Title: Entanglement Renormalization Circuits for Chiral Topological Order

Speakers: Su-Kuan Chu

Series: Perimeter Institute Quantum Discussions

Date: December 06, 2023 - 3:30 PM

URL: https://pirsa.org/23120025

Abstract: Entanglement renormalization circuits are quantum circuits that can be used to prepare large-scale entangled states. For years, it has remained a mystery whether there exist scale-invariant entanglement renormalization circuits for chiral topological order. In this paper, we solve this problem by demonstrating entanglement renormalization circuits for a wide class of chiral topologically ordered states, including a state sharing the same topological properties as Laughlin's bosonic fractional quantum Hall state at filling fraction 1/4 and eight states with Ising-like non-Abelian fusion rules. The key idea is to build entanglement renormalization circuits by interleaving the conventional multi-scale entanglement renormalization ansatz (MERA) circuit (made of spatially local gates) with quasi-local evolution. Given the miraculous power of this circuit to prepare a wide range of chiral topologically ordered states, we refer to these circuits as MERA with quasi-local evolution (MERAQLE).\

Zoom link https://pitp.zoom.us/j/98523529456?pwd=dUlNbUQyemZGNFlCUGFJNStPU0xxdz09

Entanglement Renormalization Circuits for Chiral Topological Order

Su-Kuan Chu (朱書寬)

University of Maryland, College Park

arXiv:2304.13748 [quant-ph]

Guanyu Zhu
(IBM)

Alexey Gorshkov
(UMD)

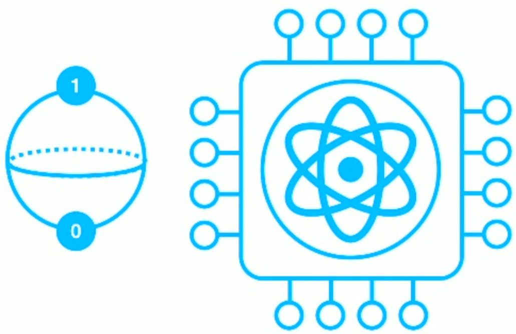
1

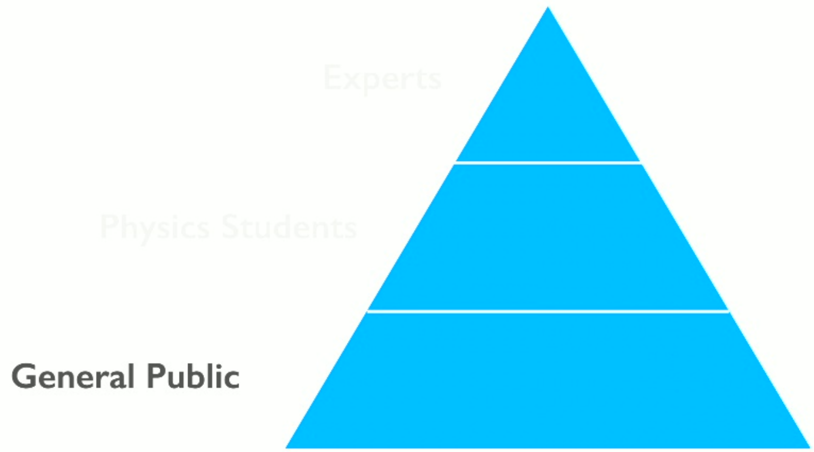
JOINT CENTER FOR QUANTUM INFORMATION AND COMPUTER SCIENCE

JOINT QUANTUM INSTITUTE

Take-home message

We solved a mathematical fun problem related to quantum computing!

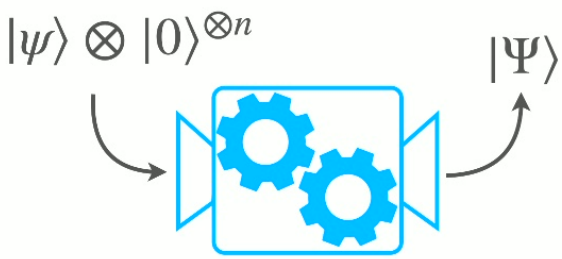


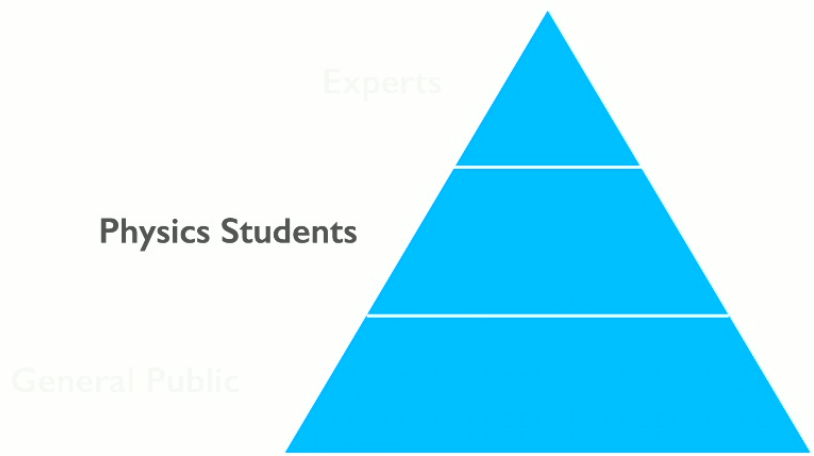


2

Take-home message

We developed a procedure to generate some **highly entangled quantum states** from some initially less entangled states!

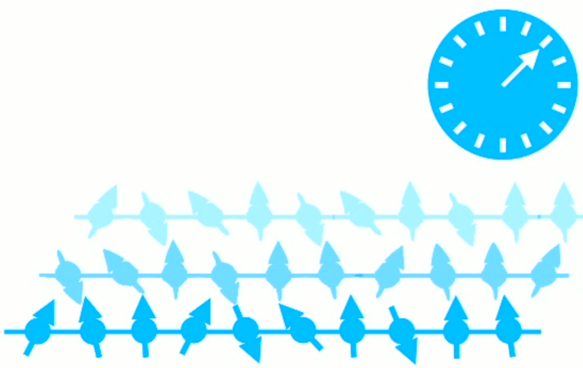




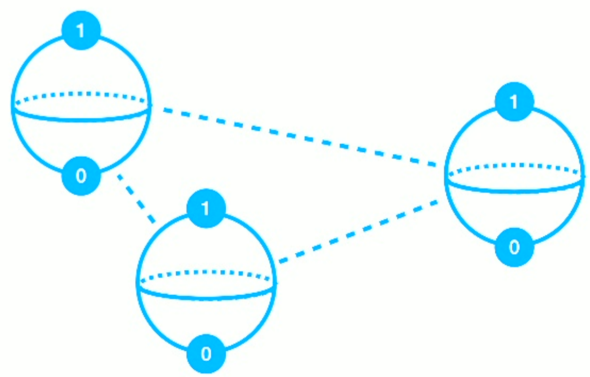
Take-Home Message

called MERA with quasi-local evolution (MERAQLE)

We constructed entanglement renormalization circuits for a class of chiral spin liquids.



It deepens our understanding of the quantum complexity and entanglement structure of quantum many-body states.



Outline

Introduction to entanglement renormalization circuits

- MERA circuit for the toric code

Chiral Topological Order

- Challenges for chiral topological order
- An entanglement renormalization circuit for the p_x+ip_y topological superconductor

Kitaev's 16-fold way chiral spin liquids

- Construction via bosonization
- Entanglement renormalization circuits (MERAQLE circuits)

Conclusions and outlook

Outline

Introduction to entanglement renormalization circuits

- MERA circuit for the toric code

Chiral Topological Order

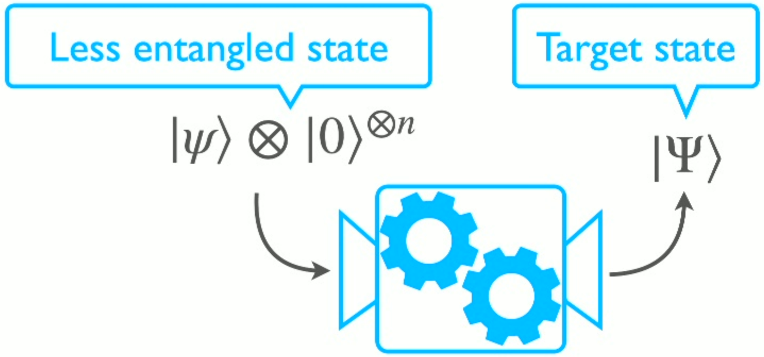
- Challenges for chiral topological order
- An entanglement renormalization circuit for the p_x+ip_y topological superconductor

Kitaev's 16-fold way chiral spin liquids

- Construction via bosonization
- Entanglement renormalization circuits (MERAQLE circuits)

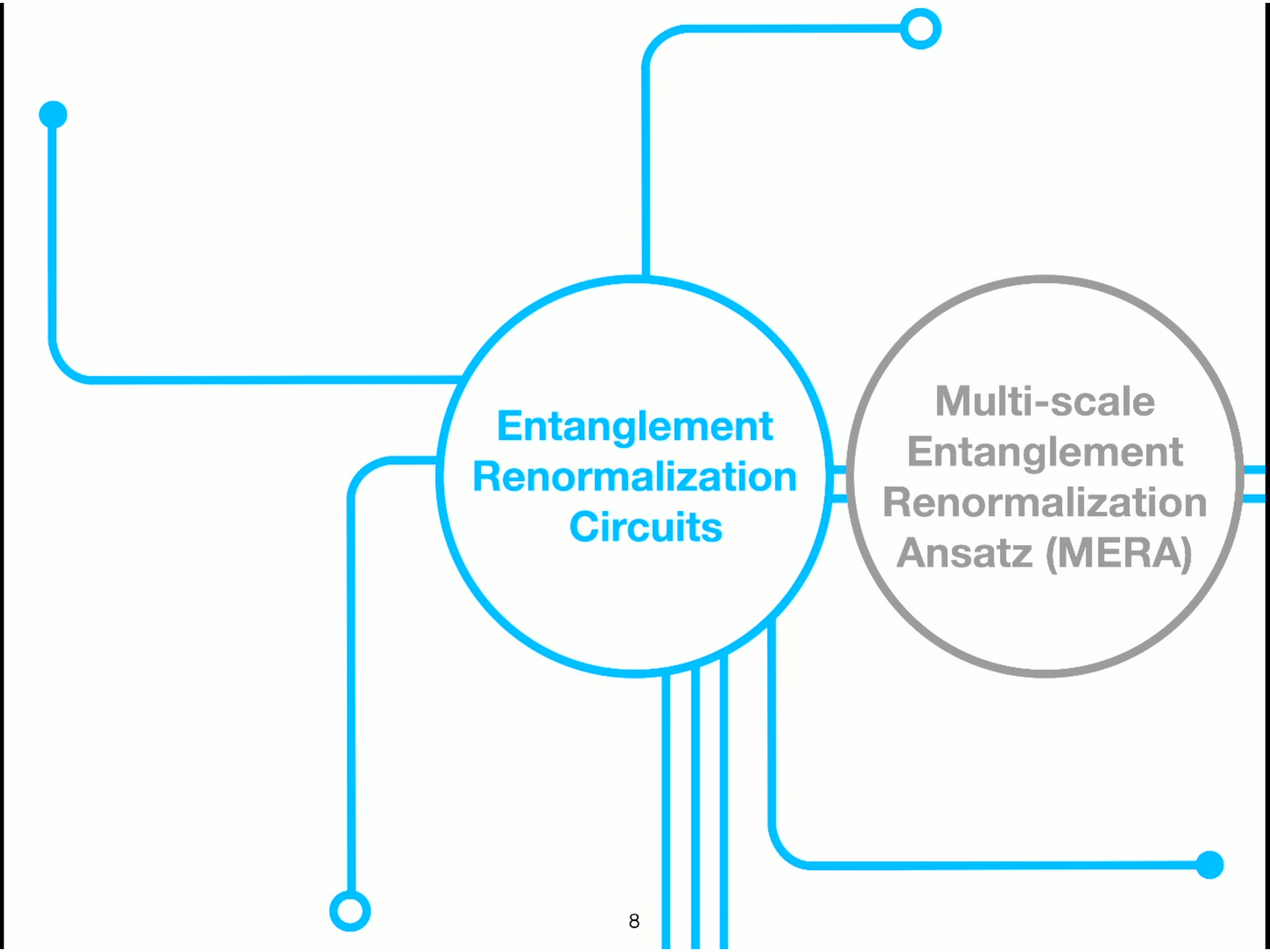
Conclusions and outlook

Entanglement Structure of Quantum Many-Body Systems



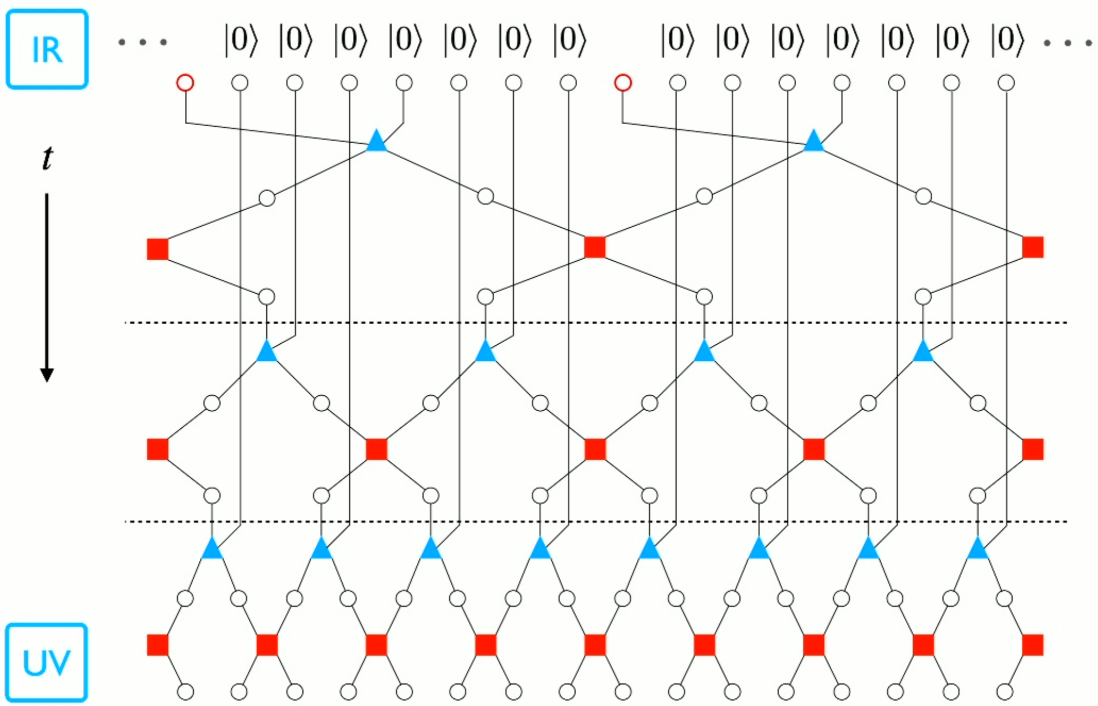
Operational Definition

Quantum Complexity


Entanglement Renormalization Circuits
Multi-scale Entanglement Renormalization Ansatz (MERA)

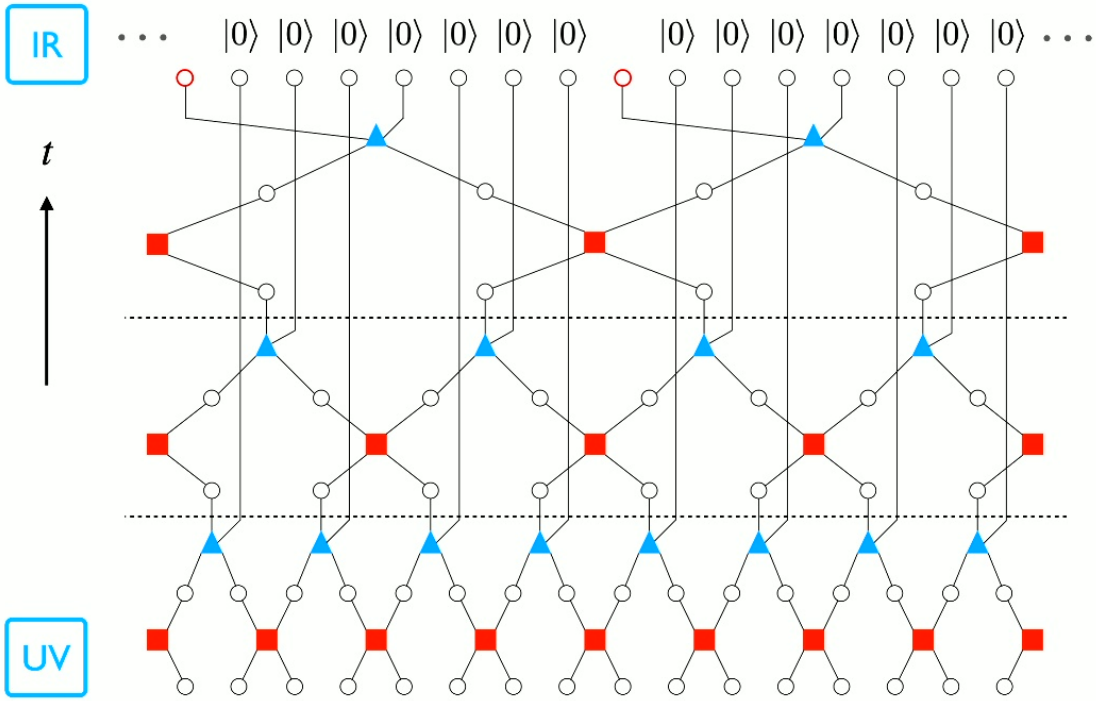
8

1-D MERA


IR
UV
t
$|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$

9

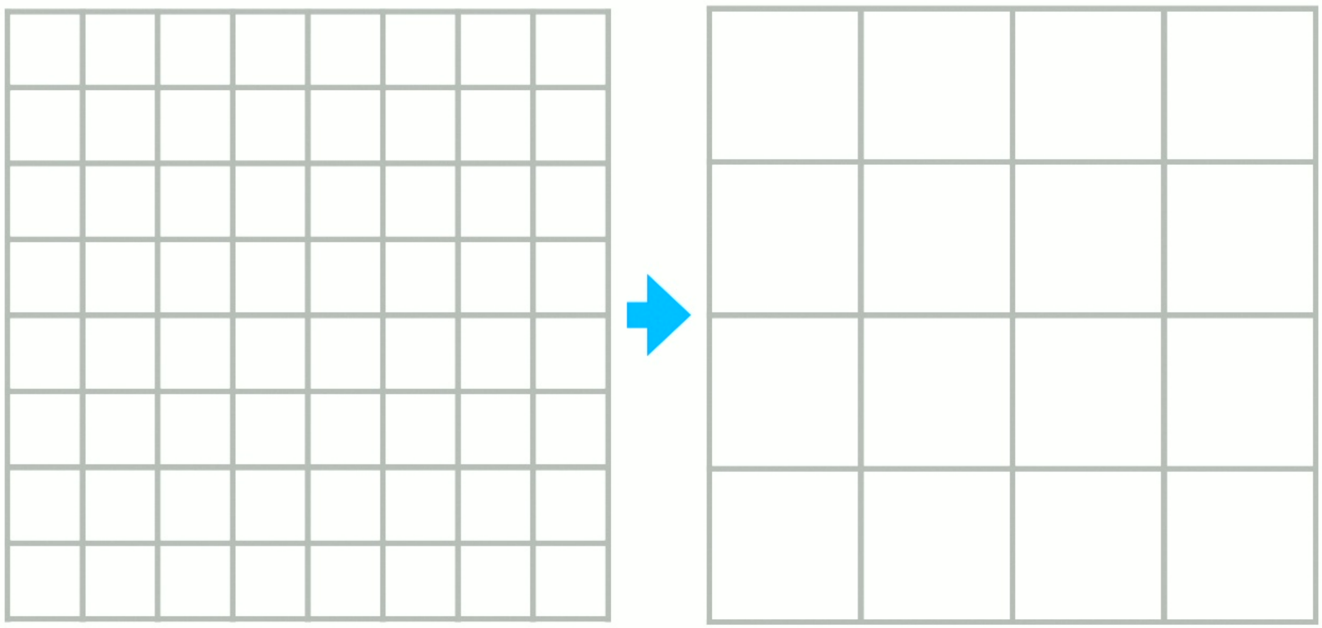
1-D MERA



Strictly local gates with respect to the coarse-grained lattice

Translation invariance

2-D MERA



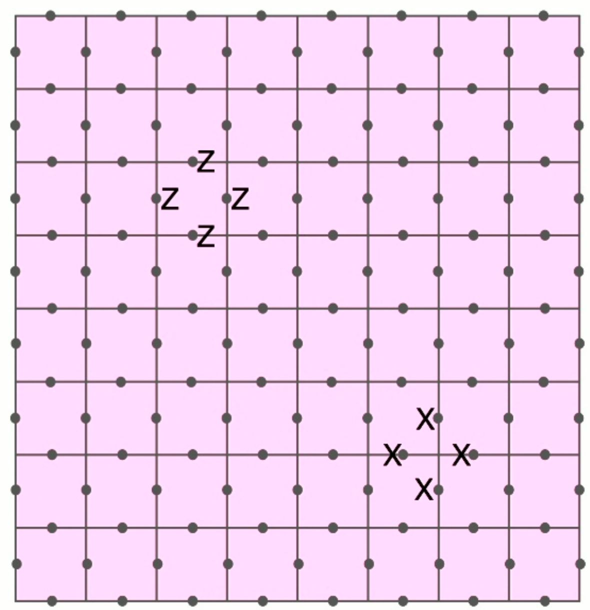
10

Toric Code

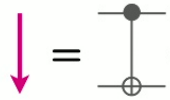
Quantum error-correcting code

Topological order

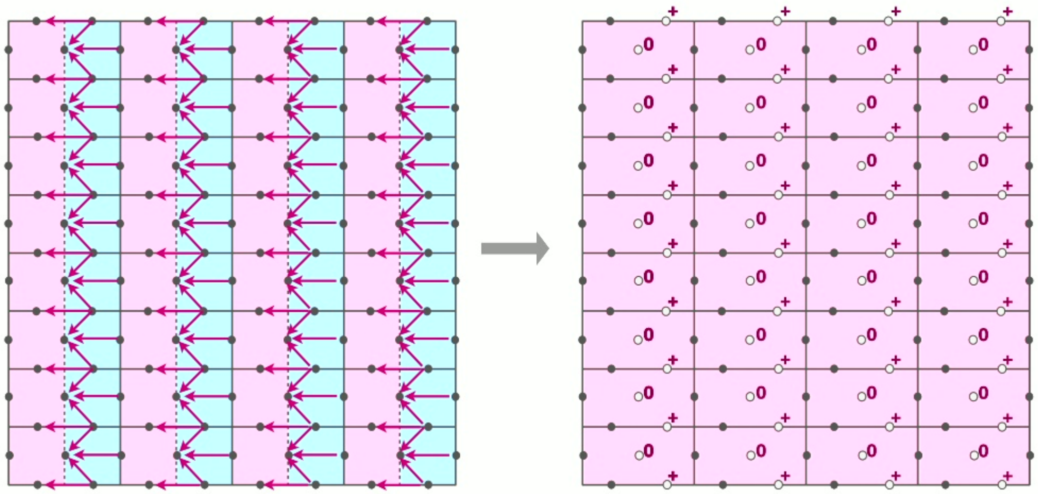
Interpretation of a Z_2 lattice gauge theory



MERA for the Toric Code



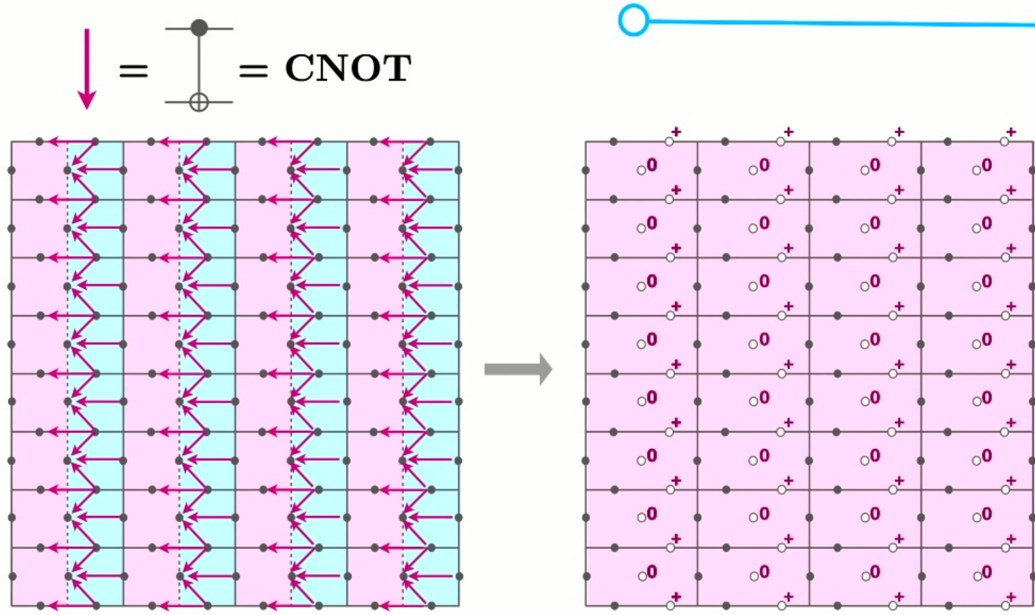
= **CNOT**



$$\mathcal{C}_{Z_2,x} = \prod_{\{c,t\}} \mathrm{CNOT}_{c\to t}$$

(M. Aguado and G. Vidal 2005,
SKC, G. Zhu, and A. Gorshkov 2023)

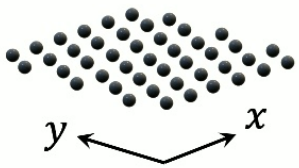
MERA for the Toric Code



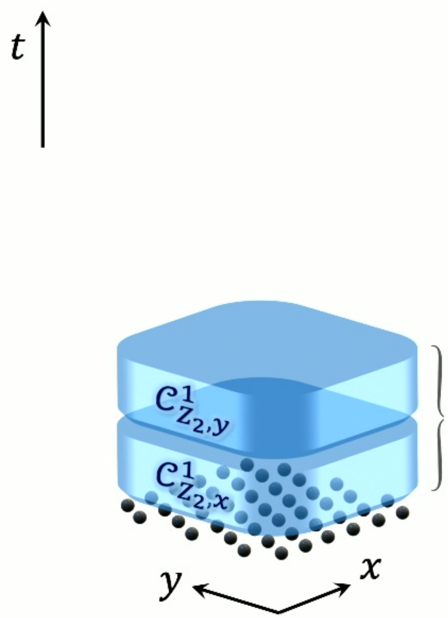
$$\mathcal{C}_{Z_2,x} = \prod_{\{c,t\}} \mathrm{CNOT}_{c\to t}$$

Z_2 lattice gauge theory

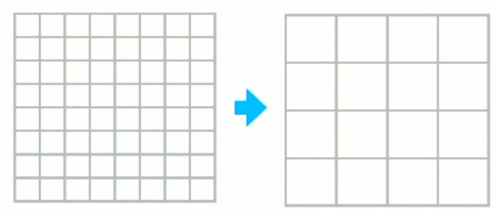
(M. Aguado and G. Vidal 2005,
SKC, G. Zhu, and A. Gorshkov 2023)



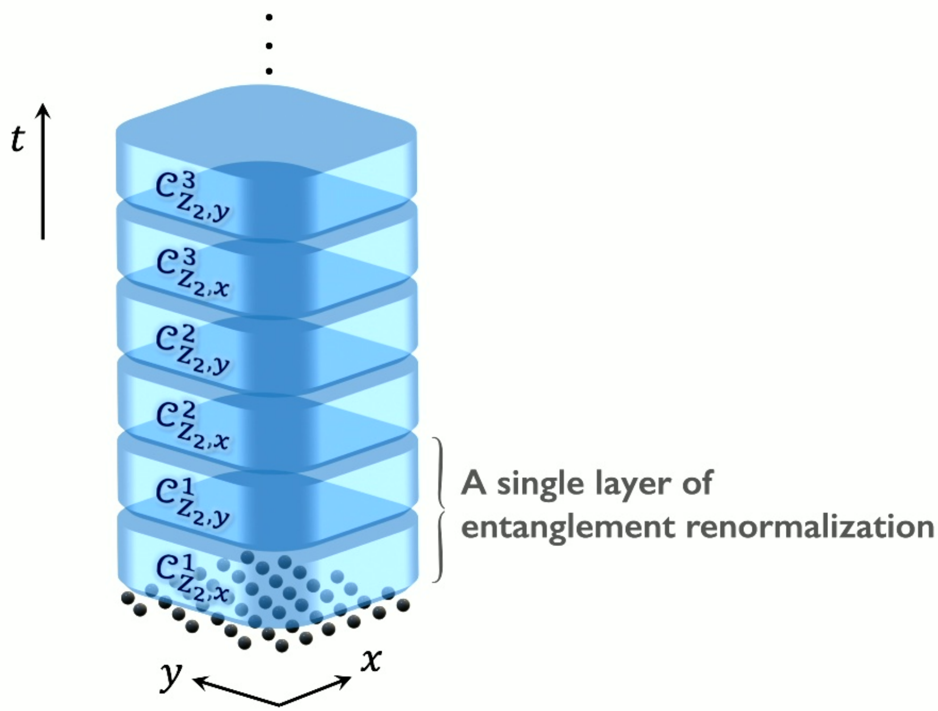
y
x

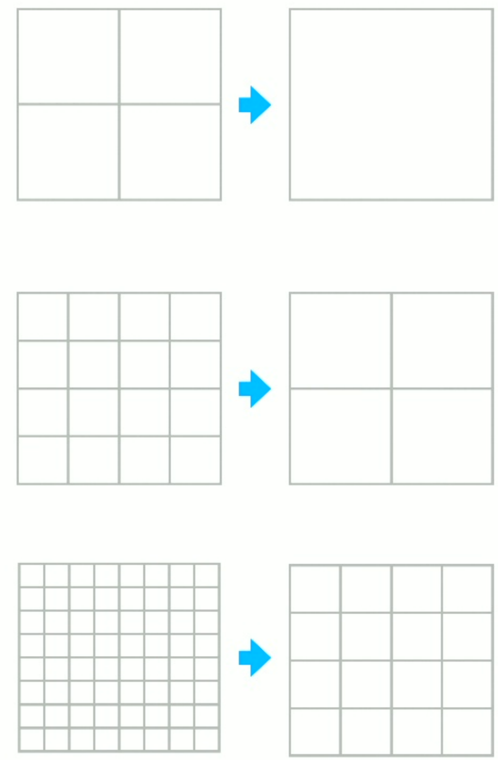


A single layer of entanglement renormalization



MERA for the Toric Code





Short Summary

We have introduced the notion of entanglement renormalization circuits, which are quantum circuits that can be used to prepare large-scale entangled states.

Short Summary

We have introduced the notion of entanglement renormalization circuits, which are quantum circuits that can be used to prepare large-scale entangled states.

MERA circuits are the earliest realization of entanglement renormalization circuits. It is often convenient to have a scale-invariant and translation-invariant MERA circuit.

We have presented a MERA circuit for the toric code model.

Short Summary

We have introduced the notion of entanglement renormalization circuits, which are quantum circuits that can be used to prepare large-scale entangled states.

MERA circuits are the earliest realization of entanglement renormalization circuits. It is often convenient to have a scale-invariant and translation-invariant MERA circuit.

We have presented a MERA circuit for the toric code model.

Outline

Introduction to entanglement renormalization circuits

- MERA circuit for the toric code

Chiral Topological Order

- Challenges for chiral topological order
- An entanglement renormalization circuit for the p_x+ip_y topological superconductor

Kitaev's 16-fold way chiral spin liquids

- Construction via gauging p_x+ip_y topological superconductors
- Entanglement renormalization circuits (MERAQLE circuits)

Conclusions and outlook

Big Question

18

Big Question

It is also known how to construct scale-invariant MERA circuits (with strictly local gates) for the Levin-Wen models (string-net models). The Levin-Wen models include the toric code model.

Big Question

It is also known how to construct scale-invariant MERA circuits (with strictly local gates) for the Levin-Wen models (string-net models). The Levin-Wen models include the toric code model.

However, all of these models are non-chiral.

Can we find scale-invariant MERA circuits (with strictly local gates) for chiral topologically ordered systems? 🧐

Chiral Topological Order

Chiral edge modes on the boundary.

Does not preserve time-reversal symmetry.

Examples:

- Integer quantum Hall states
- Fractional quantum Hall states
- The p_x+ip_y topological superconductor
- The Ising TQFT
- Chiral spin liquids

Challenges for Chiral Topological Order

Can we find scale-invariant MERA circuits (with strictly local gates) for chiral topologically ordered systems?

It's hard! 🥲

Correlation-Length-Reduction No-Go Argument

After each layer of entanglement renormalization,

$$\ell' = \ell / b$$

Factor $b > 1$

New correlation length with respect to the new coarse-grained lattice

Old correlation length with respect to the old coarse-grained lattice

(SKC, G. Zhu, and A. Gorshkov 2023)

Correlation-Length-Reduction No-Go Argument

$|\Psi'\rangle = |\Psi\rangle$

In the fixed-point scenario, the correlation length reduction formula $\ell' = \ell/b$ implies that

$$\ell = \ell/b$$
$$\Rightarrow \ell = 0 \text{ or } \ell = \infty$$

Gapless

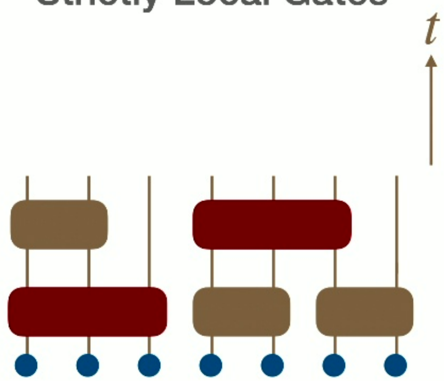
Question

Can we go beyond the conventional MERA framework (composed of strictly local and discrete gates) to develop entanglement renormalization circuits for chiral topological order?

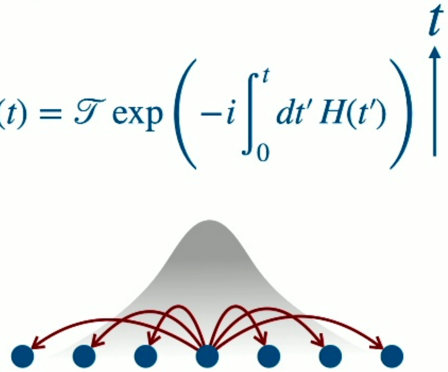
Consider quasi-local evolution!

Strictly Local Gates

Quasi-Local Evolution



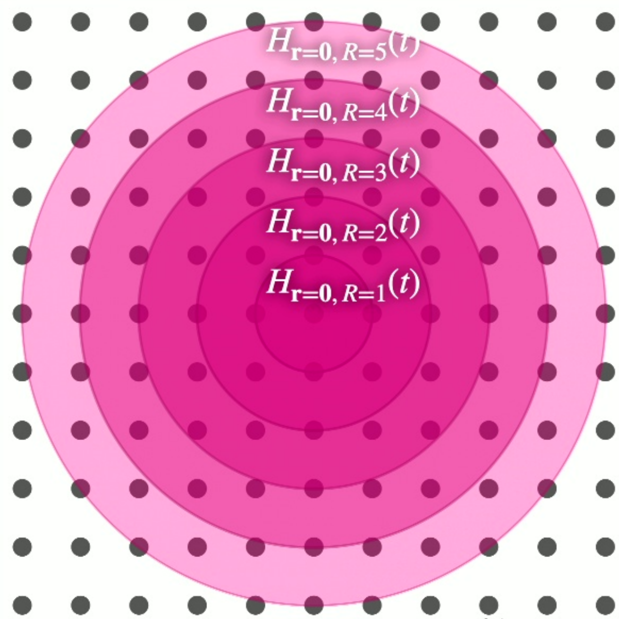
$$U(t) = \mathcal{T} \exp\left(-i\int_0^t dt'\, H(t')\right)$$



$$H(t) = \sum_{\mathbf{r}} H_{\mathbf{r}}(t)$$

$$H_{\mathbf{r}}(t) = \sum_R H_{\mathbf{r},R}(t)$$

$$\|H_{\mathbf{r},R}(t)\| = o\left(1/R^{\alpha}\right), \ \forall \alpha > 0$$

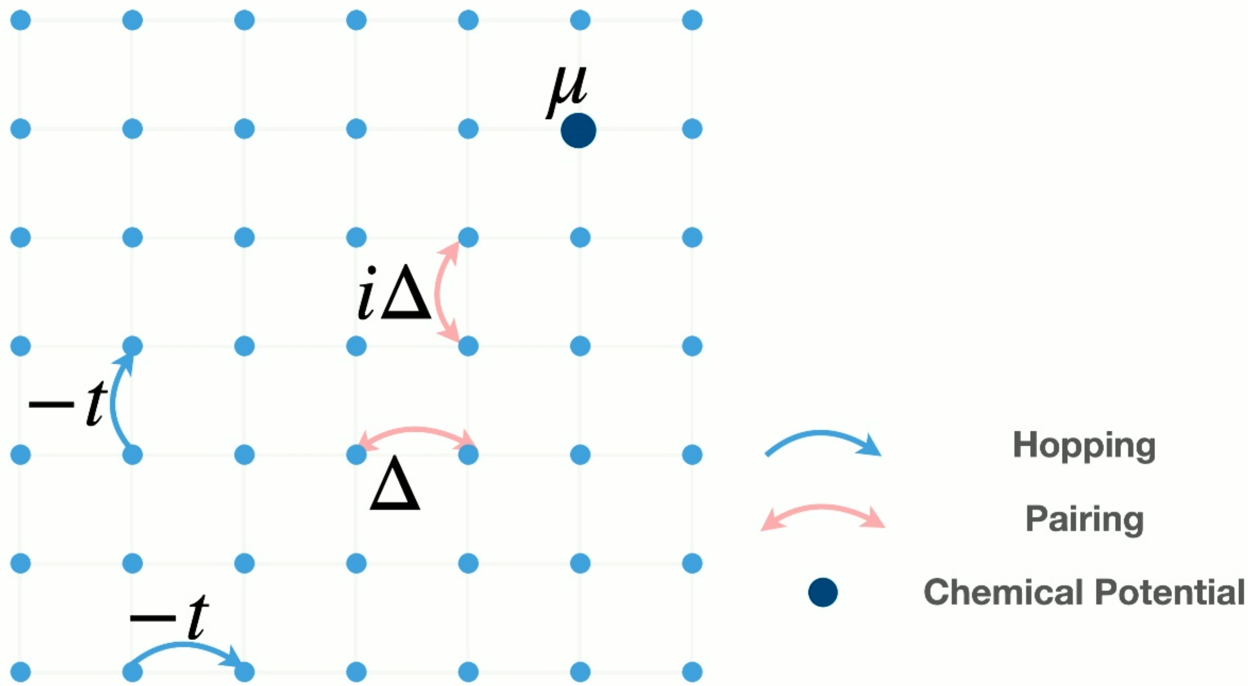


$$H_{\mathbf{r}=0}(t) = \sum_R H_{\mathbf{r}=0,R}(t)$$

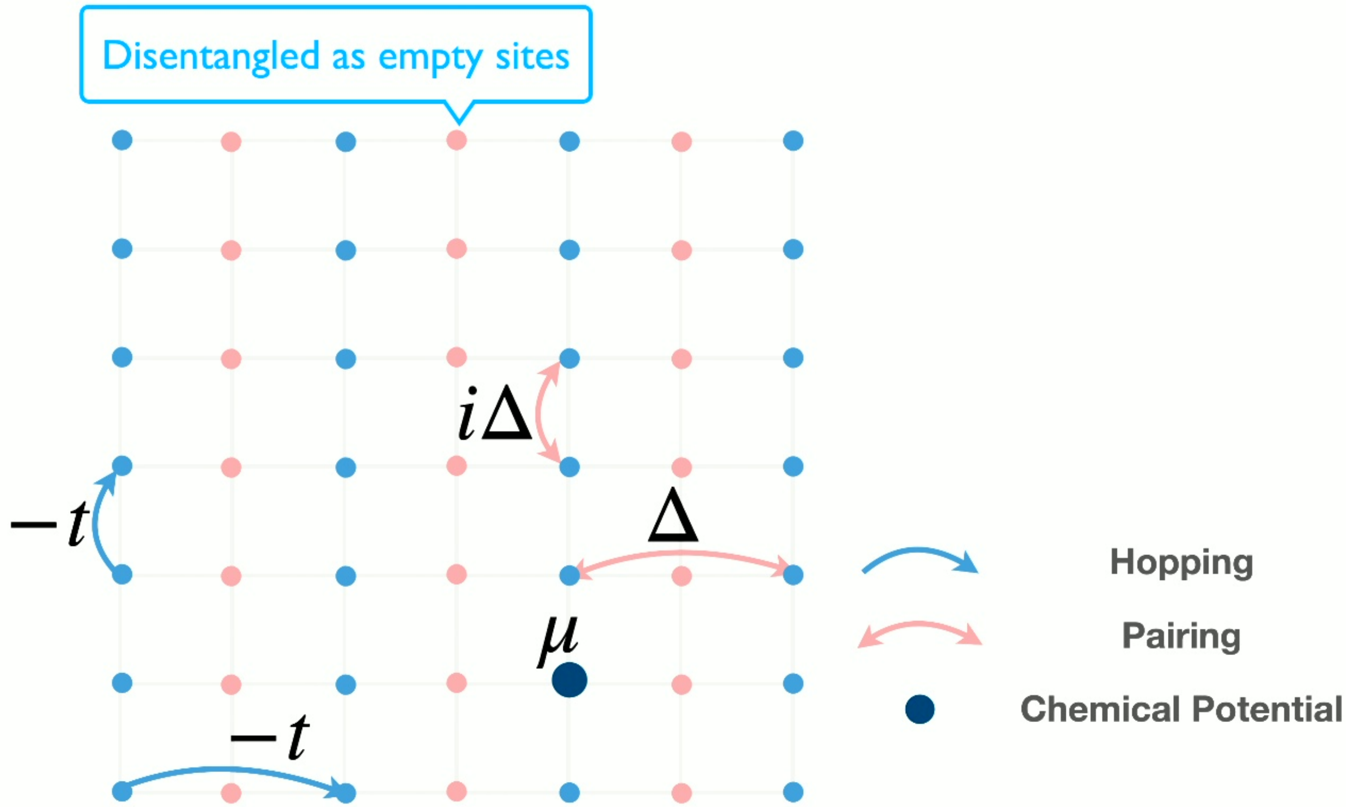
24

The p_x+ip_y Topological Superconductor

(B. Swingle and J. McGreevy 2016,
SKC, G. Zhu, and A. Gorshkov 2023)

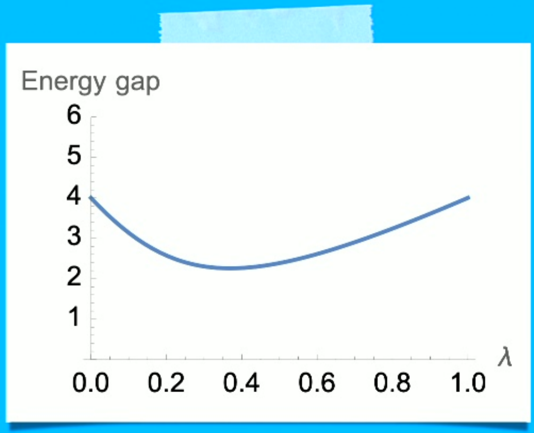


$$H = -t\sum_r \left(c^\dagger_{r+x}c_r + c^\dagger_{r+y}c_r + \text{h.c.}\right) - \mu\sum_r c^\dagger_r c_r$$
$$+\sum_r \left(\Delta c^\dagger_{r+x}c^\dagger_r + i\Delta c^\dagger_{r+y}c^\dagger_r + \text{h.c.}\right) \qquad -4t < \mu < 0$$


Disentangled as empty sites
$i\Delta$
$-t$
Δ
μ
$-t$
Hopping
Pairing
Chemical Potential

$$H(\lambda) = (1 - \lambda)\, H_{\text{initial}} + \lambda\, H_{\text{final}}$$

MIRACLE!



28

Quasi-Adiabatic Evolution

However, exact adiabatic evolution takes an infinite amount of time.

We can convert the adiabatic path to a finite-time quasi-adiabatic evolution, which is a quasi-local evolution.

(M. Hastings 2003, T. J. Osborne 2007)

Quasi-Adiabatic Evolution

$$\mathcal{U}_{\rm qa} = \mathcal{T} \exp\left(i \int_0^1 d\lambda\, \mathcal{D}(\lambda) \right)$$

Quasi-adiabatic continuation operator

$$\mathcal{D}(\lambda) = -i \int_{-\infty}^{\infty} dt\, F(E_{\rm gap} t)\, e^{iH(\lambda)t} \partial_\lambda H(\lambda) e^{-iH(\lambda)t}$$

Can be proved to be quasi-local evolution

$$\mathcal{D}(t) = \sum_{\mathbf{r}} \mathcal{D}_{\mathbf{r}}(t)$$

$$\mathcal{D}_{\mathbf{r}}(t) = \sum_{R} \mathcal{D}_{\mathbf{r},R}(t)$$

$$\|\mathcal{D}_{\mathbf{r},R}(t)\| = o\left(1/R^{\alpha}\right), \ \forall \alpha > 0$$

Short Summary

We introduced the notion of chiral topological order.

$$\ell' = \ell / b$$

We presented the correlation-length-reduction no-go argument to show why constructing conventional MERA circuits (with strictly local gates) for chiral topological order is hard.

However, we can construct an entanglement renormalization circuit for the non-interacting p_x+ip_y topological superconductor using quasi-local evolution. The quasi-local evolution helps us circumvent the correlation-length-reduction no-go argument.

Outline

Introduction to entanglement renormalization circuits

- MERA circuit for the toric code

Chiral Topological Order

  - Challenges for chiral topological order
  - An entanglement renormalization circuit for the p_x+ip_y topological superconductor

Kitaev's 16-fold way chiral spin liquids

- Construction via bosonization
- Entanglement renormalization circuits (MERAQLE circuits)

Conclusions and outlook

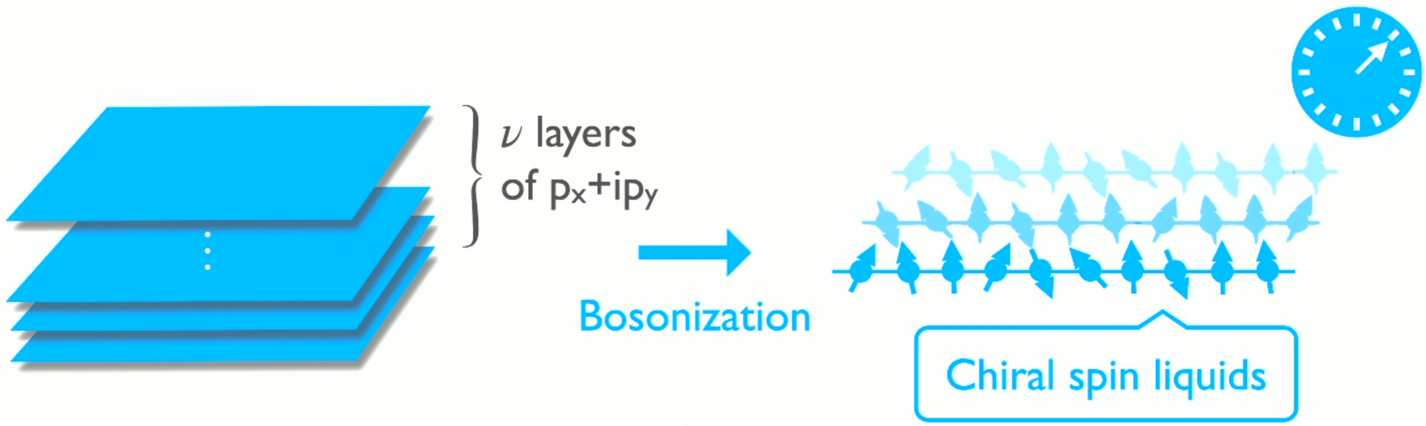
Big Question

Building entanglement renormalization circuits for interacting chiral states remains a largely unexplored area.

Now, we will use the insights we gained from previous examples to construct entanglement renormalization circuits for a class of chiral spin liquids. The circuits are called MERA with quasi-local evolution (MERAQLE).

The chiral spin liquids we are considering here are exactly solvable chiral spin liquids that we will construct through bosonizing ν layers of p_x+ip_y topological superconductors.

(Y.A. Chen, A. Kapustin, and D. Radicevic 2018)

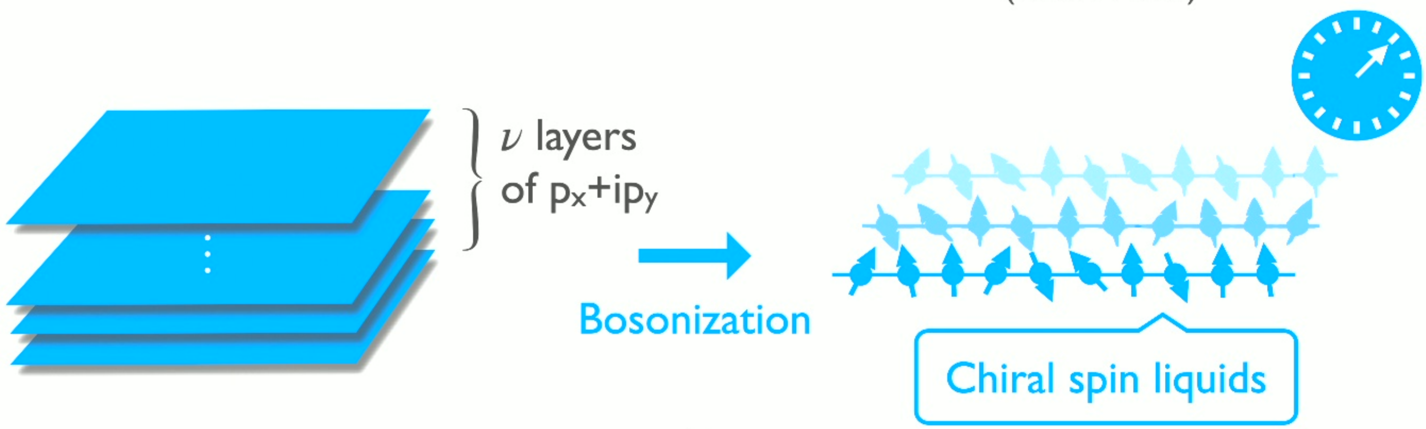


The chiral spin liquids we are considering here are exactly solvable chiral spin liquids that we will construct through bosonizing ν layers of p_x+ip_y topological superconductors.

(Y.A. Chen, A. Kapustin, and D. Radicevic 2018)

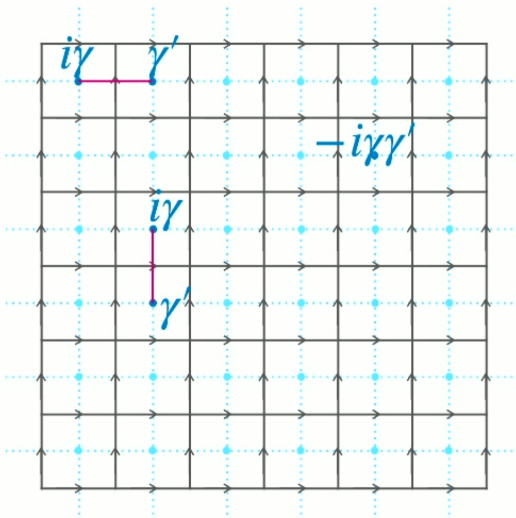
The resulting spin liquids can be thought of as layers of p_x+ip_y topological superconductors coupled to a Z_2 lattice gauge theory. It can be shown that our construction falls into Kitaev's 16-fold way classification.

(Kitaev 2005)

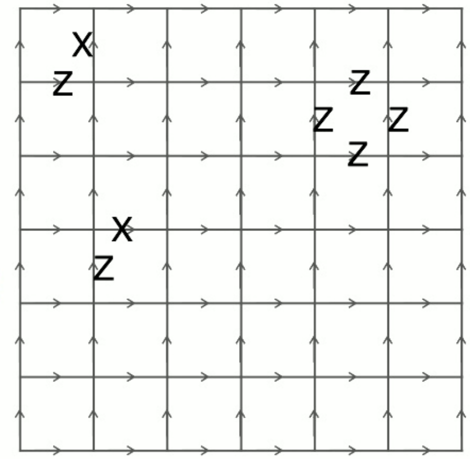


Fermion System

Spin-1/2 System



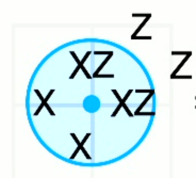
Bosonization



Majorana operators

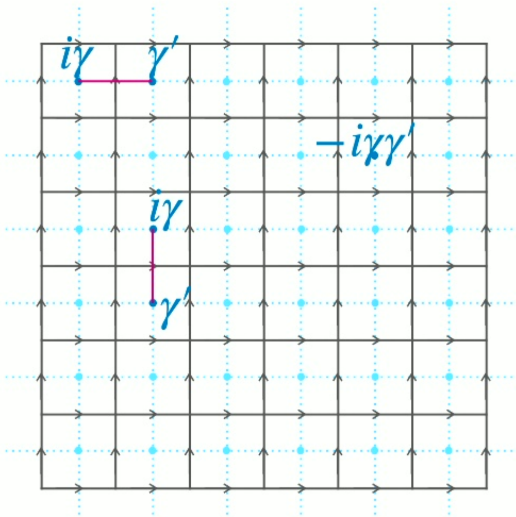
$c_f = (\gamma_f + i\gamma'_f)/2 \quad c_f^\dagger = (\gamma_f - i\gamma'_f)/2$

$\{\gamma_f, \gamma_{f'}\} = 2\delta_{f,f'} \quad \{\gamma_f, \gamma'_{f'}\} = 0$

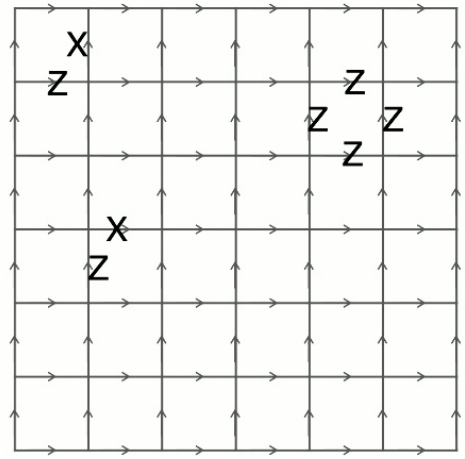
Zero-Flux Constraint: $F_v \equiv$  $= 1$

Fermion System

Spin-1/2 System



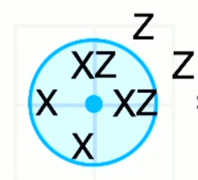
Bosonization



Majorana operators

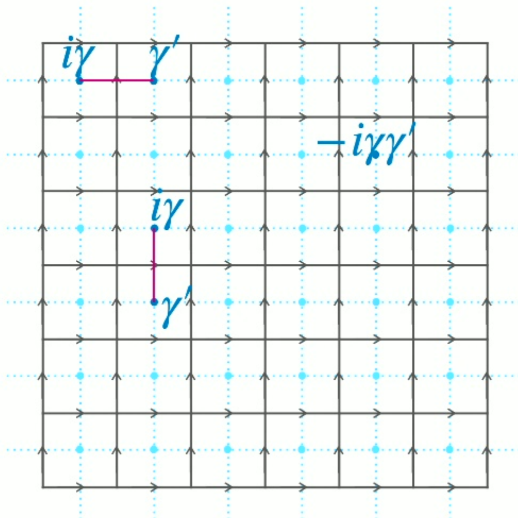
$c_f = (\gamma_f + i\gamma_f')/2 \quad c_f^\dagger = (\gamma_f - i\gamma_f')/2$

$\{\gamma_f, \gamma_{f'}\} = 2\delta_{f,f'} \quad \{\gamma_f, \gamma_{f'}'\} = 0$

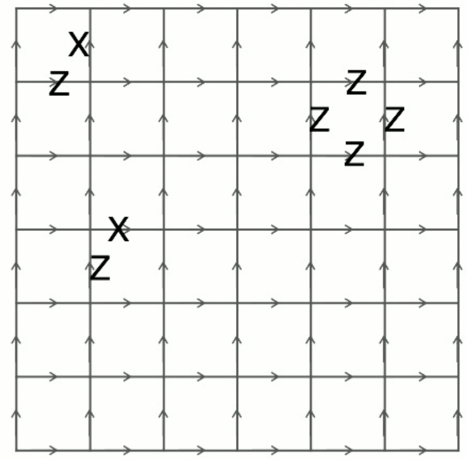
Zero-Flux Constraint: $F_v \equiv$  $= 1$

Violations ⇒Z_2 gauge fluxes ⇒vortices

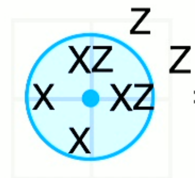
35



Bosonization

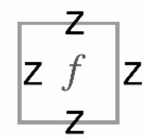


$\nu = 0$

Zero-Flux Constraint: $F_v \equiv$  $= 1$

$$H_{\text{insulator}} = -\sum_{\mathbf{r}} (1 - 2c_{\mathbf{r}}^{\dagger} c_{\mathbf{r}})$$

$$\{H_{\text{insulator}}\}^{\text{bosonized}} = -\sum_{f} \prod_{e \in f} Z_e$$

$$= -\sum_{f}$$ 

Ground state is the toric code ground state

36

The $\nu = 1$ chiral spin liquid described by the Ising TQFT

(SKC, G. Zhu, and A. Gorshkov 2023)

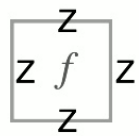
$$H_{\text{Ising CSL}} = \{H_{p_x+ip_y}\}^{\text{bosonized}}$$

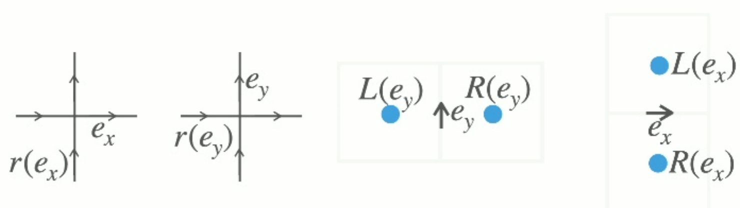
$$= \sum_{e_y} \left[-\left(\frac{t+\Delta}{2}\right) (W_{L(e_y)})(X_{e_y} Z_{r(e_y)})(W_{R(e_y)}) - \left(\frac{t-\Delta}{2}\right) X_{e_y} Z_{r(e_y)} \right]$$

$$+ \sum_{e_x} \left[-\frac{t}{2} (W_{L(e_x)})(X_{e_x} Z_{r(e_x)})(W_{R(e_x)}) - \frac{t}{2} (X_{e_x} Z_{r(e_x)}) \right.$$

$$\left. + i\frac{\Delta}{2}(W_{L(e_x)})(X_{e_x} Z_{r(e_x)}) - \frac{i\Delta}{2}(X_{e_x} Z_{r(e_x)})(W_{R(e_x)}) \right] - \mu \sum_f \left(1 - W_f\right).$$

$$W_f \equiv \prod_{e \in f} Z_e \equiv$$





Kitaev's 16-fold Way Classification

(Kitaev 2005)

By bosonizing different layers of p_x+ip_y superconductors, we can get many kinds of chiral topological orders.

However, it turns out that we can only get 16 different kinds of topological orders from the perspective of the topological properties of the anyons, which is determined by ν (mod 16).

ν odd: non-Abelian
ν even: Abelian

$\nu = 0$: Toric Code
$\nu = 1$: Ising TQFT (with Ising anyons)
$\nu = 2$: Laughlin's FQHE state at filling fraction 1/4
$\nu = 3$: Bosonic Moore-Read FQHE state at filling fraction one

⋮

Question

Can we find scale-invariant entanglement renormalization circuits such that our chiral spin liquids are the fixed-point wavefunctions?

Question

Can we find scale-invariant entanglement renormalization circuits such that our chiral spin liquids are the fixed-point wavefunctions? 🧐

Can we circumvent the correlation-length-reduction no-go argument?

MERAQLE

The entanglement renormalization circuits are MERA circuits with quasi-local evolution (MERAQLE). The very high-level idea is that:

MERAQLE circuit

Quasi-local evolution

=

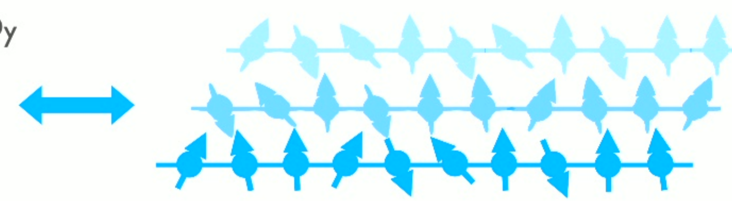
Bosonization of the quasi-adiabatic evolutions for p_x+ip_y superconductors

+

MERA circuit for the toric code

Z_2 lattice gauge theory

ν layers of p_x+ip_y



+ the Z_2 lattice gauge theory

40

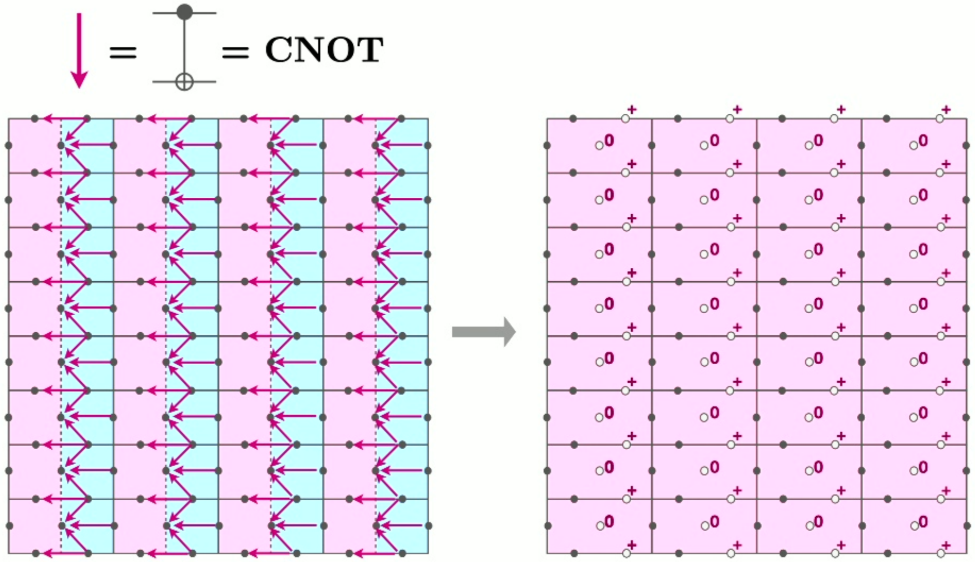
MERAQLE for the $\nu = 0$ Spin Liquid

Ground state is the toric code ground state

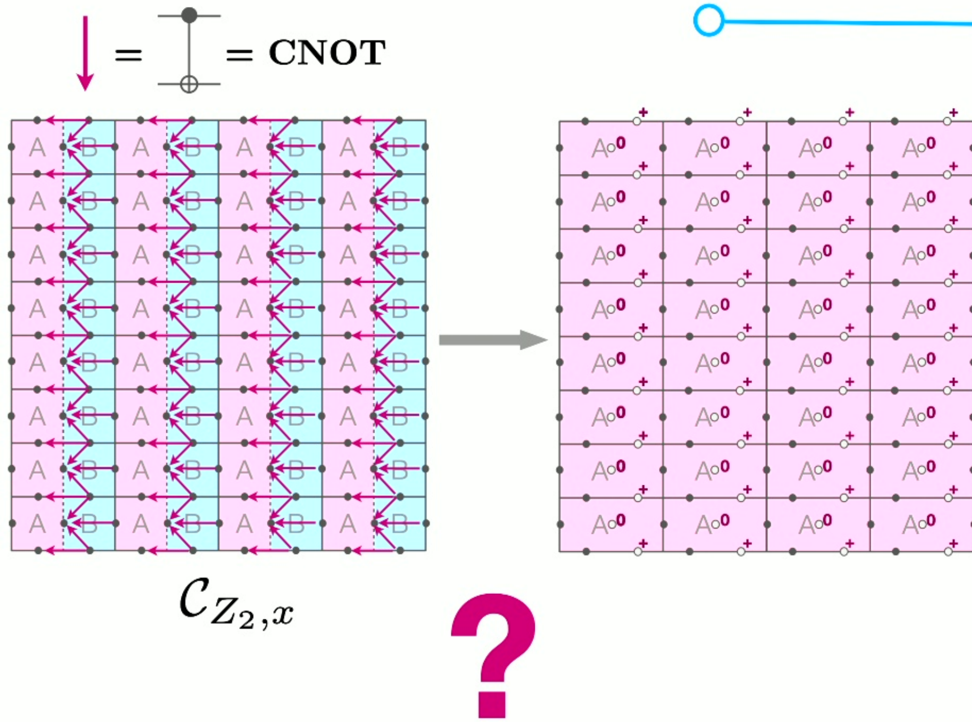
MERAQLE for the $\nu = 0$ Spin Liquid

Ground state is the toric code ground state

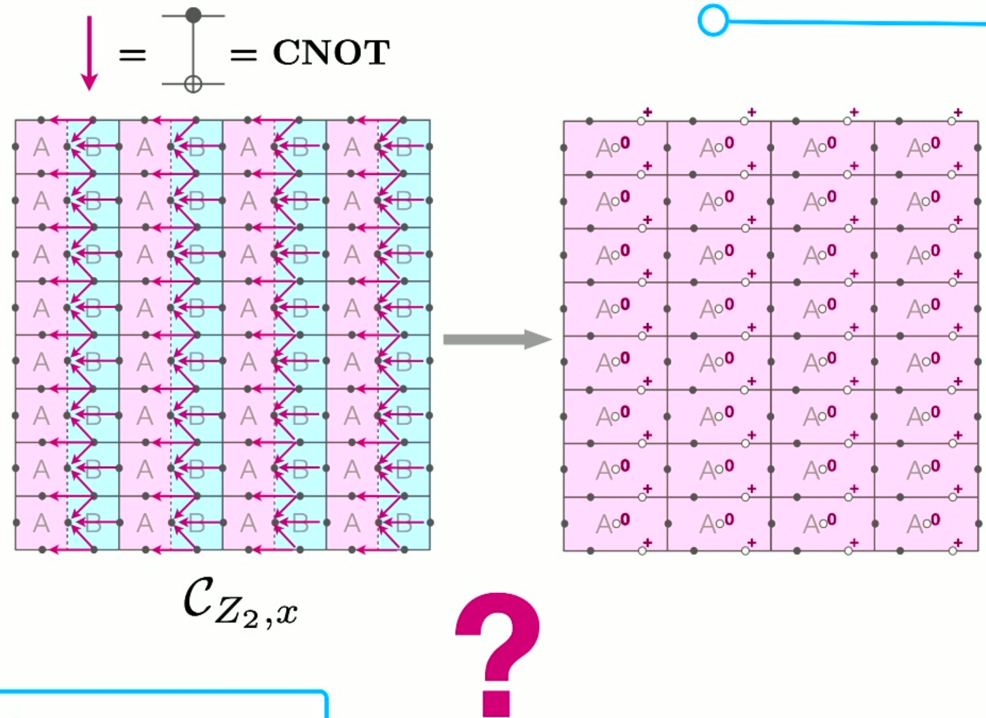
$$\mathcal{C}_{Z_2,x} = \prod_{\{c,t\}} \mathrm{CNOT}_{c \to t}$$



MERAQLE for the $\nu = 1$ Ising Chiral Spin Liquid

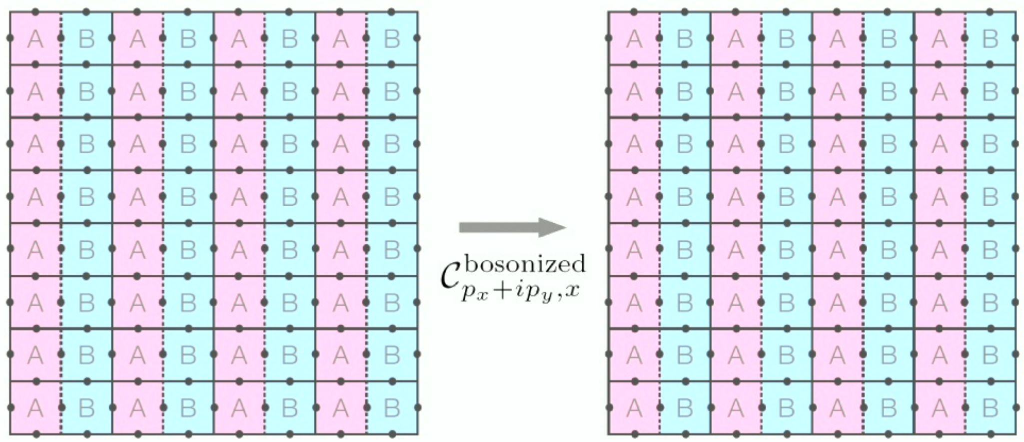


MERAQLE for the $\nu = 1$ Ising Chiral Spin Liquid



Dual fermionic sites associated with the blue B faces are not empty

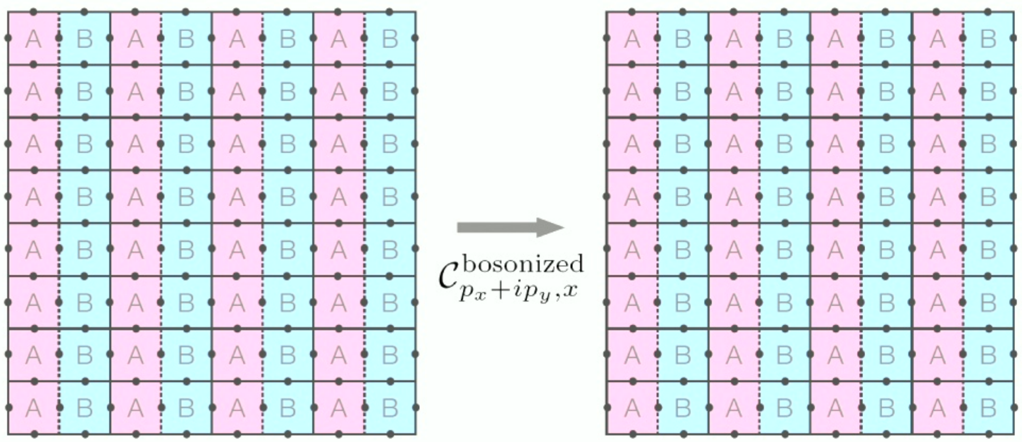
MERAQLE for the $\nu = 1$ Ising Chiral Spin Liquid



Dual fermionic sites associated with the blue B faces are empty sites

$C^{\text{bosonized}}_{p_x+ip_y,x}$: Bosonization of the quasi-adiabatic circuit for the p_x+ip_y topological superconductor

MERAQLE for the $\nu = 1$ Ising Chiral Spin Liquid



Dual fermionic sites associated with the blue B faces are empty sites

$C^{\text{bosonized}}_{p_x+ip_y,x}$: Bosonization of the quasi-adiabatic circuit for the p_x+ip_y topological superconductor

MERAQLE for the $\nu = 1$ Ising Chiral Spin Liquid

$$C_{\text{Ising CSL}, x} = C_{Z_2, x}\, C^{\text{bosonized}}_{p_x + ip_y, x}$$

MERA for the toric code

Bosonization of the quasi-adiabatic circuit for p_x+ip_y

(SKC, G. Zhu, and A. Gorshkov 2023)

MERAQLE for the $\nu = 1$ Ising Chiral Spin Liquid

$$C_{\text{Ising CSL},\,x} = C_{Z_2,\,x}\, C^{\text{bosonized}}_{p_x+ip_y,\,x}$$

MERA for the toric code

Bosonization of the quasi-adiabatic circuit for p_x+ip_y

Can easily be generalized to other 16-fold way chiral spin liquids

(SKC, G. Zhu, and A. Gorshkov 2023)

Outline

Introduction to entanglement renormalization circuits

- MERA circuit for the toric code

Chiral Topological Order

- Challenges for chiral topological order
- An entanglement renormalization circuit for the p_x+ip_y topological superconductor

Kitaev's 16-fold way chiral spin liquids

- Construction via bosonization
- Entanglement renormalization circuits (MERAQLE circuits)

Conclusions and outlook

Conclusions

We constructed entanglement renormalization circuits for chiral spin liquids that fall into Kitaev's 16-fold way classification. Some chiral spin liquids are Abelian, and some are non-Abelian.

Equipped with quasi-local evolutions, MERAQLE circuits are more powerful than MERA circuits. The quasi-local evolutions help us circumvent the correlation-length-reduction no-go argument.

Quantum Complexity

$$\text{Fixed-Pt State}_{\text{strictly local gates}} \subsetneq \text{Fixed-Pt State}_{\text{strictly local gates} + \text{quasi-local evolution}}$$

Quantum Complexity

Preparing the 16-fold way chiral spin liquids scale-invariantly is **NOT** in here
∵ Correlation-length-reduction no-go argument

$$\text{Fixed-Pt State}_{\text{strictly local gates}} \subsetneq \text{Fixed-Pt State}_{\text{strictly local gates} + \text{quasi-local evolution}}$$

- The notion of the locality of quantum circuits needs refinement!
- Chiral topological order is likely more complex than non-chiral topological order.

Outlook

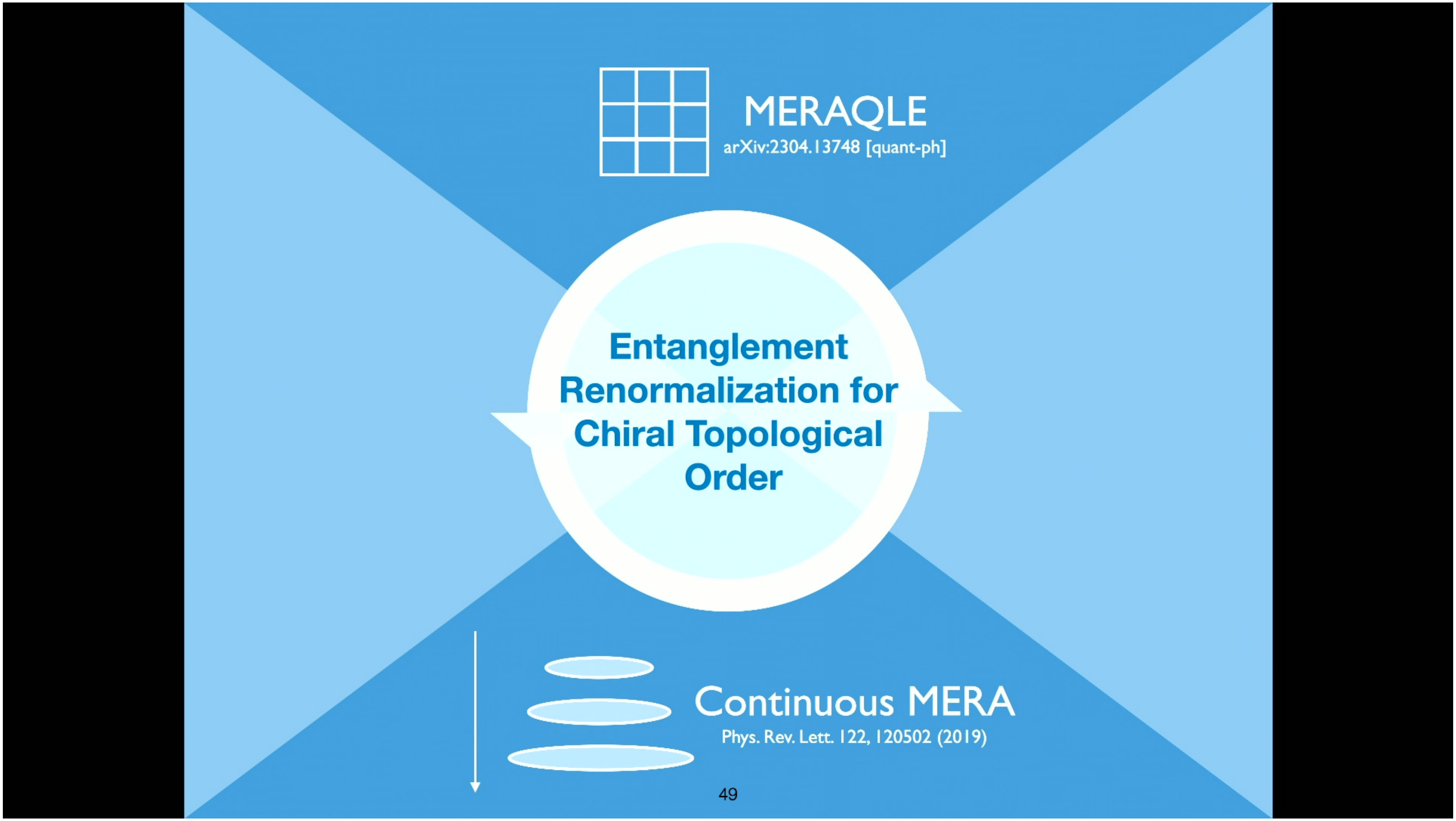
48

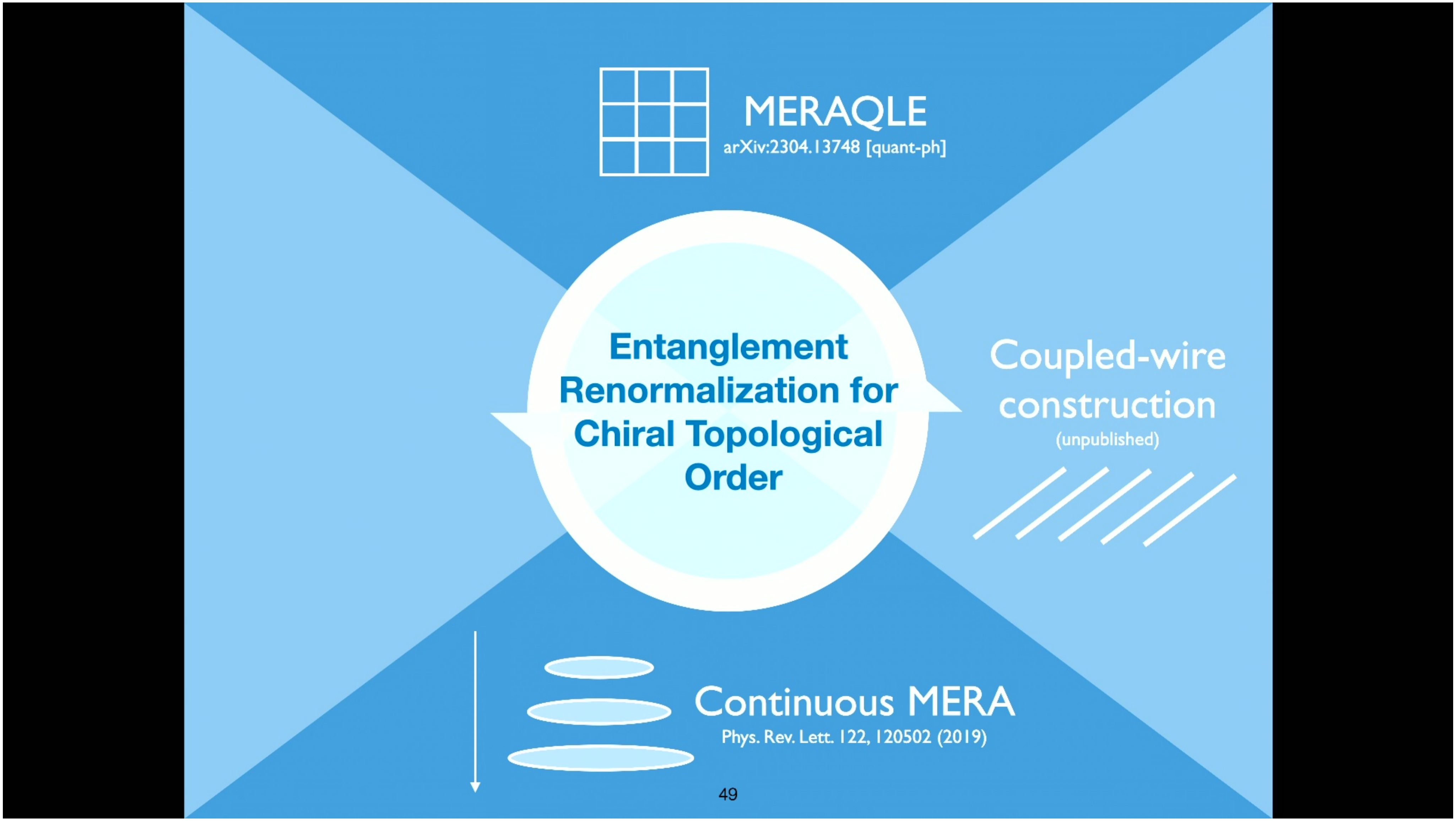
MERAQLE

arXiv:2304.13748 [quant-ph]

Entanglement Renormalization for Chiral Topological Order

49


MERAQLE
arXiv:2304.13748 [quant-ph]
Entanglement Renormalization for Chiral Topological Order
Continuous MERA
Phys. Rev. Lett. 122, 120502 (2019)
49



MERAQLE
arXiv:2304.13748 [quant-ph]
Entanglement Renormalization for Chiral Topological Order
Coupled-wire construction
(unpublished)
Continuous MERA
Phys. Rev. Lett. 122, 120502 (2019)
49

Thank you!