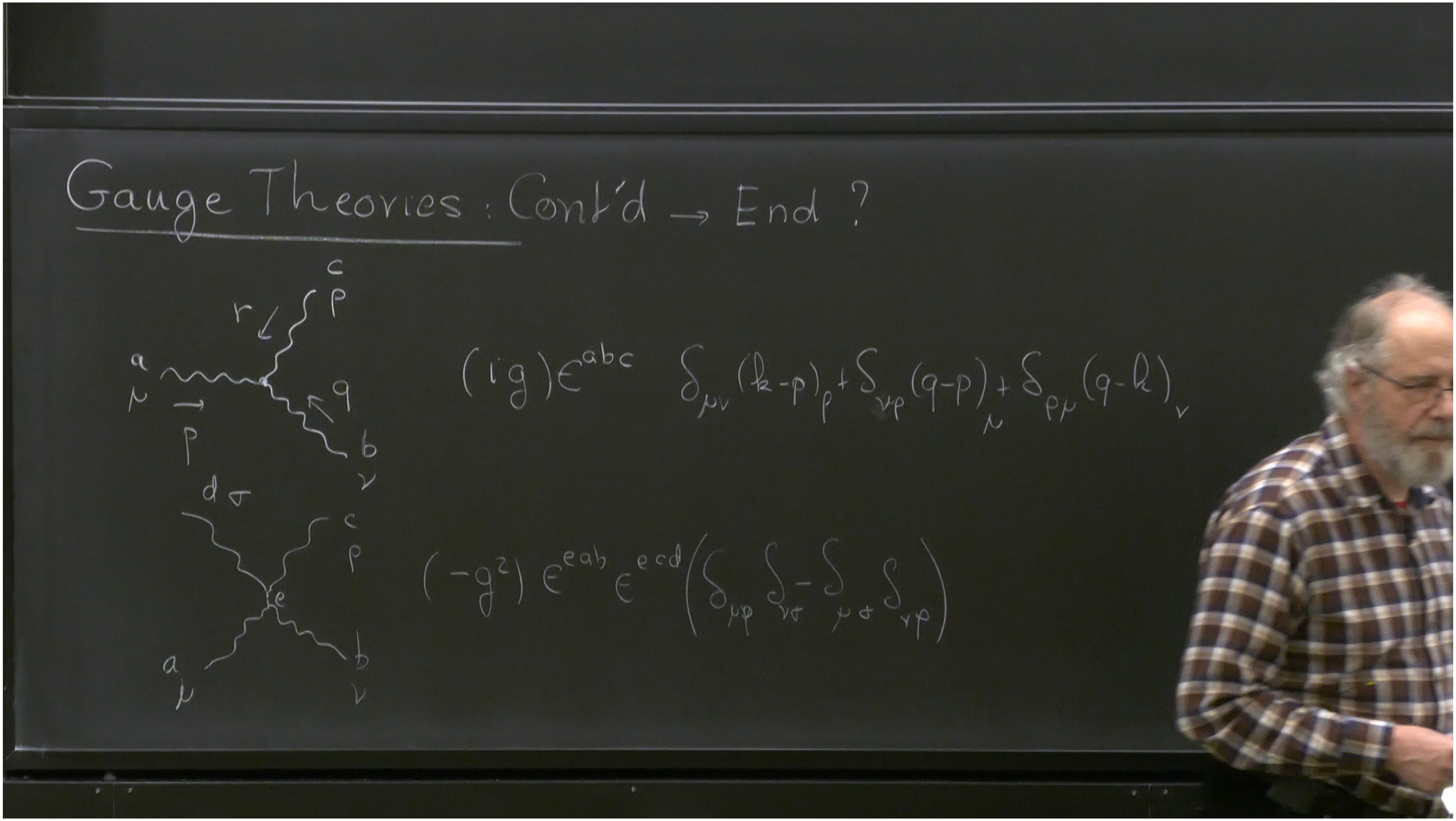
Title: QFT2 Lecture - 121223

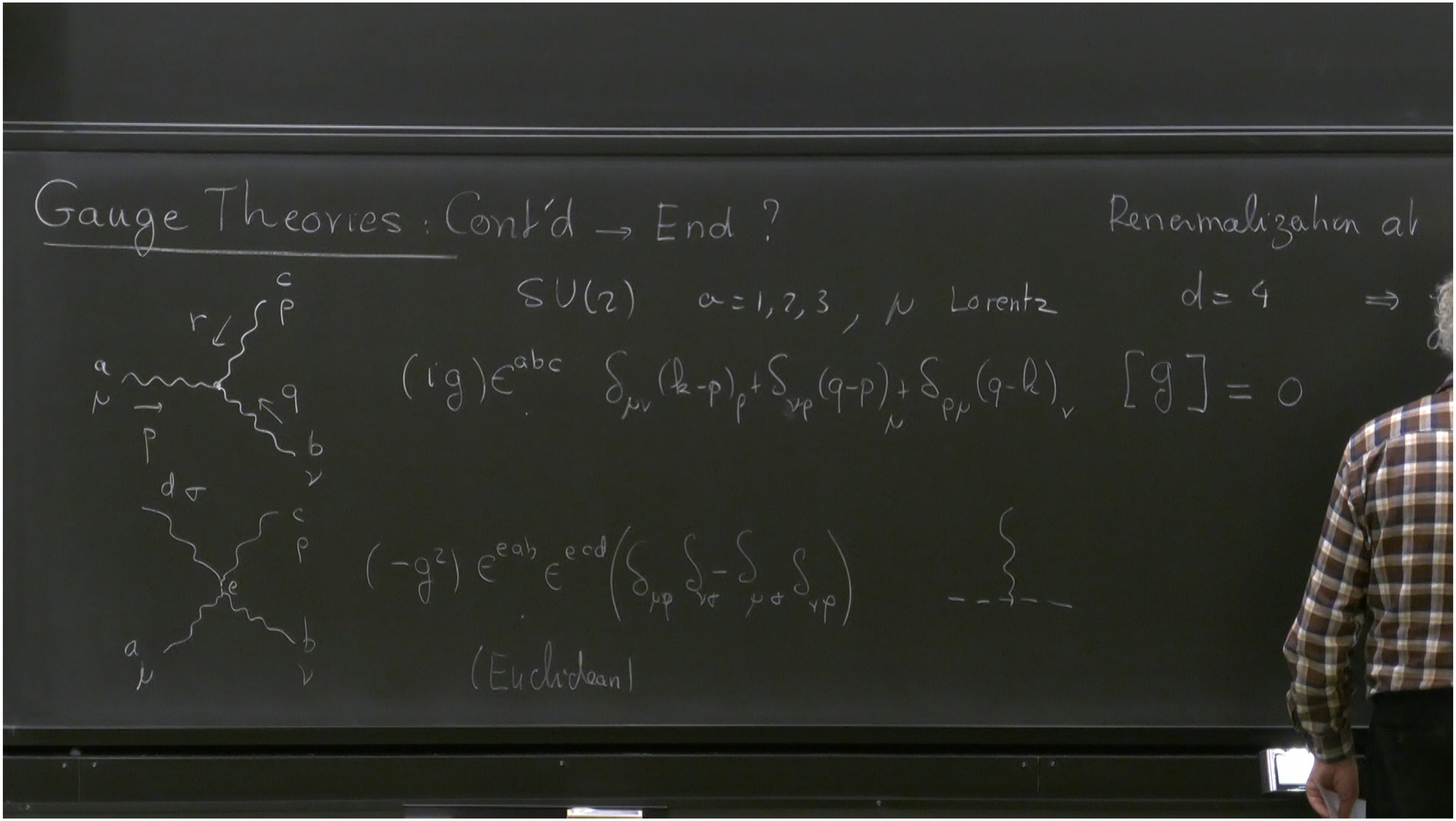
Speakers: Francois David

Collection: Quantum Field Theory 2 2023/24

Date: December 12, 2023 - 9:00 AM

URL: https://pirsa.org/23120009

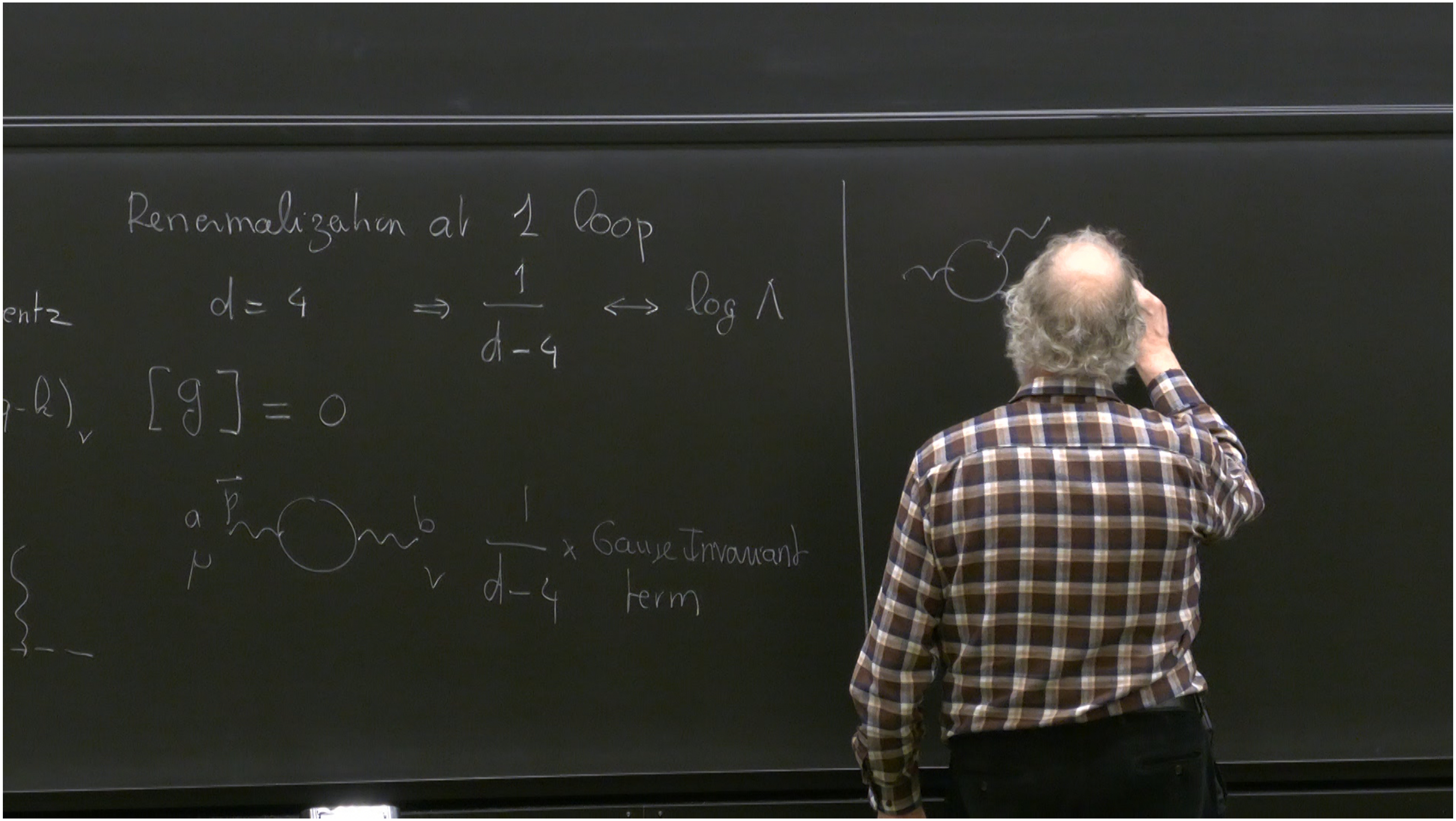

Gauge Theories: Cont'd → End ?
$(ig)\epsilon^{abc}\ \delta_{\mu\nu}(k-p)_\rho + \delta_{\nu\rho}(q-p)_\mu + \delta_{\rho\mu}(q-k)_\nu$
$(-g^2)\,\epsilon^{eab}\epsilon^{ecd}\left(\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho}\right)$

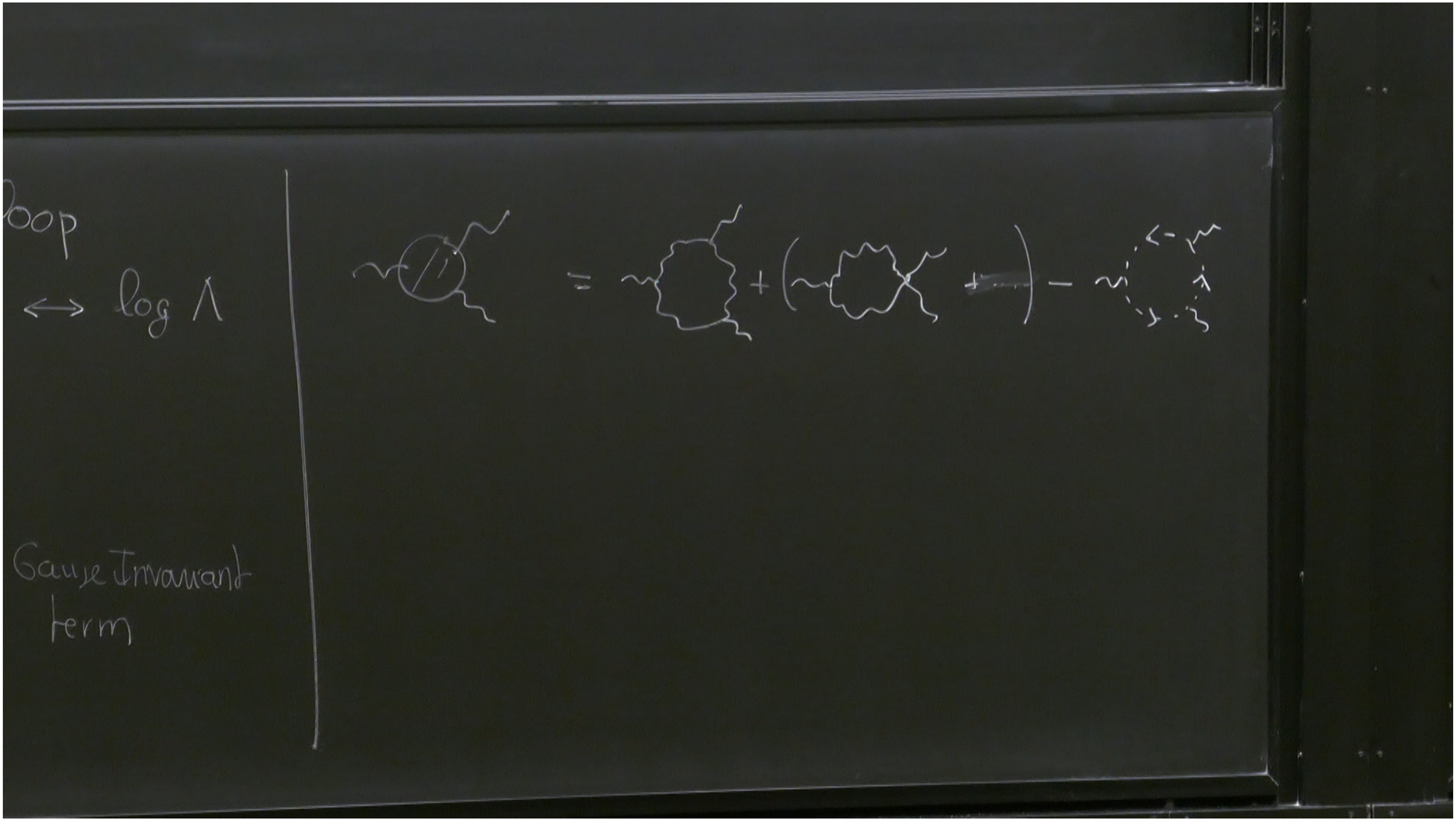

Gauge Theories: Cont'd → End ?
Renormalization at
$SU(2)$ $a = 1, 2, 3$, μ Lorentz
$d = 4$ ⇒
$(ig)\epsilon^{abc}\ \delta_{\mu\nu}(k-p)_\rho + \delta_{\nu\rho}(q-p)_\mu + \delta_{\rho\mu}(q-k)_\nu$
$[g] = 0$
$(-g^2)\,\epsilon^{eab}\epsilon^{ecd}(\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho})$
(Euclidean)

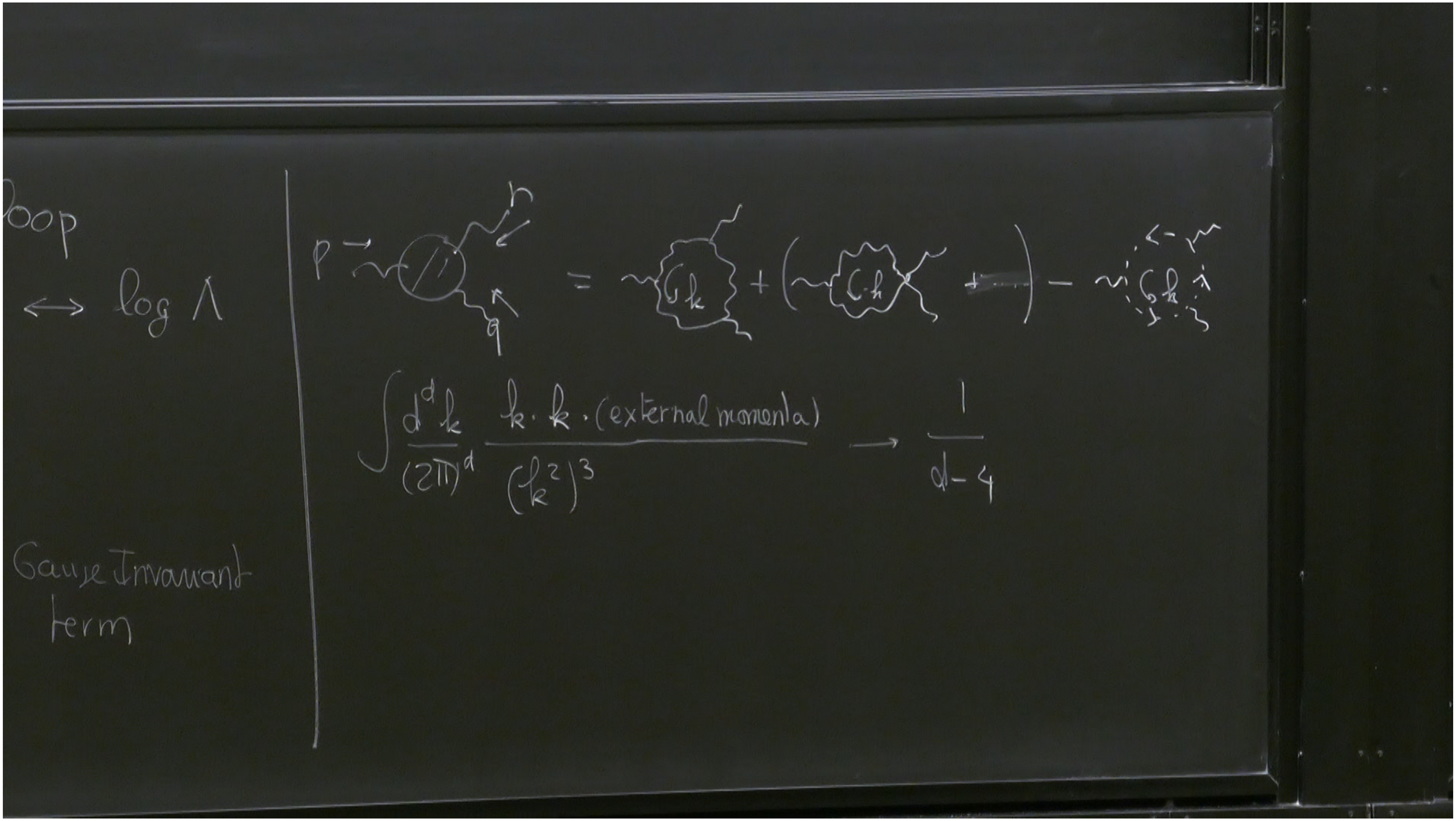
Renormalization at 1 loop

entz $\quad d = 4 \quad \Rightarrow \frac{1}{d-4} \longleftrightarrow \log \Lambda$

$q-k)_\nu \quad [g] = 0$

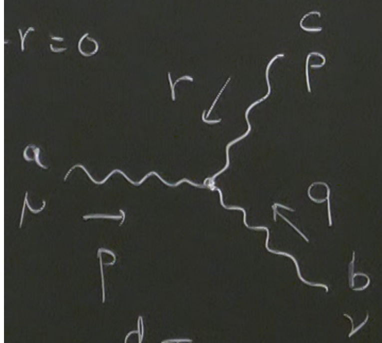

Renormalization at 1 loop
entz
d = 4 ⇒ 1/(d−4) ⟷ log Λ
[g] = 0
a p b
μ ν
1/(d−4) × Gauge Invariant term


loop
↔ log Λ
Gauge Invariant
term


loop
$\longleftrightarrow \log \Lambda$
Gauge Invariant term
p
q
r
k
$\int \frac{d^d k}{(2\pi)^d} \frac{k \cdot k \cdot (\text{external momenta})}{(k^2)^3} \longrightarrow \frac{1}{d-4}$

auge Theories : Cont'd → End ?

Renormalization ab

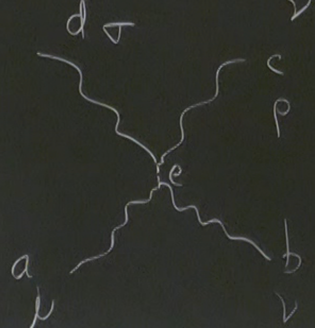


SU(2) a = 1, 2, 3 , μ Lorentz

$$\boxed{(ig)\,\epsilon^{abc}\;\delta_{\mu\nu}(k-p)_\rho + \delta_{\nu\rho}(q-p)_\mu + \delta_{\rho\mu}(q-k)_\nu}$$

d = 4 ⇒

$$[g] = 0$$



$$\boxed{(-g^2)\,\epsilon^{eab}\,\epsilon^{ecd}\left(\delta_{\mu\rho}\,\delta_{\nu\sigma} - \delta_{\mu\sigma}\,\delta_{\nu\rho}\right)}$$

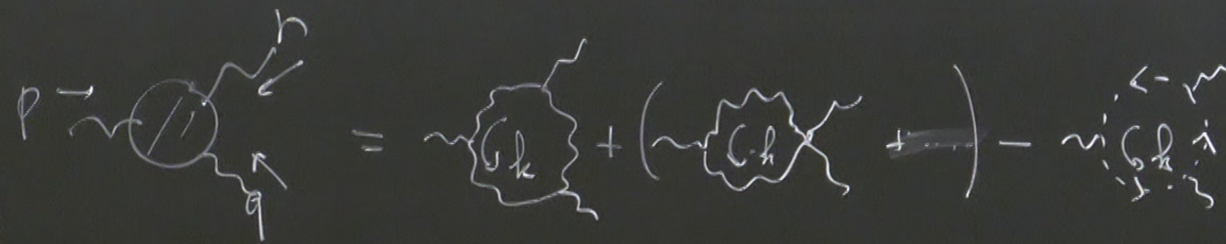
(Euclidean)

...n at 1 loop

$$\Rightarrow \frac{1}{d-4} \longleftrightarrow \log \Lambda$$

...b

$$\frac{1}{d-4} \times \text{Gauge Invariant term}$$



$$\int \frac{d^d k}{(2\pi)^d} \frac{k \cdot k \cdot (\text{external momenta})}{(k^2)^3} \longrightarrow \frac{1}{d-4}$$

$$\hookrightarrow \frac{(\ldots\ 3\ \text{vertex})}{d-4} \cdot C_3 \leftarrow \text{some coefficient}$$

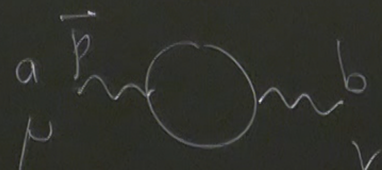
Renormalization at 1 loop

?, 3 , μ Lorentz

$$\delta_{\nu\rho}(q-p)_\mu + \delta_{\rho\mu}(q-k)_\nu$$

$$d = 4 \Rightarrow \frac{1}{d-4} \leftrightarrow \log \Lambda$$

$$[g] = 0$$

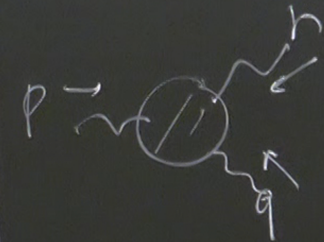


$$\frac{1}{d-4} \times (p_\mu p_\nu - g_{\mu\nu} p^2)\,\delta_{ab}$$

(— propagator)

$$C_2 = \frac{10}{3} \; (?)$$

$$+ \delta_{\nu\rho})$$



$$= \sim$$

$$\int \frac{d^d k}{(2\pi)^d} \frac{k \cdot k \cdot (ext}{(k^2)^3}$$

$$\hookrightarrow \frac{(\;\;\langle\; 3\text{ vertex})}{d-4}$$

renormalization at 1 loop

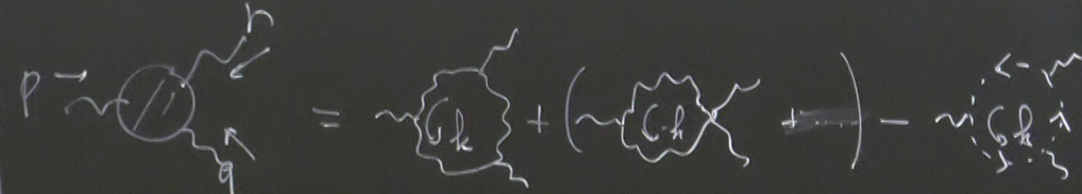
$$d = 4 \Rightarrow \frac{1}{d-4} \longleftrightarrow \log \Lambda$$

$$[g] = 0$$

$$\frac{1}{d-4} \times (\text{propagator})\,(p_\mu p_\nu - g_{\mu\nu} p^2)\,\delta_{ab}$$

$$\boxed{C_2} = \frac{10}{3} \; (?)$$

AA term



$$\int \frac{d^d k}{(2\pi)^d} \frac{k \cdot k \cdot (\text{external momenta})}{(k^2)^3} \longrightarrow \frac{1}{d-4}$$

$$\searrow \frac{(\text{3 vertex})}{d-4} \cdot \boxed{C_3} \leftarrow \text{some coefficient}$$

$$g\,AAA$$

$$AA + g\,AAA$$

$$\downarrow$$

$$\left(1 - \frac{C_2}{d-4} g^2\right) AA + g \left(1 - \frac{C_3}{d-4} g^2\right) AAA$$

$$\text{Action} = AA + g\,AAA \qquad A = Z A_B \qquad g = Z_g\, g_B$$

$$\downarrow$$

$$\underbrace{\left(1 - \frac{C_2}{d-4} g^2\right)}_{} AA + \underbrace{g\left(1 - \frac{C_3}{d-4} g^2\right)}_{g_B} AAA$$

$$\| $$

$$S_R(A)$$

Like in ϕED

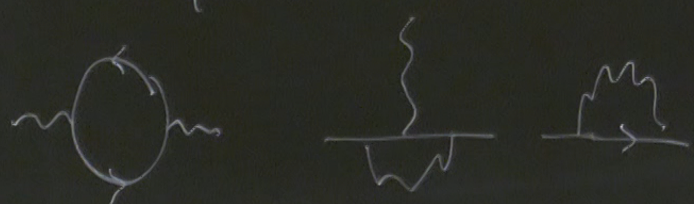
$$\text{Action} = AA + g\, AAA \qquad A = Z A_B \qquad g = Z_g\, g_B$$

$$\downarrow$$

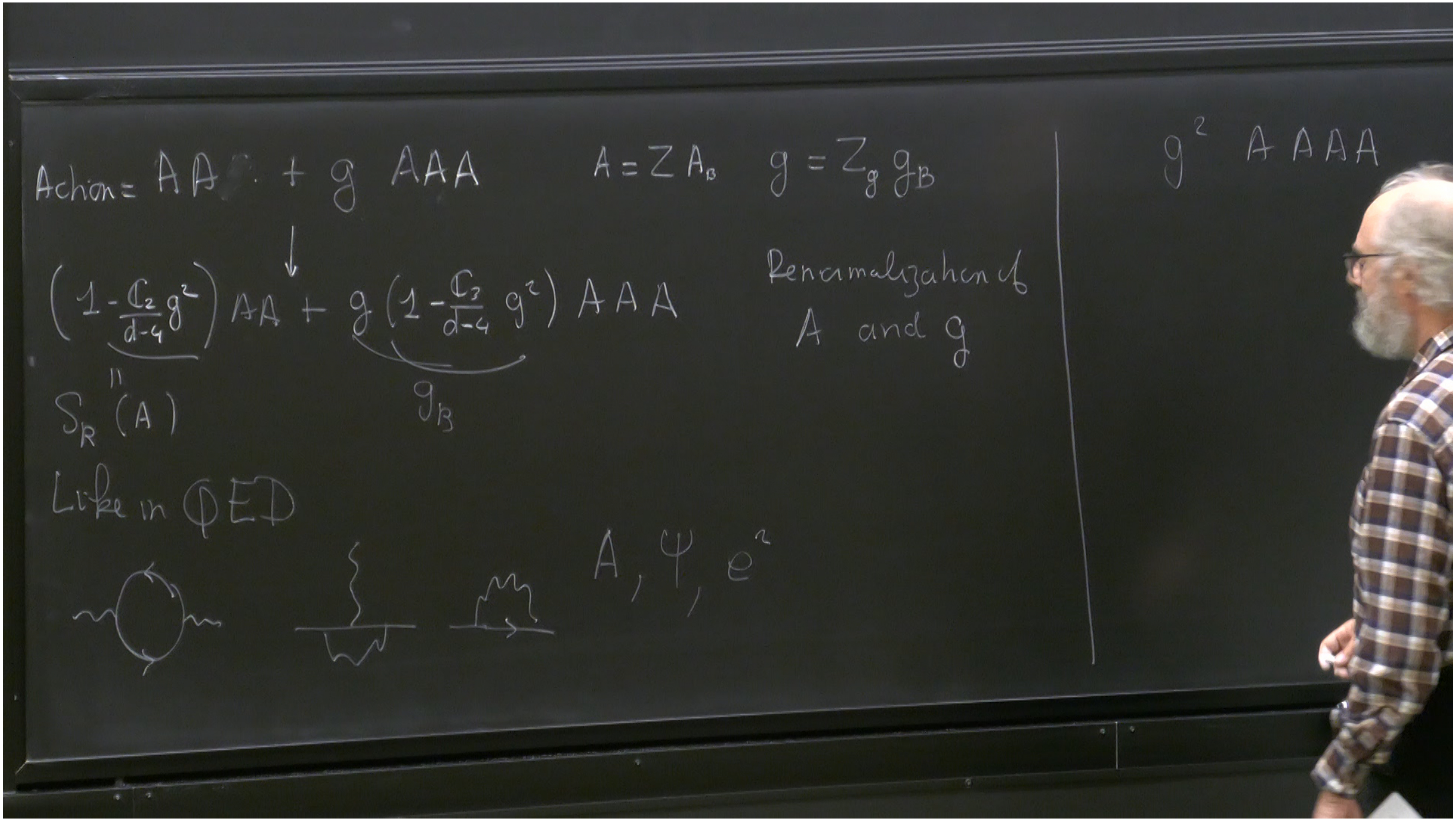
$$\left(1-\frac{C_2}{d-4}g^2\right)AA + \underbrace{g\left(1-\frac{C_3}{d-4}g^2\right)}_{g_B} AAA$$

$$= S_R(A)$$

Like in QED



$$A,\ \psi,\ e^2$$

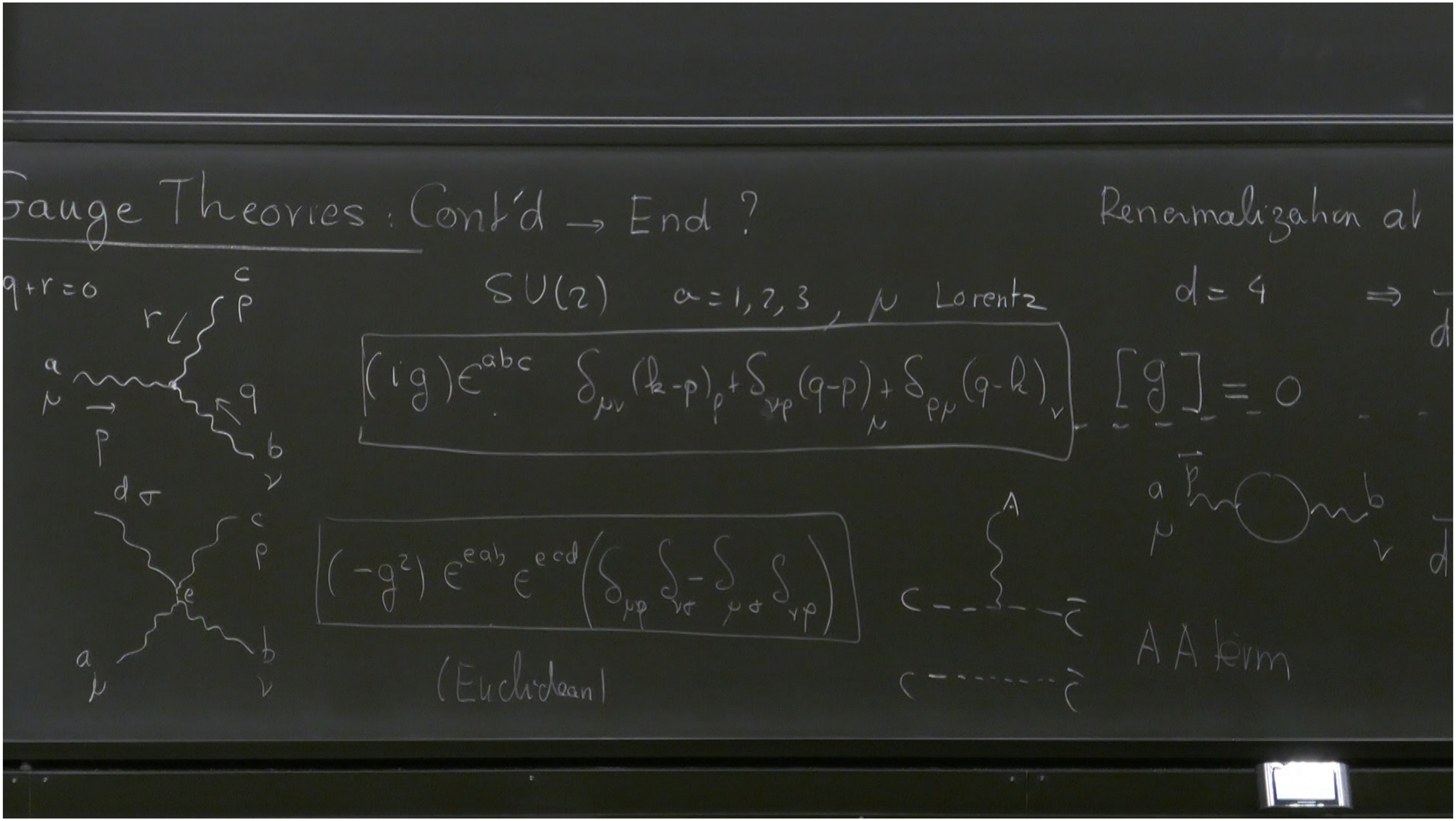

Action = AA + g AAA
A = Z A_B
g = Z_g g_B
(1 - C_2/(d-4) g^2) AA + g (1 - C_3/(d-4) g^2) AAA
Renormalization of
A and g
S_R (A)
g_B
Like in QED
A, ψ, e^2
g^2 AAAA

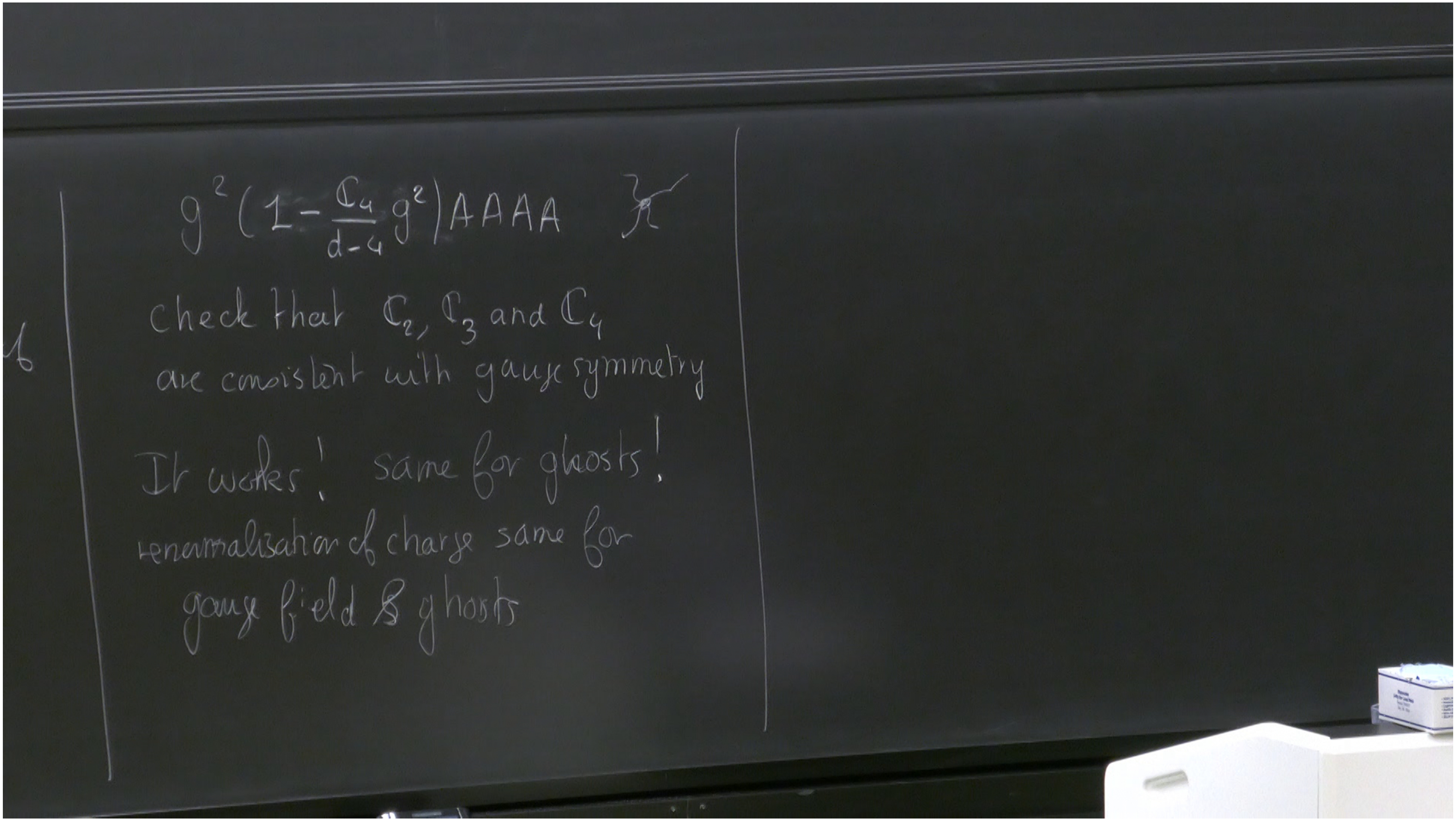
$g^2\left(1-\frac{\mathbb{C}_4}{d-4}g^2\right)AAAA$ 

Check that $\mathbb{C}_2$, $\mathbb{C}_3$ and $\mathbb{C}_4$

are consistent with gauge symmetry

It works! same for ghosts


Gauge Theories: Cont'd → End ?
$q+r=0$
$SU(2)$ $a=1,2,3$, μ Lorentz
$(ig)\epsilon^{abc}\; \delta_{\mu\nu}(k-p)_\rho + \delta_{\nu\rho}(q-p)_\mu + \delta_{\rho\mu}(q-k)_\nu$
$(-g^2)\,\epsilon^{eab}\epsilon^{ecd}\left(\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho}\right)$
(Euclidean)
Renormalization at
$d=4 \Rightarrow$
$[g]=0$
AA term


$g^2\left(1-\frac{\mathbb{C}_4}{d-4}g^2\right)AAAA$
Check that $\mathbb{C}_2$, $\mathbb{C}_3$ and $\mathbb{C}_4$
are consistent with gauge symmetry
It works! same for ghosts!
renormalisation of charge same for
gauge field & ghosts

$1-\frac{C_4}{d-4}g^2)AAAA$

that C_2, C_3 and C_4

sistent with gauge symmetry

ks! same for ghosts!

sation of charge same for

field & ghosts

'71 G 't Hooft 1 loop

↓

~74 Lee & Zinn-Justin Ward Identities

~ BRST BRST sym

(Euclidean)

Modern Techniques   Background Field Gauge Fixing Methods

effective action $\Gamma[\varphi] = S[\varphi] + \frac{1}{2}\,\mathrm{Tr.Log}\left(\frac{\delta S}{\delta\varphi\,\delta\varphi}\right) + \cdots$

$\Gamma[A] = S_{YM}[A] + \frac{1}{2}\,\mathrm{Tr.Log}\,[$

a_μ b_ν (Euclidean)

AA term

$\boxed{C_2} = \frac{10}{3}$

Modern Techniques — Background Field Gauge Fixing Methods — Heat

effective action $\Gamma[\varphi] = S[\varphi] + \frac{1}{2} \mathrm{Tr.Log}\left(\frac{\delta S}{\delta\varphi\delta\varphi}\right) + \cdots$

$$\Gamma[A] = S_{YM}[A] + \frac{1}{2}\mathrm{Tr.Log}\left[-D_A D_A + 2F_A\right] - \mathrm{Tr\,Log}\left[-D_A D_A\right]$$

$A_{Full} = A + \tilde{A}$

A: background; $\tilde{A}$: quantum

A in $\Gamma[A]$: background Field

D_A covariant derivative operator in A

F_A: Field Strength associated to A

$-\mathrm{Tr\,Log}[-D_A D_A]$: ghosts

$D^\mu_A \tilde{A} = 0$

(Euclidean)

Modern Techiques Background Field Gauge Fixing Methods

effective action $\Gamma[\varphi] = S[\varphi] + \frac{1}{2} \mathrm{Tr.Log}\left(\frac{\delta S}{\delta\varphi\delta\varphi}\right) + \cdots$

$\Gamma[A] = S_{YM}[A] + \frac{1}{2} \mathrm{Tr.Log}\left[-D_A \cdot D_A + 2F_A\right] + \cdots$

$A_{Full} = A + \tilde{A}$

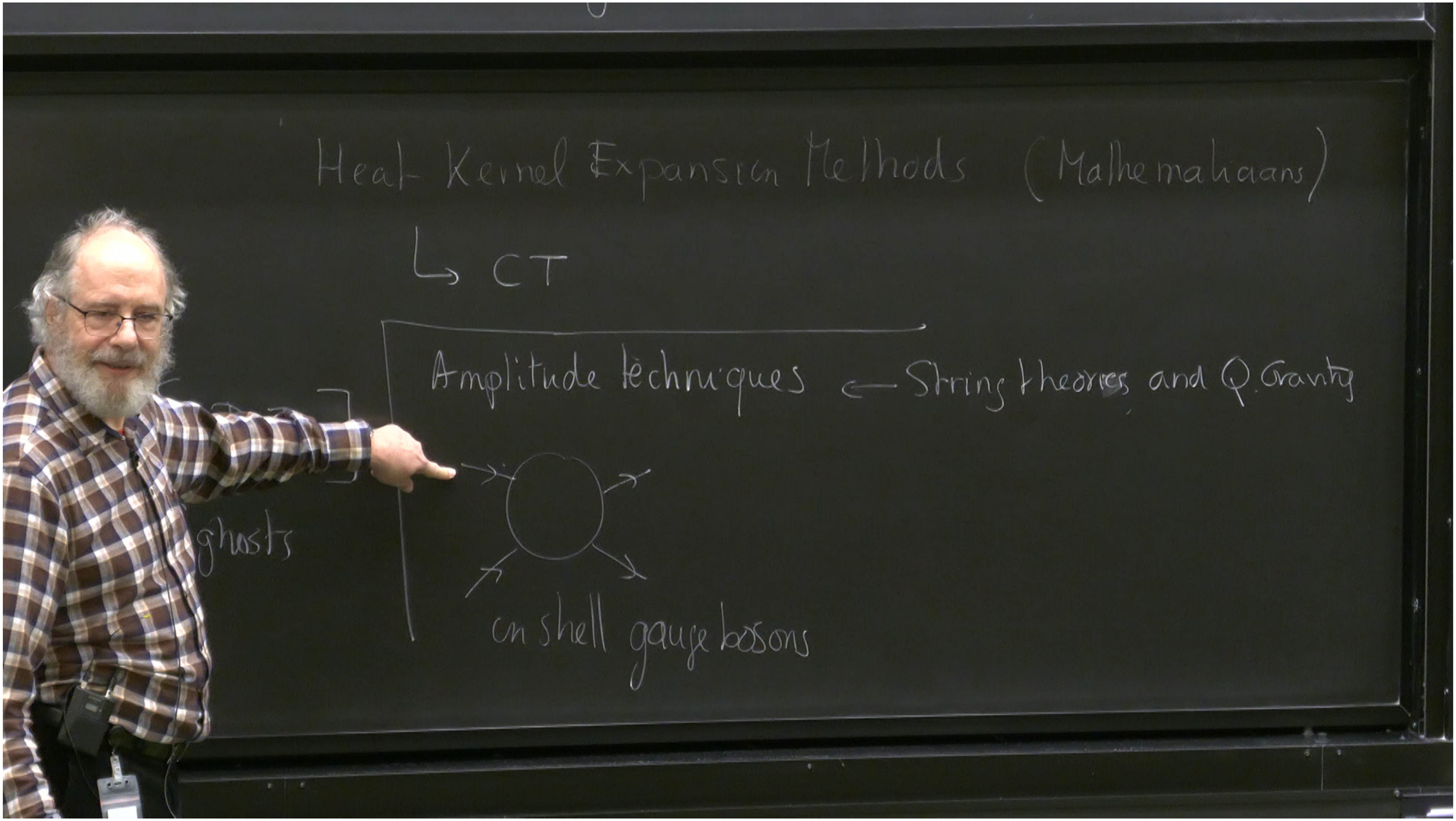
A: background; $\tilde{A}$: quantum

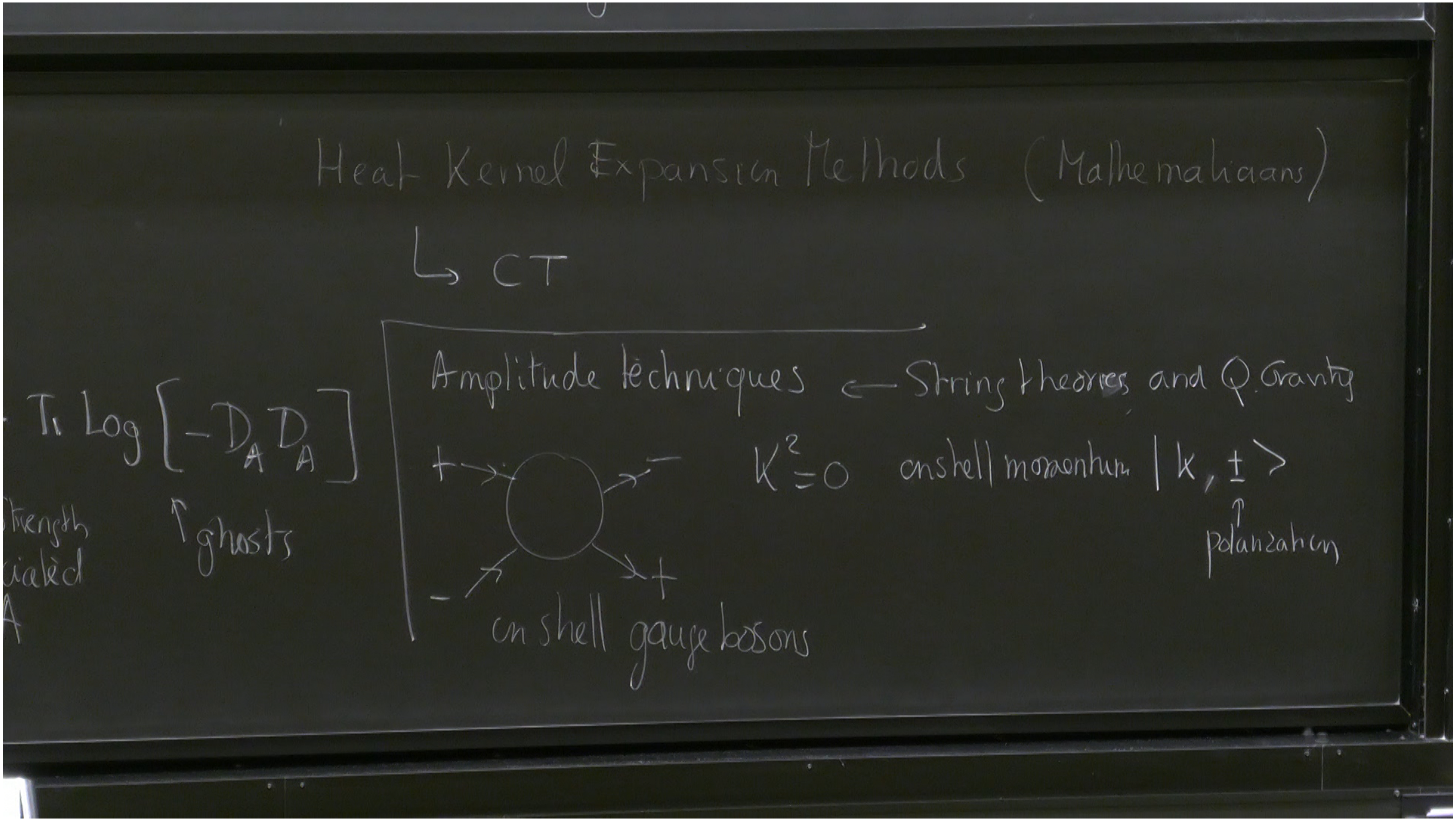
A in $\Gamma[A]$: background Field

D_A covariant derivative operator in A

F_A: Field asso... to ...

$D^{\mu}_{A} \tilde{A}_{\mu} = 0$


Heat Kernel Expansion Methods (Mathematicians)
↳ CT
Amplitude techniques ← String theories and Q. Gravity
ghosts
on shell gauge bosons


Heat Kernel Expansion Methods (Mathematicians)
↳ CT
Amplitude techniques ← String theories and Q. Gravity
Tr Log [-D_A D_A]
ghosts
strength
ciated
+
-
-
+
K^2 = 0 onshell momentum | k, ± >
polarization
on shell gauge bosons

Heat Kernel Expansion Methods (Mathematicians)

↳ CT

$\mathrm{Tr}\,\mathrm{Log}\left[-D_A D_A\right]$

↑ ghosts

trength
ciated
A

Amplitude techniques ← String theories and Q. Gravity

on shell gauge bosons

$K^2=0$ onshell momentum $|k,\pm\rangle$

↑ polarization

François Gelis Book

The β-function. 't Hooft.

SU(N) $\quad \alpha = g^2$

$$\beta(\alpha) = \alpha^2 \frac{1}{(4\pi)^2}\left(\underset{\substack{\uparrow \\ \text{gauge + ghosts}}}{-\frac{11}{3}N} + \underset{\substack{\uparrow \\ \text{matter}}}{\frac{4}{3}N_F \frac{1}{2}}\right)$$

N_F Dirac Fermions

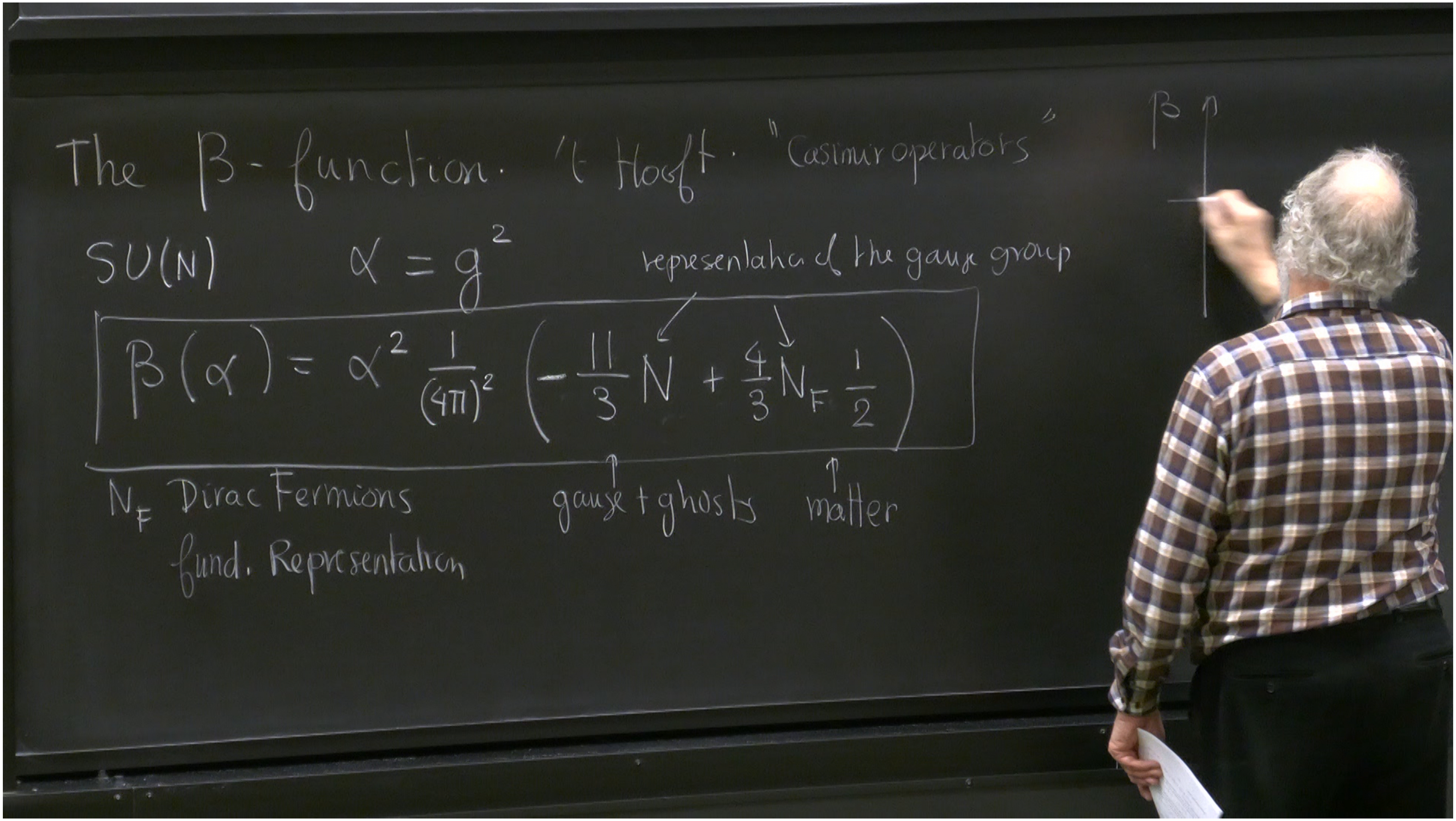
The β-function. 't Hooft. "Casimir operators"

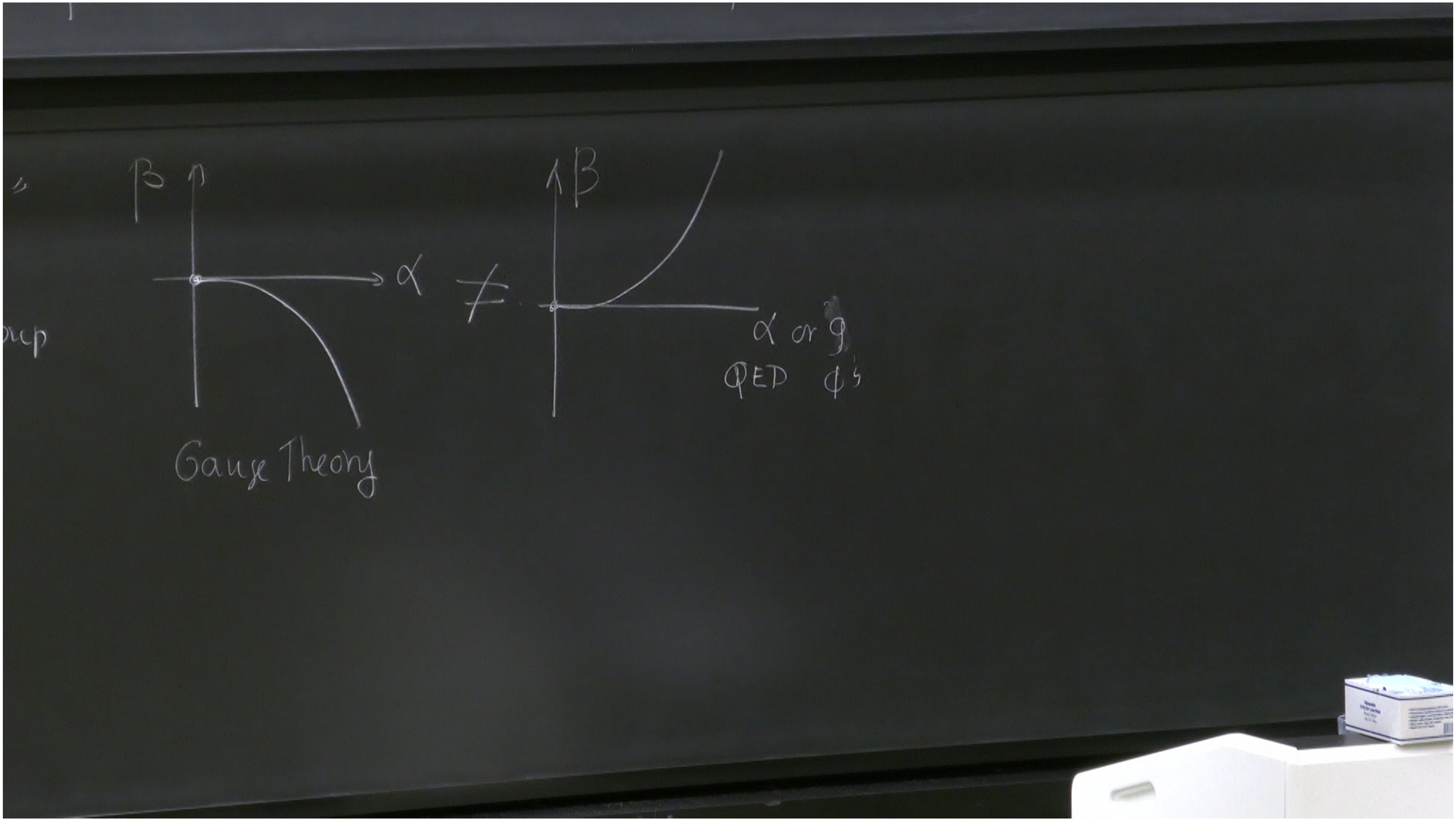
$SU(N)$ $\qquad \alpha = g^2$ $\qquad$ representation of the gauge group

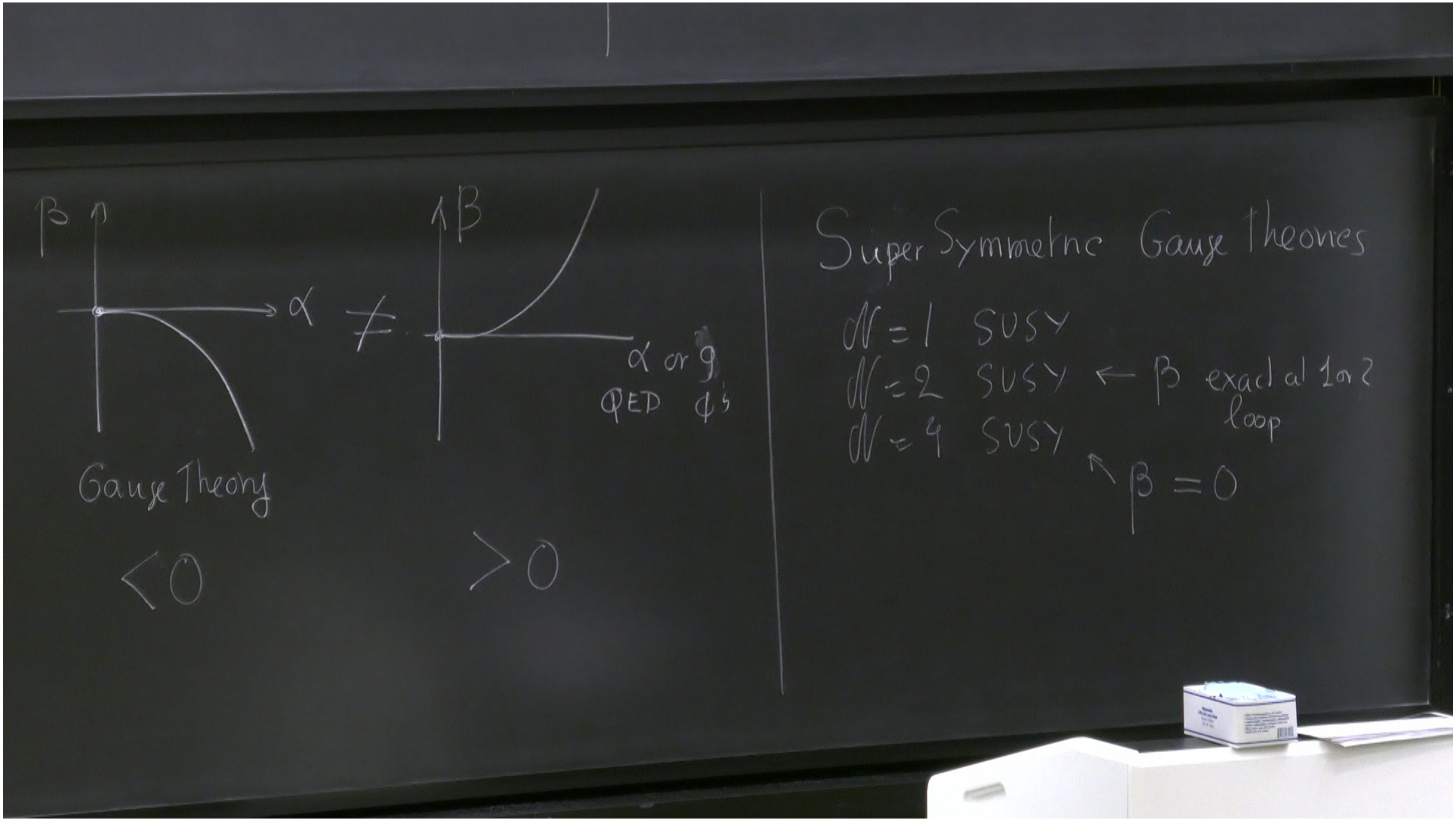
$$\beta(\alpha) = \alpha^2 \frac{1}{(4\pi)^2} \left(-\frac{11}{3} N + \frac{4}{3} N_F \frac{1}{2} \right)$$

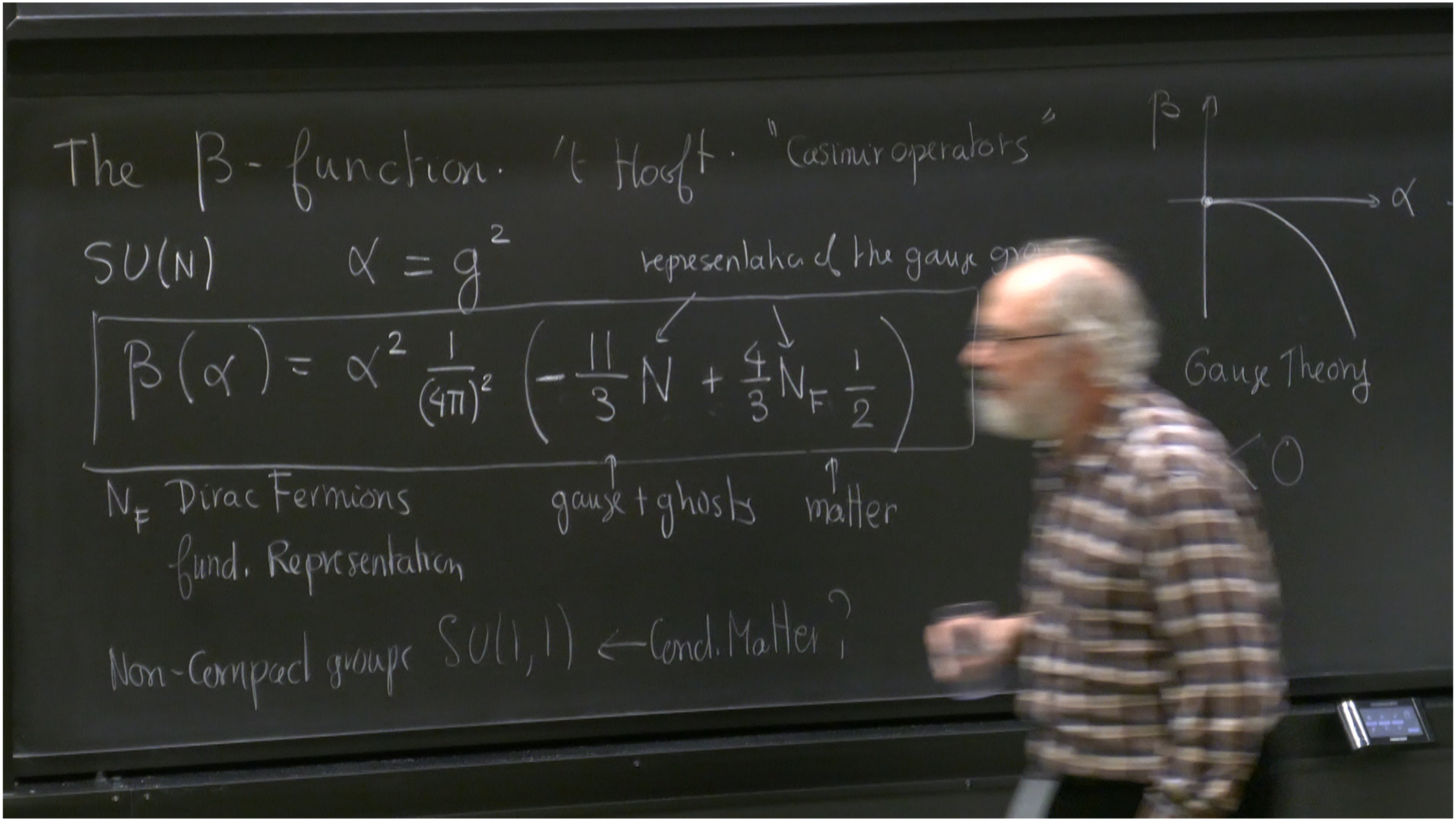
gauge + ghosts $\qquad$ matter

N_F Dirac Fermions
fund. Representation


The β-function. 't Hooft. "Casimir operators"
SU(N)
α = g²
representation of the gauge group
β(α) = α² 1/(4π)² (−11/3 N + 4/3 N_F 1/2)
gauge + ghosts
matter
N_F Dirac Fermions
fund. Representation
β


β
α
≠
β
α or g
QED ϕ⁴
Gauge Theory


β
α
≠
β
α or g
QED φ's
Gauge Theory
<0
>0
Super Symmetric Gauge Theories
N = 1 SUSY
N = 2 SUSY ← β exact at 1 or 2 loop
N = 4 SUSY
β = 0


The β-function. 't Hooft. "Casimir operators"
SU(N)
α = g²
representation of the gauge
β(α) = α² 1/(4π)² (−11/3 N + 4/3 N_F 1/2)
N_F Dirac Fermions
fund. Representation
gauge + ghosts
matter
Non-Compact groups SU(1,1) ← Cond. Matter?
β
α
Gauge Theory
< 0

A

running coupling at energy E

$$\beta(\alpha) = -C\alpha^2 < 0 \qquad E\frac{d}{dE}\alpha_{eff}(E) = \beta(\alpha_{eff}(E))$$

scale anomaly

Classically: YM is scale invariant in $d=4$

$[A] = 1 \qquad [g] = 0$

Energy dimension

... gauge bosons

y E

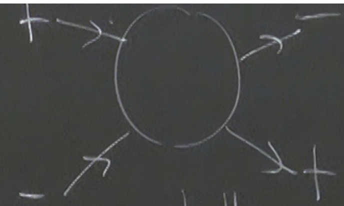
high energy $E \nearrow$, small $\alpha_s(E)$

Asymptotic Freedom (in the UV)

A

Field Strength associated to A

ghosts



on shell gauge bosons

$k^2 = 0$ onshell momentum $|k, \pm\rangle$

polarization

François Gelis Book

sy E

high energy $E \nearrow$, small $\alpha_s(E)$

Asymptotic Freedom (in the UV)

Pert. Theory is "safe" in the UV

Gauge Theory "Exists" in the continuum limit

10^{-*} F.

$\square$

tiny box

What happends at low energies?

QCD quarks ψ, gluons A

SU(3) ~~isolated~~

singlets of SU(3) exists

nergy $E \nearrow$, small $\alpha_s(E)$

nptotic Freedom (in the UV)

Theory is "safe" in the UV

eory "Exists" in the continuum limit

10^{-*} F.

$\square$

tiny box

What happend at low energies?

QCD quarks ψ, gluons A

SU(3) ~~isolated~~

might exists

singlets of SU(3) exists

di-quarks mesons

tri-quarks hadrons

Non-compact group SO(1,1) ← cond. matter

Confinement (K. Wilson)

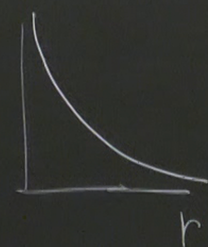
Pure gauge theory + ∞ massive matter particle

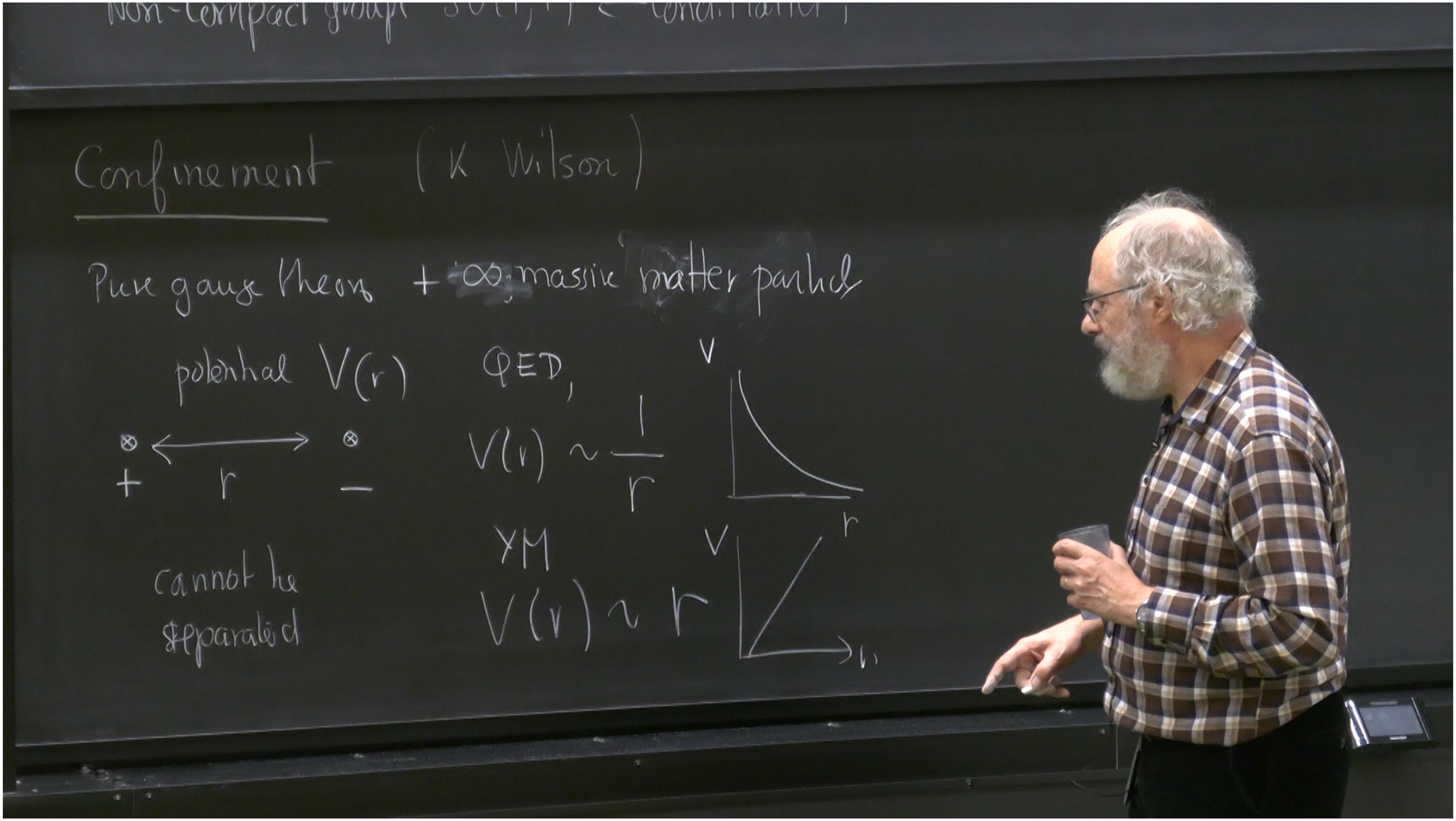
potential $V(r)$ QED,

$\otimes \longleftrightarrow \otimes$

$+ \quad r \quad -$

$$V(r) \sim \frac{1}{r}$$

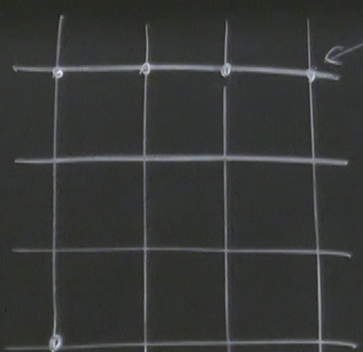



Confinement (K Wilson)
Pure gauge theory + ∞ massive matter particle
potential V(r)
QED,
V(r) ~ 1/r
+ r −
YM
V(r) ~ r
cannot be separated

<0 $\qquad$ >0

Landau Pole (Ghost)

Lattice gauge theory

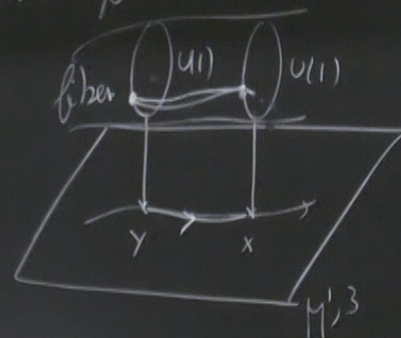


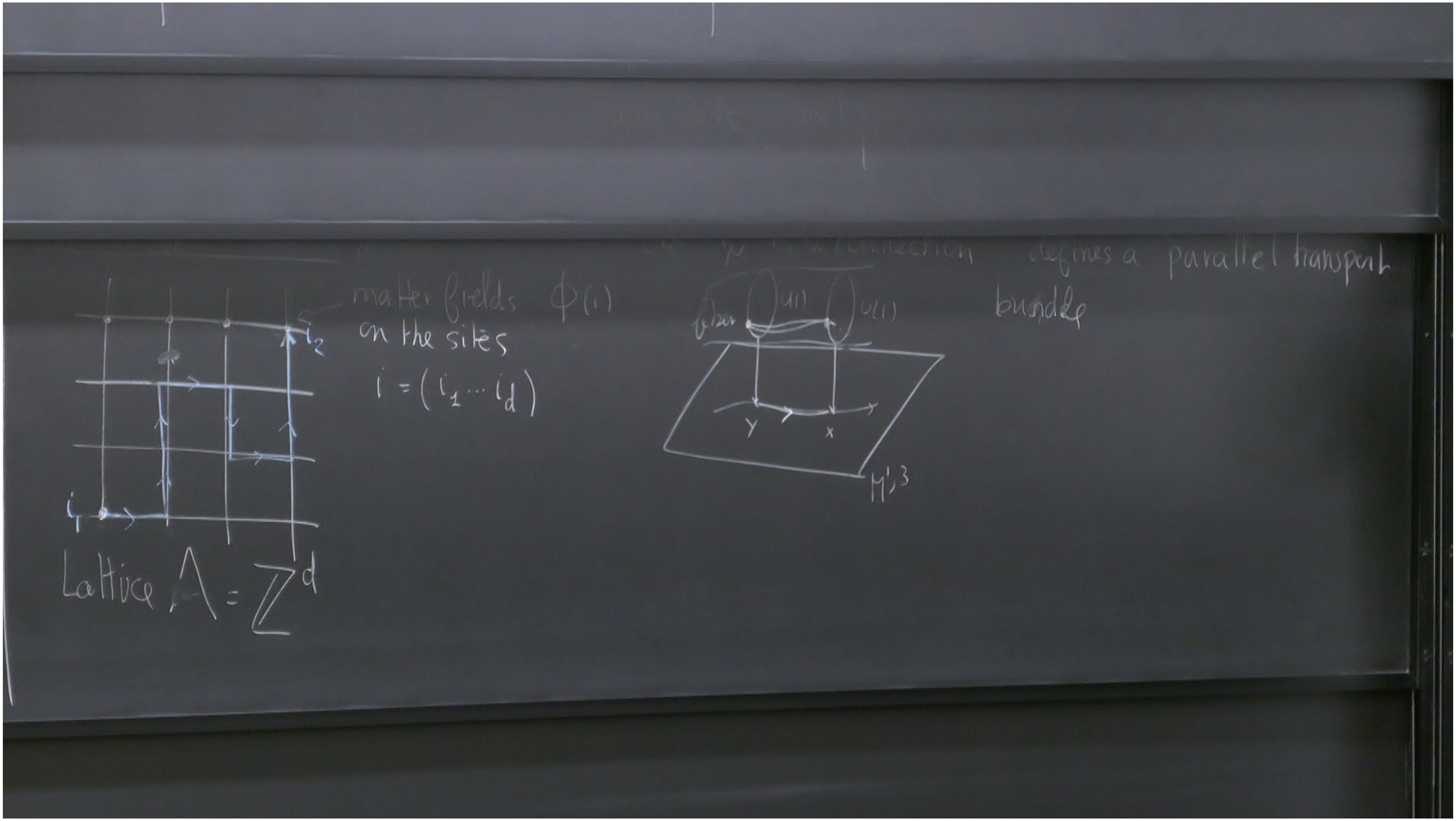
matter fields $\phi(i)$ on the sites

$i = (i_1 \cdots i_d)$

Lattice $\Lambda = \mathbb{Z}^d$

$dx^\mu A_\mu^a$ is a "connection" defines a "parallel transport" bundle



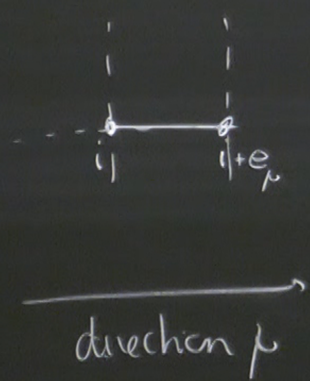

matter fields φ(i)
on the sites
i = (i1 … id)
i1
i2
Lattice Λ = Z^d
defines a parallel transport
bundle
fiber
U(1)
U(1)
y
x
M1,3

separated

$V(r) \sim r$

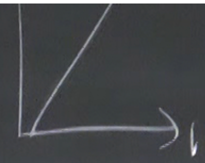
gauge field "lives" on the links of the lattice

edge $(i, i+e_\mu) = e \longrightarrow$

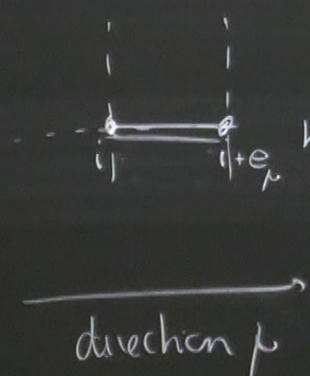


separated

$V(r) \sim r$



gauge field "lives" on the links of the lattice

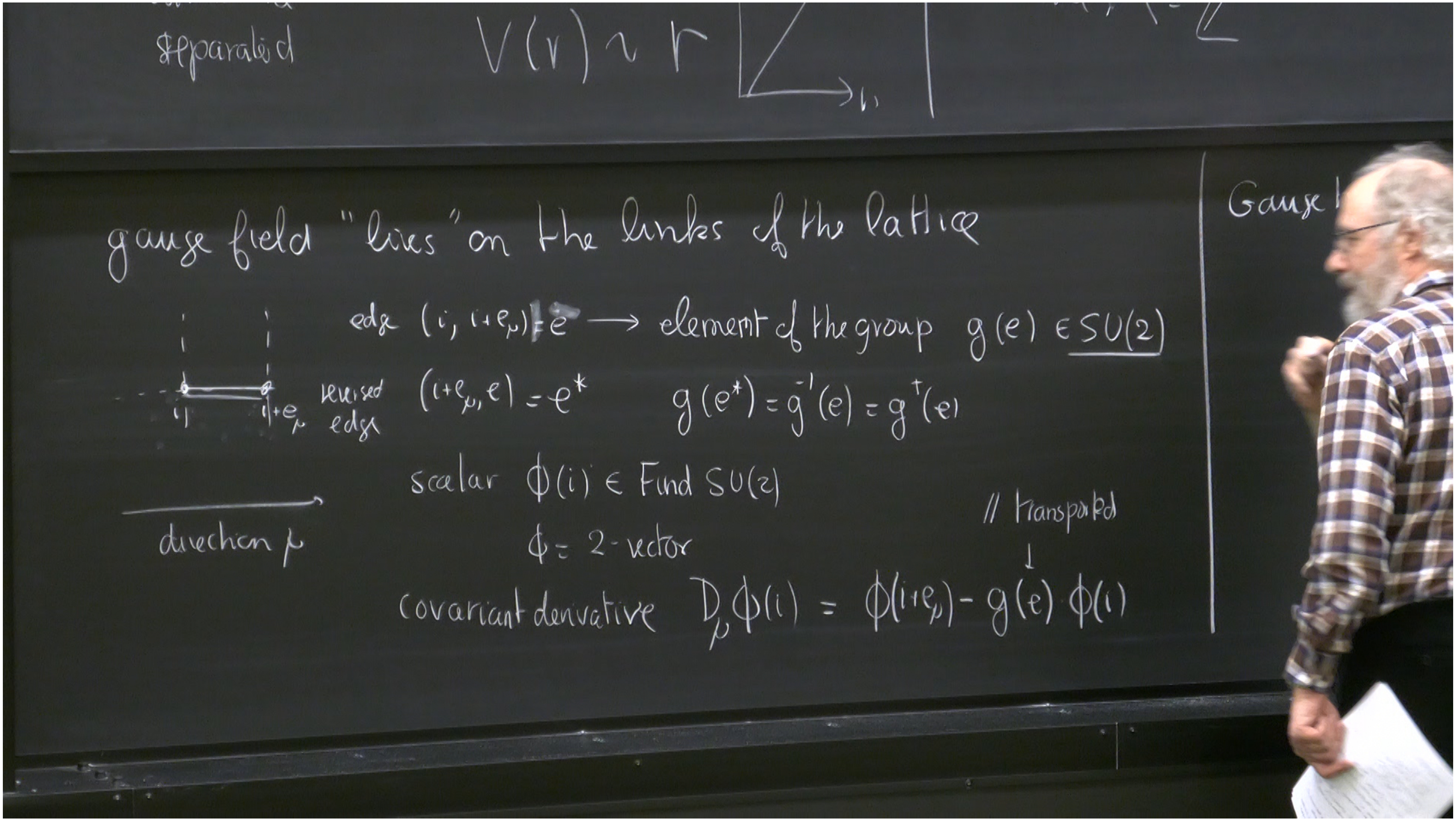


edge $(i, i+e_\mu) = e \longrightarrow$ element of the group $g(e) \in \underline{SU(2)}$

reversed edge $(i+e_\mu, e) = e^*$ $\quad g(e^*) = g^{-1}(e) = g^\dagger(e)$

scalar $\phi(i) \in$ Fund $SU(2)$

$\phi = 2\text{-vector}$


separated
$V(r) \sim r$
Gauge
gauge field "lives" on the links of the lattice
edge $(i, i+e_\mu) = e \longrightarrow$ element of the group $g(e) \in SU(2)$
reversed edge $(i+e_\mu, e) = e^*$
$g(e^*) = g^{-1}(e) = g^\dagger(e)$
i
$i+e_\mu$
direction μ
scalar $\phi(i) \in$ Fund $SU(2)$
$\phi = 2$-vector
// transported
covariant derivative $D_\mu \phi(i) = \phi(i+e_\mu) - g(e)\,\phi(i)$

$= \mathbb{Z}^d$

Gauge transformation $U(i) \in SU(2)$

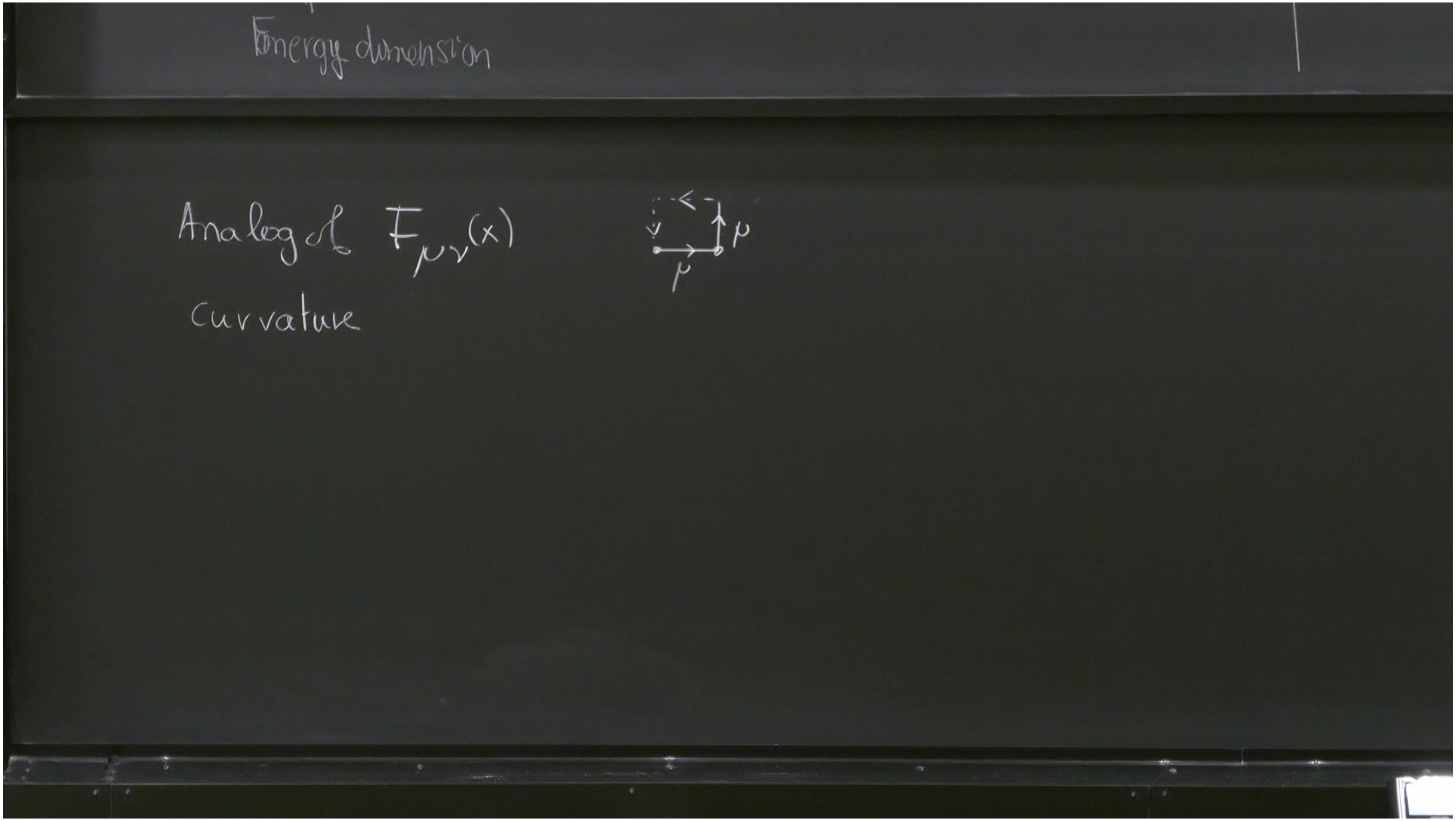
site

$$\phi(i) \longrightarrow U(i)\,\phi(i) = \phi'(i)$$

$$g(e) \longrightarrow U(j)\,g(e)\,U^{-1}(i)$$

$$e = (i, j)$$

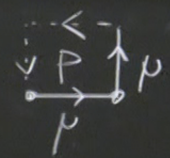
$$j = i + e_\mu$$


Energy dimension
Analog of $F_{\mu\nu}(x)$
curvature
μ
ν

Energy dimension

Analog of $F_{\mu\nu}(x)$

curvature

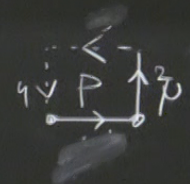


plaquette P

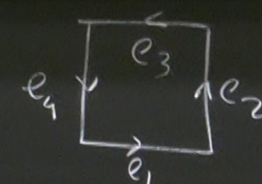
Energy dimension

Analog of $F_{\mu\nu}(x)$

curvature



plaquette P

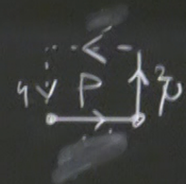


$$\prod_{\substack{\text{ordered}\\ \text{product}\\ \text{of edges along the plaquette}}} g(e) = g(e_4)\,g(e_3)\,g(e_2)\,g(e_1)$$

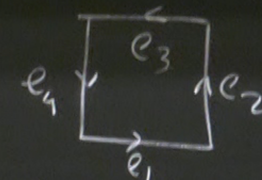
Energy dimension

Analog of $F_{\mu\nu}(x)$

curvature



plaquette P



$$\prod_{\substack{\text{ordered}\\ \text{product}\\ \text{of edges along the plaquette}}} g(e) = g(e_4)\,g(e_3)\,g(e_2)\,g(e_1)$$

<u>Pure gauge</u>

$g(e) = u(j)\,u^{-1}(i)$

for some u

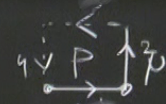
$[A] = 1 \quad [g] = 0$

energy dimension

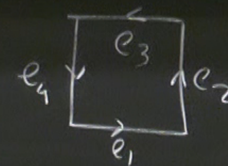
Gauge Theory "Exists" in

Analog of $F_{\mu\nu}(x)$

curvature



plaquette P



$\prod g(e) = g(e_4)\,g(e_3)\,g(e_2)\,g(e_1) = U(\text{plaquette})$

ordered product of edges along the plaquette

plaquet not a pure gauge

Pure gauge

$g(e) = u(j)\,u^{-1}(i)$

for some u

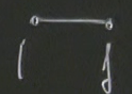
$A_\mu = \partial_\mu \alpha \quad u = e^{i\alpha}$

$U(p) = 1$

$U(p)$

$U(p) \neq 1$

$U(p) \sim e^{+i F_{\mu\nu}}$



(plaquette)

Action Wilson $S_W[g] = \sum_{\substack{\text{plaquettes} \\ p}}$

(plaquette)

Action Wilson $S_W[g] = \sum_{\substack{\text{plaquettes} \\ p}} \mathrm{Tr}(1 - U(p)$

(plaquette)

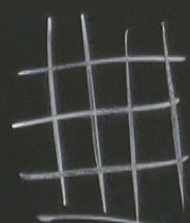
<u>Action</u>
<u>Wilson</u>

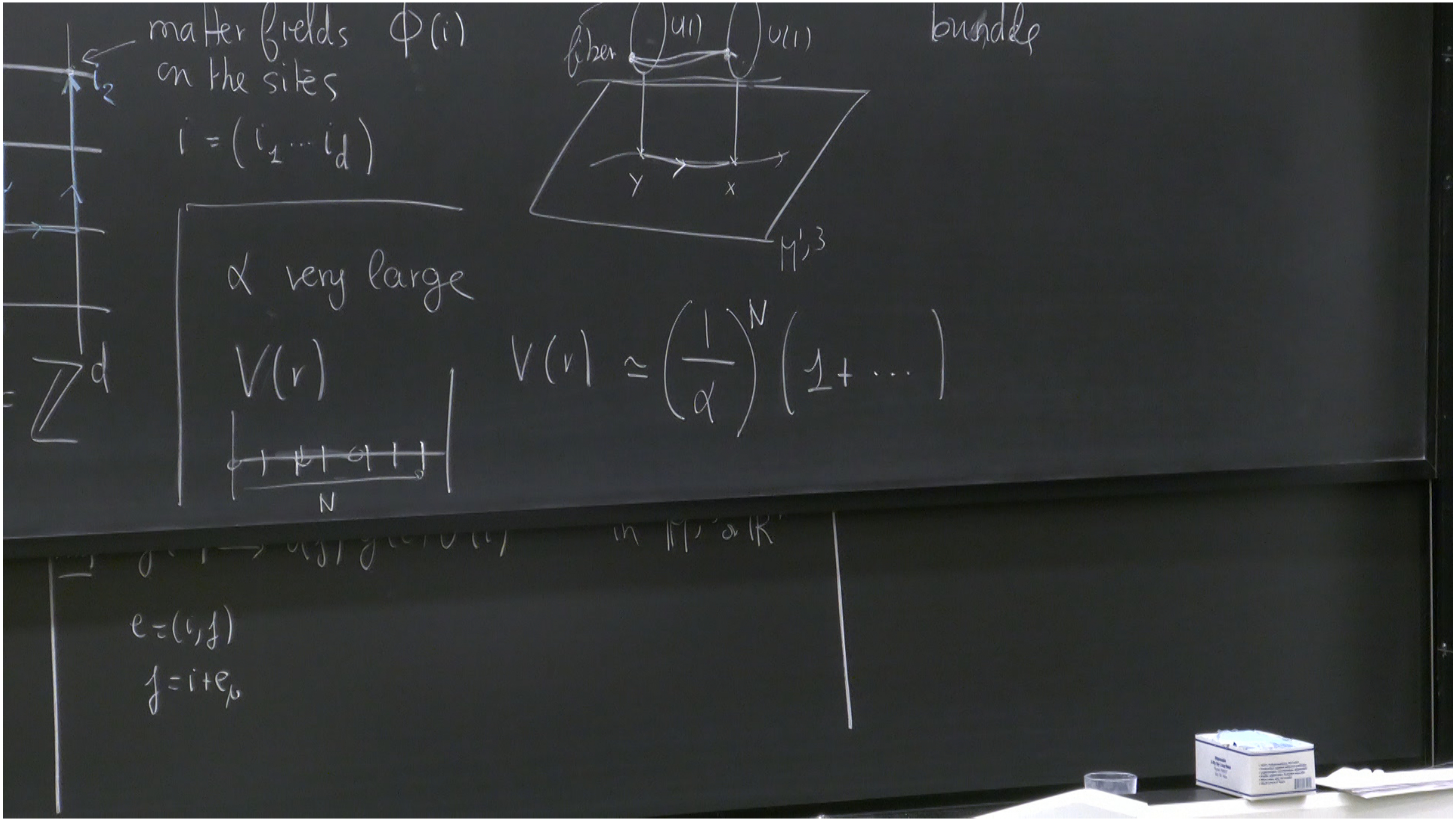
$$S_W[g] = \frac{1}{\alpha} \sum_{\substack{\text{plaquettes} \\ p}} \mathrm{Re}\left(\mathrm{Tr}\left(\mathbb{1} - U(p)\right)\right)$$

α: coupling constant

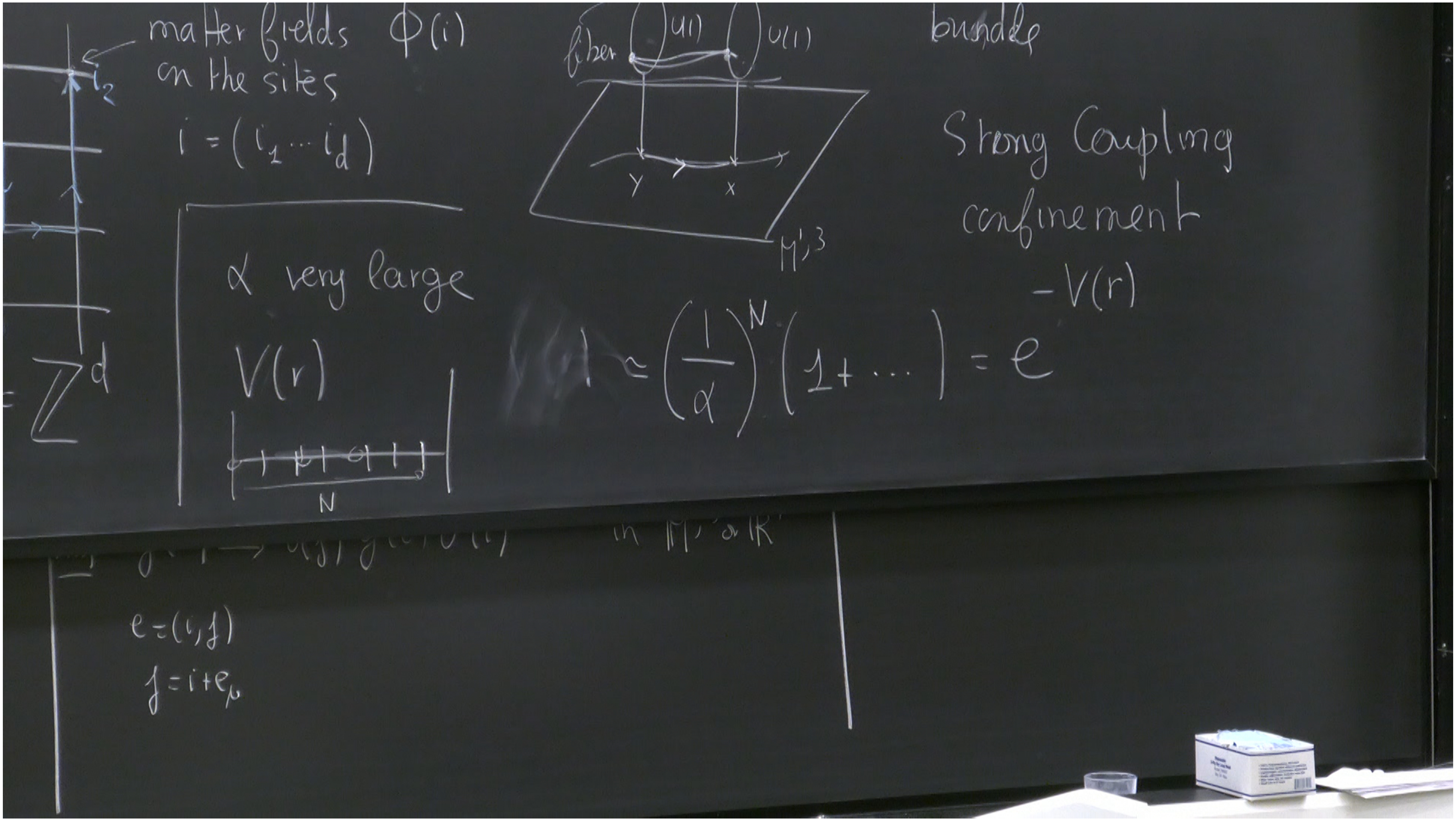
$$\int D[A]\, e^{-\frac{1}{g^2}S[A]} = \int \prod_{\text{edge}} d\mu(g(e))\, \exp\left(-\frac{1}{\alpha}\sum_p \mathrm{Re}\left(\mathrm{Tr}\left(1 - U(p)\right)\right)\right)$$

$d\mu$: Haar Measure on the group





matter fields $\Phi(i)$
on the sites
$i = (i_1 \cdots i_d)$
i_2
$\mathbb{Z}^d$
fiber
$U(1)$
$U(1)$
bundle
y
x
$M^{1,3}$
α very large
$V(r)$
N
$V(r) \simeq \left(\frac{1}{\alpha}\right)^N \left(1 + \cdots\right)$
$e = (i,j)$
$j = i + e_\mu$


matter fields Φ(i)
on the sites
i = (i1 ... id)
i2
Zd
fiber
U(1)
U(1)
bundle
Y
X
M3
Strong Coupling
confinement
α very large
V(r)
N
≃ (1/α)N (1+ ...) = e−V(r)
e = (i,j)
j = i+eμ