Title: Charting the space of ground states with matrix product states

Speakers: Marvin Qi

Series: Quantum Matter

Date: December 13, 2023 - 11:00 AM

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Abstract: In this talk I will use matrix product states (MPS) to study topological families of gapped ground states in one spatial dimension. To such families I will describe how to associate a gerbe, a mathematical structure which generalizes the line bundle associated to gapped ground states in 0d. Nontriviality of the gerbe represents an obstruction to representing the family of ground states with an MPS tensor that is continuous everywhere over parameter space. I will illustrate these constructions using an exactly solvable topological family which exhibits the key physics in a simple manner.

Zoom link https://pitp.zoom.us/j/91497524520?pwd=MkFHSW9PeGlMb2lFOTR3Qmo5clU0dz09

Charting the space of ground states with MPS

arXiv:2305.07700

Marvin Qi

Interview @ Perimeter

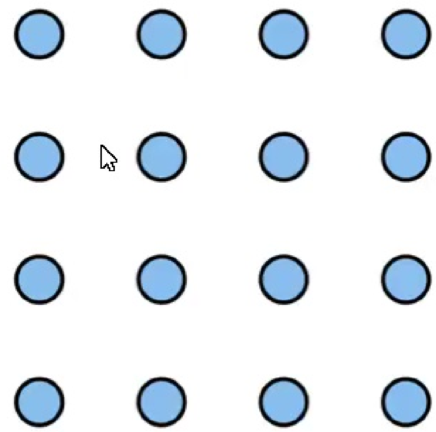
December 13 2023

Outline

- Introduction and motivation
- Parametrized systems
- Main example: Chern number pump
- Parametrized matrix product states

Quantum phases of matter

- Gapped quantum lattice models can be partitioned into equivalence classes called phases
- Two systems are in the same phase if they can be deformed into each other while preserving the gap
- Systems in the same phase have the same universal properties

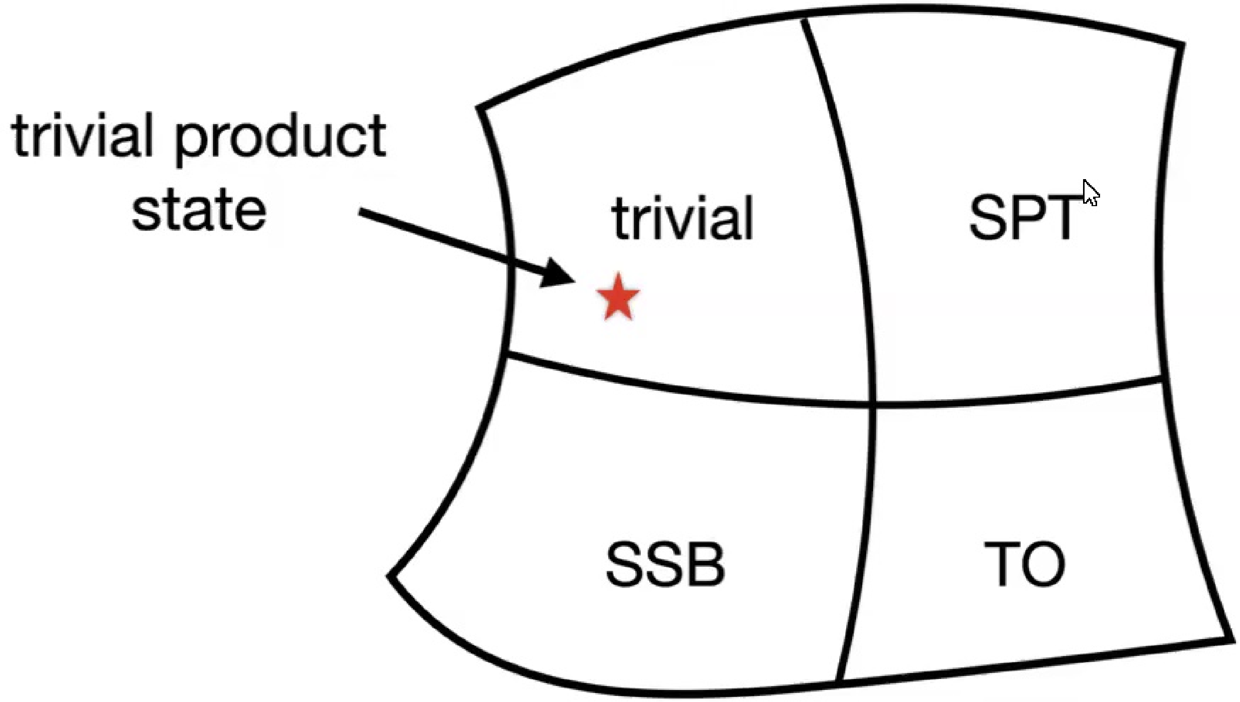


$$\mathscr{H} = \bigotimes_v \mathbb{C}^d$$

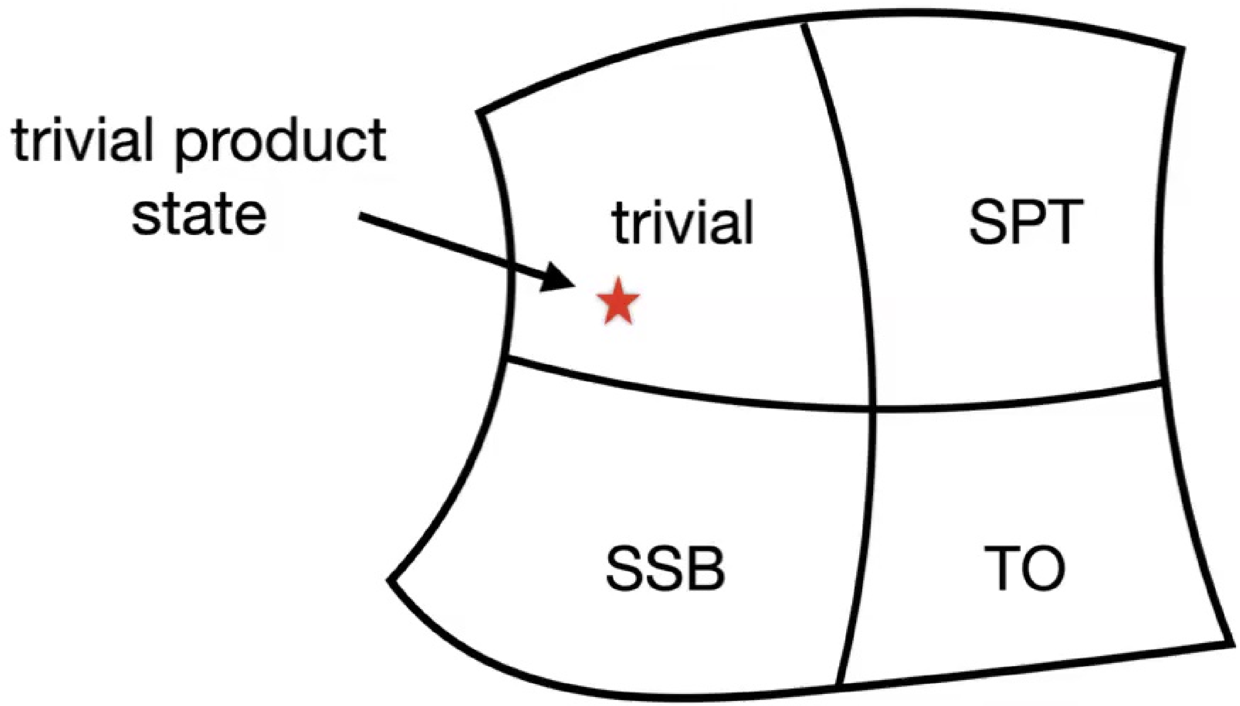
$$H = \sum_v H_v$$

Classification as topology

Classification as topology



Classification as topology



Gapped phases ↔ connected components

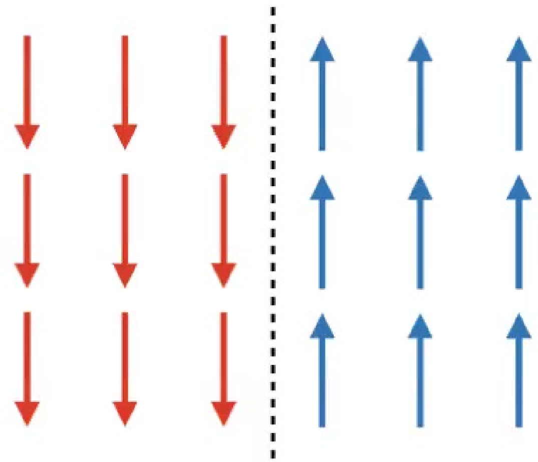
Classification as topology

- Classifying gapped phases is like finding π_0(quantum systems).
- What about higher homotopy groups of each phase?

Why do we care?

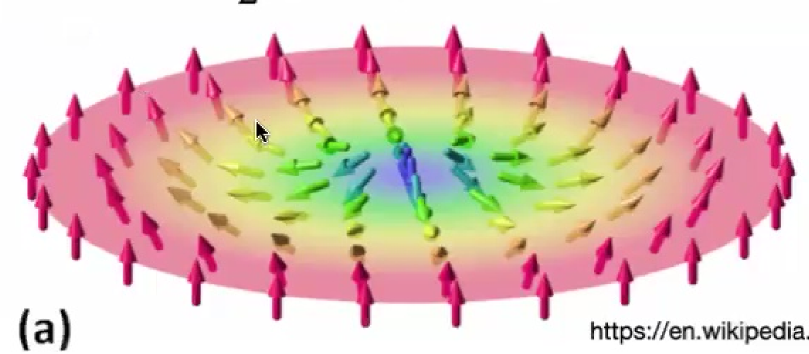
- Analogy: defects and textures in ordered phases
- Suppose symmetry G is broken down to subgroup $H \subset G$.
  - Coset space G/H labels possible ground states
  - Homotopy groups $\pi_k(G/H)$ determine codimension-$(k+1)$ defects and codimension-k textures

Why do we care?

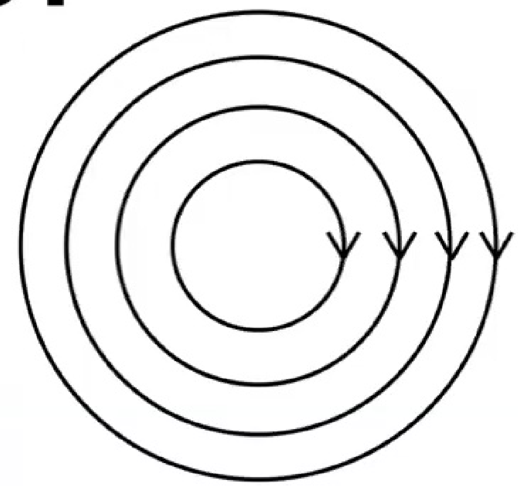


$\pi_0(\mathbb{Z}_2)$

$\pi_2(SO(3)/SO(2))$



https://en.wikipedia.org/wiki/Magnetic_skyrmion



$\pi_1(U(1))$

- Nontrivial π_k(quantum systems) can lead to interesting defects and textures!

Debray, Devalapurkar, Krulewski, Liu, Pacheco-Tallaj, Thorgren

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Parametrized systems

Wen, MQ, Beaudry, Moreno, Pflaum, Spiegel, Vishwanath, Hermele

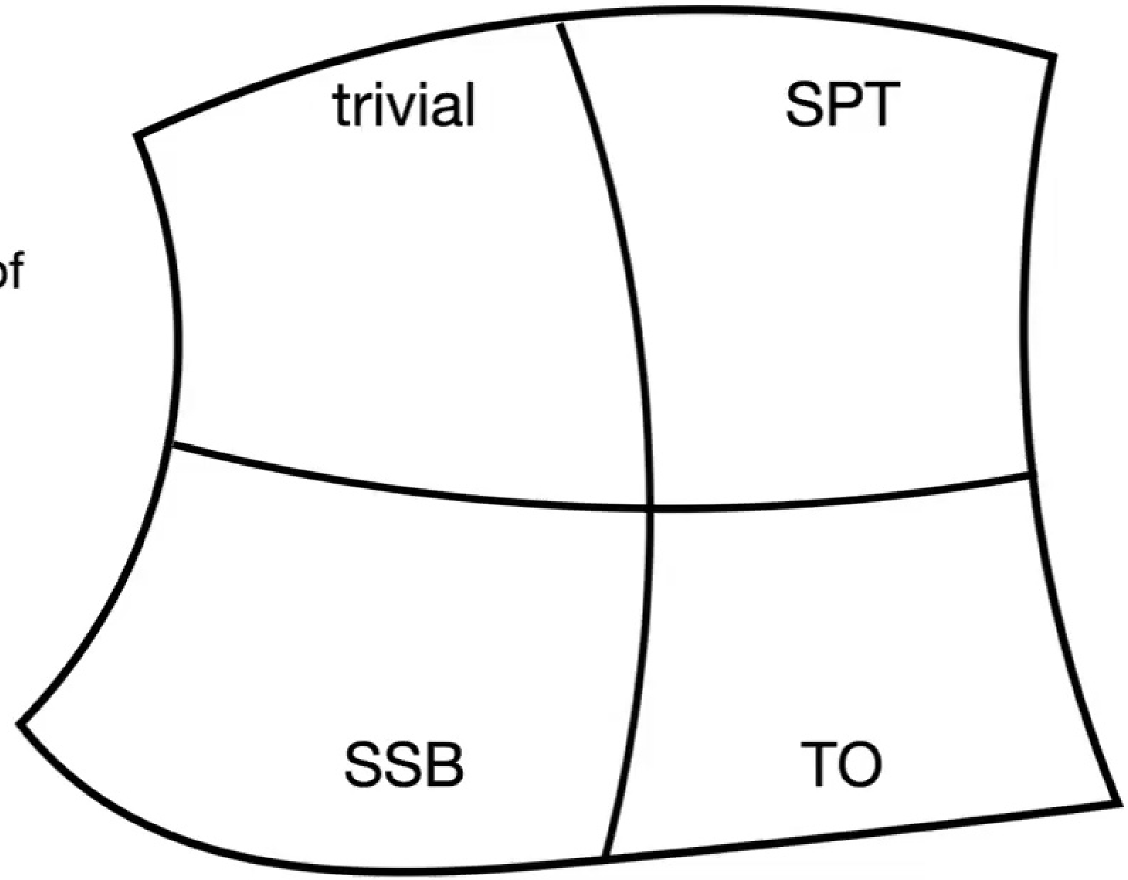
A system over X is a map

$$H : X \to \begin{array}{c}\text{gapped Hamiltonians} \\ \text{(with symmetry } G\text{)}\end{array}$$

- H_1 and H_2 are in the same phase over X if they can be deformed into one another while preserving the gap everywhere over X.
- A system over X is trivial if it is constant as a function of X, *and* the ground state is a product state.

Parametrized systems

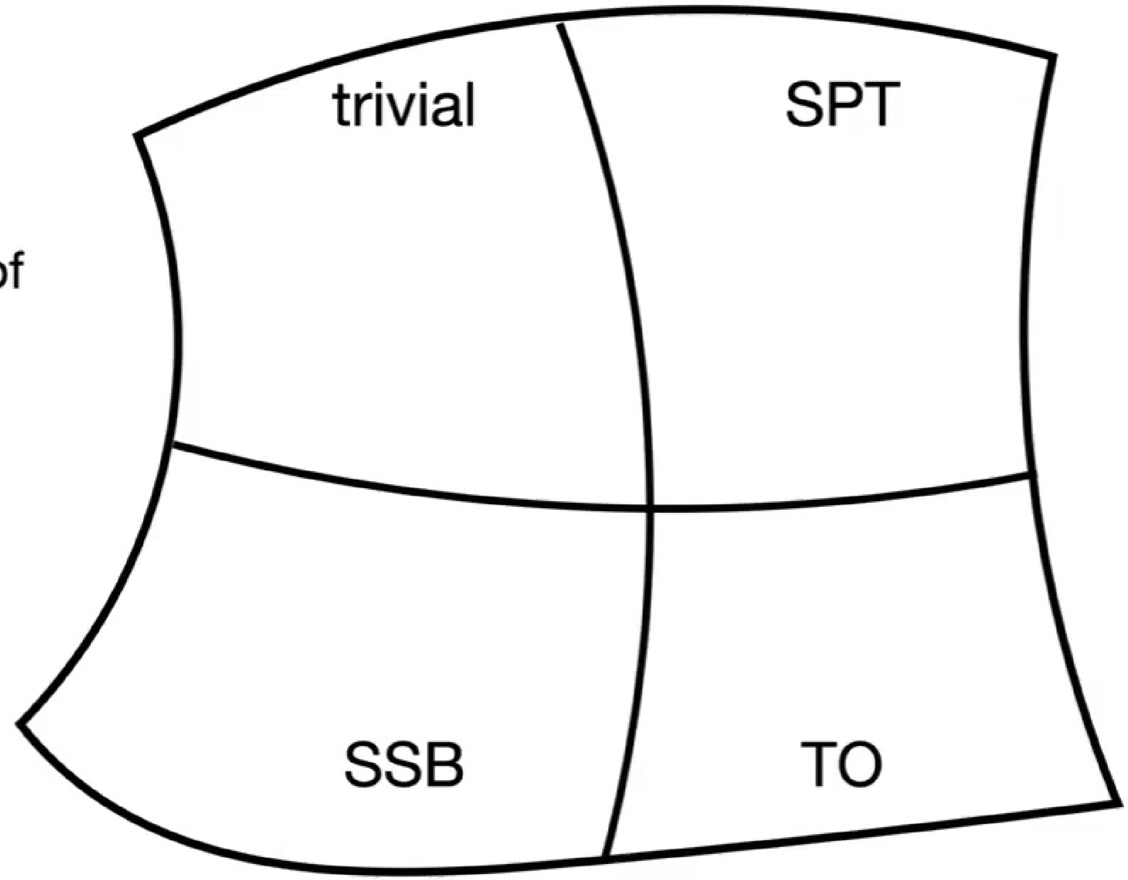
- $X = pt$: ordinary classification of phases



Parametrized systems

- $X = pt$: ordinary classification of phases

- $X = S^1$: quantum pumps (e.g. Thouless charge pump)



Parametrized systems

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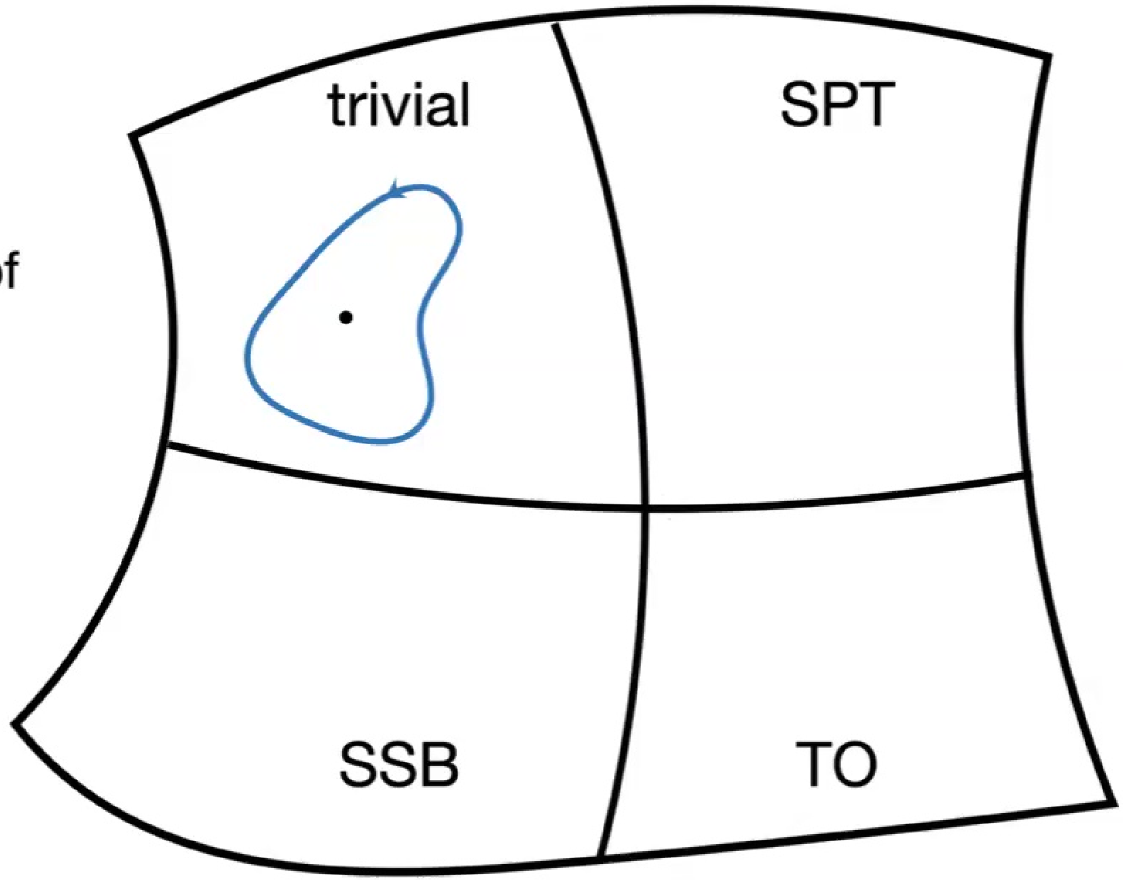


Berry phase in 0d

Berry, Simon

$$H(\overrightarrow{w}) = \overrightarrow{w} \cdot \vec{\sigma}$$

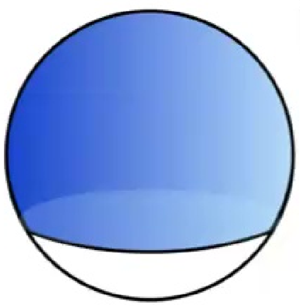
- For $X = S^2$, spin-1/2 in magnetic field gives a nontrivial parametrized system
- Phase invariant is given by Chern number, which takes on a quantized value

$$c_1 = \int_{S^2} \mathscr{F}^{(2)} = 2\pi$$

- Impossible to deform system to a constant family over S^2, e.g. $H(\overrightarrow{w}) = \sigma^z$

Berry phase in 0d

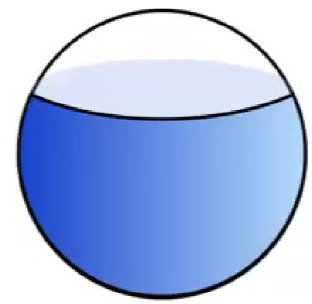
Consider the wavefunction on the northern and southern hemispheres of S^2 :



$$|\psi_N(\theta,\phi)\rangle = -\sin\frac{\theta}{2}e^{-i\phi}|\uparrow\rangle + \cos\frac{\theta}{2}|\downarrow\rangle$$

$e^{i\phi}$

$$|\psi_S(\theta,\phi)\rangle = -\sin\frac{\theta}{2}|\uparrow\rangle + \cos\frac{\theta}{2}e^{i\phi}|\downarrow\rangle$$



On the overlap, wavefunctions are related by a phase $e^{i\phi}$, which has nontrivial winding around the equator.

Obstruction to finding a globally well-defined wavefunction!

Line bundle of ground states

- Ground state wavefunctions of 0d parametrized system form a mathematical structure called a line bundle
- Base space is the collection of ground state density matrices
- Fiber over each basepoint is the set of ground state wavefunctions
- Nontriviality of the line bundle is an obstruction to a globally well-defined ground state wavefunction

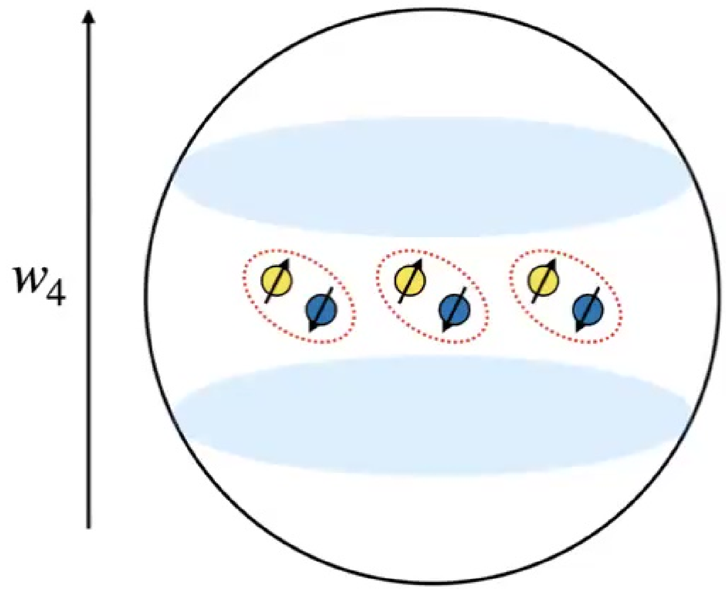
Higher Berry curvature in 1d

Kapustin, Spodyneiko

Wen, MQ, Beaudry, Moreno, Pflaum, Spiegel, Vishwanath, Hermele

$X = S^3$

$|\vec{w}|^2 + w_4^2 = 1$



$$H(\vec{w}, |w_4| \leq 1/2) = \sum_i \vec{w} \cdot \vec{\sigma}_{i,A} - \vec{w} \cdot \vec{\sigma}_{i,B}$$

Higher Berry curvature in 1d

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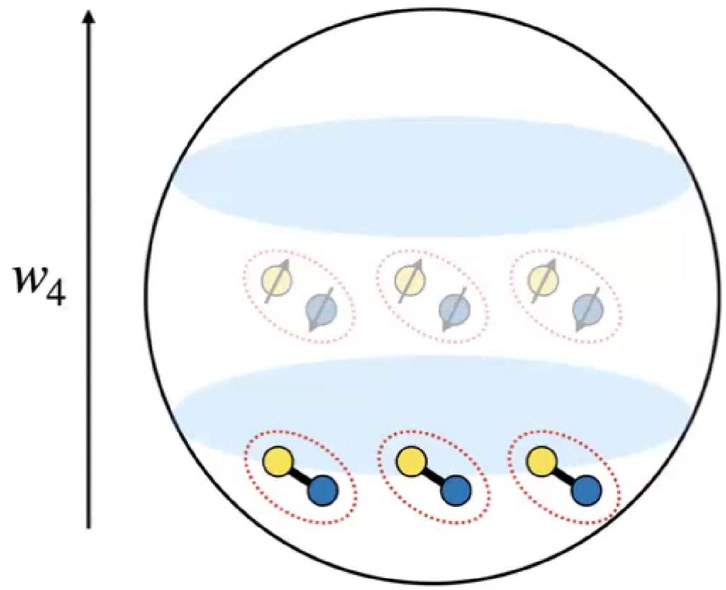

$$H(\vec{w}, |w_4| \leq 1/2) = \sum_i \vec{w} \cdot \vec{\sigma}_{i,A} - \vec{w} \cdot \vec{\sigma}_{i,B}$$

$$H(\vec{w}, w_4 < -1/2) = \sum_i \vec{w} \cdot \vec{\sigma}_{iA} - \vec{w} \cdot \vec{\sigma}_{iB} + g^-(w_4)\, \vec{\sigma}_{i,A} \cdot \vec{\sigma}_{i,B}$$

Higher Berry curvature in 1d

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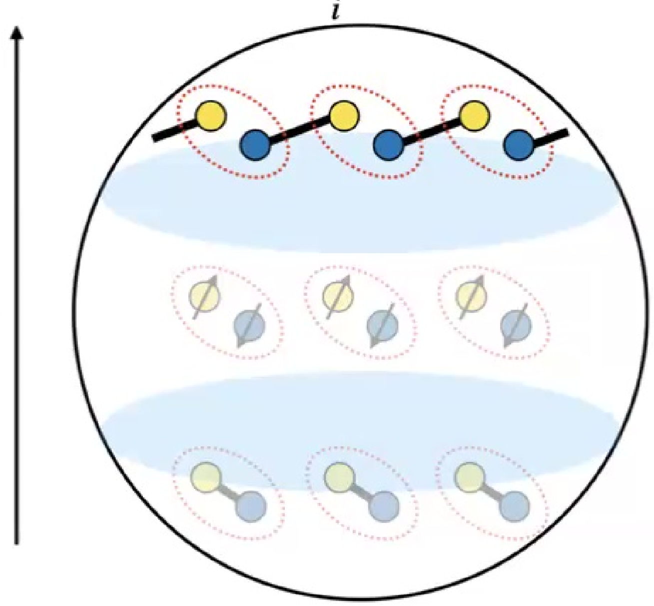
$$H(\overrightarrow{w}, w_4 > 1/2) = \sum_i \overrightarrow{w} \cdot \vec{\sigma}_{iA} - \overrightarrow{w} \cdot \vec{\sigma}_{iB} + g^+(w_4)\, \vec{\sigma}_{i,B} \cdot \vec{\sigma}_{i+1,A}$$

$$X = S^3$$
$$|\overrightarrow{w}|^2 + w_4^2 = 1$$

w_4



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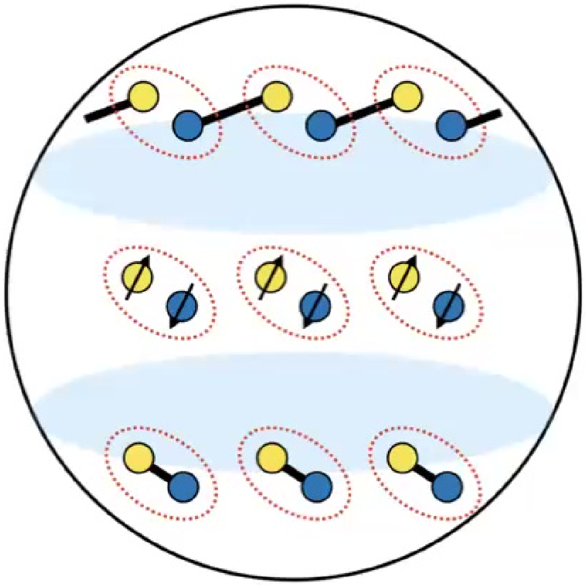
Higher Berry curvature in 1d

Wen, MQ, Beaudry, Moreno, Pflaum, Spiegel, Vishwanath, Hermele



- Nontrivial 1d system over $X = S^3$
- Edge mode: 0d boundary has a single Weyl point over S^3
- Phase invariant is given by the Kapustin-Spodyneiko higher Berry curvature invariant which takes value 2π

What does the higher Berry curvature obstruct?

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What does the higher Berry curvature obstruct?

What is the mathematical structure associated to parametrized 1d systems?

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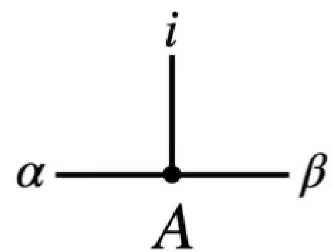
Matrix product states

- One dimensional ground states can be efficiently represented using MPS

$$|\psi\rangle = \sum_{i_1 \ldots i_n} \mathrm{Tr}\left(A^{i_1} A^{i_2} \ldots A^{i_n}\right) |i_1 i_2 \ldots i_n\rangle$$

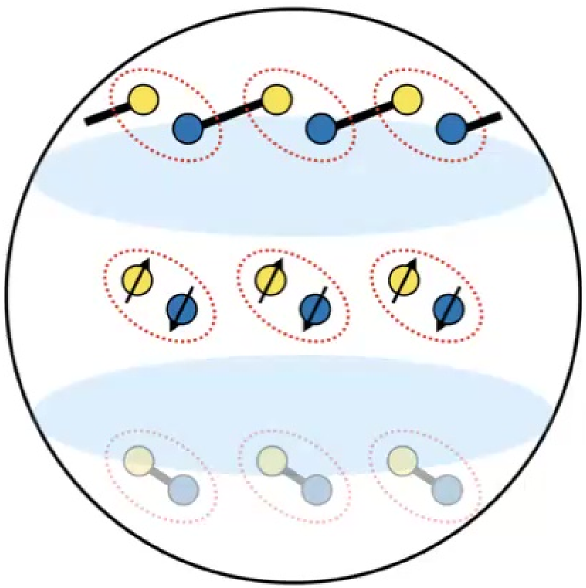
- $A^i_{\alpha\beta}$ is an MPS tensor with bond dimension χ if α, β run from $1, \ldots, \chi$



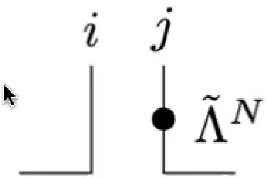
- (Minimum) bond dimension quantifies how much entanglement is present

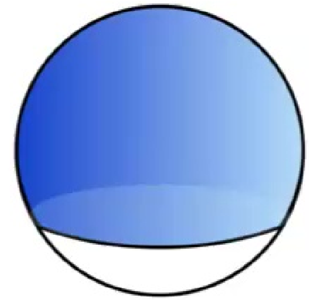
Parametrized MPS over S^3

MQ, Stephen, Wen, Spiegel, Pflaum, Beaudry, Hermele



$$A_N^{ij} =$$

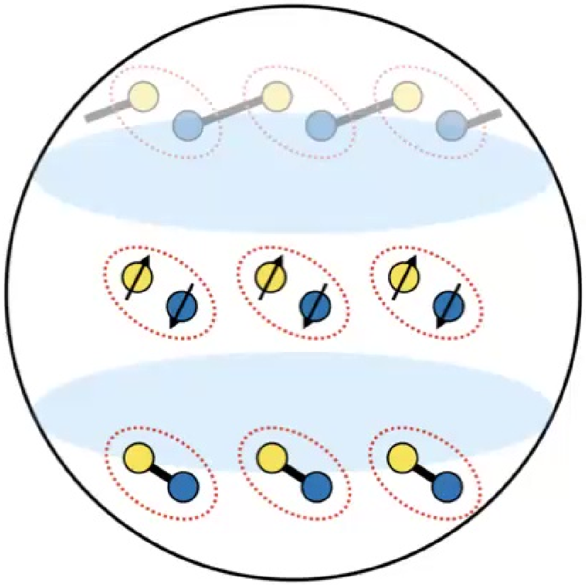


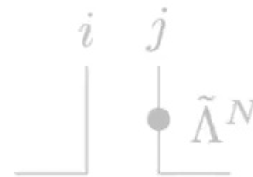


$$= |i\rangle\langle j| U(\vec{w}) \Lambda^N(\vec{w}) U(\vec{w})^T$$

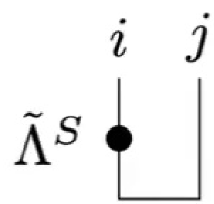
Parametrized MPS over S^3

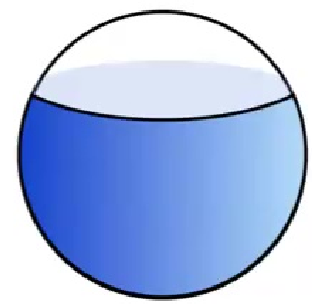
MQ, Stephen, Wen, Spiegel, Pflaum, Beaudry, Hermele



$A_N^{ij} =$ 

$= |i\rangle\langle j|U(\vec{w})\Lambda^N(\vec{w})U(\vec{w})^T$

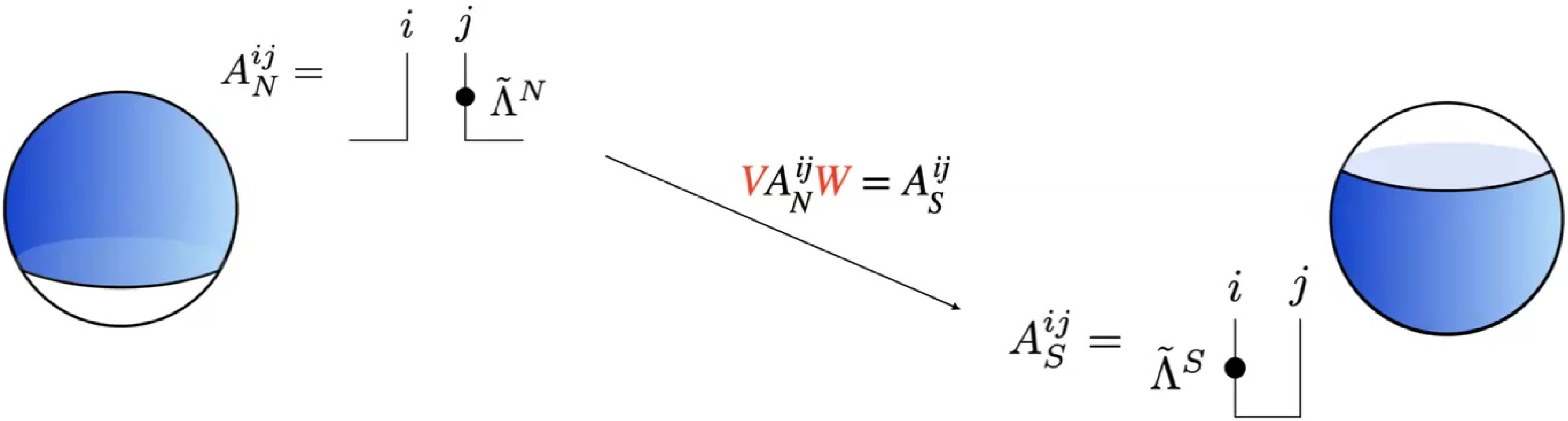
$A_S^{ij} =$ 



$= \langle i|U(\vec{w})\Lambda^S(\vec{w})U(\vec{w})^T|j\rangle$

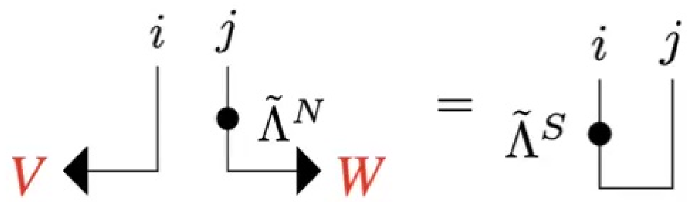
Parametrized MPS over S^3

Compare the MPS representations on the double overlap $\simeq S^2$:



On the double overlap, the MPS are related by reduction matrices V and W which are only well-defined up to a phase.

Chern number of reduction



$$W = \cos\frac{\theta}{2}\,|\uparrow\rangle - e^{-i\phi}\sin\frac{\theta}{2}\,|\downarrow\rangle$$

W carries Chern number!

- Phase ambiguity for W is identical to the phase ambiguity of the ground state wavefunction of spin-1/2 particle in magnetic field

- Reduction carries nonzero Chern number

Obstruction to choosing a globally well-defined MPS respresentation!

General structure

- More generally, the collection of MPS representations for the ground states of a parametrized 1d system assemble into the data for a gerbe

- Given a suitable open cover $\{U_\alpha\}$ for parameter space X, assign:

  - continuous MPS tensors $A(x)$ for each U_a;

  - transition line bundles L_{ab} on double overlaps U_{ab} which relate MPS tensors $A(x)$ and $B(x)$

  - isomorphisms $L_{ab} \otimes L_{bc} \to L_{ac}$ on triple overlaps U_{abc} satisfying a cocycle condition

General structure

- A gerbe over X has an associated invariant called the Dixmier-Douady class which lives in $H^3(X; \mathbb{Z})$

- In some cases this class can also be obtained by integrating the higher Berry curvature 3-form over X

- Nontrivial higher Berry curvature $\leftrightarrow$ obstruction to global MPS representation

Summary

- MPS provide powerful framework analyzing 1d parametrized systems
- Nontriviality of Chern number pump is captured by Chern number of transition line bundle
- Gerbe structure associated to MPS representations of parametrized family of 1d states
- Further directions: gerbe connections, classifying space of MPS, …
- See also work by Ohyama, Shiozaki, et al. arXiv:2303.04252, arXiv:2304.05356, arXiv:2305.08109 for complementary perspective

Other related directions

- Families of topological orders parametrized by S^1 have appeared in constructions of Floquet codes
    - Possible extensions to more general families?
- Many constructions of parametrized systems involve "pumping" lower dimensional invertible defects across the system
    - Can this make sense for non-invertible defects?
- The higher Berry curvature example can arise from perturbing a deconfined quantum critical point
    - New constructions of deconfined criticality?