

Title: General Relativity for Cosmology Lecture - 120523

Speakers: Achim Kempf

Collection: General Relativity for Cosmology

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Abstract: Zoom: <https://ptp.zoom.us/j/91640855624?pwd=dWVWV2doSnBhUS9JUkhjQVBwY0h0dz09>

GR for Cosmology, Achim Kempf

Lecture 23

More on singularity theorems

- Assume a set of symmetries of matter and spacetime has been chosen.
- Assume an exact solution or at least its asymptotic properties at early times have been found.
- Assume, we choose a timelike congruence e.g. of geodesics.

⇒ We can now explicitly calculate the **twist**, **shear** and **expansion** along the congruence:

$$\Theta_{\mu\nu} := \overset{\text{shear}}{\sigma_{\mu\nu}} + \overset{\text{expansion scalar}}{\frac{1}{3}\theta h_{\mu\nu}}$$

symmetric part of $\Theta_{\mu\nu}$
projector \perp to the timelike u -field

- $\Theta_{\mu\nu}$ is fully spacelike and symmetric $\Rightarrow \Theta_{\mu\nu}$ can be diagonalized in suitable ON frame $\{e_0, e_1, e_2, e_3\}$:

$$\Theta_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \theta_1 & 0 \\ 0 & 0 & \theta_3 \end{pmatrix}$$

3 spacelike directions.

with the traditional expansion being the trace (because

The Hubble functions:

In particular, we can see how the expansion or contraction of the universe behave dynamically, e.g. when the condition of perfect isotropy is relaxed:

- Now we have different expansions in different directions, nonlinearly influencing another.

□ Recall:

The expansion in one direction can be enhanced by shear, as long as shear shrinks other directions.

□ Definition:

We define a rate of expansion tensor that includes shear:

□ Definition:

We use H_i, H to define local directional and general scale factors l_i, l :

The l_i, l are defined as the solutions to:

$$\frac{\dot{l}_i}{l_i} = H_i$$

$$\frac{\dot{l}}{l} = H$$

Assume an exact solution or at least its asymptotic properties at early times have been found.

Assume, we choose a timelike congruence e.g. of geodesics.

⇒ We can now explicitly calculate the **twist**, **shear** and **expansion** along the congruence:

$$\Theta_{\mu\nu} := \overset{\text{shear}}{\sigma_{\mu\nu}} + \frac{1}{3} \overset{\text{expansion scalar}}{\theta} h_{\mu\nu}$$

symmetric part of $\sigma_{\mu\nu}$ projector \perp to the timelike u -field

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3 spacelike directions.

with the traditional expansion being the trace (because $\sigma_{\mu\nu}$ is traceless):

$$\theta = \theta_1 + \theta_2 + \theta_3$$

why $\frac{1}{3}$? Recall that $\text{Tr}(h_{\mu\nu}) = 3$ ⇒ is not quite projector

Definition: $H_i := \frac{1}{3} \theta_i$ Local Hubble expansion function in direction e_i .
 $H := \frac{1}{3} \theta$ Overall local Hubble expansion function.

Now we have different expansions in different directions, nonlinearly influencing another.

Recall:

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The l_i, l are defined as the solutions to:

$$\frac{\dot{l}_i}{l_i} = H_i$$

$$\frac{\dot{l}}{l} = H$$

Here, the time derivative is defined as:

$$\dot{l} = u(\dot{l}) = u^\alpha \frac{\partial}{\partial x^\alpha} l$$

recall: u is timelike.

What behavior can occur in the far past?

$$\theta_{\mu\nu} = \begin{pmatrix} 0 & \theta_1 & 0 \\ 0 & \theta_2 & 0 \\ 0 & 0 & \theta_3 \end{pmatrix}$$

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- What behavior can occur in the far past?

Full set of cases not yet known.

But:

Explicit examples are known where e.g:

- All $l_i \rightarrow 0$ as in FL cosmologies
- $l_1, l_2 \rightarrow 0, l_3 \rightarrow \infty$ "cigar singularity"
- $l_1, l_2 \rightarrow 0, l_3 \rightarrow \text{const}$ "barrel singularity"
- $l_1, l_2 \rightarrow \text{const}, l_3 \rightarrow 0$ "pancake singularity"

- Note:** For homogeneous, isotropic FL models, H is the regular Hubble parameter and l is its scale factor.

Singularity theorems for black holes

In 1915, Schwarzschild found this black hole solution to the Einstein equation:

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} = \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Mass of black hole

Singularity: $r = 0$

Horizon: $r = 2M$

Here, $x = (t, r, \theta, \phi)$ are called the Schwarzschild coordinates.

Schwarzschild coordinates long misled intuition:

- The singularity at $r = 2M$ is not real: it disappears in other coordinate systems. The curvature is smooth across $r = 2M$.

Full set of cases not yet known.

But:

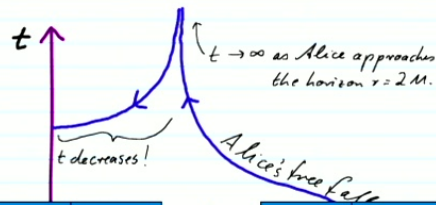
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□ Note: For homogeneous, isotropic FL models, H is the regular Hubble parameter and ℓ is its scale factor.

□ Due to the sign changes across $r = 2M$, for $r < 2M$ dt is spacelike and dr is timelike for $r < 2M$.

□ Consider, for example, a traveler, Alice, who is freely falling from $r = r_0$ to $r = 0$:



$$r(\omega) = \frac{r_0}{2} (1 + \cos(\omega))$$

$$t(\omega) = \left(\frac{r_0}{2} + 2M\right) \omega + \frac{r_0}{2} \omega \sin(\omega) + 2M \log \left| \frac{\omega + \tan(\omega/2)}{\omega - \tan(\omega/2)} \right|$$

$$\tau(\omega) = \frac{r_0}{2} \left(\frac{r_0}{2M}\right)^{1/2} (\omega + \sin(\omega))$$

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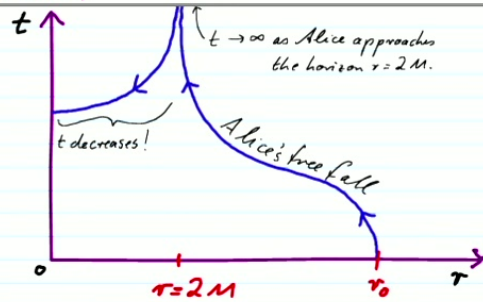
Simplification:

For now, we drop the φ and θ coordinates.

First design of a new eds (T, R) - Alice's choice (for $r_0 = 2M$):

□ Require $g_{\mu\nu}(T, R)$ to be regular across $r = 2M$.

□ Require $g_{\mu\nu}(0, 0) = \eta_{\mu\nu}$ at $r = 2M$. If there's really no singularity at $r = 2M$ this must be possible.



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Here, $0 < \omega < \pi$ and $\omega = \left(\frac{r_0}{2M} - 1\right)^{1/2}$

⇒ need better choices of coordinate systems!

- Require $g_{\mu\nu}(T,R)$ to be regular across $r = 2M$.
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- Extend this cds so that $g_{\mu\nu}(T,R) = f(T,R) \eta_{\mu\nu}$

⇒ Alice's choice are the Kruskal-Szekeres coordinates (T,R) :

$$T(t,r) := 4M \left| \frac{r}{2M} - 1 \right|^{1/2} e^{\frac{r-2M}{4M}} \left(\sinh\left(\frac{t}{4M}\right) \theta(r-2M) + \cosh\left(\frac{t}{4M}\right) \theta(2M-r) \right)$$

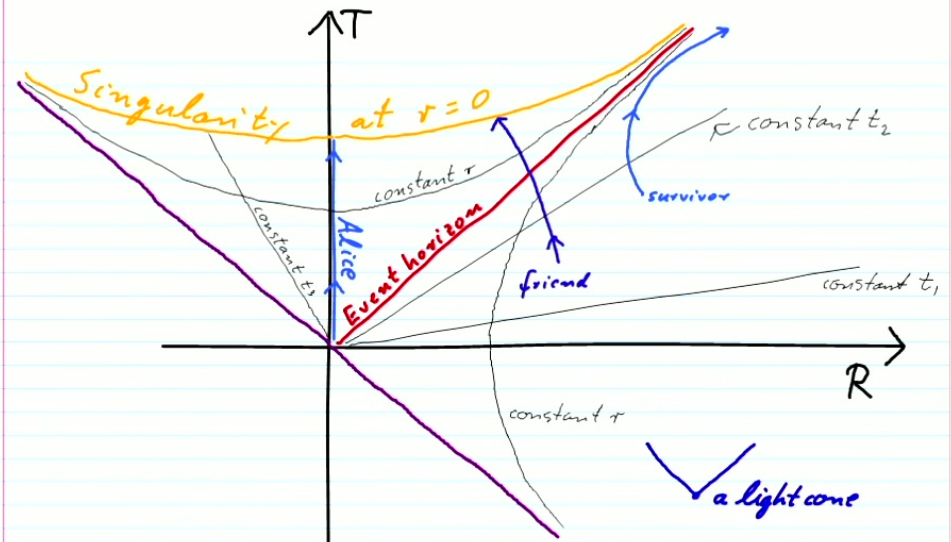
$$R(t,r) := 4M \left| \frac{r}{2M} - 1 \right|^{1/2} e^{\frac{r-2M}{4M}} \left(\cosh\left(\frac{t}{4M}\right) \theta(r-2M) + \sinh\left(\frac{t}{4M}\right) \theta(2M-r) \right)$$

This map is, in principle, invertible, to obtain $t(T,R)$, $r(T,R)$.

The Schwarzschild metric now takes this form:

$$ds^2 = \frac{2M}{r(T,R)} e^{1 - \frac{r(T,R)}{2M}} \underbrace{\left(dT^2 - dR^2 \right)}_{\eta_{\mu\nu}} \quad \text{Obeys all conditions!}$$

Conformal prefactor = 1 as $r = 2M$



- Alice was at rest at the event horizon.
- The singularity is at $T(R) = \left(R^2 + \frac{16M^2}{c}\right)^{1/2}$ and is spacelike.

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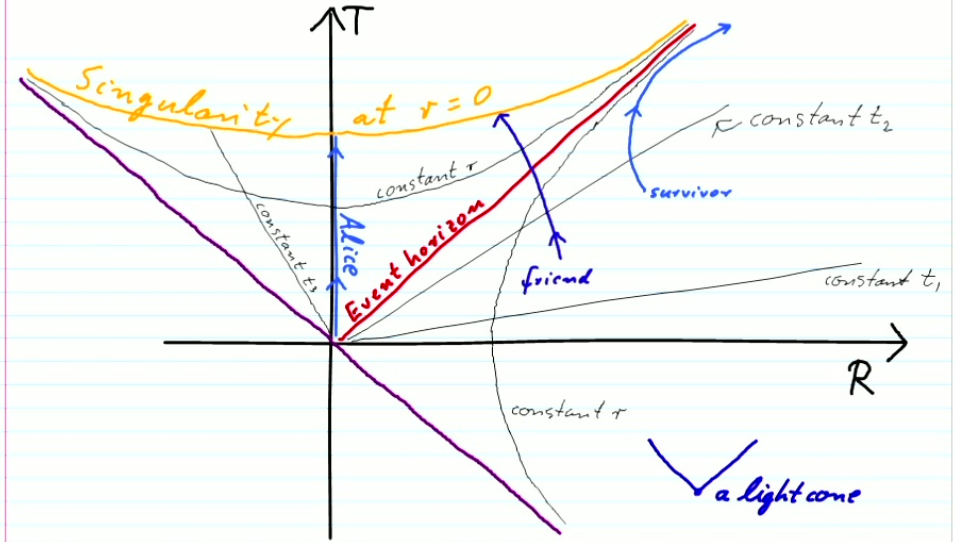
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Alice's light cone coordinates:

$$u := T - R, \quad v := T + R$$

$$\text{Metric: } ds^2 = \frac{2M}{r(u, v)} e^{1 - \frac{r(u, v)}{2M}} du dv$$

conformal factor (which is 1 at horizon) light cone Minkowski

⇒ The action $S[\phi] = \frac{1}{2} \int g^{\alpha\beta} \phi_{,\alpha} \phi_{,\beta} \sqrt{g} d^2x$ becomes:

Bob's choice of coordinate system

Bob is far from the black hole.

He wants a cds in which:

- $g_{\mu\nu}(x) \rightarrow \eta_{\mu\nu}$ as $r \rightarrow \infty$.
- $g_{\mu\nu}(x) = f(x) \eta_{\mu\nu}$ everywhere.

$$R(t, r) := 4M \left| \frac{r}{2M} - 1 \right|^{-1} e^{+M \left(\cosh\left(\frac{r}{4M}\right) \theta(r-2M) + \sinh\left(\frac{r}{4M}\right) \theta(2M-r) \right)}$$

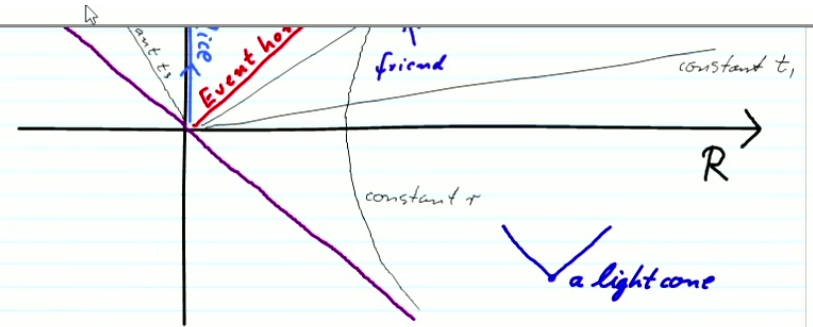
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$$= 2 \int_{-\infty}^{\infty} \int_0^{\infty} (\partial_u \phi(u, v)) (\partial_v \phi(u, v)) dv du$$

← 1/2 region $T > -R$ means $T+R > 0, \dots, v > 0$.

⇒ Eqn of motion: $\partial_u \partial_v \phi(u, v) = 0$

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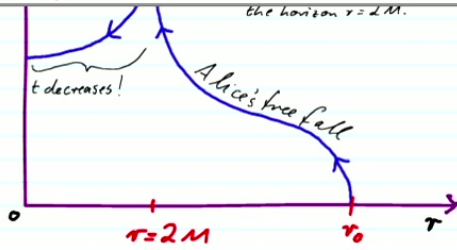
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This is so that in his cds too

- photons travel at 45°
- equations of motion of matter fields will be simple (useful in QFT!)

⇒ Bob's choice is the Tortoise coordinate system.



the horizon $r = 2M$.

$$t(\lambda) = \left(\frac{r_0}{2} + 2M\right) + 2M \log \left| \frac{w + \tan(\lambda/2)}{w - \tan(\lambda/2)} \right|$$

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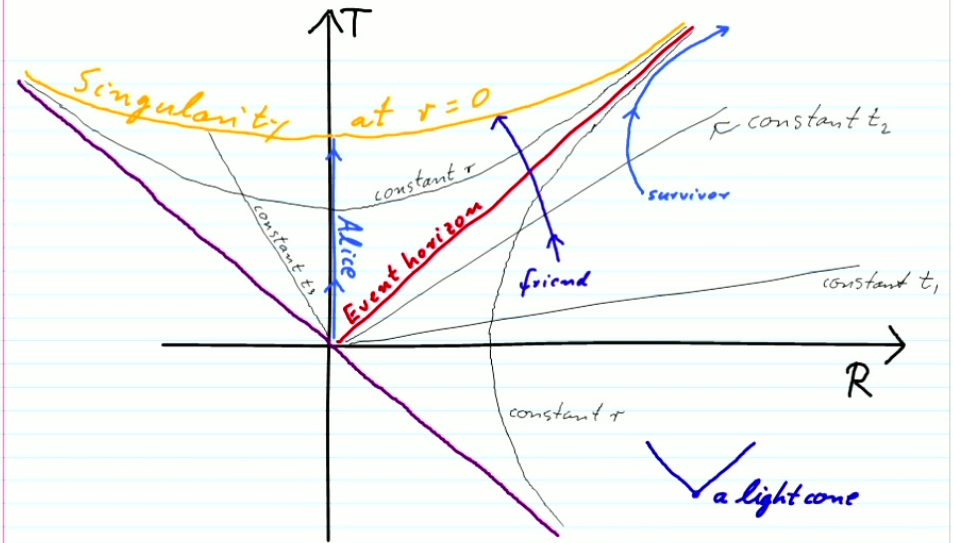
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(which is 1 at horizon)

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- equations of motion of matter fields will be simple (useful in QFT!)

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Tortoise cds (t^*, r^*):

□ In terms of the Schwarzschild cds:

$t^* := t$

$r^* := r - 2M + 2M \log\left(\frac{r}{2M} - 1\right)$

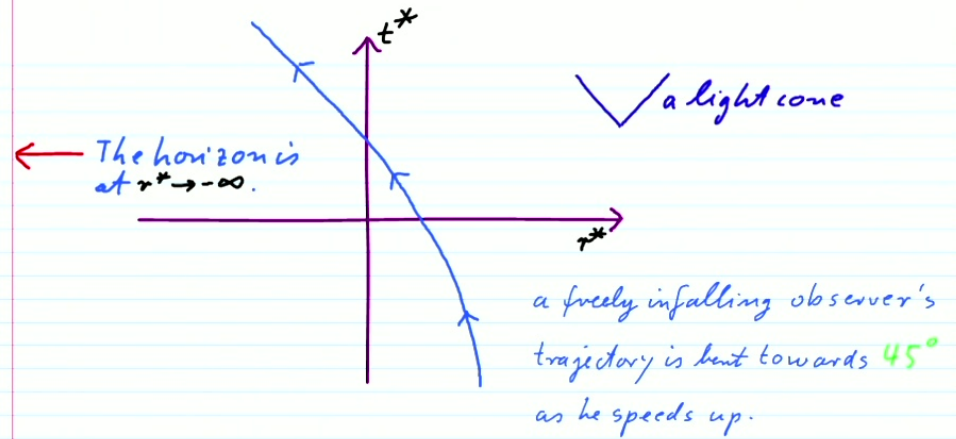
must require $r > 2M$!

⇒ Important: This is in principle invertible, to obtain $r = r(r^*)$ but only for $r > 2M$!

⇒ The tortoise cds only cover the BH's outside!

Metric: $ds^2 = \left(1 - \frac{2M}{r(r^*)}\right) (dt^{*2} - dr^{*2})$

Conformal factor $\rightarrow 1$ as $r \rightarrow \infty$, as planned but $\rightarrow 0$ at horizon.



Bob's light cone coordinates: $\bar{u} := t^* - r^*$, $\bar{v} := t^* + r^*$

The metric is then: $ds^2 = \left(1 - \frac{2M}{r(\bar{u}, \bar{v})}\right) d\bar{u} d\bar{v}$

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a freely infalling observer's trajectory is bent towards 45° as he speeds up.

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⇒ The action:

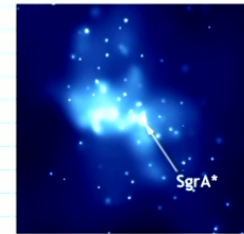
$$\begin{aligned} S[\phi] &= \frac{1}{2} \int g^{\alpha\beta} \phi_{,\alpha} \phi_{,\beta} \sqrt{|g|} d^2x \text{ becomes:} \\ &= \frac{1}{2} \int_{\mathbb{R}^2} (\partial_{\bar{u}} \phi(t^*, r^*))^2 - (\partial_{\bar{v}} \phi(t^*, r^*))^2 d\bar{u} d\bar{v} \\ &= 2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (\partial_{\bar{u}} \phi(\bar{u}, \bar{v})) (\partial_{\bar{v}} \phi(\bar{u}, \bar{v})) d\bar{v} d\bar{u} \end{aligned}$$

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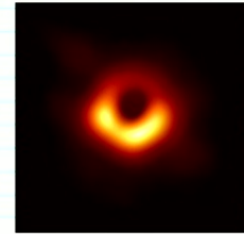
Do real black holes possess a singularity?

Sagittarius A*

- 4 Mio stellar masses
- Diameter 44 Mio km
- 26000 light years away at centre of Milky Way.



→ Observation of M87 by the Event Horizon Telescope (in mm band) with enough resolution to see the event horizon:



$$= \frac{1}{2} \int_{\mathbb{R}^2} (\partial_{\bar{\alpha}} \phi(\bar{u}, \bar{v})) (\partial_{\bar{\nu}} \phi(\bar{u}, \bar{v})) d\bar{v} d\bar{u}$$

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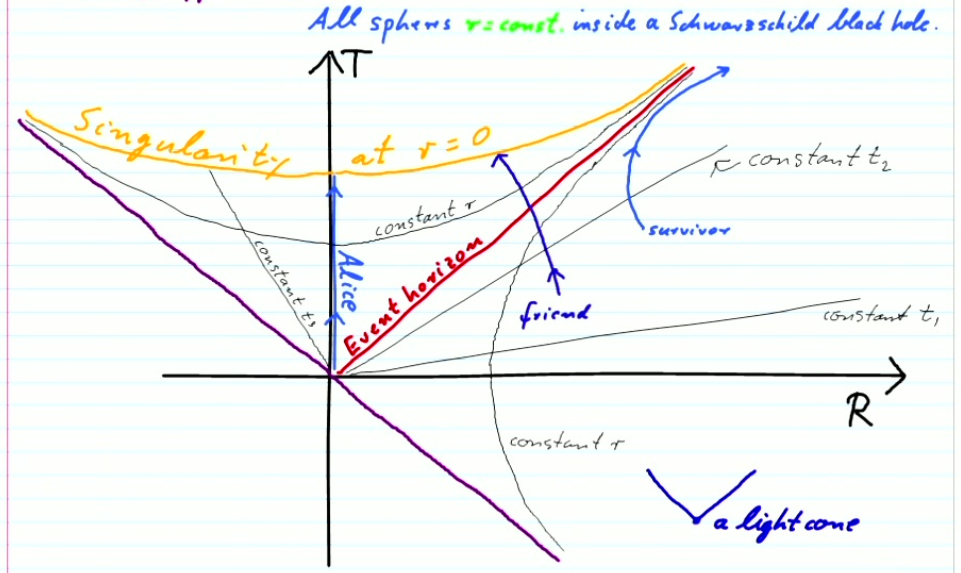
How to model properties of real black holes roughly?

Singularity theorems suitable for black holes involve the concept and assumption of a trapped surface:

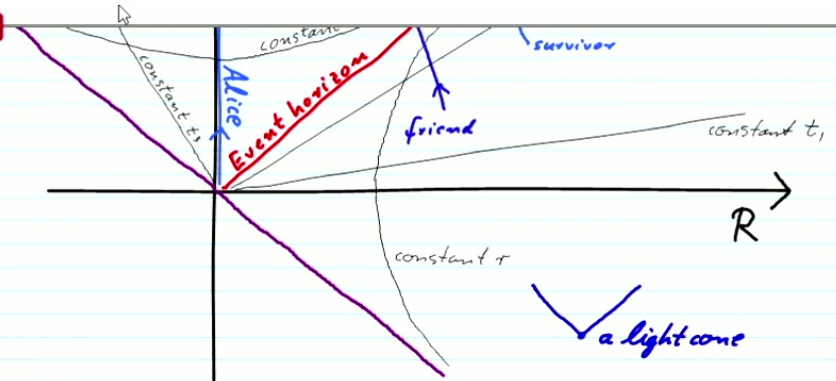
Def:

- Let Σ be a spacelike hypersurface. (Note: Σ is 3-dimensional)
- Let $T \subset \Sigma$ be a compact, 2-dimensional smooth spacelike submanifold of Σ . Consider the ingoing and the outgoing future-directed null geodesics that are orthogonal to T .
- If all these geodesics possess negative expansion, $\theta < 0$, then T is called a trapped surface.

Examples of trapped surfaces:

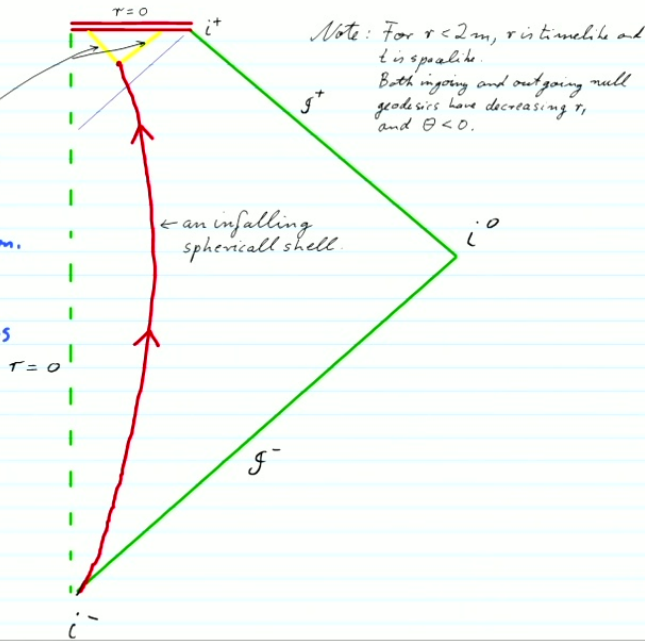


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Generally:

The in- and outgoing null geodesics both have negative expansion. Can't see it here b/c the neighboring geodesics are neighbors in the suppressed angular directions.



Note: For $r < 2m$, r is timelike and t is spacelike. Both ingoing and outgoing null geodesics have decreasing r , and $\theta < 0$.

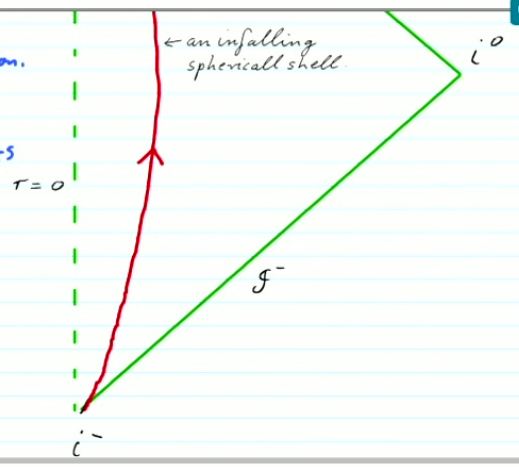
Def: Let Σ be a spacelike hypersurface. Then, the (3-dim. spacelike) union, \mathcal{T} , of all trapped surfaces $T \subset \Sigma$ is called the trapped region of Σ .

Def: The boundary $\partial\mathcal{T} \subset \Sigma$ is called the apparent horizon of the spacelike hypersurface Σ .

Note: $\partial\mathcal{T}$ is 2-dimensional.

Def: If we foliate spacetime into spacelike hypersurfaces $\Sigma_u, u \in I \subset \mathbb{R}$ each with its apparent horizon, \mathcal{H}_u , then their union $\mathcal{A} := \bigcup_u \mathcal{H}_u$ is called the Trapping horizon of the spacetime.

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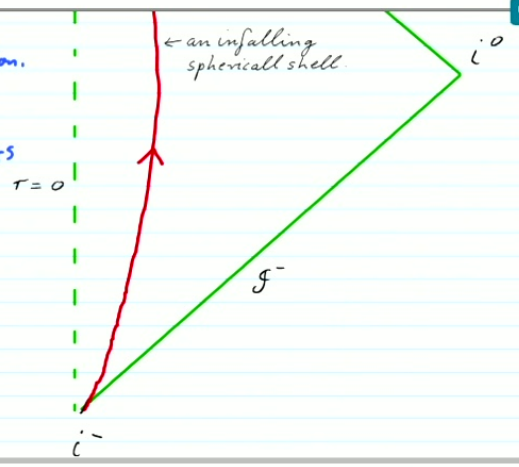
Remarks:

- To check for the existence of an event horizon j^- (worldline to i^+) in principle requires knowledge of the full future.
- But one can check for the existence of an apparent horizon in any spacelike hypersurface by calculating the expansion: only at that time!
- The notion of apparent horizons is dependent on the choice of foliation of spacetime into spacelike hypersurfaces.

- For static Schwarzschild black holes the event and apparent horizons coincide.
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