Title: QFT2 Lecture - 121123

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Collection: Quantum Field Theory 2 2023/24

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Renormalization of Gauge Theories

Diagra

Gauge Symmetry

$SU(2)$ $\quad A_\mu^a \quad a = 1, 2, 3$ spin 1 Bosons

Adj Representation $\bar{C}^a, C^a$ ghosts spin 0 Fermions

ξ- Gauge

$\partial^\mu A_\mu = 0$

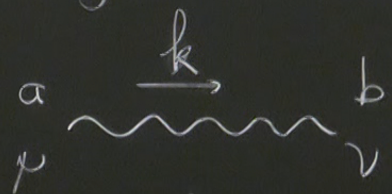
a

μ

...res

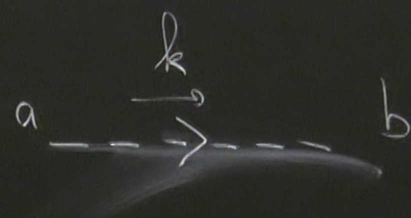
Diagrammatics (Euclidean $++++$) when $\xi = 1$

Bosons



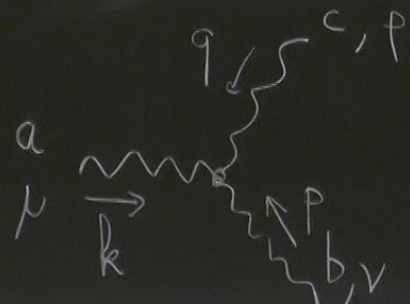
$$\delta_{ab}\,\frac{1}{k^2}\left(\delta_{\mu\nu} + \big(\underset{\text{G.F}}{\xi} - 1\big)\frac{k_\mu k_\nu}{k^2}\right) = \delta_{ab}\,\delta_{\mu\nu}\,\frac{1}{k^2}$$

Fermions



$$\delta_{ab}\,\frac{1}{k^2}$$

Rotation/Lorentz + gauge



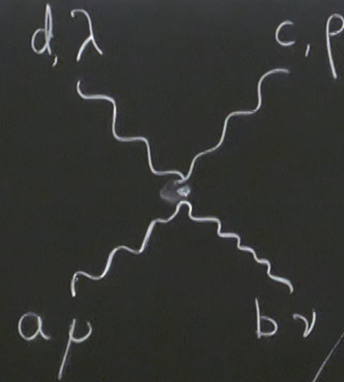
$$k + p + q = 0$$

$$(ig)\left(\delta_{\mu\nu}(k-p)_\rho + \delta_{\nu\rho}(p-q)_\mu + \delta_{\rho\mu}(q-k)_\nu\right)\epsilon_{abc}$$

$++\Big)$ when $\xi = 1$

$\frac{k_\mu k_\nu}{k^2}\Big) = \delta_{ab}\,\delta_{\mu\nu}\,\frac{1}{k^2}$

…ion/Lorentz + gauge



$$(-g^2)\,\epsilon^{eab}\,\epsilon^{ecd}$$

$$\left(\delta_{\mu\rho}\,\delta_{\nu\sigma} - \delta_{\mu\sigma}\,\delta_{\nu\rho}\right)$$

$$\epsilon^{eab}\,\epsilon^{ecd} = \delta_{ac}\,\delta_{bd} - \delta_{ad}\,\delta_{bc}$$

$L^{[\rho,\sigma]}_{\mu\nu}$ Generators of $SO(d)$

$SO(3) \longleftrightarrow SU(2)$

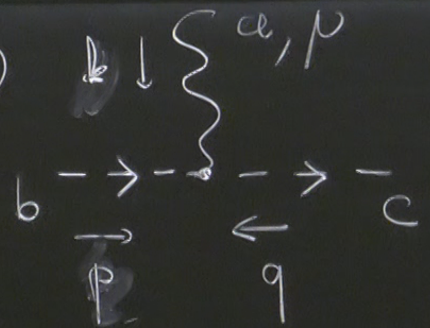
$L^{[bd]}_{ac} = SO(3)$

1

$(p-q)_\mu + \delta_{\rho\mu}(q-k)_\nu\big)\,\epsilon_{abc}$

2

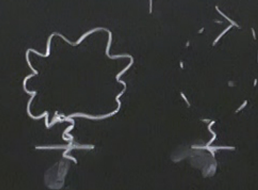
(2)



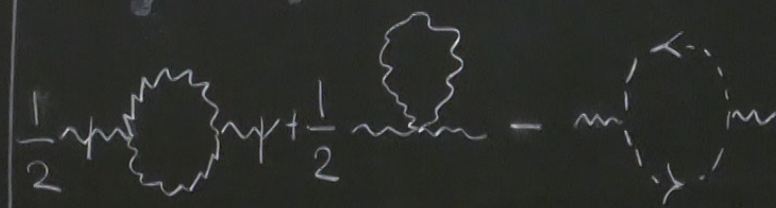
$$(ig)\,\epsilon_{abc}\, q_\mu$$

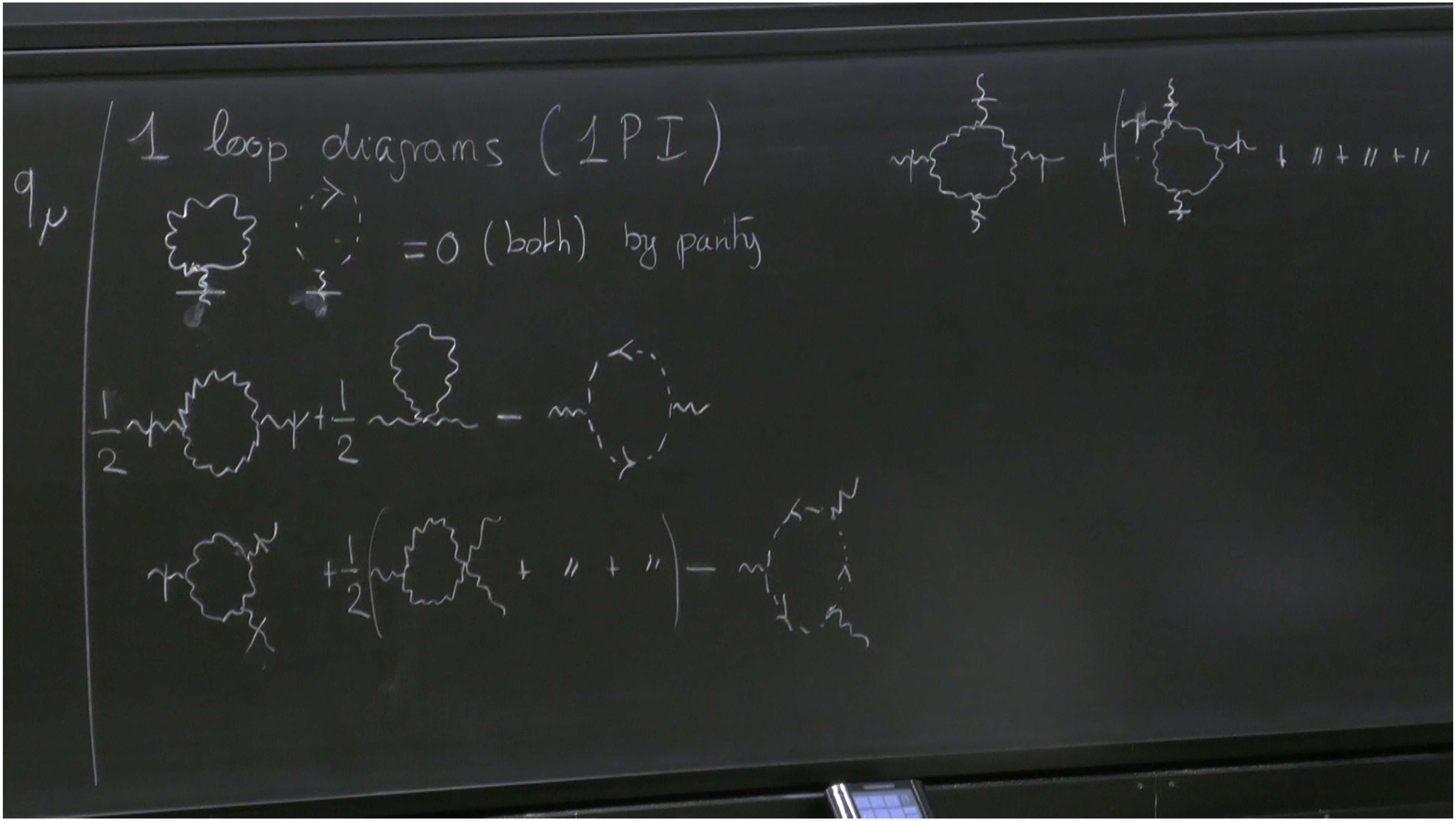
$$p+q+k=0$$

1 loop diagrams (1PI)

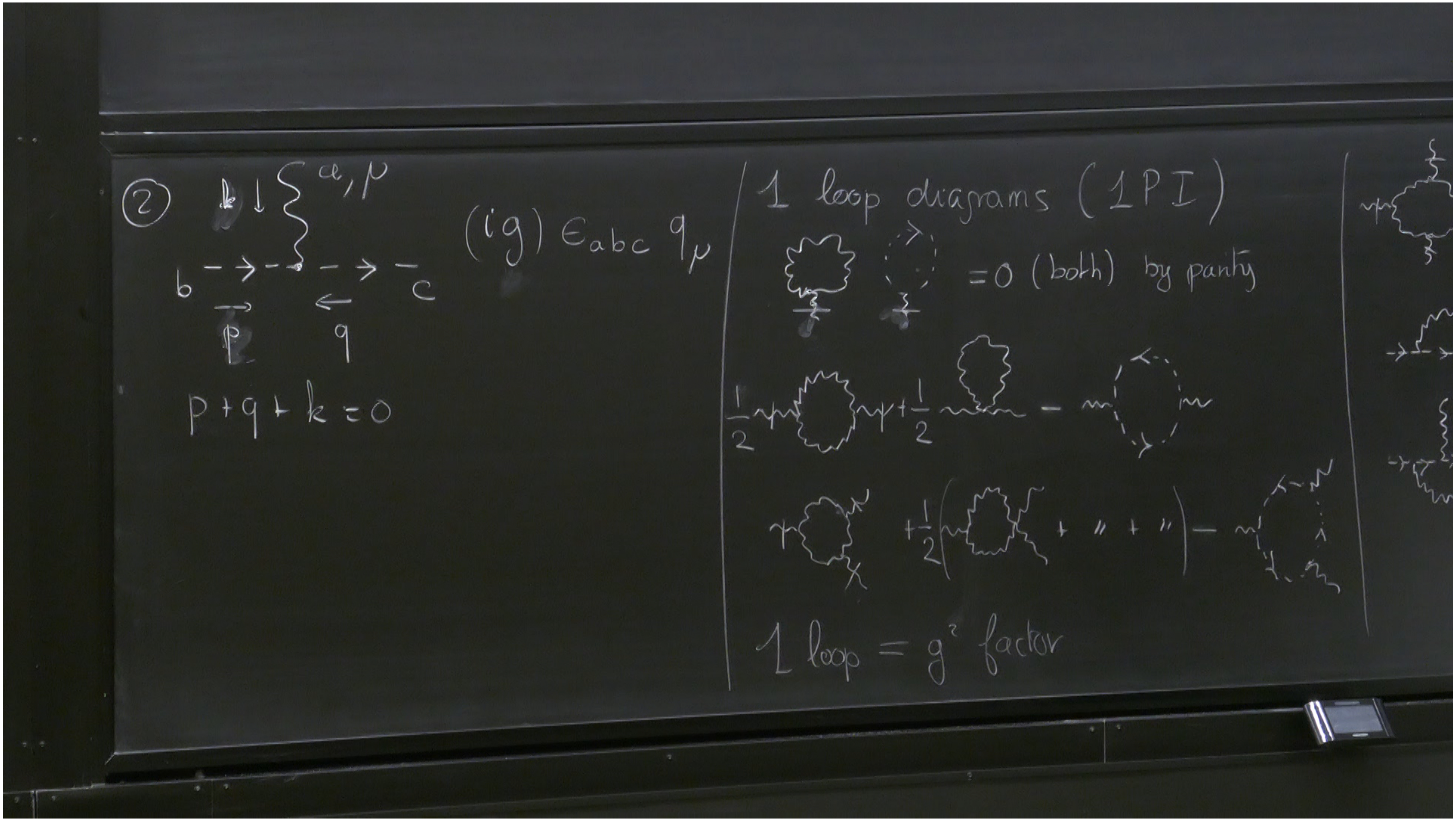


$= 0$ (both) by parity

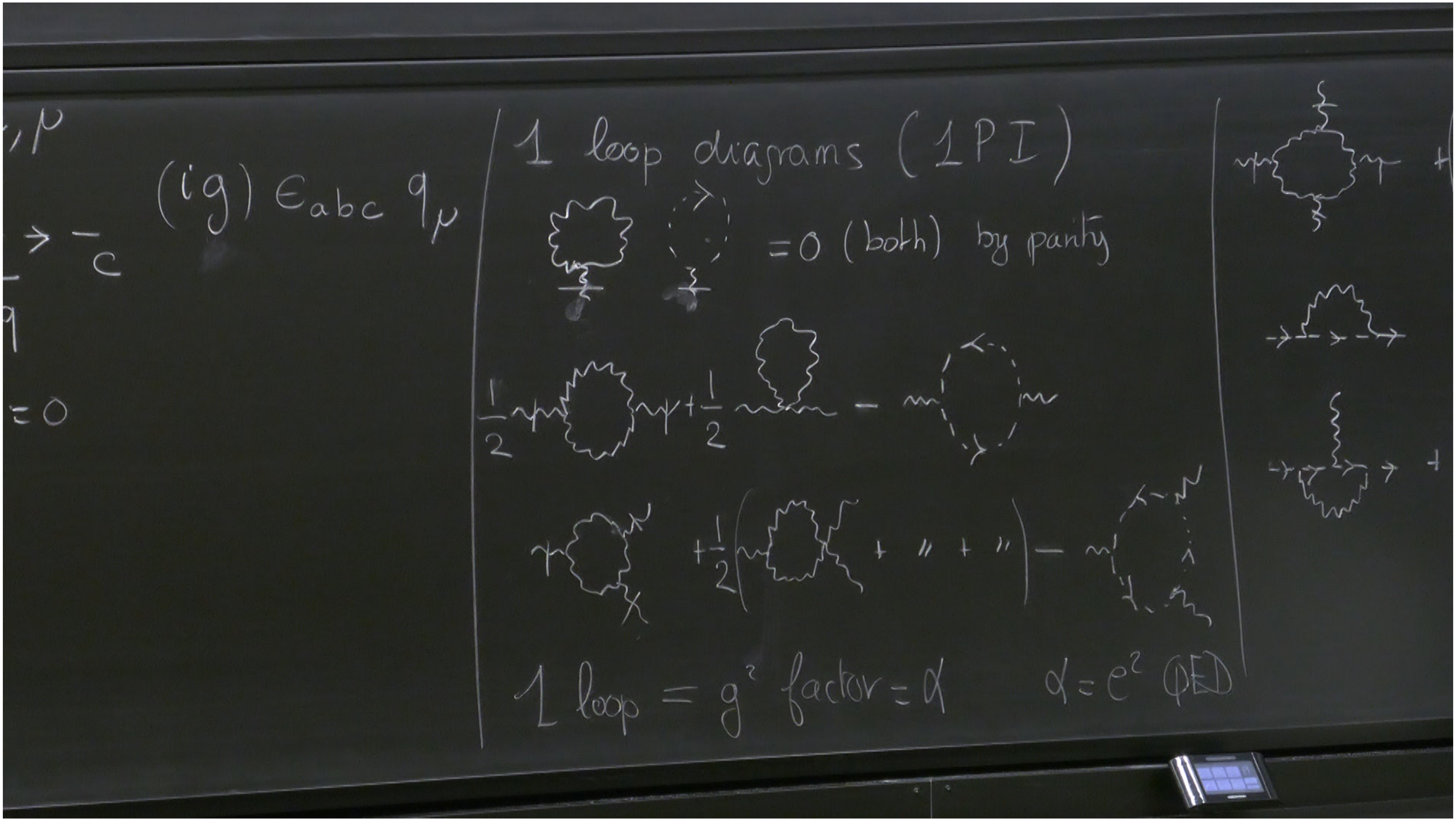




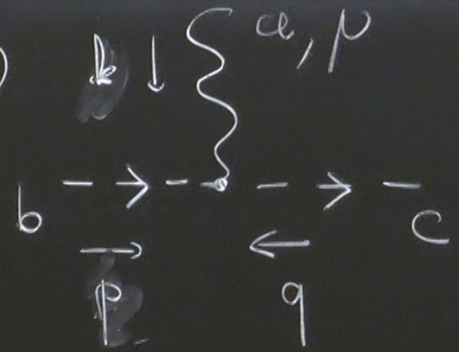
q_μ
1 loop diagrams (1PI)
= 0 (both) by parity
$\frac{1}{2}$
$+\frac{1}{2}$



②
k
a, μ
(ig) ε abc q μ
b
c
p
q
p + q + k = 0
1 loop diagrams (1PI)
= 0 (both) by parity
1/2
+ 1/2
+ 1/2
+ " + "
1 loop = g² factor


(ig) $\epsilon_{abc}\, q_\mu$
= 0
1 loop diagrams (1PI)
= 0 (both) by parity
$\frac{1}{2}$
+ $\frac{1}{2}$
+ $\frac{1}{2}$ (+ " + ")
1 loop = g^2 factor = α
$\alpha = e^2$ QED

(2)

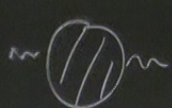


$$(ig)\,\epsilon_{abc}\, q_\mu$$

$$p+q+k=0$$

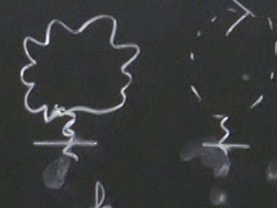
possible UV singularities

$d=4$ potential div $d \geq 2$ pot div $d \geq 4$

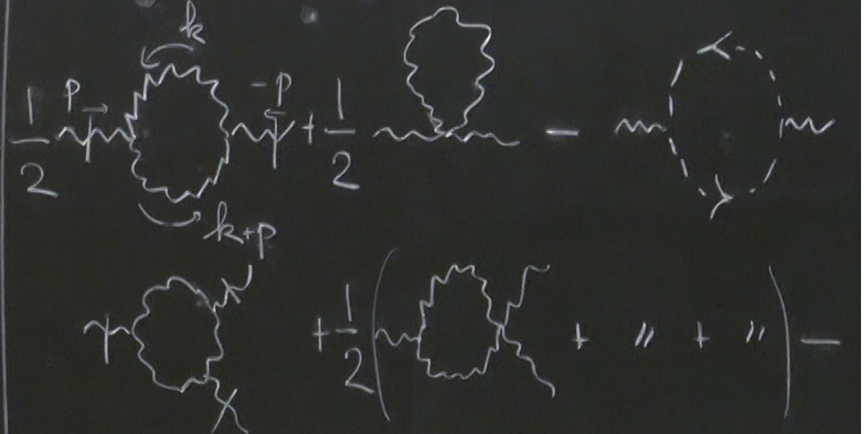


$$\int \frac{d^d k}{(k^2)^2}\left(k\cdot k + \cancel{k\cdot p} + p\cdot p\right)$$

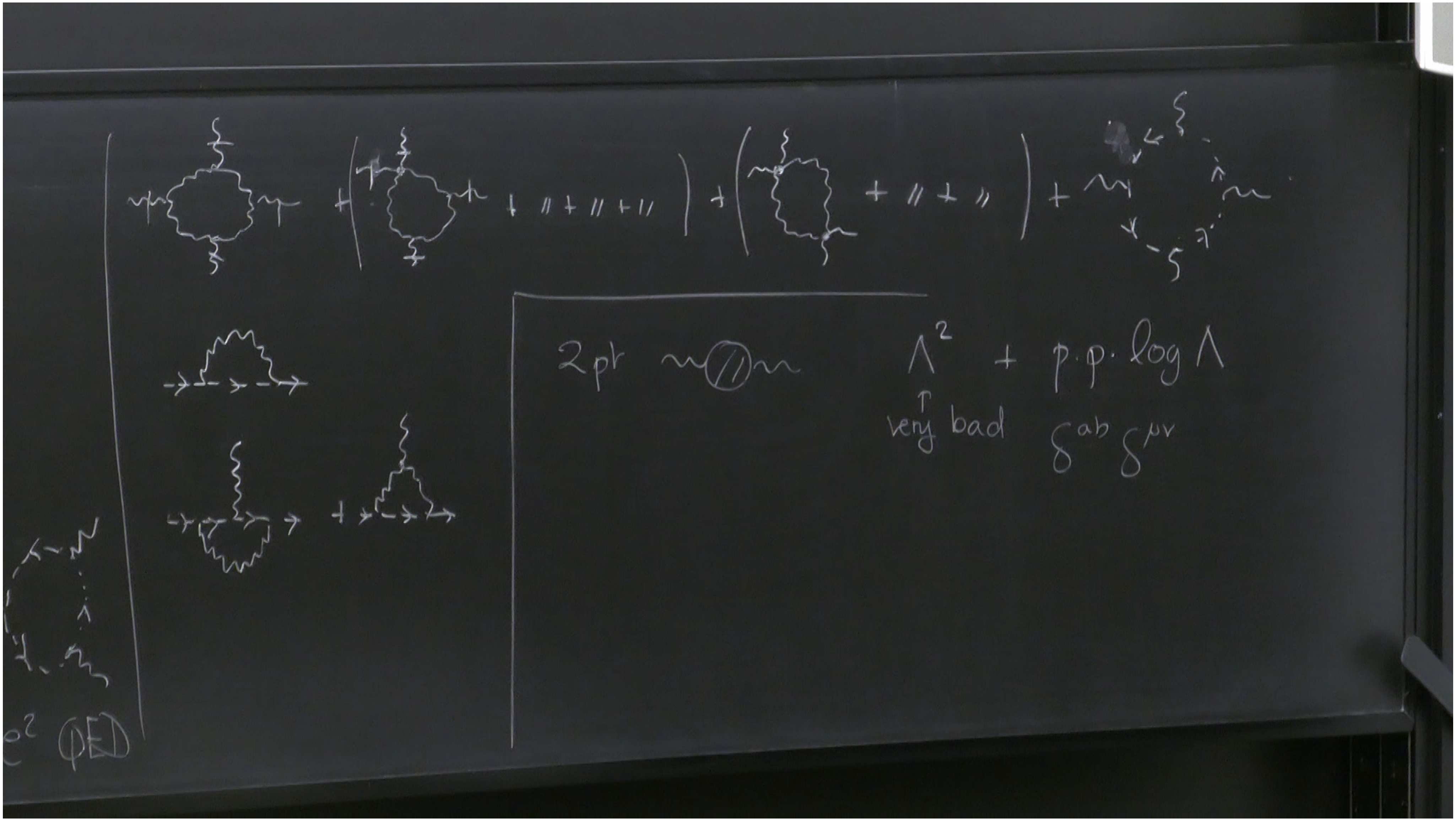
1 loop diagrams (1PI)

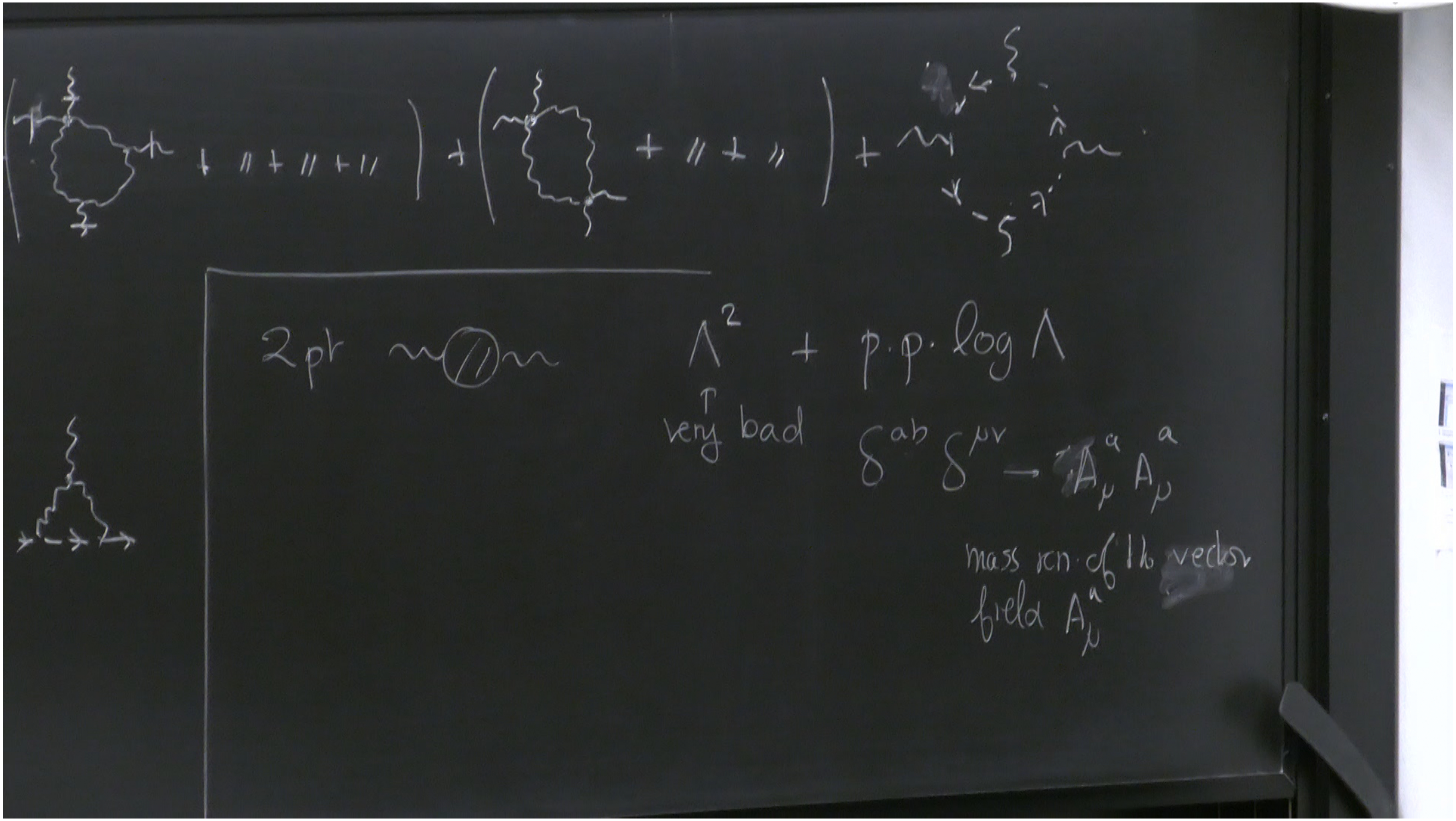


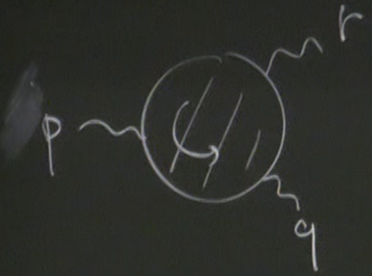
$= 0$ (both) by par



1 loop $= g^2$ factor $= \alpha$ α


2pt
Λ² + p·p· log Λ
very bad
δ^ab δ^μν
QED


2pt
Λ² + p·p· log Λ
very bad
δ^ab δ^μν → A^a_μ A^a_μ
mass ren. of the vector
field A^a_μ



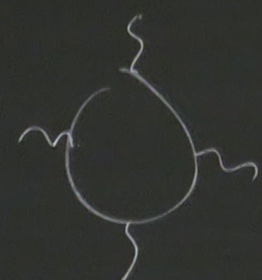
$$\int d^d k \, \frac{1}{(k^2)^3} \left(k \cdot k \cdot k + k \cdot k \cdot (\text{external momenta}) + \right.$$

div if $d \geq 3$
but 0 by symm.

din if $d \geq 4$

$\partial A \cdot A \cdot A$

$p + q + r = 0$ d-vectors



$$\int d^d k \, \frac{1}{(k^2)^4} \, k \cdot k \cdot k \cdot k + \ldots$$

div if $d \geq 4$

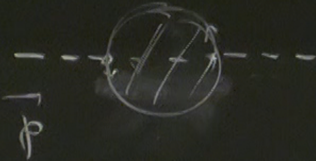
xternal momenta) +

$\uparrow$

un if $d \geq 4$

$\downarrow$

$A \cdot A \cdot A$



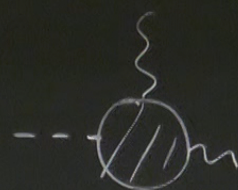
$$\int d^d k \, \frac{1}{(k^2)^2} \left(\cancel{k \cdot k} + \cancel{k\, p} + p\, p + \cdots \right)$$

$d \geq 2$ $\qquad$ $d \geq 4$

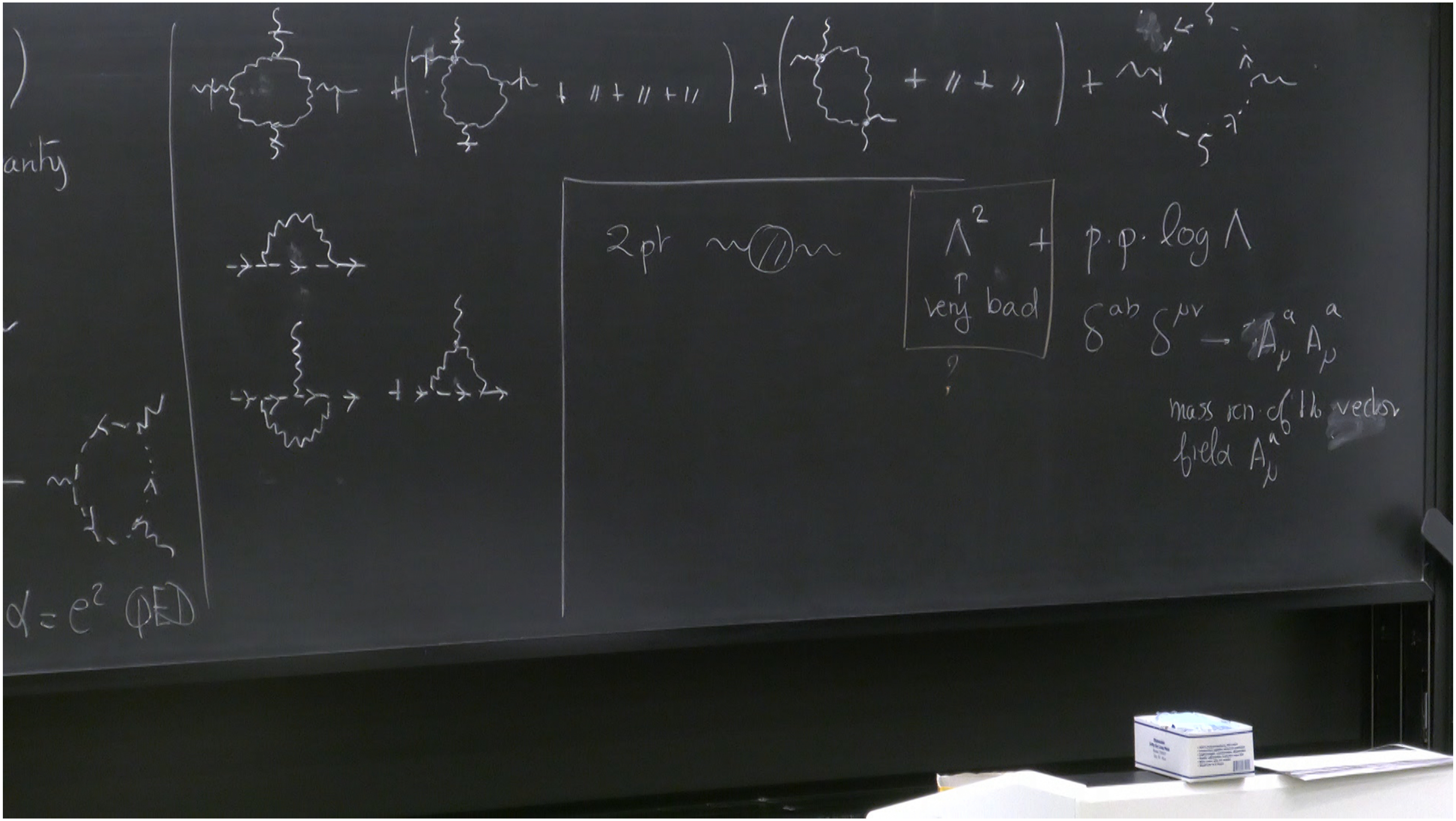
ghost are <u>massless</u>

gauge invariance

$\partial \bar{c} \; \partial c$



$d \geq 4 \qquad c A \partial \bar{c}$


2pt
Λ² + p·p·log Λ
very bad
δ^ab δ^μν → A_μ^a A_μ^a
mass ren. of the vector field A_μ^a
α = e² QED

$\cancel{k} + \cancel{k\,p} + p\,p + \ldots$

$d \geq 4$

$\partial \bar{c} \;\; \partial c$

Gauge Symm. preserved a 1 loop ?

Lorentz invariance

div at $\alpha \to 0$

dim d as a parameter

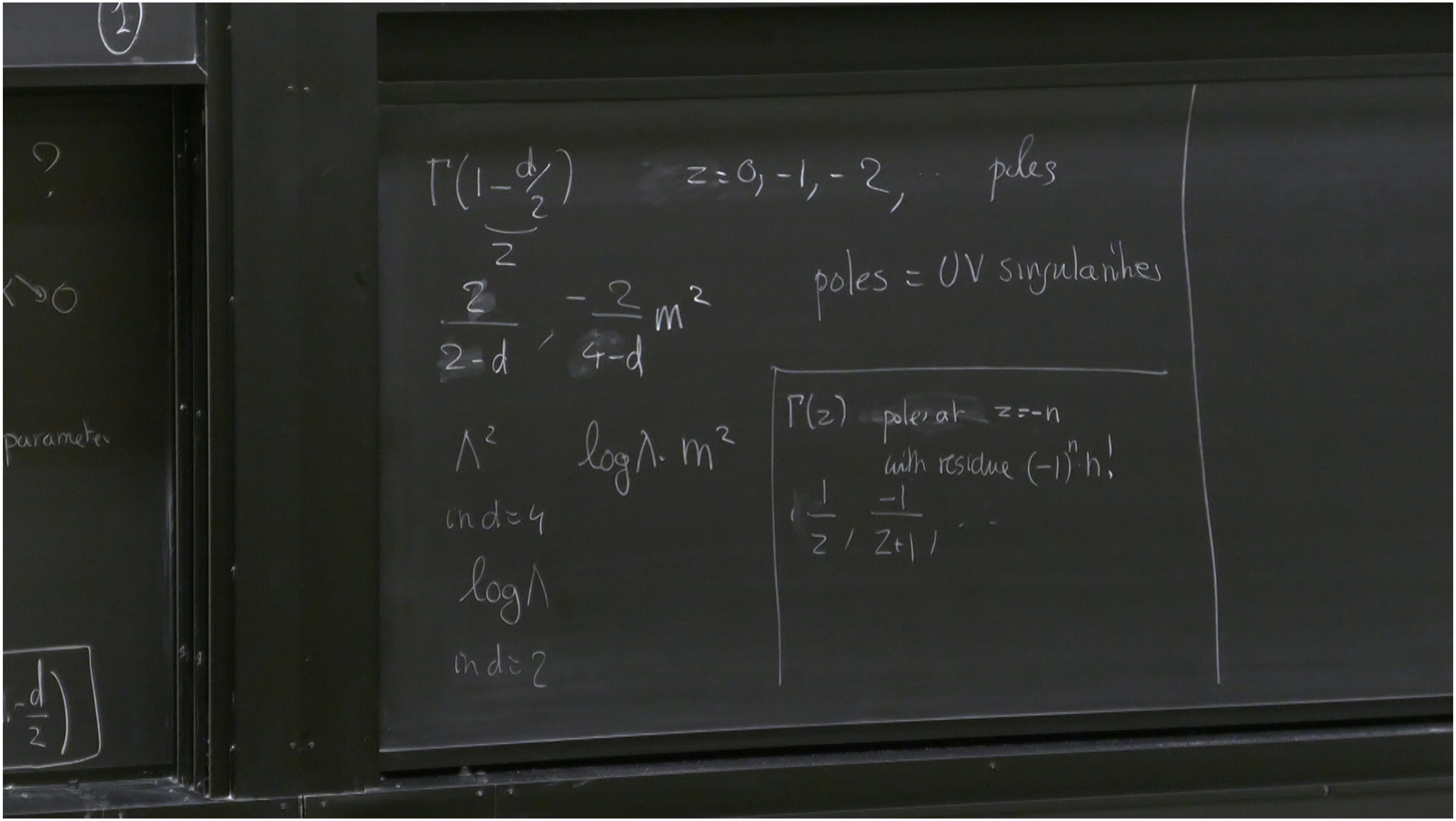
$$\int d^d k \, \frac{1}{k^2} \longrightarrow \text{pole } \frac{1}{2-d}$$

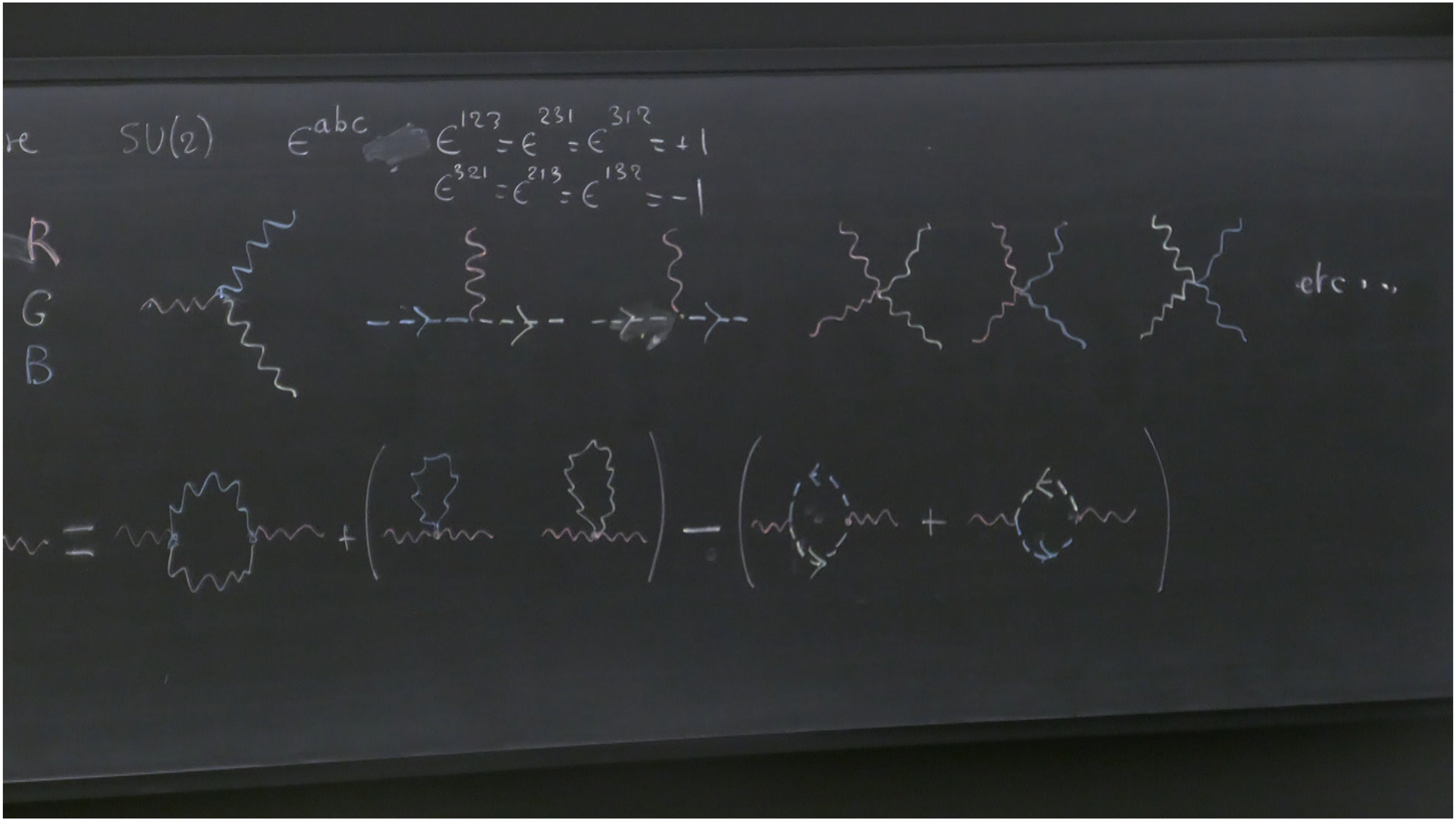
α Schwinger parameter "proper time"

$d \simeq 2$

$$\int \frac{d^d \ell}{(2\pi)^d} \frac{1}{\ell^2 + m^2} = \int_0^\infty d\alpha \, e^{-\alpha(\ell^2+m^2)} \int \frac{d^d \ell}{(2\pi)^d}$$

$$= \frac{1}{(4\pi)^{d/2}} \int_0^\infty d\alpha \, \alpha^{-\frac{d}{2}} e^{-\alpha m^2} = \boxed{\frac{1}{(4\pi)^{d/2}} (m^2)^{\frac{d-2}{2}} \Gamma\left(1 - \frac{d}{2}\right)}$$

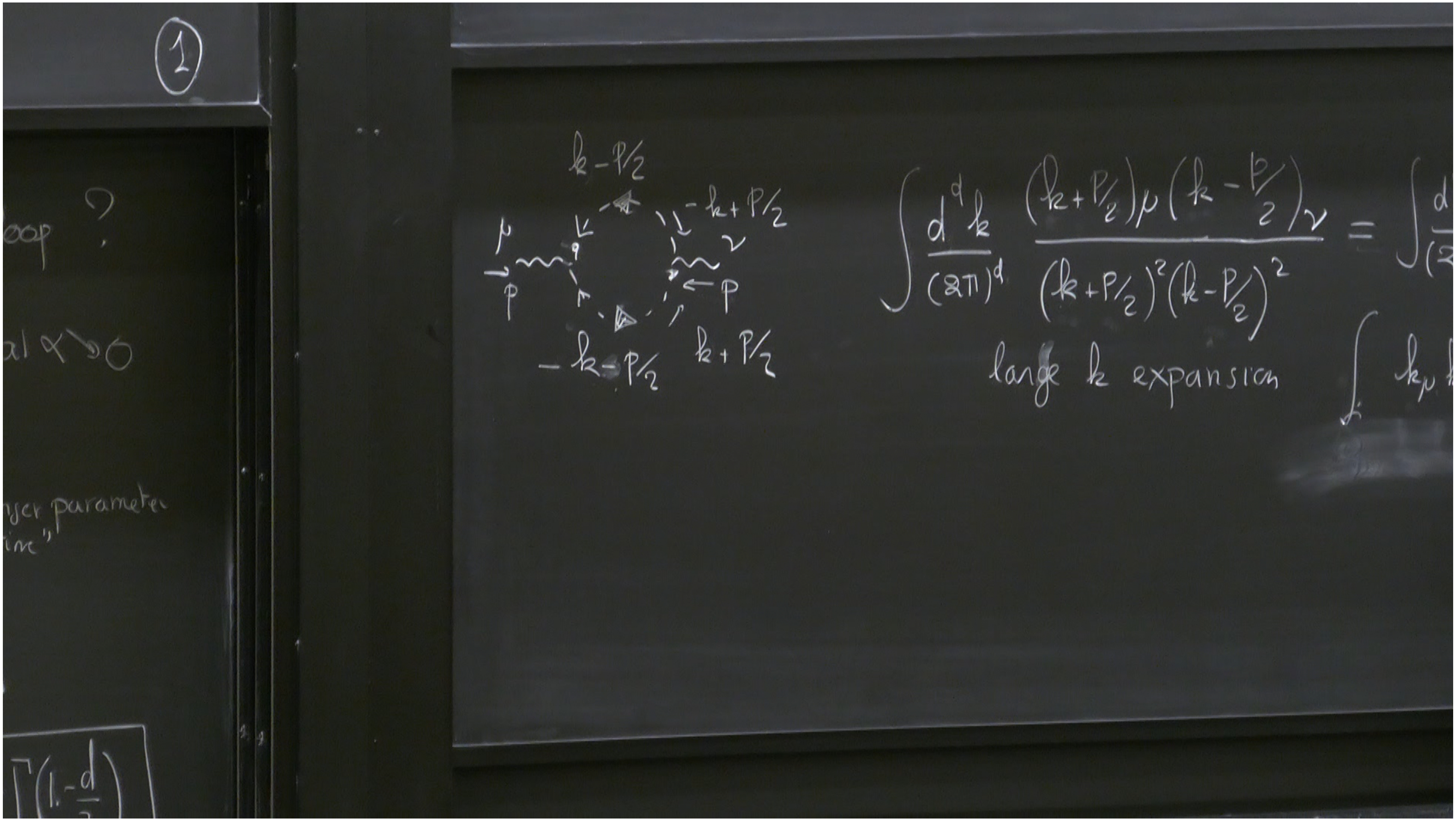

?
parameter
Γ(1 - d/2) with z under (1 - d/2)
z = 0, -1, -2, ... poles
2/(2-d), -2/(4-d) m^2
poles = UV singularities
Λ^2   log Λ · m^2
in d=4
log Λ
in d=2
Γ(z) poles at z=-n
with residue (-1)^n · n!
1/z, -1/(z+1), ...


re
SU(2)
ϵ^{abc}
$\epsilon^{123} = \epsilon^{231} = \epsilon^{312} = +1$
$\epsilon^{321} = \epsilon^{213} = \epsilon^{132} = -1$
R
G
B
etc...

$$\int \frac{d^d k}{(2\pi)^d} \frac{(k+p/2)_\mu (k-p/2)_\nu}{(k+p/2)^2 (k-p/2)^2} = \int \frac{d^d k}{(2\pi)^d} \left(\frac{k_\mu k_\nu}{(k^2)^2} + \frac{k_\mu k_\nu (k\cdot p)^2}{(k^2)^4} - \frac{1}{2} \frac{k^\mu k^\nu p^2}{(k^2)^3} - \frac{1}{4} \right.$$

large k expansion

$$\int k_\mu k_\nu \to \int \delta_{\mu\nu} k^2/d \qquad \underbrace{k_\mu k_\nu k_\rho k_\sigma}\, p_\rho p_\sigma$$

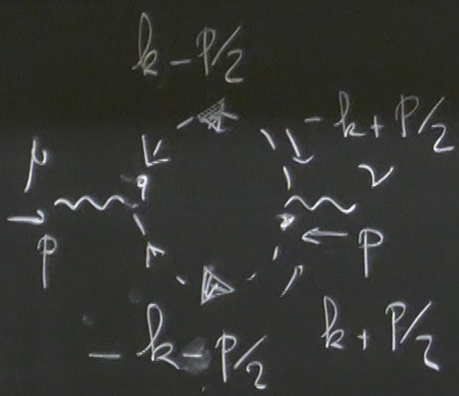

1
oop ?
al α → 0
nger parameter
ime"
$k - P/2$
$-k + P/2$
μ
ν
p
p
$-k - P/2$
$k + P/2$
$\int \frac{d^d k}{(2\pi)^d} \frac{(k+P/2)_\mu (k-P/2)_\nu}{(k+P/2)^2 (k-P/2)^2} =$
large k expansion
$\int k_\mu$

$$= \int \frac{d^d k}{(2\pi)^d} \left(\frac{k_\mu k_\nu}{(k^2)^2} + \frac{k_\mu k_\nu (k\cdot p)^2}{(k^2)^4} - \frac{1}{2} \frac{k^\mu k^\nu p^2}{(k^2)^3} - \frac{1}{4} \frac{p_\mu p_\nu}{(k^2)^2} + \cdots \right)$$

$$\int k_\mu k_\nu \longrightarrow \int \delta_{\mu\nu} k^2/d$$

$$\underbrace{k_\mu k_\nu k_\rho k_\sigma} p_\rho p_\sigma$$

$$(\delta_{\mu\nu}\delta_{\rho\sigma} + \delta_{\mu\rho}\delta_{\nu\sigma} + \delta_{\mu\sigma}\delta_{\nu\sigma})(k^2)^2 \cdot \frac{1}{d(d+2)}$$



$$\int \frac{d^d k}{(2\pi)^d} \frac{(k+P/2)_\mu (k - P/2)_\nu}{(k+P/2)^2 (k-P/2)^2} = \int \frac{d^d k}{(2\pi)^d} \left(\frac{k_\mu k_\nu}{(k^2)^2} + \frac{k_\mu \cdot k_\nu}{(k^2)^4} \right.$$

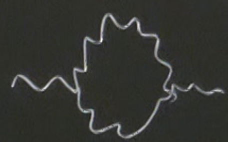
large k expansion

$$\int k_\mu k_\nu \longrightarrow \int \delta_{\mu\nu}\, k^2/d$$

$$(k^2)^2 \cdot p^2$$

div at $d = 2$

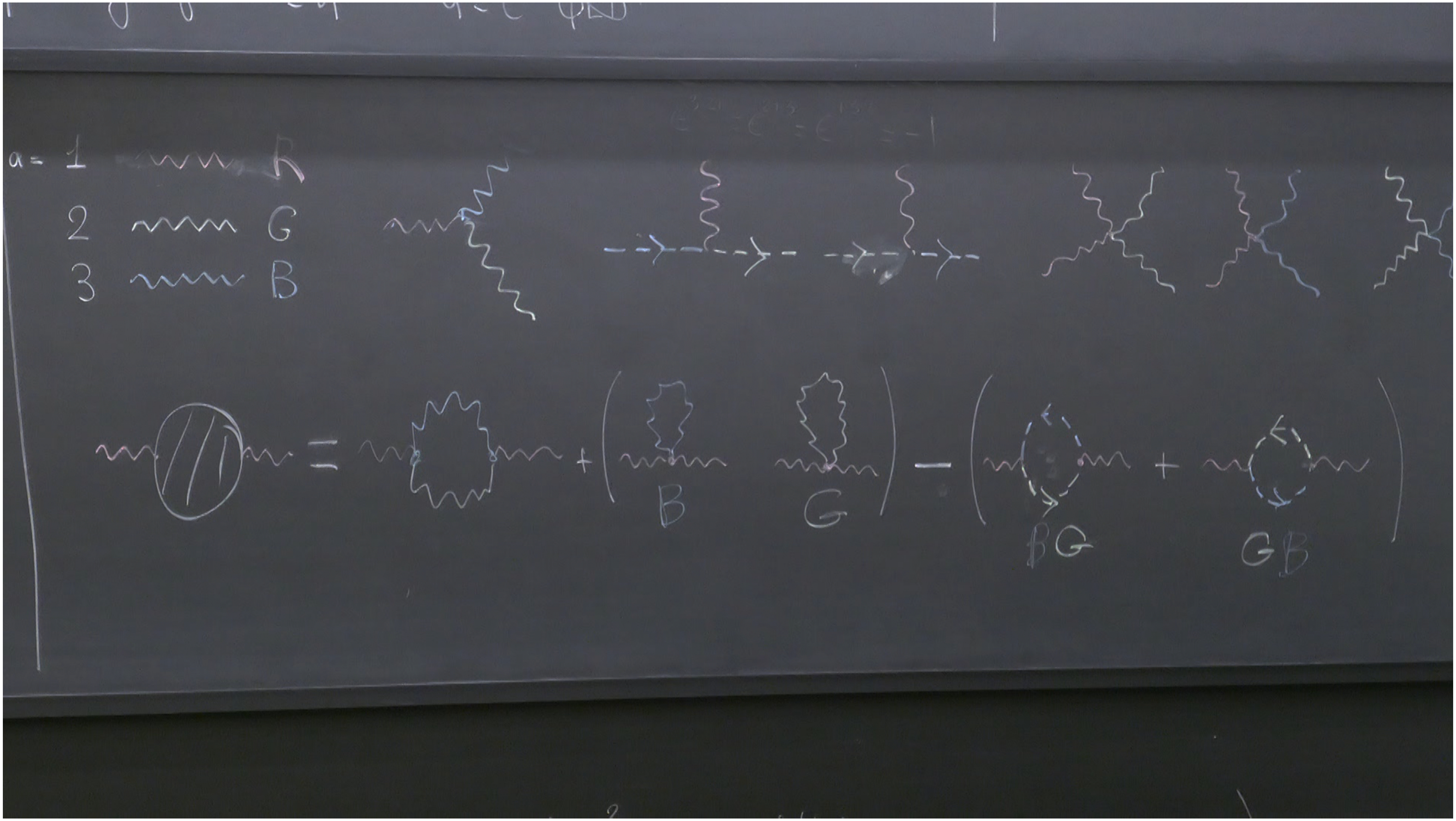
$$\int \frac{d^d k}{(2\pi)^d} \frac{k_\mu k_\nu}{(k^2)^2} \longrightarrow \delta_{\mu\nu} \int \frac{d^d k}{(2\pi)^d} \frac{1}{d} \frac{1}{k^2} \to \frac{\delta_{\mu\nu}}{2-d}$$

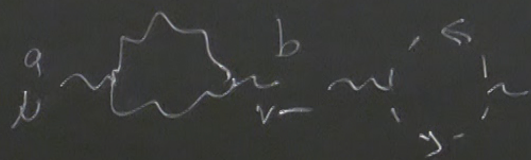
 $\longrightarrow \dfrac{3}{2-d}$  $\longrightarrow -\dfrac{1}{2-d}$  $\longrightarrow \dfrac{1}{2}\,\dfrac{1}{2-d}$

$d=2$

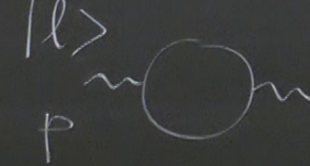
$$3 - 2\times 1 \boxed{-\,2\times \tfrac{1}{2}} = 3-2-1 = 0$$

no $A_\mu^a A_\mu^a$ divergence


a = 1 R
2 G
3 B
B
G
BG
GB

$d=4$ 

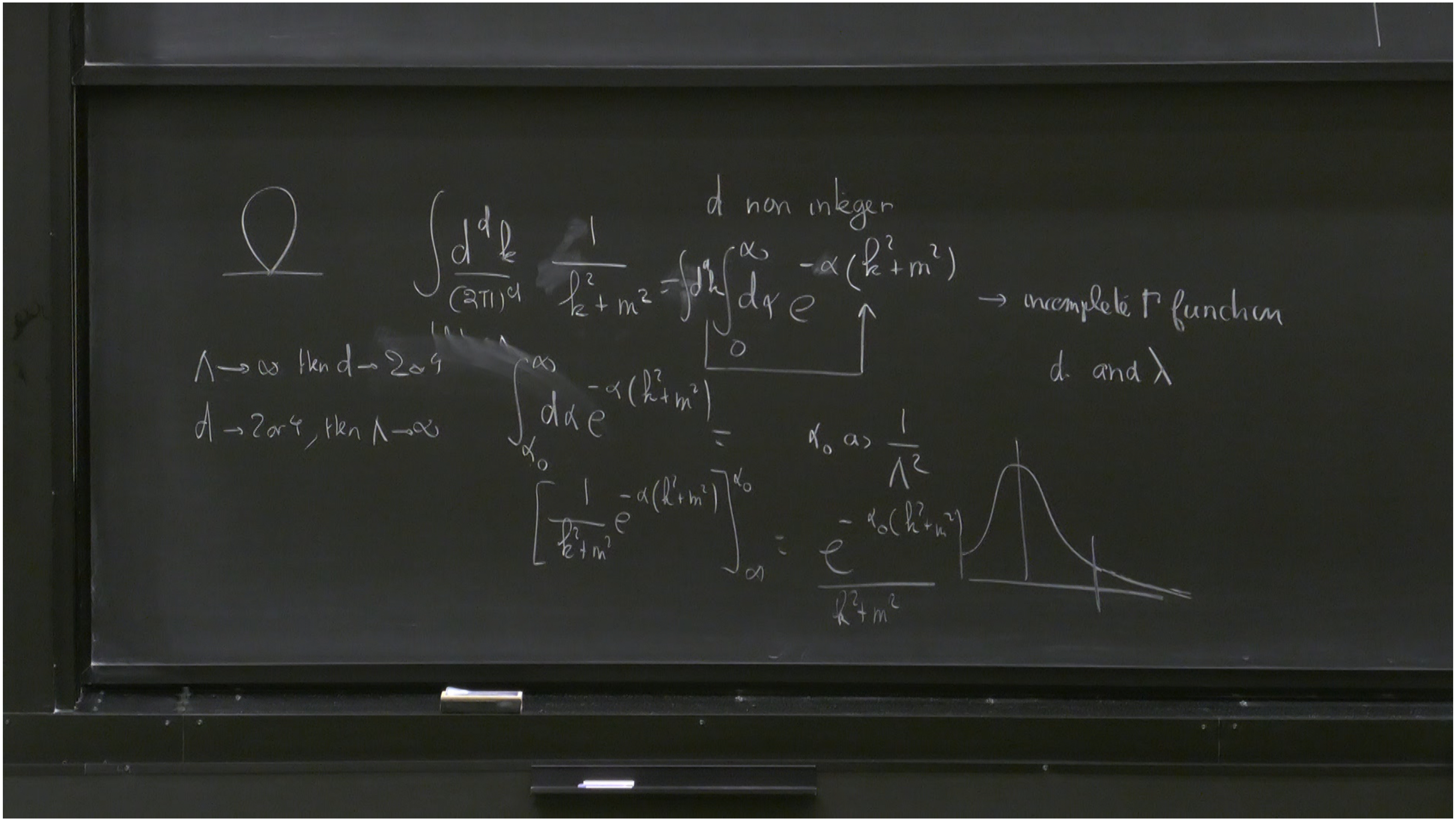
gauge invariant



$= 0$

$$\frac{1}{4-d}\;\frac{10}{3}\left(\delta_{\mu\nu}p^2 - p_\mu p_\nu\right)\delta^{ab} \qquad p_\mu\left(\quad\right)_{\mu\nu} = 0$$

$$\left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right)^2$$



d non integer
$\int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2+m^2} = \int d^d k \int_0^\infty d\alpha\, e^{-\alpha(k^2+m^2)}$
→ incomplete Γ function
d and λ
$\Lambda \to \infty$ then $d \to 2$ or 4
$d \to 2$ or 4, then $\Lambda \to \infty$
$\int_{\alpha_0}^\infty d\alpha\, e^{-\alpha(k^2+m^2)} = \left[\frac{1}{k^2+m^2} e^{-\alpha(k^2+m^2)}\right]_\infty^{\alpha_0} = \frac{e^{-\alpha_0(k^2+m^2)}}{k^2+m^2}$
α_0 as $\frac{1}{\Lambda^2}$

ger

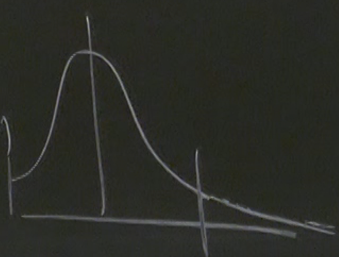
(k^2+m^2)

$\rightarrow$ incomplete Γ function

α and λ

$\rightarrow \frac{1}{\Lambda^2}$

$\alpha_0(k^2+m^2)$



$+m^2$

poles in $\alpha \longleftrightarrow \log \Lambda$

$\alpha \geq \alpha_0$ small $\longleftrightarrow |k^2| \leq \Lambda^2$ large d non integer

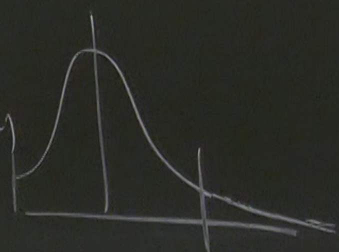
$\int d^d k \int_0^{\alpha_0} d\alpha \, e^{-\alpha(k^2+m^2)}$ → incomplete Γ function

d. and λ

$m^2)$

$=$ $\qquad \alpha_0$ as $\frac{1}{\Lambda^2}$

$(k^2+m^2)\Big]_{\alpha}^{\alpha_0} = \frac{e^{-\alpha_0(k^2+m^2)}}{k^2+m^2}$



poles in d ⟵ log Λ

$\alpha \geq \alpha_0$ small ⟵ $|k^2| \leq \Lambda^2$ large

$d \geq 4$ or 2

Heat Kernel regularization