Title: Topological quantum phase transitions in exact two-dimensional isometric tensor networks - VIRTUAL

Speakers: Yu-Jie Liu

Series: Machine Learning Initiative

Date: December 08, 2023 - 2:30 PM

URL: https://pirsa.org/23120036

Abstract: Isometric tensor networks (isoTNS) form a subclass of tensor network states that have an additional isometric condition, which implies that they can be efficiently prepared with a linear-depth quantum circuit. In this work, we introduce a procedure to construct isoTNS encoding of certain 2D classical partition functions. By continuously tuning a parameter in the isoTNS, the many-body ground state undergoes quantum phase transitions, exhibiting distinct 2D topological order. We illustrate this by constructing an isoTNS path with bond dimension $D = 2$ interpolating between distinct symmetry-enriched topological (SET) phases. At the transition point, the isoTNS wavefunction is related to a gapless point in the classical six-vertex model. Furthermore, the critical wavefunction supports a power-law correlation along one spatial direction while remains long-range ordered in the other spatial direction. We provide an exact linear-depth parametrized local quantum circuit that realizes the path. The above features can therefore be efficiently realized on a programmable quantum device. In the second part of my talk, I will show how to discover efficiently measurable order parameters for quantum phases using model-independent training of quantum circuit classifiers. The possibility of the efficient realization of phase transition path is useful for benchmarking quantum phase recognition methods in higher than one dimension.

Zoom link https://pitp.zoom.us/j/93183360141?pwd=RVdYeUxUbE1aZ1dUbzRSL3lBb0lHZz09

Topological quantum phase transitions in 2D isometric tensor networks

MLI seminar, Perimeter Institute, 8 Dec, 2023

Yu-Jie Liu
Technical University of Munich


Michael Knap


Frank Pollmann

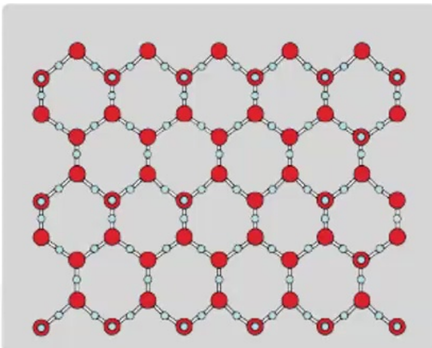

Kirill Shtengel

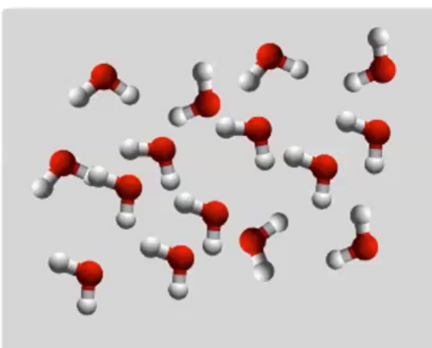

Adam Smith

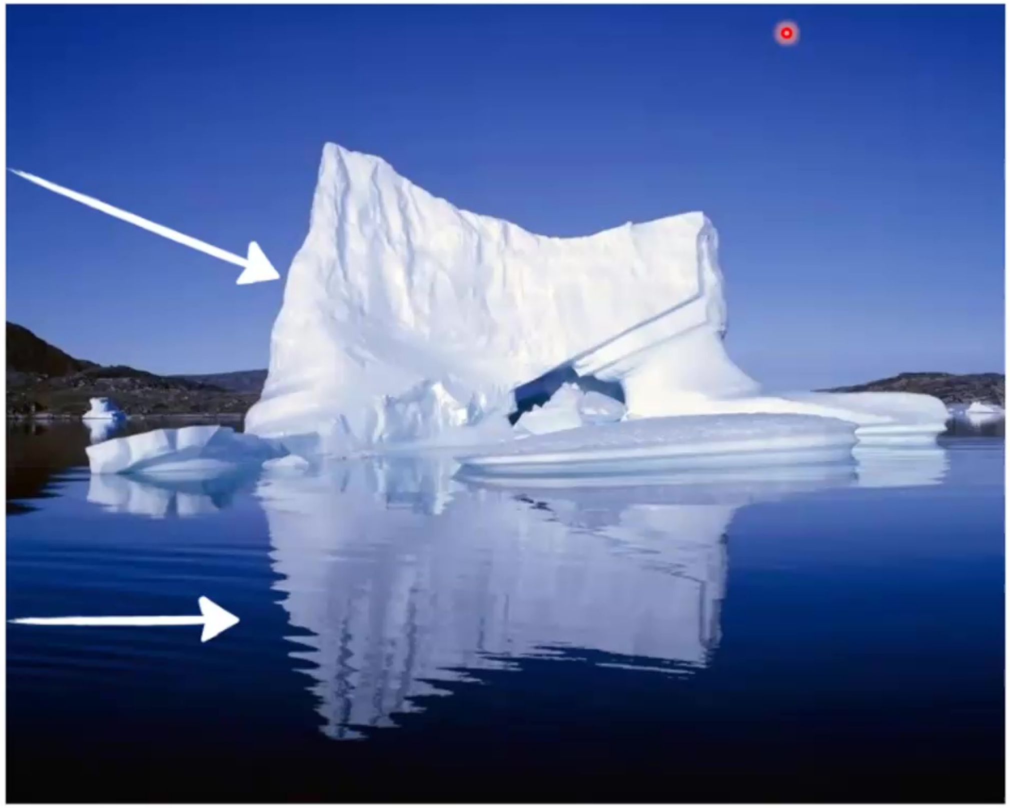
References:
[1] **YJL**, K. Shtengel and F. Pollmann, *Topological quantum phase transitions in 2D isometric tensor networks* (2023).
[2] **YJL**, A. Smith, M. Knap and F. Pollmann, Phys. Rev. Lett. 130. 220603 (2023).

Matter occurs in different phases

Symmetry breaking, universal scaling, topological invariants….







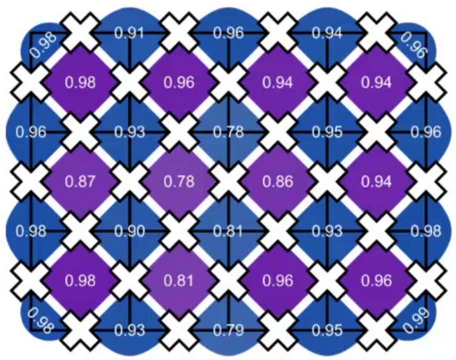
Challenges:
Realization and detection of novel quantum phases.

Realization of topologically ordered states

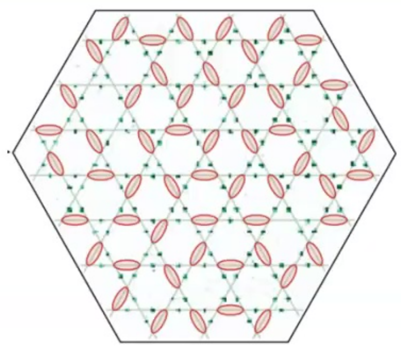
The search for topologically ordered/spin liquid states

- Fractional Quantum Hall Effect. [Tsui et al (1982)]
- Various candidates for QSL, hard to confirm.
- Trial states, theoretical understanding. [Anderson (1973), Laughlin (1983)]

Unprecedented control of quantum systems



Satzinger et al, Science (2021)



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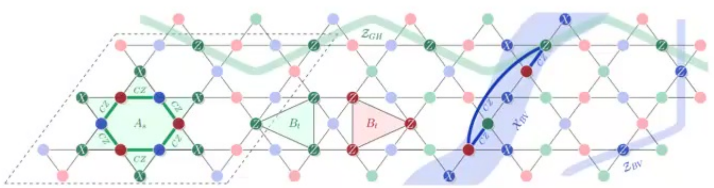

Iqbal et al, arXiv2305:03766 (2023)

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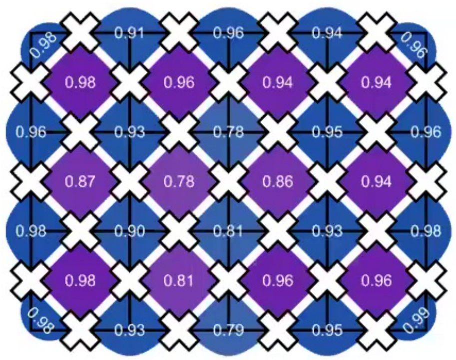

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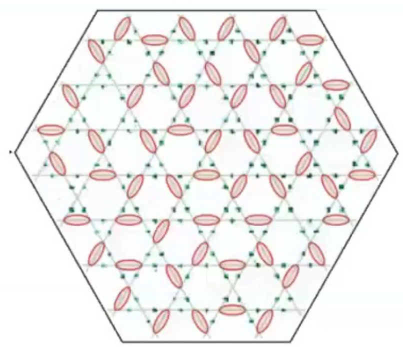

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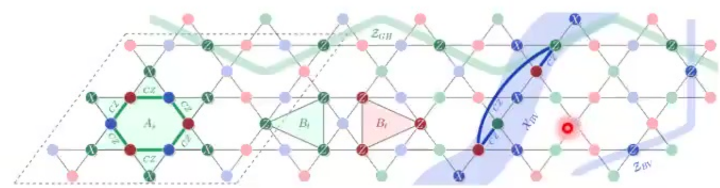

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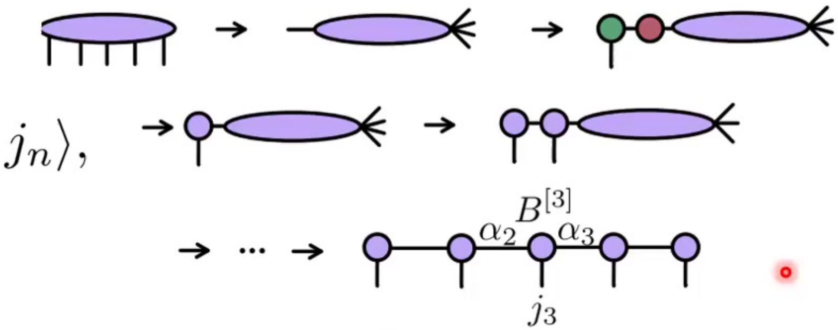
Probing trial model states with intrinsic topological order!

Efficient description of ground states by tensor networks

1D matrix-product states (MPS)

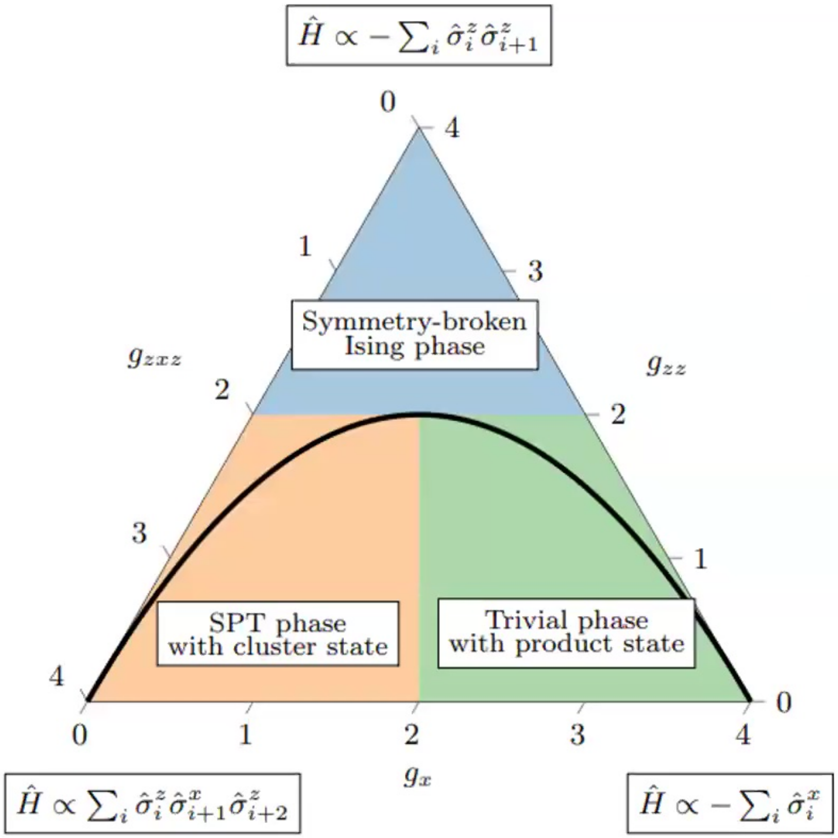


$$|\psi\rangle = \sum_{\{j_k\}} \sum_{\{\alpha_l\}} B^{[1]j_1}_{\alpha_1} B^{[2]j_2}_{\alpha_1\alpha_2} \dots B^{[n]j_n}_{\alpha_{n-1}} |j_1, j_2, \dots, j_n\rangle,$$

where B in the bulk is some $d \times \chi \times \chi$ tensor.
Area-law entanglement → Good approximation of ground states for finite χ.

Quantum phase transition in matrix product states

Engineer 1D quantum phase transitions from MPS. [Wolf et al (2006), Jones et al (2021)]



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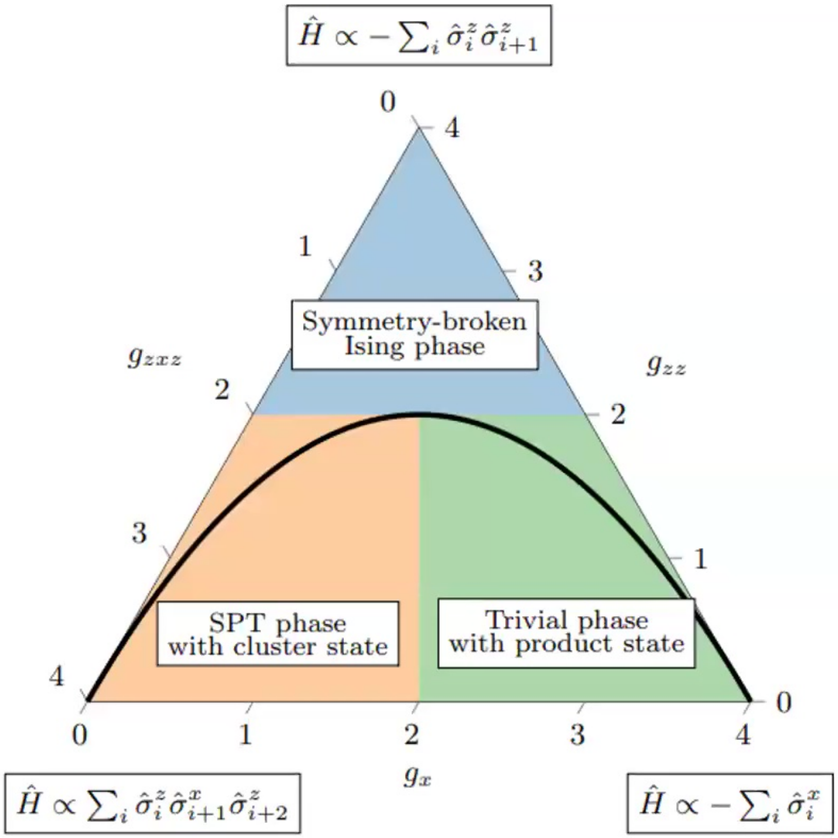
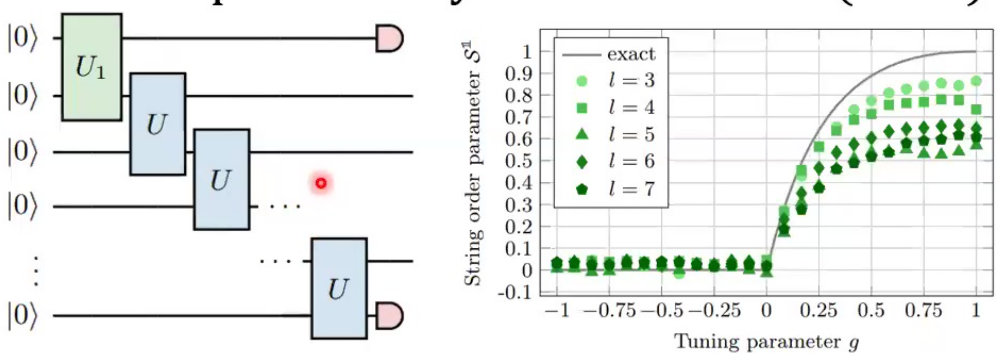


Analytical MPS $\chi = 2$

$$M^{[0]} = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}, \qquad M^{[1]} = \begin{pmatrix} 1 & g \\ 0 & 0 \end{pmatrix},$$

SPT at $g = -1$ Trivial at g $= 1$

Experiment by Smith et al PRR (2022)



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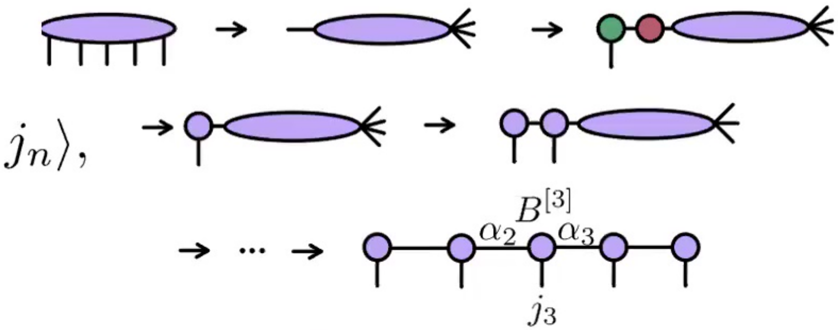

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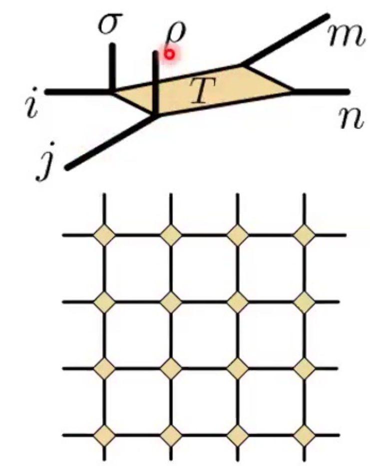
2D tensor-network states (TNS/PEPS)



$$|\psi\rangle = \sum_{\sigma_1,\cdots,\sigma_N} \mathrm{tTr}\left(\{T^{\sigma_1\sigma_2}, \cdots, T^{\sigma_{N-1}\sigma_N}\}\right) |\sigma_1, \cdots, \sigma_N\rangle,$$

Contracting neighbouring tensors of shape $d^2 \times D \times D \times D \times D$.

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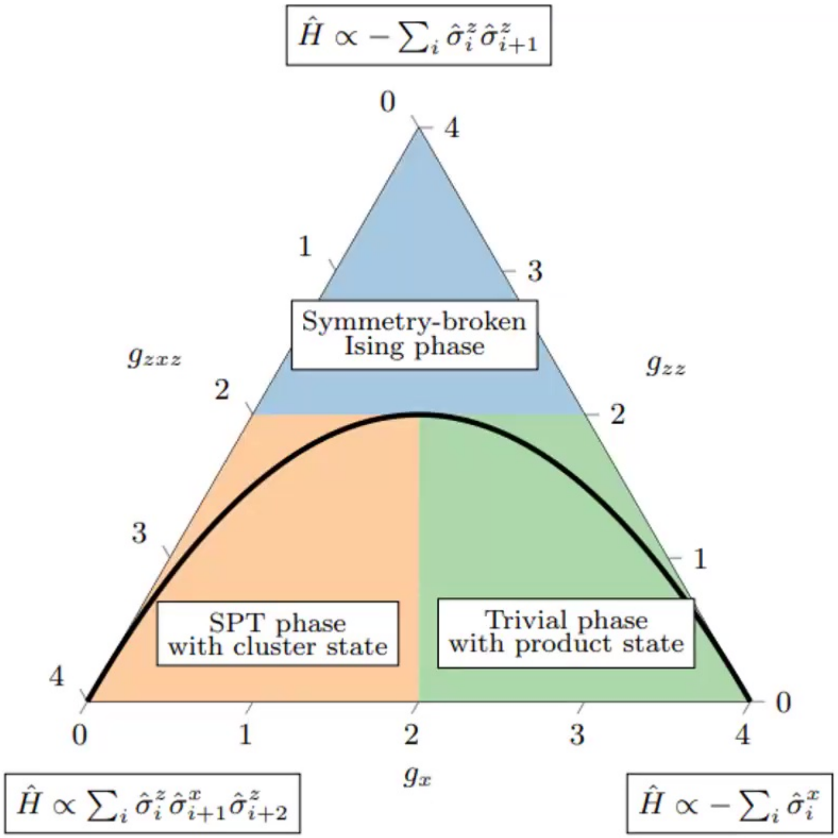

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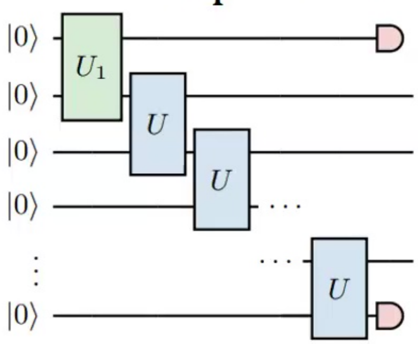
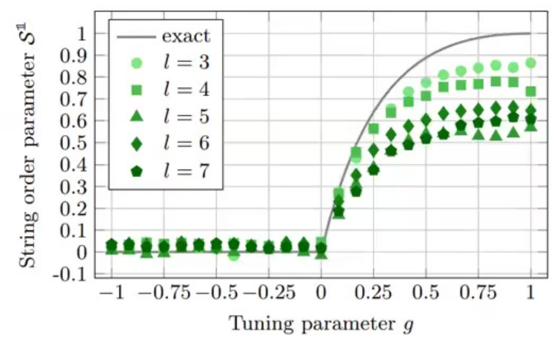
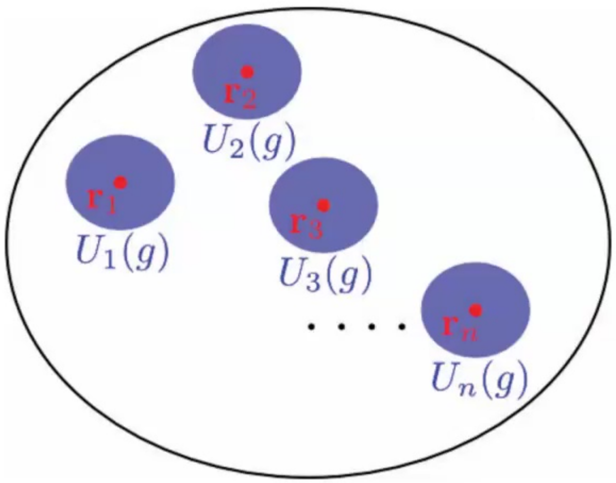




Quantum phase transition in 2D TNS

E.g. Quantum phase transition between symmetry-enriched topological (SET) phases

Symmetry fractionlizes over anyons.



Mesaros and Ran, PRB (2013)

Haller, Xu, **YJL** and Pollmann PRR (2023)

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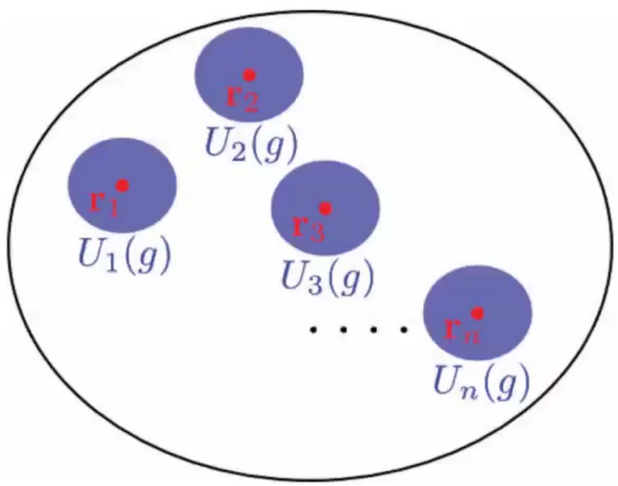
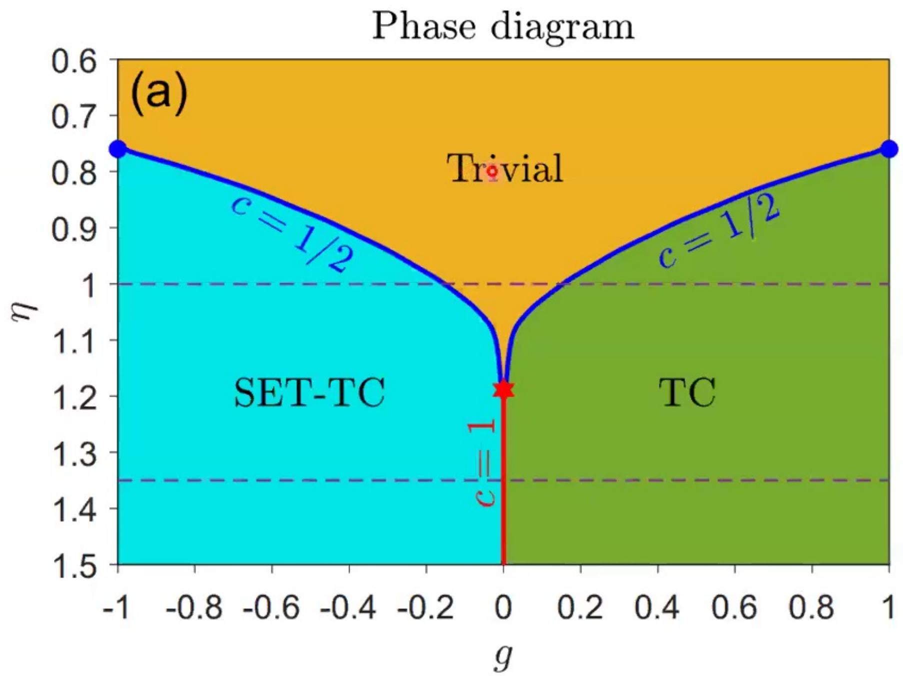


Mesaros and Ran, PRB (2013)

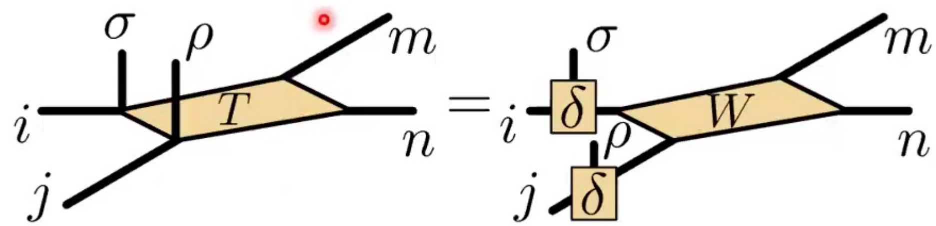
Tensor-network solvable ground states $D = 3$



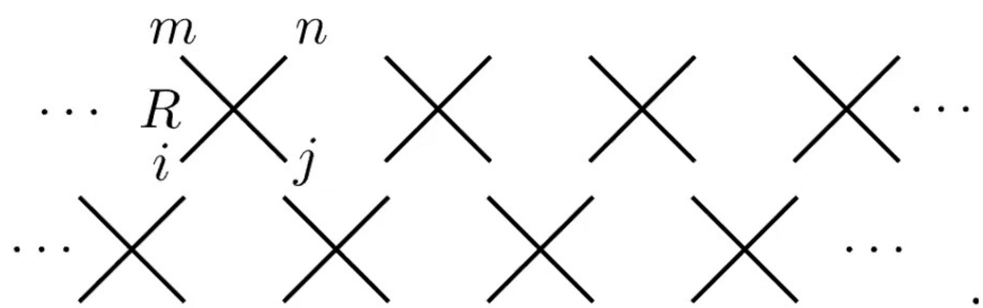
Haller, Xu, **YJL** and Pollmann PRR (2023)

Classical partition functions in isoTNS

"Plumbing" to encode classical partition functions in TNS



The local weight matrix $R_{ijmn} = |W_{ijmn}|^2$ encodes the classical Boltzmann weights.



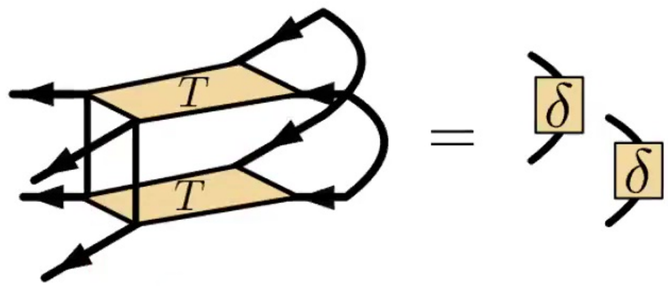
Squared norm of the wavefunction coefficients = classical Boltzmann weights

[Laughlin, PRL (1983), Verstraete et al, PRL (2006)]

YJL, Shtengel and Pollmann (to appear) (2023)

Classical partition functions in isoTNS

Impose isometric condition → isometric tensor netowrks (isoTNS)



Analogous to the canonical form of MPS → Linear-depth sequential quantum circuit.

[Pollmann & Zaletel, PRL (2020), Wei et al, PRL (2022)]

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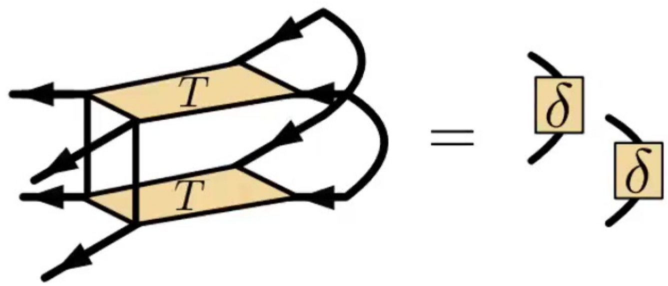

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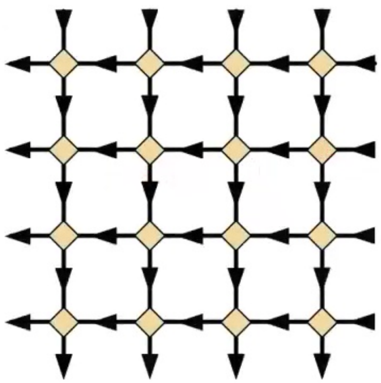

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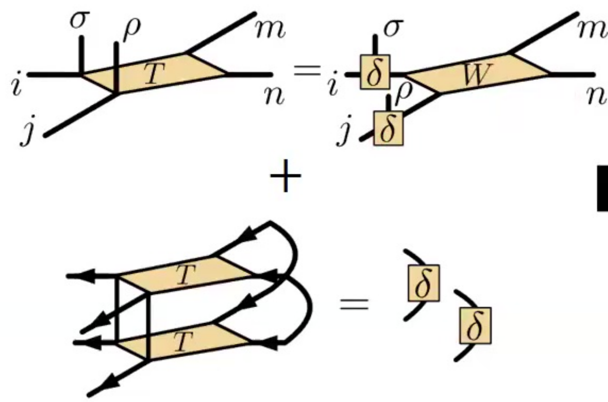
Classical partition functions in isoTNS

Visualizing with the $\boldsymbol{W}$-matrix

Classical PF:

IsoTNS:



$$\sum_{mn} \left|W_{ijmn}\right|^2 = 1$$

YJL, Shtengel and Pollmann (to appear) (2023)

Classical partition functions in isoTNS

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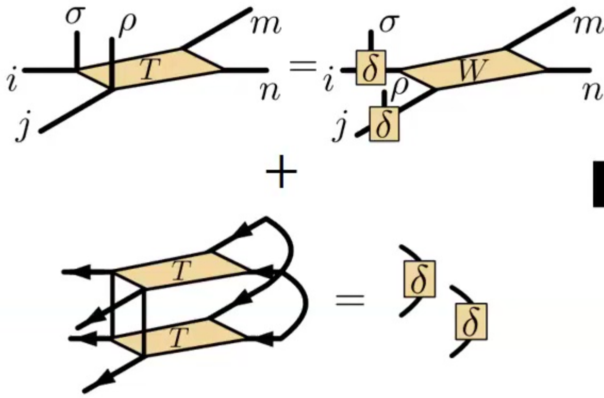
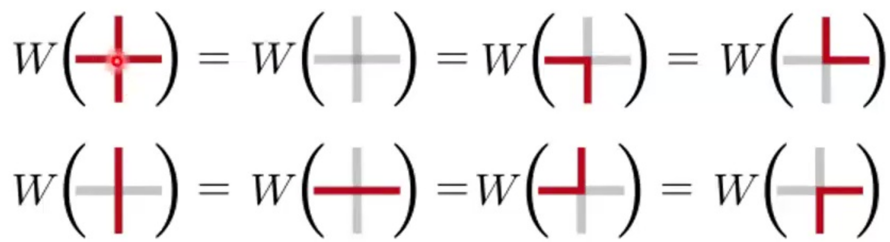


$$\sum_{mn} \left|W_{ijmn}\right|^2 = 1$$

Toric code ground state [Kitaev (1997)] falls into this class → **Eight-vertex model**

$$W^{(\mathrm{TC})} = \begin{array}{c} \begin{array}{cccc} |00\rangle & |01\rangle & |10\rangle & |11\rangle \end{array} \\ \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \end{pmatrix} \end{array} \begin{array}{l} \\ |00\rangle \\ |01\rangle \\ |10\rangle, \\ |11\rangle \end{array}$$



YJL, Shtengel and Pollmann (to appear) (2023)

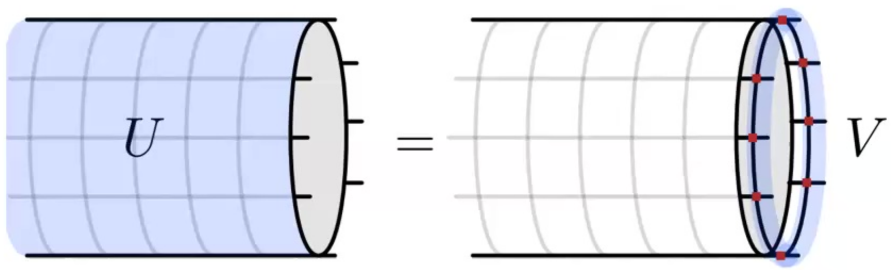
A continuous isoTNS path between distinct SET phases

Consider the W-matrix

$$W(g) = \begin{pmatrix} \frac{1}{\sqrt{1+|g|}} & 0 & 0 & \mathrm{sign}(g)\sqrt{\frac{|g|}{1+|g|}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \sqrt{\frac{|g|}{1+|g|}} & 0 & 0 & \frac{1}{\sqrt{1+|g|}} \end{pmatrix},$$

where $g \in [-1,1]$. The path has Z_2 symmetry $U = \prod_i X_i$. Classification $H^{(2)}(Z_2, Z_2) = Z_2$.

[Essin & Hermele, PRB (2013)]



$$U = \prod_i X_i \to V(g)$$

$$V(g)^2 = \mathrm{sign}(g)^P$$

YJL, Shtengel and Pollmann (to appear) (2023)

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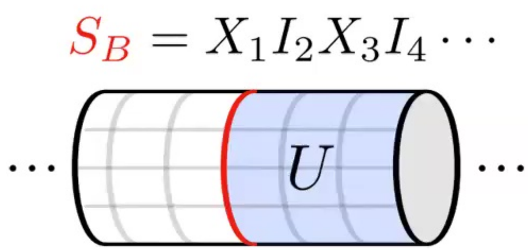

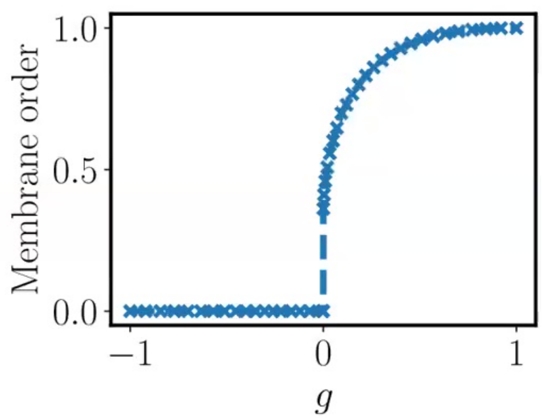

Superselection rules when SF happens → zero membrane order.

[Pollmann & Turner, PRB (2012)]

YJL, Shtengel and Pollmann (to appear) (2023)

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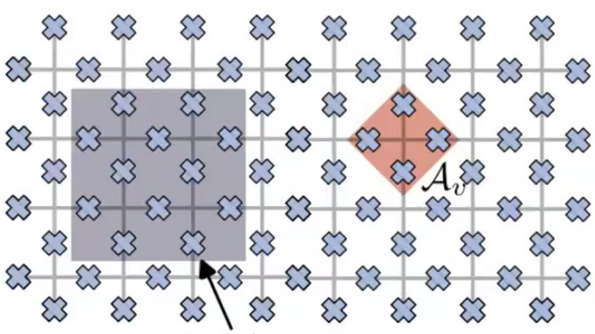
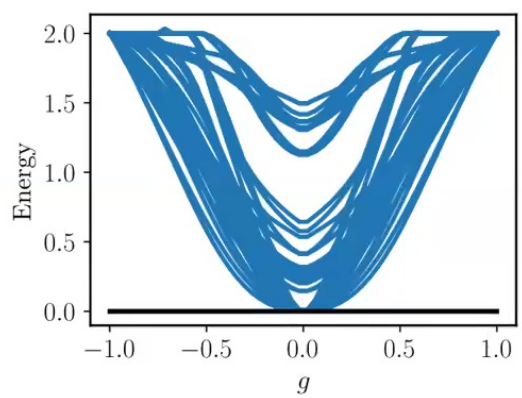




$$H(g) = \sum_v \mathcal{A}_V + \sum_p \mathcal{B}_p(g) P_{p(v)}$$

YJL, Shtengel and Pollmann (to appear) (2023)

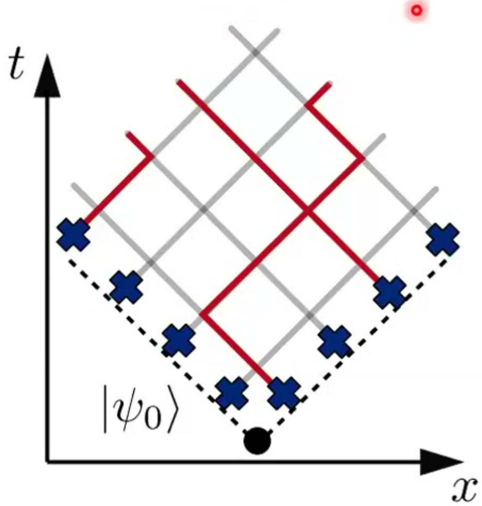
Power-law correlation and 1D classical dynamics

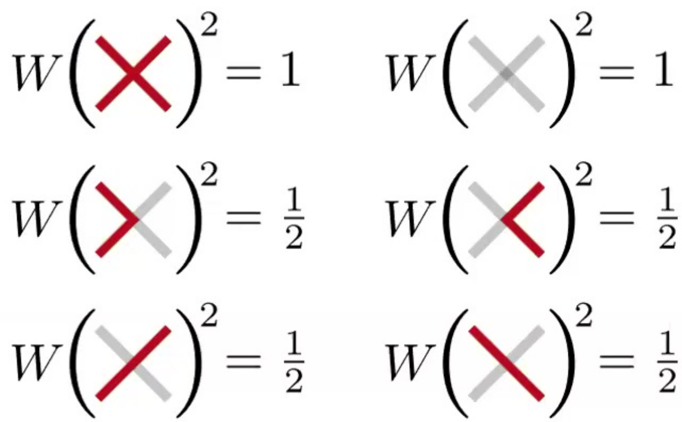
Quantum critical point at $g = 0$.

Eight-vertex → Six-vertex model.

Boundary condition matters! Convenient to rotate the lattice by 45 degrees.

→ The lines of states $|1\rangle$ are conserved across any horizontal slice, unless terminating on the boundary.





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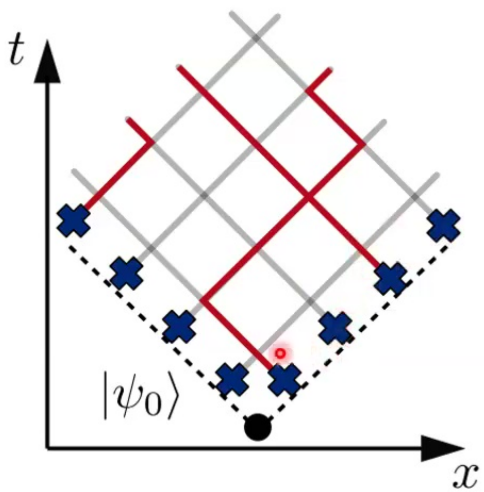
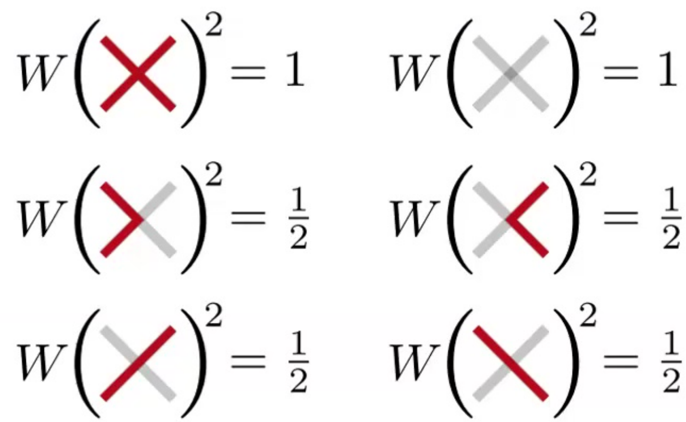


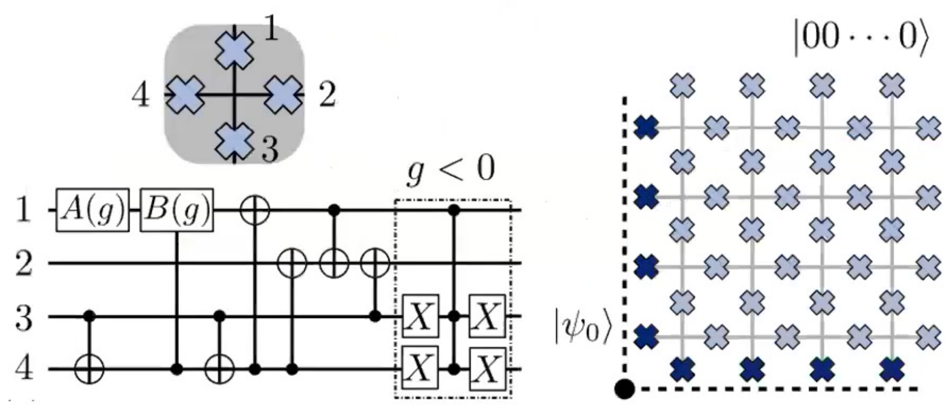


Random walk of 1D particles with exclusion.
$|\psi\rangle$ = a superposition of worldlines of 1D particles emanating from the boundary

YJL, Shtengel and Pollmann (to appear) (2023)

An efficient quantum circuit representation

Map to sequentially generated quantum circuit with depth $O(L)$.



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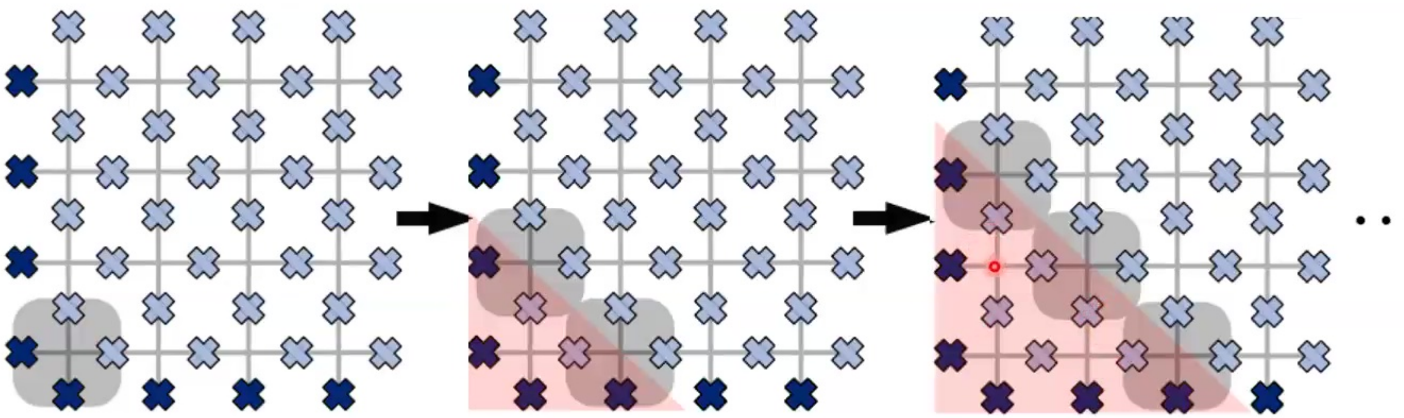
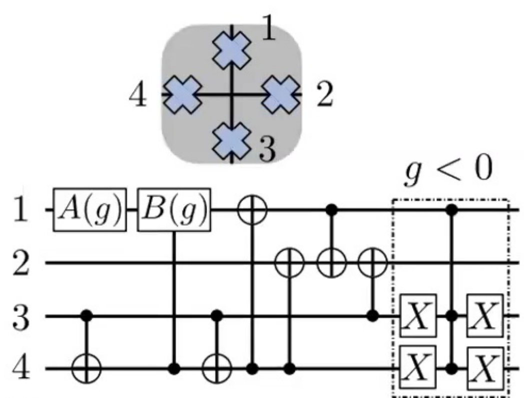
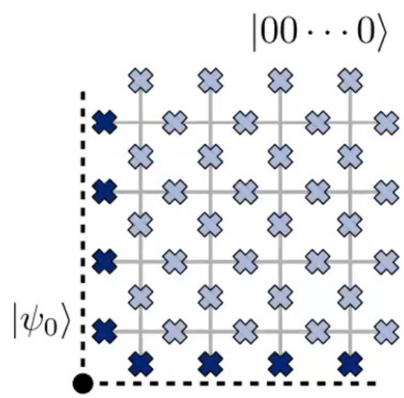


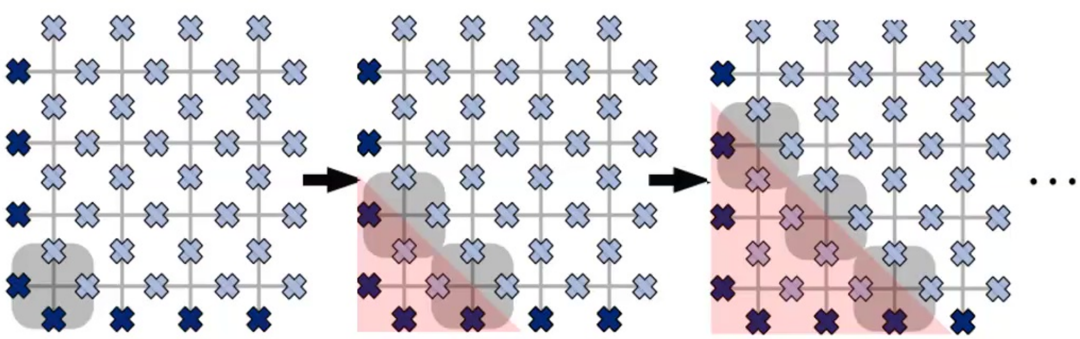




YJL, Shtengel and Pollmann (to appear) (2023)

Summary

- Propose a "plumbing" procedure to encode classical PF in isoTNS.
- A continuous isoTNS path between two SET phases, crossing a QCP.
- ~~Open question~~: power-law correlation in isoTNS.
- Efficient realization of the path with a linear-depth quantum circuit.

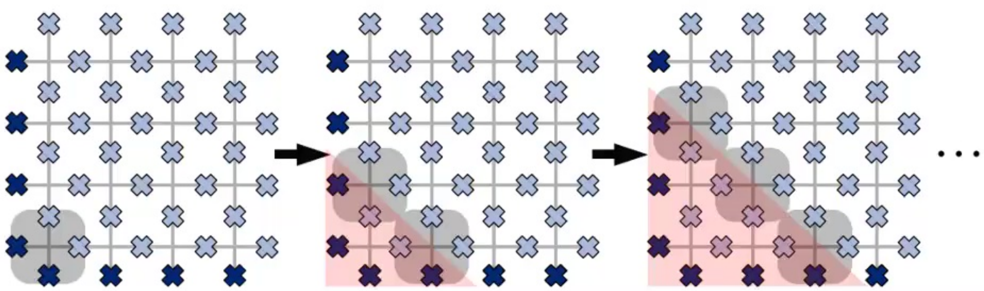


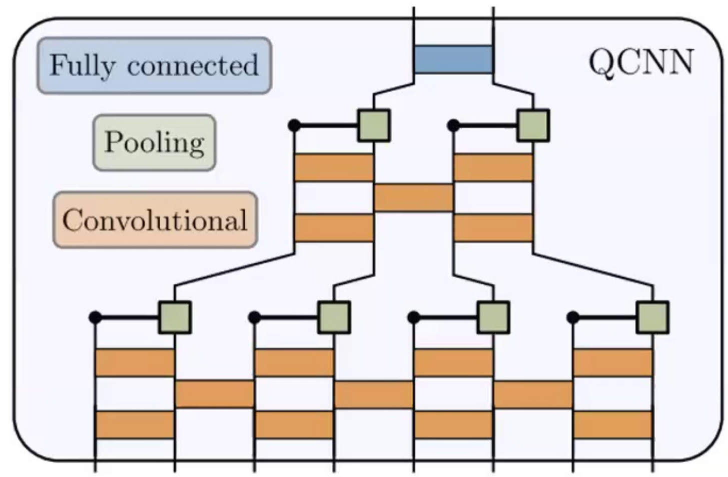
YJL, Shtengel and Pollmann (to appear) (2023)

Overview

Realization of quantum phase transitions in isoTNS

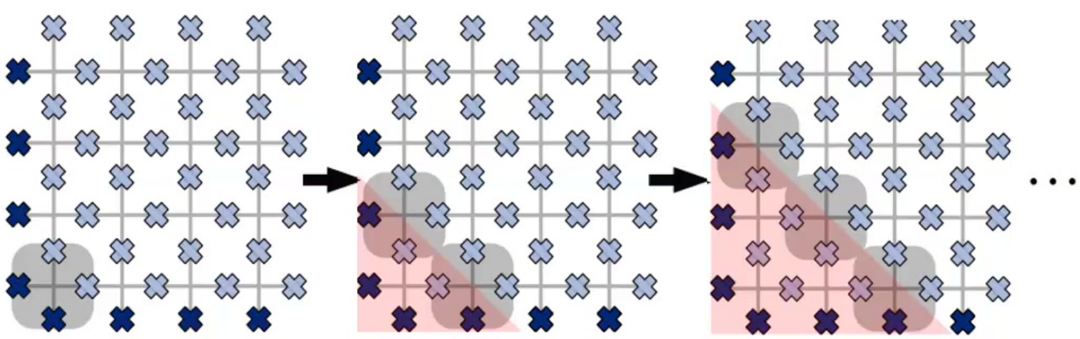
Recognizing quantum phases





Summary

- Propose a "plumbing" procedure to encode classical PF in isoTNS.
- A continuous isoTNS path between two SET phases, crossing a QCP.
- ~~Open question~~: power-law correlation in isoTNS.
- Efficient realization of the path with a linear-depth quantum circuit.



YJL, Shtengel and Pollmann (to appear) (2023)

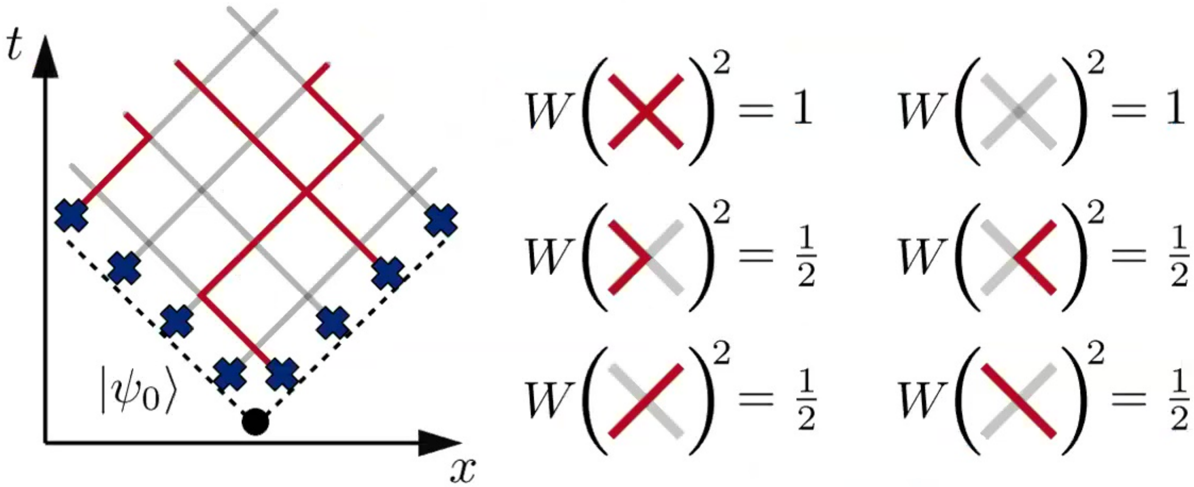
Power-law correlation and 1D classical dynamics

Quantum critical point at $g = 0$.

Eight-vertex → Six-vertex model.

Boundary condition matters! Convenient to rotate the lattice by 45 degrees.

→ The lines of states $|1\rangle$ are conserved across any horizontal slice, unless terminating on the boundary.



YJL, Shtengel and Pollmann (to appear) (2023)

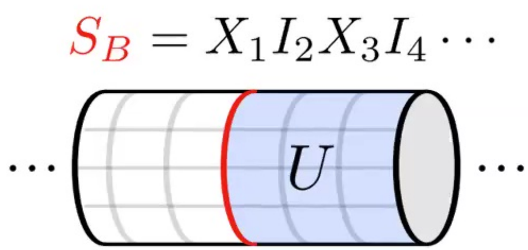
A continuous isoTNS path between distinct SET phases

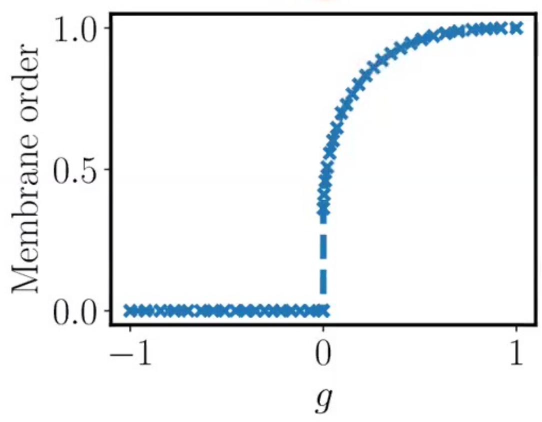
Consider the $\boldsymbol{W}$-matrix

$$W(g) = \begin{pmatrix} \frac{1}{\sqrt{1+|g|}} & 0 & 0 & \mathrm{sign}(g)\sqrt{\frac{|g|}{1+|g|}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \sqrt{\frac{|g|}{1+|g|}} & 0 & 0 & \frac{1}{\sqrt{1+|g|}} \end{pmatrix},$$

where $g \in [-1,1]$. The path has Z_2 symmetry $U = \prod_i X_i$. Classification $H^{(2)}(Z_2, Z_2) = Z_2$.

[Essin & Hermele, PRB (2013)]





Superselection rules when SF happens → zero membrane order.

[Pollmann & Turner, PRB (2012)]

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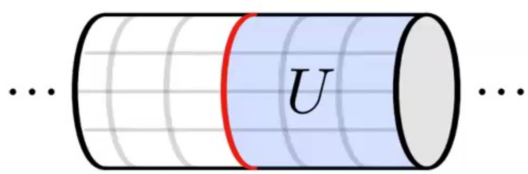
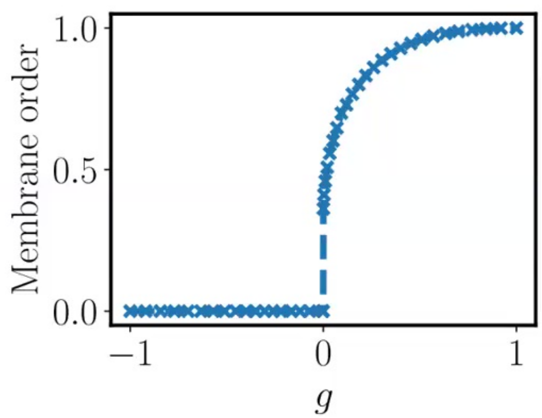
$S_B = X_1 I_2 X_3 I_4 \cdots$





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Overview

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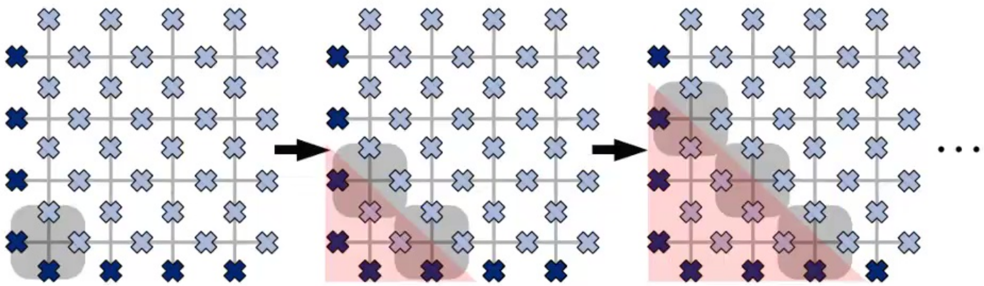

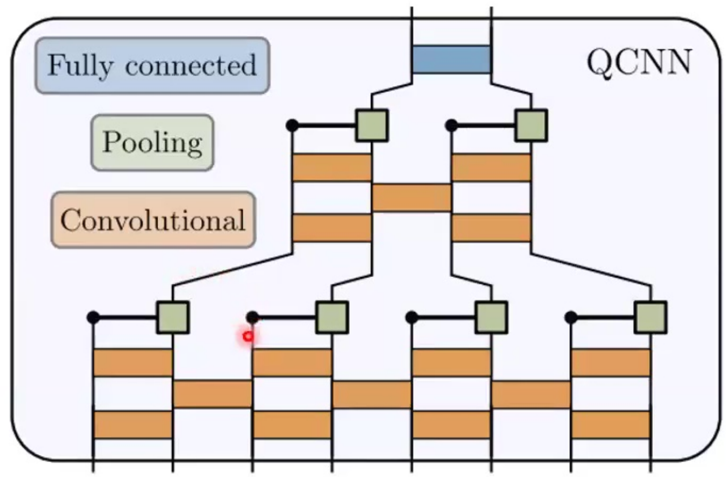

Gapped quantum phases of matter

We can realize fixed points of gapped phases!

Task: Given an input ground state $|\psi\rangle$, which known phases does it belong to?

Can we automate the discovery of order parameters using fixed points?

Q: When does $|\psi\rangle$ belong to phase A?

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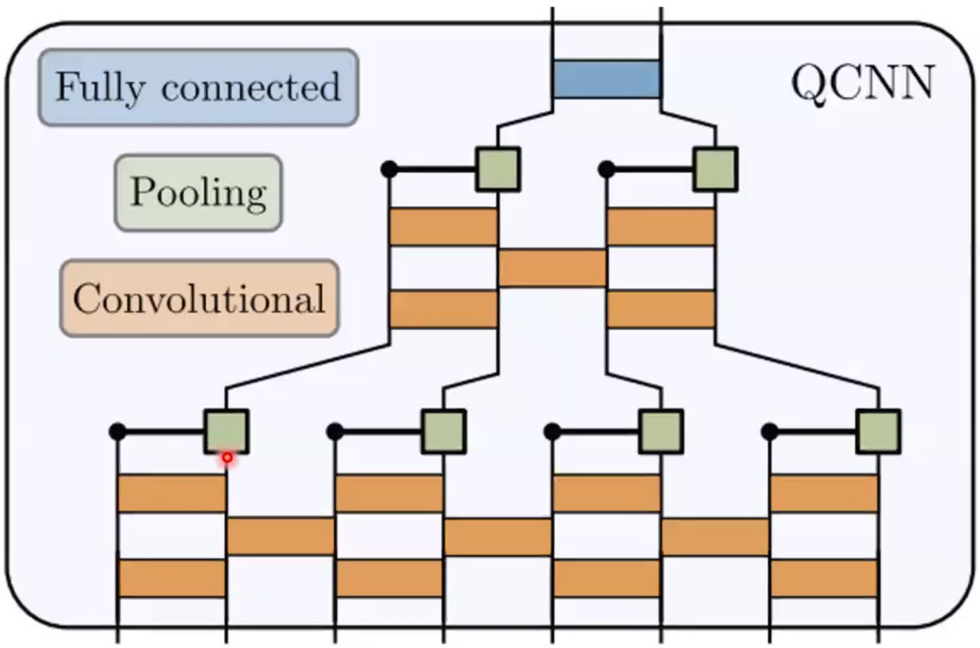
$$|\psi\rangle \xrightarrow{U} |\text{Fixed Point}\rangle$$

Minimal information: A fixed-point wavefunction and the symmetry group.

Quantum machine learning with quantum data



Quantum convolutional neural networks:

- Introduced as a phase classifier. [Cong et al (2019)]
- Mimicking RG flow.
- Advantages in gradient optimization. [Pesah et al (2021)]

Challenges: Hard to train in practice!

Model-independent training

Protocol to generate large training data set:

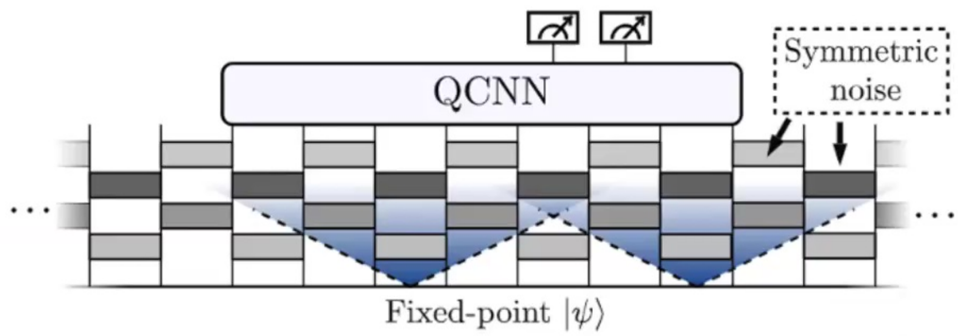
1. Prepare a fixed-point state for the phase.
2. Apply local symmetric unitary gates.
3. Supervised training of QCNN.

$|00\rangle \rightarrow SB$
$|01\rangle \rightarrow PM$
$|10\rangle \rightarrow SPT$
$|11\rangle \rightarrow Fail$

Enforce translational invariance --- non-existence of the solution without additional symmetires

Advantages:

1. Independent of models.
2. Fixed-points are easy to prepare.
3. Unitary controls correlation length and mask irrelevant short-range physics



YJL, Smith, Knap and Pollmann, PRL (2023)

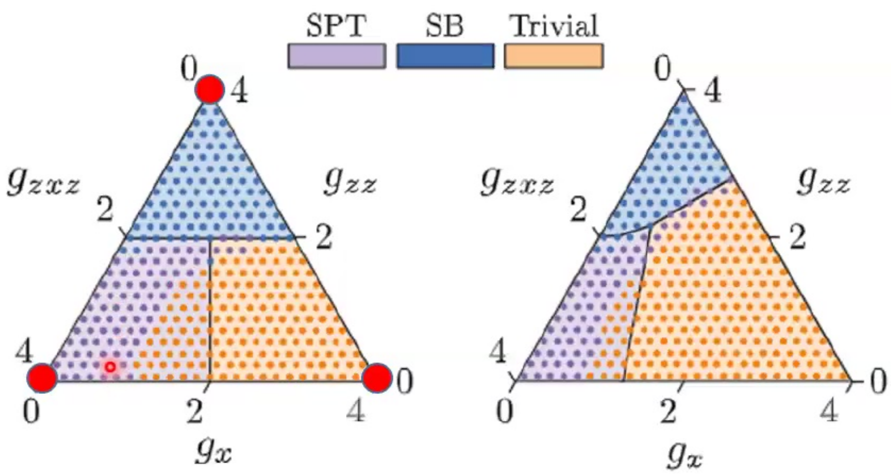
Time-reversal cluster-Ising model ($K \prod_i X_i$)

Consider the cluster-Ising model

$$H_{CI} = g_{zxz} \sum_i Z_{i-1} X_i Z_{i+1} - g_{zz} \sum_i Z_i Z_{i+1} - g_x \sum_i X_i,$$

time-reversal → 3 phases (SPT, Trivial and SB)

4-qubit QCNN



H_{CI}

$H_{CI} - \frac{g_x}{2} \sum_i X_i X_{i+1}$

YJL, Smith, Knap and Pollmann, PRL (2023)

1D time-reversal cluster-Ising model $(K \prod_i X_i)$

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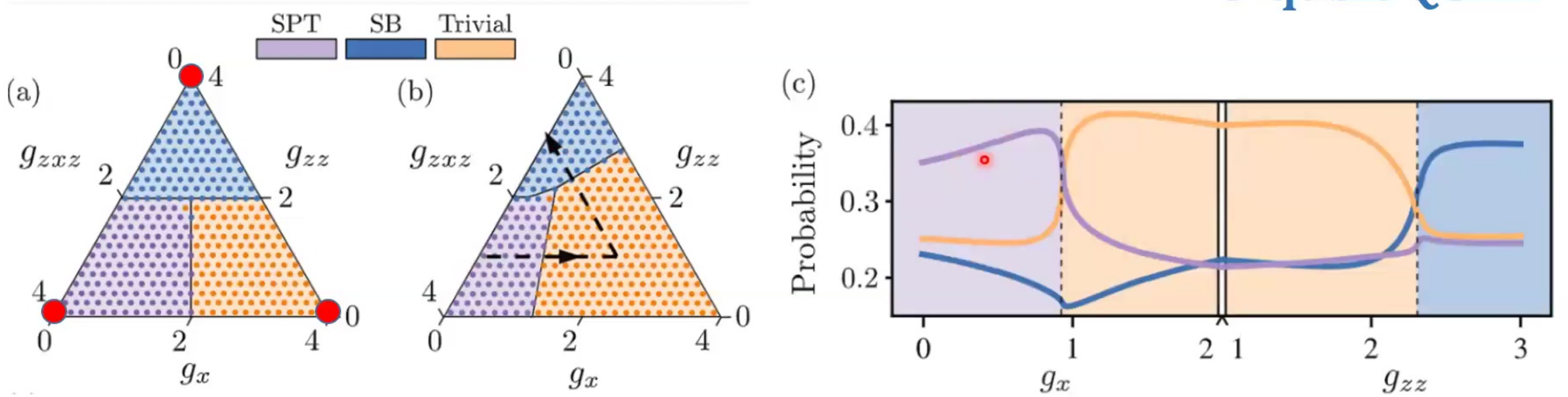
8-qubit QCNN



$$H_{CI} - \frac{g_x}{2} \sum_i X_i X_{i+1}$$

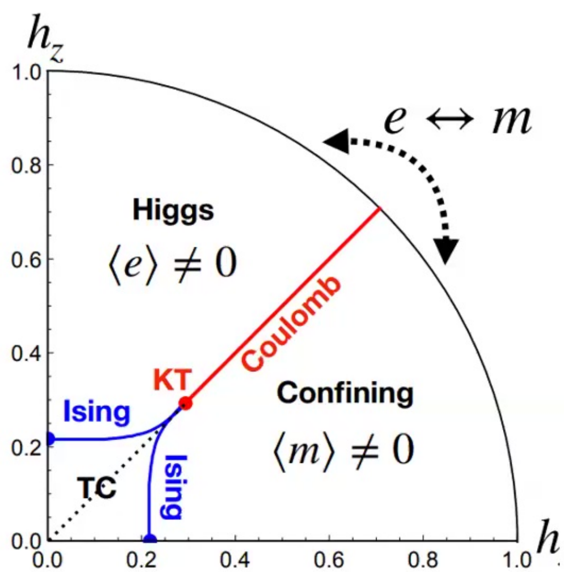
YJL, Smith, Knap and Pollmann, PRL (2023)

Beyond 1D models --- intrinsic topological order

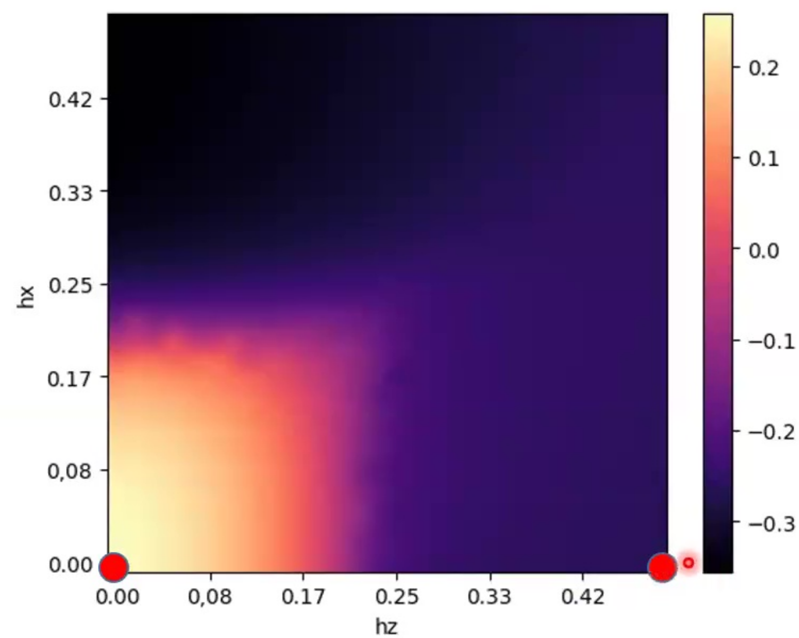
Can we discover an order parameter for 2D topological order?

Work in progress!

Target phase diagram



Model-independent training



Conclusion and outlook

- Training quantum circuit classifiers using only a reference state.
- Learning universal features of quantum phases.
- Importance of symmetry in the quantum data.

To be explored…

- Can we discover order parameters for topological phases in higher dimensions?
- Benchmarking topological quantum phase transitions in experiment.
- Extension to phase transitions between more generic gapped quantum phases.
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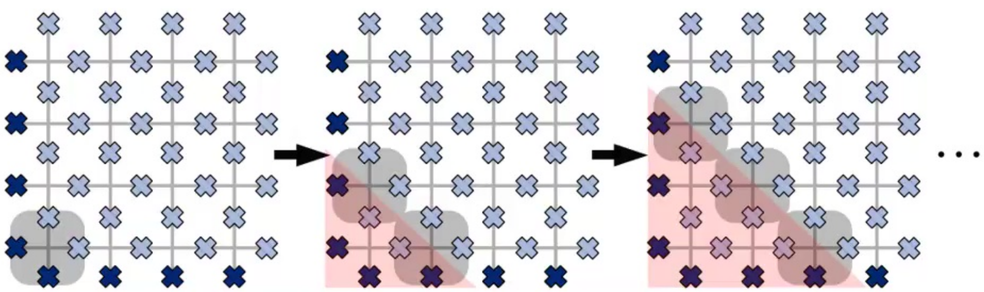
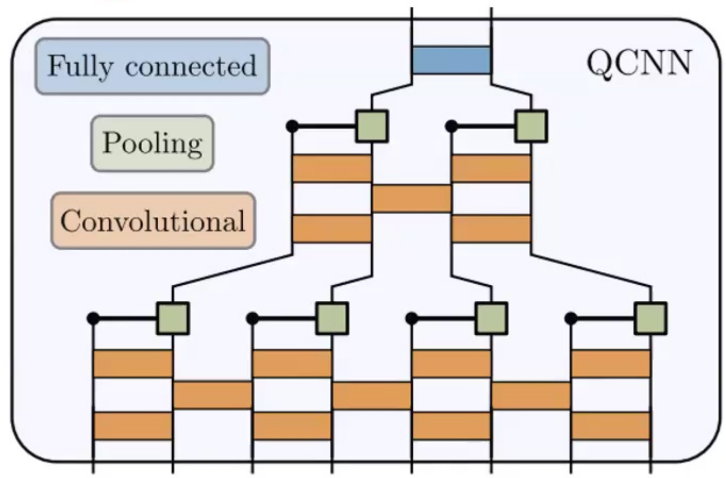


YJL, K. Shtengel and F. Pollmann, (*to appear*) (2023).



YJL, A. Smith, M. Knap and F. Pollmann,
Phys. Rev. Lett. 130. 220603 (2023).

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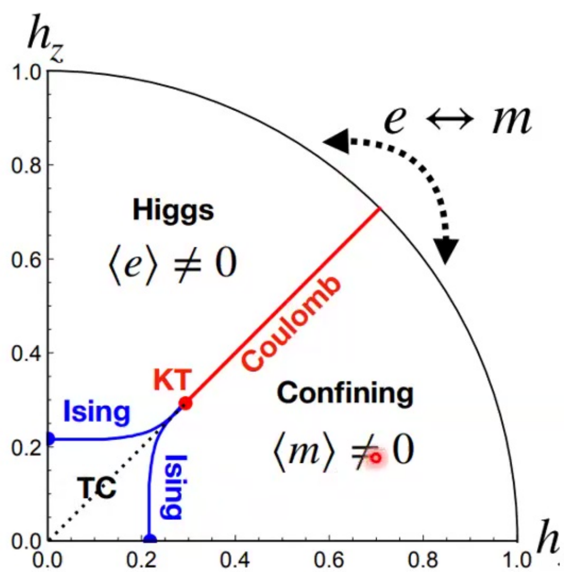

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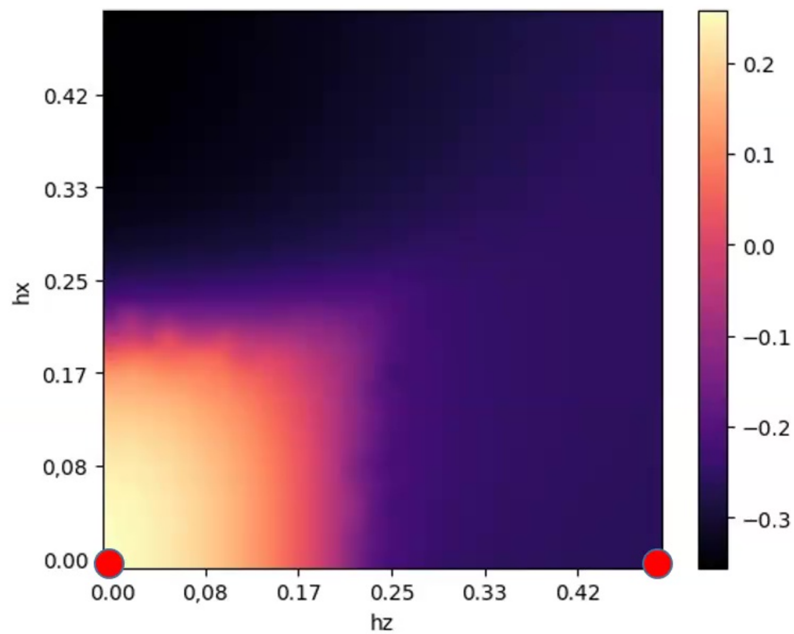

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