Title: Twisted Holography Mini-Course - Lecture 20231214

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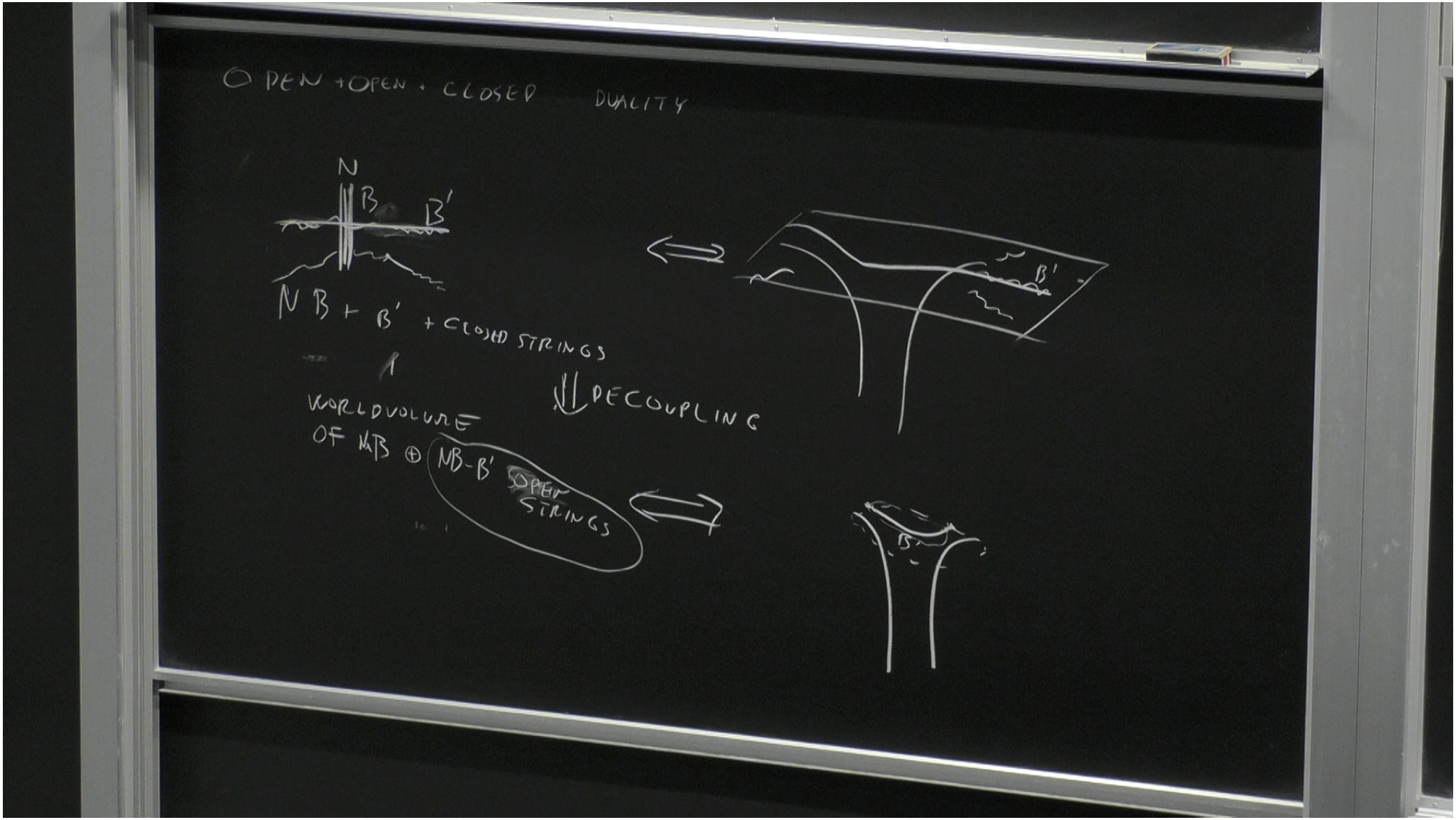
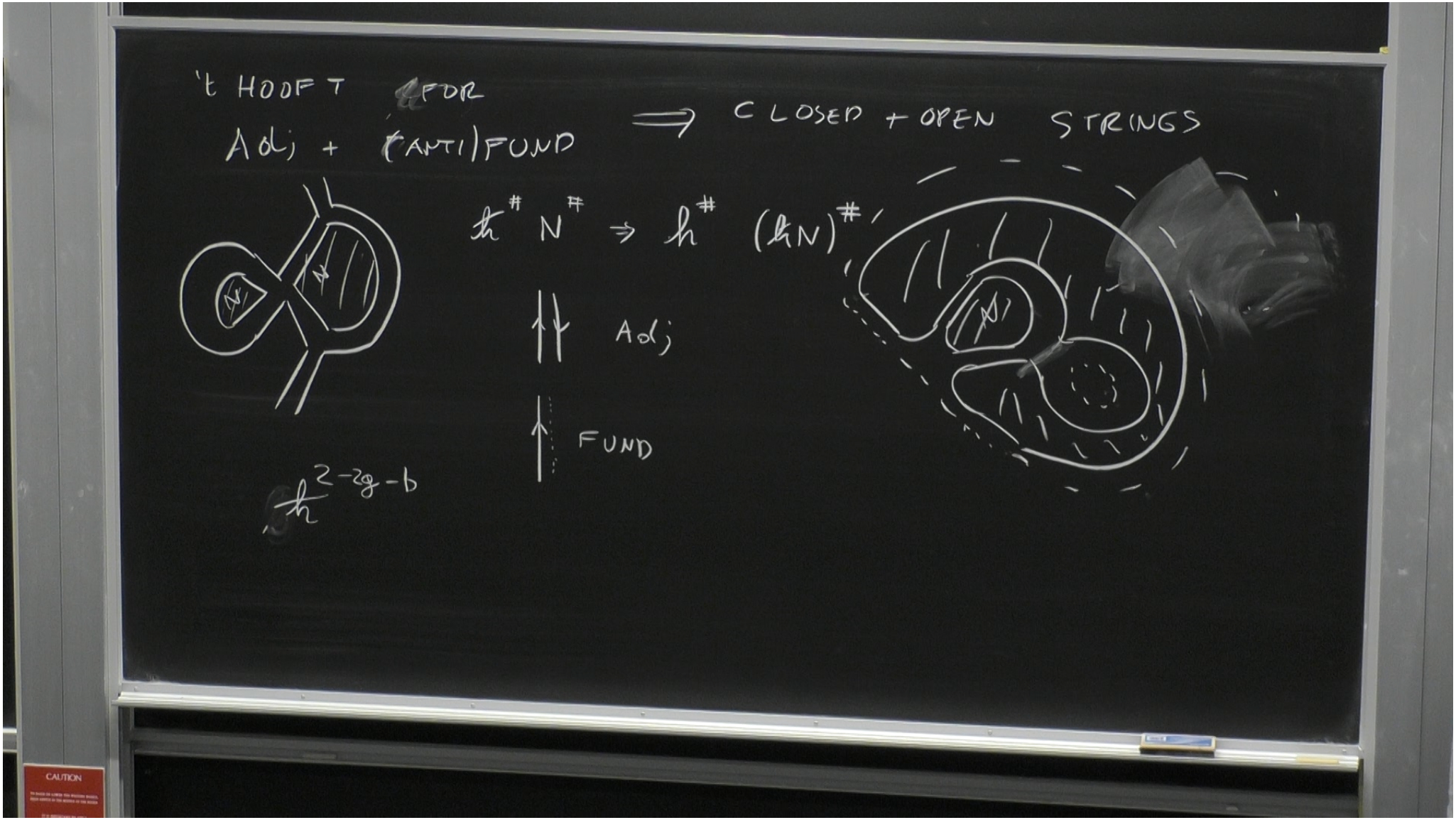
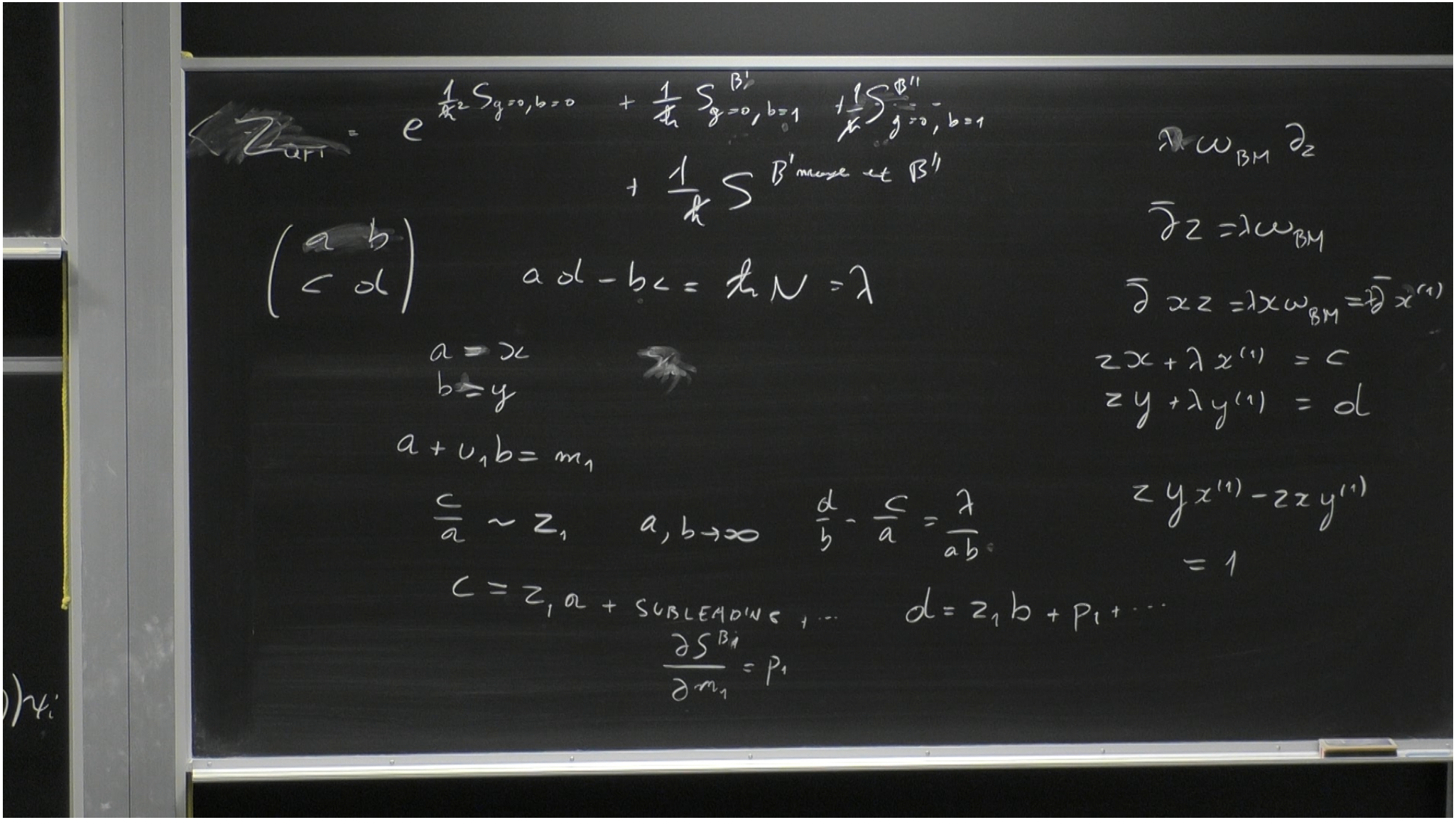
Collection: Twisted Holography Mini-Course

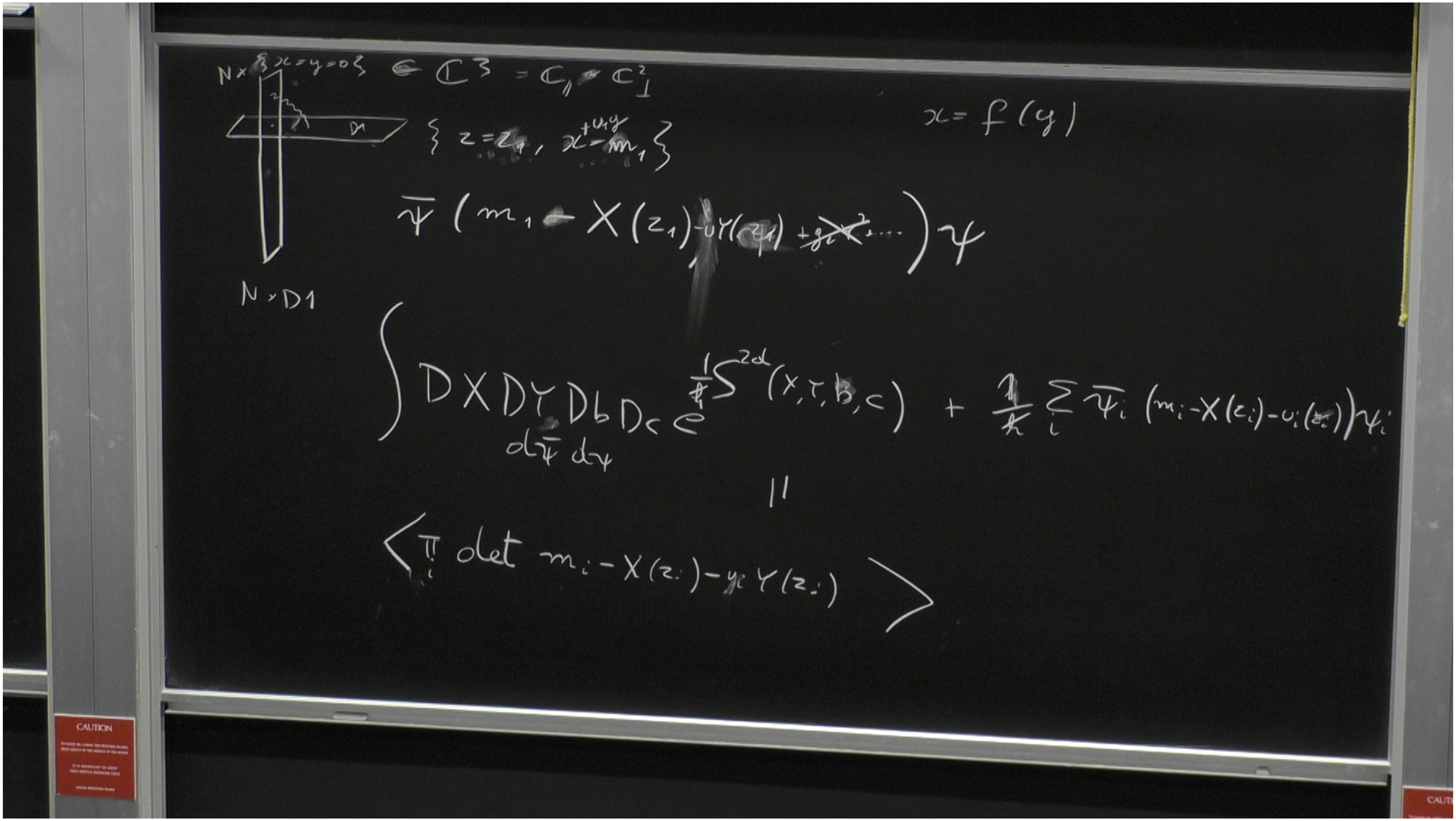
Date: December 14, 2023 - 9:00 AM

URL: https://pirsa.org/23120018

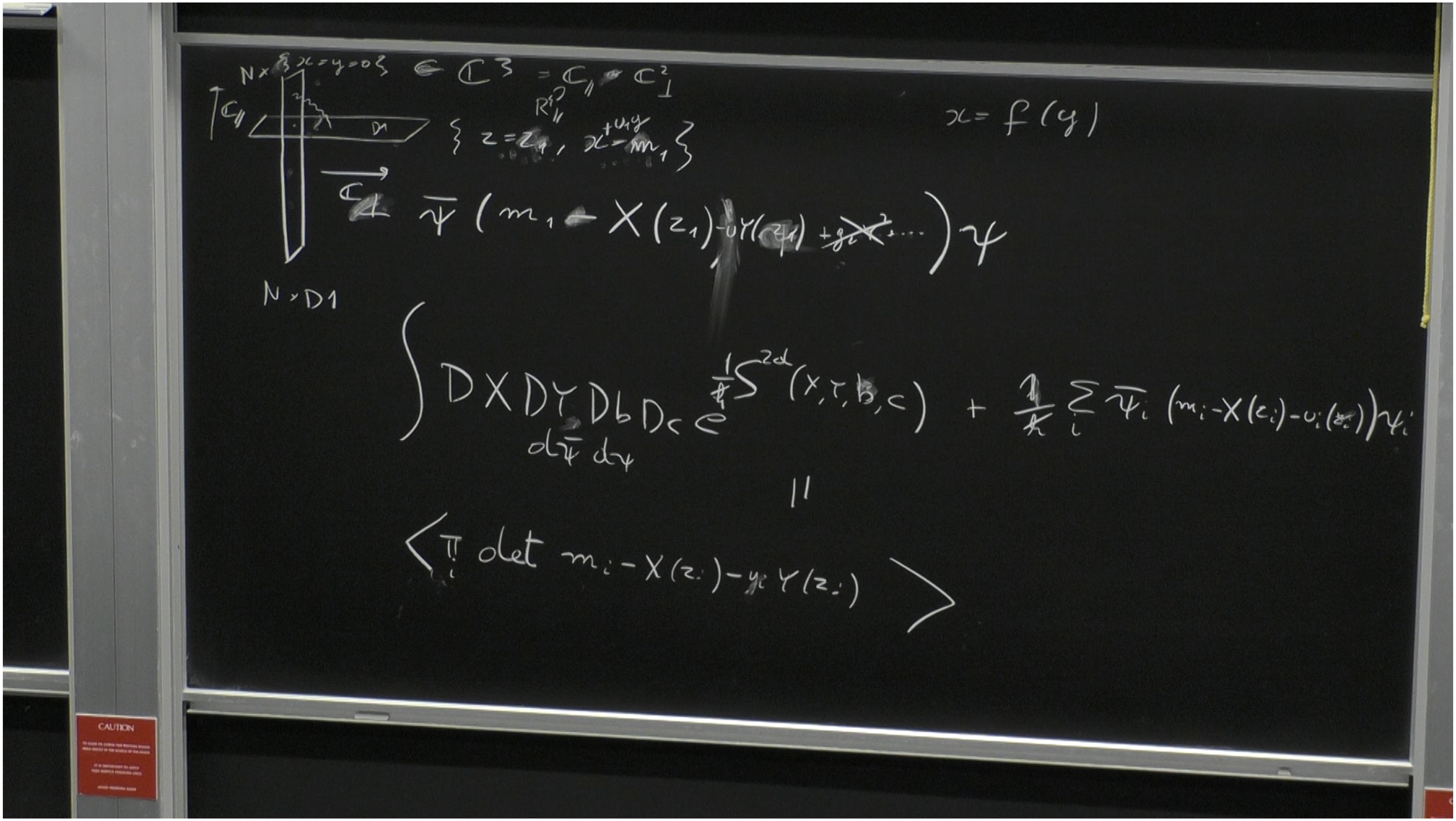

OPEN + OPEN + CLOSED DUALITY
N
B
B'
N B + B' + CLOSED STRINGS
WORLDVOLUME OF NB ⊕ NB-B'
OPEN STRINGS
DECOUPLING
B'
B


't HOOFT FOR
Adj + (ANTI)FUND
CLOSED + OPEN STRINGS
$t^{\#} N^{\#} \Rightarrow h^{\#}$ $(tN)^{\#}$
Adj
FUND
t^{2-2g-b}


$Z_{QFT} = e^{\frac{1}{\hbar^2} S_{g=0,b=0} + \frac{1}{\hbar} S^{B'}_{g=0,b=1} + \frac{1}{\hbar} S^{B''}_{g=0,b=1} + \frac{1}{\hbar} S^{B' \text{ mixte et } B''}}$
$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $ad - bc = \hbar N = \lambda$
$a = x$
$b = y$
$a + v_1 b = m_1$
$\frac{c}{a} \sim z_1$ $a, b \to \infty$ $\frac{d}{b} - \frac{c}{a} = \frac{\lambda}{ab}$
$c = z_1 a + \text{SUBLEADING} + \cdots$ $d = z_1 b + p_1 + \cdots$
$\frac{\partial S^{B_1}}{\partial m_1} = p_1$
$\omega_{BM} \partial z$
$\bar{\partial} z = \lambda \omega_{BM}$
$\bar{\partial} x z = \lambda x \omega_{BM} = \bar{\partial} x^{(1)}$
$zx + \lambda x^{(1)} = c$
$zy + \lambda y^{(1)} = d$
$zy x^{(1)} - zx y^{(1)} = 1$



$N \times \{x=y=0\} \in \mathbb{C}^3 = \mathbb{C}_{\parallel} \times \mathbb{C}^2_{\perp}$
$\{z = z_1,\ x + u_1 y = m_1\}$
$x = f(y)$
$\bar{\Psi}(m_1 - X(z_1) - u_1 Y(z_1) + \dots)\Psi$
$N \times D1$
$\int DX\, DY\, Db\, Dc\, d\bar{\Psi}\, d\Psi\ e^{\frac{1}{\hbar} S^{2d}(X, Y, b, c) + \frac{1}{\hbar} \sum_i \bar{\Psi}_i (m_i - X(z_i) - u_i Y(z_i)) \Psi_i}$
$\parallel$
$\langle \prod_i \det\, m_i - X(z_i) - u_i Y(z_i) \rangle$


N × {x = y = 0} ∈ C³ = C∥ × C²⊥
x = f(y)
{ z = z₁ , x + u₁y = m₁ }
ψ̄ (m₁ − X(z₁) − u₁Y(z₁) + ...) ψ
N × D1
∫ DX DY Db Dc dψ̄ dψ e^(1/ħ ∫²ᵈ (X, Y, b, c) + 1/ħ Σᵢ ψ̄ᵢ (mᵢ − X(zᵢ) − uᵢ(zᵢ)) ψᵢ)
=
⟨ ∏ᵢ det mᵢ − X(zᵢ) − yᵢ Y(zᵢ) ⟩
CAUTION

$$\frac{\partial S^{Bi}}{\partial m_1} = P_1$$

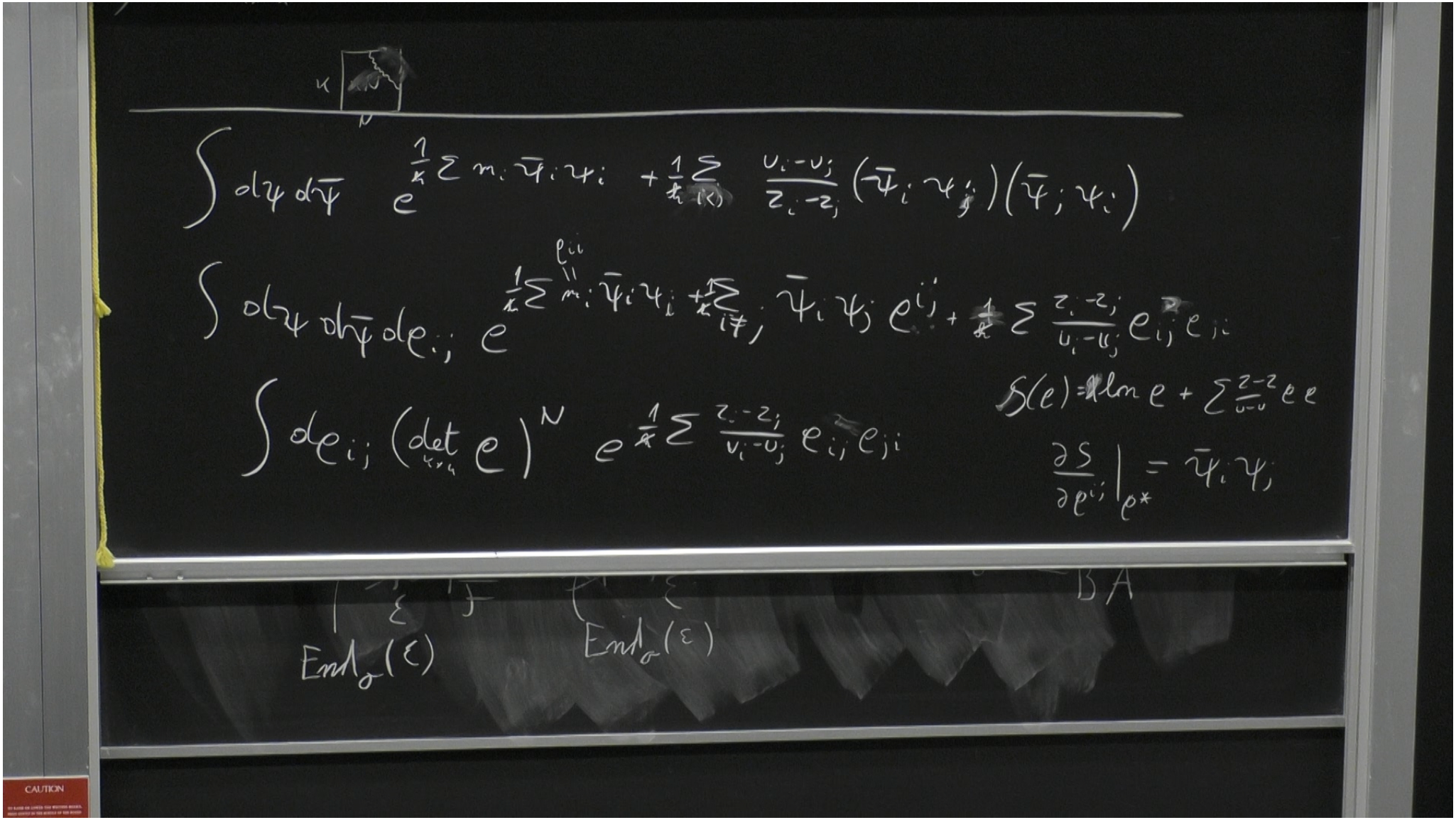
$$\det_{N_k \times N_k} \left(I_{K\times K} \otimes I_{N\times N} \; K_{K\times K} \otimes X_{N\times N} - S_{K\times K} \otimes I_{N\times N} \right)$$

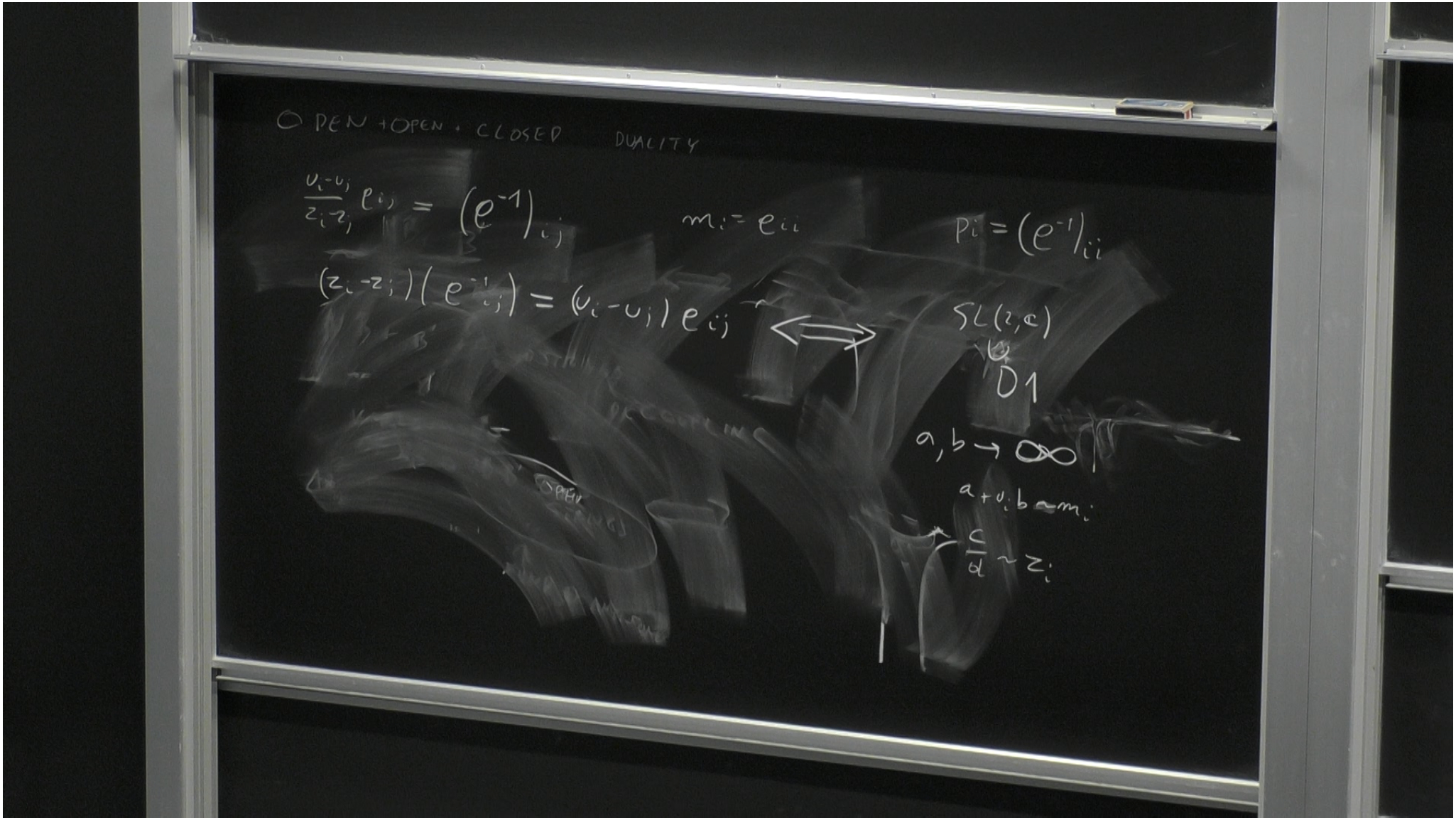
$$\int d\psi\, d\bar\psi \; e^{\frac{1}{\hbar}\sum m_i \bar\psi_i \psi_i + \frac{1}{\hbar}\sum_{i\neq j} \frac{v_i - v_j}{z_i - z_j} (\bar\psi_i \psi_j)(\bar\psi_j \psi_i)}$$

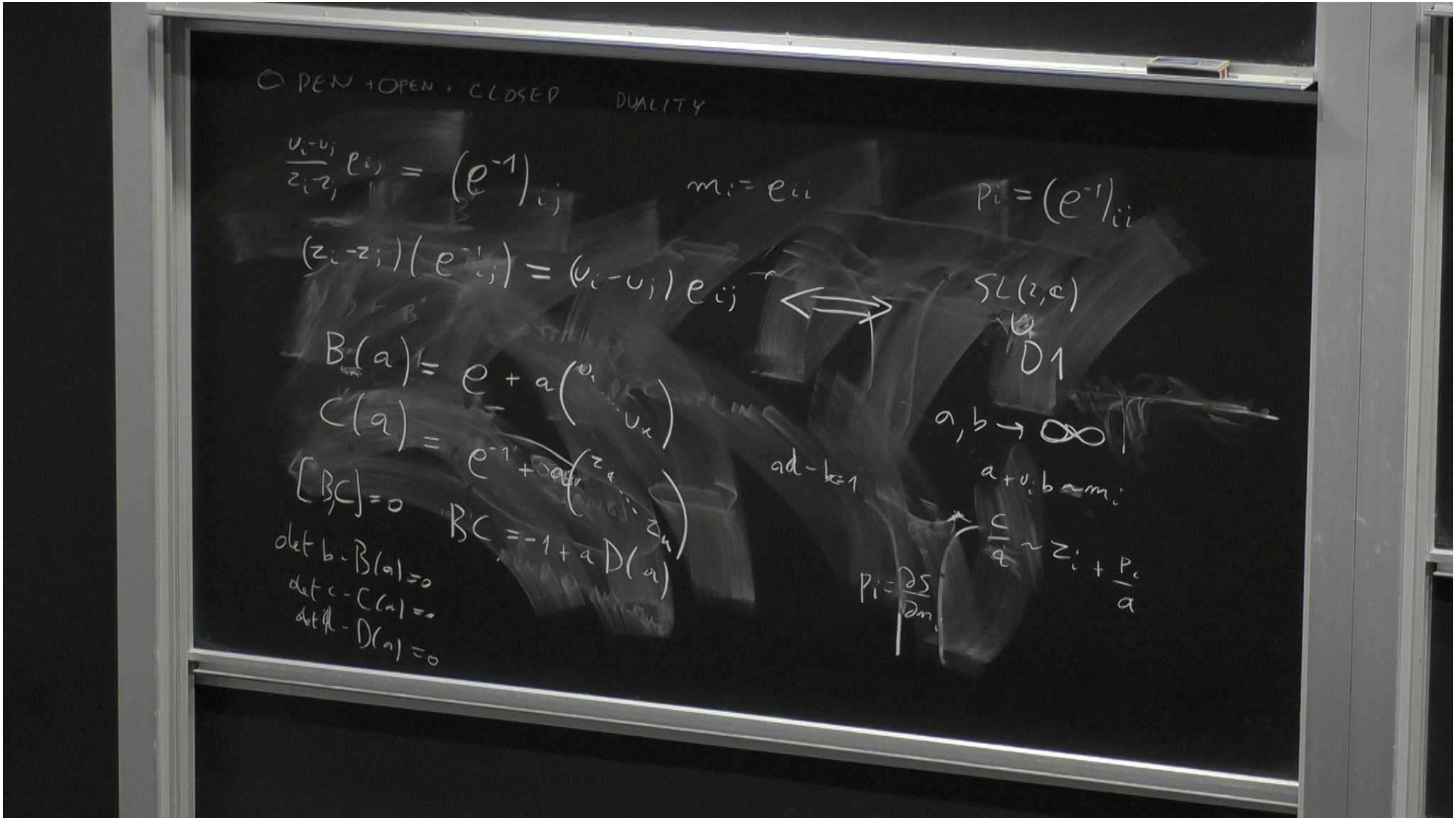
$$\int d\psi\, d\bar\psi\, de_{ij} \; e^{\frac{1}{\hbar}\sum \overbrace{e_{ii}}^{} m_i \bar\psi_i \psi_i + \frac{1}{\hbar}\sum_{i\neq j} \bar\psi_i \psi_j e^{ij} + \frac{1}{\hbar}\sum \frac{z_i - z_j}{v_i - v_j} e_{ij} e_{ji}}$$

$$\int de_{ij} \, (\det e)^N \; e^{\frac{1}{\hbar}\sum \frac{z_i - z_j}{v_i - v_j} e_{ij} e_{ji}}$$

$$S(e) = N \ln e + \sum \frac{z-z}{v-v} e e$$


$\int d\psi\, d\bar\psi\; e^{\frac{1}{\hbar}\sum m_i \bar\psi_i \psi_i + \frac{1}{\hbar}\sum_{i<j} \frac{v_i - v_j}{z_i - z_j}(\bar\psi_i \psi_j)(\bar\psi_j \psi_i)}$
$\int d\psi\, d\bar\psi\, de_{ij}\; e^{\frac{1}{\hbar}\sum m_i \bar\psi_i \psi_i + \frac{1}{\hbar}\sum_{i\neq j} \bar\psi_i \psi_j e^{ij} + \frac{1}{\hbar}\sum \frac{z_i - z_j}{v_i - v_j} e_{ij} e_{ji}}$
$e_{ii} = m_i$
$\int de_{ij} (\det_{i,j} e)^N e^{\frac{1}{\hbar}\sum \frac{z_i - z_j}{v_i - v_j} e_{ij} e_{ji}}$
$S(e) = \hbar \ln e + \sum \frac{z-z}{v-v} e e$
$\frac{\partial S}{\partial e^{ij}}\Big|_{e^*} = \bar\psi_i \psi_j$
$End_{\mathcal{O}}(\mathcal{E})$
$End_{\mathcal{O}}(\mathcal{E})$
BA


OPEN + OPEN + CLOSED DUALITY
$\frac{u_i - u_j}{z_i - z_j} e_{ij} = (e^{-1})_{ij}$
$m_i = e_{ii}$
$p_i = (e^{-1})_{ii}$
$(z_i - z_j)(e^{-1}_{ij}) = (u_i - u_j) e_{ij}$
$SL(2;\mathbb{C})$
D1
$a, b \to \infty$
$a + u_i b \sim m_i$
$\frac{c}{d} \sim z_i$


OPEN + OPEN + CLOSED DUALITY
$\frac{v_i - v_j}{z_i - z_j} e_{ij} = (e^{-1})_{ij}$
$m_i = e_{ii}$
$p_i = (e^{-1})_{ii}$
$(z_i - z_j)(e^{-1}_{ij}) = (v_i - v_j) e_{ij}$
$SL(2,\mathbb{C})$
$D1$
$B(a) = e + a \begin{pmatrix} v_1 & & \\ & \ddots & \\ & & v_N \end{pmatrix}$
$C(a) = e^{-1} + a \begin{pmatrix} z_1 & & \\ & \ddots & \\ & & z_N \end{pmatrix}$
$a, b \to \infty$
$a + v_i b \sim m_i$
$ad - bc = 1$
$[B, C] = 0$
$BC = -1 + a D(a)$
$\det b \cdot B(a) = 0$
$\det c \cdot C(a) = 0$
$\det ac \cdot D(a) = 0$
$p_i = \frac{\partial S}{\partial m_i}$
$\frac{c}{a} \sim z_i + \frac{p_i}{a}$