar geometries

$$Z(\beta) = \int dA dT f_\beta(T) e^{iS}, \quad \text{with} \quad S = -iA[\gamma]/4G - E_\xi T \qquad (5)$$

Some comments:

- The integrand is not pure phase, but that is okay
- To get convergence, we first perform the integral over T
- E_ξ is also a function of A
- This gives the usual results in simple cases, e.g. Euclidean BHs are saddles (this is only possible because we included metrics with conical singularities)

Boundary conditions for Lorentzian axion wormholes

We now apply this same analysis to the Lorentzian axion wormhole in AAdS, $\Lambda < 0$.

- In Euclidean signature

$$ds^2 = dr^2 + a^2(r)d\Omega^2_{d-1} \tag{6}$$

and Wick rotating to Lorentzian

$$ds^2 = dr^2 + a^2(r)d\Sigma^2_{d-1} \tag{7}$$

where $d\Sigma^2_d$ is the static patch de Sitter line element. Eventually we'll take a quotient.

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- We'll also take (real) Dirichlet boundary conditions for the axion on the two boundaries
$$S_L = \frac{1}{16\pi G} \int_{\mathcal{M}} \sqrt{-g} \left(R - \Lambda - \partial_\mu \chi \partial^\mu \chi \right) + S_{\text{bndy.}} \tag{8}$$

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Note that the kinetic term has the correct sign! Looks like an ordinary scalar field...

Equations of motion for Lorentzian axion wormholes

- Since the axion is constant on the boundaries, we set $\chi(r, x^i) = \chi(r)$. The constant flux is

$$q \equiv \int_{r=\text{constant}} d^{d-1}x\sqrt{h}\partial_r\chi(r) = a^{d-1}(r)\partial_r\chi(r). \qquad (9)$$

  relating χ and the scale factor.

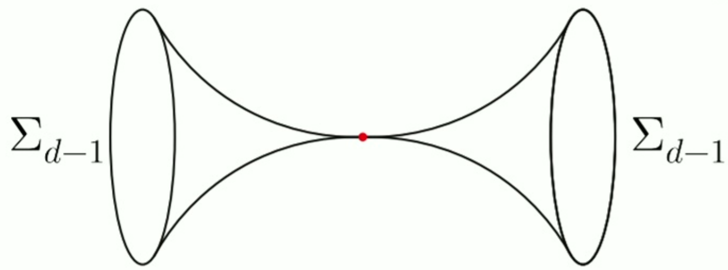
- The rr-component of Einstein's equations gives

$$\left(\partial_r a(r)\right)^2 = 1 + \frac{q^2}{(d-1)(d-2)a^{2(d-2)}} - \frac{2a^2\Lambda}{(d-1)(d-2)} \qquad (10)$$

- $\implies$ with $\Lambda < 0$, $\partial_r a(r)$ never vanishes. What does this mean?

No fixed-area Lorentzian axion wormholes

- No positive minimal value of $a(r)$
- Not really a two boundary solution (no "bounce") but two disconnected one-boundary solutions with a singularity in the deep bulk



- Curvature grows polynomially in a^{-1} approaching the singularity

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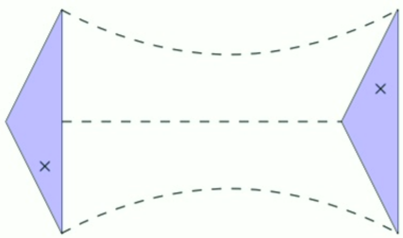
Imaginary values for the axion?

- We could consider imaginary boundary conditions for the axion, then q^2 changes sign and

$$\left(\partial_r a(r)\right)^2 = 1 - \frac{q^2}{(d-1)(d-2)a^{2(d-2)}} - \frac{2a^2\Lambda}{(d-1)(d-2)} \quad (11)$$

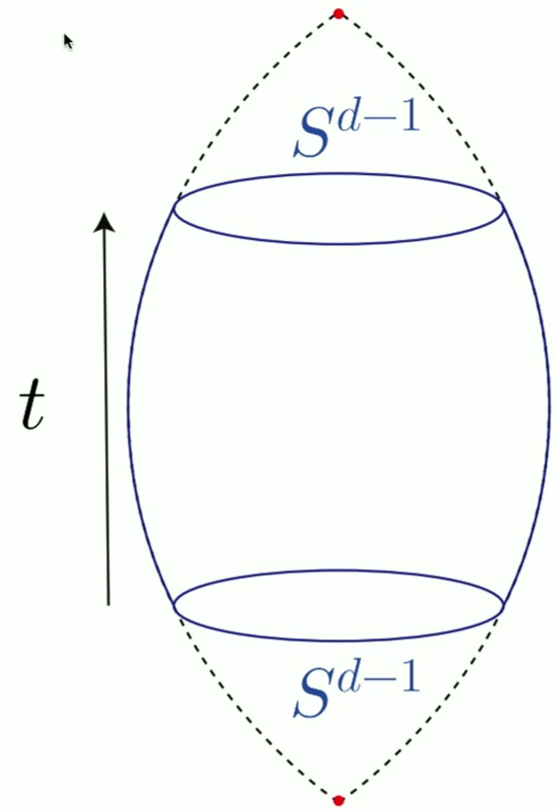
  does give a wormhole solution.
- Perhaps, via analytic continuation, this tells us something about dual CFT correlators for real $\Delta\chi$
- But, the wormhole is traversable $\implies$ non-zero commutators between the two boundaries, which does not match the CFT calculation



Fixed-Length Wormholes

Revisiting Loges et. al.

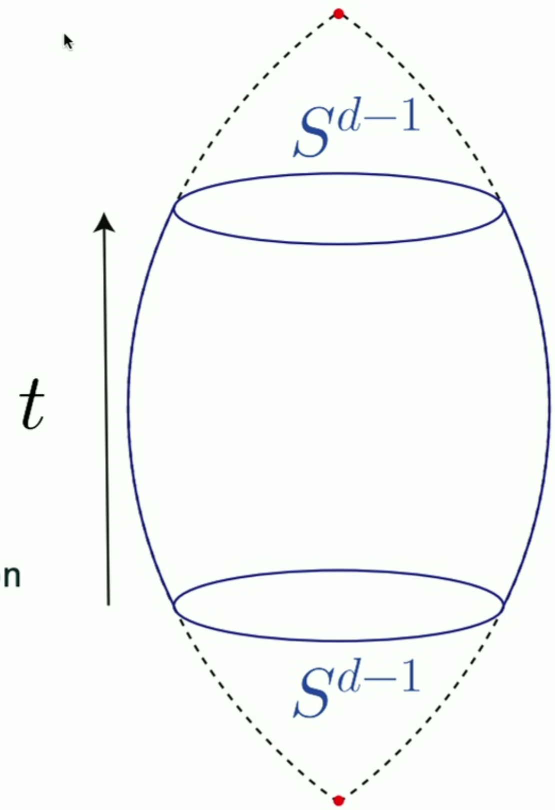
- Two boundary transition amplitude in axion gravity, with a Lorentzian GPI, Dirichlet boundary conditions for the metric, and Neumann boundary conditions for the axion



Fixed-Length Wormholes

Revisiting Loges et. al.

- Two boundary transition amplitude in axion gravity, with a Lorentzian GPI, Dirichlet boundary conditions for the metric, and Neumann boundary conditions for the axion
- Explore fixed-"length" saddles, integrating over the lapse N
- Find family of bang/crunch cosmologies with imaginary saddle point
- These aren't relevant to our calculation (results depend on our particular choice of calculation!), but we could use the idea of fixed-length saddles...



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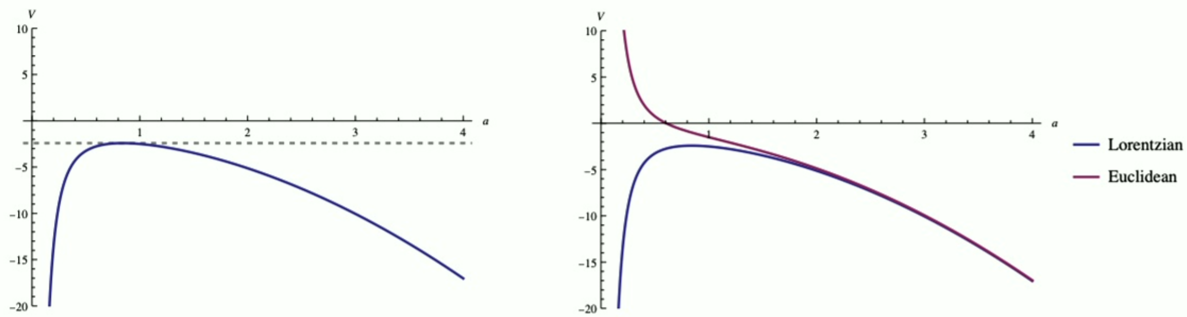
A new equation of motion

To vary over the wormhole length we must go off-shell:

$$(\partial_r a)^2 \underbrace{- \left(1 + \frac{q^2}{(d-1)(d-2)a^{2(d-2)}} - \frac{2a^2\Lambda}{(d-1)(d-2)}\right)}_{\text{potential } V(a)} = E \qquad (12)$$



Imposing the same boundary conditions as before, we find the scale factor.

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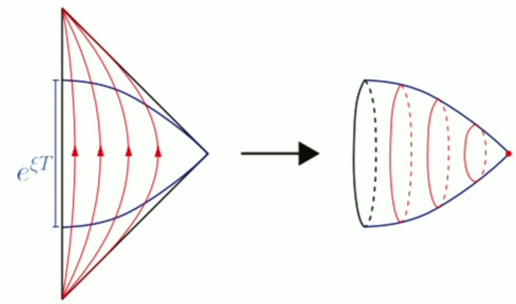
Computing the action

Using this scale factor solution, we can solve for the action

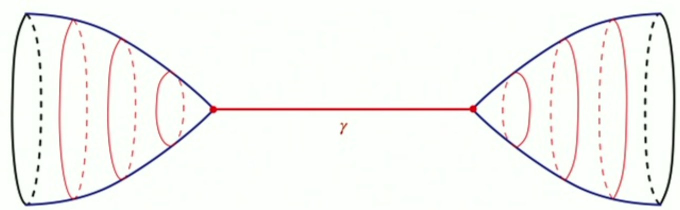
$$S = -iA[\gamma]/4G - E_\xi T \quad (13)$$

- E_ξ is the ADM energy of the boundary KVF ξ
- What is γ?

On each boundary we have:



The conical defect is a codimension-2 surface connecting the boundaries:



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Contour analysis

- Once we have the action, we plug into the GPI

$$Z(\beta) = \int dLdAdTf_\beta(T)e^{A/4G}e^{-iE_\xi T} \tag{14}$$

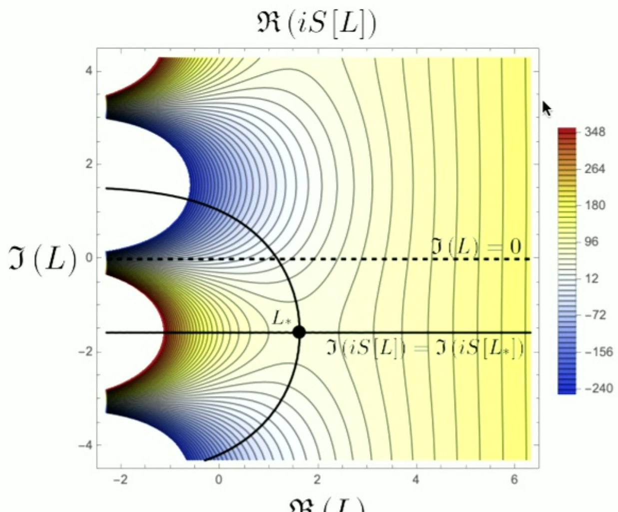
  and perform the T integral, setting $T = -i\beta$.

- We then perform a saddle point and contour analysis for the remaining integral. By Morse theory:

$$I = \sum n_p \mathcal{J}_p \tag{15}$$

  So we look for complex saddles for which the steepest ascent contour intersects the contour over reals

Preliminary results



$l = h_0 = \chi = 1 \quad T = 2\pi$

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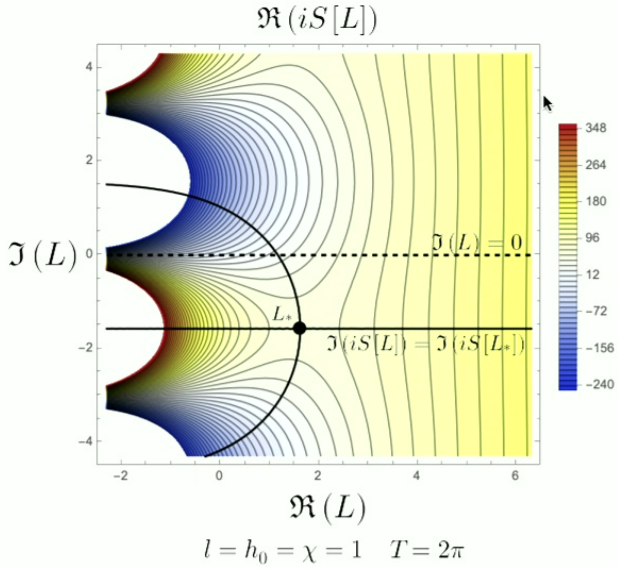

- In $2+1$-dim, our results so far suggest that these fixed-length wormholes are irrelevant
- Different results from [Loges et. al. '22], reinforces the point that **the relevance of the axion wormhole is inextricably linked to the particular quantity we are computing**

Conclusion & Future Directions

Future work

- Understanding what parts of our analysis here can be applied to other Euclidean wormholes supported by gradients, like those in [Marolf, Santos '21][Chandra, Hartman '22][Stanford '20]. The argument against allowing imaginary $\Delta\chi$ doesn't seem to hold for wormholes supported by gradients
- Extending our fixed-length wormhole results beyond $2+1$-dimensions
- Applying [Marolf '22] to other contexts where a Lorentzian approach would be enlightening, e.g. [Chua, Hartman '23]

Summary

- The main moral: the relevance of the axion wormhole is highly dependent on the exact question we are asking
- In the fixed-area calculation, we found there are no Lorentzian axion wormholes with real BC's for χ
- We argued against considering axion wormholes with imaginary BC's for χ
- We considered "off-shell" fixed-length wormholes, with preliminary results suggesting that these do not contribute to the GPI (in $2+1$-dim)

Thank you!

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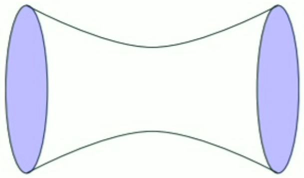

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Factorization problem

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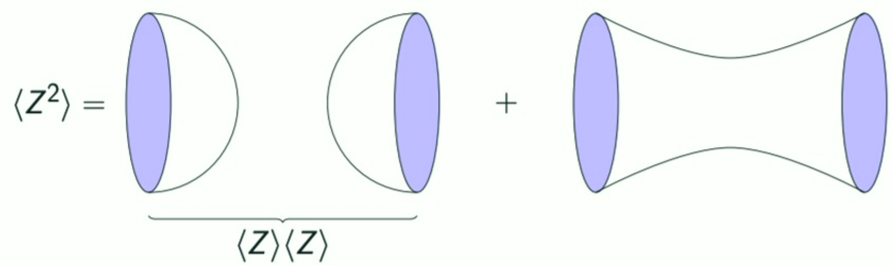

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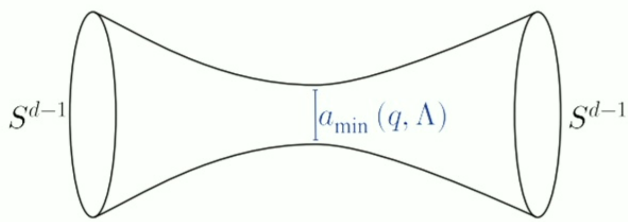

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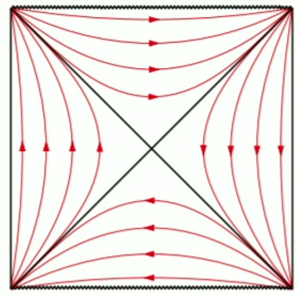

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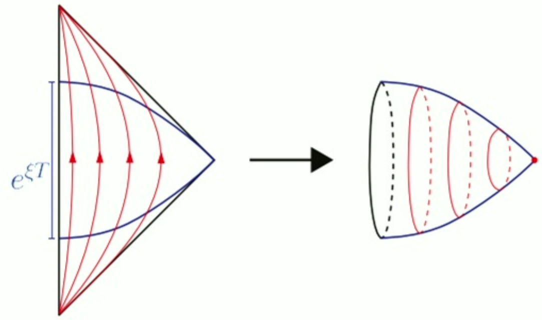

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Fixed-Area Calculation

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  where $d\Sigma^2_d$ is the static patch de Sitter line element. Eventually we'll take a quotient.
- We'll also take (real) Dirichlet boundary conditions for the axion on the two boundaries

$$S_L = \frac{1}{16\pi G}\int_{\mathcal{M}}\sqrt{-g}\left(R - \Lambda - \partial_\mu\chi\partial^\mu\chi\right) + S_{\text{bndy.}} \tag{8}$$

Note that the kinetic term has the correct sign! Looks like an ordinary scalar field...

Equations of motion for Lorentzian axion wormholes

- Since the axion is constant on the boundaries, we set $\chi(r, x^i) = \chi(r)$. The constant flux is

$$q \equiv \int_{r=\text{constant}} d^{d-1}x\sqrt{h}\partial_r\chi(r) = a^{d-1}(r)\partial_r\chi(r). \tag{9}$$

  relating χ and the scale factor.
- The rr-component of Einstein's equations gives

$$\left(\partial_r a(r)\right)^2 = 1 + \frac{q^2}{(d-1)(d-2)a^{2(d-2)}} - \frac{2a^2\Lambda}{(d-1)(d-2)} \tag{10}$$

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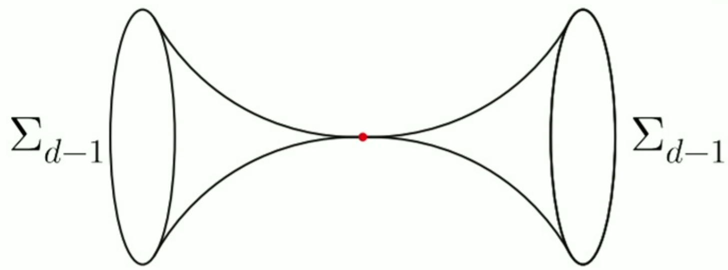
- $\Longrightarrow$ with $\Lambda < 0$, $\partial_r a(r)$ never vanishes. What does this mean?

No fixed-area Lorentzian axion wormholes

- No positive minimal value of $a(r)$
- Not really a two boundary solution (no "bounce") but two disconnected one-boundary solutions with a singularity in the deep bulk



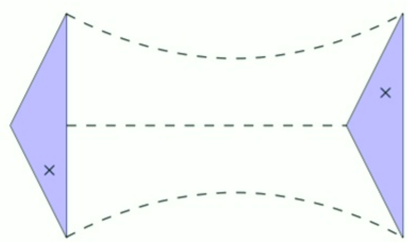
- Curvature grows polynomially in a^{-1} approaching the singularity

Imaginary values for the axion?

- We could consider imaginary boundary conditions for the axion, then q^2 changes sign and

$$\left(\partial_r a(r)\right)^2 = 1 - \frac{q^2}{(d-1)(d-2)a^{2(d-2)}} - \frac{2a^2\Lambda}{(d-1)(d-2)} \quad (11)$$

  does give a wormhole solution.
- Perhaps, via analytic continuation, this tells us something about dual CFT correlators for real $\Delta\chi$
- But, the wormhole is traversable $\implies$ non-zero commutators between the two boundaries, which does not match the CFT calculation

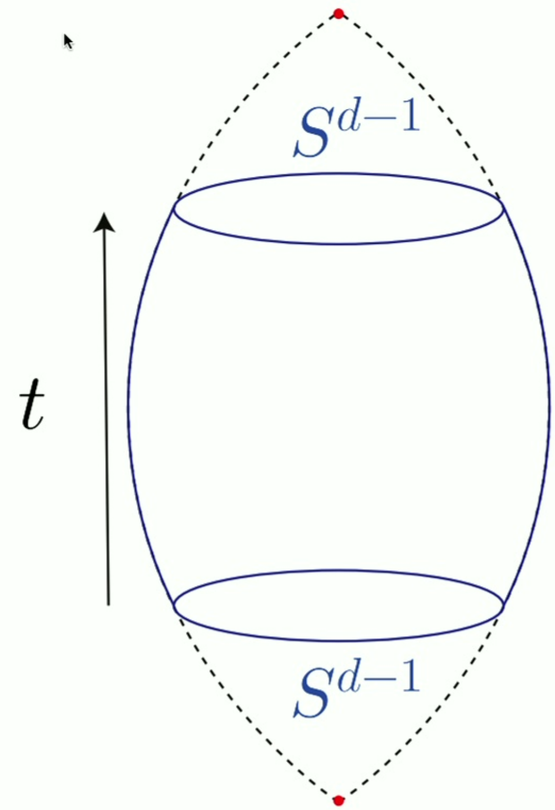


- Another oddity: action does not change under continuation, but no classical solution

Fixed-Length Wormholes

Revisiting Loges et. al.

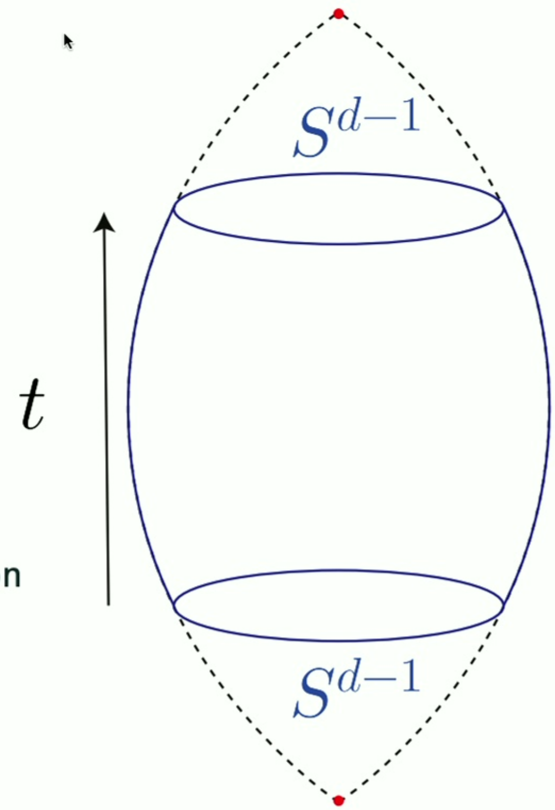
- Two boundary transition amplitude in axion gravity, with a Lorentzian GPI, Dirichlet boundary conditions for the metric, and Neumann boundary conditions for the axion



Fixed-Length Wormholes

Revisiting Loges et. al.

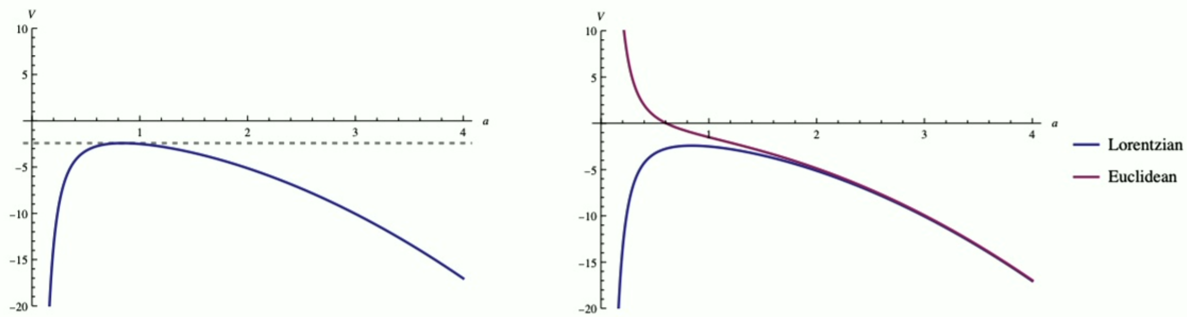
- Two boundary transition amplitude in axion gravity, with a Lorentzian GPI, Dirichlet boundary conditions for the metric, and Neumann boundary conditions for the axion
- Explore fixed-"length" saddles, integrating over the lapse N
- Find family of bang/crunch cosmologies with imaginary saddle point
- These aren't relevant to our calculation (results depend on our particular choice of calculation!), but we could use the idea of fixed-length saddles...



A new equation of motion

To vary over the wormhole length we must go off-shell:

$$(\partial_r a)^2 \underbrace{- \left(1 + \frac{q^2}{(d-1)(d-2)a^{2(d-2)}} - \frac{2a^2\Lambda}{(d-1)(d-2)}\right)}_{\text{potential } V(a)} = E \qquad (12)$$



Imposing the same boundary conditions as before, we find the scale factor.

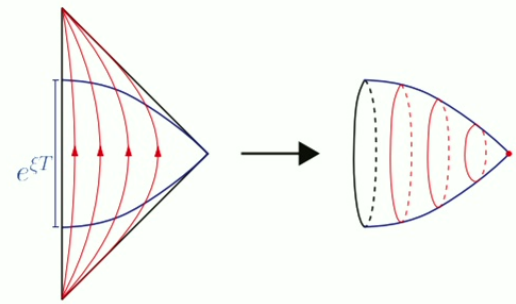
Computing the action

Using this scale factor solution, we can solve for the action

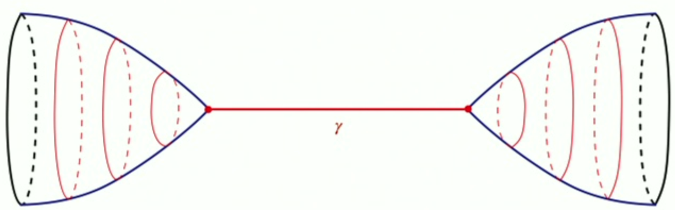
$$S = -iA[\gamma]/4G - E_\xi T \tag{13}$$

- E_ξ is the ADM energy of the boundary KVF ξ
- What is γ?

On each boundary we have:



The conical defect is a codimension-2 surface connecting the boundaries:



Contour analysis

- Once we have the action, we plug into the GPI

$$Z(\beta) = \int dLdAdTf_\beta(T)e^{A/4G}e^{-iE_\xi T} \tag{14}$$

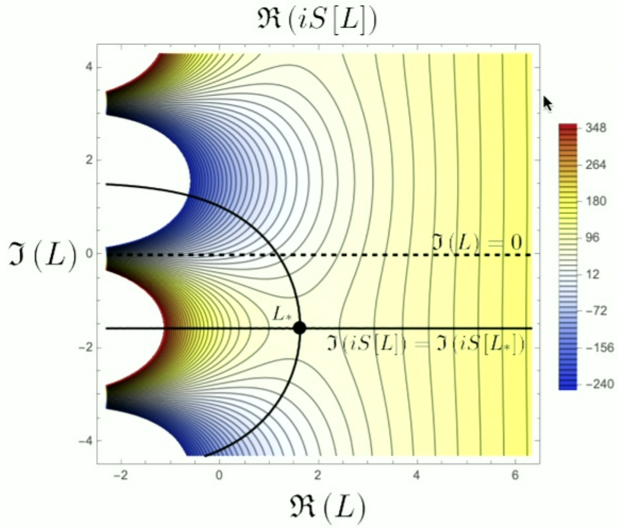
  and perform the T integral, setting $T = -i\beta$.

- We then perform a saddle point and contour analysis for the remaining integral. By Morse theory:

$$I = \sum n_p \mathcal{J}_p \tag{15}$$

  So we look for complex saddles for which the steepest ascent contour intersects the contour over reals

Preliminary results



$l = h_0 = \chi = 1 \quad T = 2\pi$

- In $2+1$-dim, our results so far suggest that these fixed-length wormholes are irrelevant
- Different results from [Loges et. al. '22], reinforces the point that **the relevance of the axion wormhole is inextricably linked to the particular quantity we are computing**

Conclusion & Future Directions

Future work

- Understanding what parts of our analysis here can be applied to other Euclidean wormholes supported by gradients, like those in [Marolf, Santos '21][Chandra, Hartman '22][Stanford '20]. The argument against allowing imaginary $\Delta\chi$ doesn't seem to hold for wormholes supported by gradients
- Extending our fixed-length wormhole results beyond $2+1$-dimensions
- Applying [Marolf '22] to other contexts where a Lorentzian approach would be enlightening, e.g. [Chua, Hartman '23]

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Summary

- The main moral: the relevance of the axion wormhole is highly dependent on the exact question we are asking
- In the fixed-area calculation, we found there are no Lorentzian axion wormholes with real BC's for χ
- We argued against considering axion wormholes with imaginary BC's for χ
- We considered "off-shell" fixed-length wormholes, with preliminary results suggesting that these do not contribute to the GPI (in $2+1$-dim)

Thank you!