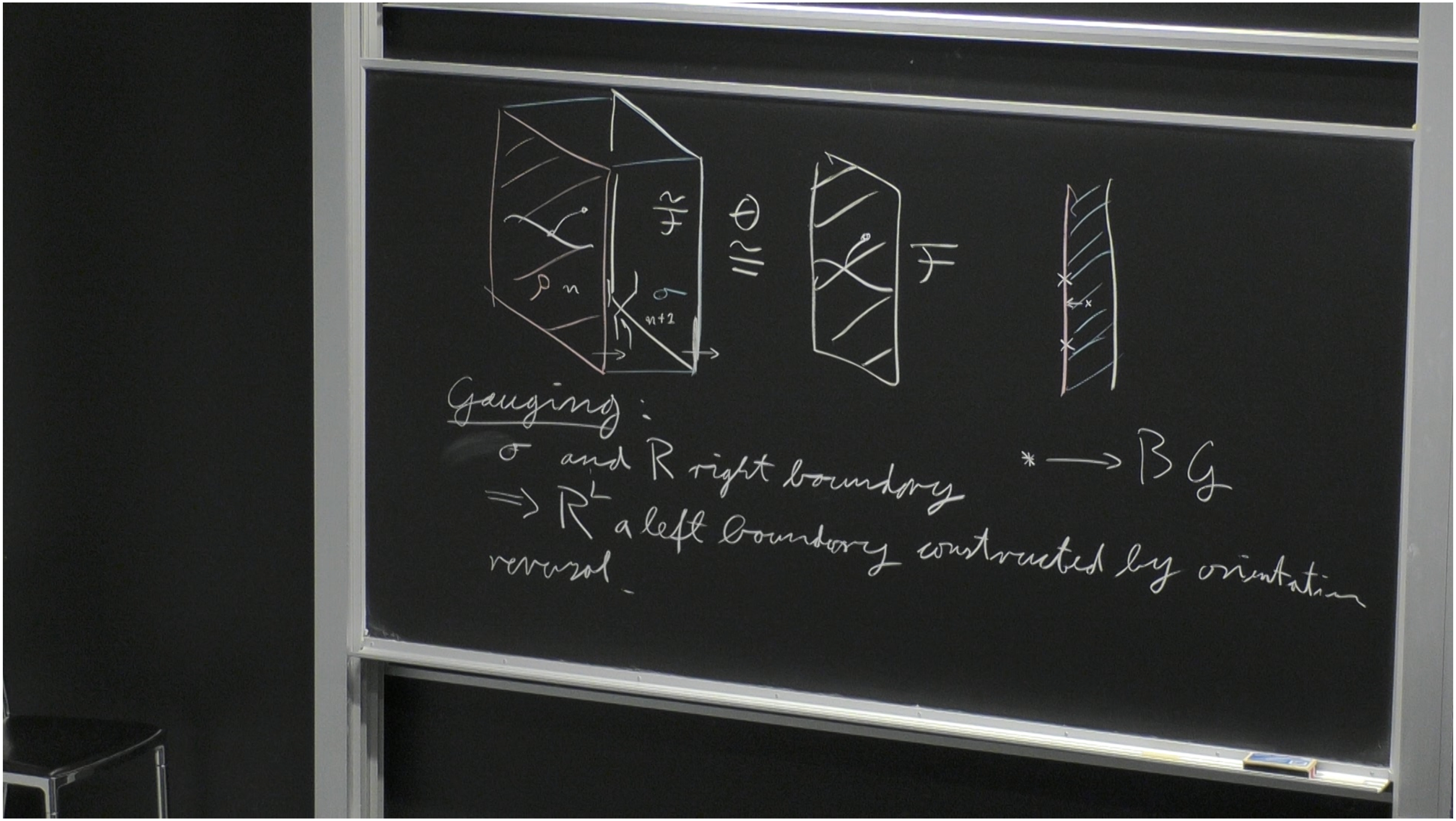
Title: Topological Quantum Field Theories Lecture 20231208

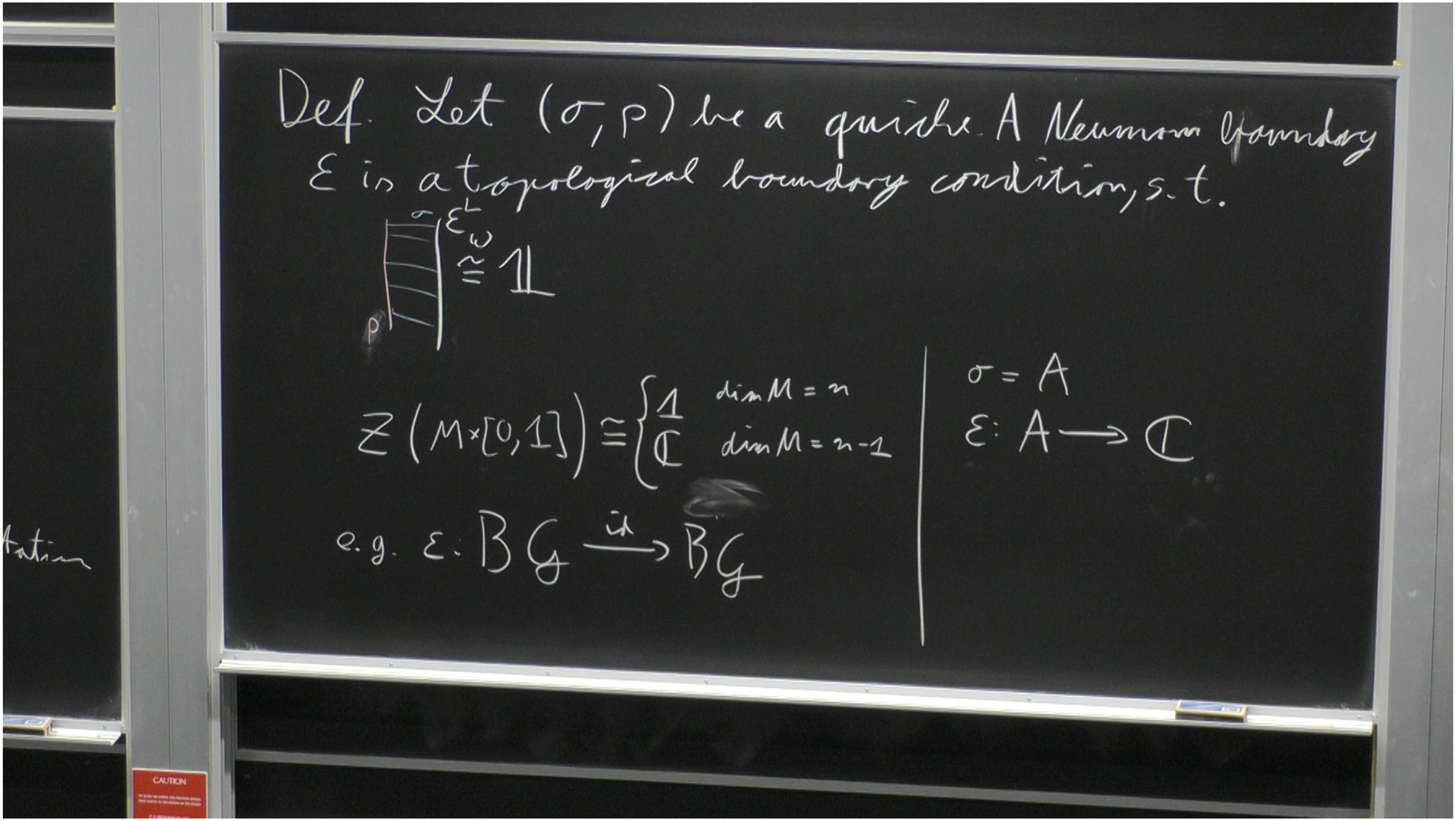
Speakers: Lukas Mueller

Collection: Topological Quantum Field Theories - mini-course

Date: December 08, 2023 - 2:00 PM

URL: https://pirsa.org/23120017


F̃
Θ ≅
F
ρ n
σ n+1
Gauging:
σ and R right boundary
* ⟶ BG
⟹ R^L a left boundary constructed by orientation reversal.


Def. Let (σ, ρ) be a quiche. A Neumann boundary
ε is a topological boundary condition, s.t.
≅ 𝟙
Z(M×[0,1]) ≅ { 1 dim M = n
ℂ dim M = n-1
e.g. ε: BG —id→ BG
σ = A
ε: A ⟶ ℂ

Def. Let (σ, ρ) be a quiche. A Neumann boundary $\mathcal{E}$ is a topological boundary condition, s.t.

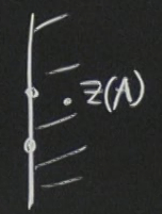
$$\mathcal{E}^{L}_{\omega} \cong \mathbb{1}$$

$$Z(M \times [0,1]) \cong \begin{cases} \mathbb{1} & \dim M = n \\ \mathbb{C} & \dim M = n-1 \end{cases}$$

e.g. $\mathcal{E} : BG \xrightarrow{id} BG$

$(\sigma, \rho) = A \qquad A \otimes_A \mathbb{C}$

$\mathcal{E} : A \longrightarrow \mathbb{C} = A \otimes \mathbb{C} / \{ a \triangleleft b \otimes \lambda \sim a \otimes \iota(b) \lambda \}$



$A \otimes M \to M$

$A \otimes_A \mathbb{C}$

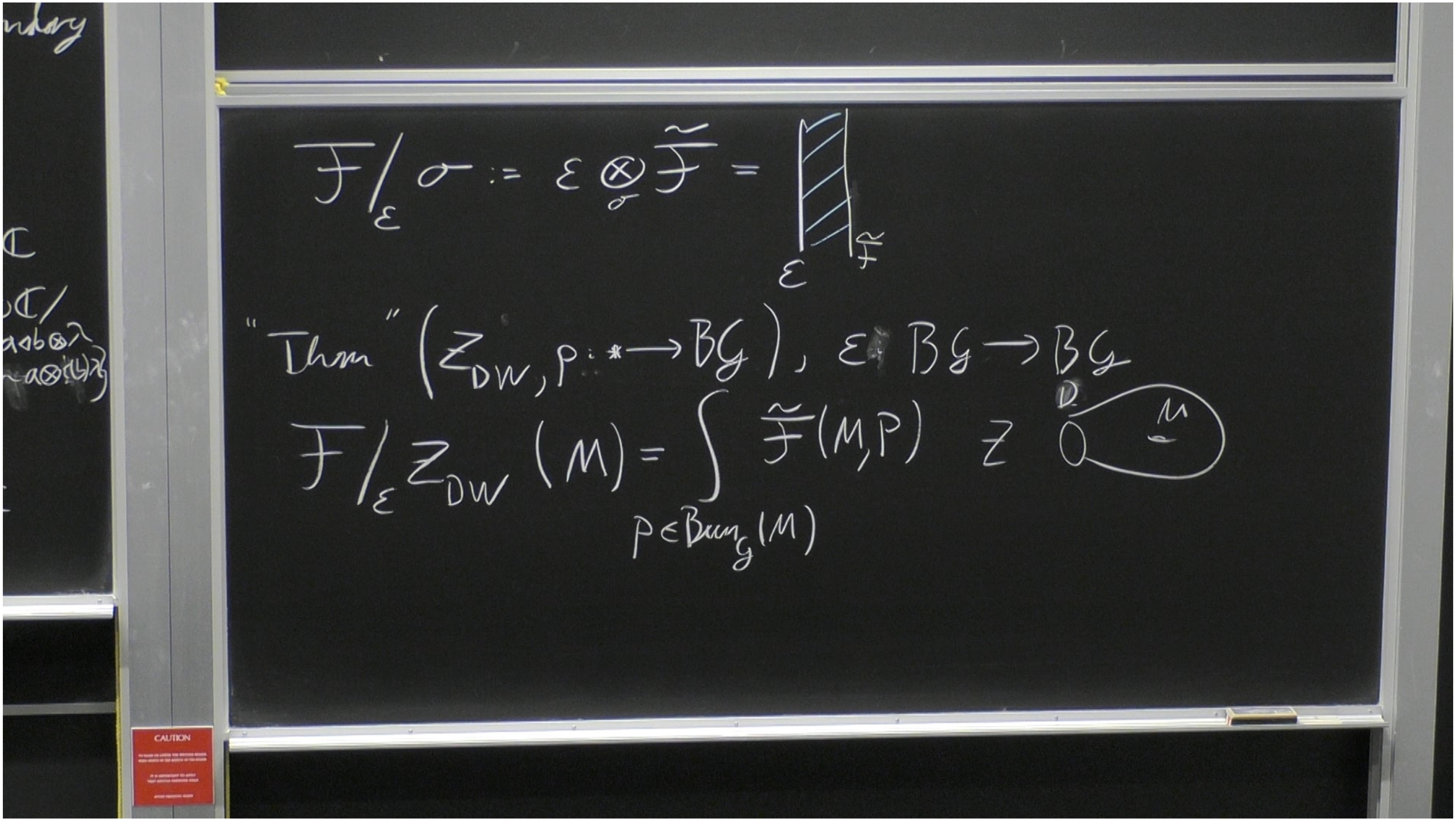
Proof:

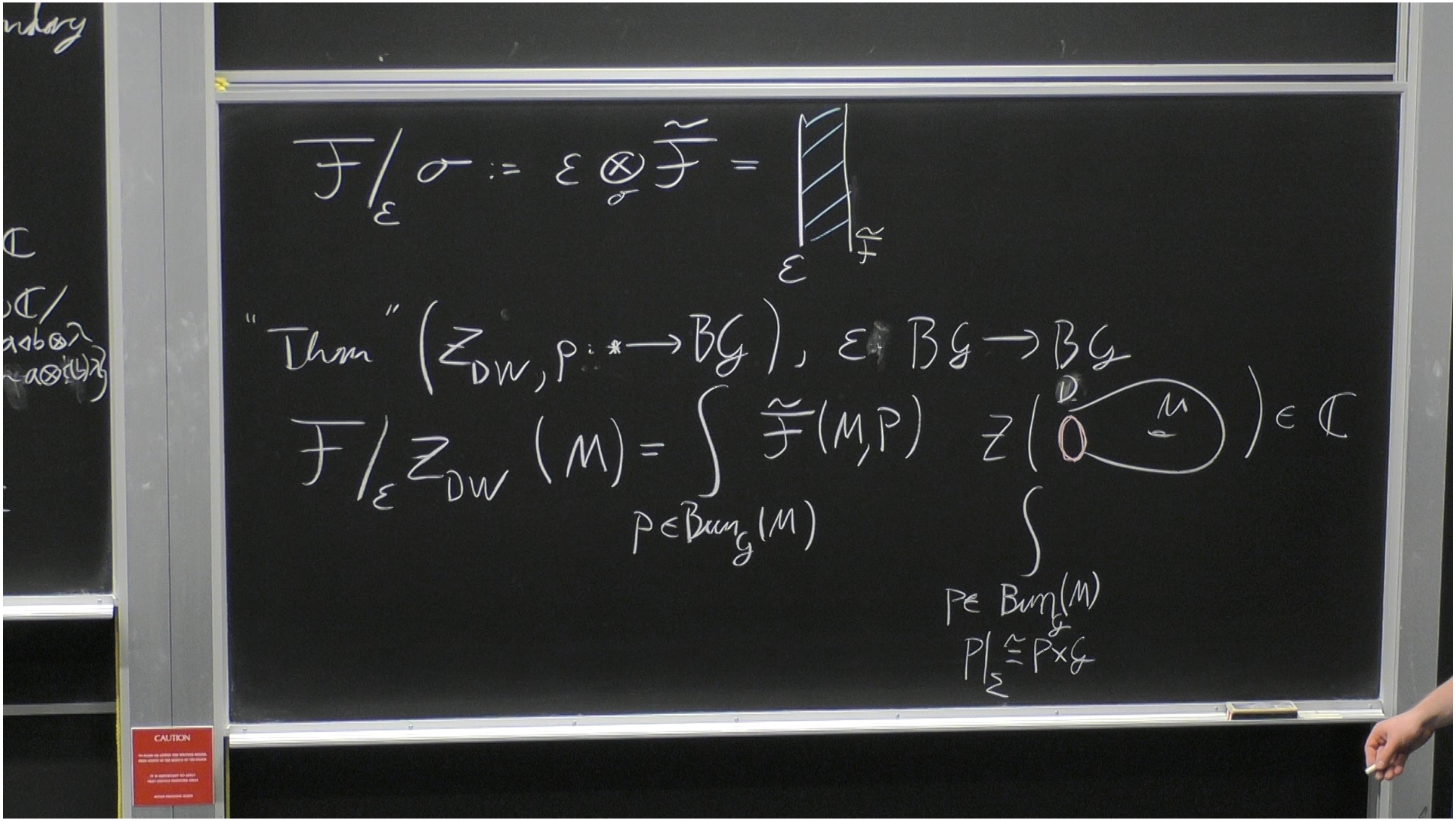
$$Z_{DW}\left(\Sigma \times M\right) \in Z_{DW}(M) = \mathrm{Fun}\left(\mathrm{Bun}_G(M), \mathbb{C}\right)$$

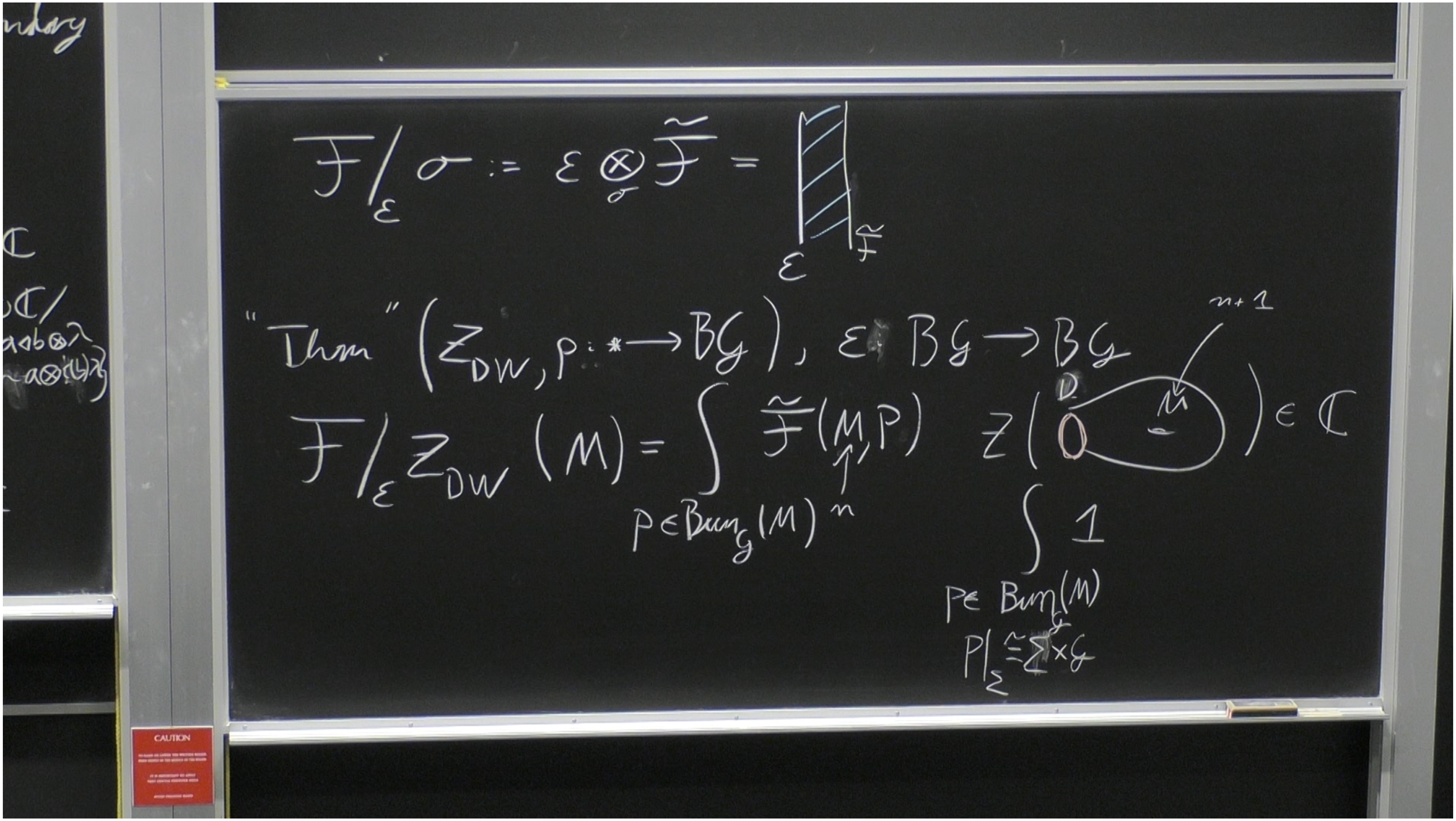
$$\Psi_\Sigma : P \longmapsto 1$$

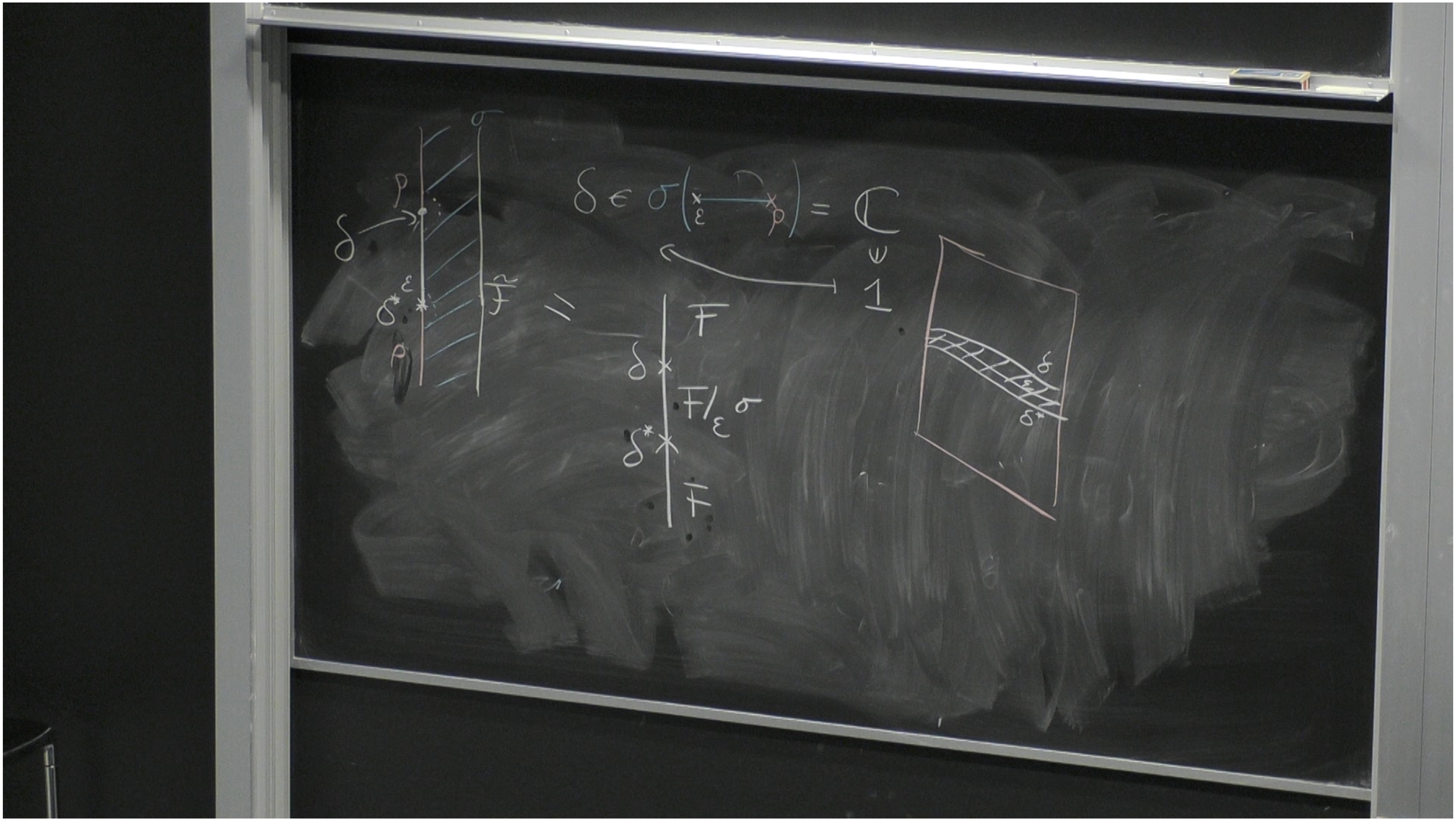
$$\langle \Psi_\Sigma, \tilde{\mathcal{F}} \rangle = \int_{P \in \mathrm{Bun}_G(M)} \Psi_\Sigma(P)\, \tilde{\mathcal{F}}(P, M) = 1$$

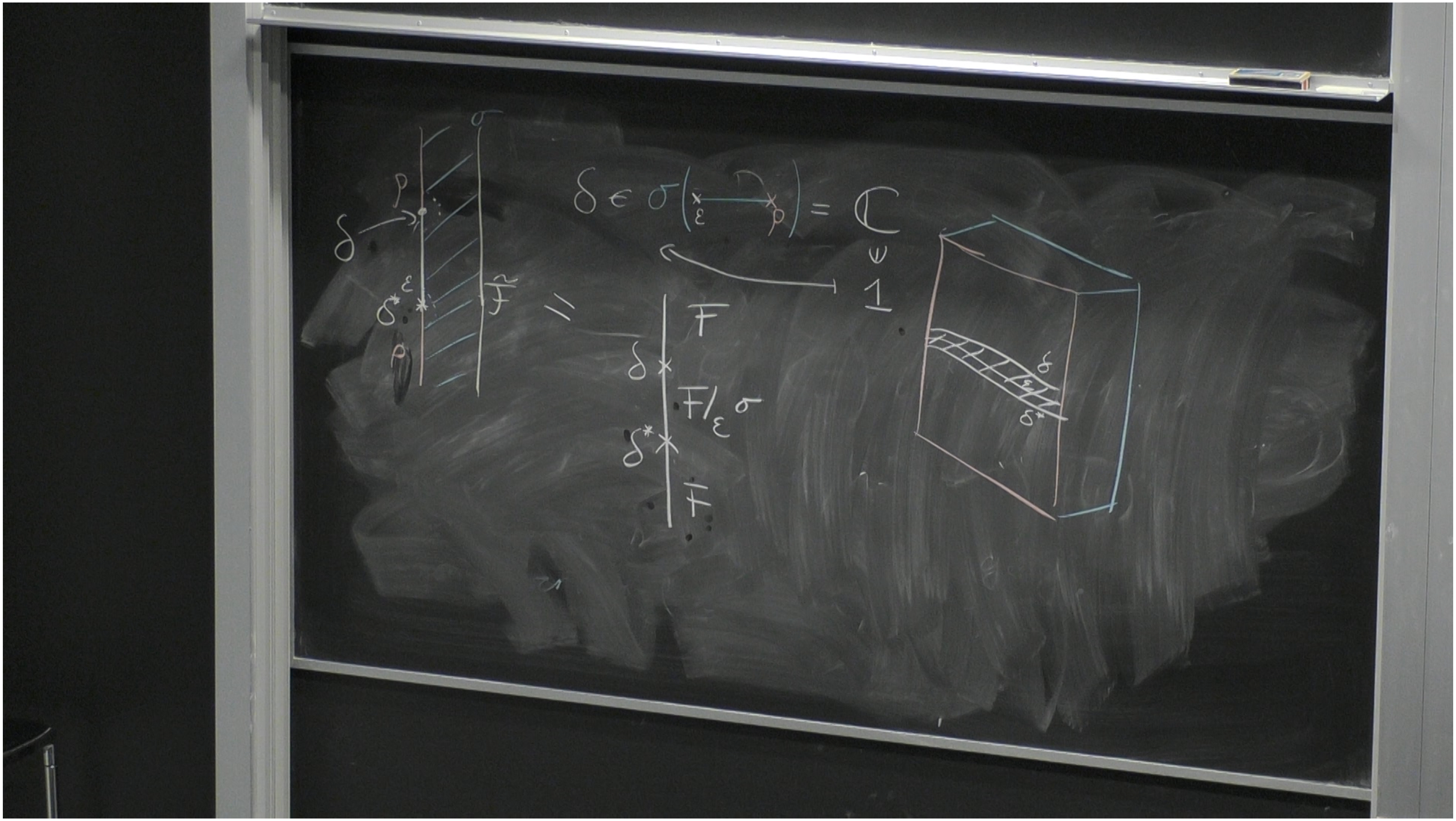
$\square$

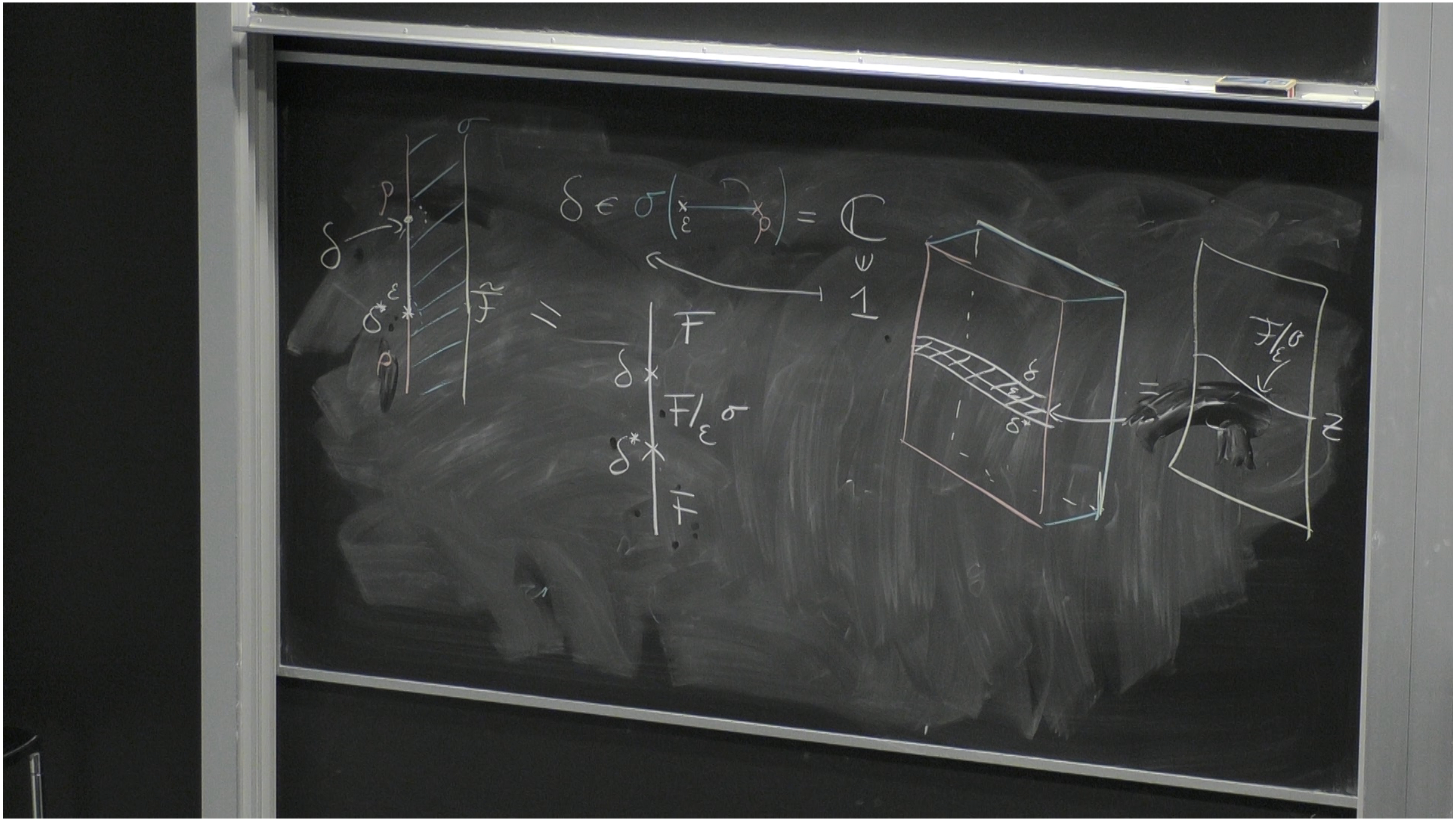

F/_ε σ := ε ⊗_σ F̃ =
ε
F̃
"Thm" (Z_DW, p: * → BG), ε: BG → BG
F/_ε Z_DW (M) = ∫_{p∈Bun_G(M)} F̃(M,P) Z
M


F/ε σ := ε ⊗σ F̃ =
ε
F̃
"Thm" (ZDW, ρ: * → BG), ε: BG → BG
F/ε ZDW (M) = ∫ F̃(M,P) Z(D M) ∈ ℂ
P ∈ BunG(M)
P ∈ BunG(M)
P|Σ ≅ P×G

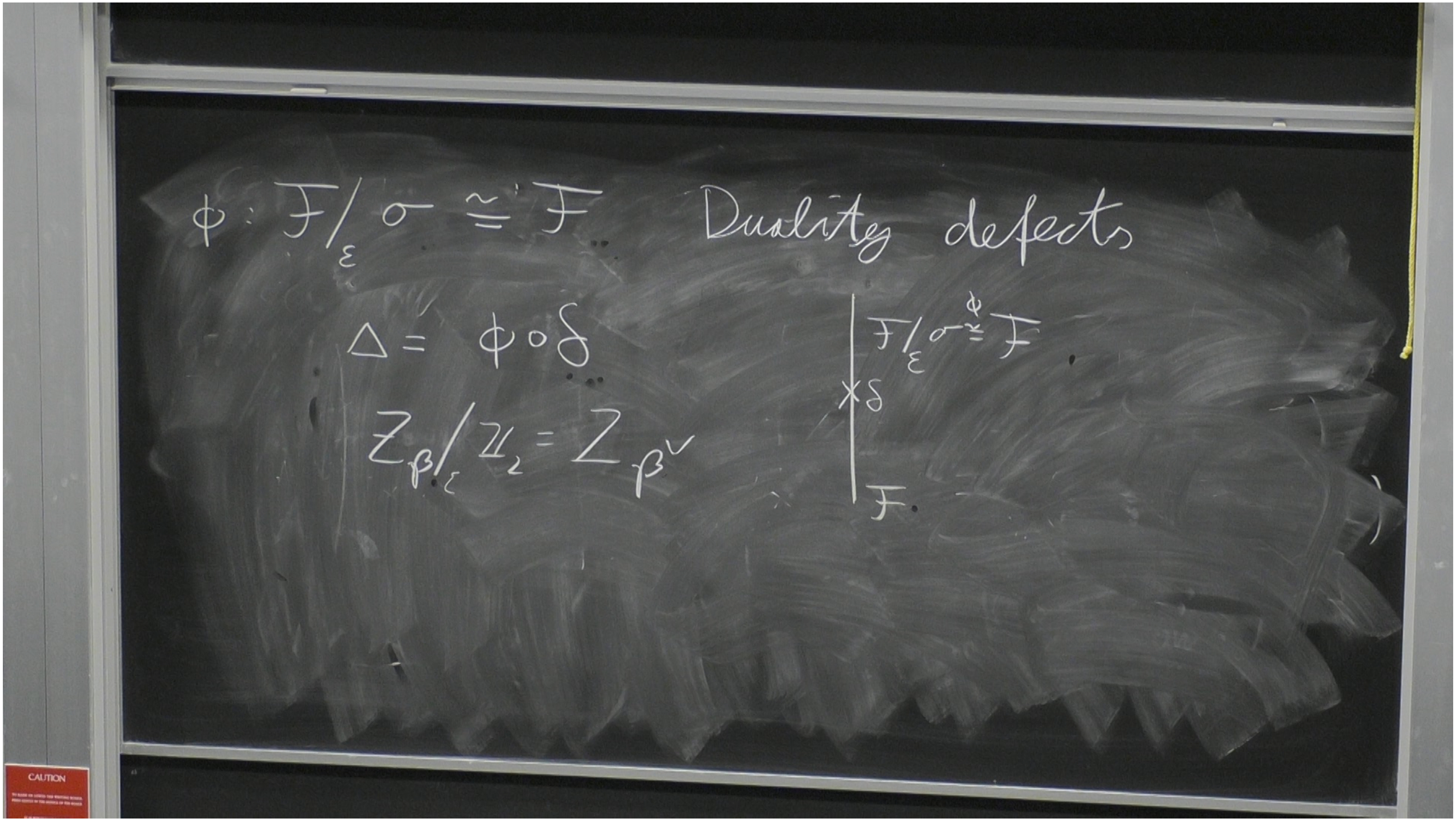

F/ε σ := ε ⊗σ F̃ =
ε
F̃
"Thm" (Z_DW, p: * → BG), ε: BG → BG
n+1
F/ε Z_DW (M) = ∫ F̃(M,P)
p ∈ Bun_G(M)
n
Z(M) ∈ ℂ
∫ 1
P ∈ Bun_G(M)
P|Σ ≅ Σ × G
CAUTION

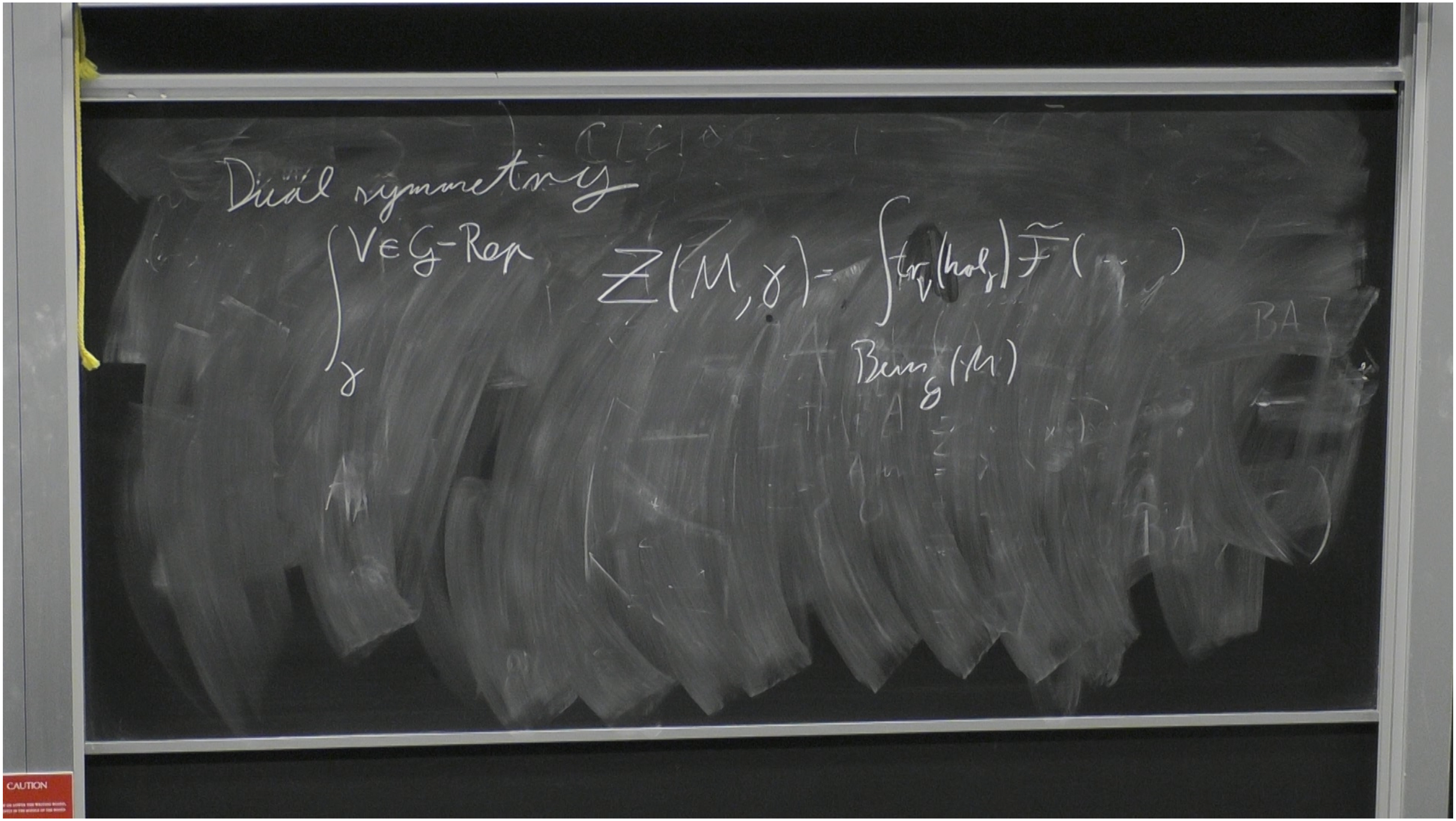

σ
p
δ
ε
δ*
F̃
=
δ ∈ σ (ε p) = ℂ
1
F
δ
F/ε σ
δ*
F
δ
ε
δ*

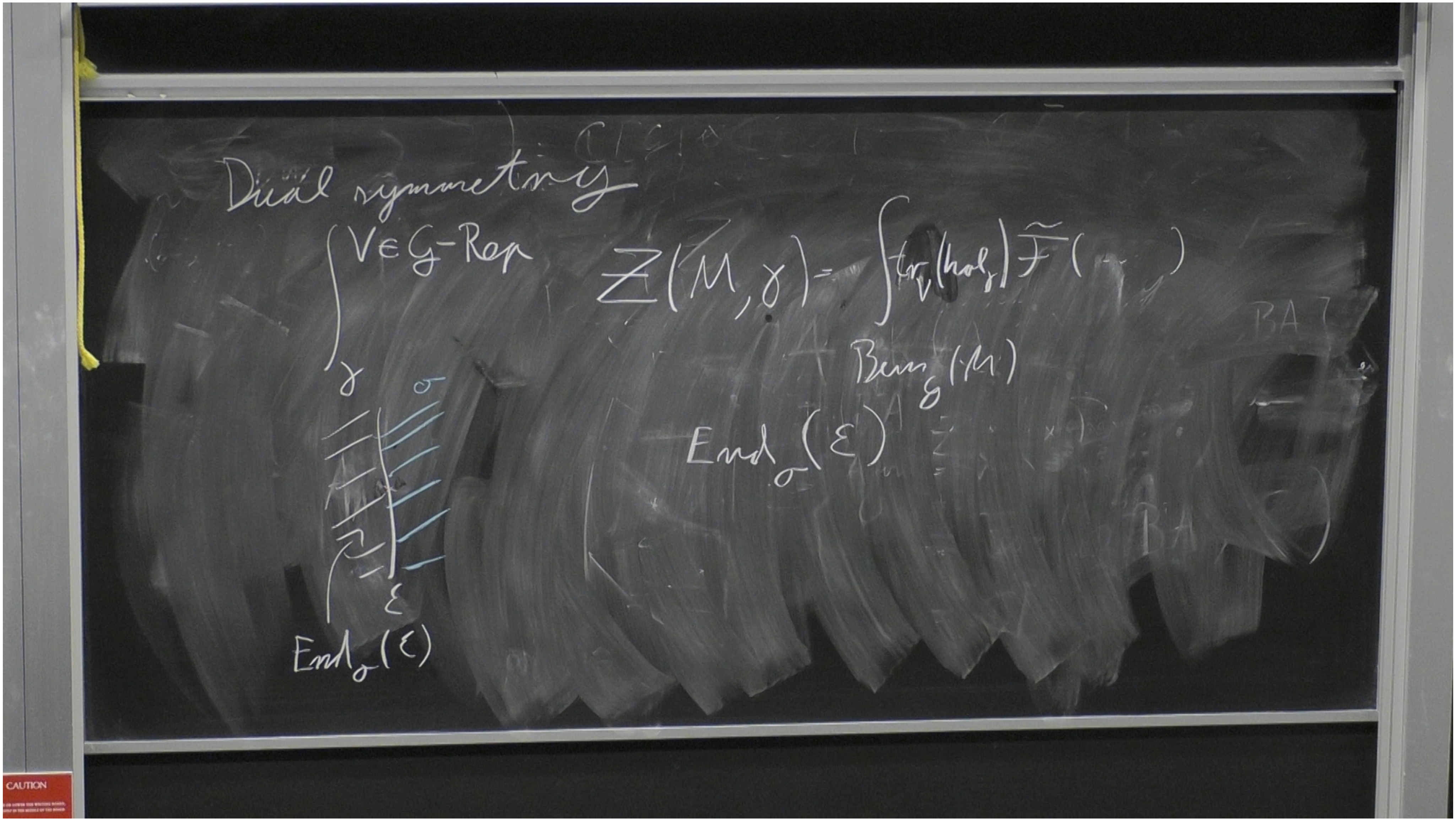


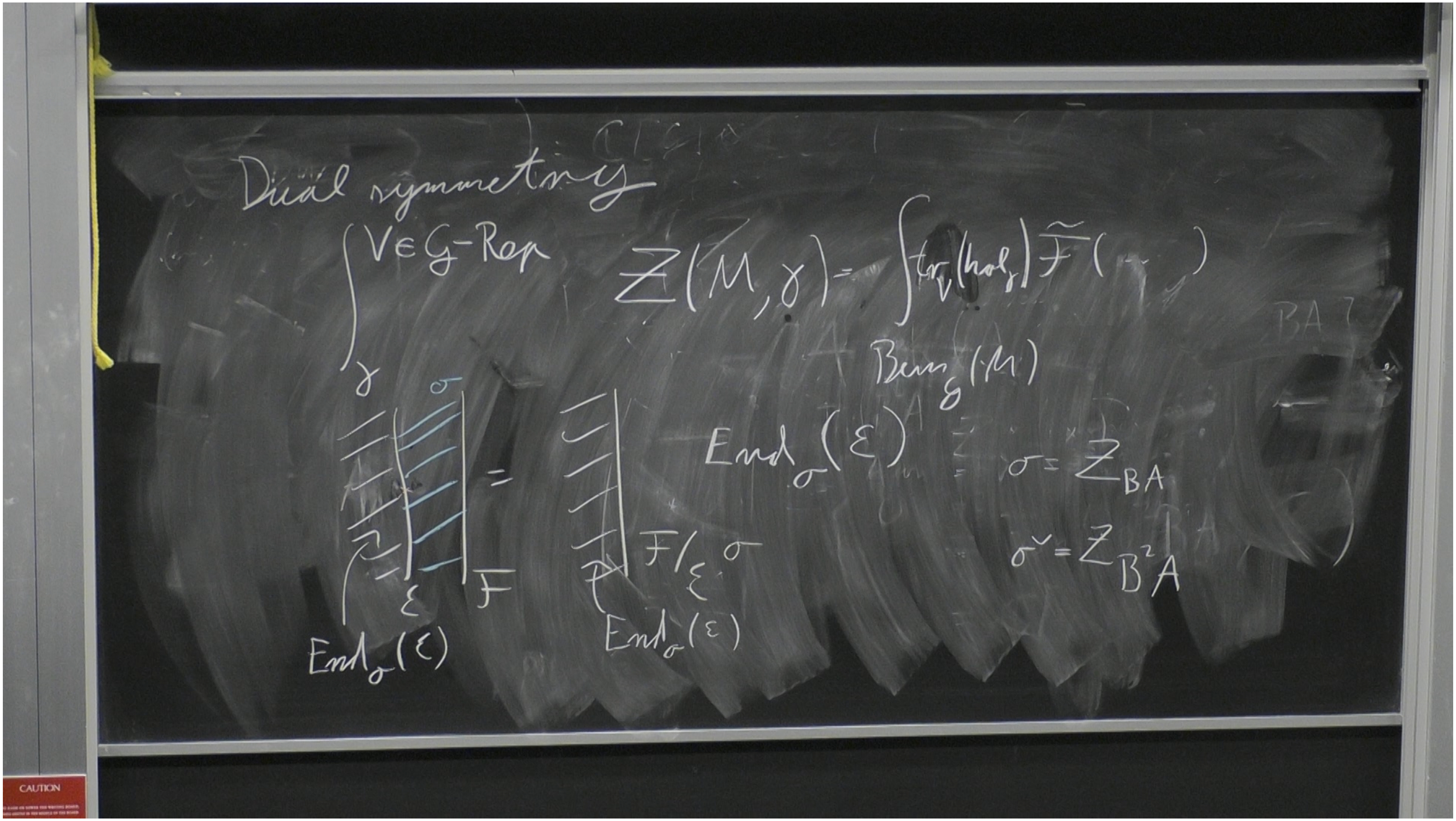


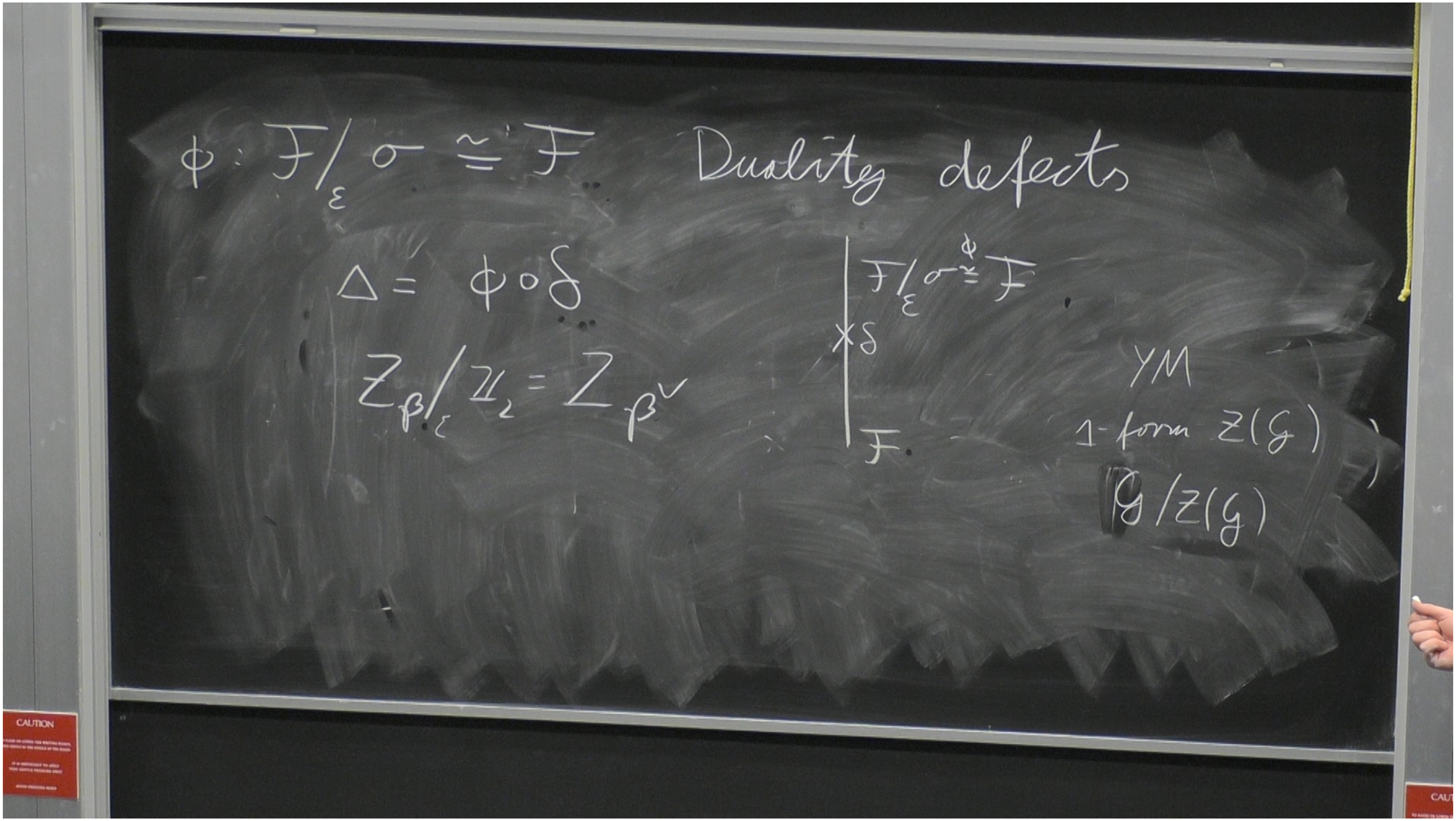
$\delta \in \sigma\left(\overset{\times}{\varepsilon} \longrightarrow \overset{\times}{\rho} \right) = \mathbb{C}$
1
$\tilde{F}$
F
$F/_{\varepsilon}\sigma$
$F/_{\varepsilon}\sigma$
z


$\phi : \mathcal{F}/_{\varepsilon}\sigma \cong \mathcal{F}$
Duality defects
$\Delta = \phi \circ \delta$
$Z_{\beta}/_{\varepsilon}\mathbb{Z}_2 = Z_{\beta^\vee}$
$\mathcal{F}/_{\varepsilon}\sigma \overset{\phi}{\cong} \mathcal{F}$
δ
$\mathcal{F}$
CAUTION


Dual symmetry
V ∈ G-Rep
Z(M, γ) = ∫ tr_V(hol_γ) F̃( )
Bun_G(M)
CAUTION


Dual symmetry
V ∈ G-Rep
Z(M, γ) = ∫ tr_V(hol_γ) F̃( )
Bun_G(M)
γ
σ
ε
End_σ(ε)
End_σ(ε)
CAUTION

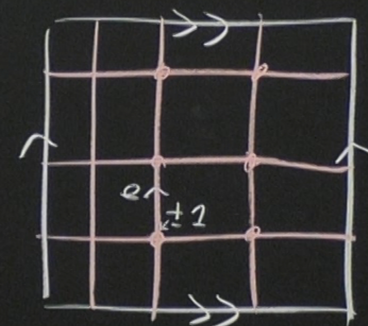

Dual symmetry
$V \in G\text{-Rep}$
$Z(M,\gamma) = \int tr_V(hol_\gamma)\,\widetilde{\mathcal{F}}(\;\;)$
$Bun_G(M)$
$End_\sigma(\mathcal{E})$
$\sigma = Z_{BA}$
$\sigma^\vee = Z_{B^2A}$
$\mathcal{E}$
$\mathcal{F}$
$\mathcal{F}/_{\mathcal{E}}\ \sigma$
$End_\sigma(\mathcal{E})$
$End_\sigma(\mathcal{E})$
CAUTION


$\phi : \mathcal{F}/_{\varepsilon}\sigma \cong \mathcal{F}$
Duality defects
$\Delta = \phi \circ \delta$
$Z_{\beta}/_{\varepsilon}\mathbb{Z}_2 = Z_{\beta^\vee}$
$\mathcal{F}/_{\varepsilon}\sigma \overset{\phi}{\cong} \mathcal{F}$
δ
$\mathcal{F}$
YM
1-form $Z(G)$
$G/Z(G)$

Ising model

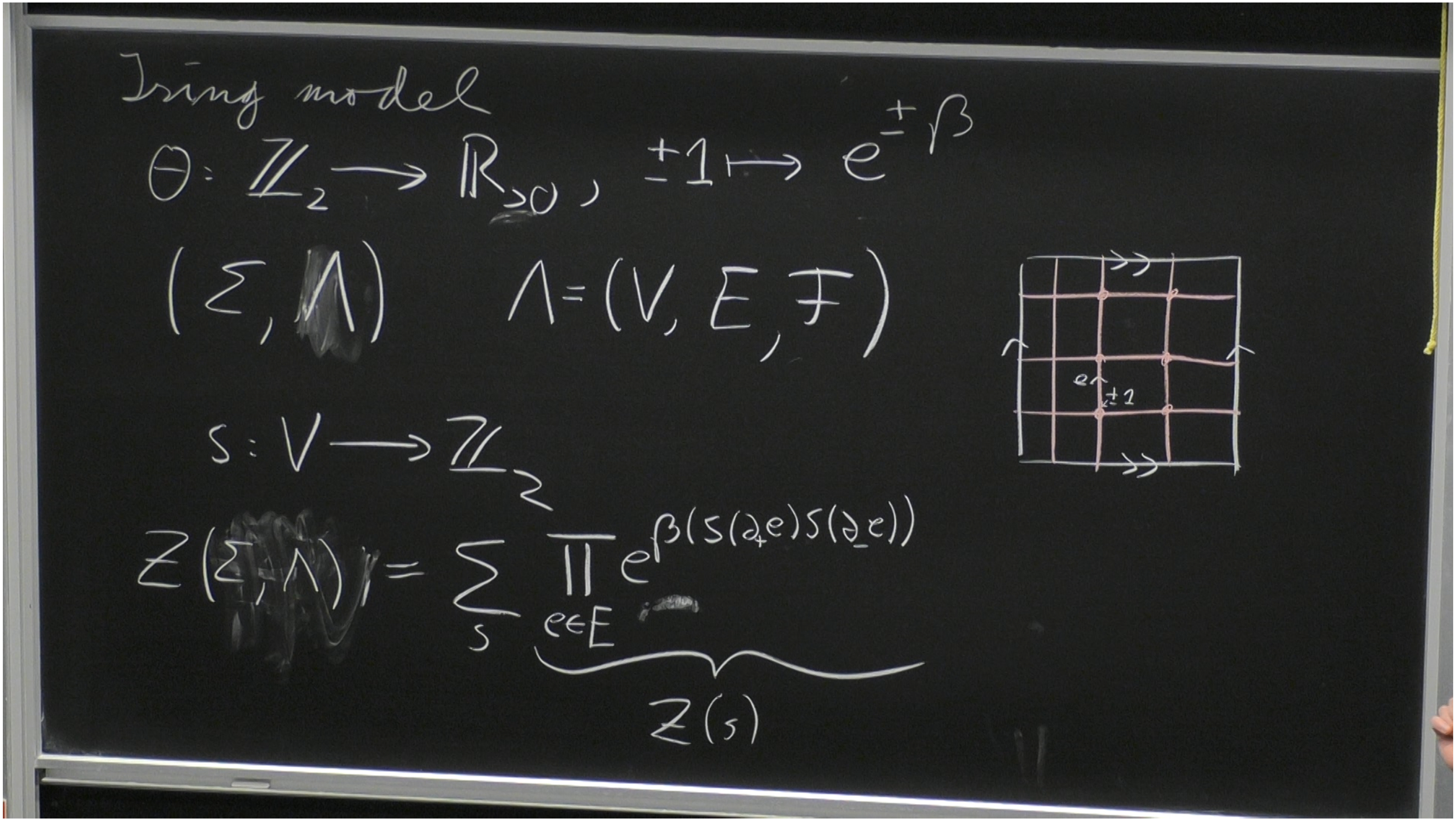
$$\Theta : \mathbb{Z}_2 \longrightarrow \mathbb{R}_{\geq 0}, \quad \pm 1 \longmapsto e^{\pm \beta}$$

$$(\Sigma, \Lambda) \qquad \Lambda = (V, E, F)$$



$$s : V \longrightarrow \mathbb{Z}_2$$

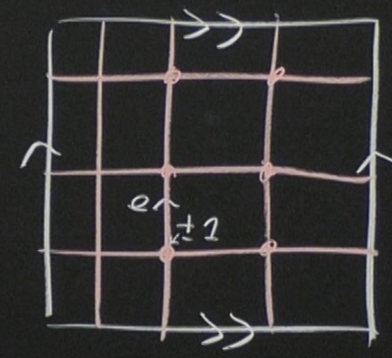
$$Z(\Sigma, \Lambda) = \sum_{s} \underbrace{\prod_{e \in E} e^{\beta(s(\partial_+ e) s(\partial_- e))}}_{Z(s)}$$


Ising model
$\Theta : \mathbb{Z}_2 \longrightarrow \mathbb{R}_{\geq 0}, \quad \pm 1 \longmapsto e^{\pm\beta}$
$(\Sigma, \Lambda) \qquad \Lambda = (V, E, F)$
$s : V \longrightarrow \mathbb{Z}_2$
$Z(\Sigma, \Lambda) = \sum_s \prod_{e \in E} e^{\beta(s(\partial_+ e) s(\partial_- e))}$
$Z(s)$
e
± 1

Ising model

$$\Theta : \mathbb{Z}_2 \longrightarrow \mathbb{R}_{\geq 0}, \quad \pm 1 \longmapsto e^{\pm\beta}$$

$$(\Sigma, \Lambda) \qquad \Lambda = (V, E, F)$$



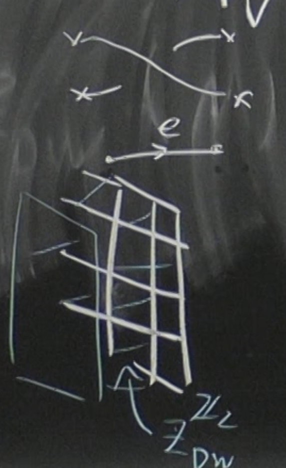
$$s : V \longrightarrow \mathbb{Z}_2$$

$$Z(\Sigma, \Lambda) = \sum_s \underbrace{\prod_{e \in E} e^{\beta(s(\partial_+ e)s(\partial_- e))}}_{Z(s)}$$

Let $P \xrightarrow{\pi} \Sigma$ be $\mathbb{Z}_2$-bundle on Σ.

$$Z(M, P, \Lambda) = \sum_s \prod_e \Theta\left(pt_e(s(\partial_- e), s(\partial_+ e))\right)$$

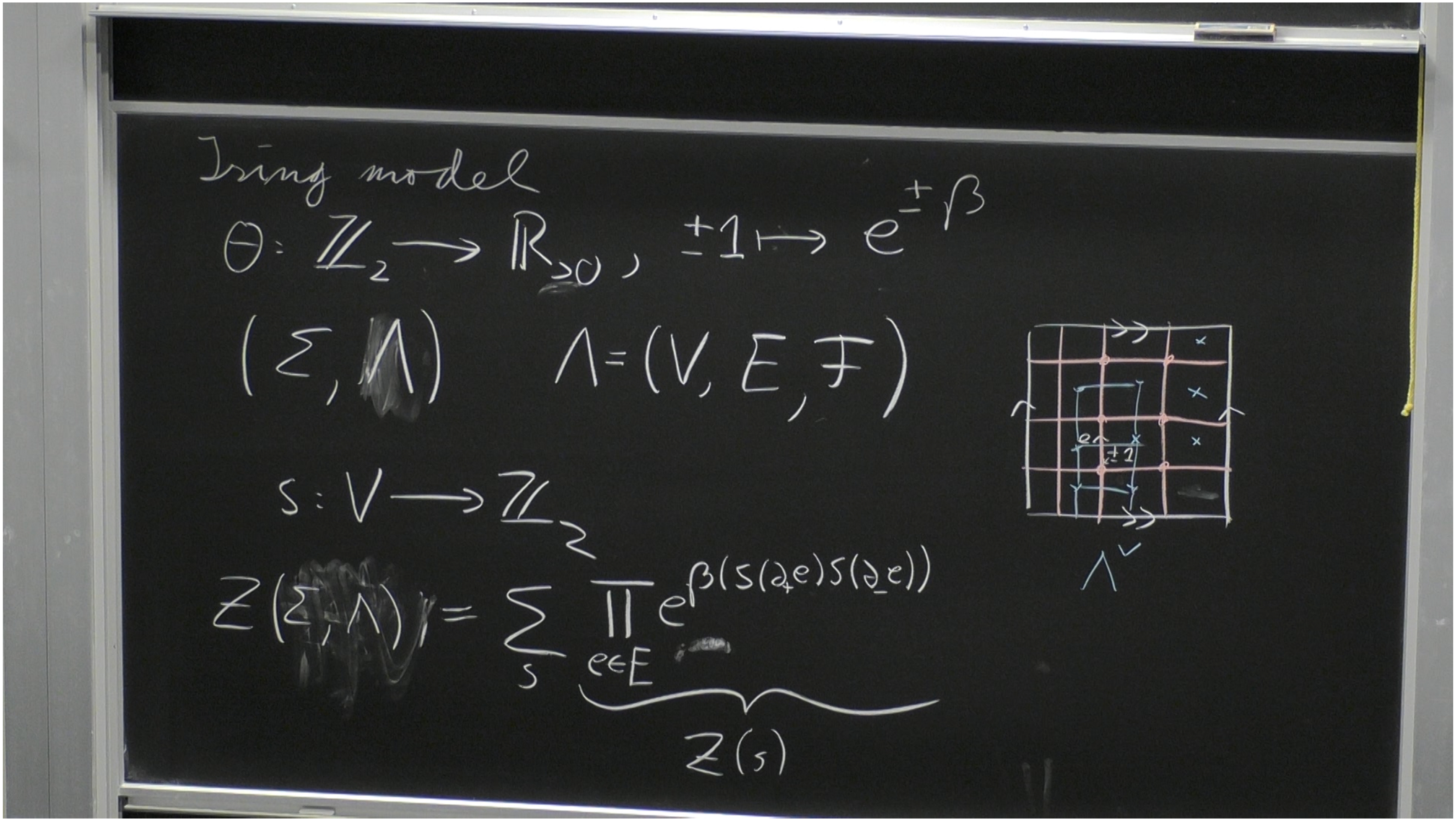
$s : V \longrightarrow P|_V$ s.t. $\pi \circ s = id$

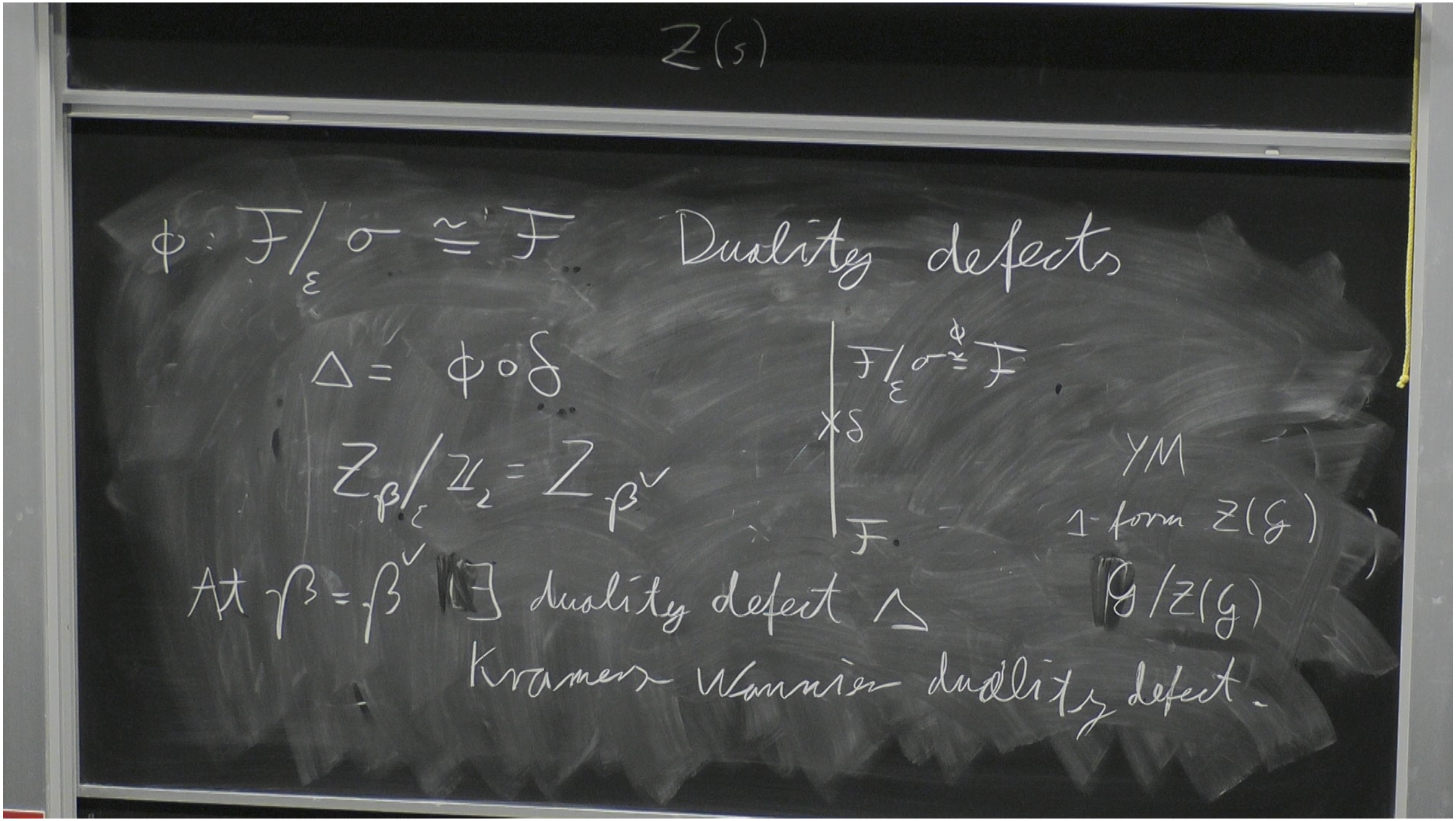


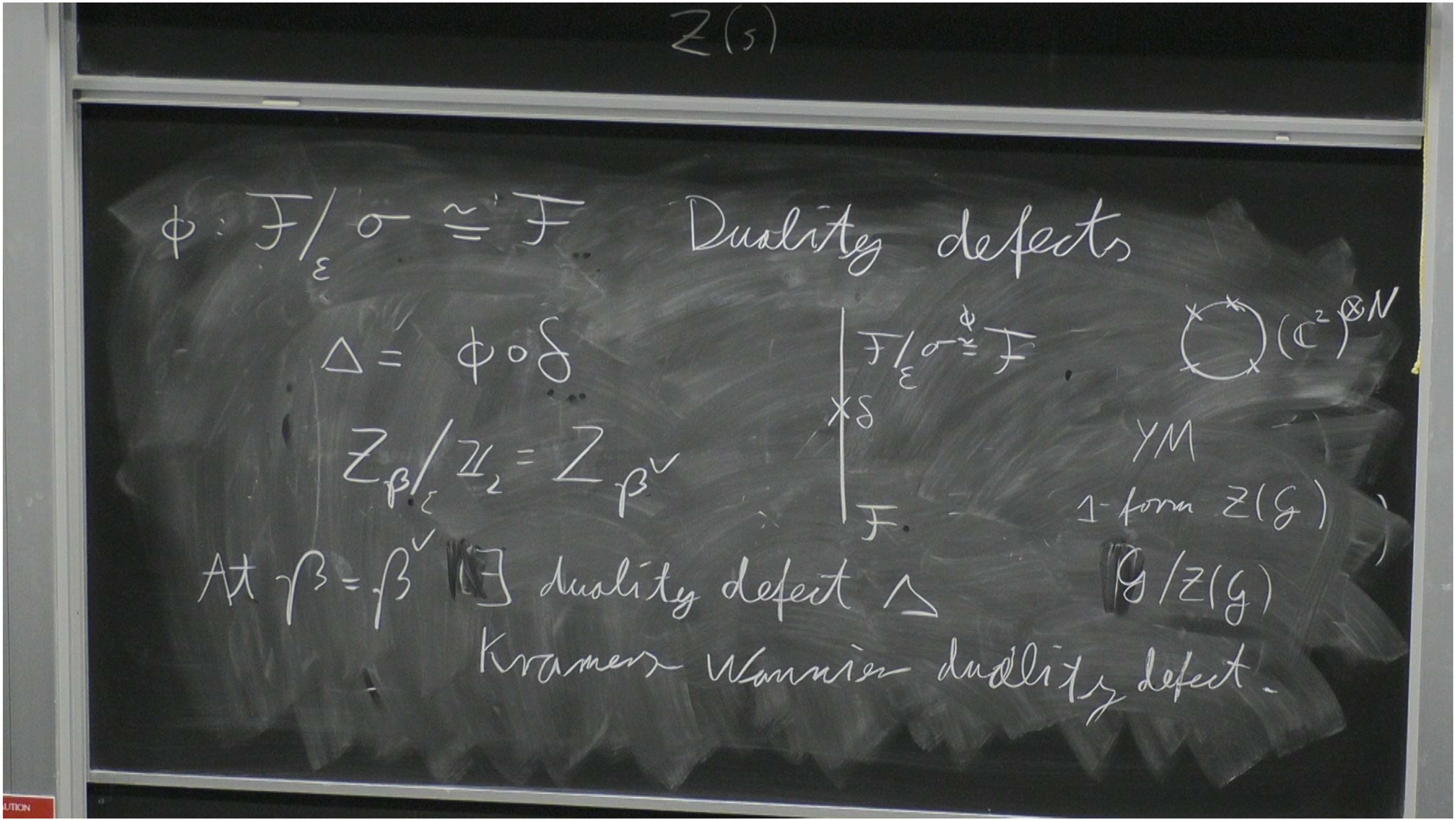
Exercise

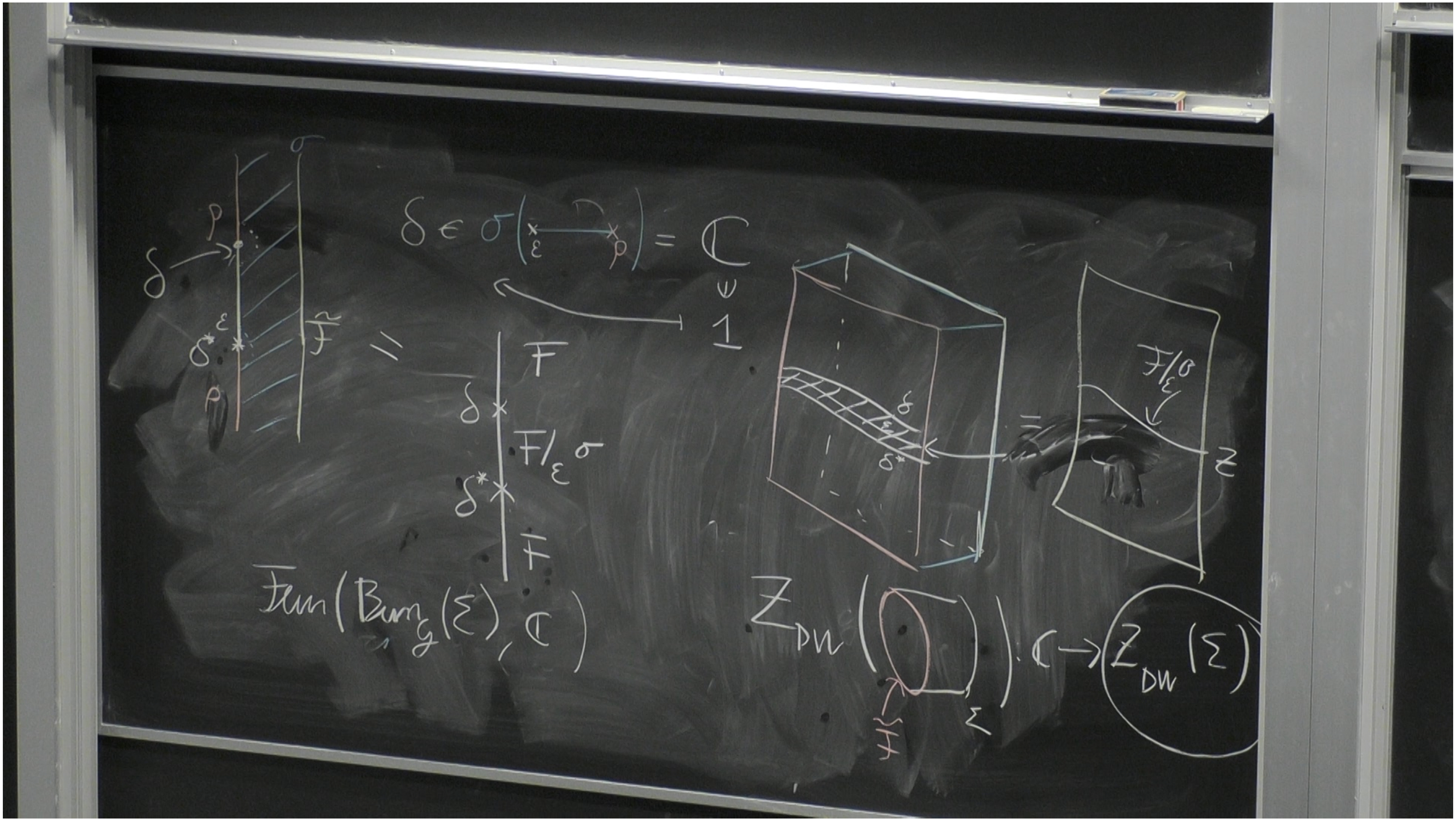
$$Z^{\Lambda}_{\beta} /_{\Sigma} \mathbb{Z}_2 = Z^{\Lambda^\vee}_{\beta^\vee}$$

$$\sinh(2\beta)\sinh(2\beta^\vee) = 1$$


Ising model
$\Theta : \mathbb{Z}_2 \longrightarrow \mathbb{R}_{>0}$, $\pm 1 \longmapsto e^{\pm\beta}$
(Σ, Λ)
$\Lambda = (V, E, F)$
$s : V \longrightarrow \mathbb{Z}_2$
$Z(\Sigma, \Lambda) = \sum_s \prod_{e \in E} e^{\beta(s(\partial_+ e)s(\partial_- e))}$
$Z(s)$
± 1
$\Lambda^\vee$


$Z(S)$
$\phi : \mathcal{F}/_\varepsilon \sigma \cong \mathcal{F}$
Duality defects
$\Delta = \phi \circ \delta$
$Z_\beta / \mathbb{Z}_2 = Z_{\beta^\vee}$
$\mathcal{F}/_\varepsilon \sigma \overset{\phi}{\cong} \mathcal{F}$
$\times \delta$
$\mathcal{F}$
YM
1-form $Z(G)$
At $\beta = \beta^\vee$ $\exists$ duality defect Δ
$G/Z(G)$
Kramers Wannier duality defect.


Z(s)
φ : F/ε σ ≅ F
Duality defects
Δ = φ∘δ
Z_β / Z_2 = Z_β∨
At β = β∨ ∃ duality defect Δ
Kramers Wannier duality defect.
F/ε σ ≅ F (φ)
δ
F
(C²)^⊗N
YM
1-form Z(G)
G/Z(G)


σ
δ ∈ σ(ε → ρ) = ℂ
1
F̃ = F
δ
F/ε σ
δ*
F̄
Fun(Bun_G(Σ), ℂ)
δ
δ*
F/ε σ
Z_DW( ), ℂ → Z_DW(Σ)
Σ