Title: QFT2 Lecture - 120523

Speakers: Francois David

Collection: Quantum Field Theory 2 2023/24

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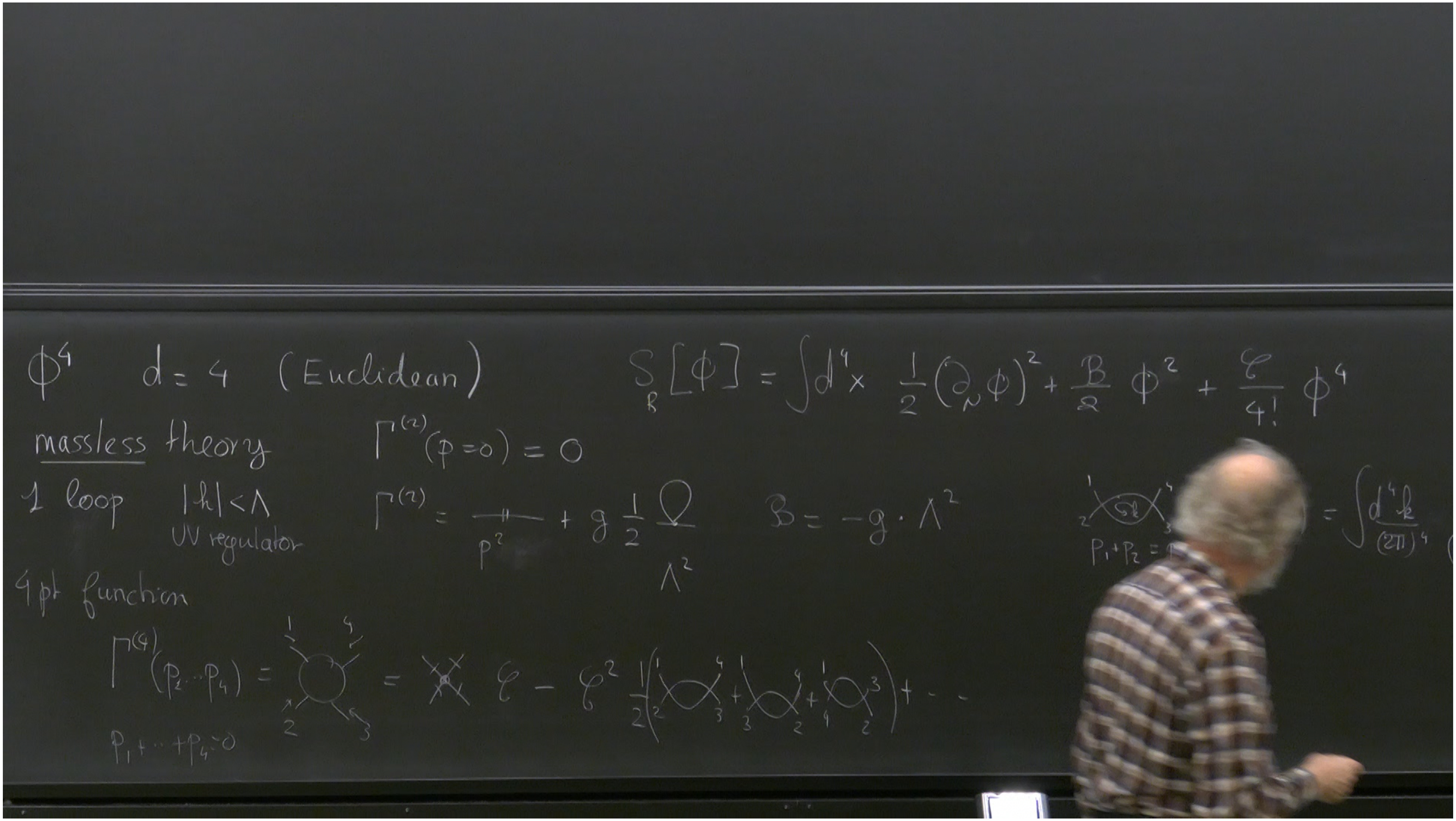


ϕ^4 $d = 4$ (Euclidean)

$$S_R[\phi] = \int d^4x \ \frac{1}{2}(\partial_\mu \phi)^2 + \frac{B}{2}\phi^2 + \frac{g}{4!}\phi^4$$

massless theory $\Gamma^{(2)}(\phi=0) = 0$

1 loop $|k| < \Lambda$

UV regulator

$\Gamma^{(2)} = p^2 + g\,\frac{1}{2}\,\Lambda^2$ $B = -g \cdot \Lambda^2$

$p_1 + p_2 =$ $= \int \frac{d^4k}{(2\pi)^4}$

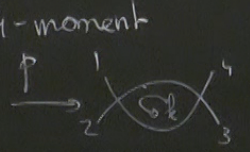
4 pt function

$\Gamma^{(4)}(p_1 \cdots p_4) = \ = g - g^2\,\frac{1}{2}\left(\ + \ + \ \right) + \cdots$

$p_1 + \cdots + p_4 = 0$

$$\times \quad \frac{1}{2}(\partial_\nu \phi)^2 + \frac{B}{2}\phi^2 + \frac{C}{4!}\phi^4$$

$$g \cdot \Lambda^2$$



$$= B(p) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k - p/_2)^2 (k + p/_2)^2}$$

$p_1 + p_2 = p$

if $p \neq 0$ IR safe

$\uparrow$
IR sing. at $k = \pm p$
not a divergence

$) + \cdots$

dim. argument

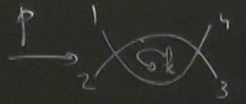
$$\times \quad \frac{1}{2}\left(\partial_\nu \phi\right)^2 + \frac{B}{2}\,\phi^2 + \frac{g}{4!}\,\phi^4$$

UV divergence

$|k| \gg |p|$

$-g \cdot \Lambda^2$

4-moment



$$= B(p) = \int \frac{d^4k}{(2\pi)^4}\,\frac{1}{\left(k-P/2\right)^2\left(k+P/2\right)^2}$$

$p_1 + p_2 = p$

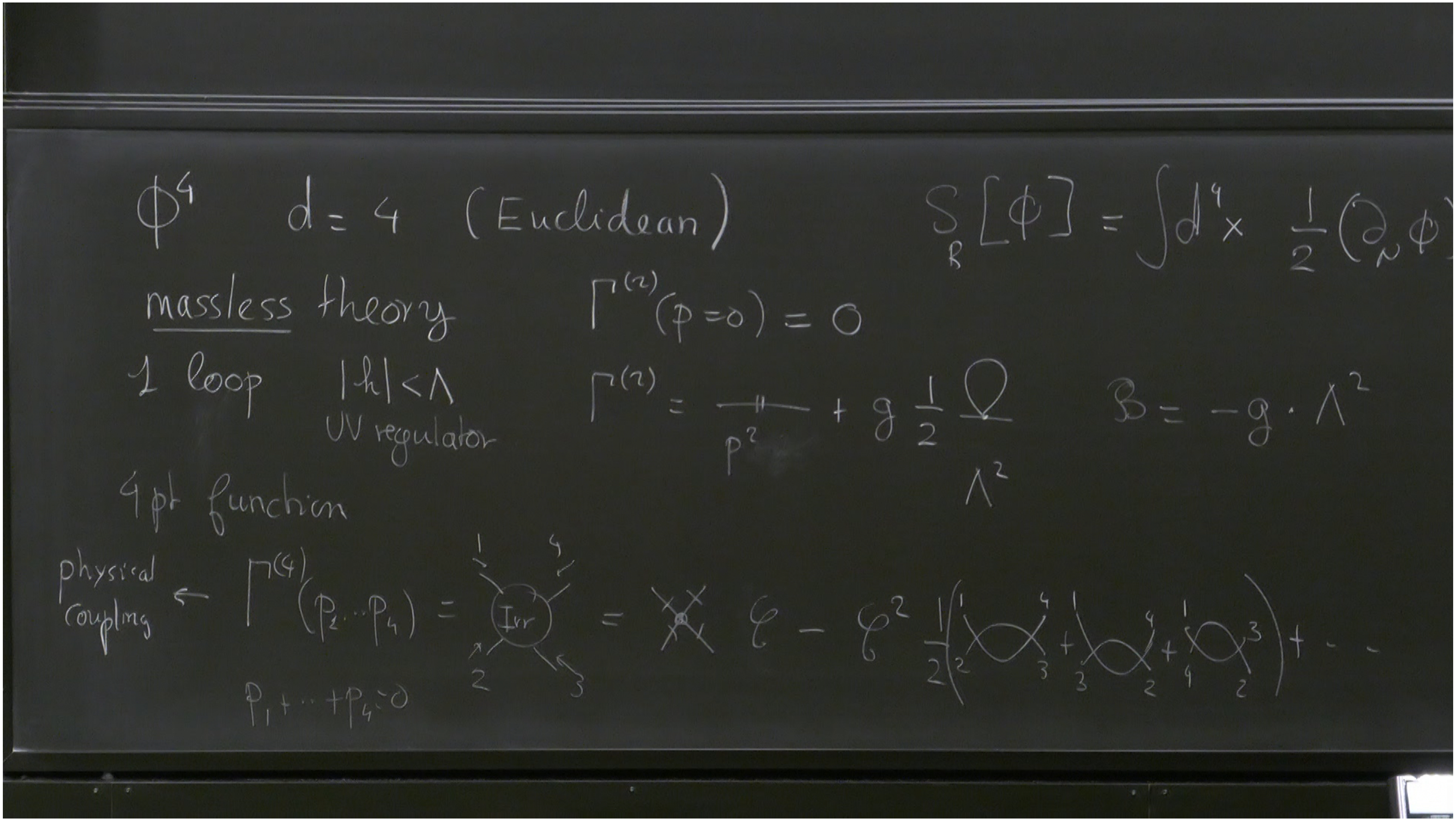
if $p \neq 0$ IR safe

IR sing. at $k = \pm p$
not a divergence

$$x_2^3\Big) + \cdots$$

$$\underset{\substack{\uparrow\\ \text{UV regulator}}}{B(p;\Lambda)} = \frac{1}{(4\pi)^2}\log\left(\frac{\Lambda^2}{p^2}\right) + \underline{cst} + \left(\frac{p^2}{\Lambda^2}\log\left(\frac{\Lambda^2}{p^2}\right) + \cdots\right) \to 0 \quad \Lambda \to \infty$$

depends on the cutoff



ϕ^4 $d = 4$ (Euclidean) $S_R[\phi] = \int d^4x \; \frac{1}{2}(\partial_\mu \phi$

massless theory $\Gamma^{(2)}(p=0) = 0$

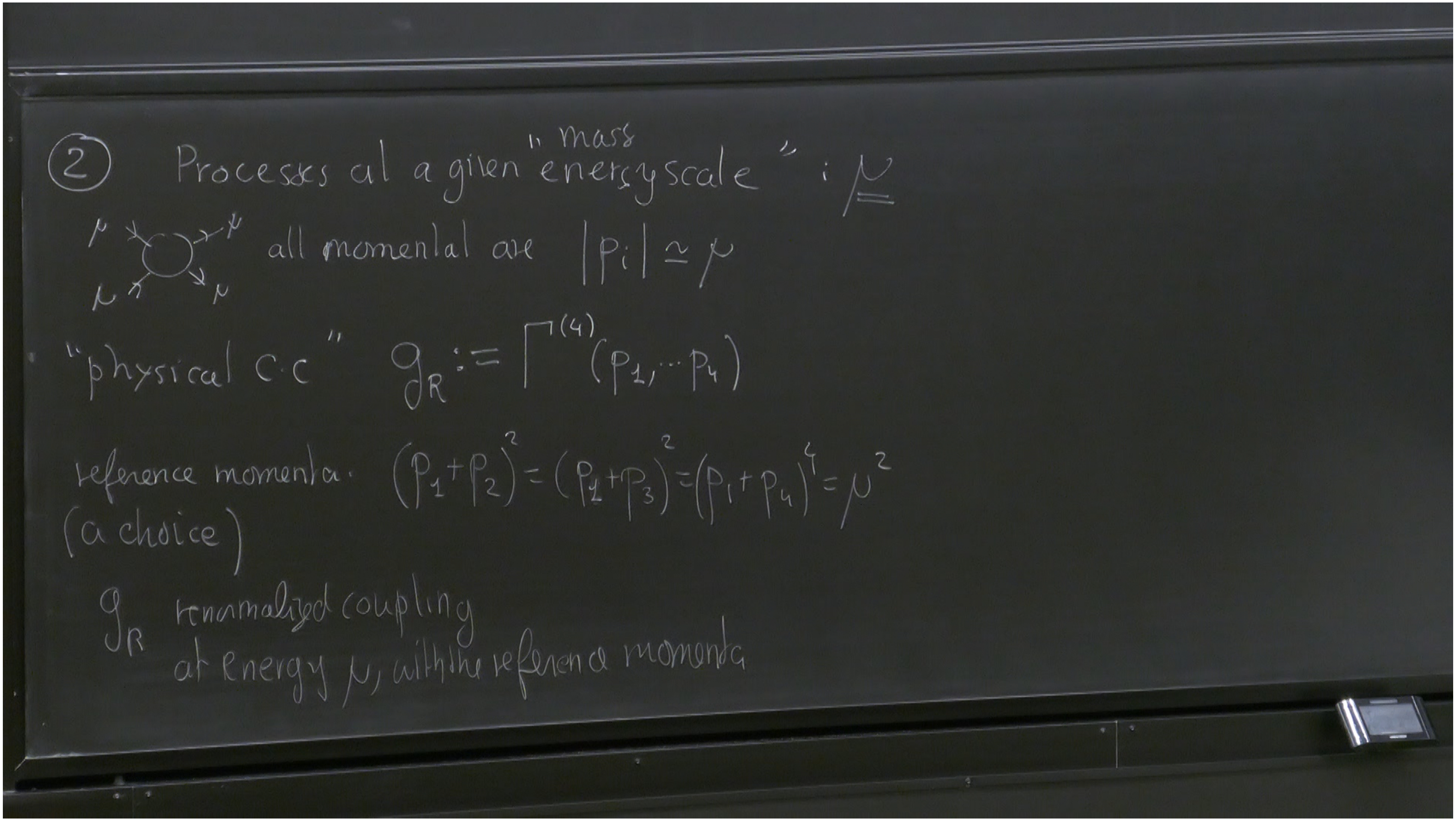
1 loop $|k| < \Lambda$ UV regulator

$\Gamma^{(2)} = \underset{p^2}{\text{—}} + g\,\frac{1}{2}\,\underset{\Lambda^2}{\Omega}$ $\mathcal{B} = -g \cdot \Lambda^2$

4pt function

physical coupling $\leftarrow$ $\Gamma^{(4)}(p_1 \ldots p_4) = \text{Irr} = g - g^2\,\frac{1}{2}\left(+ + \right) + \ldots$

$p_1 + \cdots + p_4 = 0$


② Processes at a given "$\overset{\text{mass}}{\text{energy}}$ scale" ; $\underline{\underline{\mu}}$

all momenta are $|p_i| \simeq \mu$

"physical c.c" $g_R := \Gamma^{(4)}(p_1, \cdots p_4)$

reference momenta. $(p_1+p_2)^2 = (p_1+p_3)^2 = (p_1+p_4)^2 = \mu^2$
(a choice)

g_R renormalized coupling
at energy μ, with the reference momenta

② Processes at a given "$\overset{\text{mass}}{\text{energy}}$ scale" : $\underline{\underline{\mu}}$

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(a choice)

(Euclidean momenta region)

but not a problem

g_R renormalized coupling
at energy μ, with the reference momenta

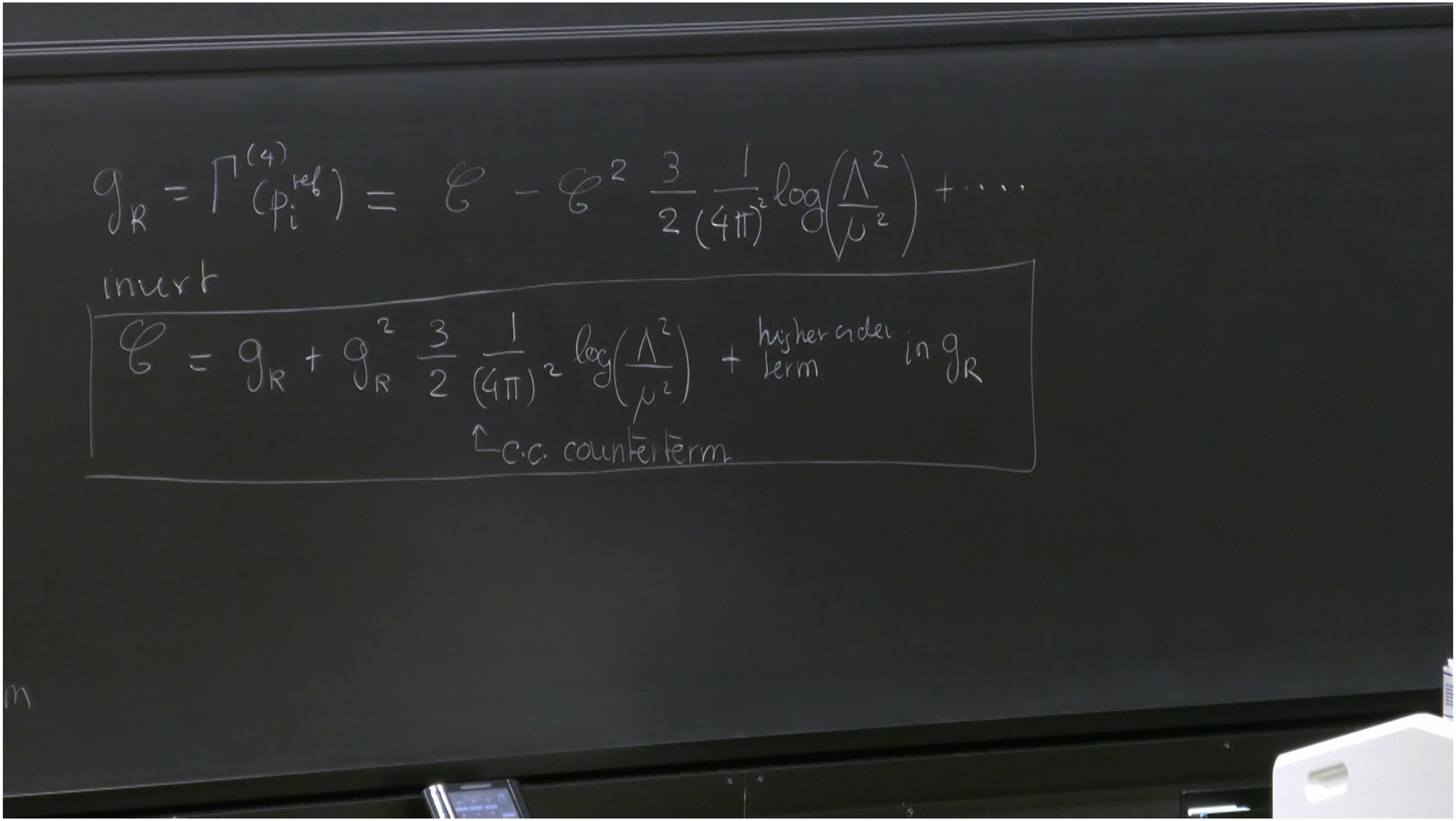
$$g_R = \Gamma^{(4)}(p_i^{ref}) = \mathcal{G} - \mathcal{G}^2 \frac{3}{2} \frac{1}{4\pi} \log\left(\frac{\Lambda^2}{\mu^2}\right) + \cdots$$

m

$$g_R = \Gamma^{(4)}(\phi_i^{\text{ref}}) = \mathcal{G} - \mathcal{G}^2 \frac{3}{2} \frac{1}{(4\pi)^2} \log\left(\frac{\Lambda^2}{\mu^2}\right) + \cdots$$

invert

$$\mathcal{G} = g_R + g_R^2 \frac{3}{2} \frac{1}{(4\pi)^2} \log\left(\frac{\Lambda^2}{\mu^2}\right) + \text{higher order term in } g_R$$



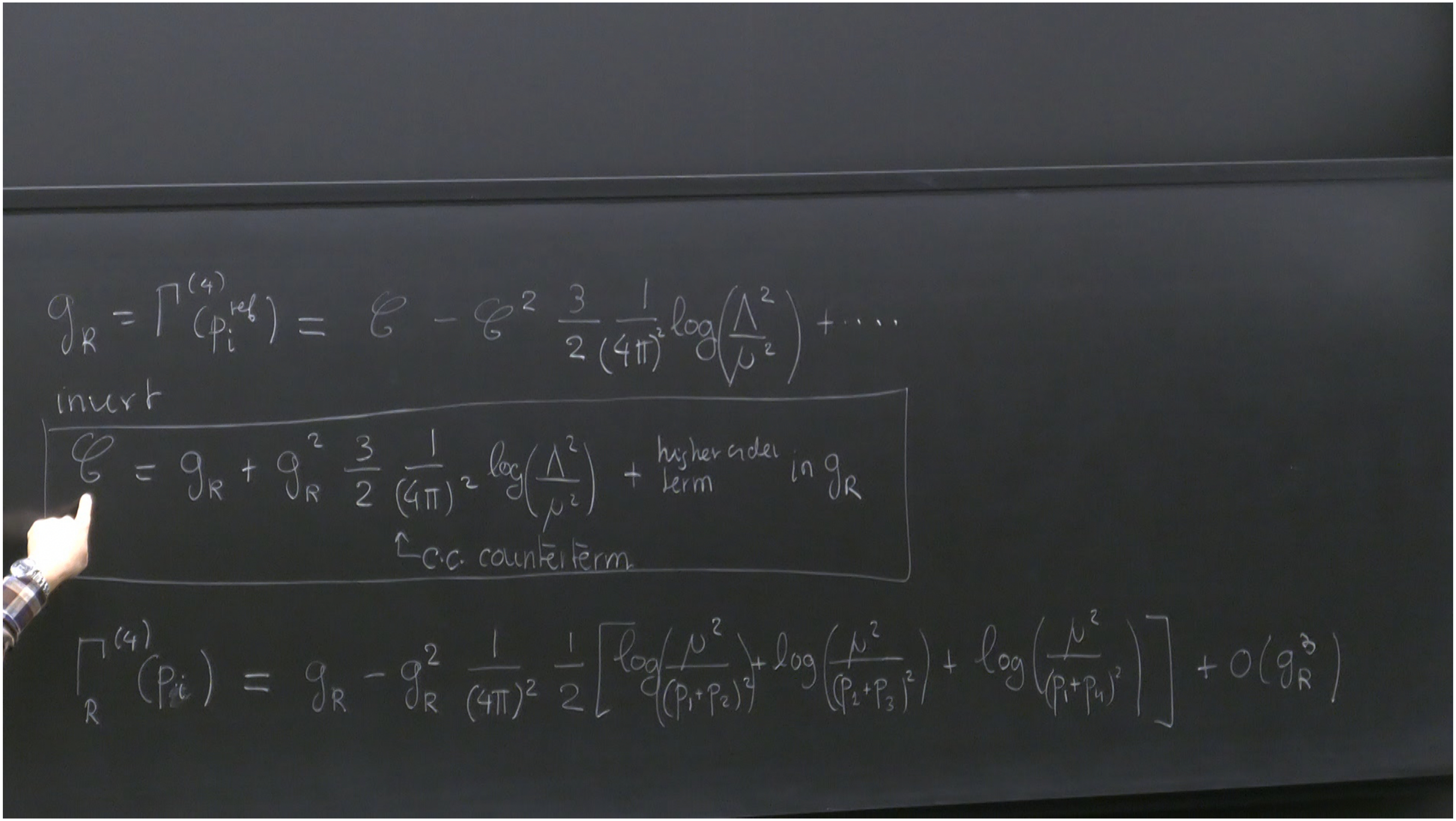
$$g_R = \Gamma^{(4)}(\phi_i^{\text{ref}}) = \mathcal{C} - \mathcal{C}^2 \, \frac{3}{2} \frac{1}{(4\pi)^2} \log\left(\frac{\Lambda^2}{\mu^2}\right) + \cdots$$

invert

$$\boxed{\mathcal{C} = g_R + g_R^2 \, \frac{3}{2} \frac{1}{(4\pi)^2} \log\left(\frac{\Lambda^2}{\mu^2}\right) + \text{higher order term in } g_R}$$

↰ c.c. counterterm

m



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$\uparrow$ c.c. counterterm

$$\Gamma_R^{(4)}(p_i) = g_R - g_R^2 \, \frac{1}{(4\pi)^2} \, \frac{1}{2}\left[\log\left(\frac{\mu^2}{(p_1+p_2)^2}\right) + \log\left(\frac{\mu^2}{(p_2+p_3)^2}\right) + \log\left(\frac{\mu^2}{(p_1+p_4)^2}\right)\right] + O(g_R^3)$$

$$g_R = \Gamma^{(4)}(p_i^{\text{ref}}) = \mathcal{G} - \mathcal{G}^2 \frac{3}{2}\frac{1}{(4\pi)^2}\log\left(\frac{\Lambda^2}{\mu^2}\right) + \cdots$$

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$$\boxed{\mathcal{G} = g_R + g_R^2 \frac{3}{2}\frac{1}{(4\pi)^2}\log\left(\frac{\Lambda^2}{\mu^2}\right) + \text{higher order term in } g_R}$$

↖ c.c. counterterm

Renorma

$$\boxed{\Gamma^{(4)}_R(p_i) = g_R - g_R^2 \frac{1}{(4\pi)^2}\frac{1}{2}\left[\log\left(\frac{\mu^2}{(p_1+p_2)^2}\right) + \log\left(\frac{\mu^2}{(p_1+p_3)^2}\right) + \log\left(\frac{\mu^2}{(p_1+p_4)^2}\right)\right] + O(g_R^3)}$$

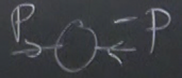
All 4pt functions UV finite, for all momenta, at 1 loop order = 1st non classical order in g_R

$$) = \mathcal{C} - \mathcal{C}^2 \frac{3}{2} \frac{1}{(4\pi)^2} \log\left(\frac{\Lambda^2}{\mu^2}\right) + \cdots$$

$$\boxed{g_R^2 \frac{3}{2} \frac{1}{(4\pi)^2} \log\left(\frac{\Lambda^2}{\mu^2}\right) + \text{higher order term in } g_R}$$

↳ c.c. counterterm

2 pt function



$$\Gamma_R^{(2)}(p) = p^2 + g_R \cdot \emptyset + O(g_R^2)$$

Renormalized 4pt function

$$= g_R - g_R^2 \frac{1}{(4\pi)^2} \frac{1}{2} \left[\log\left(\frac{\mu^2}{(p_1+p_2)^2}\right) + \log\left(\frac{\mu^2}{(p_1+p_3)^2}\right) + \log\left(\frac{\mu^2}{(p_1+p_4)^2}\right) \right] + O(g_R^3)$$

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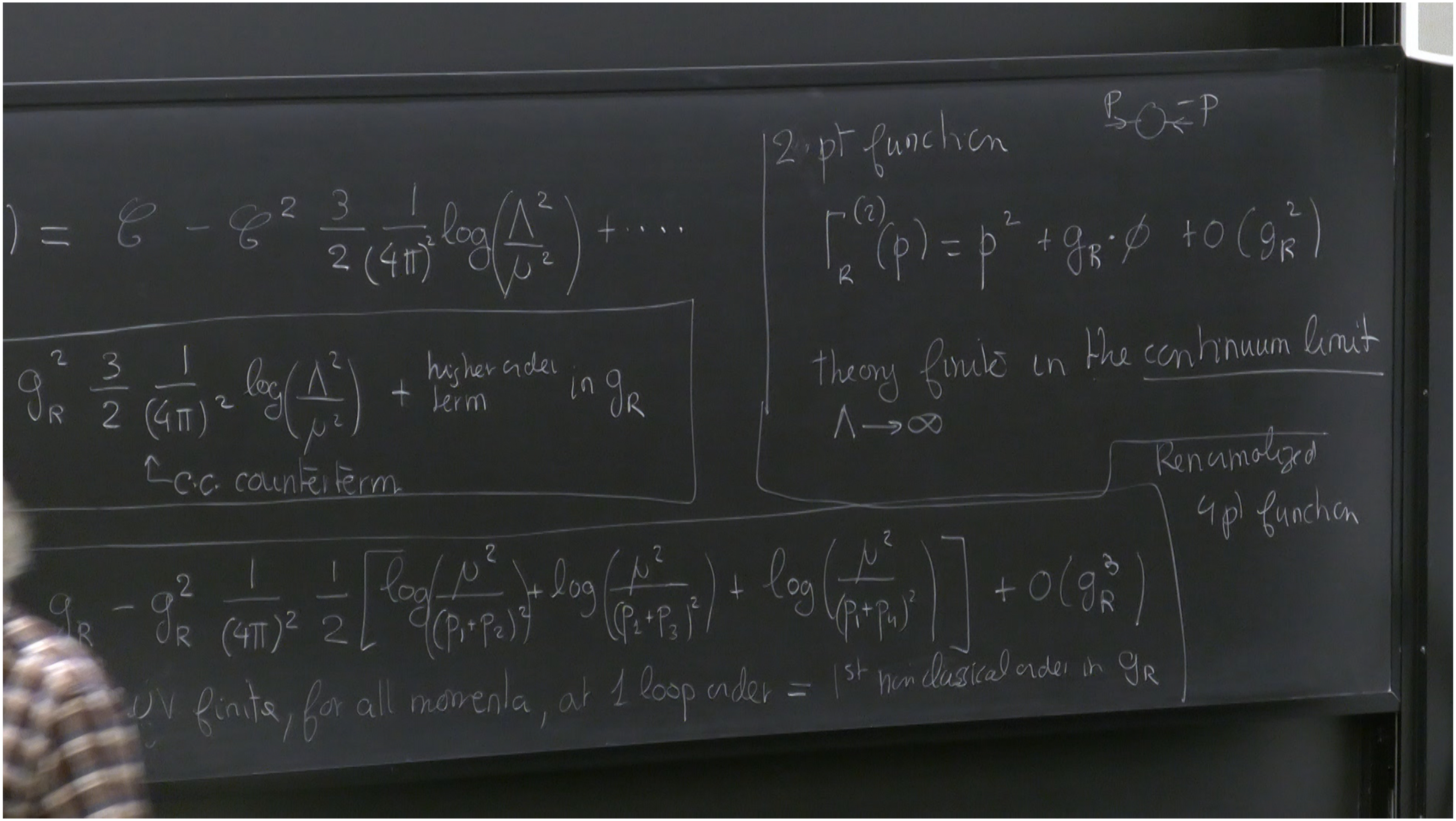


$$) = \mathcal{C} - \mathcal{C}^2 \frac{3}{2} \frac{1}{(4\pi)^2} \log\left(\frac{\Lambda^2}{\mu^2}\right) + \cdots$$

$$\boxed{g_R^2 \frac{3}{2} \frac{1}{(4\pi)^2} \log\left(\frac{\Lambda^2}{\mu^2}\right) + \text{higher order term in } g_R}$$

↑ c.c. counterterm

2 pt function

$$\Gamma_R^{(2)}(p) = p^2 + g_R \cdot \emptyset + O(g_R^2)$$

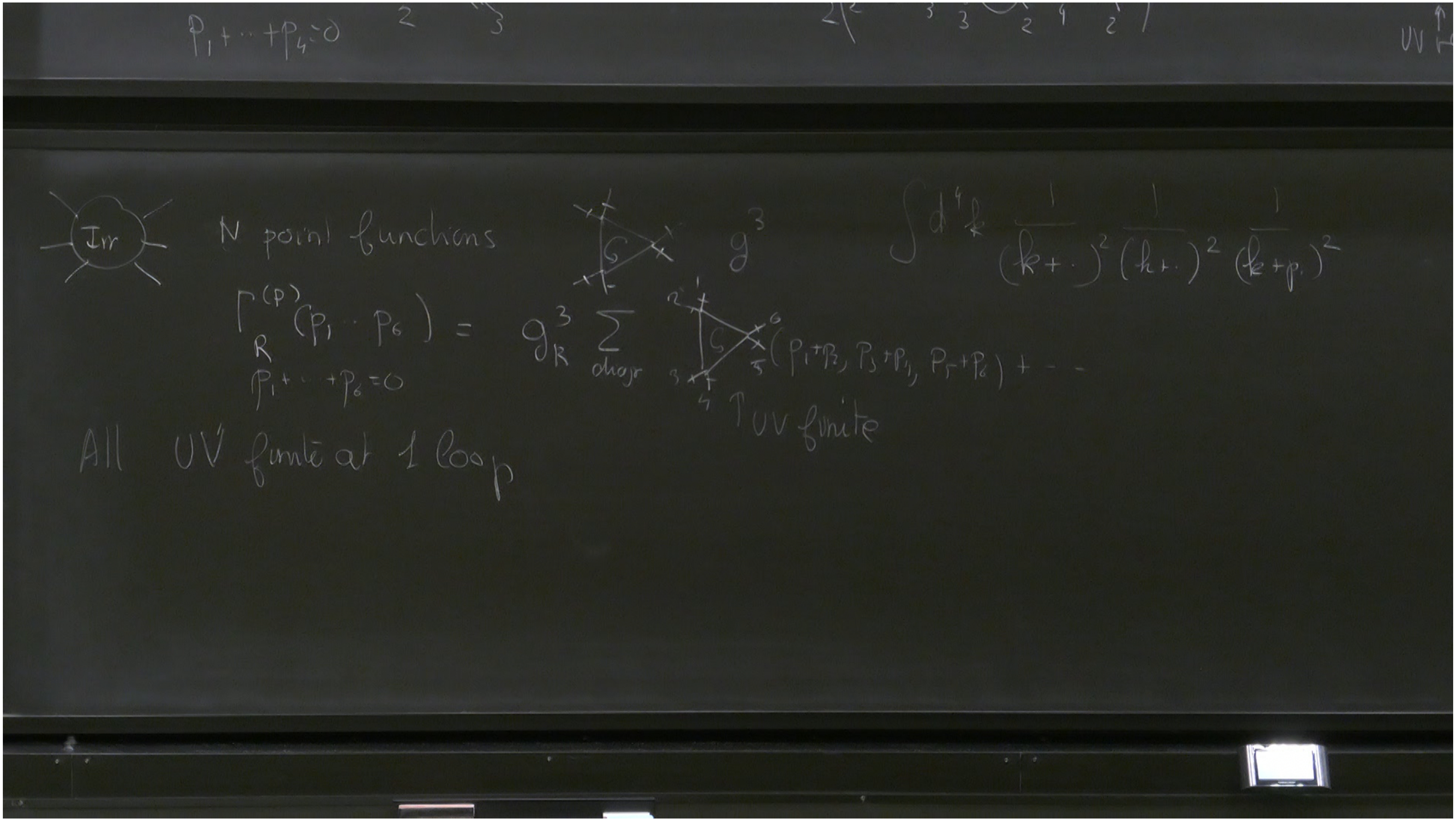
theory finite in the continuum limit

$\Lambda \to \infty$

Renormalized 4 pt function

$$g_R - g_R^2 \frac{1}{(4\pi)^2} \frac{1}{2} \left[\log\left(\frac{\mu^2}{(p_1+p_2)^2}\right) + \log\left(\frac{\mu^2}{(p_1+p_3)^2}\right) + \log\left(\frac{\mu^2}{(p_1+p_4)^2}\right) \right] + O(g_R^3)$$

UV finite, for all momenta, at 1 loop order = 1st non classical order in g_R



$p_1+\cdots+p_4=0$

Irr

N point functions

g^3

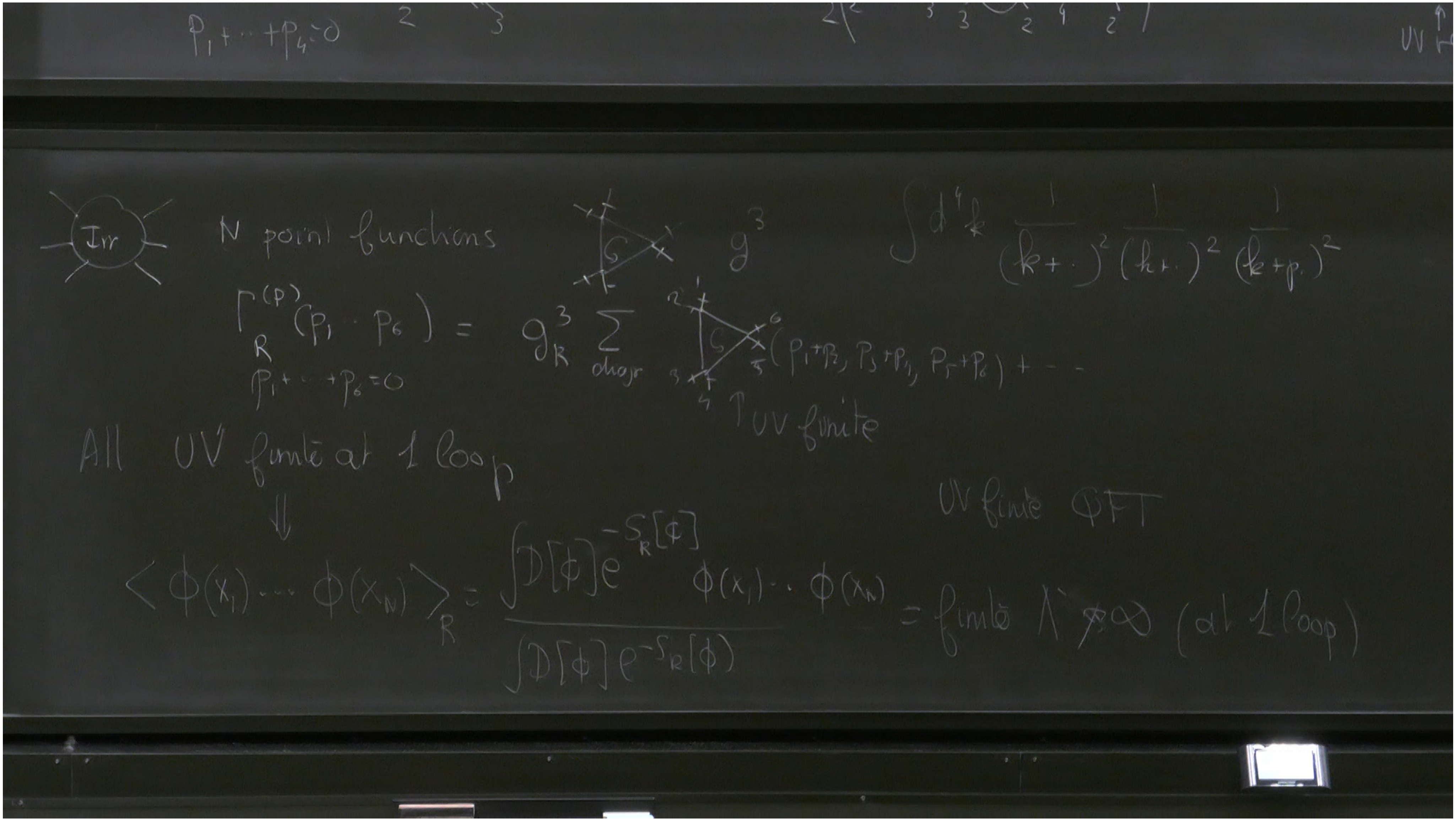
$$\int d^4k \, \frac{1}{(k+\cdot)^2} \, \frac{1}{(k+\cdot)^2} \, \frac{1}{(k+p_\cdot)^2}$$

$$\Gamma^{(p)}_R(p_1 \cdots p_6) = g_R^3 \sum_{\text{diagr}} (p_1+p_2,\ p_3+p_4,\ p_5+p_6) + \cdots$$

$p_1+\cdots+p_6=0$

↑ UV finite

All UV finite at 1 loop



$p_1 + \dots + p_4 = 0$ 2 3

2 3 3 2 4 2

UV

Irr

N point functions

g^3

$$\int d^4k \, \frac{1}{(k+\cdot)^2} \frac{1}{(k+\cdot)^2} \frac{1}{(k+p_\cdot)^2}$$

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$p_1 + \cdots + p_6 = 0$

↑ UV finite

All UV finite at 1 loop

$\Downarrow$

UV finite QFT

$$\langle \phi(x_1) \cdots \phi(x_N) \rangle_R = \frac{\int \mathcal{D}[\phi] e^{-S_R[\phi]} \phi(x_1) \cdots \phi(x_N)}{\int \mathcal{D}[\phi] e^{-S_R[\phi]}} = \text{finite } \Lambda \to \infty \text{ (at 1 loop)}$$

$)^2$

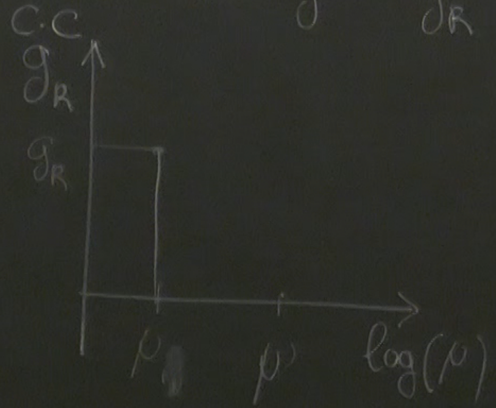
oop)

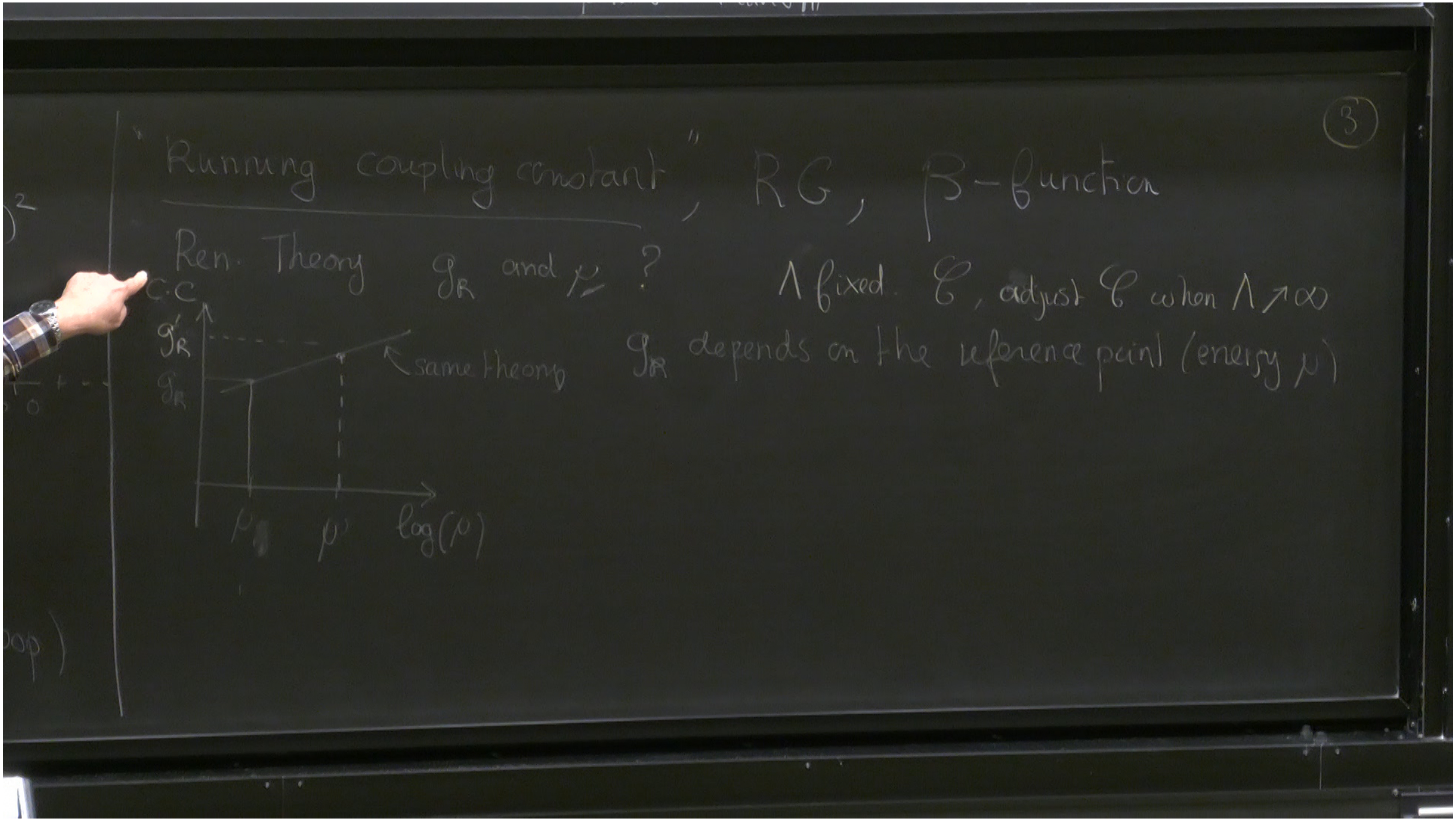
③

"Running coupling constant", RG, β-function

Ren. Theory g_R and μ ?

Λ fixed. $\mathcal{C}$, adjust $\mathcal{C}$ when $\Lambda \nearrow \infty$





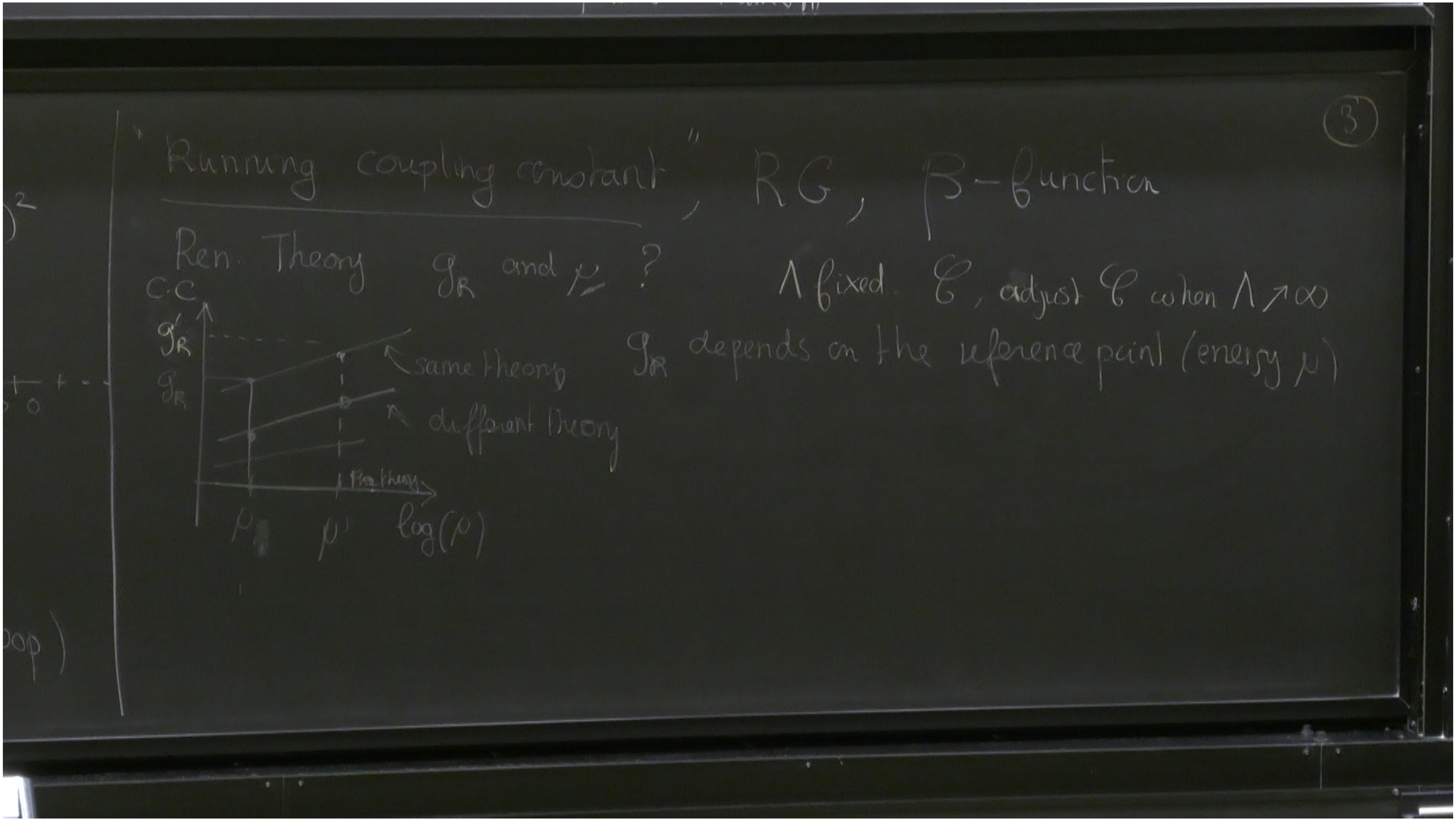
③

"Running coupling constant", RG, β-function

Ren. Theory g_R and μ ?

Λ fixed. $\mathcal{C}$, adjust $\mathcal{C}$ when $\Lambda \nearrow \infty$

g_R depends on the reference point (energy μ)



"Running coupling constant", RG, β-function

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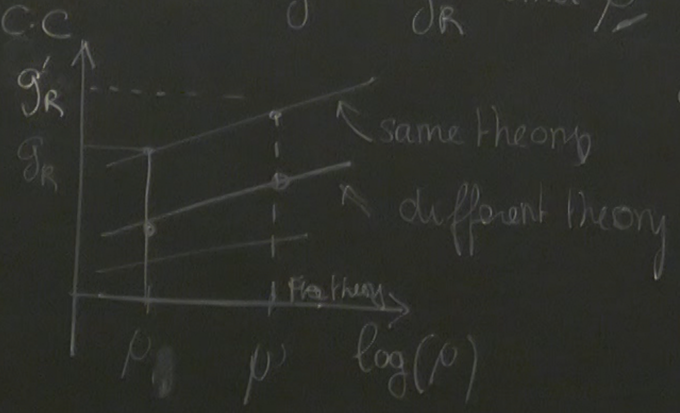
$)^2$

oop)

③

"Running coupling constant", RG, β-function

Ren. Theory g_R and μ ?



Λ fixed $\mathcal{C}$, adjust $\mathcal{C}$ when $\Lambda \nearrow \infty$

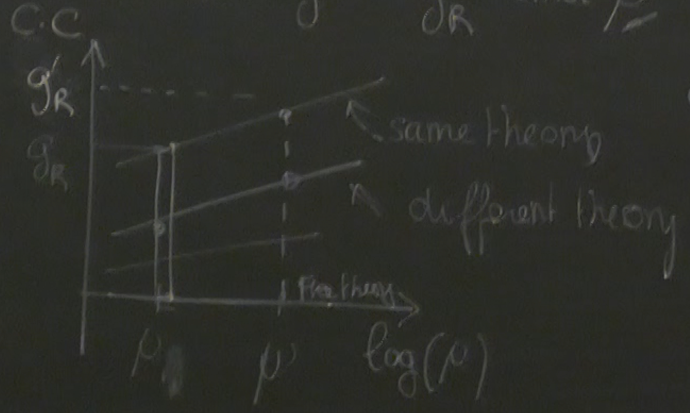
g_R depends on the reference point (energy μ)

How does g_R depends on μ ? $\mathcal{C}, \Lambda$ being fixed but $\Lambda \to \infty$

③

"Running coupling constant", RG, β-function

Ren. Theory g_R and μ ?



Λ fixed. $\mathcal{C}$, adjust $\mathcal{C}$ when $\Lambda \nearrow \infty$

g_R depends on the reference point (energy μ)

How does g_R depends on μ ? $\mathcal{C}, \Lambda$ being fixed but $\Lambda \to \infty$

Better take a differential form

④ $g_R \leftrightarrow \mu \quad g_R' \leftrightarrow \mu' = \mu + \delta\mu \quad g_R' = g_R + \delta g_R$

$$\delta g_R = \frac{3}{(4\pi)^2}\, g_R^2\, \frac{\delta\mu}{\mu}$$

④ $g_R \leftrightarrow \mu_0$: $g_R' \leftrightarrow \mu_0' = \mu_0 + \delta\mu$ $\quad g_R' = g_R + \delta g_R$

$$\delta g_R = \frac{3}{(4\pi)^2}\, g_R^2\, \frac{\delta\mu}{\mu}$$

"Beta - Function"

$$g_R \leftrightarrow \mu_0 \qquad g_R(\mu) \; : \; \boxed{\mu \frac{d}{d\mu} g_R(\mu)} = \frac{3}{(4\pi)^2}\, g_R^2(\mu) = \boxed{\beta\left(g_R(\mu)\right)}$$

general definition

1 loop

$$g_R \quad g_R \; (4\pi)^2 \; 2 \Big[\; (p_1+p_2)^2 \quad (p_2+p_3)^2 \quad (p_1+p_4)^2 \Big] \Big] + O(g_R \;)$$

functions UV finite, for all momenta, at 1 loop order = 1st non classical order in g_R

$$\Phi_4 \quad \beta(g) = \frac{3}{(4\pi)^2} g^2 + \cdot\, g^3 + \cdot\, g^4 \cdots\cdots$$

"perturb. theory" at
all theorem
$\Uparrow$
Th on Renormalization

$\left((p_1+p_2)^2\right) \qquad \left((p_2+p_3)^2\right) \qquad (p_1+p_4)^2 \Big] \Big] + O(g_R$

functions UV finite, for all momenta, at 1 loop order = 1st non classical order in g_R

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$\uparrow$

1 loop

"perturb. theory" at all theorem

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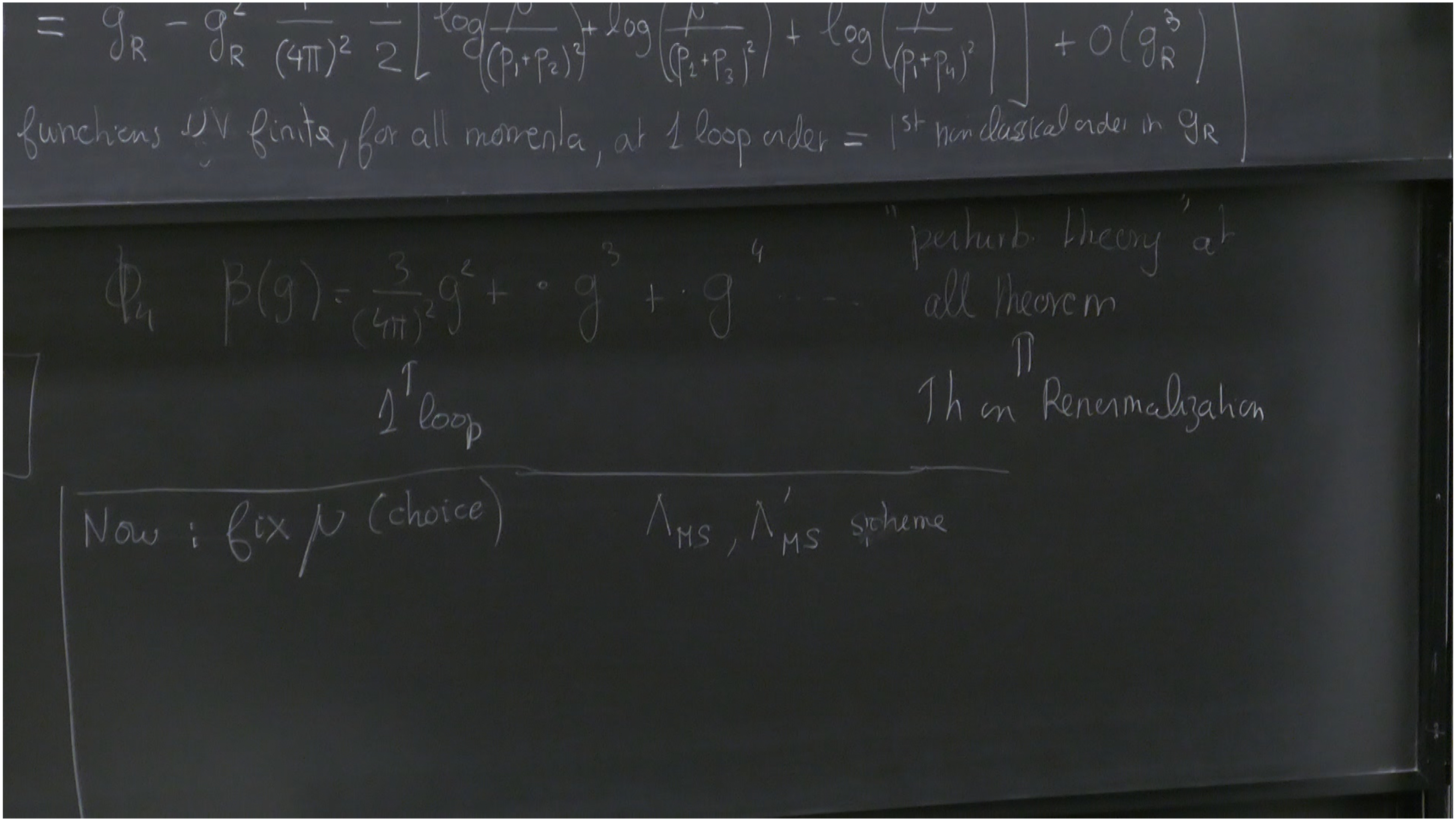
Th on Renormalization



$$= g_R - g_R^2 \frac{1}{(4\pi)^2}\frac{1}{2}\left[\log\left(\frac{\mu}{(p_1+p_2)^2}\right)+\log\left(\frac{\mu}{(p_2+p_3)^2}\right)+\log\left(\frac{\mu}{(p_1+p_4)^2}\right)\right] + O(g_R^3)$$

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$\uparrow$ 1 loop

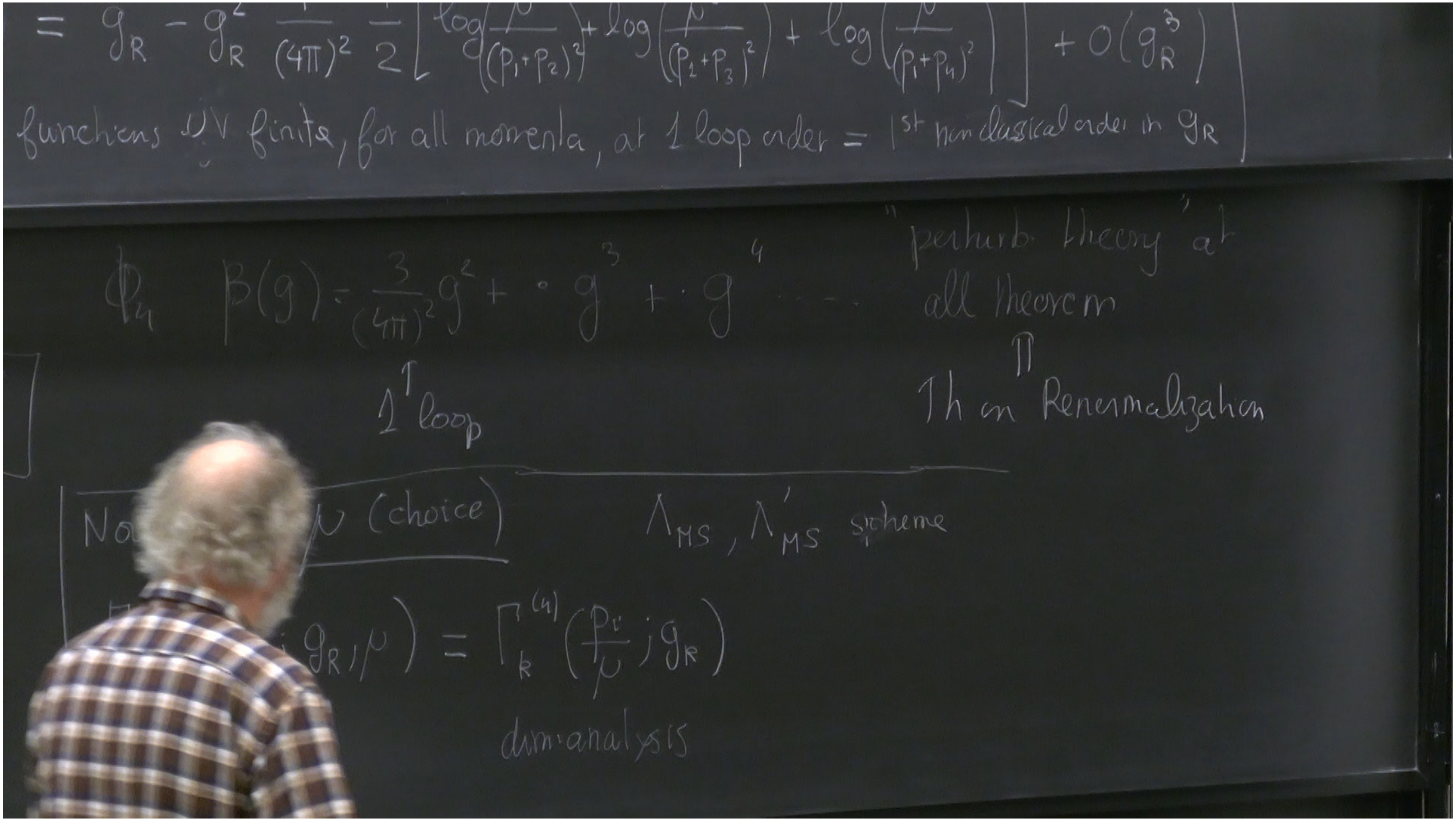
"perturb. theory" at all theorem

$\Uparrow$

Th on Renormalization

Now : fix μ (choice)

$\Lambda_{MS}, \Lambda'_{MS}$ scheme



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$\uparrow$ 1 loop

"perturb. theory" at all theorem

$\Uparrow$

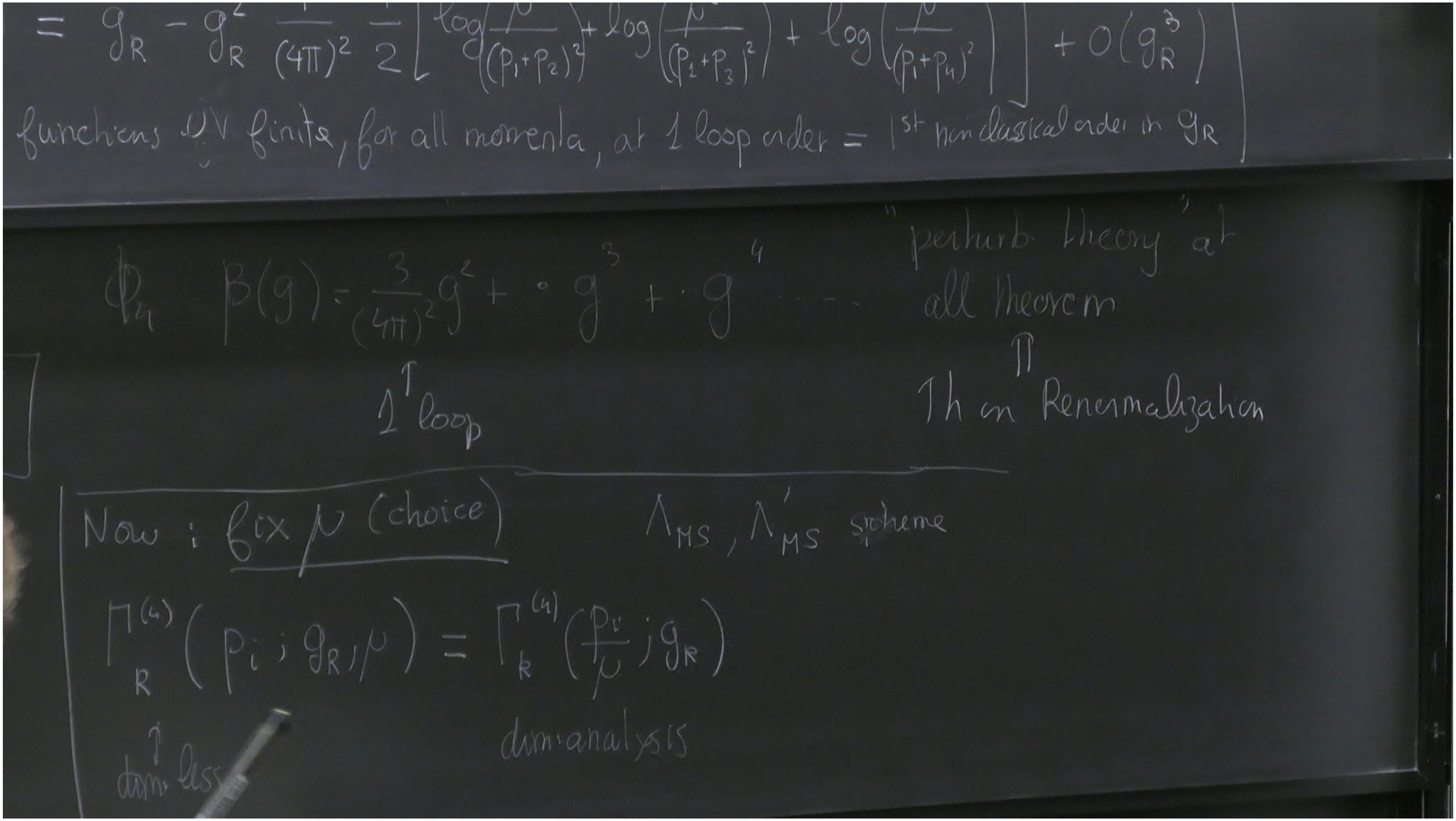
Th on Renormalization

No [illegible] μ (choice)

$\Lambda_{MS}, \Lambda'_{MS}$ scheme

[illegible] $; g_R, \mu) = \Gamma_k^{(4)}\left(\frac{p_i}{\mu}; g_R\right)$

dim. analysis



$$= g_R - g_R^2 \frac{1}{(4\pi)^2}\frac{1}{2}\left[\log\left(\frac{\mu^2}{(p_1+p_2)^2}\right)+\log\left(\frac{\mu^2}{(p_2+p_3)^2}\right)+\log\left(\frac{\mu^2}{(p_1+p_4)^2}\right)\right] + O(g_R^3)$$

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$\uparrow$ 1 loop

"perturb. theory" at all theorem

$\Uparrow$

Th on Renormalization

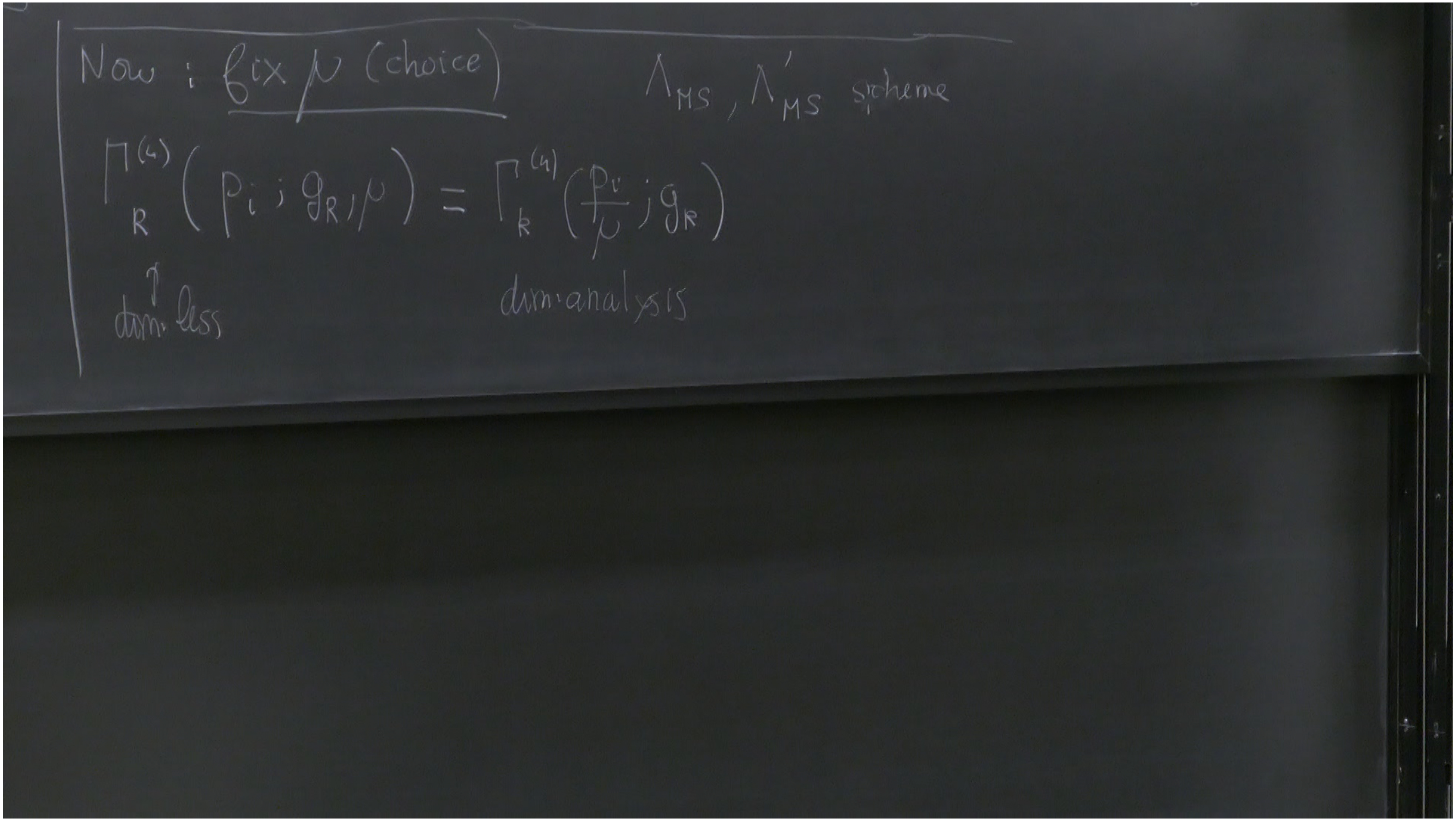
Now : fix μ (choice)

Λ_{MS}, Λ'_{MS} scheme

$$\Gamma_R^{(4)}(p_i ; g_R, \mu) = \Gamma_R^{(4)}\left(\frac{p_i}{\mu} ; g_R\right)$$

$\uparrow$ dim. less

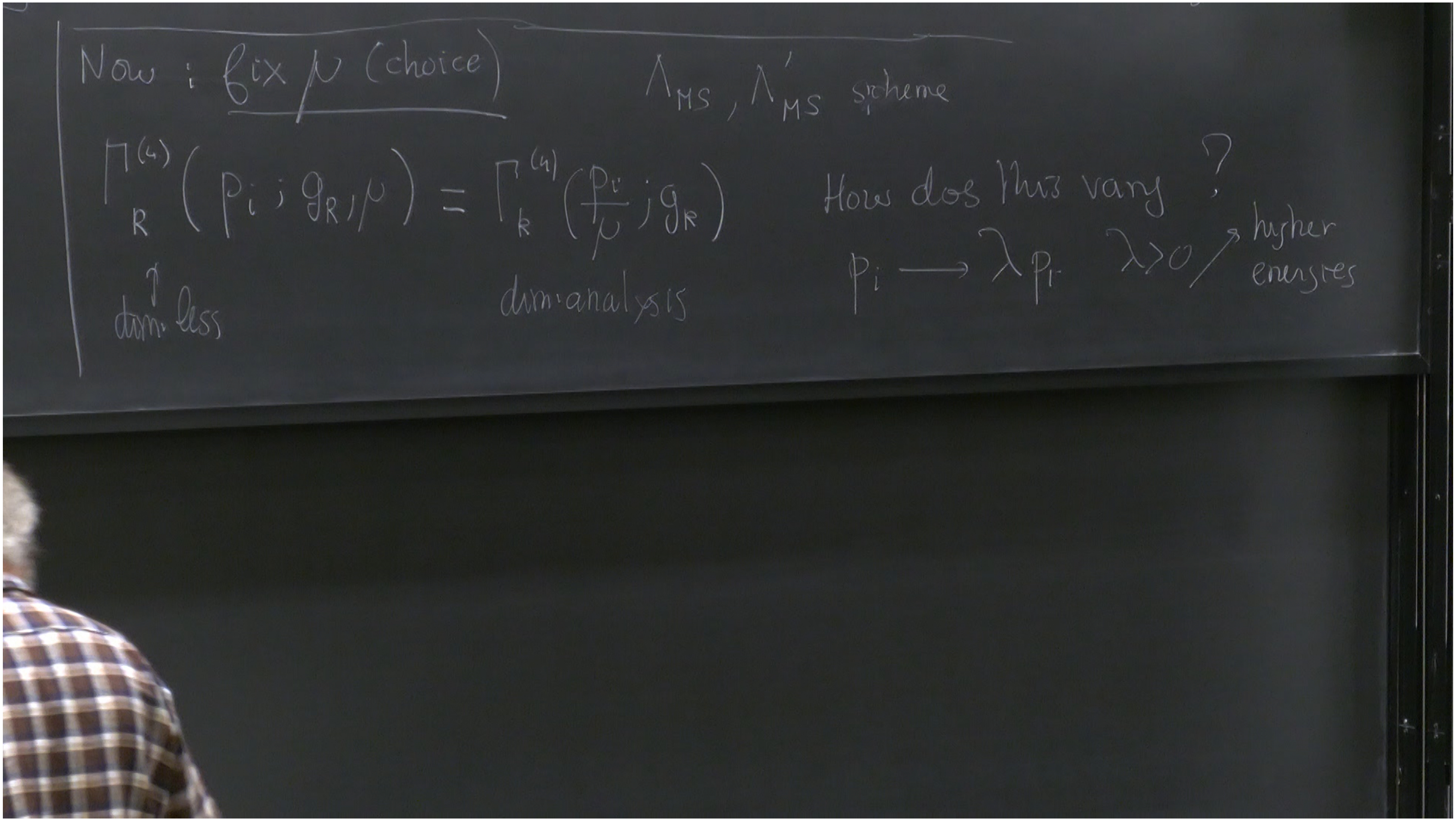
dim. analysis



Now : $\underline{\text{fix } \mu \text{ (choice)}}$ Λ_{MS} , Λ'_{MS} scheme

$$\Gamma^{(4)}_R \left(p_i ; g_R , \mu \right) = \Gamma^{(4)}_R \left(\frac{p_i}{\mu} ; g_R \right)$$

$\uparrow$ dim. less dim. analysis



Now: fix μ (choice)

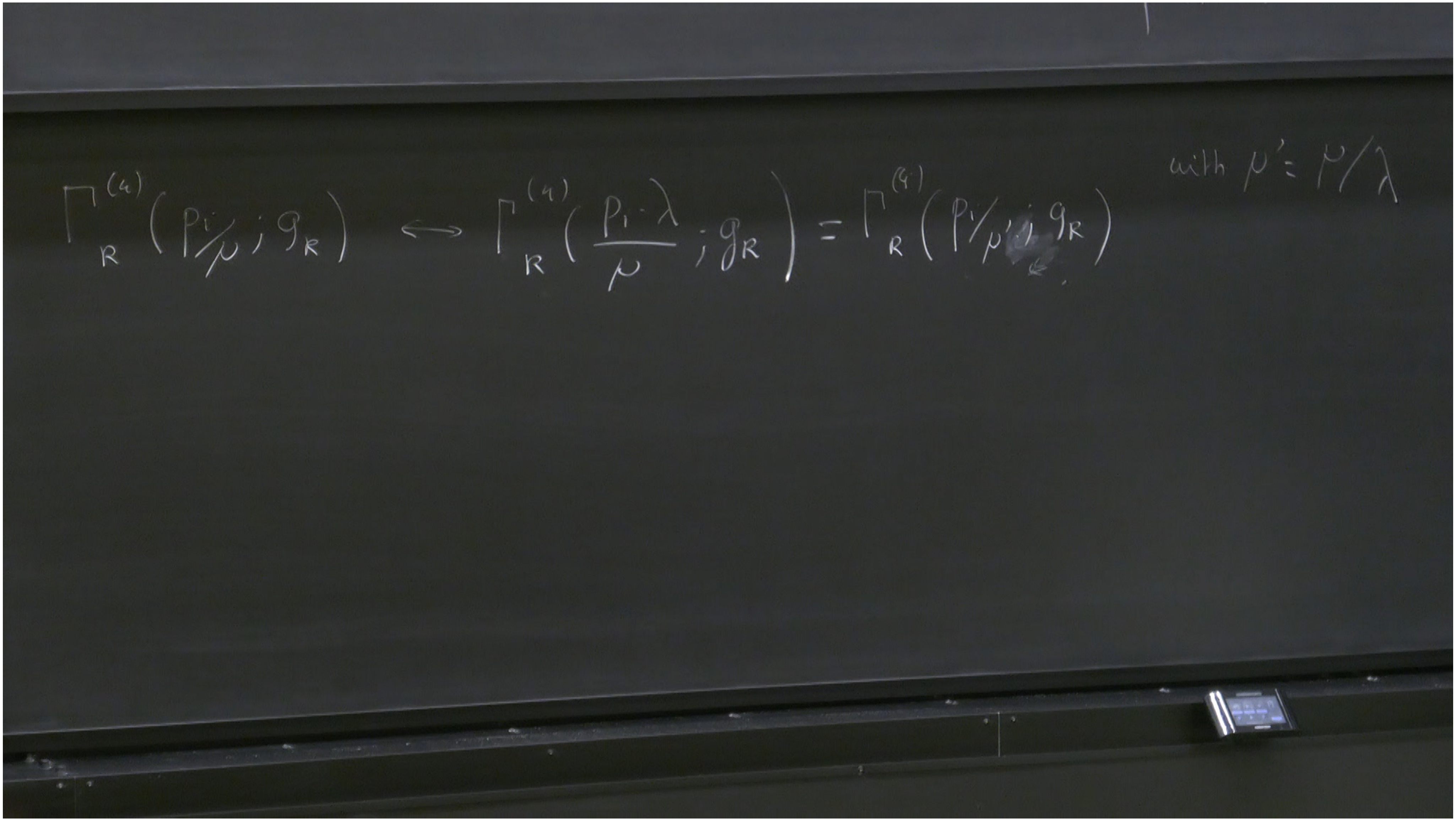
Λ_{MS}, Λ'_{MS} scheme

$$\underset{\substack{\uparrow \\ \text{dim. less}}}{\Gamma^{(4)}_R}(p_i; g_R, \mu) = \Gamma^{(4)}_R\left(\frac{p_i}{\mu}; g_R\right)$$

dim. analysis

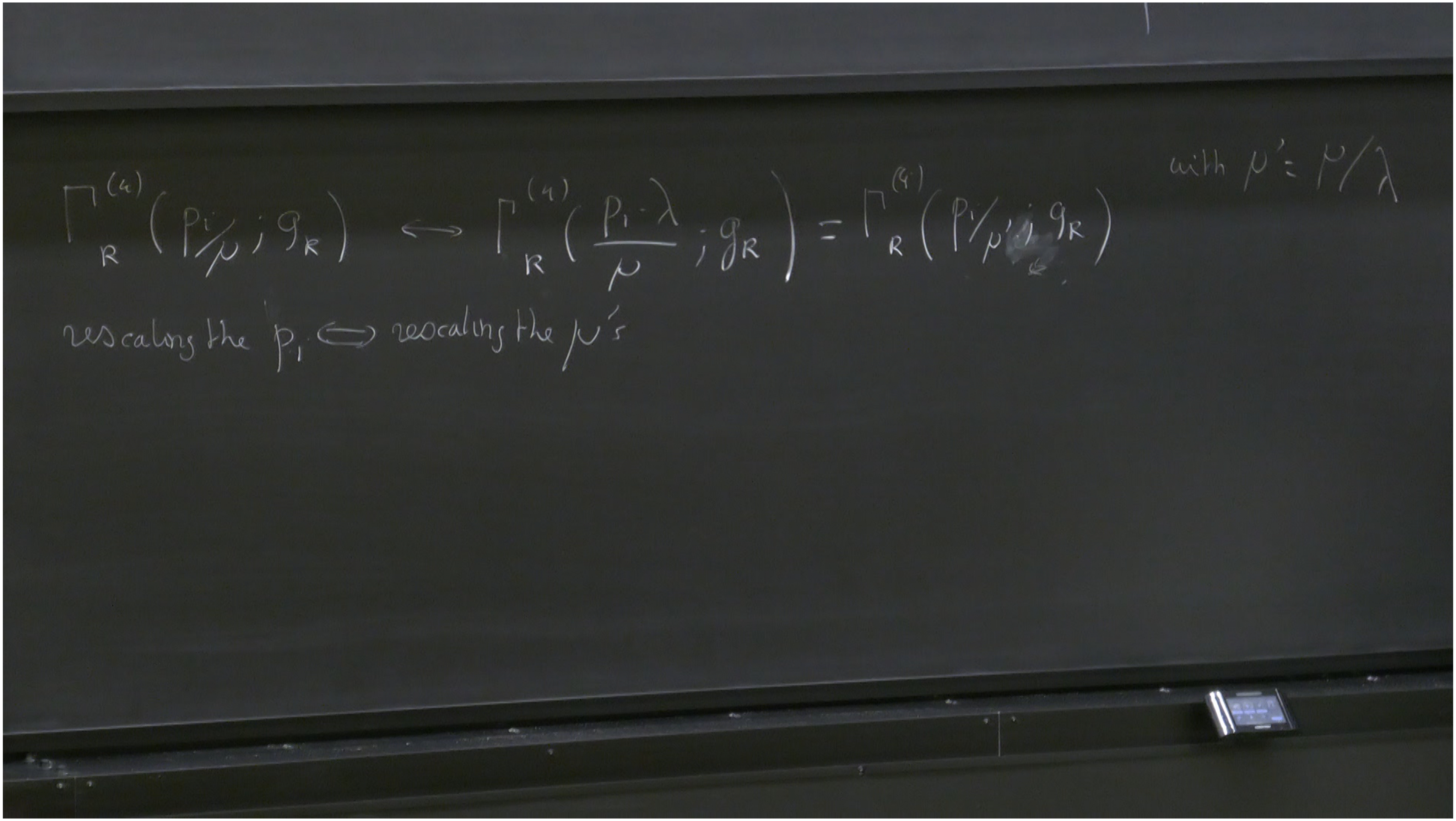
How does this vary ?

$p_i \longrightarrow \lambda p_i \quad \lambda > 0 \nearrow$ higher energies



$$\Gamma_R^{(4)}\left(p_i/\mu\,;g_R\right) \longleftrightarrow \Gamma_R^{(4)}\left(\frac{p_i\cdot\lambda}{\mu}\,;g_R\right) = \Gamma_R^{(4)}\left(p_i/\mu'\,;g_R\right)$$

with $\mu' = \mu/\lambda$



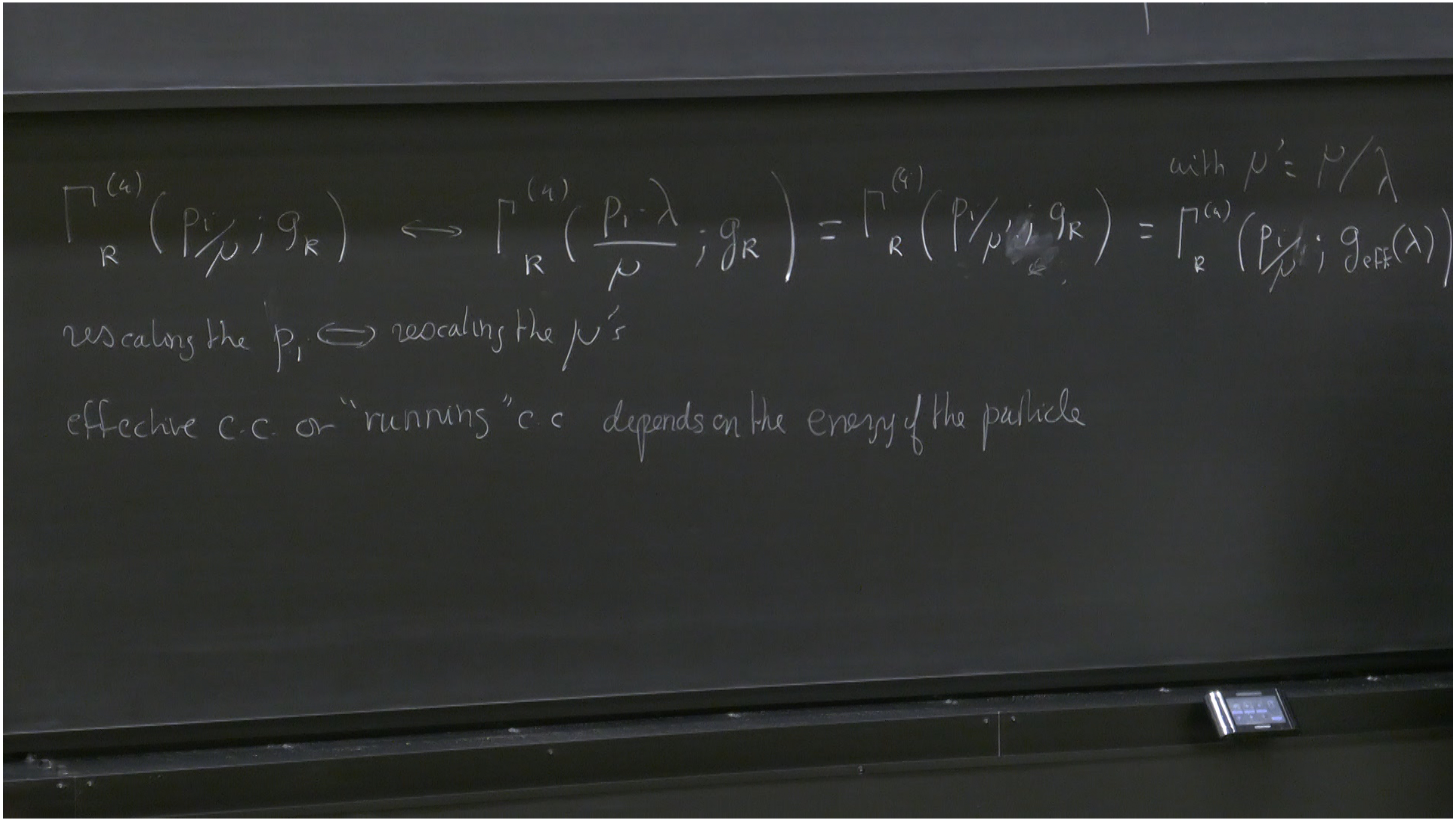
$$\Gamma^{(4)}_R\left(p_i/\mu\,;g_R\right) \longleftrightarrow \Gamma^{(4)}_R\left(\frac{p_i\cdot\lambda}{\mu}\,;g_R\right) = \Gamma^{(4)}_R\left(p_i/\mu'\,;g_R\right)$$

with $\mu' = \mu/\lambda$

rescaling the $p_i \Longleftrightarrow$ rescaling the μ's with $\mu' = \mu/\lambda$

$$\Gamma_R^{(4)}\left(p_i/\mu \,;\, g_R\right) \longleftrightarrow \Gamma_R^{(4)}\left(\frac{p_i\cdot\lambda}{\mu} \,;\, g_R\right) = \Gamma_R^{(4)}\left(p_i/\mu' \,;\, g_R\right) = \Gamma_R^{(4)}\left(p_i/\mu \,;\, g_{\text{eff}}(\lambda)\right)$$

rescaling the $p_i \Longleftrightarrow$ rescaling the μ's

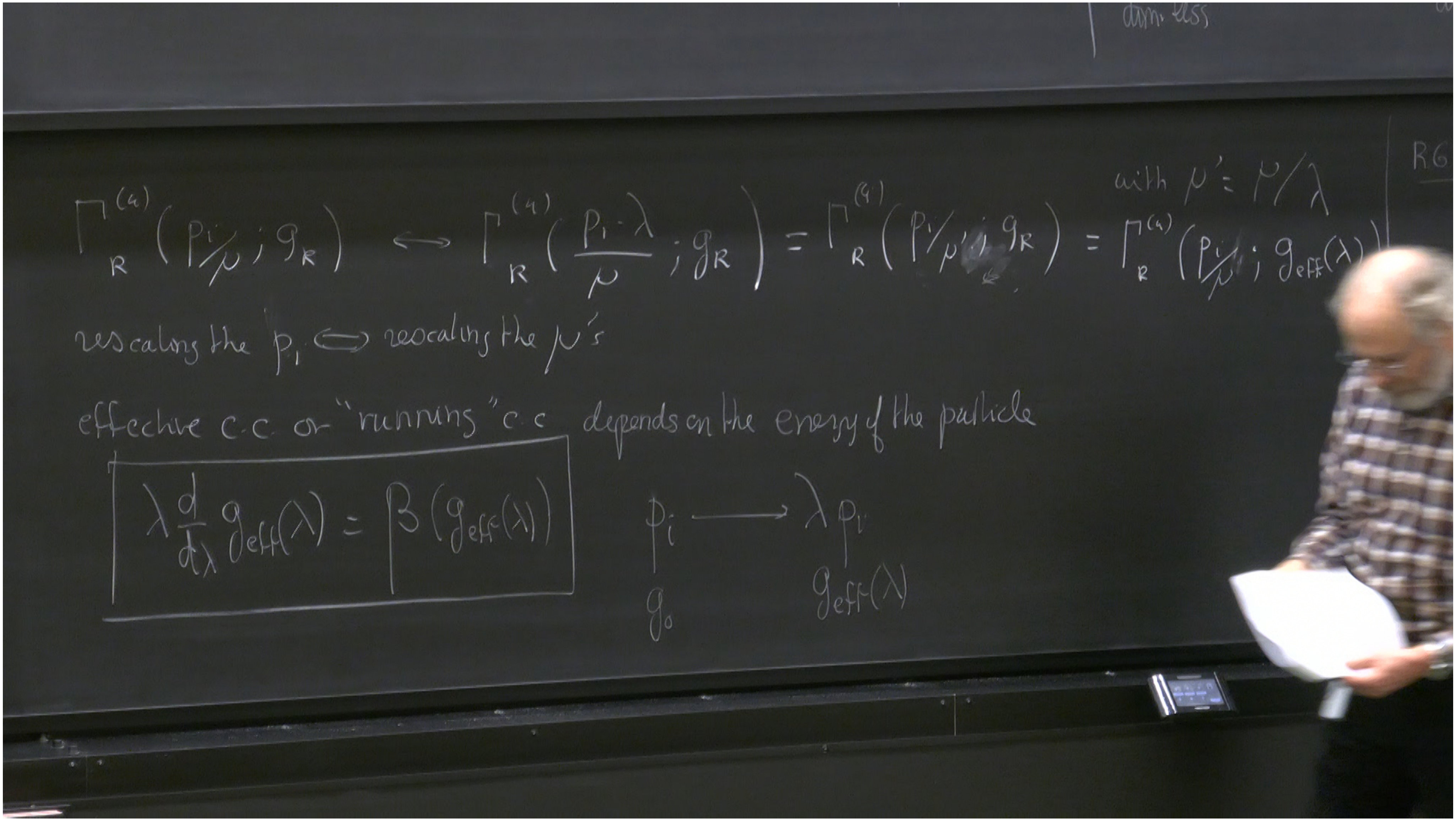


with $\mu' = \mu/\lambda$

$$\Gamma^{(4)}_R(p_i/\mu ; g_R) \longleftrightarrow \Gamma^{(4)}_R\left(\frac{p_i\cdot\lambda}{\mu} ; g_R\right) = \Gamma^{(4)}_R(p_i/\mu' ; g_R) = \Gamma^{(4)}_R(p_i/\mu ; g_{eff}(\lambda))$$

rescaling the $p_i \Longleftrightarrow$ rescaling the μ's

effective c.c. or "running" c.c depends on the energy of the particle



dim. less

RG

with $\mu' = \mu/\lambda$

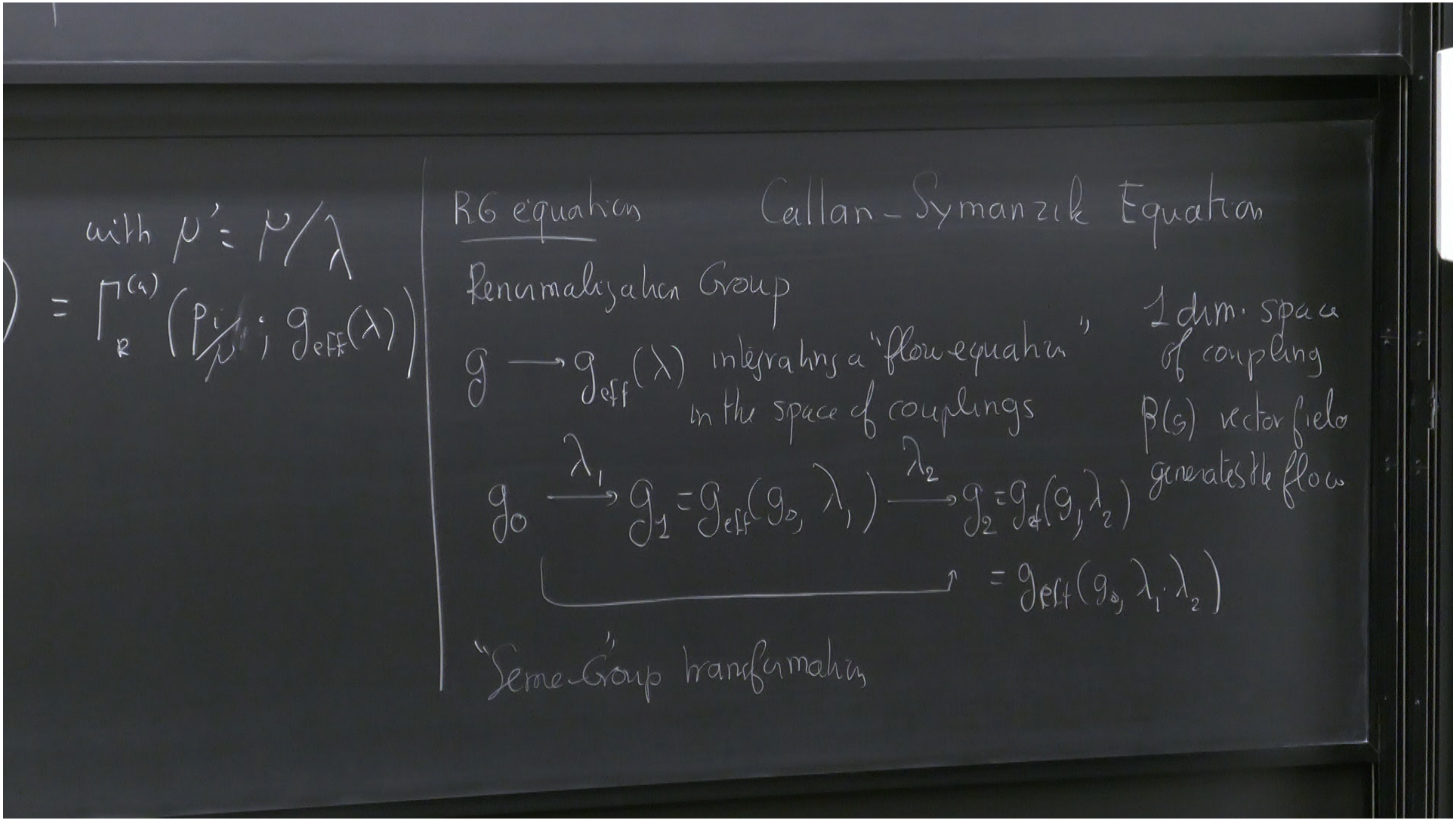
$$\Gamma_R^{(n)}(p_i/\mu\,; g_R) \longleftrightarrow \Gamma_R^{(n)}\left(\frac{p_i\cdot\lambda}{\mu}\,; g_R\right) = \Gamma_R^{(n)}(p_i/\mu'\,; g_R) = \Gamma_R^{(n)}(p_i/\mu\,; g_{\text{eff}}(\lambda))$$

rescaling the $p_i \Longleftrightarrow$ rescaling the μ's

effective c.c. or "running" c.c. depends on the energy of the particle

$$\boxed{\lambda \frac{d}{d\lambda} g_{\text{eff}}(\lambda) = \beta(g_{\text{eff}}(\lambda))}$$

$$\begin{array}{ccc} p_i & \longrightarrow & \lambda p_i \\ | & & | \\ g_0 & & g_{\text{eff}}(\lambda) \end{array}$$



with $\mu' = \mu/\lambda$

$$) = \Gamma_R^{(n)}\left(p_i/\mu' ; g_{\text{eff}}(\lambda)\right)$$

<u>RG equation</u> Callan-Symanzik Equation

Renormalization Group

$g \longrightarrow g_{\text{eff}}(\lambda)$ integrating a "flow equation" in the space of couplings

1 dim. space of coupling

$\beta(g)$ vector field generates the flow

$$\begin{tikzcd} g_0 \arrow[r, "\lambda_1"] \arrow[rr, bend right, swap, ""] & g_1 = g_{\text{eff}}(g_0, \lambda_1) \arrow[r, "\lambda_2"] & g_2 = g_{\text{eff}}(g_1, \lambda_2) \end{tikzcd}$$

$$= g_{\text{eff}}(g_0, \lambda_1 \cdot \lambda_2)$$

"Semi-Group" transformation

$$\int D[\phi]\, e^{-S_R[\phi]}$$

= finite $\Lambda \to \infty$ (at 2 loop)

$$\boxed{\phi^4} \qquad \beta(g) = g^2 \frac{3}{(4\pi)^2} + \cdots$$

$$\lambda \frac{d}{d\lambda} g(\lambda) = g(\lambda)^2 \cdot \frac{3}{(4\pi)^2}$$

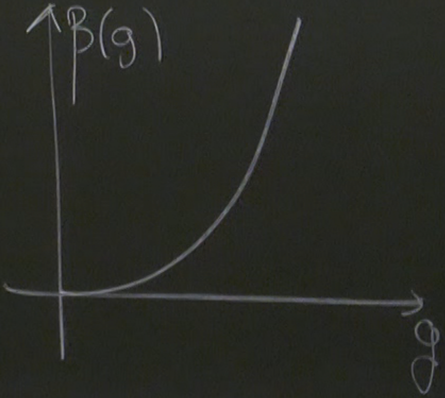
+ sign

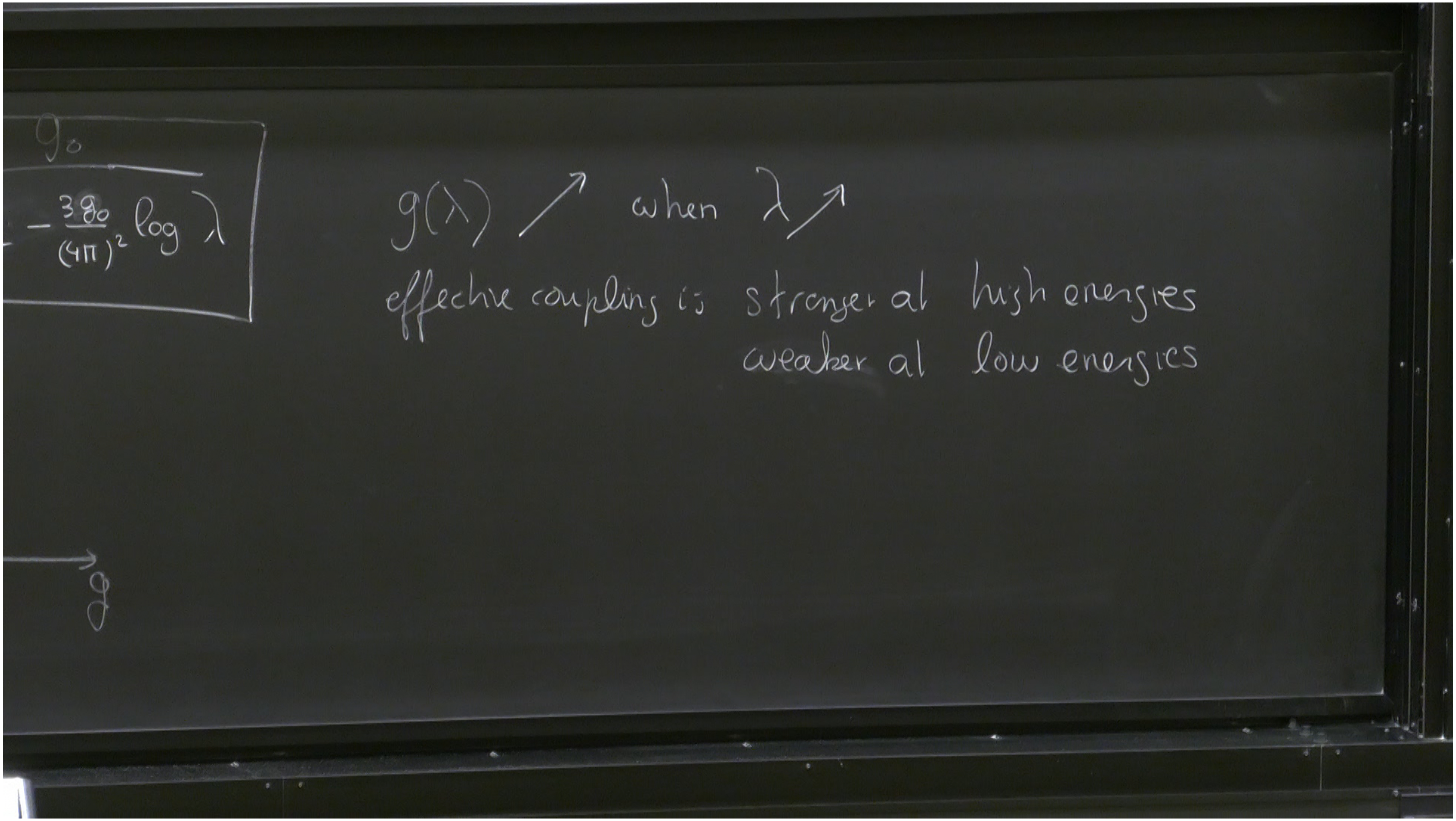
$\lambda = 1$ by $g = g_0$

$$\frac{dg(\lambda)}{g^2(\lambda)} = \frac{3}{(4\pi)^2} d(\log \lambda)$$

$$-\frac{1}{g(\lambda)} + \frac{1}{g_0} = \frac{3}{(4\pi)^2} \log \lambda$$

$$\boxed{g(\lambda) = \frac{g_0}{1 - \frac{3 g_0}{(4\pi)^2} \log \lambda}}$$





$$\boxed{\frac{g_0}{-\frac{3g_0}{(4\pi)^2}\log\lambda}}$$

$g(\lambda) \nearrow$ when $\lambda \nearrow$

effective coupling is stronger at high energies
weaker at low energies

$\longrightarrow$ g

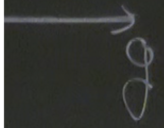
$$\boxed{\frac{g_0}{\;-\frac{3g_0}{(4\pi)^2}\log\lambda}}$$

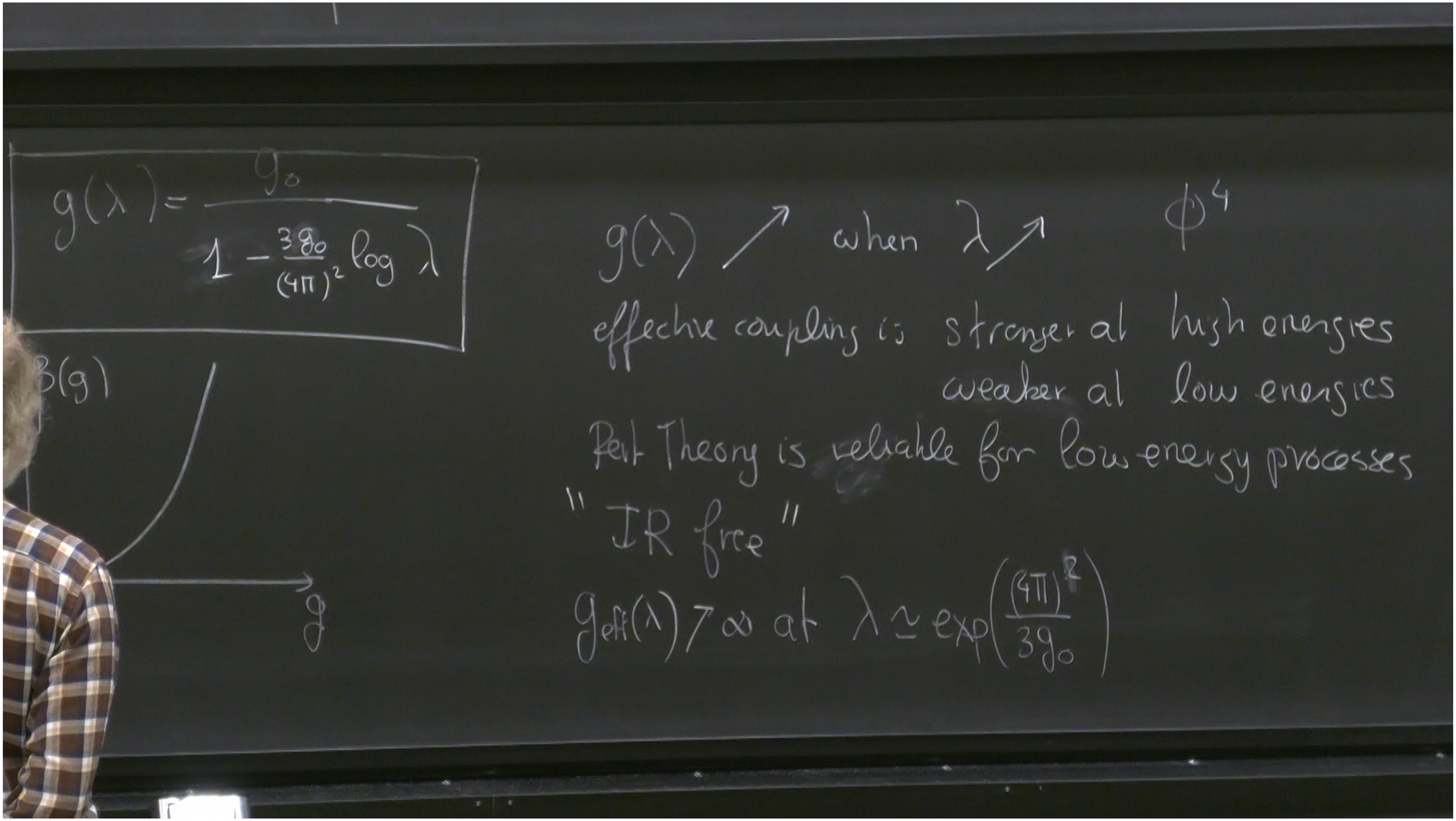
$g(\lambda)\nearrow$ when $\lambda\nearrow$ $\qquad\phi^4$

effective coupling is stronger at high energies

weaker at low energies

Pert Theory is reliable for low energy processes





$$\boxed{g(\lambda) = \frac{g_0}{1 - \frac{3g_0}{(4\pi)^2}\log\lambda}}$$

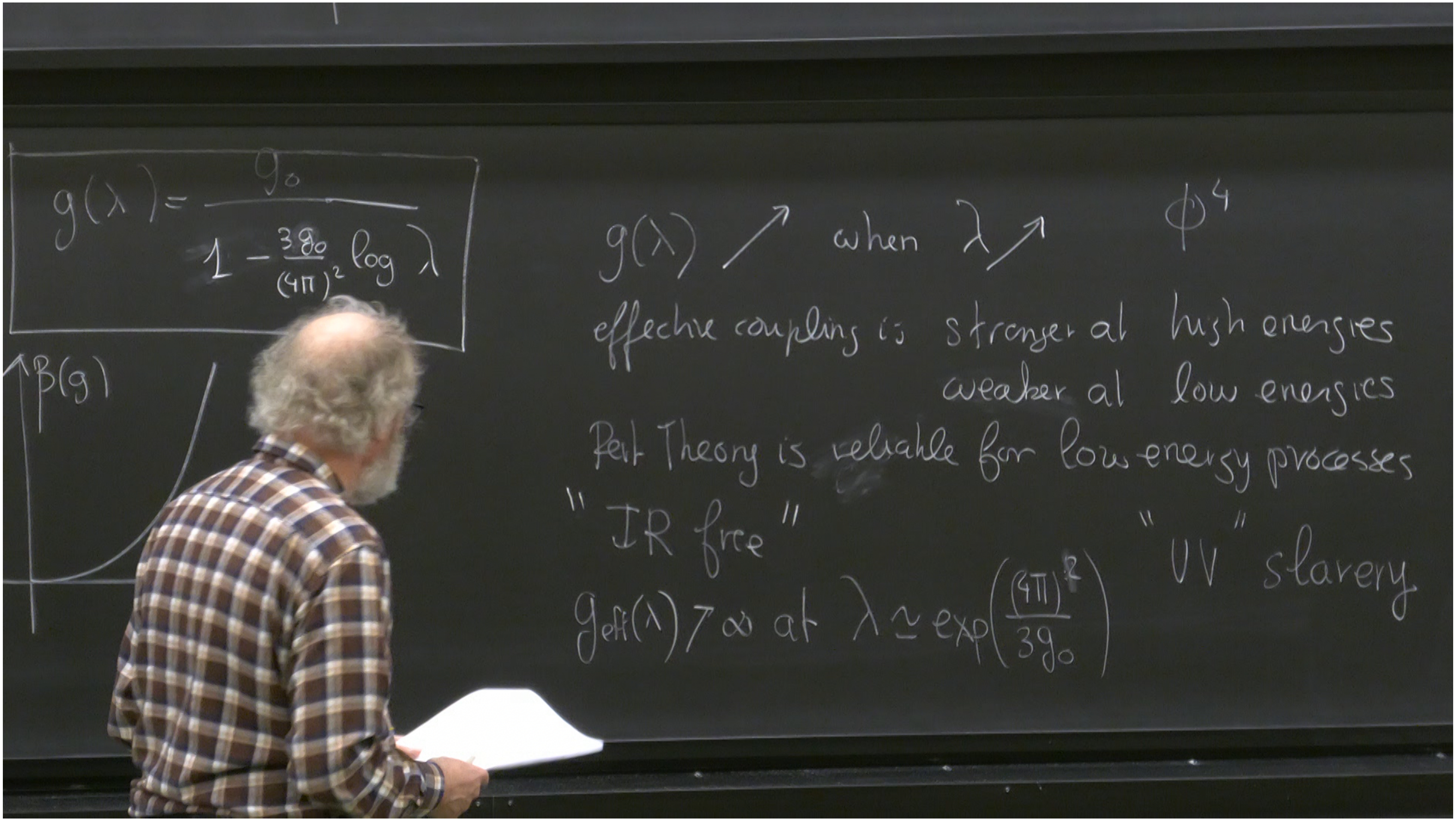
$g(\lambda) \nearrow$ when $\lambda \nearrow$ $\qquad \phi^4$

effective coupling is stronger at high energies
weaker at low energies

Pert Theory is reliable for low energy processes

"IR free"

$$g_{\text{eff}}(\lambda) \to \infty \text{ at } \lambda \simeq \exp\left(\frac{(4\pi)^2}{3g_0}\right)$$



$$\boxed{g(\lambda) = \frac{g_0}{1 - \frac{3g_0}{(4\pi)^2}\log\lambda}}$$

$g(\lambda) \nearrow$ when $\lambda \nearrow$ ϕ^4

effective coupling is stronger at high energies

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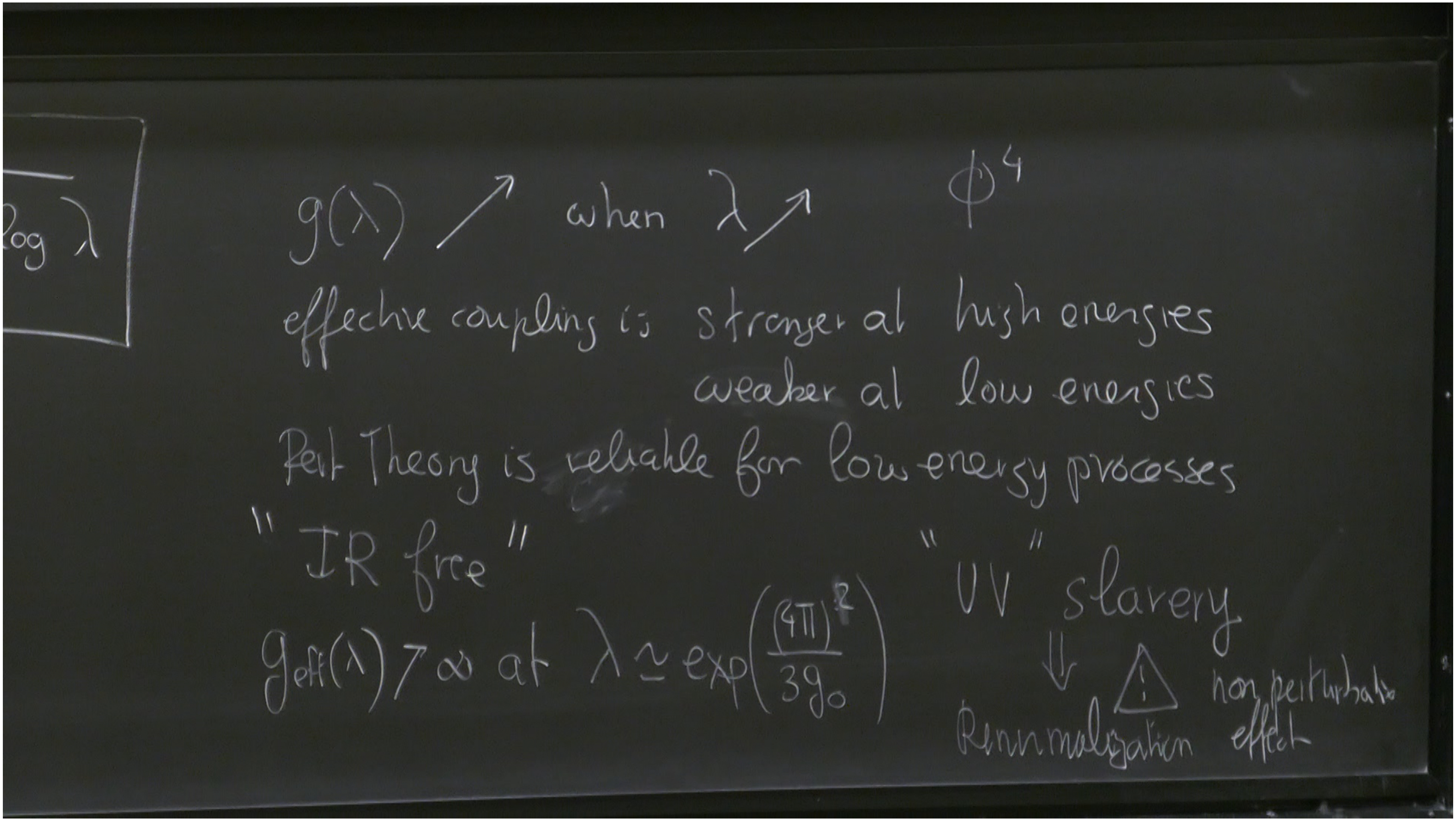
Pert Theory is reliable for low energy processes

" IR free " "UV" slavery

$g_{\text{eff}}(\lambda) \nearrow \infty$ at $\lambda \simeq \exp\left(\frac{(4\pi)^2}{3g_0}\right)$



$\log \lambda$

$g(\lambda) \nearrow$ when $\lambda \nearrow$ $\qquad \phi^4$

effective coupling is stronger at high energies
weaker at low energies

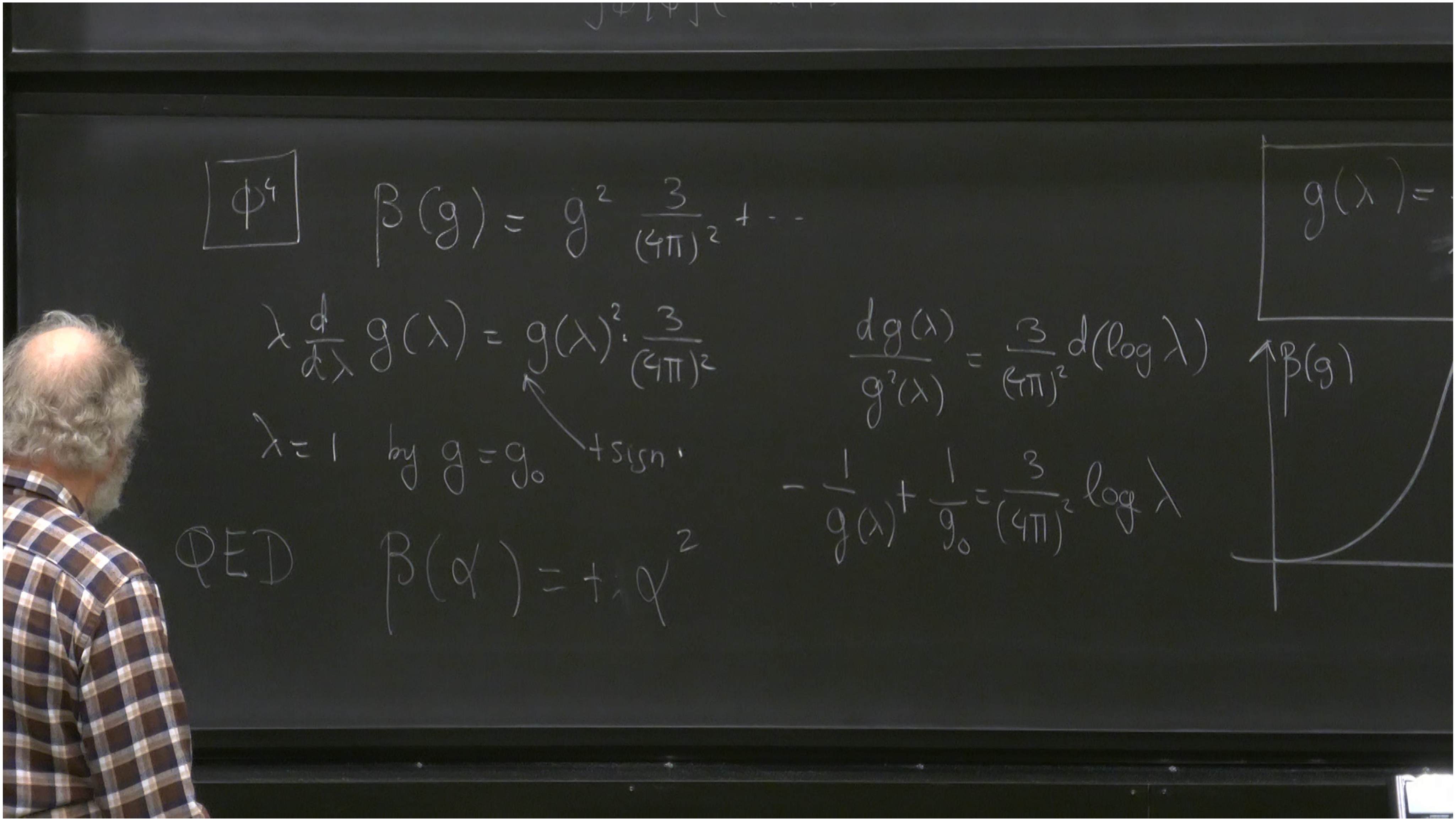
Pert Theory is reliable for low energy processes

"IR free" $\qquad$ "UV" slavery

$g_{eff}(\lambda) \to \infty$ at $\lambda \simeq \exp\left(\frac{(4\pi)^2}{3g_0}\right)$

$\Downarrow$ $\triangle$ non perturbative effect

Renormalization



$$\boxed{\phi^4} \qquad \beta(g) = g^2 \frac{3}{(4\pi)^2} + \cdots$$

$$\lambda \frac{d}{d\lambda} g(\lambda) = g(\lambda)^2 \cdot \frac{3}{(4\pi)^2}$$

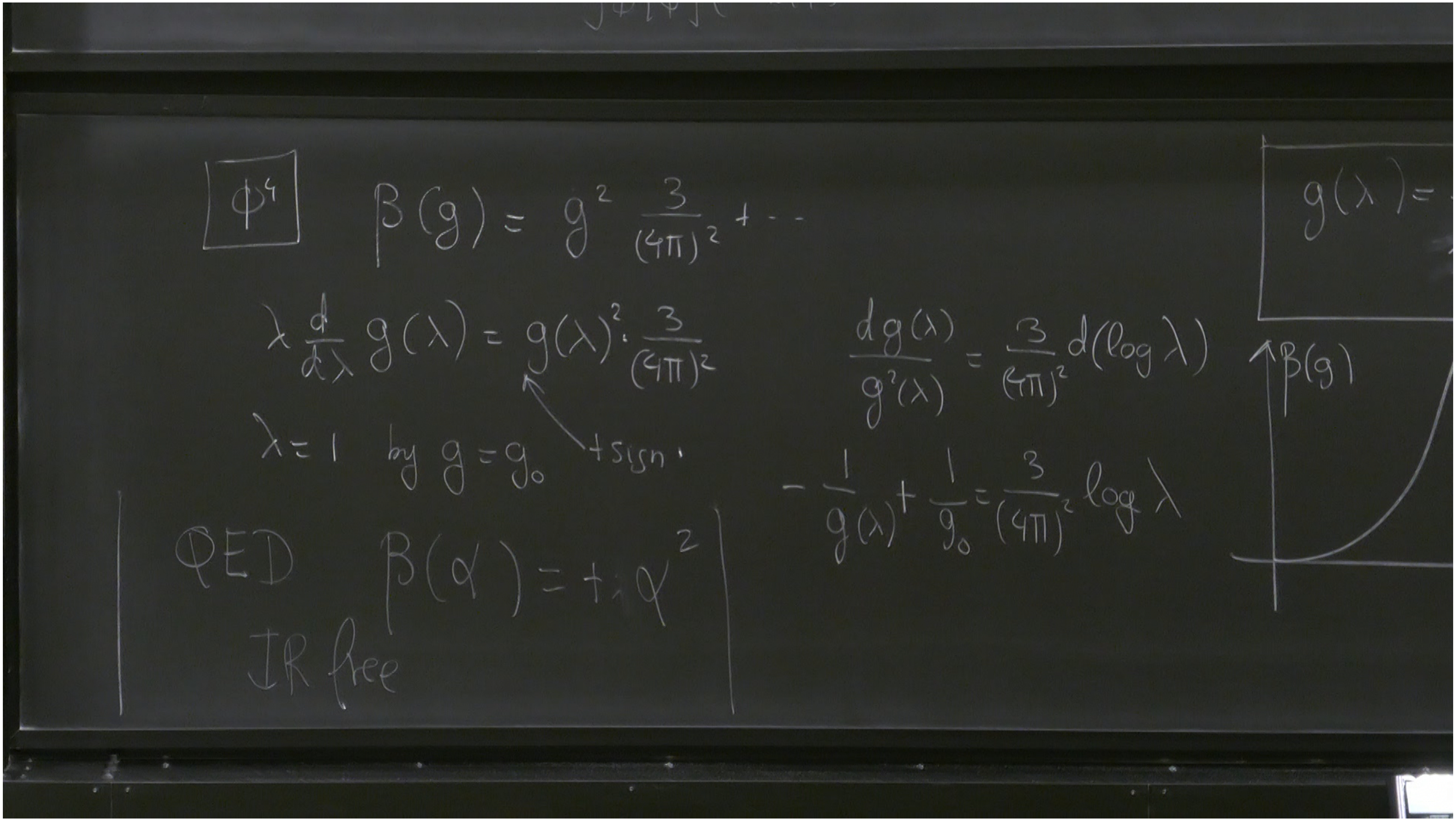
$\lambda = 1$ by $g = g_0$ +sign

QED $\beta(\alpha) = + \cdot \alpha^2$

$$\frac{dg(\lambda)}{g^2(\lambda)} = \frac{3}{(4\pi)^2} d(\log \lambda)$$

$$-\frac{1}{g(\lambda)} + \frac{1}{g_0} = \frac{3}{(4\pi)^2} \log \lambda$$

$g(\lambda) =$



$\boxed{\phi^4}$ $\beta(g) = g^2 \frac{3}{(4\pi)^2} + \cdots$

$$\lambda \frac{d}{d\lambda} g(\lambda) = \overset{+\text{sign}}{g(\lambda)^2} \cdot \frac{3}{(4\pi)^2}$$

$\lambda = 1$ by $g = g_0$

$$\frac{dg(\lambda)}{g^2(\lambda)} = \frac{3}{(4\pi)^2} d(\log \lambda)$$

$$-\frac{1}{g(\lambda)} + \frac{1}{g_0} = \frac{3}{(4\pi)^2} \log \lambda$$

QED $\beta(\alpha) = + \alpha^2$

IR free

$g(\lambda) =$