Title: The Large-Scale Structure Renormalization Group: Bridging the Gap Between QFT and Cosmology - VIRTUAL

Speakers: Henrique Rubira

Series: Cosmology & Gravitation

Date: December 05, 2023 - 11:00 AM

URL: https://pirsa.org/23120032

Abstract: Major improvements in the theoretical aspects of Cosmology have been possible in recent years due to QFT-inspired methods, such as the effective field theory of large-scale structure (EFTofLSS). In this talk, I will explore further connections between high-energy physics and cosmology. I will present a systematic approach to renormalizing the galaxy bias parameters using path integrals and a finite cutoff scale ?. I will derive the differential equations of the Wilson-Polchinski renormalization group that describe the evolution of the finite-scale bias parameters with ?, analogous to the ?-function running in QFT. I will then discuss how the RG-flow of EFTofLSS can lead to improvements in the extraction of cosmological parameters and also serve as a tool for sanity checks.

Zoom link https://pitp.zoom.us/j/93248279658?pwd=Z1M1UnI3eVh3NTZ3NCt4NUtGaW5idz09

The Large-Scale Structure Renormalization Group

Bridging the Gap Between QFT and Cosmology

Henrique Rubira

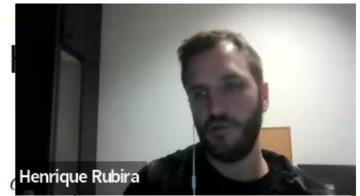
In collaboration with Fabian Schmidt (MPA)

Perimeter, Dec 5

henrique.rubira@tum.de

ArXiv:
2307.15031,
2312.xxxxx

Message to take home

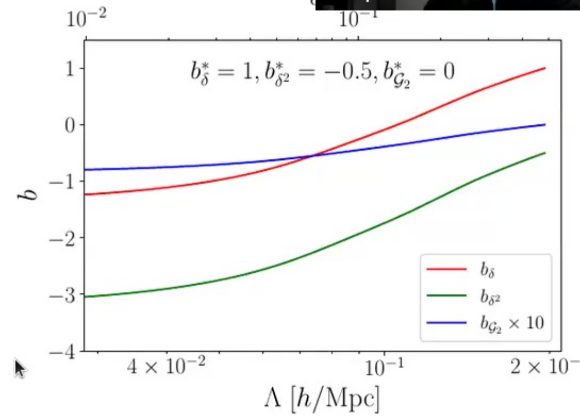


We derive the Callan-Symanzik equation for the galaxy bias parameters:

$$\frac{db_\delta}{d\Lambda} = -\left[\frac{68}{21}b_{\delta^2} + 3b^*_{\delta^3} - \frac{4}{3}b^*_{\mathcal{G}_2\delta}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}\,,$$

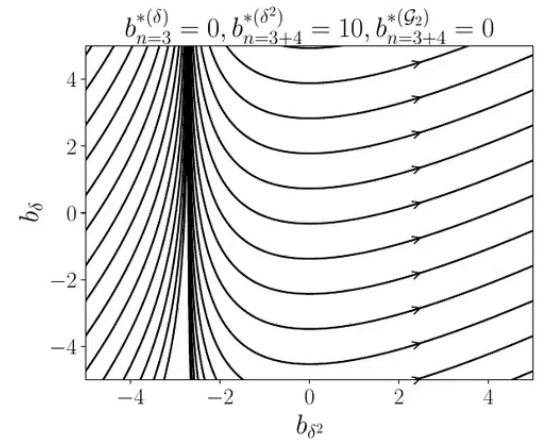
$$\frac{db_{\delta^2}}{d\Lambda} = -\left[\frac{8126}{2205}b_{\delta^2} + \frac{17}{7}b^*_{\delta^3} - \frac{376}{105}b^*_{\mathcal{G}_2\delta} + b^{*(\delta^2)}_{n=4}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}\,,$$

$$\frac{db_{\mathcal{G}_2}}{d\Lambda} = -\left[\frac{254}{2205}b_{\delta^2} + \frac{116}{105}b^*_{\mathcal{G}_2\delta} + b^{*(\mathcal{G}_2)}_{n=4}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}\,.$$



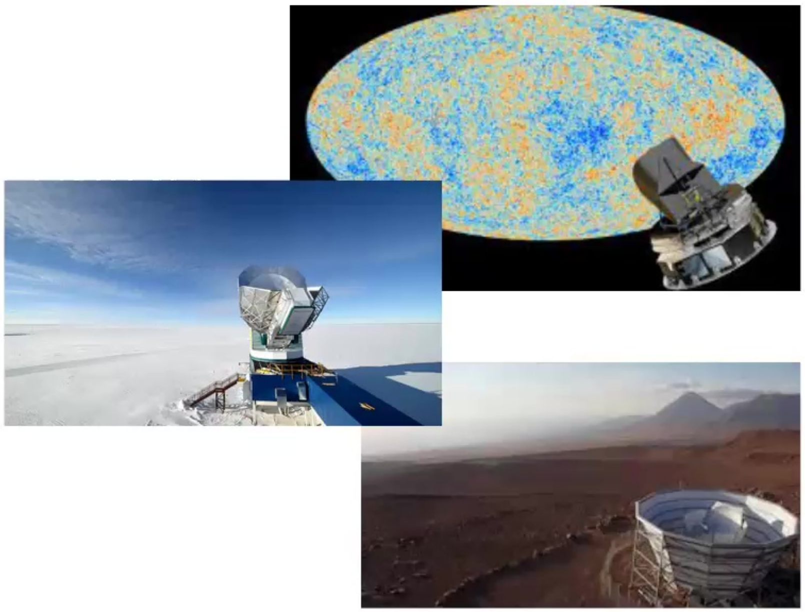
Many things to explore:

- systematic construction of operator basis and their priors,
- systematic renormalization of n-point functions,
- extra cross-checks,
- more information from galaxy clustering (to be investigated)

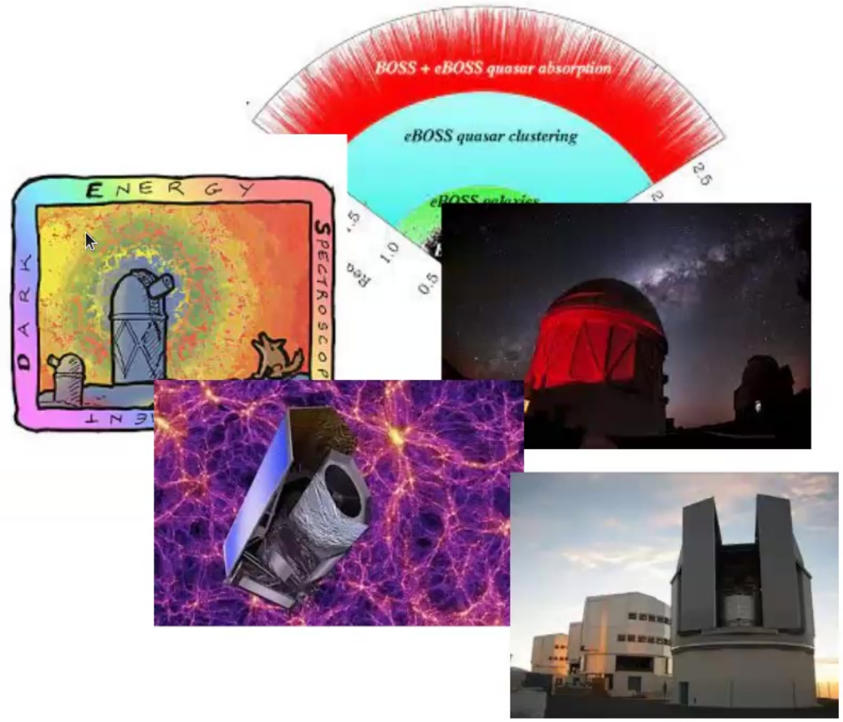


Cosmology in the era of (subpercent) precise c

CMB
(Planck, ACT, SPT, SO, CMB-S4)

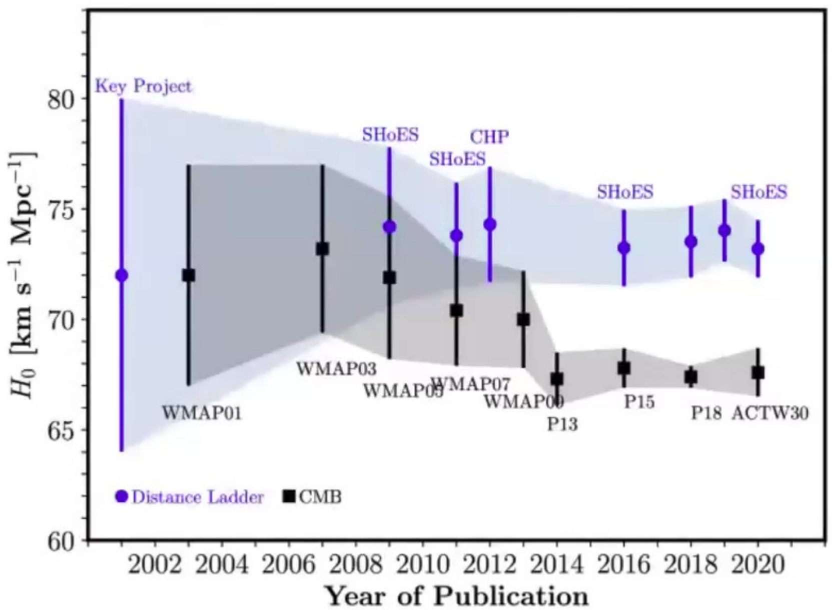


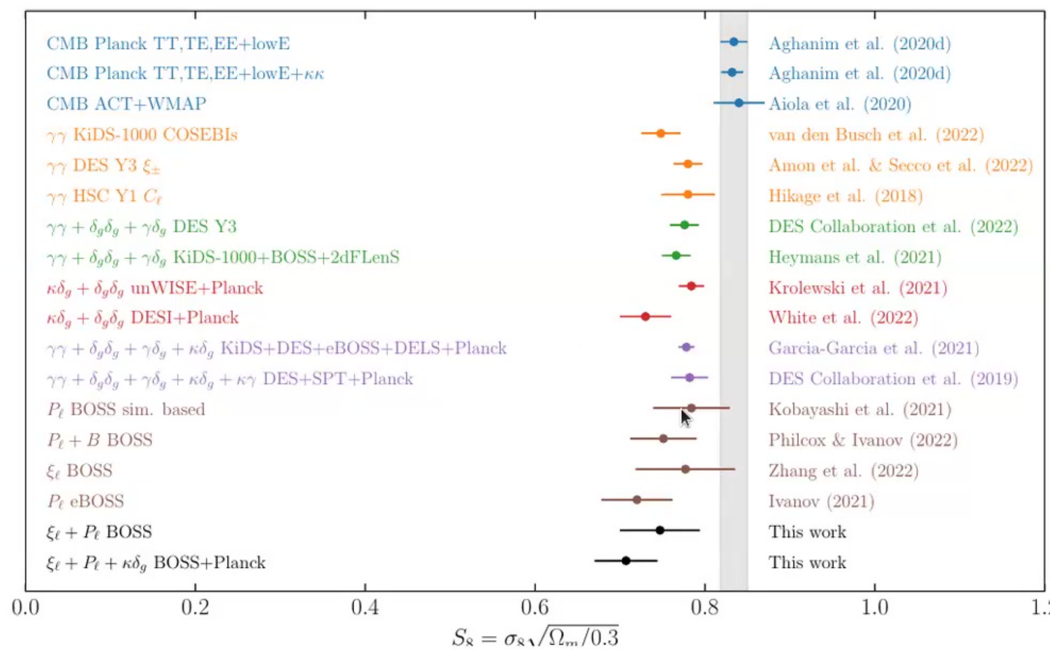
LSS
(BOSS, eBOSS, DESI, Euclid, DES, Kids)



More data is leading to more tensions

Chen, White, +, 22





Freedman 21

Can theory catch up?

We should be able to provide:

1) **Models** to be easily checked. How to address those tensions (once we take them seriously)?

- Hard to be consistent with CMB, Supernova, LSS;
- What is well-motivated from the particle-physics perspective?

2) Model-independent **tools** to analyze data.

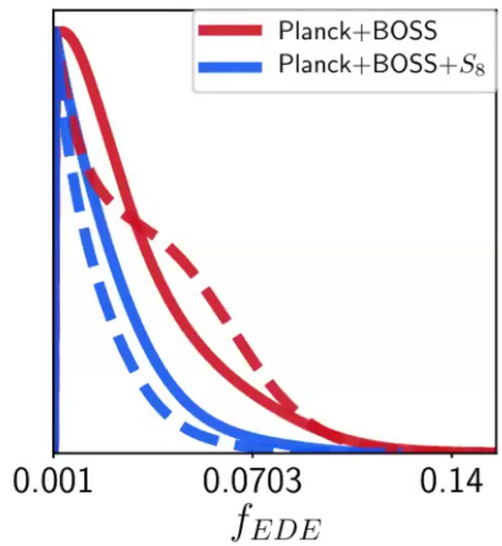
- We learnt a lot from HEP analysis in the last 50yrs. Now that cosmology is becoming a precise science, can we import HEP those methods?

Part 1: Models

The H0 Tension

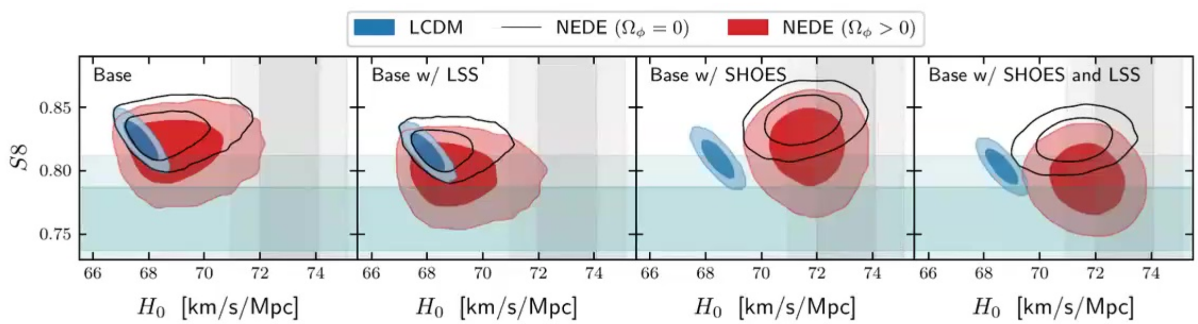
Early DE (canonical) models

$$V(\varphi) = V_0 \left[1 - \cos\left(\frac{\varphi}{f}\right)\right]^n$$



Mcdonough, Hill, Ivanov, Posta, Toomey

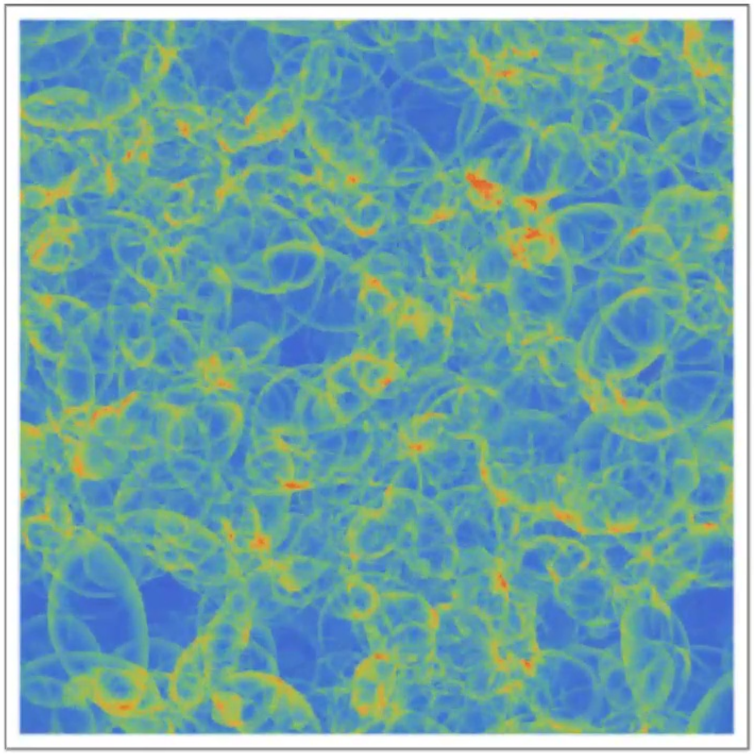
Beyond-canonical: e.g., Phase transitions in the early Universe



Cruz, Niedermann, Sloth; ++
Garny, Niedermann, Sloth, **HR,** (To appear)

Interesting connection with GWs (NANOGrav, ...)!

Aside



Higgsless simulations of PTs

New simulation scheme for GWs from PTs

Advantages: fast, shocks, non-linear, Higgsless, different frequencies…

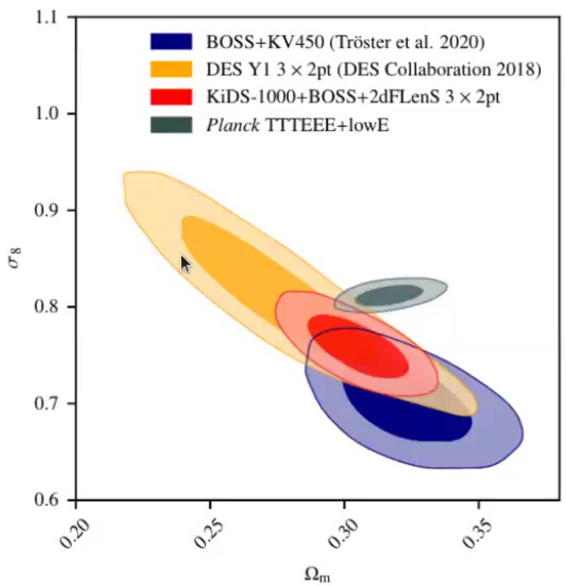
$$\Omega_{\rm GW} \propto \frac{(q/q_0)^3}{1+(q/q_0)^2[1+(q/q_1)^4]}$$

Ryusuke Jinno,
Thomas Konstandin, **HR**, Isaac Stromberg

see 2002.11083, 2010.00971, 2108.11947, 2209.04369, 2302.06952

The S8 Tension

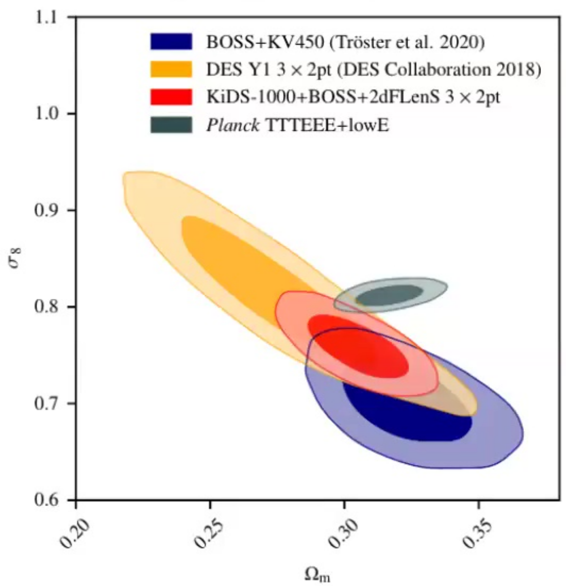
Heymans, Tröster +, 20



$$\sigma^2 = \frac{1}{2\pi^2}\int dk\, k^2 P(k,z)\,|W(kR)|^2$$

The S8 Tension

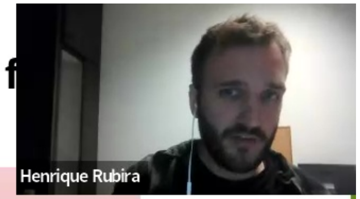
Heymans, Tröster +, 20

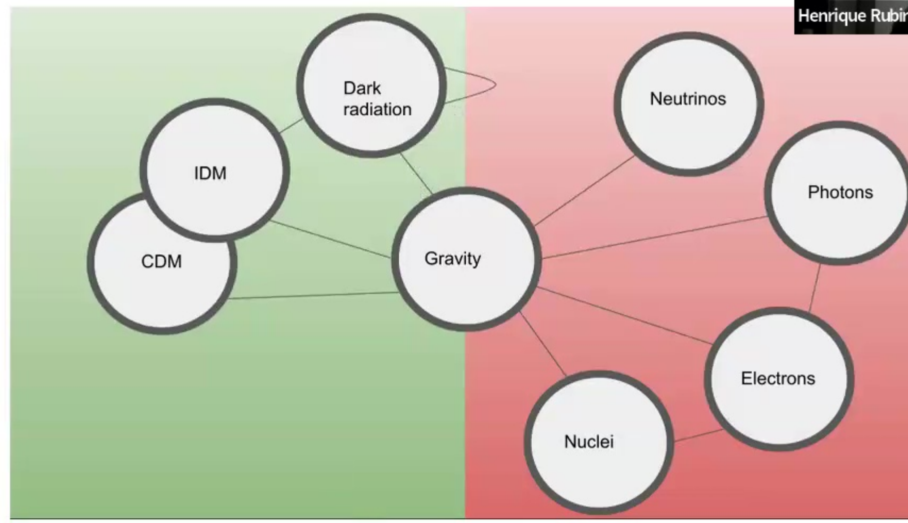


$$\sigma^2 = \frac{1}{2\pi^2}\int dk\, k^2 P(k,z)\,|W(kR)|^2$$

ETHOS (Effective theory of structure f

Cyr-Racine+ 1512.05344





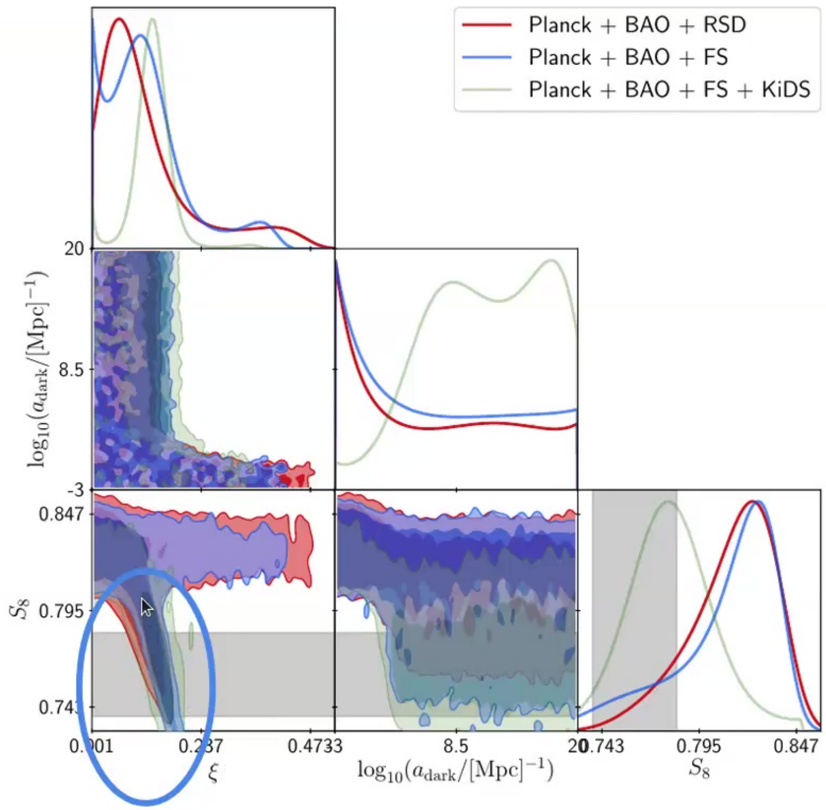
$$f \equiv \frac{\Omega_{\rm IDM}}{\Omega_{\rm CDM}+\Omega_{\rm IDM}} \qquad \xi \equiv \left.\frac{T_{\rm DR}}{T_{\rm CMB}}\right|_{z=0} \qquad \Gamma \propto a_n (1+z)^n$$

$$\dot\delta_{\rm IDM} + \theta_{\rm IDM} - 3\dot\phi_g = 0$$
$$\dot\theta_{\rm IDM} - c^2_{\rm IDM}k^2\delta_{\rm IDM} + \mathcal{H}\theta_{\rm IDM} - k^2\psi_g = \Gamma_{\rm IDM-DR}(\theta_{\rm IDM} - \theta_{\rm DR})$$

What is the reason why IDM-DR lifts S8?

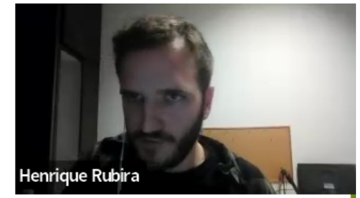
HR, Mazoun, Garny

$$n = 0$$



This feature in the MCMC shows one parameter direction that addresses S8!

What is the reason why IDM-DR lifts S8?



HR, Mazoun, Garny

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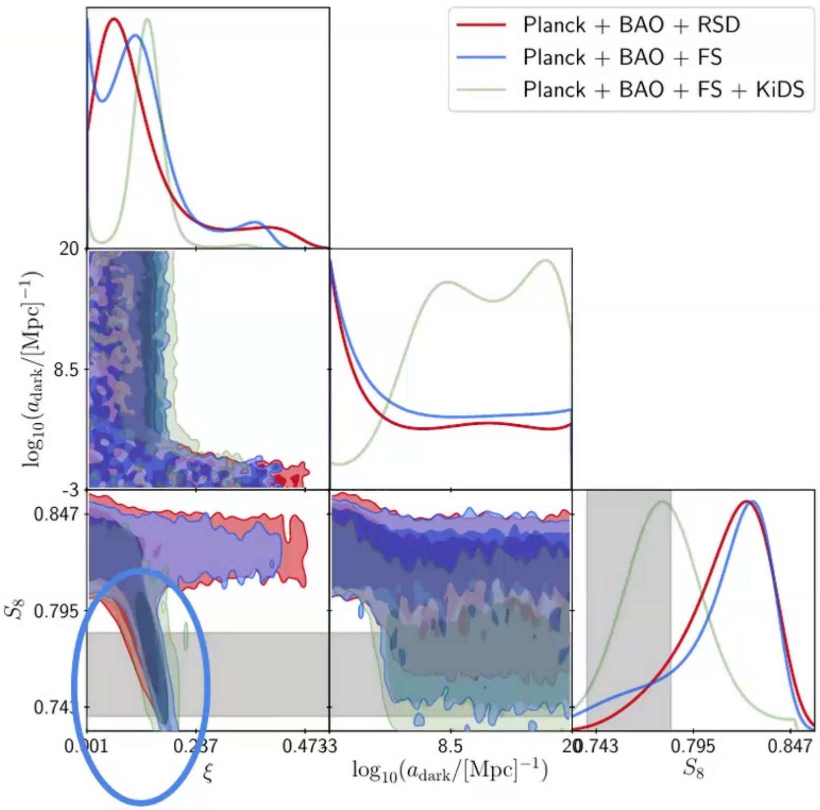

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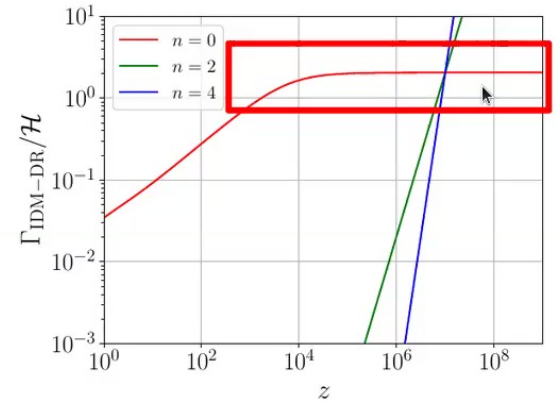
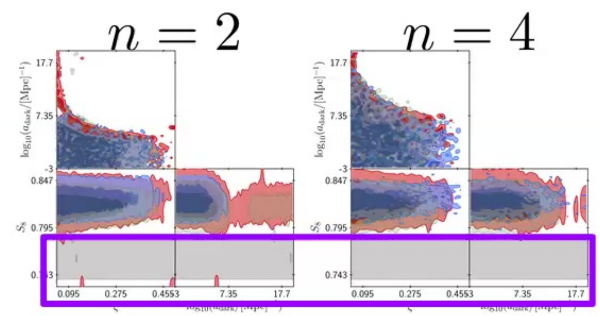
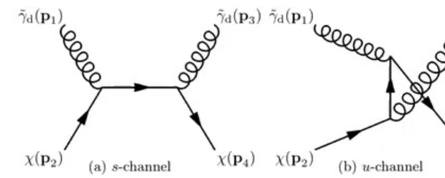
Milder suppression for longer times

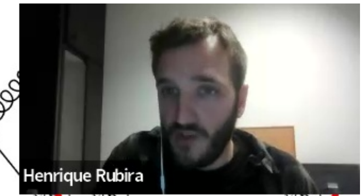




No solution to S8

Connecting COSMO to HEP





$$\mathcal{L} = -\frac{1}{2}\mathrm{tr}(F_{\mu\nu}F^{\mu\nu}) + \bar{\chi}(i\gamma^\mu D_\mu - m_\chi)\chi$$

t-diagram introduce a log-divergence

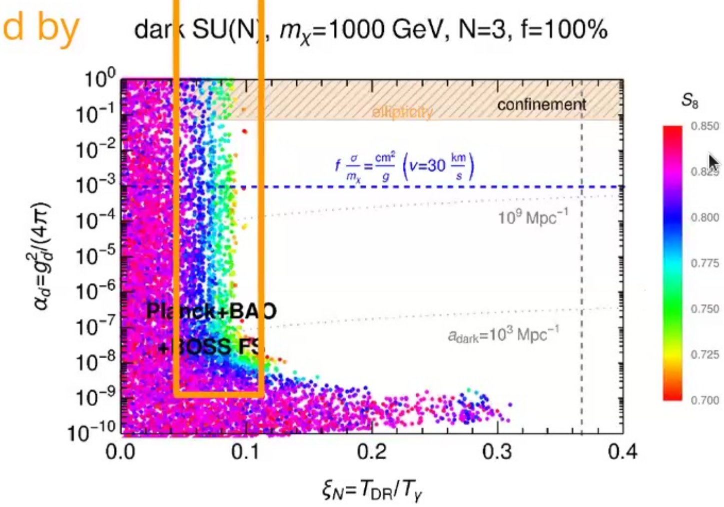
Non-relativistic limit $T_{\mathrm{DR}} \ll m_\chi$

Weak coupling limit $g_d \ll 1$

$$\Gamma_{\mathrm{IDM-DR}} = -a\frac{\pi}{18}\frac{\alpha_d^2}{m_\chi}\eta_{\mathrm{DR}}\left\{T_{\mathrm{DR}}^2\left[\ln\alpha_d^{-1} + c_0 + c_1 g_d + \mathcal{O}(g_d^2)\right] + \mathcal{O}\left(\frac{T_{\mathrm{DR}}^4}{m_\chi^2}\right)\right\}$$

For the first time those contributions were calculated in that context

Favored by KiDS

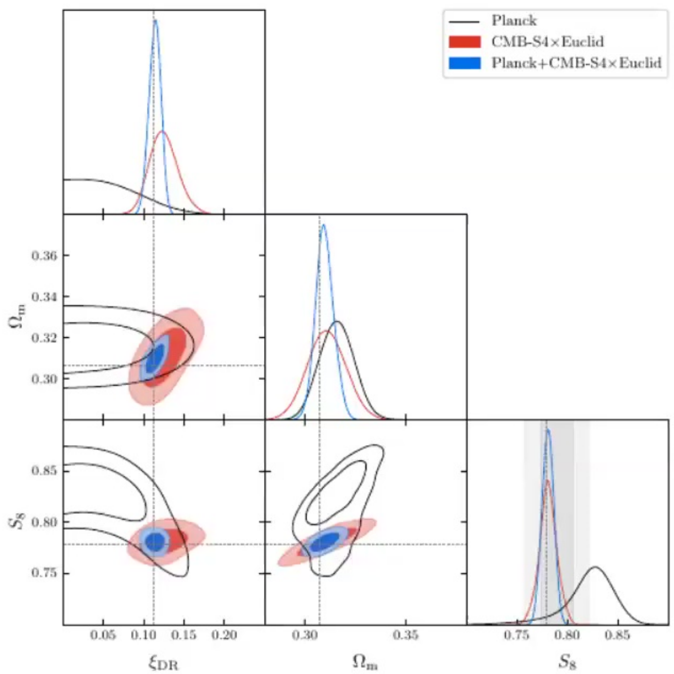


HR, Mazoun, Garny

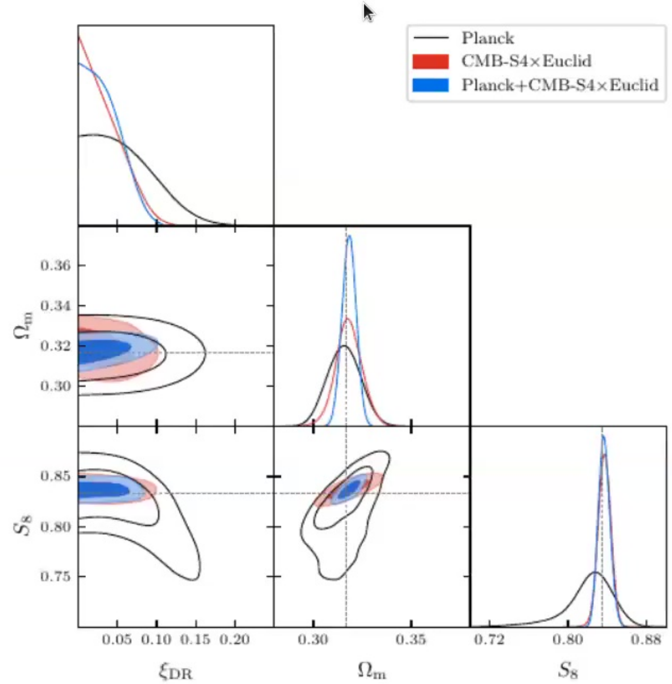
Interacting DM-DR (with Mathias Garny, Asmaa Mazou

Testing with clusters: Preliminary (Mazoun, Bocquet, Garny, Mohr, **HR**, Vogt)

IDM mock



LCDM mock



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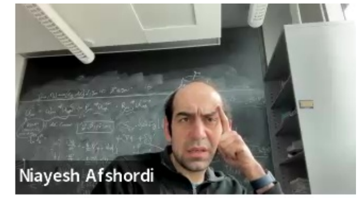

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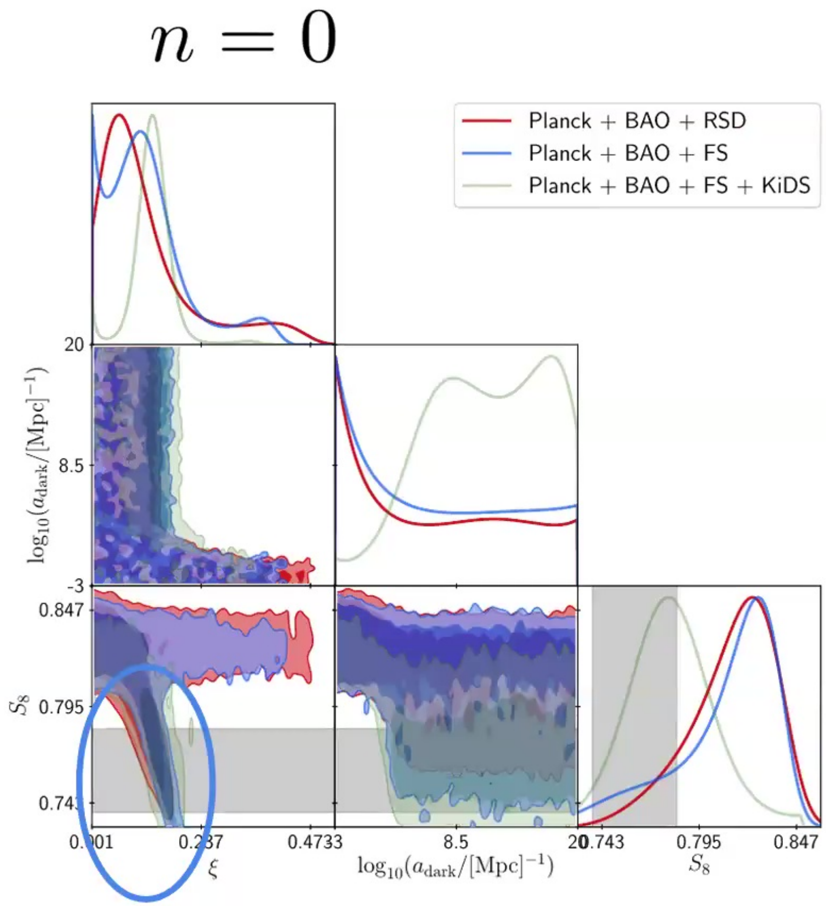

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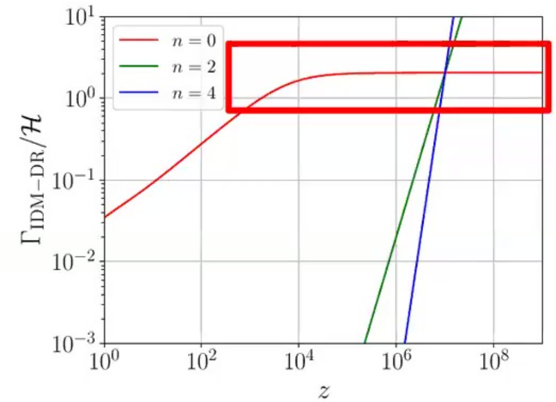

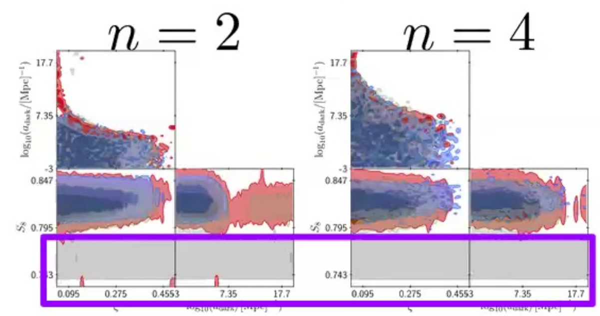

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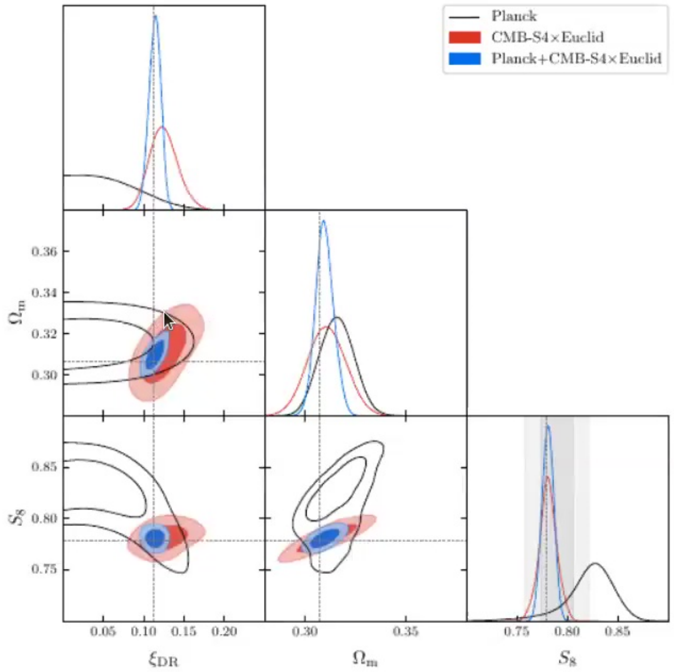
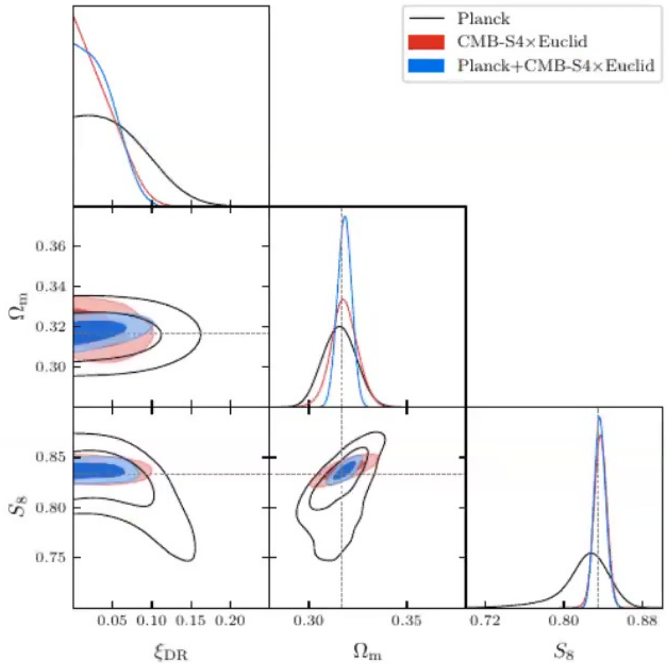


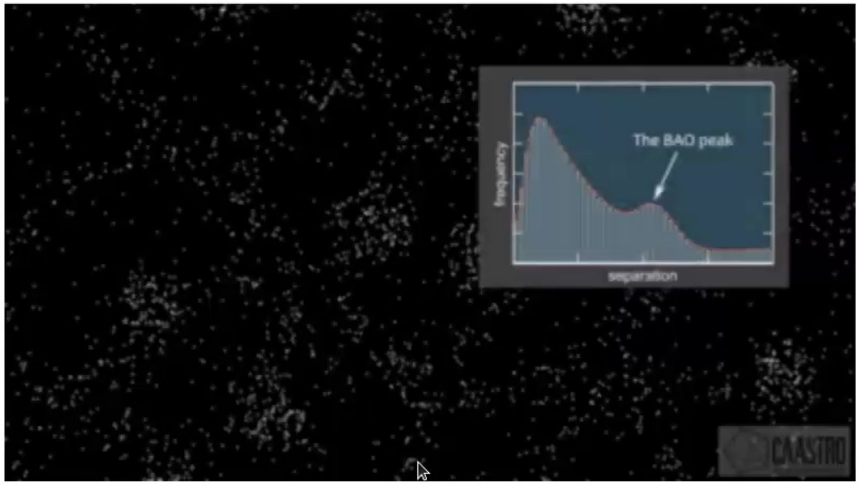
LCDM mock

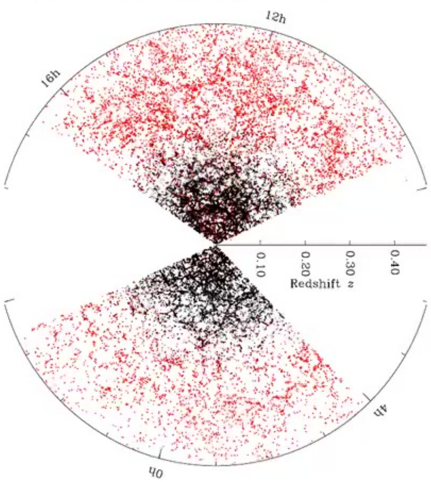


Part 2: Tools (EFTofLSS)

also a pause to have some water

On why (EFTof)LSS is so powerful

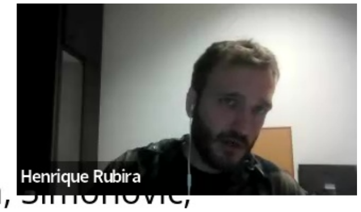


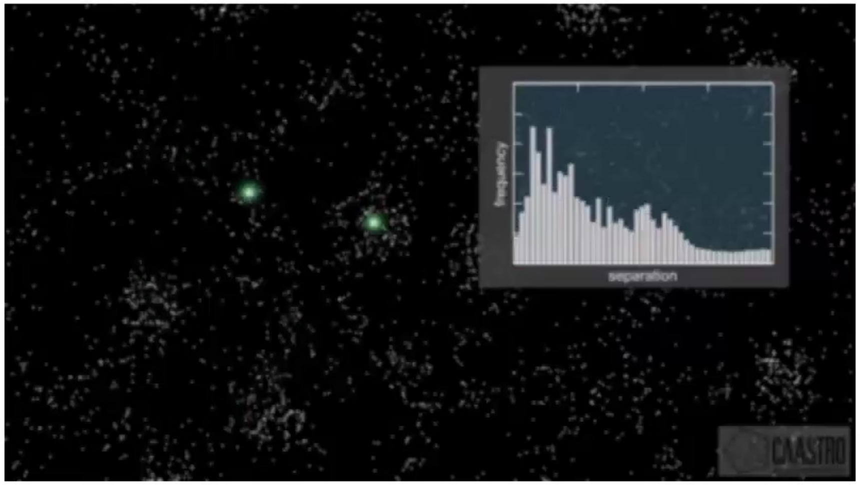


CAASTRO

SDSS
collaboration

On why (EFTof)LSS is so powerful



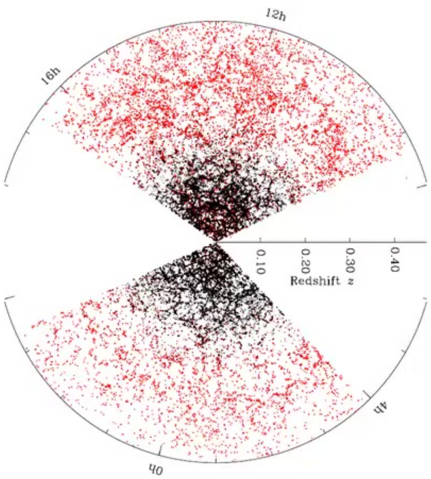


Baumann, Nicolis, Senatore, Zaldarriaga, Simonović, Ivanov, Philcox, Garny, Vlah, Schmidt, D'Amico, Zhang, Kokron, Wadekar, Chen, Scoccimarro, **HR**, (many others)

$$P^{gg}(z,k) = (b_1)^2 [P_{\rm lin}(z,k) + P_{\rm 1L}(z,k)] + b_1 b_2 \mathcal{I}_{\delta^2}(z,k) + 2 b_1 b_{\mathcal{G}_2} \mathcal{I}_{\mathcal{G}_2}(z,k) + \left(2 b_1 b_{\mathcal{G}_2} + \frac{4}{5} b_1 b_{\Gamma_3}\right) \mathcal{F}_{\mathcal{G}_2}(z,k) + \frac{1}{4}(b_2)^2 \mathcal{I}_{\delta^2\delta^2}(z,k) + (b_{\mathcal{G}_2})^2 \mathcal{I}_{\mathcal{G}_2\mathcal{G}_2}(z,k) + b_2 b_{\mathcal{G}_2} \mathcal{I}_{\delta^2\mathcal{G}_2}(z,k) + P_{\nabla^2\delta}(z,k) + P_{\varepsilon\varepsilon}(z,k)\,,$$

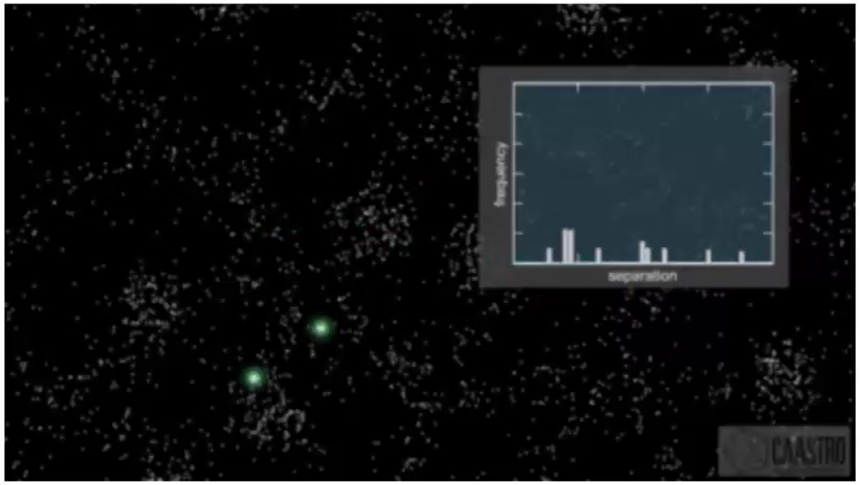
Bias expansion Counter-terms Stochasticity

CAASTRO



SDSS collaboration

On why (EFTof)LSS is so powerful



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\begin{aligned}
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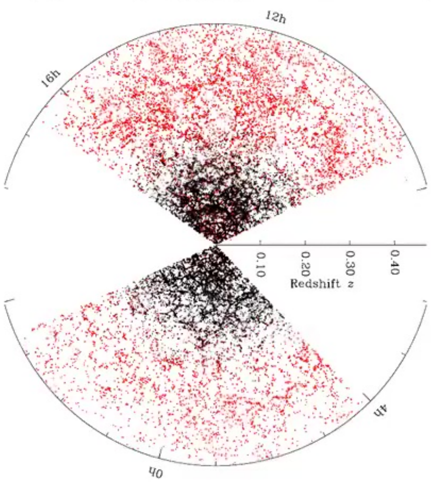
&+ \left(2b_1 b_{\mathcal{G}_2} + \frac{4}{5} b_1 b_{\Gamma_3}\right)\mathcal{F}_{\mathcal{G}_2}(z,k) + \frac{1}{4}(b_2)^2 \mathcal{I}_{\delta^2\delta^2}(z,k) \\
&+ (b_{\mathcal{G}_2})^2 \mathcal{I}_{\mathcal{G}_2\mathcal{G}_2}(z,k) + b_2 b_{\mathcal{G}_2}\mathcal{I}_{\delta^2\mathcal{G}_2}(z,k) + P_{\nabla^2\delta}(z,k) + P_{\varepsilon\varepsilon}(z,k),
\end{aligned}
$$

Bias expansion | Counter-terms | Stochasticity

CAASTRO



SDSS collaboration

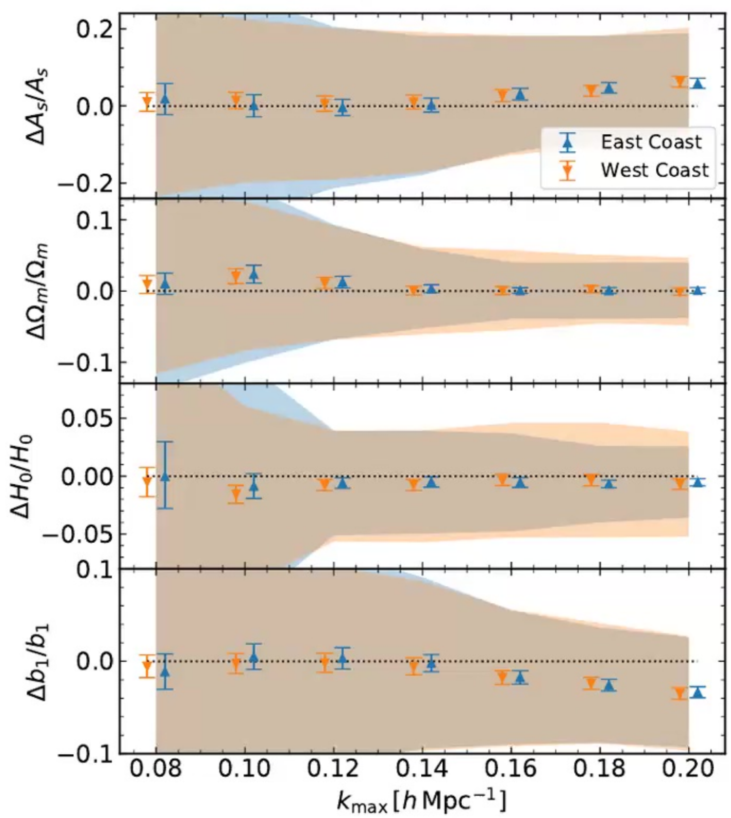
Cons: many free parameters

Pros:

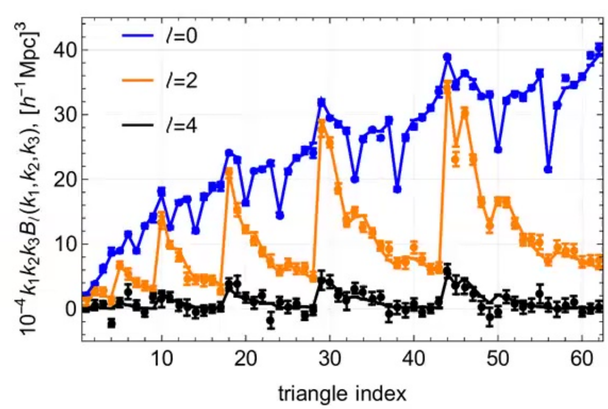
- full (correct) non-linear parametrization
- trivially generalized to higher-order n-point function and other models

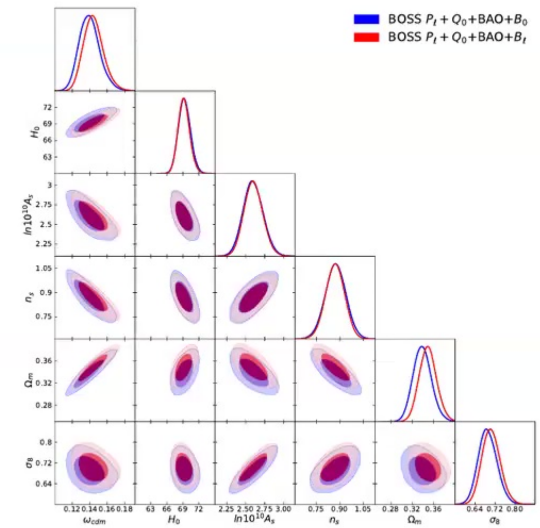
On why (EFTof)LSS is so powerful

Success in blinded analysis
(Nishimichi et al)



State of the art of BOSS analysis:
Bispectra multipoles
(Ivanov, Philcox, +)





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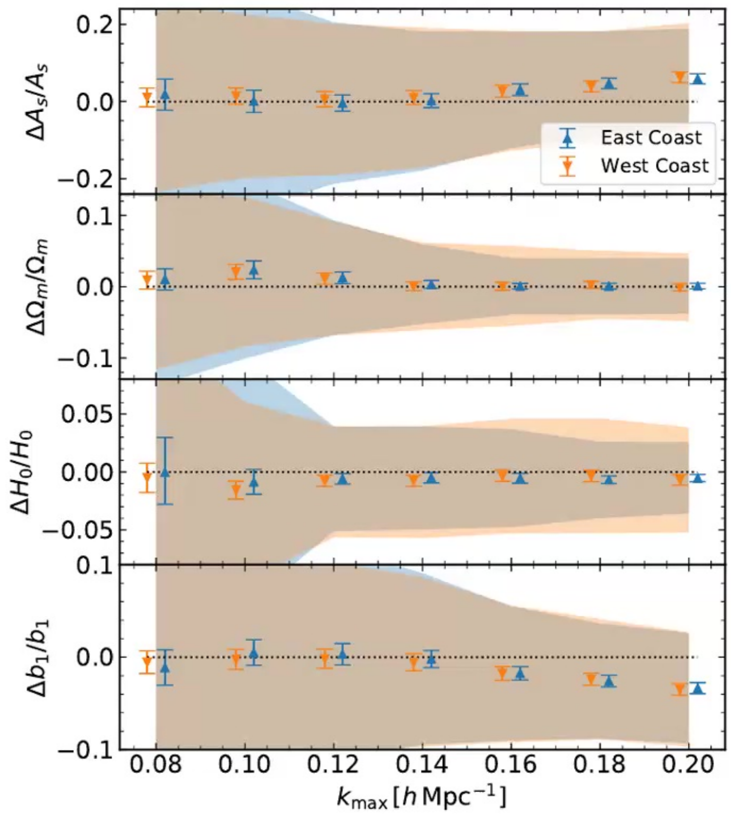

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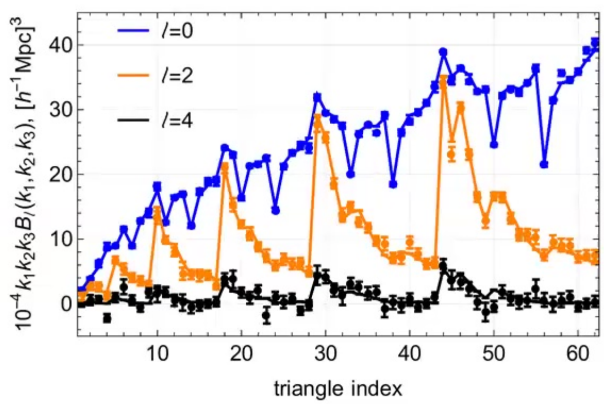
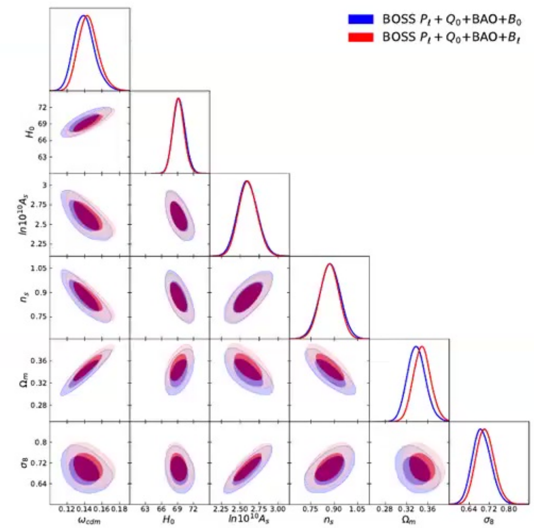
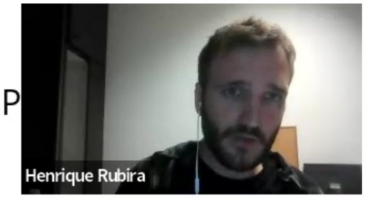


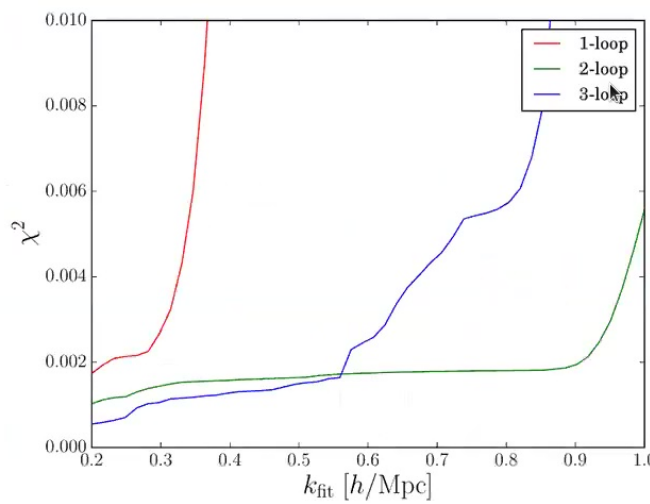


The 3-loop calculation

(In collaboration with Thomas Konstandin + Rafael P



$$
\begin{aligned}
P^{\text{EFT}}_{\text{3-loop}} = P^{\text{SPT}}_{\text{3-loop}} &- 2(2\pi)(c^2_{s(1)} + c^2_{s(2)} + c^2_{s(3)})k^2 P_0 - 2(2\pi)(c^2_{s(1)} + c^2_{s(2)})k^2 P_1 \\
&- 2(2\pi)c^2_{s(1)}k^2 P_2 + (2\pi)^2\Big((c^2_{s(1)})^2 + 2c^2_{s(2)}c^2_{s(1)}\Big)k^4 P_0 + (2\pi)^2(c^2_{s(1)})^2 k^4 P_1 \\
&- 2(2\pi)(c_{2,\text{quad}(1)} + c_{2,\text{quad}(2)})k^2 P_{\text{quad}} - 2(2\pi)c_{2,\text{quad}(1)}k^2 P_{\text{quad}(2)} \\
&- 2(2\pi)^2(c_{4(1)} + c_{4(2)})k^4 P_0 - 2(2\pi)^2 c_{4(1)}k^4 P_1 + 2(2\pi)^3 c^2_{s(1)}c_{4(1)}k^6 P_0 \\
&+ 2(2\pi)^2 c^2_{s(1)}c_{2,\text{quad}(1)}k^4 P_{\text{quad}} + (2\pi)^2 c_{\text{stoch}}k^4 - 2(2\pi)^2 c_{4,\text{quad}}k^4 P_{\text{quad}} \\
&- 2(2\pi)^3 c_6 k^6 P_0.
\end{aligned}
$$

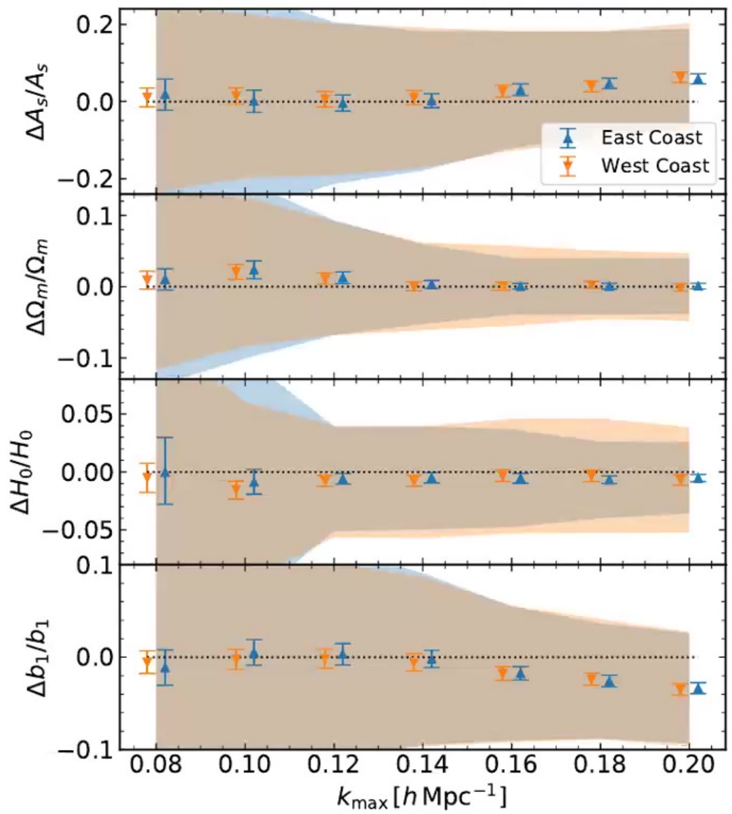


EFT as an asymptotic series

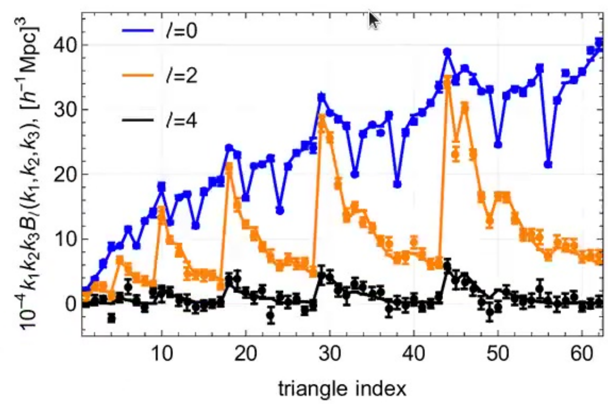
On why (EFTof)LSS is so powerful

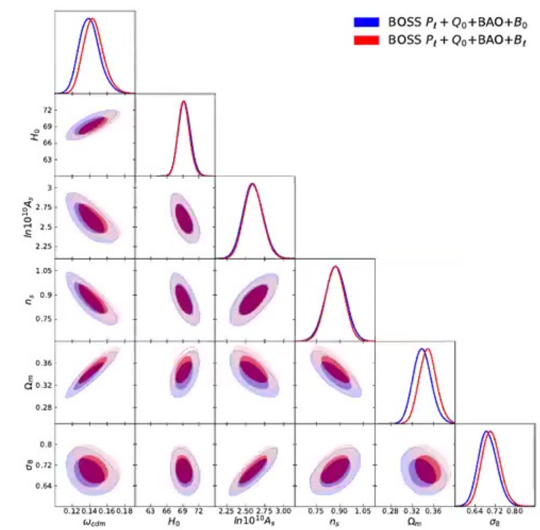
Henrique Rubira

Success in blinded analysis
(Nishimichi et al)



State of the art of BOSS analysis:
Bispectra multipoles
(Ivanov, Philcox, +)

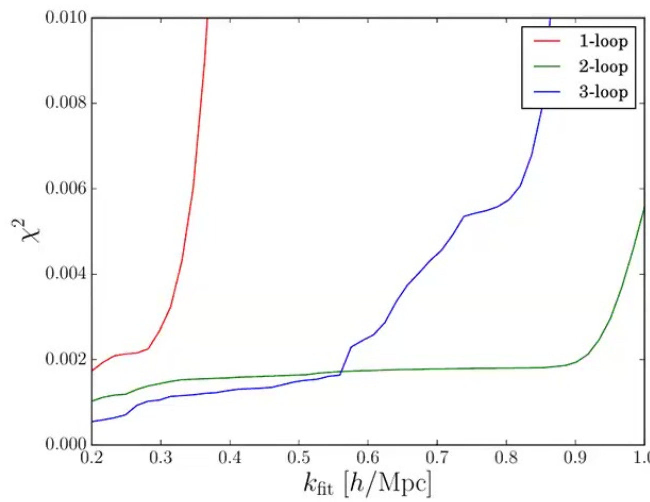




The 3-loop calculation

(In collaboration with Thomas Konstandin + Rafael P

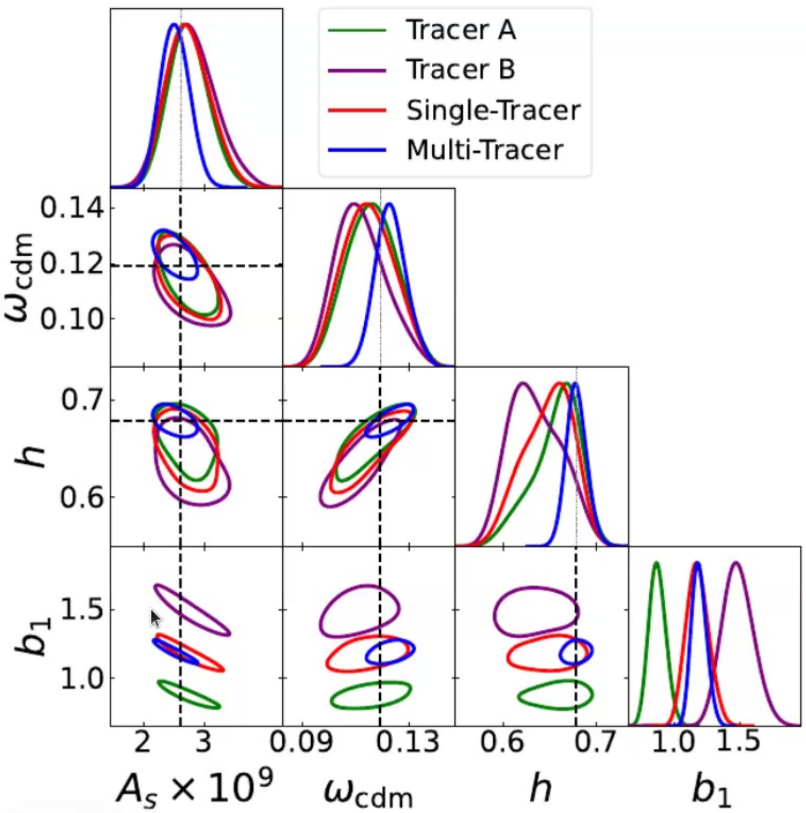
$$
\begin{aligned}
P^{\text{EFT}}_{\text{3-loop}} = {} & P^{\text{SPT}}_{\text{3-loop}} - 2(2\pi)(c^2_{s(1)} + c^2_{s(2)} + c^2_{s(3)})k^2 P_0 - 2(2\pi)(c^2_{s(1)} + c^2_{s(2)})k^2 P_1 \\
& - 2(2\pi)c^2_{s(1)}k^2 P_2 + (2\pi)^2\Big((c^2_{s(1)})^2 + 2c^2_{s(2)}c^2_{s(1)}\Big)k^4 P_0 + (2\pi)^2 (c^2_{s(1)})^2 k^4 P_1 \\
& - 2(2\pi)(c_{2,\text{quad}(1)} + c_{2,\text{quad}(2)})k^2 P_{\text{quad}} - 2(2\pi)c_{2,\text{quad}(1)}k^2 P_{\text{quad}(2)} \\
& - 2(2\pi)^2(c_{4(1)} + c_{4(2)})k^4 P_0 - 2(2\pi)^2 c_{4(1)}k^4 P_1 + 2(2\pi)^3 c^2_{s(1)}c_{4(1)}k^6 P_0 \\
& + 2(2\pi)^2 c^2_{s(1)}c_{2,\text{quad}(1)}k^4 P_{\text{quad}} + (2\pi)^2 c_{\text{stoch}}k^4 - 2(2\pi)^2 c_{4,\text{quad}}k^4 P_{\text{quad}} \\
& - 2(2\pi)^3 c_6 k^6 P_0 .
\end{aligned}
$$



EFT as an asymptotic series

Multi-tracer

2306.05474, 2108.11363, In collaboration with Mergulhão, Voivc



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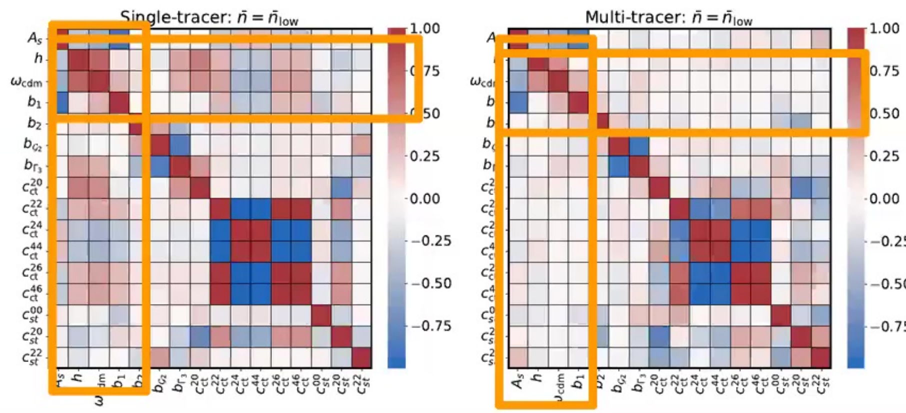
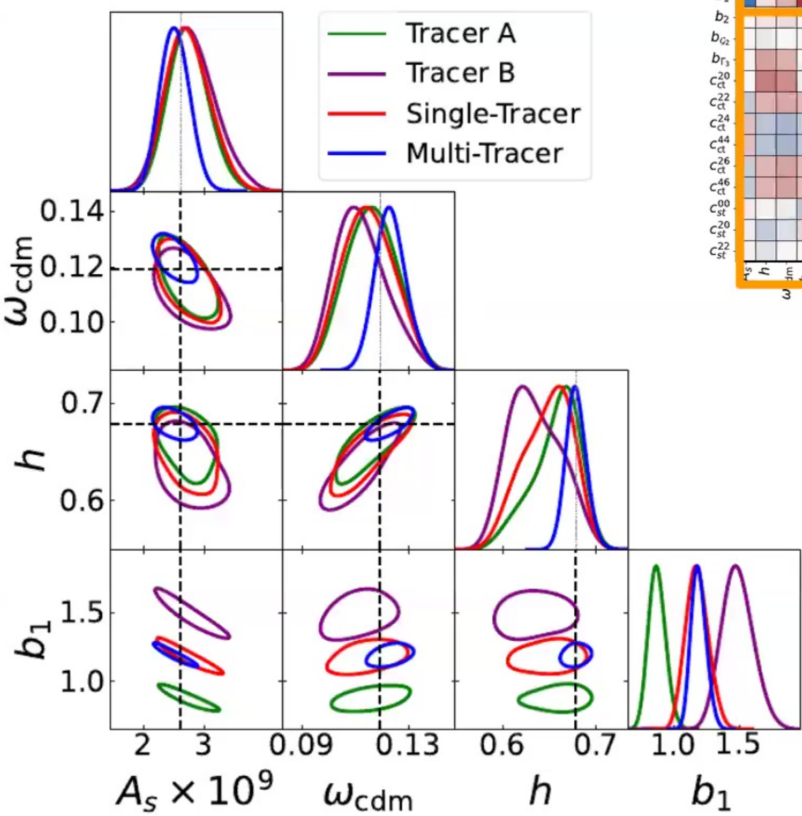
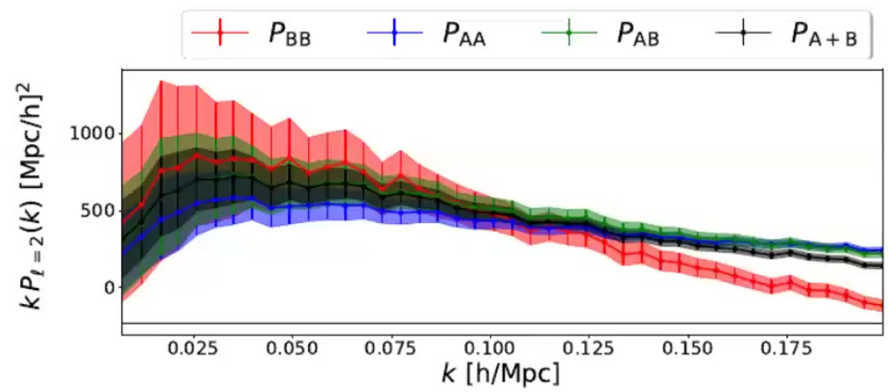




Information from cross-spectra: $b^A_{\mathcal{O}} b^B_{\mathcal{O}}$



EFTofLSS via Wilson Polchinski

(base
Leiche
13)

$$Z[J] = \int \mathcal{D}\phi_{\rm in} \ \exp\left(S_0[\phi_{\rm in}] + J_i \phi^i[\phi_{\rm in}]\right)$$ with $$S_0[\phi_{\rm in}] = -\frac{1}{2}\phi^i [P(\Lambda)^{-1}]_{ij}\phi^j$$

We use $$\left.\frac{\partial \mathcal{Z}}{\partial J_\Lambda \ldots \partial J_\Lambda}\right|_{J_\Lambda = 0}$$ Since $$\langle \phi^{i_1} \cdots \phi^{i_n} \rangle = \int \mathcal{D}\phi_{\rm in} \ \phi^{i_1}[\phi_{\rm in}] \cdots \phi^{i_n}[\phi_{\rm in}] e^{S_0[\phi_{\rm in}]}$$

EFTofLSS via Wilson Polchinski

(basec
Leiche
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$$\phi^i_{\rm SPT} \equiv K^i_{{\rm SPT}\,j}\,\phi^j_{\rm in} + \frac{1}{2} K^i_{{\rm SPT}\,jk}\,\phi^j_{\rm in}\phi^k_{\rm in} + \cdots$$

$$\frac{d}{d\Lambda} K^{j_1 \cdots j_n}_{i_1 \cdots i_m} = -\frac{1}{2}\left(\frac{dP^{ij}}{d\Lambda} K^{j_1 \cdots j_n}_{iji_1 \cdots i_m} + \frac{dP^{ij}}{d\Lambda}\sum_{k=0}^{m}\sum_{l=0}^{n}\binom{m}{k}\binom{n}{l} K^{j_1 \cdots j_l}_{ii_1 \cdots i_k} K^{j_{l+1} \cdots j_n}_{ji_{k+1} \cdots i_m}\right)$$

EFTofLSS via Wilson Polchinski

(basec
Leiche
13)

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Advantages:

- Path integral formulation
- Systematic generation EFT structure (coefficients are closed under RG flow)
- Keeps small (yet-perturbative) modes in the theory

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The LSS Renormalization Group

also another pause before showing more dense calculations

EFTofLSS via Wilson Polchinski

(basec
Leiche
13)

$$Z[J] = \int \mathcal{D}\phi_{\rm in}\ \exp\left(S_0[\phi_{\rm in}] + J_i \phi^i[\phi_{\rm in}]\right) \quad \text{with} \quad S_0[\phi_{\rm in}] = -\frac{1}{2}\phi^i [P(\Lambda)^{-1}]_{ij}\phi^j$$

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Since
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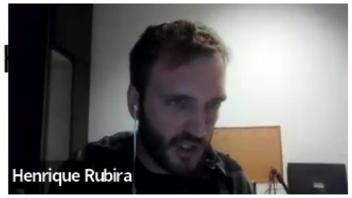
The LSS Renormalization Group

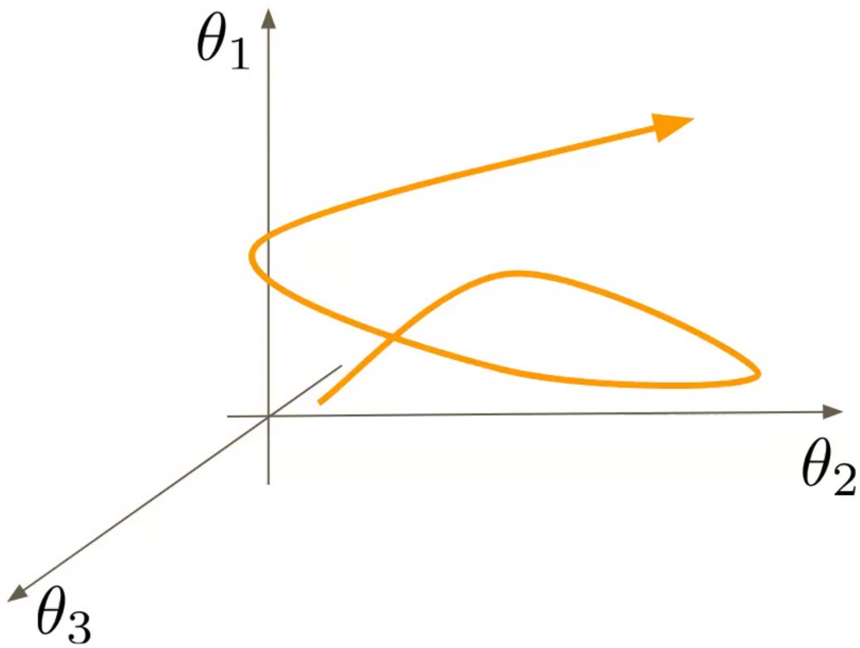
also another pause before showing more dense calculations

Preamble 1... QFT101



Renormalization group: coupling constants evolve with the cutoff ("flow").

Observables don't depend on the cutoff!



Callan-Symanzik equation:

$$\frac{\partial g}{\partial \ln \mu} = \beta(g)$$

QED: $$\beta(e) = \frac{e^3}{12\pi^2}$$

QCD: $$\beta(g) = -\left(11 - \frac{n_s}{6} - \frac{2n_f}{3}\right)\frac{g^3}{16\pi^2}$$

Preamble 2... Historical overview and frameworks

- **Dim Reg, scale transformations and applications to QED**: Stueckelberg, Petermann, Gell-Mann, Low ~1953

- **RG in condensed matter**: Kadanoff, 1966

- **RG in the continuum, derivation of RG equations and critical phenomena**: Callan and Symanzik 1970, Kenneth Wilson, 1970/71 (Nobel Prize 1982)

- **RG via path integrals**: Polchinski, 1984

Framework 1 (a la Wilson/Polchinski):

$$\Lambda \frac{d}{d\Lambda} Z[J] = 0$$

Sliding cutoff,
integrate out modes between cutoffs

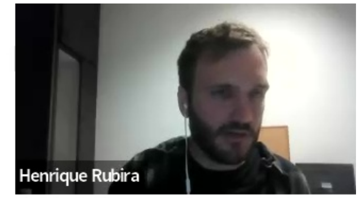
$$\Lambda \to \Lambda'$$

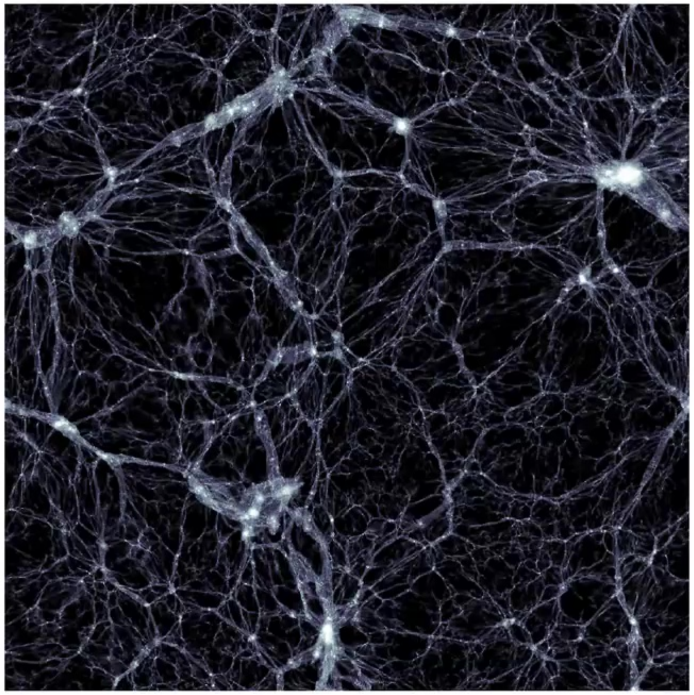
Framework 2:
Sliding renormalization conditions (e.g. Dim Reg), no UV regulator

$$\frac{\partial g}{\partial \ln \mu} = \beta(g)$$

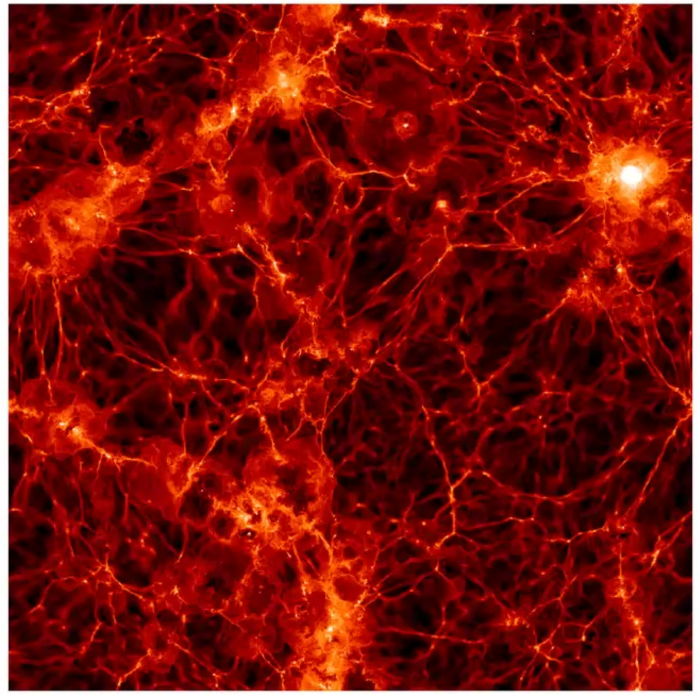
- More practical for computations

Preamble 3... The galaxy bias expansion





(a) dark matter



(b) baryons

From Illustris simulation, Haiden, Steinhauser, Vogelsberger, Genel, Springel, Torrey, Hernquist, 15

$$\delta_g(\boldsymbol{x},\tau) \equiv \frac{n_g(\boldsymbol{x},\tau)}{\bar{n}_g(\tau)} - 1 = \sum_O \left[b_O(\tau) + c_{\epsilon,O}(\tau)\epsilon(\boldsymbol{x},\tau)\right] O(\boldsymbol{x},\tau) + \epsilon(\boldsymbol{x},\tau)$$

Preamble 3… The galaxy bias expansion

Bias and stochastic parameters

Stochastic field

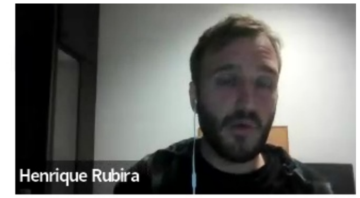
$$\delta_g(\boldsymbol{x},\tau) \equiv \frac{n_g(\boldsymbol{x},\tau)}{\bar{n}_g(\tau)} - 1 = \sum_O \left[b_O(\tau) + c_{\epsilon,O}(\tau)\epsilon(\boldsymbol{x},\tau)\right] O(\boldsymbol{x},\tau) + \epsilon(\boldsymbol{x},\tau)$$

Operators:

First order:	$\delta\,;$
Second order:	$\delta^2,\ \mathcal{G}_2\,;$
Third order:	$\delta^3,\ \delta\,\mathcal{G}_2,\ \Gamma_3,\ \mathcal{G}_3\,;$

Bias review: Desjacques, Jeong, Schmidt

Renormalizing the bias parameters



$$\delta_g(\boldsymbol{x},\tau) \equiv \frac{n_g(\boldsymbol{x},\tau)}{\bar{n}_g(\tau)} - 1 = \sum_O \left[b_O(\tau) + c_{\epsilon,O}(\tau)\epsilon(\boldsymbol{x},\tau)\right] O(\boldsymbol{x},\tau) + \epsilon(\boldsymbol{x},\tau)$$

$$O[\delta](\boldsymbol{k}) = \int_{\boldsymbol{p}_1,\ldots,\boldsymbol{p}_n} \delta_{\rm D}(\boldsymbol{k} - \boldsymbol{p}_{1\ldots n}) S_O(\boldsymbol{p}_1,\ldots\boldsymbol{p}_n)\delta(\boldsymbol{p}_1)\cdots\delta(\boldsymbol{p}_n)$$

First order: $\delta\,;$

Second order: $\delta^2,\ \mathcal{G}_2\,;$

Third order: $\delta^3,\ \delta\,\mathcal{G}_2,\ \Gamma_3,\ \mathcal{G}_3\,;$

Contribution from arbitrarily small scales!

Renormalizing the bias parameters

$$\delta_g(\boldsymbol{x},\tau) \equiv \frac{n_g(\boldsymbol{x},\tau)}{\bar{n}_g(\tau)} - 1 = \sum_O \left[b_O^{\Lambda}(\tau) + c_{\epsilon,O}^{\Lambda}(\tau)\epsilon^{\Lambda}(\boldsymbol{x},\tau)\right] O^{\Lambda}(\boldsymbol{x},\tau) + \epsilon^{\Lambda}(\boldsymbol{x},\tau)$$

+ counter-terms (Λ)

$$O[\delta](\boldsymbol{k}) = \int_{\boldsymbol{p}_1,\ldots,\boldsymbol{p}_n}^{\Lambda} \delta_{\mathrm{D}}(\boldsymbol{k} - \boldsymbol{p}_{1\ldots n}) S_O(\boldsymbol{p}_1,\ldots \boldsymbol{p}_n)\delta(\boldsymbol{p}_1)\cdots\delta(\boldsymbol{p}_n)$$

First order: δ ;

Second order: $\delta^2, \ \mathcal{G}_2$;

Third order: $\delta^3, \ \delta\,\mathcal{G}_2, \ \Gamma_3, \ \mathcal{G}_3$;

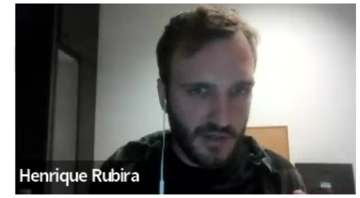
Notation:

$$[\![O]\!] = O^{\Lambda} + \text{counter-terms } (\Lambda)$$

How to determine the renormalization condition?

Contribution from arbitrarily small scales!

The n-point function renormalized bias
(Assassi, Baumann, Green, Zaldarriaga, 2014)



Intuition: Define the bias parameter of order "n" as the large-scale limit of "n+1"-point functions

Example 1: Define the linear bias in the large-scale limit of P(k):

$$b_\delta = \lim_{k \to 0} \frac{\langle \delta_g \delta \rangle}{\langle \delta \delta \rangle}$$

Example 2: Define the 2nd-order bias parameters in the large-scale limit of B(k1,k2,k3)

More formally:

$$\langle \delta^{(1)}(\boldsymbol{k}_1) \cdots \delta^{(1)}(\boldsymbol{k}_m) [\![O]\!](\boldsymbol{k}) \rangle \xrightarrow{k_i \to 0} \langle \delta^{(1)}(\boldsymbol{k}_1) \cdots \delta^{(1)}(\boldsymbol{k}_m) O[\delta](\boldsymbol{k}) \rangle_{\rm LO}$$

Example:

$$[\![\delta^2]\!] = \delta^2 - \sigma^2_\infty \left(1 + \frac{68}{21}\delta + \frac{8126}{2205}\delta^2 + \frac{254}{2205}\mathcal{G}_2 \right)$$

Main motivation

RENORMALIZATION AND EFFECTIVE LAGRANGIANS

Joseph POLCHINSKI*

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

Received 27 April 1983

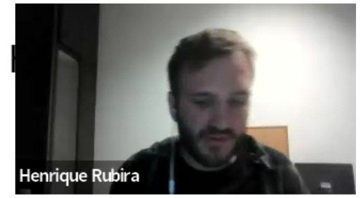
There is a strong intuitive understanding of renormalization, due to Wilson, in terms of the scaling of effective lagrangians. We show that this can be made the basis for a proof of perturbative renormalization. We first study renormalizability in the language of renormalization group flows for a toy renormalization group equation. We then derive an exact renormalization group equation for a four-dimensional $\lambda\phi^4$ theory with a momentum cutoff. We organize the cutoff dependence of the effective lagrangian into relevant and irrelevant parts, and derive a linear equation for the irrelevant part. A lengthy but straightforward argument establishes that the piece identified as irrelevant actually is so in perturbation theory. This implies renormalizability. The method extends immediately to any system in which a momentum-space cutoff can be used, but the principle is more general and should apply for any physical cutoff. Neither Weinberg's theorem nor arguments based on the topology of graphs are needed.

1. Introduction

The understanding of renormalization has advanced greatly in the past two decades. Originally it was just a means of removing infinities from perturbative calculations. The question of why nature should be described by a renormalizable theory was not addressed. These were simply the only theories in which calculations could be done.

A great improvement comes when one takes seriously the idea of a physical cutoff at a very large energy scale Λ. The theory at energies above Λ could be another field

Motivation (for different tastes)



Extend the renormalization picture constructing the Wilson-Polchinski renormalization group that describe the evolution of the finite-scale bias parameters with the cutoff.

Lattice person: "At field level you smooth out over your cutoff and those bias parameters have to be defined at a fixed scale!"

HEP person: "Everything is an EFTs and RG-flow is the next thing to do. "

Cosmo-MCMC person: "How can we be sure we are not messing up with the priors in my EFT analysis? Maybe extract more information..."

EFT-negationist person: "You have a bunch of free parameters. How can you trust them?"

The bias partition function (based on Carroll, Leichenauer

$$\mathcal{Z}[J_\Lambda] = \int \mathcal{D}\delta_\Lambda^{(1)} \mathcal{P}[\delta_\Lambda^{(1)}] \exp\left(\int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) \left[\sum_O b_O^\Lambda O[\delta_\Lambda^{(1)}](-\boldsymbol{k}) \right] + \frac{1}{2} P_\epsilon^\Lambda \int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) J_\Lambda(-\boldsymbol{k}) + \mathcal{O}[J_\Lambda^2 \delta_\Lambda^{(1)},\ J_\Lambda^3] \right)$$

Single-current term

Path-integral over linear-smoothed density, normalized

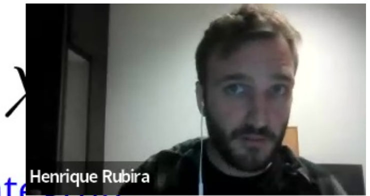
Double-current term captures stochasticity source

N-point correlators evaluated as:

$$\left. \frac{\partial \mathcal{Z}}{\partial J_\Lambda \ldots \partial J_\Lambda} \right|_{J_\Lambda = 0}$$

See Cabass, Schmidt 19

The shell expansion (Wilson formalism)



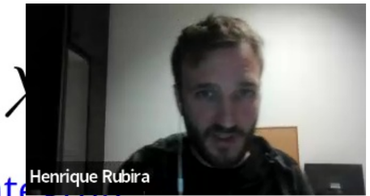
Consider a very thin shell with width: $\Lambda = \Lambda' - \lambda$

$$\delta^{(1)}_{\Lambda'}(\boldsymbol{k}) = \delta^{(1)}_{\Lambda}(\boldsymbol{k}) + \boxed{\delta^{(1)}_{\text{shell}}(\boldsymbol{k})}$$

Idea: Integrate out the shell!

$$\begin{aligned}
\mathcal{Z}[J_\Lambda] = \int \mathcal{D}\delta^{(1)}_\Lambda \mathcal{P}[\delta^{(1)}_\Lambda] \exp\Bigg(\int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) \Bigg[\sum_O b_O^{\Lambda'} O[\delta^{(1)}_\Lambda](-\boldsymbol{k}) \Bigg] \\
+ \frac{1}{2} P_\epsilon^{\Lambda'} \int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) J_\Lambda(-\boldsymbol{k}) + \mathcal{O}[J_\Lambda^2 \delta^{(1)}_\Lambda,\ J_\Lambda^3] \Bigg) \\
\times \Bigg(1 + \int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) \Bigg[\sum_O b_O^{\Lambda'} \Big(\mathcal{S}^1_O[\delta^{(1)}_\Lambda](-\boldsymbol{k}) + \mathcal{S}^2_O[\delta^{(1)}_\Lambda](-\boldsymbol{k}) + \ldots \Big) \Bigg] \\
+ \frac{1}{2} \int_{\boldsymbol{k},\boldsymbol{k}'} J_\Lambda(\boldsymbol{k}) J_\Lambda(\boldsymbol{k}') \sum_{O,O'} b_O^{\Lambda'} b_{O'}^{\Lambda'} \Big[\mathcal{S}^{11}_{OO'}[\delta^{(1)}_\Lambda](\boldsymbol{k},\boldsymbol{k}') + \ldots \Big] + \mathcal{O}[J_\Lambda^2 \delta^{(1)}_\Lambda,\ J_\Lambda^3] \Bigg)
\end{aligned}$$

The shell expansion (Wilson formalism)



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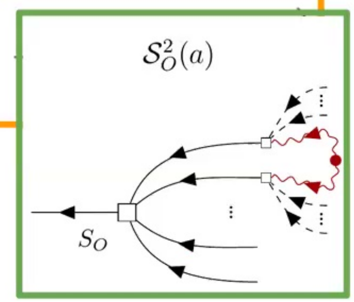
The running of the bias/stochastic operators is done connecting both cutoff

$$\times \left(1 + \int_{\boldsymbol{k}} J_\Lambda(\boldsymbol{k}) \left[\sum_O b_O^{\Lambda'} \left(\mathcal{S}^1_O[\delta^{(1)}_\Lambda](-\boldsymbol{k}) + \mathcal{S}^2_O[\delta^{(1)}_\Lambda](-\boldsymbol{k}) + \ldots \right) \right] + \frac{1}{2} \int_{\boldsymbol{k},\boldsymbol{k}'} J_\Lambda(\boldsymbol{k}) J_\Lambda(\boldsymbol{k}') \sum_{O,O'} b_O^{\Lambda'} b_{O'}^{\Lambda'} \left[\mathcal{S}^{11}_{OO'}[\delta^{(1)}_\Lambda](\boldsymbol{k}, \boldsymbol{k}') + \ldots \right] \ldots \right)$$

What appears after integrating out the shell

Bias corrections



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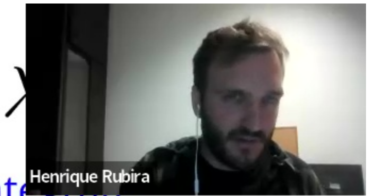

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The running of the bias/stochastic operators is done connecting both cutoff

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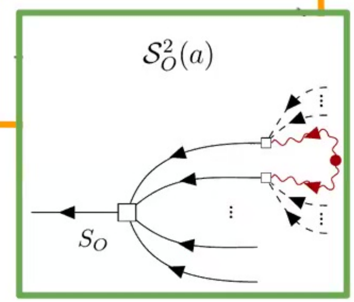
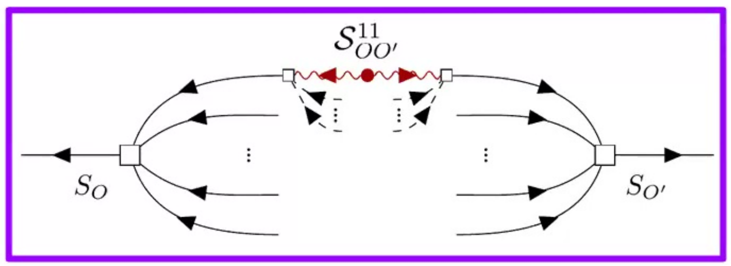
What appears after integrating out the shell

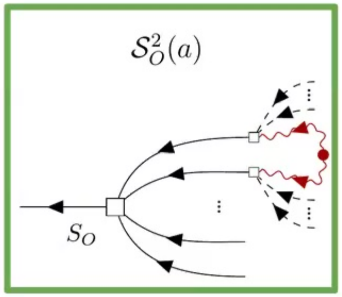
Bias corrections



Stochastic corrections



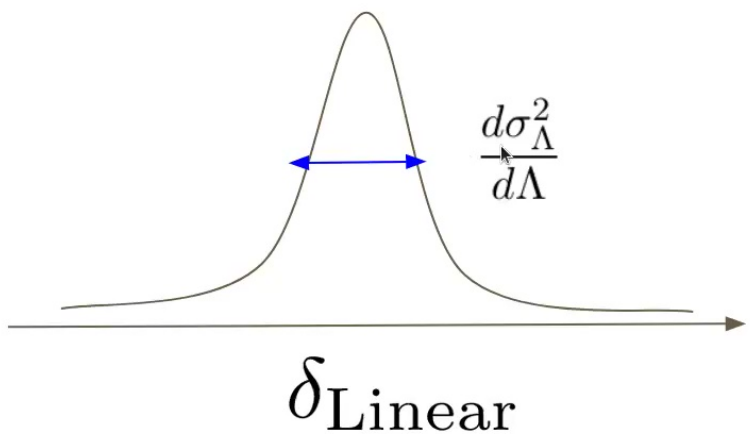
Example...



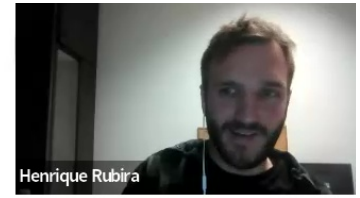
Correction to those operators!

$$\mathcal{S}^2_{\delta^2}[\delta^{(1)}_\Lambda](\boldsymbol{k}) = \left[\frac{68}{21}\delta^{(1+2)}(\boldsymbol{k}) + \frac{8126}{2205}\left[\delta^2(\boldsymbol{k})\right]^{(2)} + \frac{254}{2205}\mathcal{G}_2^{(2)}(\boldsymbol{k})\right]\int \frac{p^2 dp}{2\pi^2}P_{\rm shell}(p)$$
$$+ \text{higher derivative (h.d.)} + \mathcal{O}\left[\left(\delta^{(1)}_\Lambda\right)^3\right],$$

$$\int_{\boldsymbol{p}} P_{\rm shell}(p) = \int_\Lambda^{\Lambda+\lambda}\frac{p^2 dp}{2\pi^2}P_{\rm L}(p) = \frac{d\sigma^2_\Lambda}{d\Lambda}\Big|_\Lambda \lambda + \mathcal{O}(\lambda^2),$$



Results



Wilson-Polchinski RG-equations

$$\frac{db_\delta}{d\Lambda} = -\left[\frac{68}{21}b_{\delta^2} + 3b^*_{\delta^3} - \frac{4}{3}b^*_{\mathcal{G}_2\delta}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}\,,$$

$$\frac{db_{\delta^2}}{d\Lambda} = -\left[\frac{8126}{2205}b_{\delta^2} + \frac{17}{7}b^*_{\delta^3} - \frac{376}{105}b^*_{\mathcal{G}_2\delta} + \boxed{b^{*(\delta^2)}_{n=4}}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}\,,$$

$$\frac{db_{\mathcal{G}_2}}{d\Lambda} = -\left[\frac{254}{2205}b_{\delta^2} + \frac{116}{105}b^*_{\mathcal{G}_2\delta} + \boxed{b^{*(\mathcal{G}_2)}_{n=4}}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}\,.$$

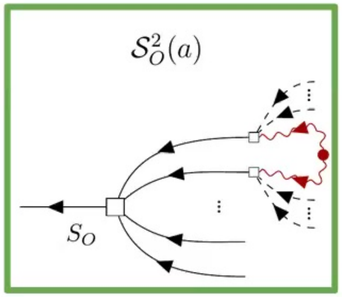
We would need 4th-order terms to calculate the running of 2nd order bias

Comments:

- Linear ODE system;
- "n"-order bias parameter connected to "n+2"-order
- How to close this hierarchy?

Example...

Henrique Rubira

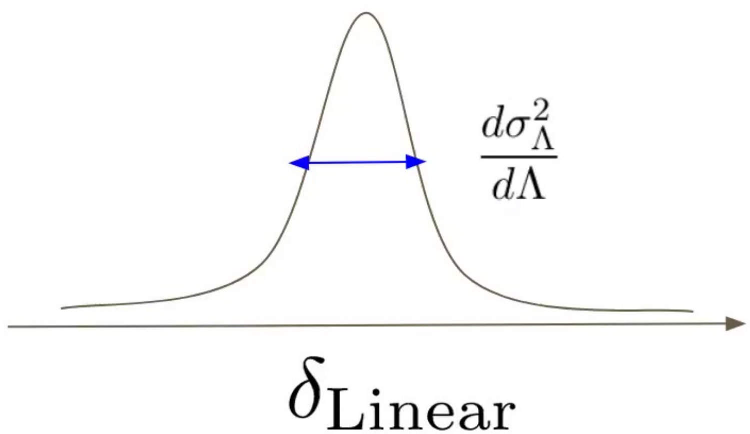


Correction to those operators!

$$\mathcal{S}^2_{\delta^2}[\delta^{(1)}_\Lambda](\boldsymbol{k}) = \left[\frac{68}{21}\delta^{(1+2)}(\boldsymbol{k}) + \frac{8126}{2205}\left[\delta^2(\boldsymbol{k})\right]^{(2)} + \frac{254}{2205}\mathcal{G}_2^{(2)}(\boldsymbol{k})\right]\int \frac{p^2 dp}{2\pi^2} P_{\rm shell}(p)$$

$$+ \text{higher derivative (h.d.)} + \mathcal{O}\left[\left(\delta^{(1)}_\Lambda\right)^3\right],$$

$$\int_{\boldsymbol{p}} P_{\rm shell}(p) = \int_\Lambda^{\Lambda+\lambda} \frac{p^2 dp}{2\pi^2} P_{\rm L}(p) = \frac{d\sigma^2_\Lambda}{d\Lambda}\bigg|_\Lambda \lambda + \mathcal{O}(\lambda^2),$$



Results

Wilson-Polchinski RG-equations

$$\frac{db_\delta}{d\Lambda} = -\left[\frac{68}{21} b_{\delta^2} + 3 b^*_{\delta^3} - \frac{4}{3} b^*_{\mathcal{G}_2\delta}\right] \frac{d\sigma^2_\Lambda}{d\Lambda}\,,$$

$$\frac{db_{\delta^2}}{d\Lambda} = -\left[\frac{8126}{2205} b_{\delta^2} + \frac{17}{7} b^*_{\delta^3} - \frac{376}{105} b^*_{\mathcal{G}_2\delta} + \boxed{b^{*(\delta^2)}_{n=4}}\right] \frac{d\sigma^2_\Lambda}{d\Lambda}\,,$$

$$\frac{db_{\mathcal{G}_2}}{d\Lambda} = -\left[\frac{254}{2205} b_{\delta^2} + \frac{116}{105} b^*_{\mathcal{G}_2\delta} + \boxed{b^{*(\mathcal{G}_2)}_{n=4}}\right] \frac{d\sigma^2_\Lambda}{d\Lambda}\,.$$

We would need 4th-order terms to calculate the running of 2nd order bias

Comments:

- Linear ODE system;
- "n"-order bias parameter connected to "n+2"-order
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Solutions

Wilson-Polchinski RG-equations

$$\frac{db_\delta}{d\Lambda} = -\left[\frac{68}{21} b_{\delta^2} + 3 b^*_{\delta^3} - \frac{4}{3} b^*_{\mathcal{G}_2\delta}\right] \frac{d\sigma^2_\Lambda}{d\Lambda} \,,$$

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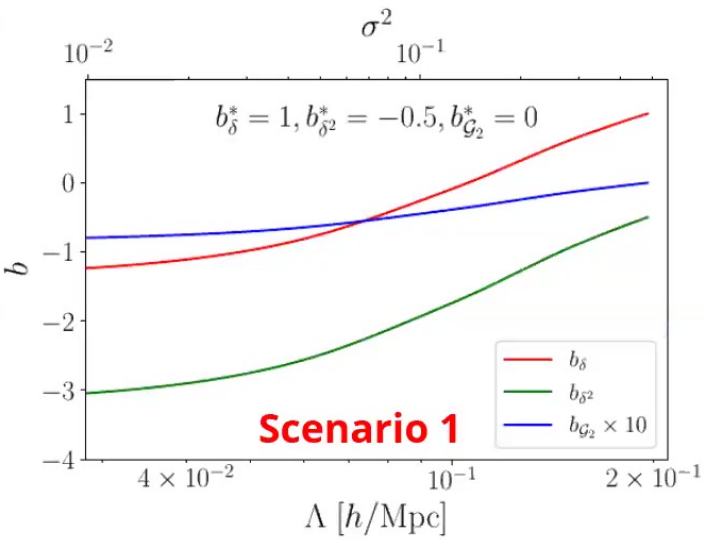
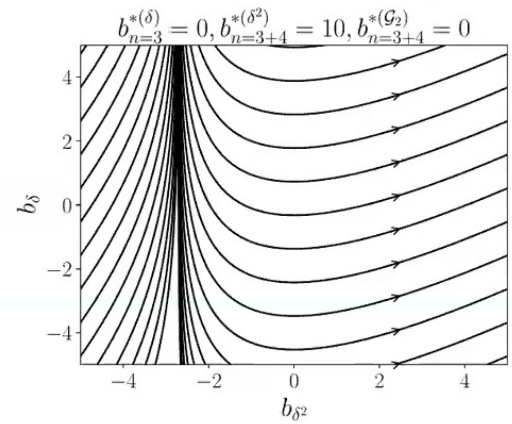
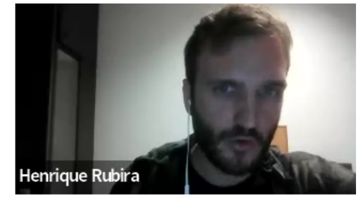


Notice that:

- **Bias parameter that are zero, may be sourced;**
- **Bias parameters may change sign!**



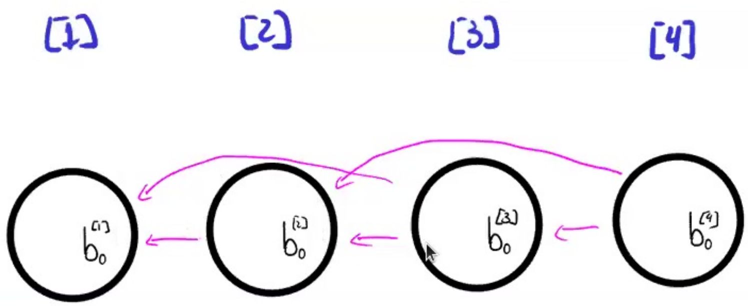
Diagrammatic understanding



$$\frac{db_\delta}{d\Lambda} = -\left[\frac{68}{21}b_{\delta^2} + 3b^*_{\delta^3} - \frac{4}{3}b^*_{\mathcal{G}_2\delta}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}\,,$$

$$\frac{db_{\delta^2}}{d\Lambda} = -\left[\frac{8126}{2205}b_{\delta^2} + \frac{17}{7}b^*_{\delta^3} - \frac{376}{105}b^*_{\mathcal{G}_2\delta} + b^{*(\delta^2)}_{n=4}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}\,,$$

$$\frac{db_{\mathcal{G}_2}}{d\Lambda} = -\left[\frac{254}{2205}b_{\delta^2} + \frac{116}{105}b^*_{\mathcal{G}_2\delta} + b^{*(\mathcal{G}_2)}_{n=4}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}\,.$$



J^1

Diagrammatic understanding

$$\frac{db_\delta}{d\Lambda} = -\left[\frac{68}{21}b_{\delta^2} + 3b^*_{\delta^3} - \frac{4}{3}b^*_{\mathcal{G}_2\delta}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}\,,$$

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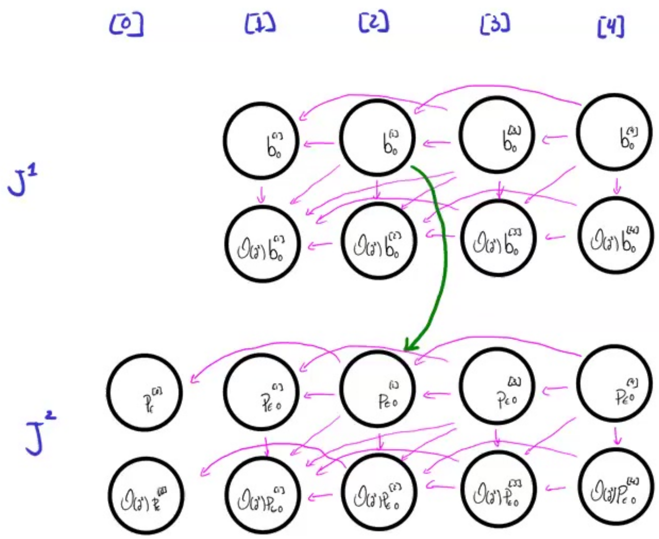
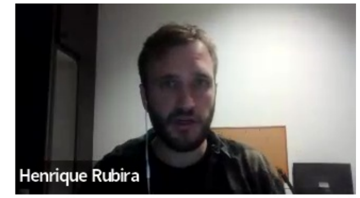
$$\frac{db_{\mathcal{G}_2}}{d\Lambda} = -\left[\frac{254}{2205}b_{\delta^2} + \frac{116}{105}b^*_{\mathcal{G}_2\delta} + b^{*(\mathcal{G}_2)}_{n=4}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}\,.$$



Coming soon
2312.xxxxx

How to relate the renormalization schemes?

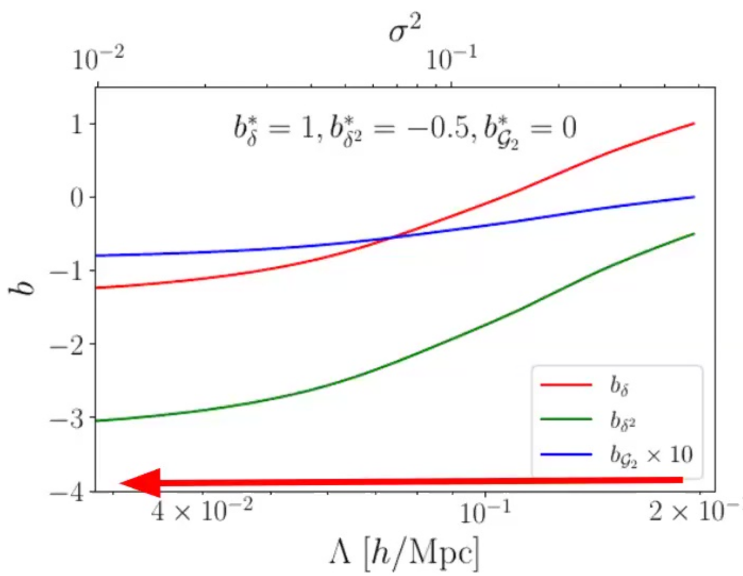


N-point function renormalized bias
(Assassi, Baumann, Green, Zaldarriaga)

Finite cutoff bias
(This work)

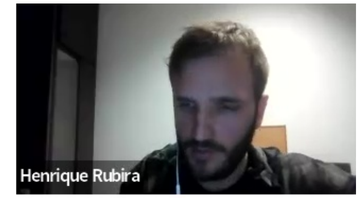
How to connect both?

$$[\![O']\!](\boldsymbol{k}') \longleftrightarrow O'[\delta_\Lambda^{(1)}](\boldsymbol{k}')$$



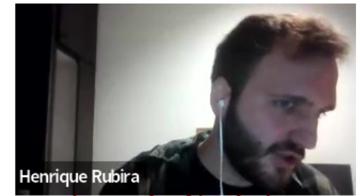
Solution: Run the bias towards

$$\Lambda \to 0$$



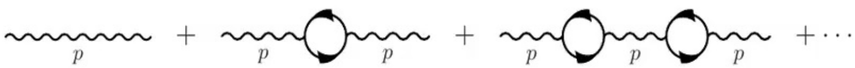
Final remarks

Logs in QFT



Logs in QFT: Arise when we have a hierarchy of scales $\lim_{E\to\infty} \Gamma(E,m) = E^d \Gamma(1, \frac{m}{E}) \times O\left[\ln\left(\frac{E}{m}\right)\right]$

Approach 1) Resum diagrams by hand
(if you can)



$$\tilde{V}(p^2) = \frac{e_R^2}{p^2}\left[1 + \frac{e_R^2}{12\pi^2}\ln\frac{p^2}{p_0^2} + \left(\frac{e_R^2}{12\pi^2}\ln\frac{p^2}{p_0^2}\right)^2 + \cdots\right] = \frac{1}{p^2}\left[\frac{e_R^2}{1 - \frac{e_R^2}{12\pi^2}\ln\frac{p^2}{p_0^2}}\right]$$

$$e_{\text{eff}}^2(p^2) = \frac{e_R^2}{1 - \frac{e_R^2}{12\pi^2}\ln\frac{p^2}{p_0^2}}$$

Approach 2) Direct from the RGE

$$p_0^2 \frac{d}{dp_0^2}\tilde{V}(p^2) = 0$$

$$p_0^2 \frac{de_{\text{eff}}}{dp_0^2} = \frac{e_{\text{eff}}^3}{24\pi^2}$$

$$e_{\text{eff}}^2(p^2) = \frac{e_R^2}{1 - \frac{e_R^2}{12\pi^2}\ln\frac{p^2}{p_0^2}}$$

Extracted from Schwartz's and Weinberg's books

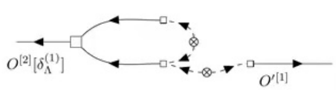
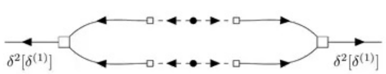
Logs in LSS

$$\Delta^2_{1-loop} = \left(\frac{k}{k_{NL}}\right)^{3+n} + \left(\frac{k}{k_{NL}}\right)^{2(3+n)} \left[\alpha(n) + \tilde{\alpha}(n) \ln\left(\frac{k}{k_{NL}}\right)\right]$$

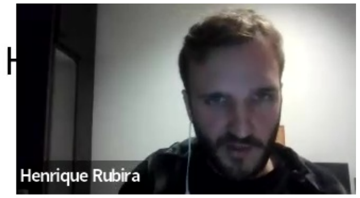
n	-2	-3/2	-1	-1/2	0	1/2	1	3/2	2	5/2	3
α_{13}	$\frac{5\pi^2}{112}$	$\frac{992\pi}{6{,}615}$	$\ldots$	$-\frac{416\pi}{8{,}085}$	$-\frac{\pi^2}{336}$	$\ldots$	$\ldots$	$\ldots$	$-\frac{\pi^2}{168}$	$\ldots$	$\ldots$
α_{22}	$\frac{75\pi^2}{784}$	-0.232	$\ldots$	.698	$\frac{29\pi^2}{784}$	$\ldots$	$\ldots$	$\ldots$	$\frac{\pi^2}{392}$	$\ldots$	$\ldots$
$\tilde{\alpha}_{13}$	0	0	$\frac{61}{315}$	0	0	0	$-\frac{4}{105}$	0	0	0	$\frac{20}{1{,}323}$
$\tilde{\alpha}_{22}$	0	0	0	0	0	$-\frac{9}{98}$	0	$\frac{31}{16{,}464}$	0	$-\frac{359}{26{,}880}$	0
α	1.38	.239	$\ldots$	.537	.336	$\ldots$	$\ldots$	$\ldots$	$-.0336$	$\ldots$	$\ldots$
$\tilde{\alpha}$	0	0	.194	0	0	$-.0918$	.0381	$-.00188$	0	$-.0134$	.0151

Pajer+Zaldarriaga, 2013

Differences between renormalization schemes

	N-point renormalization	Finite Λ
In practice, one has to:	Subtract all UV-dep part	Nothing to subtract. Bias params run... and we know how
Potential to:	Error-prone, missing finite contributions	Extra sanity-checks
$\langle(\delta)^{(1)}(\delta^2)^{(3)}\rangle$ 	Completely removed by c.t., but missing sub-leading $k^2 P_{\rm L}(k)\int_{\boldsymbol{p}} p^{-2} P_{\rm L}(p)$	Finite: $4 b_\delta^\Lambda b_{\delta^2}^\Lambda \int_p F_2(\boldsymbol{p}, \boldsymbol{k}-\boldsymbol{p}) P_{\rm L}^\Lambda(p) P_{\rm L}^\Lambda(k)$
$\langle(\delta^2)^{(2)}(\delta^2)^{(2)}\rangle$ 	Subtracted to the stochastic term, but missing sub-leading $k^2 \int_{\boldsymbol{p}} p^{-2}[P_{\rm L}(p)]^2$	Finite and contributes to the stochastic running: $2\,(b_{\delta^2}^\Lambda)^2 \int_{\boldsymbol{p}} P_{\rm L}^\Lambda(p) P_{\rm L}^\Lambda(\lvert\boldsymbol{k}-\boldsymbol{p}\rvert)$

Why you should care



- Additional cross-check for EFT inference;
- Systematic renormalization of bias and stochastic parameters;
- Completely absorb cutoff dependence in the counter-terms keeping also sub-leading contributions;
- Systematic renormalization of n-point functions. Self-consistent renormalization for P(k), B(k1,k2,k3), ...
- Priors for EFT analysis in $\Lambda \to 0$

Thanks a lot!