

Title: Models of deconfined criticality for square and triangular lattice antiferromagnets

Speakers: Henry Shackleton

Series: Quantum Matter

Date: December 04, 2023 - 11:00 AM

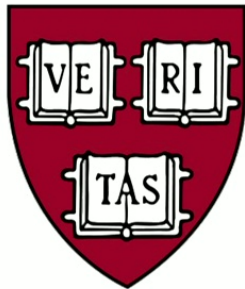
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Abstract: Frustrated quantum magnets provide a promising platform for realizing exotic phase transitions known as deconfined quantum critical points (DQCPs), where a conventional Landau-Ginzburg description fails and the resulting description involves emergent gauge fields. In the first part of my talk, I will propose a unified theory for describing a pair of continuous phase transitions numerically observed in the frustrated square lattice Heisenberg antiferromagnet, where a spin liquid phase appears to emerge in between Neel and valence bond solid (VBS) phases. The proposed DQCPs exhibit a plethora of unconventional phenomena, including anisotropic fixed points and dangerously irrelevant perturbations. In the second part of my talk, I will describe recent work analyzing an effective model of triangular lattice antiferromagnetism which supports coplanar magnetic order as well as VBS and spin liquid phases. We show that this effective model is sign-problem-free and amenable to large-scale Monte Carlo simulations, which reveal a direct transition between magnetic and VBS phases.

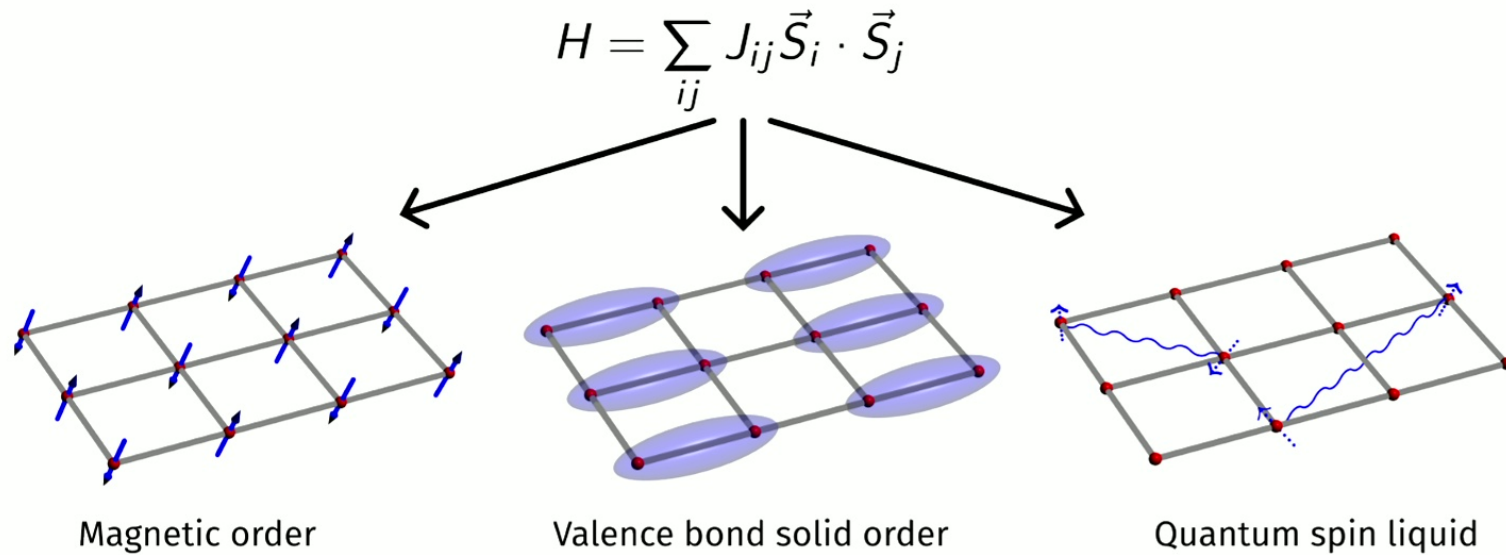
Zoom link <https://pitp.zoom.us/j/98562300020?pwd=OXYrL0dJTGkzNk5memlVM0tqY3hNQQT09>

Models of deconfined criticality on square and triangular lattice antiferromagnets

Henry Shackleton
December 4th, 2023
Harvard University



Quantum magnetism as a platform for exotic phases

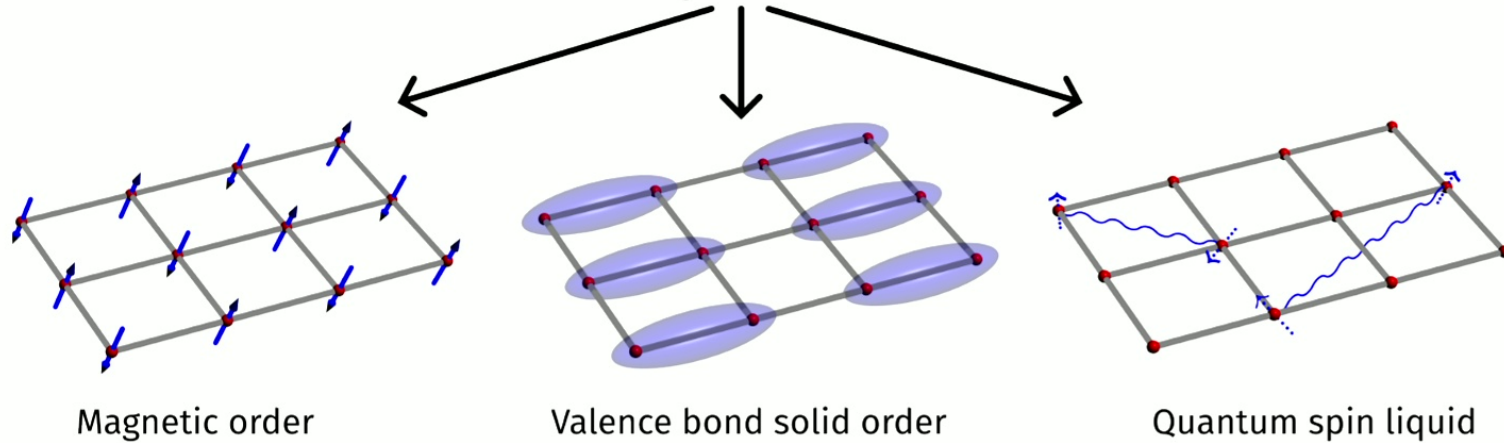


Parton construction: versatile theoretical tool

$$\vec{S}_i \rightarrow b_{i\alpha}^\dagger \vec{\sigma}^{\alpha\beta} b_{i\beta}, f_{i\alpha}^\dagger \vec{\sigma}^{\alpha\beta} f_{i\beta}$$

Quantum magnetism as a platform for exotic phases

$$H = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Square lattice: fermionic spinons for unifying numerically-observed Néel/spin liquid/VBS transitions

Triangular lattice: bosonic spinons for effective sign-problem-free model of triangular lattice DQCP

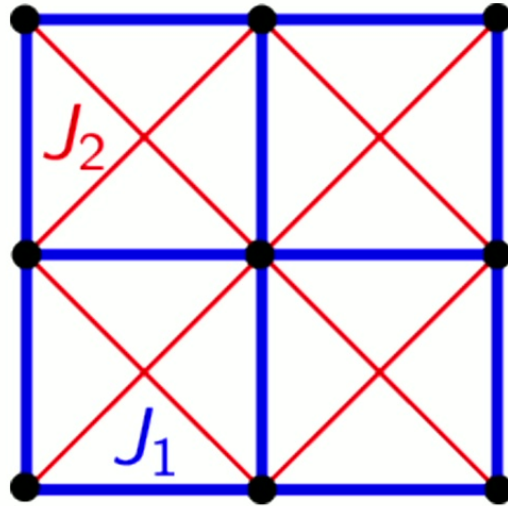
Deconfined criticality on the square lattice antiferromagnet



H. Shackleton and S. Sachdev, Journal of High Energy Physics 2022 (7), 1-35

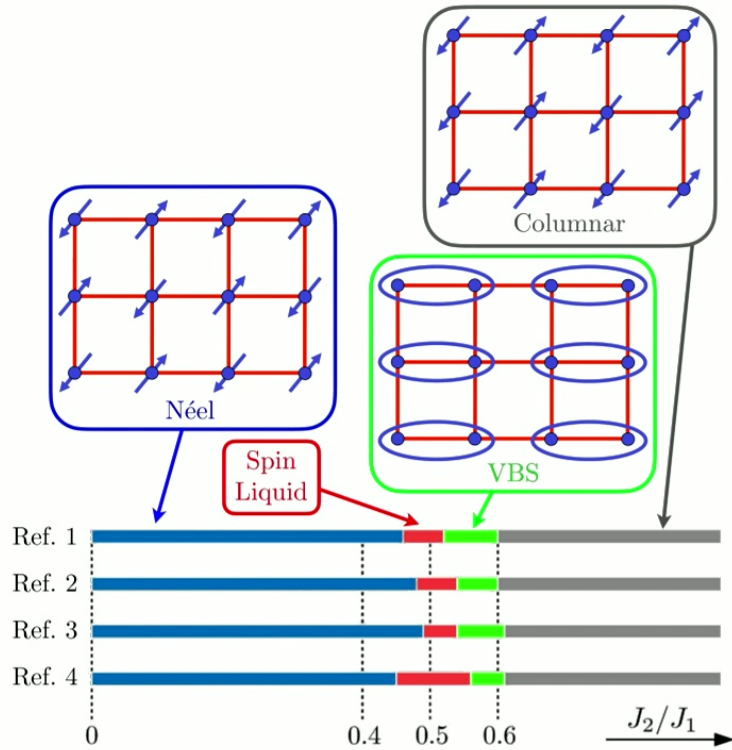
H. Shackleton, A. Thomson, S. Sachdev, Physical Review B 104 (4), 045110

Multimethod studies on $J_1 - J_2$ model indicate spin liquid phase



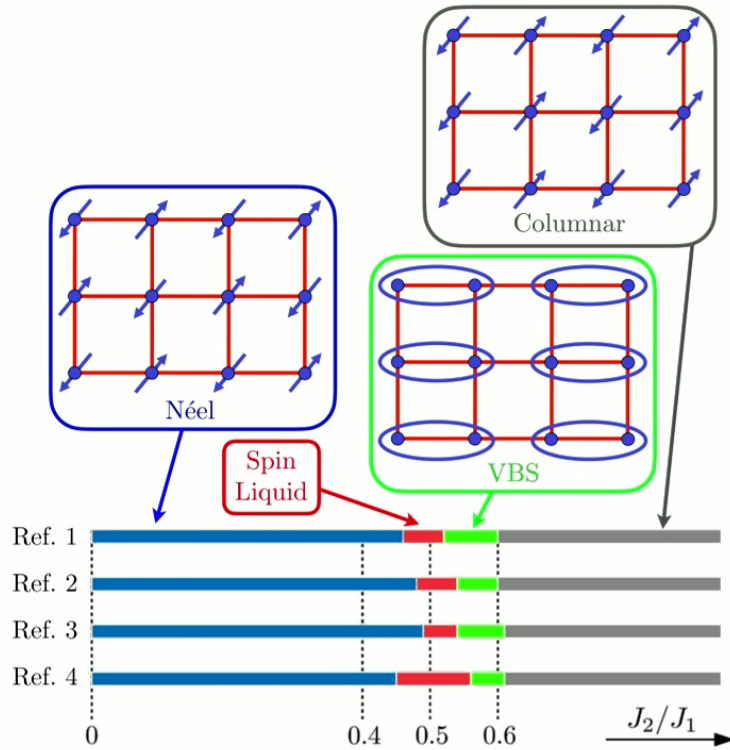
$$H = J_1 \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{\langle\langle ij \rangle\rangle} \vec{S}_i \cdot \vec{S}_j$$

Multimethod studies on $J_1 - J_2$ model indicate spin liquid phase



¹Wang and Sandvik, *Phys. Rev. Lett.*, 2018 ²Ferrari and Becca, *Phys. Rev. B.*, 2020, ³Nomura and Imada, *Phys. Rev. X.*, 2021 ⁴Liu et al., *Phys. Rev. X.*, 2022

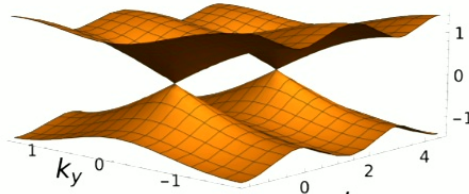
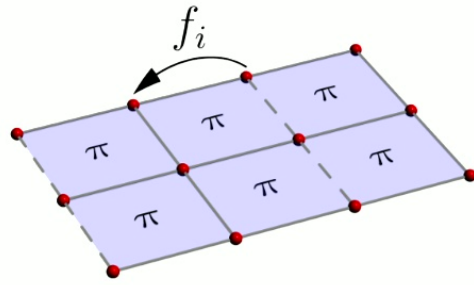
Multimethod studies on $J_1 - J_2$ model indicate spin liquid phase



Assume VMC description of spin liquid, gapless fermionic spinons with d-wave pairing (Z2Azz13)

¹Wang and Sandvik, *Phys. Rev. Lett.*, 2018 ²Ferrari and Becca, *Phys. Rev. B.*, 2020, ³Nomura and Imada, *Phys. Rev. X.*, 2021 ⁴Liu et al., *Phys. Rev. X.*, 2022

π -flux as a “parent” phase of a \mathbb{Z}_2 spin liquid



$N_f = 2$ QCD₃, emergent SO(5) symmetry

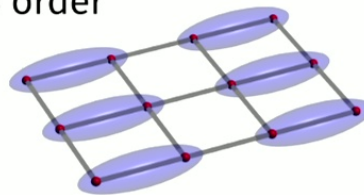
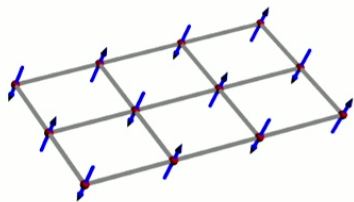
Higgs Φ_1^a, Φ_2^a



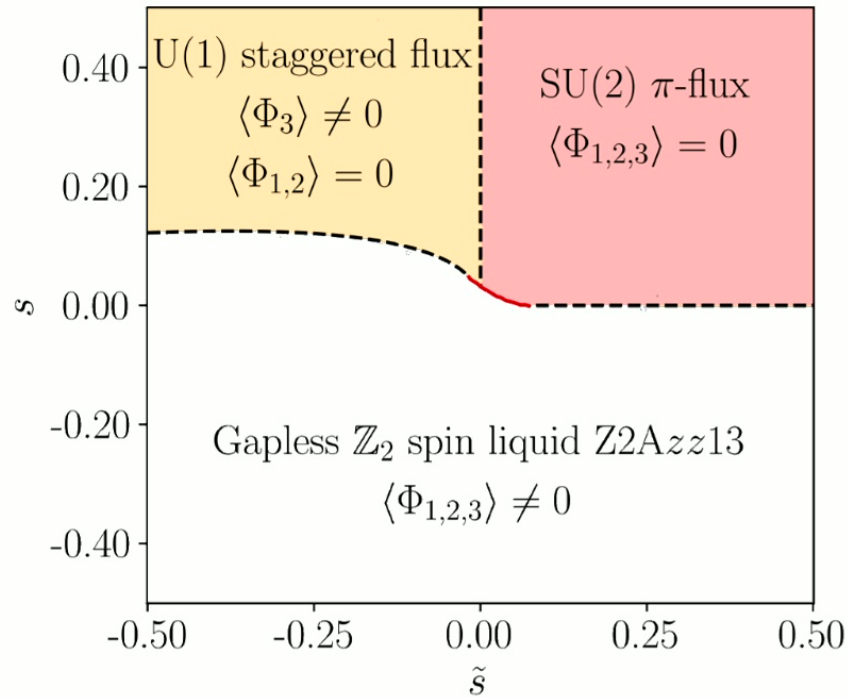
Z2Azz13

$SU(2) \rightarrow \mathbb{Z}_2$

Unstable to Néel/VBS order

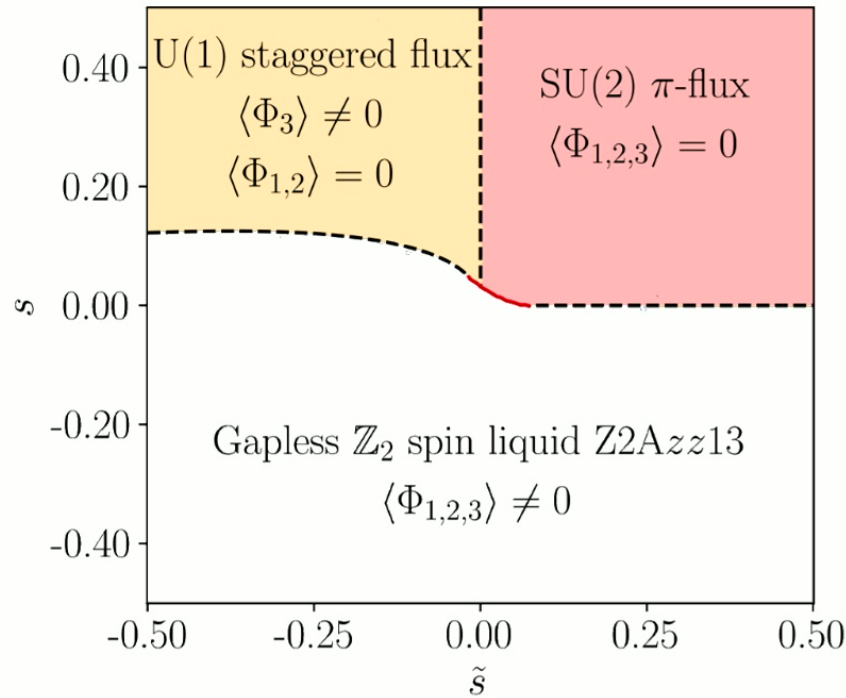


Multiple instabilities captured by proximity to Dirac spin liquid



Unique third Higgs field ϕ_3^a ,
condense-able with mass \tilde{s} along with
 ϕ_1^a, ϕ_2^a with mass s

Multiple instabilities captured by proximity to Dirac spin liquid



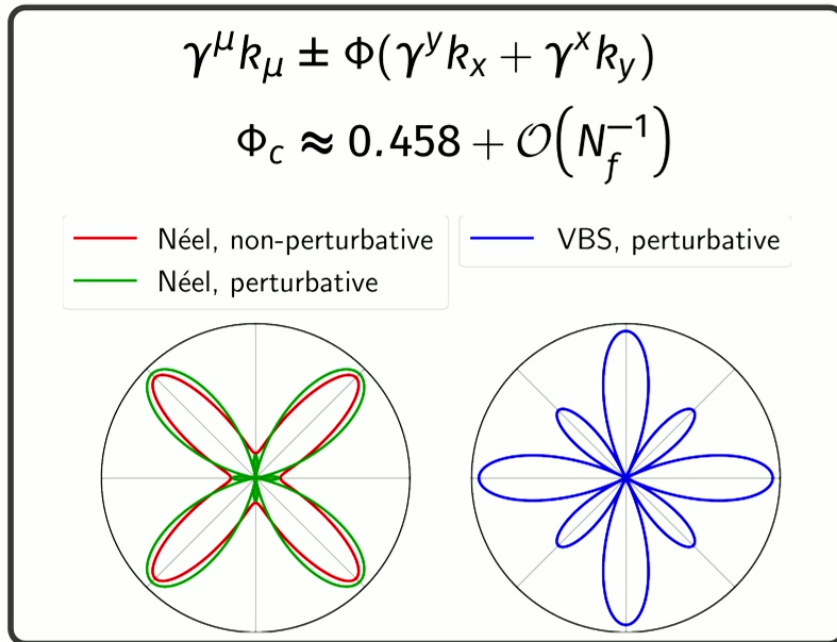
Unique third Higgs field ϕ_3^a ,
condense-able with mass \tilde{s} along with
 ϕ_1^a, ϕ_2^a with mass s

Which phase transitions correspond to which instability?

U(1) → Z₂ transition has fixed spinon anisotropy

Pure QED₃: fermion anisotropy irrelevant, emergent Lorentz symmetry⁶

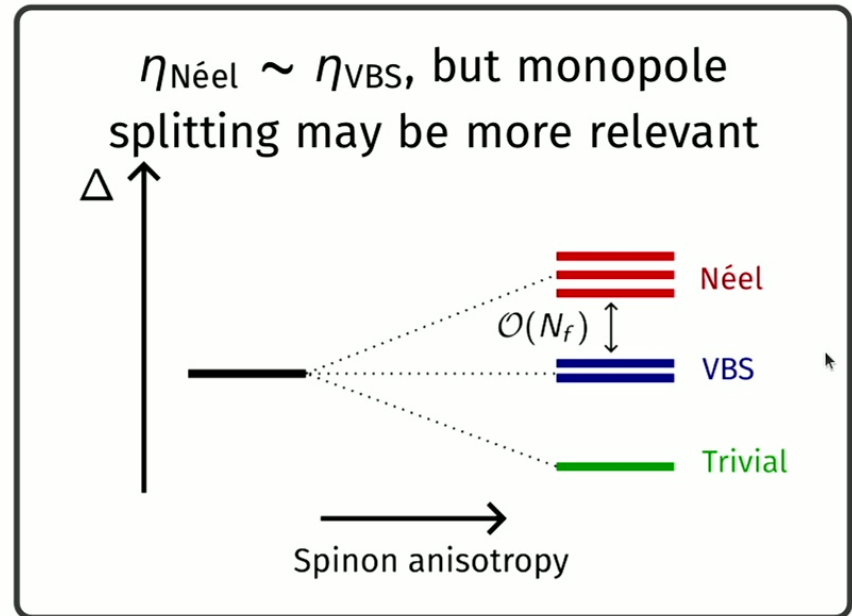
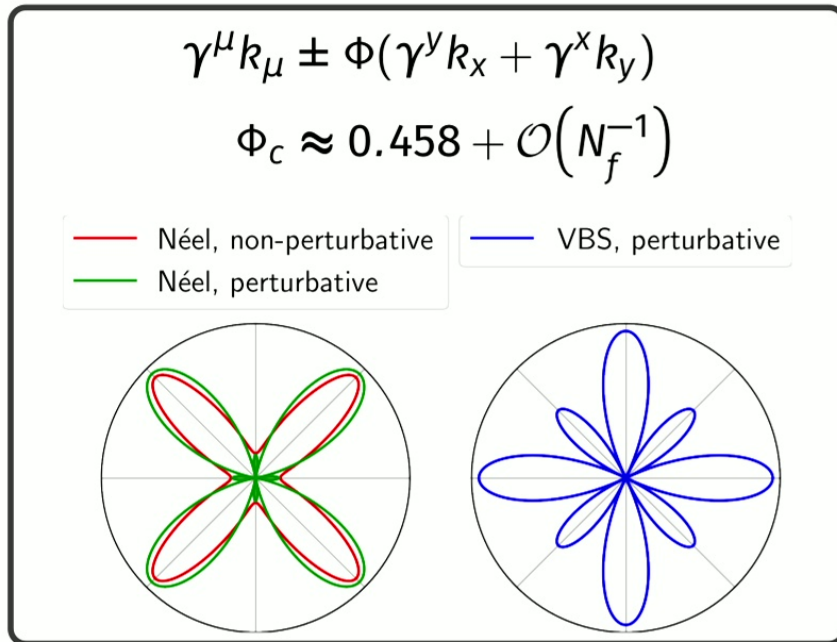
QED₃ + critical Higgs: fixed point with non-zero anisotropy



⁶Hermele, Senthil, and Fisher, *Phys. Rev. B.*, 2005

U(1) → Z₂ transition has fixed spinon anisotropy

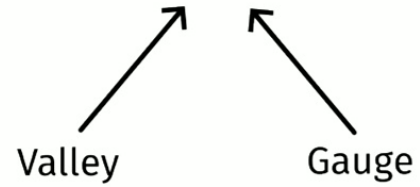
Pure QED₃: fermion anisotropy irrelevant, emergent Lorentz symmetry⁶
 QED₃ + critical Higgs: fixed point with non-zero anisotropy



⁶Hermele, Senthil, and Fisher, *Phys. Rev. B*, 2005

UV/IR mixing in $SU(2) \rightarrow \mathbb{Z}_2$ transition

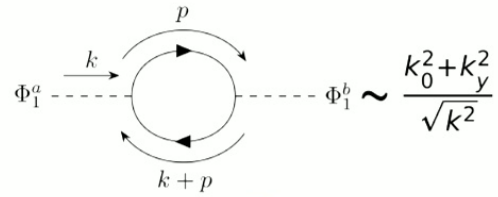
$$\mathcal{L} = \mathcal{L}_{N_f=2 \text{ QCD}_3} + \lambda \left(\Phi_1^a \bar{\psi} \gamma^x \mu^z \sigma^a \psi + \Phi_2^a \bar{\psi} \gamma^y \mu^x \sigma^a \psi \right)$$



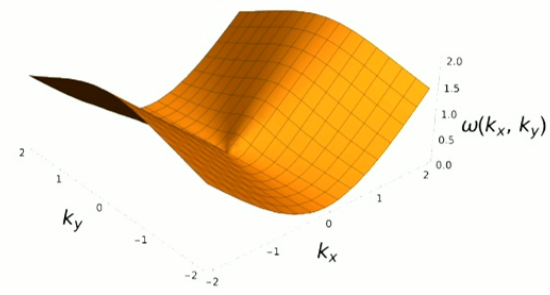
UV/IR mixing in $SU(2) \rightarrow \mathbb{Z}_2$ transition

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Conserved "currents"



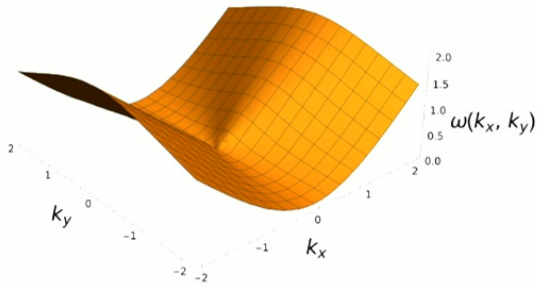
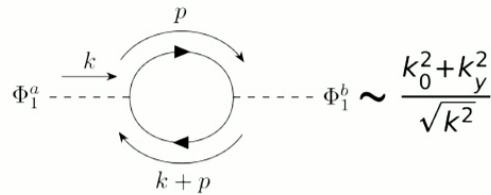
Emergent "Higgs Bose liquid," extensive gapless modes regulated by (irrelevant) $\Phi \partial^2 \Phi$ term



UV/IR mixing in $SU(2) \rightarrow \mathbb{Z}_2$ transition

$$\mathcal{L} = \mathcal{L}_{N_f=2 \text{ QCD}_3} + \lambda \left(\Phi_1^a \bar{\psi} \gamma^x \mu^z \sigma^a \psi + \Phi_2^a \bar{\psi} \gamma^y \mu^x \sigma^a \psi \right)$$

Conserved "currents"



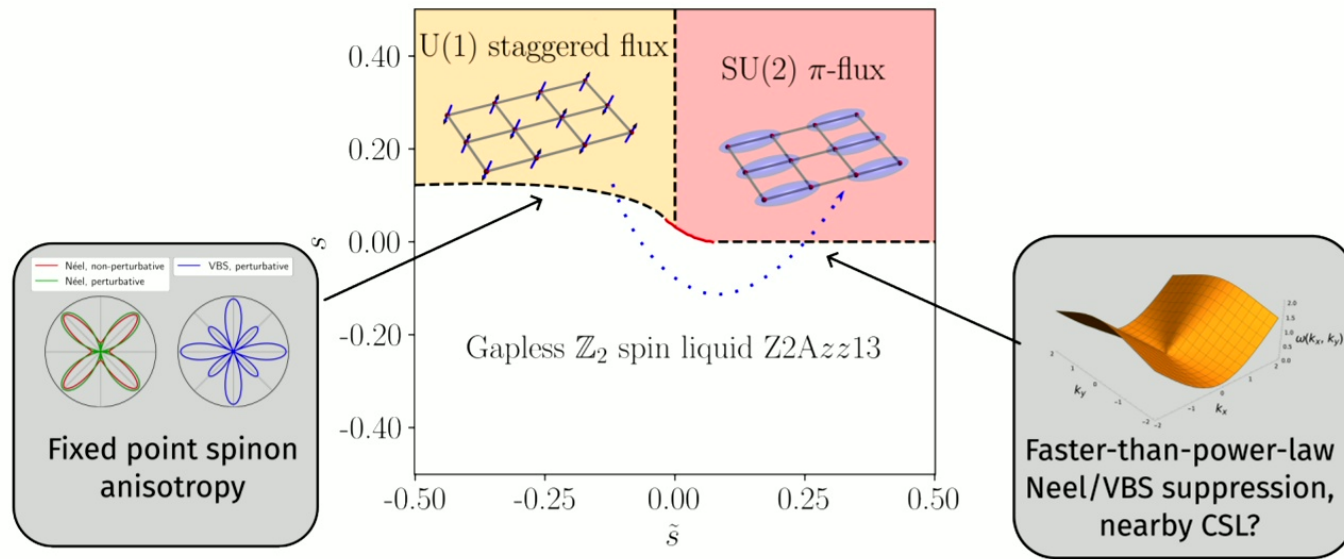
Emergent "Higgs Bose liquid," extensive gapless modes regulated by (irrelevant) $\Phi \partial^2 \Phi$ term

$$G_{\text{Néel}}(r) \sim \exp[-\eta_{\text{Néel}} \ln^2(r/a)]$$

$$G_{\text{VBS}}(r) \sim \exp[-\eta_{\text{VBS}} \ln^2(r/a)]$$

$$\eta_{\text{Néel}} > \eta_{\text{VBS}}$$

Summary and outlook

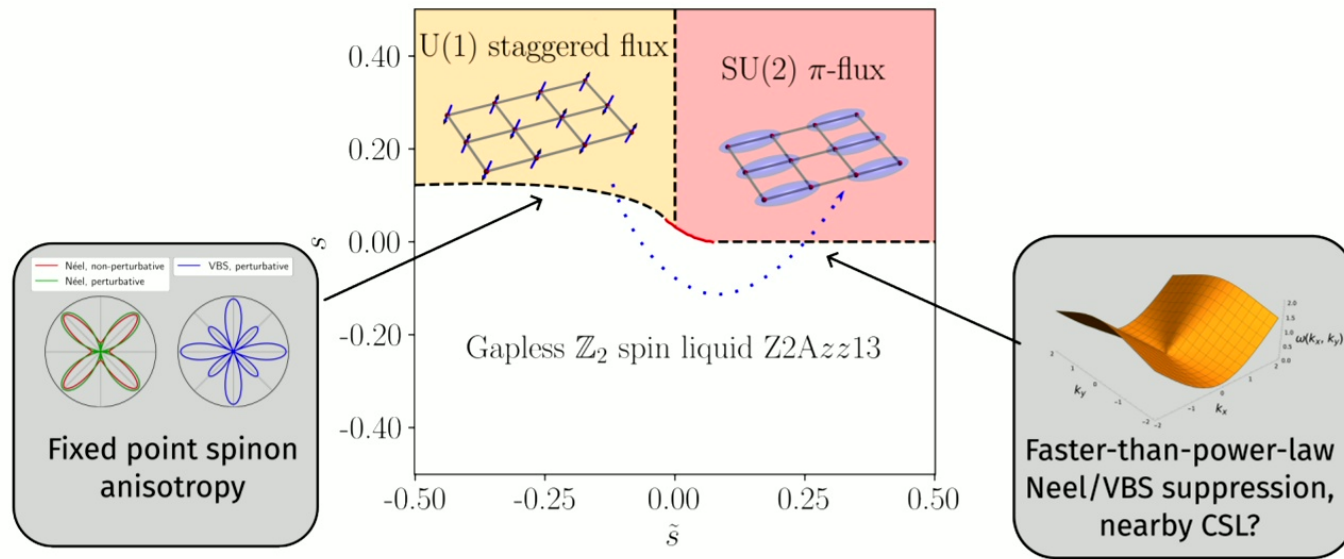


- Are \log^2 predictions accurate? Can we find a minimal model? With numerics?

⁷Lake and Senthil, *Phys. Rev. Lett.*, 2023.

⁸Gomes et al., *Phys. Rev. D.*, 1991.

Summary and outlook



- Are \log^2 predictions accurate? Can we find a minimal model? With numerics?
- Similar ideas in engineering NFLs⁷, Thirring models⁸...

⁷Lake and Senthil, *Phys. Rev. Lett.*, 2023.

⁸Gomes et al., *Phys. Rev. D.*, 1991.

Effective models for triangular lattice quantum antiferromagnets

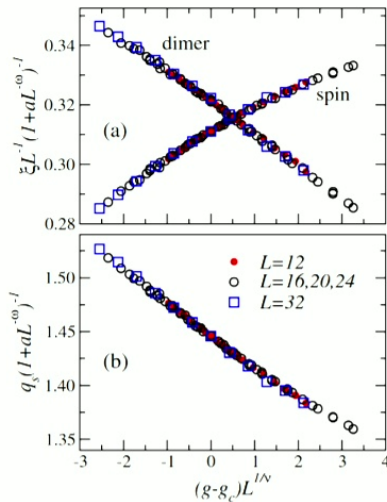


H. Shackleton and S. Sachdev, arXiv:2311.01572

Frustrated magnetism on non-bipartite lattices: a difficult problem

Bipartite lattices

Marshall sign rule allows for non-trivial “designer Hamiltonians”⁹



Non-bipartite lattice

Primarily restricted to variational ansatzes (DMRG, PEPS, NQS...) or ED
Candidate AF/VBS DQCP¹⁰ remains unexplored numerically

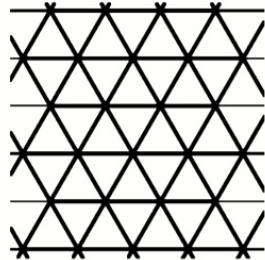
Goal: construct an effective model amenable to large-scale QMC simulations

⁹Sandvik, *Phys. Rev. Lett.*, 2007

¹⁰Jian et al., *Phys. Rev. B.*, 2018

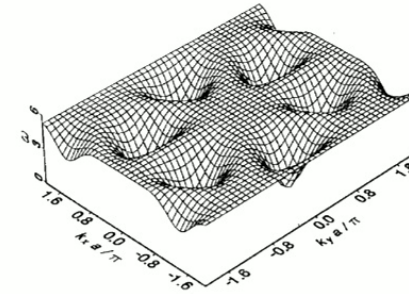
Effective models for triangular lattice quantum antiferromagnets

$$H = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Effective model of bosonic spinons, U(1) gauge fluctuations
Higgsed to \mathbb{Z}_2

$$\vec{S}_i \equiv b_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} b_{i\beta}$$

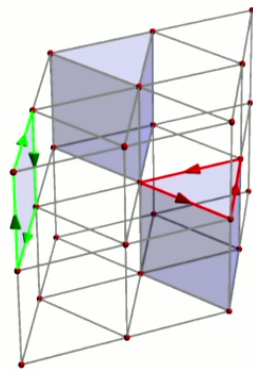


$$H = -J \sum_{j,\mu,\alpha} (z_{j,\alpha}^* z_{j+\hat{\mu},\alpha} + \text{c.c.})$$

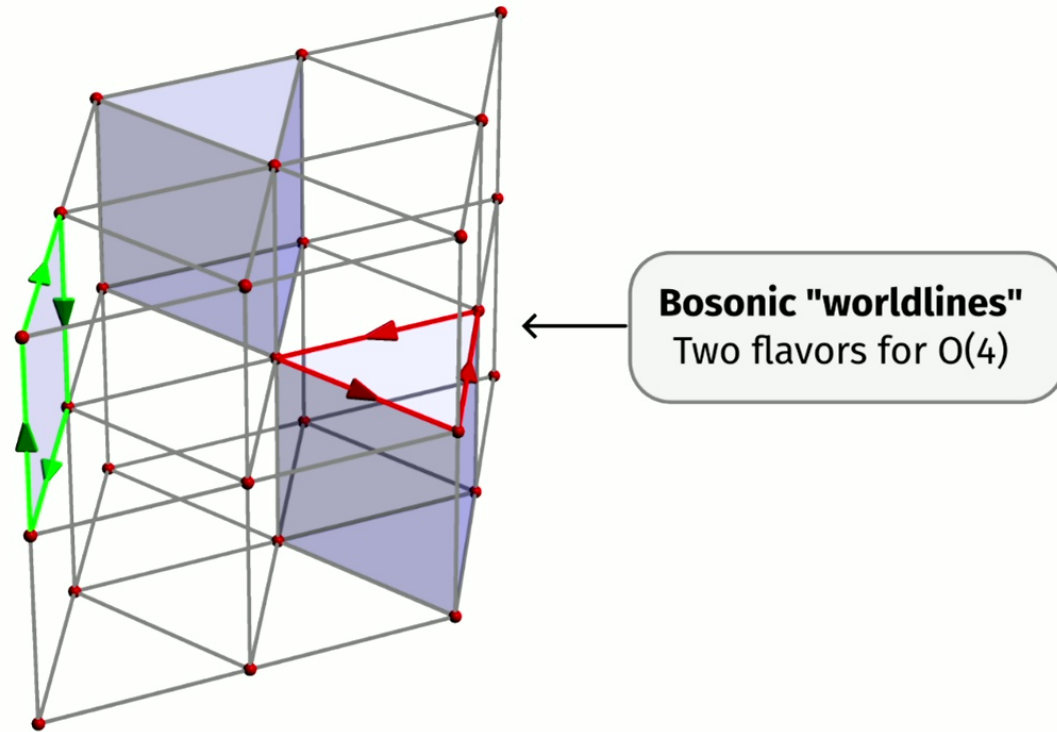
Couple to \mathbb{Z}_2 gauge field, mutual statistics captured by Berry phase

Exact sign-problem-free mapping, preserves emergent O(4) symmetry

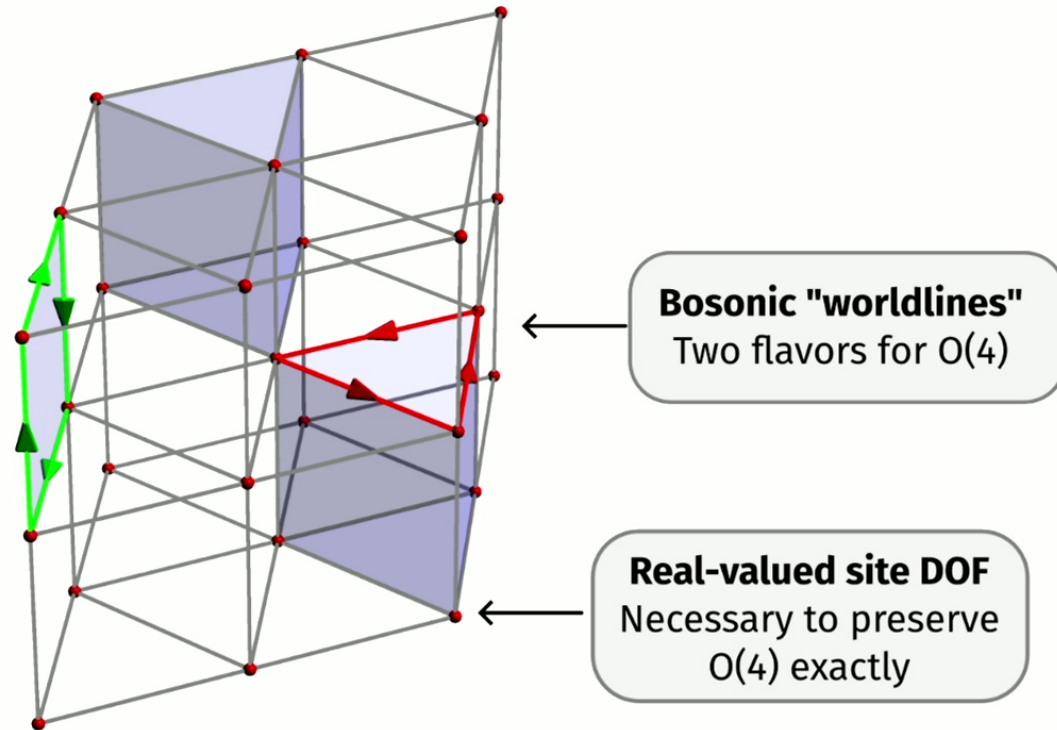
$$H = -J \sum_{j,\mu,\alpha} s_{j,j+\hat{\mu}} (z_{j,\alpha}^* z_{j+\hat{\mu},\alpha} + \text{c.c.}) - K \sum_{\Delta,\square} \prod_{\Delta,\square} s_{j,j+\hat{\mu}} + i\pi \sum_j s_{j,j+\hat{t}}$$



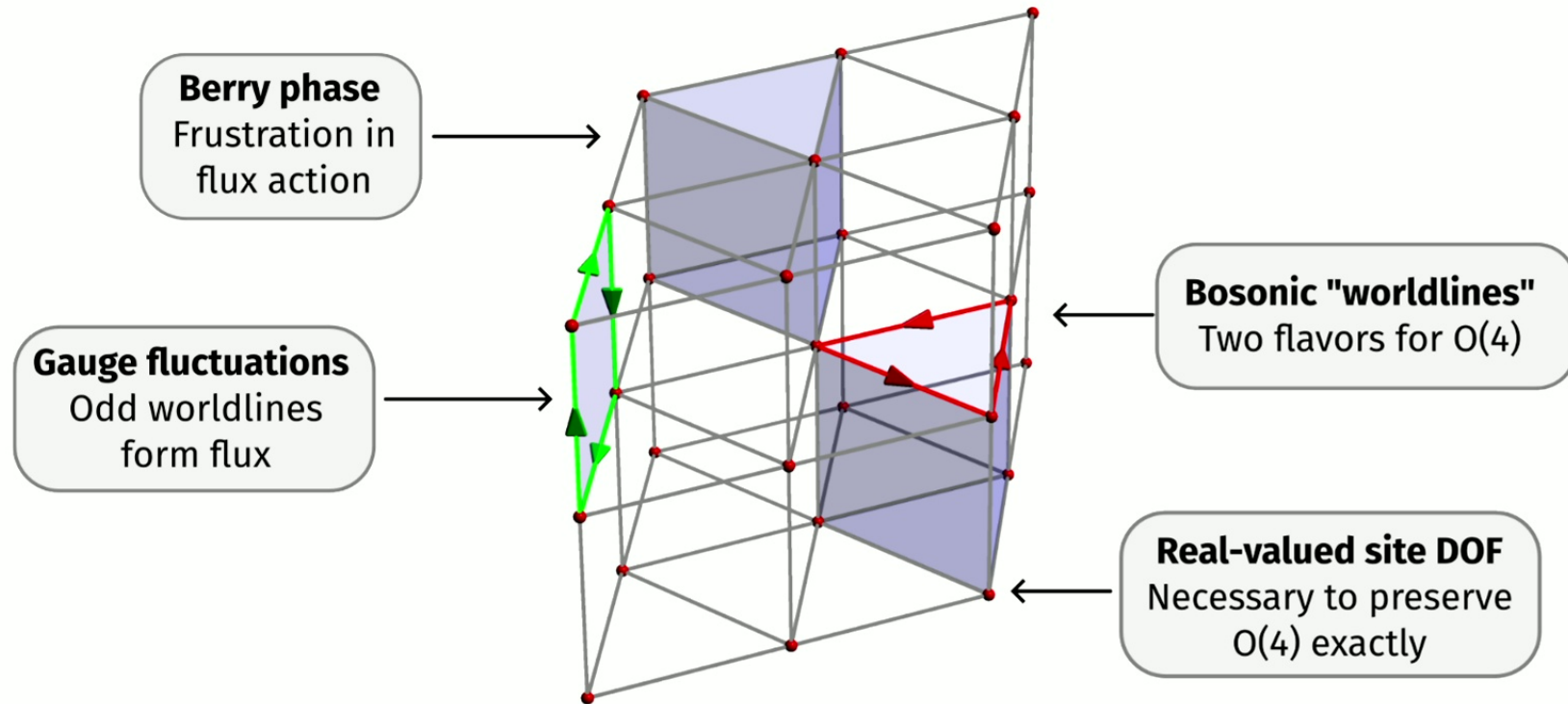
Duality transformation for bosons coupled to \mathbb{Z}_2 gauge fields



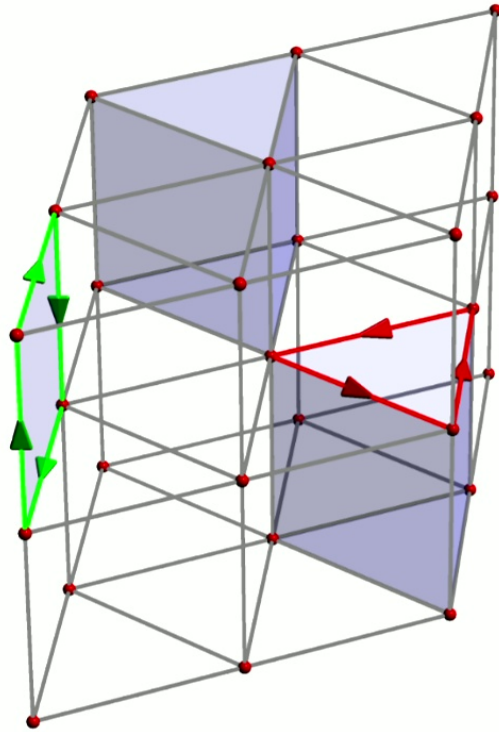
Duality transformation for bosons coupled to \mathbb{Z}_2 gauge fields



Duality transformation for bosons coupled to \mathbb{Z}_2 gauge fields



Phases identifiable in dual representation



Magnetic order
(Asymmetric) proliferation
of current loops

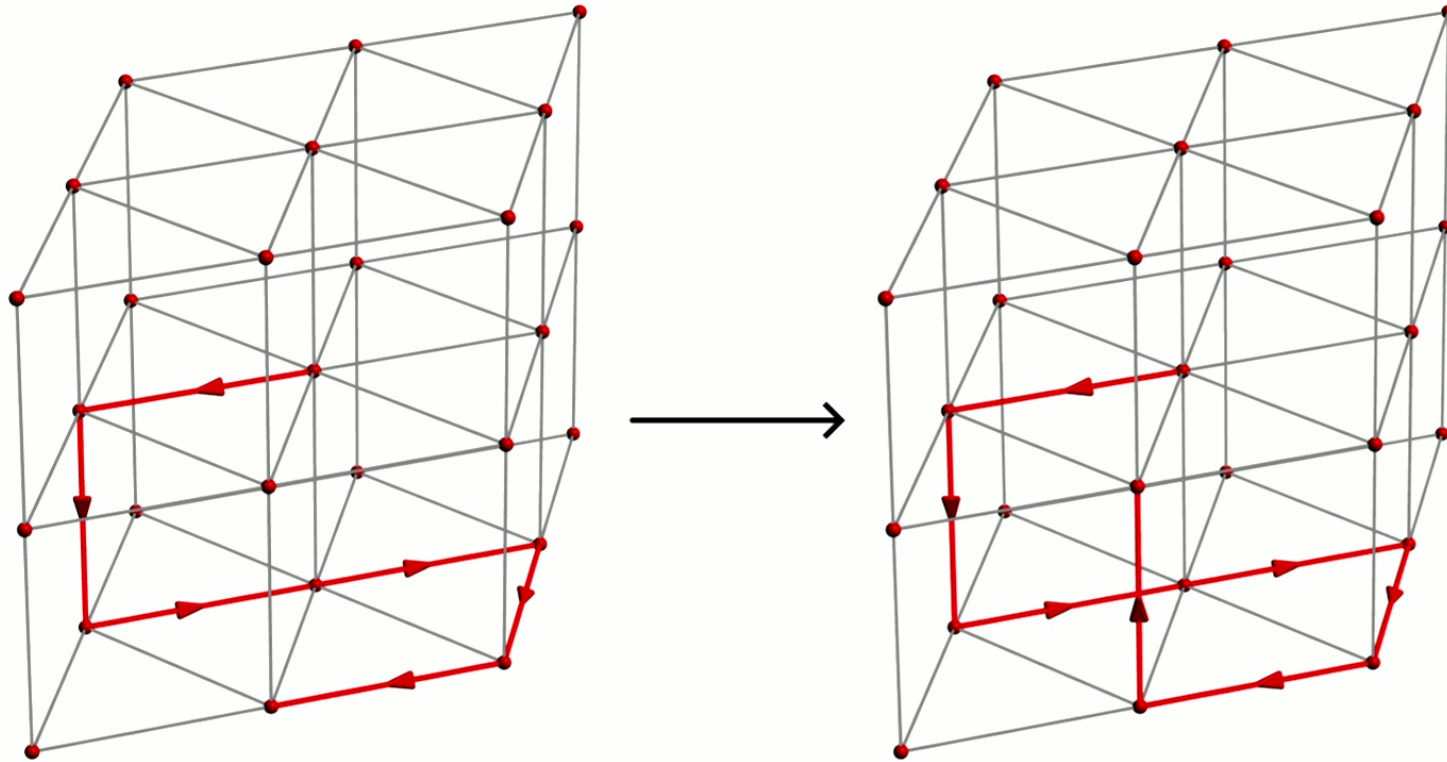
A diagram of a triangular lattice with magnetic moments represented by arrows. The arrows are colored black, blue, and red, showing a pattern of proliferation.

VBS order
Trans. symmetry breaking
of flux configurations

A diagram of a triangular lattice with flux configurations represented by red shaded regions between the lattice sites.

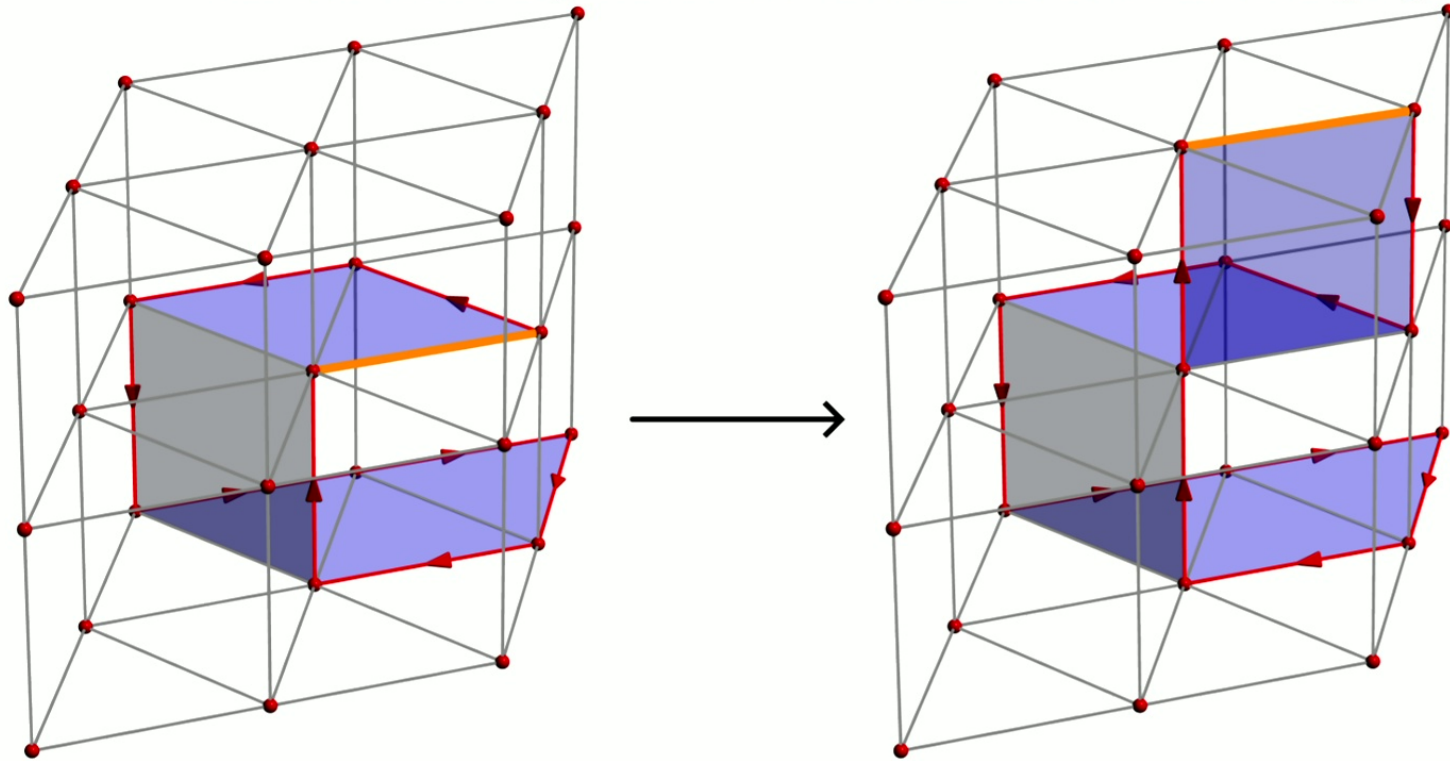
Worm algorithms difficult with gauge fluctuations

“Classical worm algorithm” effective without gauge fluctuations

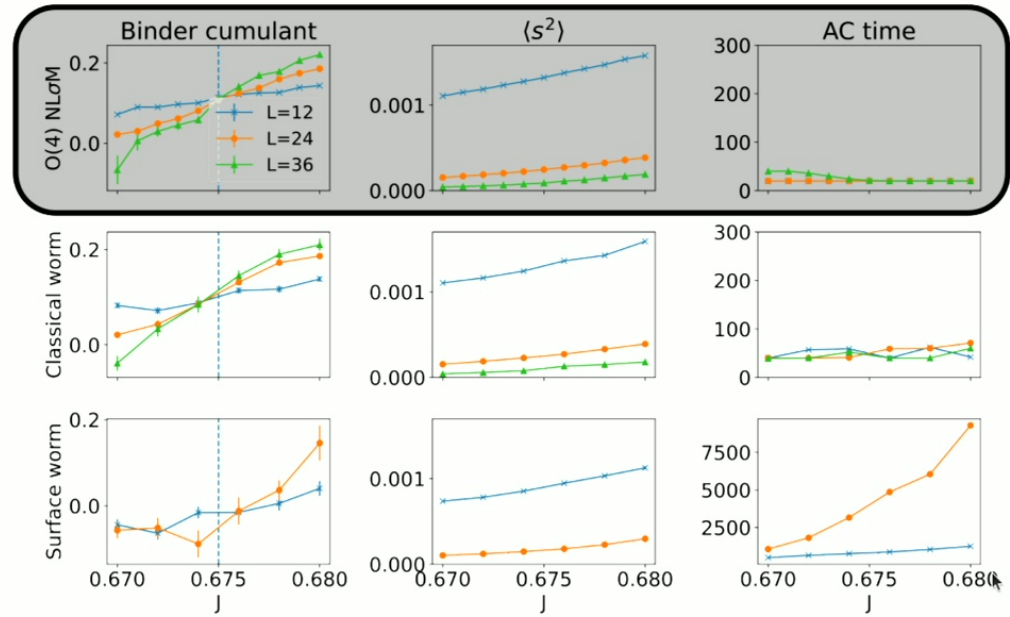
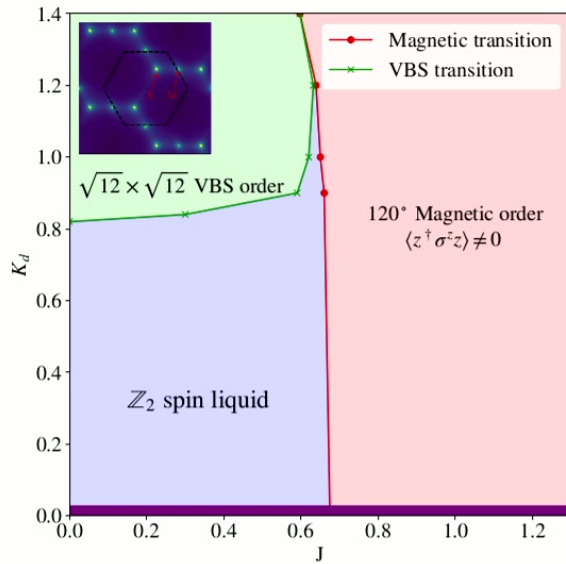


Worm algorithms difficult with gauge fluctuations

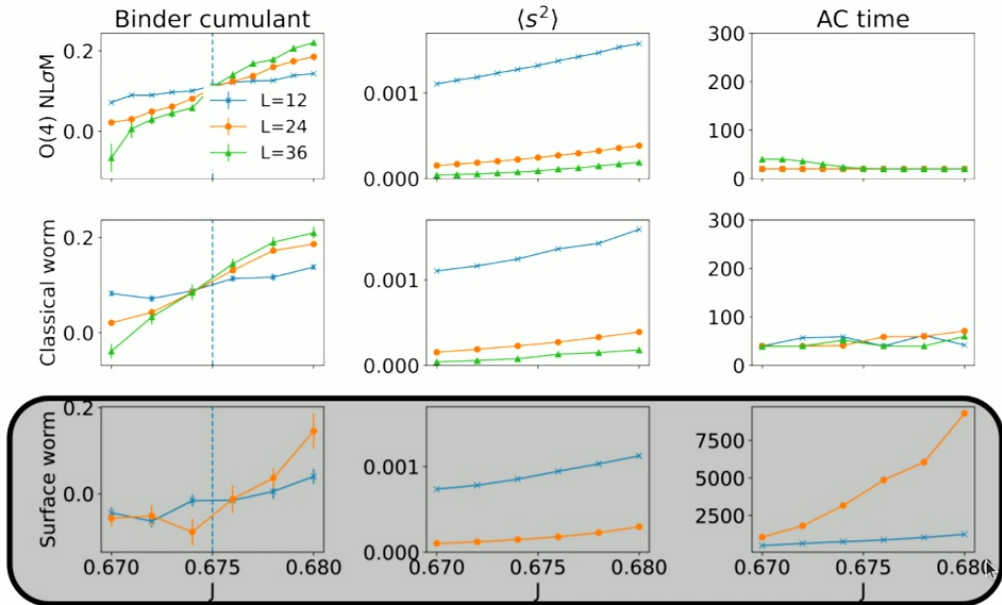
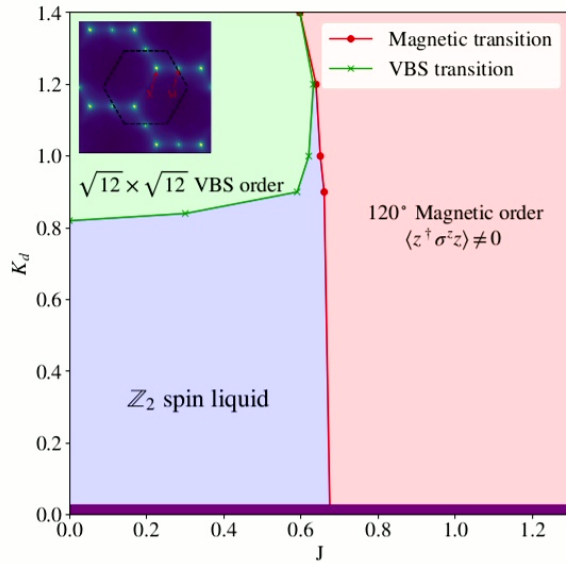
“Surface worm algorithm” works well but still has diverging AC



Monte Carlo simulations establish AF, VBS, and spin liquid phases

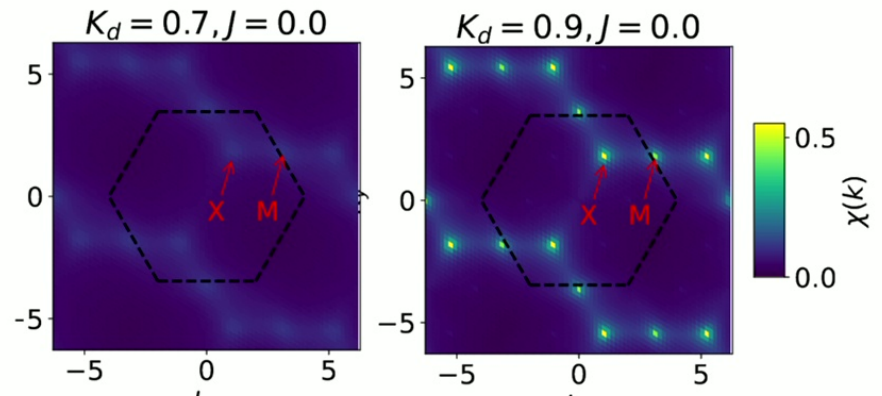
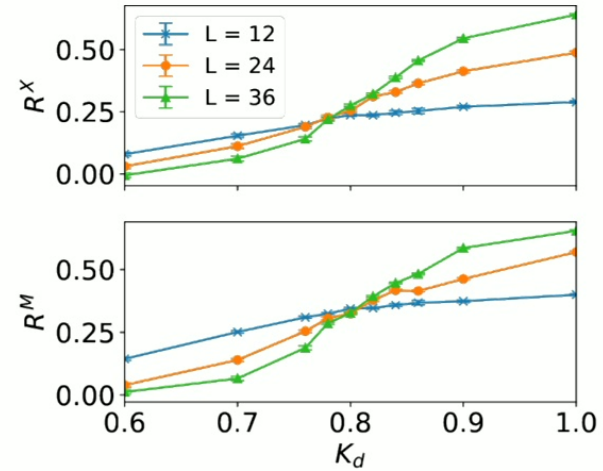
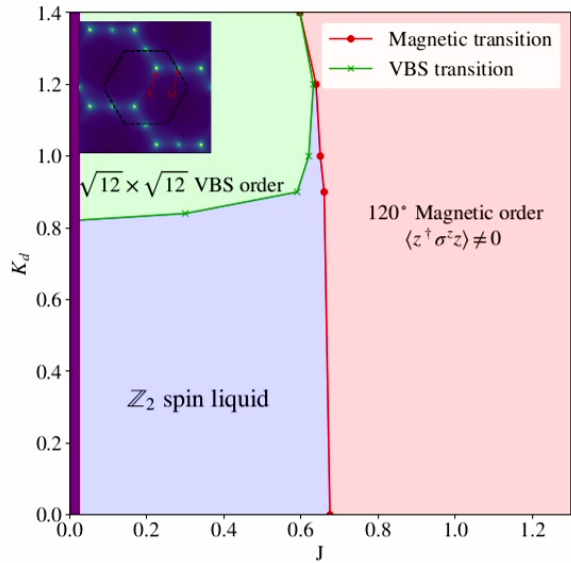


Monte Carlo simulations establish AF, VBS, and spin liquid phases

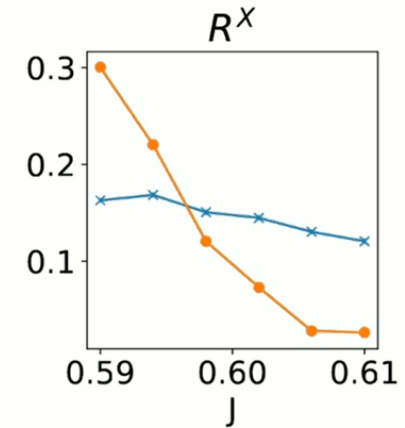
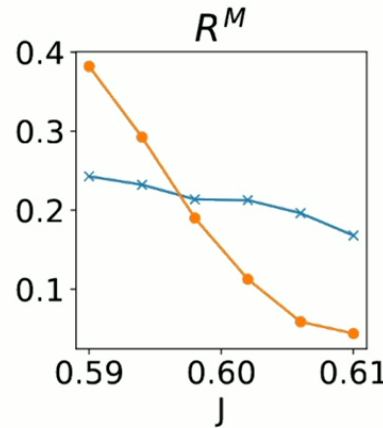
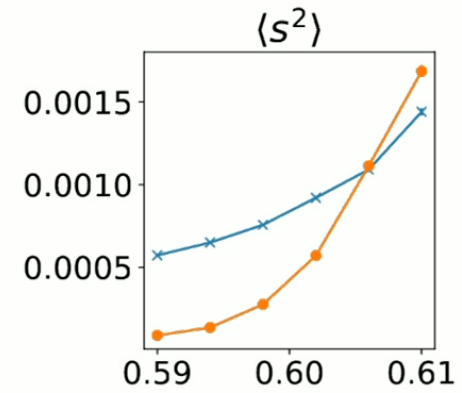
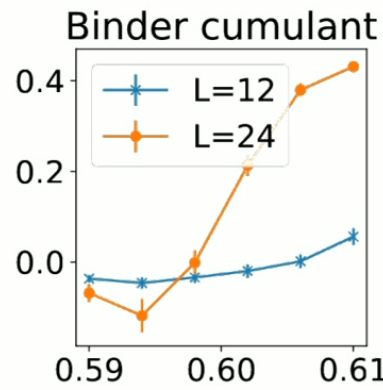
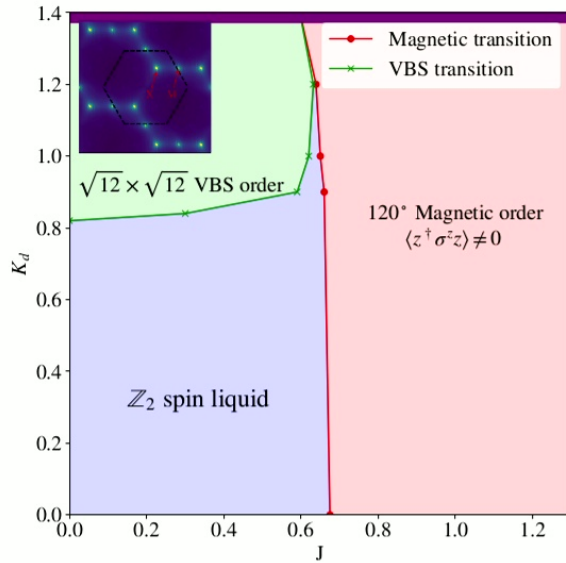


SWA still identifies transition, although restricted to small systems

Monte Carlo simulations establish AF, VBS, and spin liquid phases

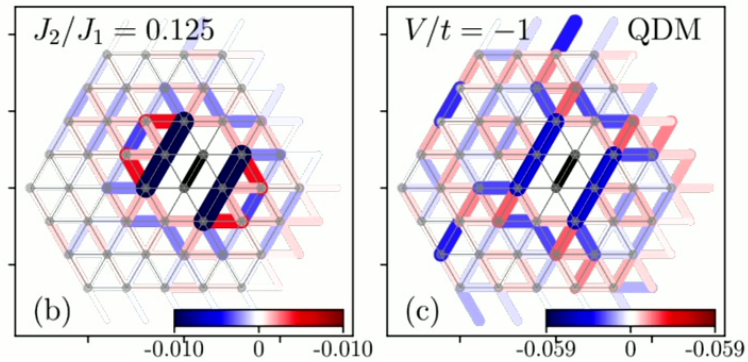
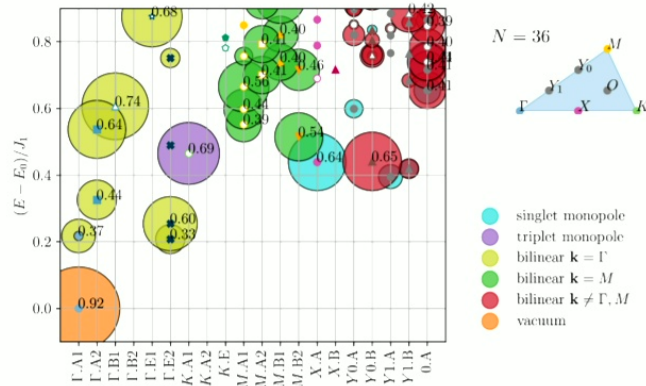


Monte Carlo simulations establish AF, VBS, and spin liquid phases



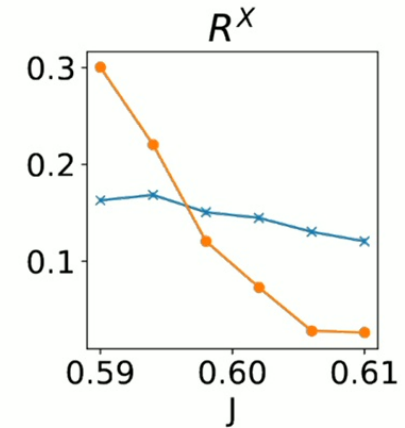
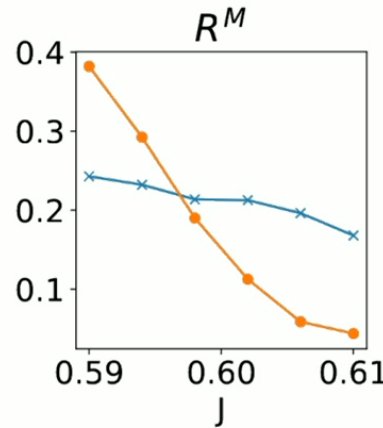
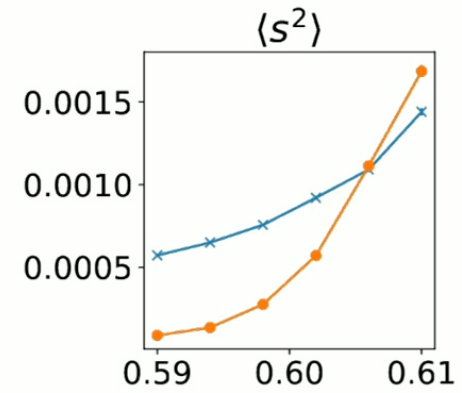
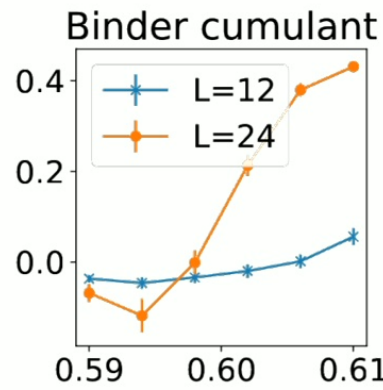
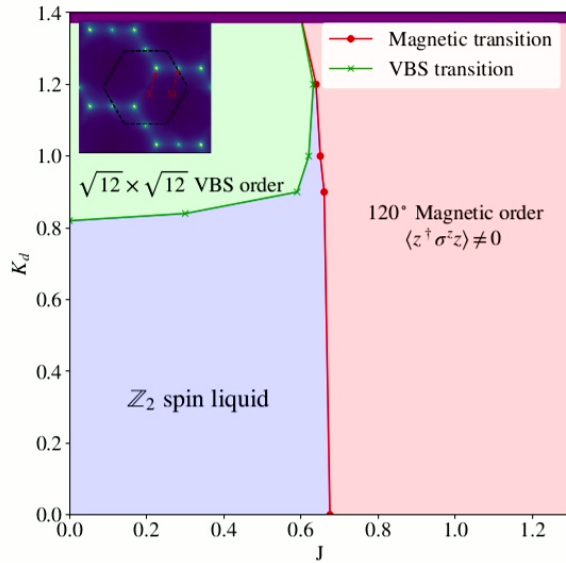
Applications to Heisenberg models

Low-energy spectrum of $J_1 - J_2$ model has high overlap with Dirac spin liquid and $\sqrt{12} \times \sqrt{12}$ VBS¹¹



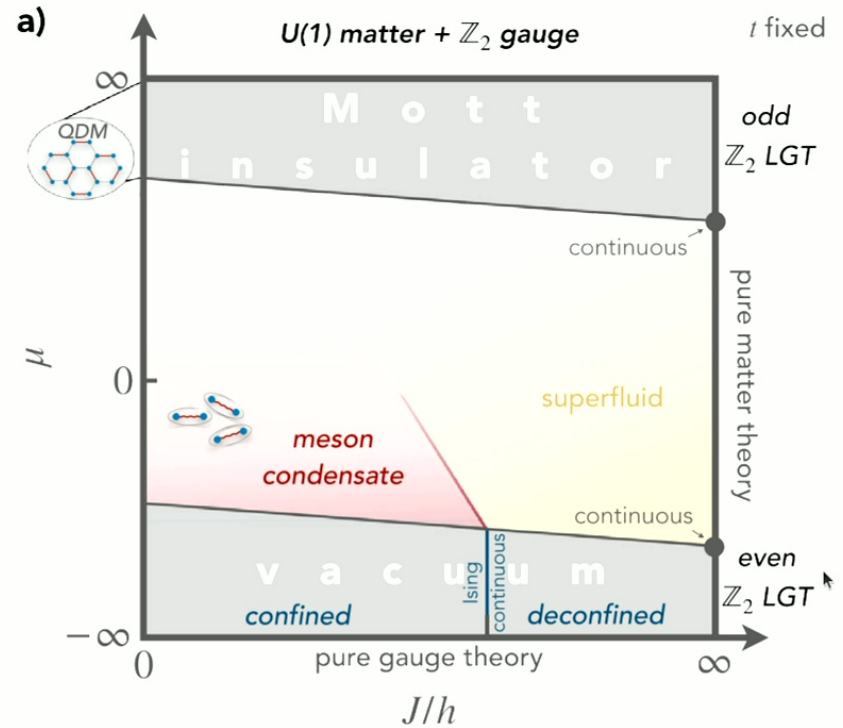
¹¹Wietek, Capponi, and Läuchli, *arXiv e-prints*, 2023.

Monte Carlo simulations establish AF, VBS, and spin liquid phases



Outlook and future directions

- Bosons coupled to discrete gauge fields remains a relatively unexplored research direction, also relevant for quantum simulators ¹²
- PIMC formulation is rather rudimentary, can this mapping be applied to continuous time? SSE?



¹²Homeier et al., *Commun. Phys.*, 2023