Title: Equilibrium dynamics of infinite-range quantum spin glasses in a field - VIRTUAL

Speakers: Maria Tikhanovskaya

Series: Quantum Matter

Date: November 29, 2023 - 10:30 AM

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Abstract: We determine the low-energy spectrum and Parisi replica symmetry breaking function for the spin glass phase of the quantum Ising model with infinite-range random exchange interactions and transverse and longitudinal (h) fields. We show that, for all h, the spin glass state has full replica symmetry breaking, and the local spin spectrum is gapless with a spectral density which vanishes linearly with frequency. These results are obtained using an action functional - argued to yield exact results at low frequencies - that expands in powers of a spin glass order parameter, which is bilocal in time, and a matrix in replica space. We also present the exact solution of the infinite-range spherical quantum p-rotor model at nonzero h: here, the spin glass state has one-step replica symmetry breaking, and gaplessness only appears after imposition of an additional marginal stability condition.

Zoom link https://pitp.zoom.us/j/98757418107?pwd=U1hiQnpKTDI4ajUyL04zRmQ4dVg3UT09

Equilibrium dynamics of infinite-range quantum spin glasses in a field

arXiv: 2309.03255

Maria Tikhanovskaya, Subir Sachdev and Rhine Samajdar

Research summary

1. High energy theory (Steklov Mathematical Institute, Russia)

2. Sachdev-Ye, random t-J models and quantum chaos (Harvard)

3. Random quantum circuits (KITP)

4. Quantum spin glasses (Harvard)

Outline

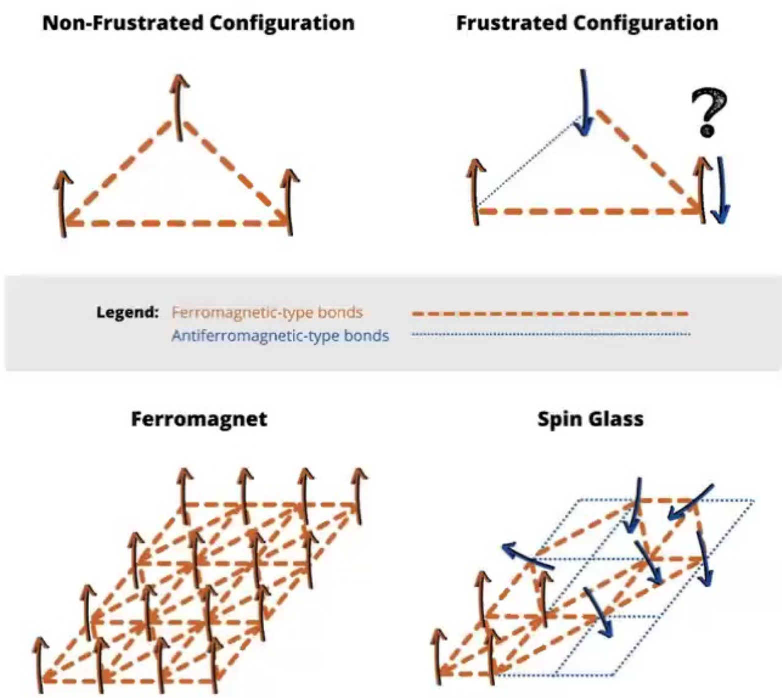
I. Introduction and motivation

II. Overview of the method to obtain a solution

III. Results: quantum Ising and p=3 – rotor models

IV. Conclusion and outlook

Spin glass - definition



Spin glass – magnetic state with randomness and frustration (mixture of non-magnetic host metal and small fraction of magnetic atoms)

https://phys.org/news/2022-11-mathematics-dont-behavior-glasses.html

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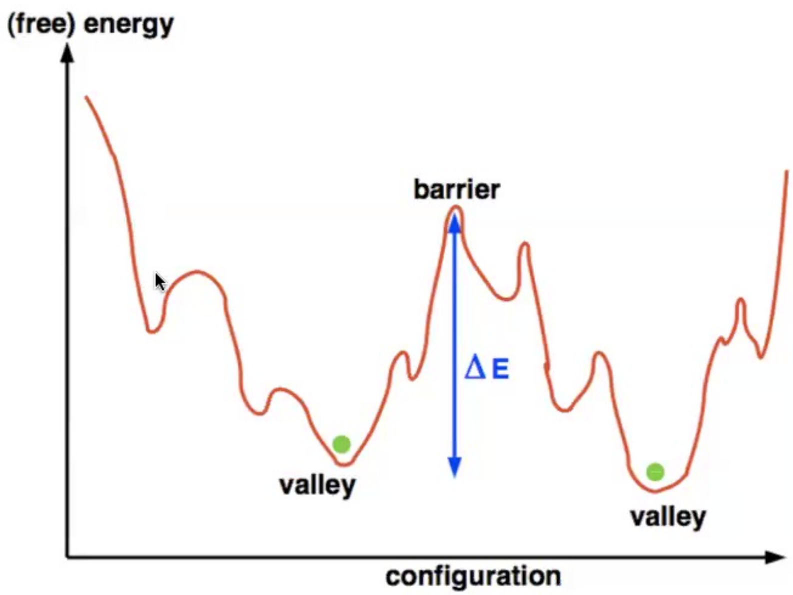

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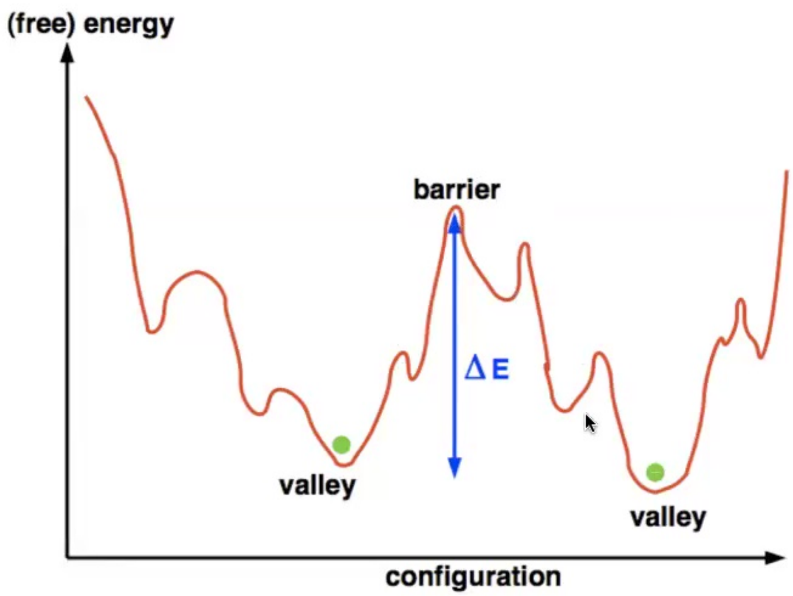

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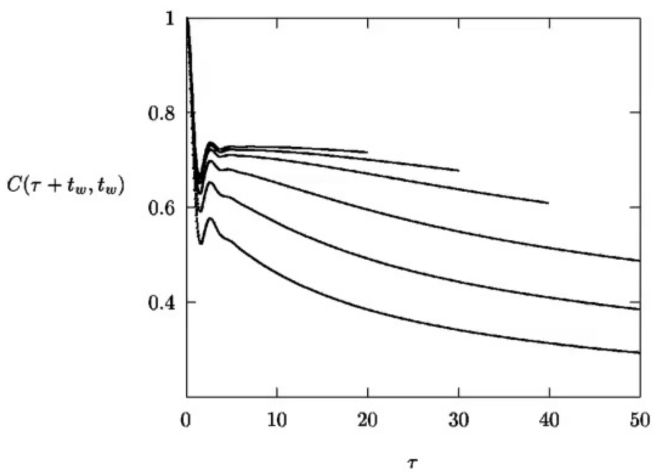

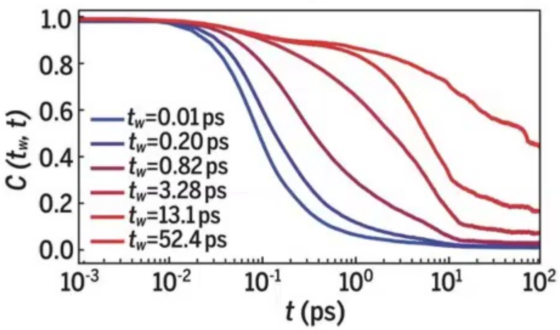

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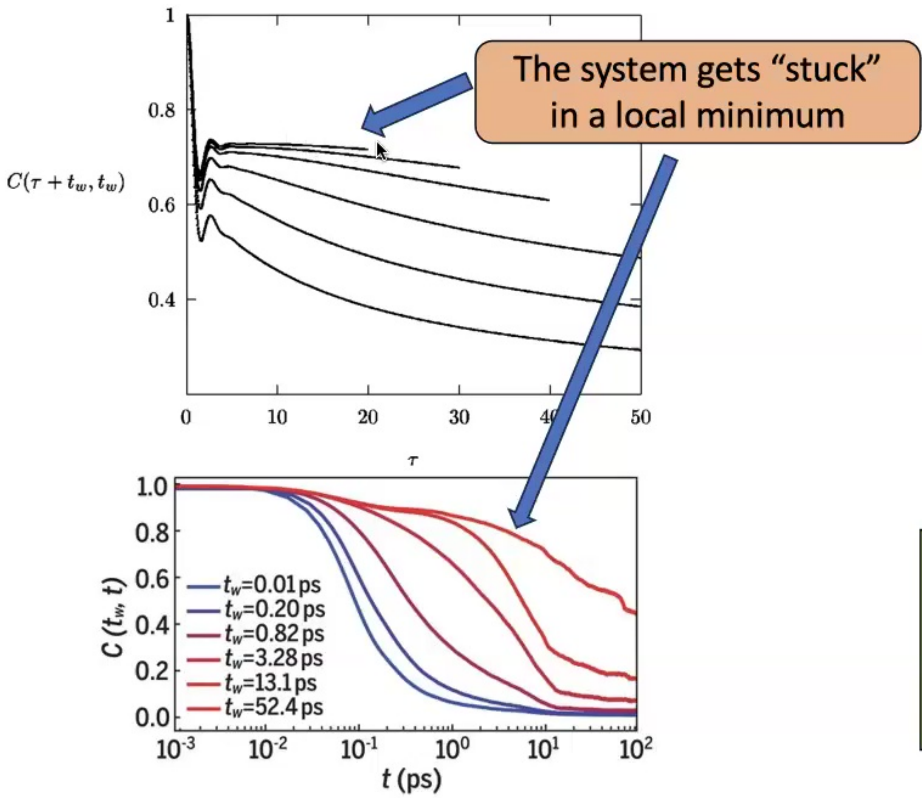

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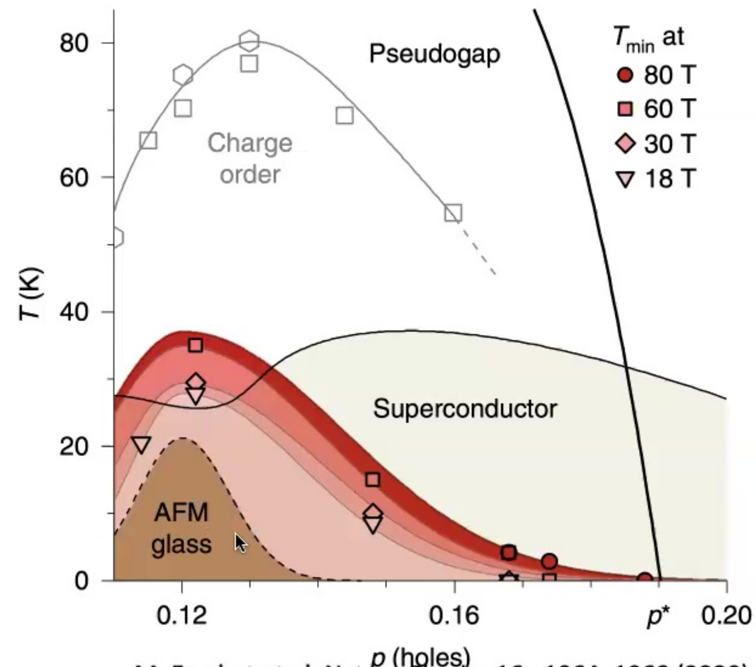
Why are spin glasses interesting?

- (Potential) region in the phase diagram of cuprates



M. Frachet et al, Nature Physics 16, 1064–1068 (2020)

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- Mechanism for spin glasses in finite dimension is poorly understood
- It is an NP-hard problem – in a class of combinatorial optimization problems

Why are spin glasses interesting?

- Tightly connected to the optimization problem

Interaction: $-J_{ij} s_i s_j$ couplings are random

Find the ground state?

2^N configurations of N spins → NP-hard problem (very hard to solve on a classical computer)

Marc Mezard "Spin glasses and optimization in complex systems" (2022)

Is there a quantum algorithm that solves it faster?

Why are spin glasses interesting?

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Interaction: $-J_{ij}s_is_j$ couplings are random

Find the ground state?

With current impressive capabilities of quantum simulators such as Rydberg platforms, the solution of this problem becomes feasible!

S. Ebadi, A. Keesling, M. Cain et al, Science 376, 1209 (2022)

Approaches to study spin glasses

- Deep learning [e.g. Andriushchenko et al, Entropy (Basel). 2022 May; 24(5): 697]
- Monte Carlo method [the most used one, although inefficient since one needs to generate many states to achieve equilibrium]
- Theory: large-N expansion, Schwinger-Keldysh (out-of-equilibrium), replica symmetry breaking (equilibrium), droplet model etc

Replica trick

Replica symmetry breaking (RSB) and the order parameter

Used to evaluate free energy of a system with quenched and annealed disorder:

$$\overline{\log\, Z} = \lim_{n\to 0} \frac{1}{n} \log\, \overline{Z^n}$$

n -> promote to be an integer number, then

$$\overline{Z^n} = \int D\sigma_1 \ldots D\sigma_n \; \overline{e^{-\beta H(\sigma_1,J)\cdots-\beta H(\sigma_n,J)}}$$

We replicated the system n times → take n->0 limit

Evaluating this free energy is much simpler than the original expression. However, replica trick does not always work… (one, in principle, needs to check whether the obtained solution is stable)

Review: arXiv: 0505032 [cond-mat]

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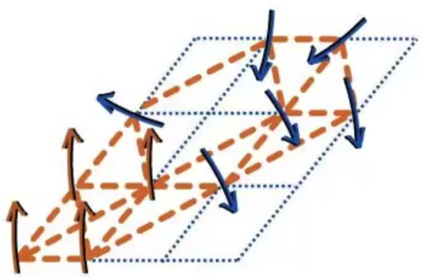
Order parameter for disordered magnetic systems – **Edward-Anderson (EA) order parameter**



$$q_{EA} = \frac{1}{N} \sum_{i=1}^{N} \overline{\langle \sigma_i \rangle}^2$$

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Example: p-rotor model

Model of p spins interacting with each other according to

$$H = -\sum_{i_1 > \ldots > i_p = 1}^{N} J_{i_1 \ldots i_p} \sigma_{i_1} \ldots \sigma_{i_p} \qquad p \geq 3 \qquad \text{with constraint} \quad \sum_{i=1}^{N} {\sigma_i}^2 = N$$

Average over disorder (*annealed*):

$$\overline{Z} = \int D\sigma \int \prod_{i<j<k} dJ_{ijk} \, \exp\left[-J_{ijk}^2 \frac{N^p}{p!} + J_{ijk} \beta \sigma_i \sigma_j \sigma_k\right] =$$

$$\int D\sigma \, \exp\left[\frac{\beta^2}{4N^{p-1}} \left(\sum_i \sigma_i^2\right)^p\right] =$$

$$\exp\left[N \frac{\beta^2}{4}\right] \Omega \,,$$

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Introduce: $1 = \int dQ_{ab}\,\delta\left(NQ_{ab} - \sum_i \sigma_i^a\sigma_i^b\right)$

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Obtain: $$\cdot\ \exp\left[\frac{\beta^2 N}{4}\sum_{ab} Q_{ab}^p + N\sum_{ab}\lambda_{ab}Q_{ab} - \sum_i\sum_{ab}\sigma_i^a\lambda_{ab}\sigma_i^b\right] =$$

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Obtained: $$\overline{Z^n} = \int DQ_{ab}\, D\lambda_{ab}\, \exp\left[-N\, S(Q,\lambda)\right]$$

where $$S(Q,\lambda) = -\frac{\beta^2}{4}\sum_{ab} Q_{ab}^p - \sum_{ab}\lambda_{ab}Q_{ab} + \frac{1}{2}\log\det(2\lambda_{ab})$$

Use saddle point (steepest descent) method to solve!

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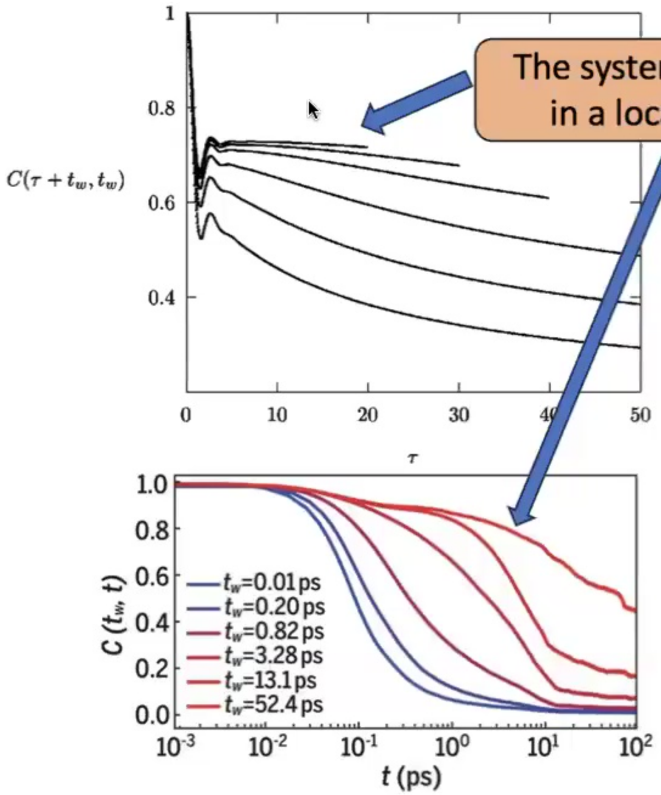

The system gets "stuck" in a local minimum

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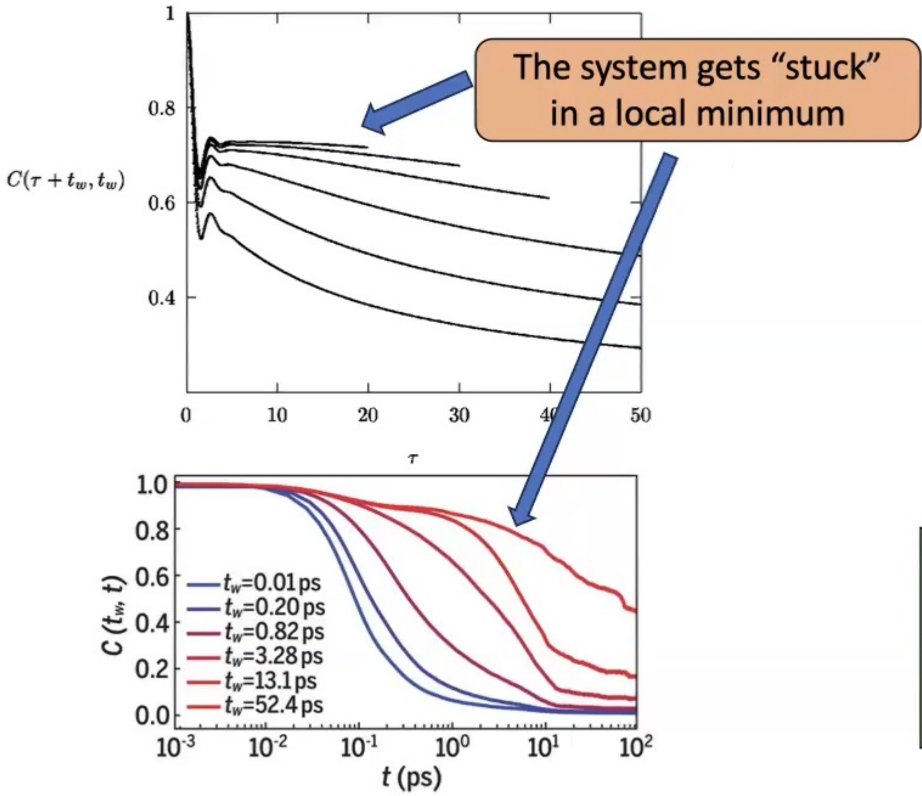

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Quantum Ising spin glass in a field

The model: $$H = \sum_{i<j} J_{ij} Z_i Z_j - g \sum_i X_i - h \sum_i Z_i$$

$$P(J_{ij}) \propto \exp\left[-\frac{N}{2}\frac{J_{ij}^2}{J^2}\right]$$

Laser-induced Rabi flipping

Laser detuning

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Not that many results with finite field

Quantum Ising spin glass in a field

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Bilocal field: $$Q_{ab}(\tau_1, \tau_2) = \frac{1}{N} \sum_i Z_{ia}(\tau_1) Z_{ib}(\tau_2)$$

Equilibrium: $$Q_{ab}(\tau_1, \tau_2) = \frac{1}{\beta} \sum_{\nu_n} Q_{ab}(i\nu_n) e^{i\nu_n(\tau_1 - \tau_2)}$$

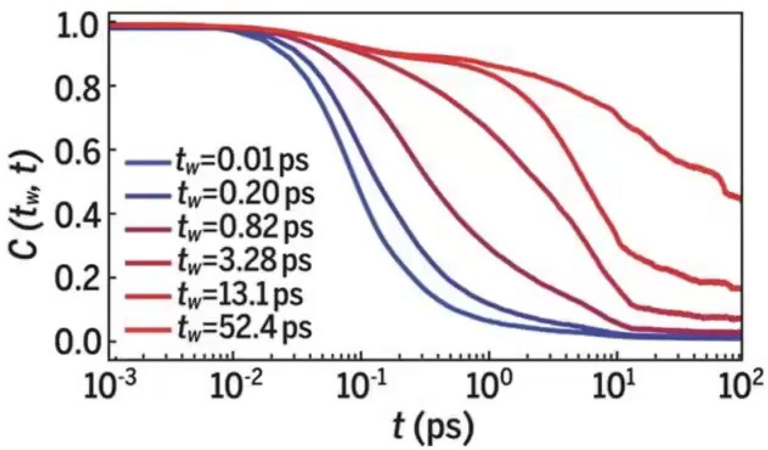
Assume ansatz: $$Q_{ab}(i\nu_n) = \begin{cases} Q_r(i\nu_n) + \beta q_{EA} \delta_{\nu_n,0}, & a = b \\ \beta q_{ab} \delta_{\nu_n,0}, & a \neq b \end{cases} .$$

arXiv: 2309.03255

Quantum Ising spin glass in a field

$h \neq 0$

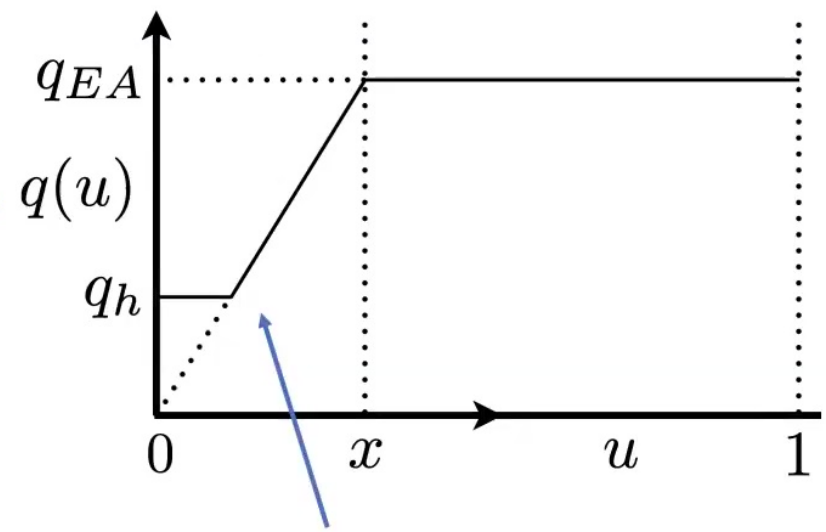
$$Q_{ab}(i\nu_n) = \begin{cases} Q_r(i\nu_n) + \beta q_{EA}\delta_{\nu_n,0}, \\ \beta q_{ab}\delta_{\nu_n,0}, \end{cases}$$



Long time

???

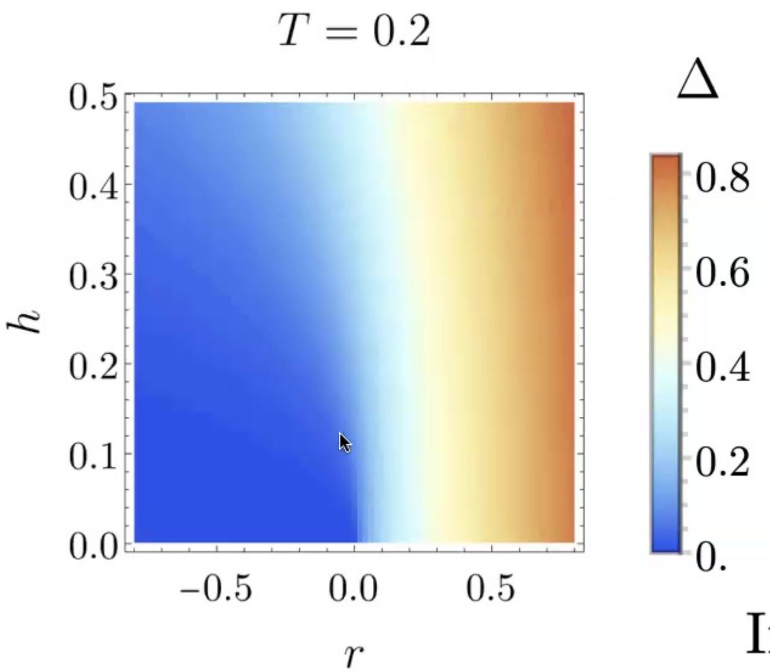
Structure of the order parameter



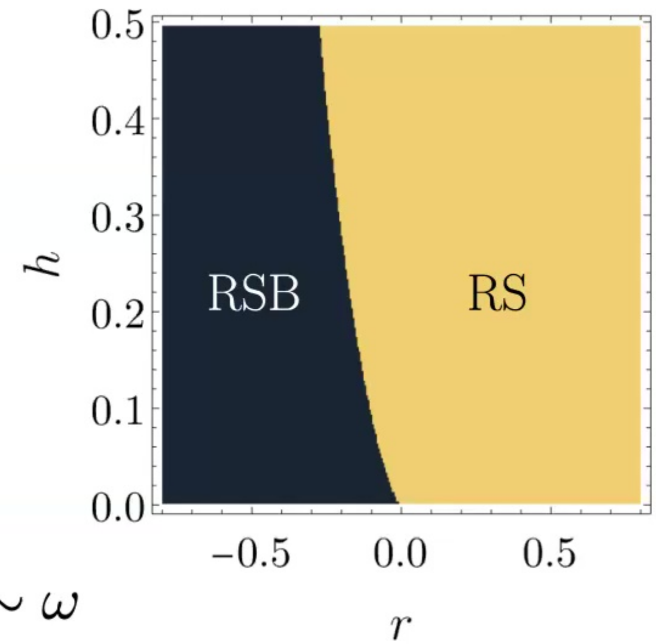
Due to replica symmetry breaking

Quantum Ising spin glass in a field

Phase diagram



$$\mathrm{Im}\, Q_r(\omega) \sim \omega$$



Landau theory. r-parameter that tunes across the SG phase transition

N. Read, S. Sachdev, and J. Ye, Phys. Rev. B 52, 384 (1995)

Quantum Ising spin glass in a field

$h \neq 0$

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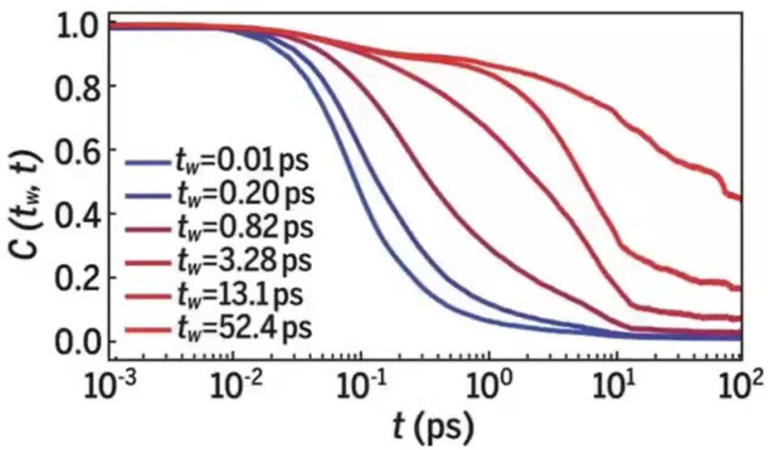

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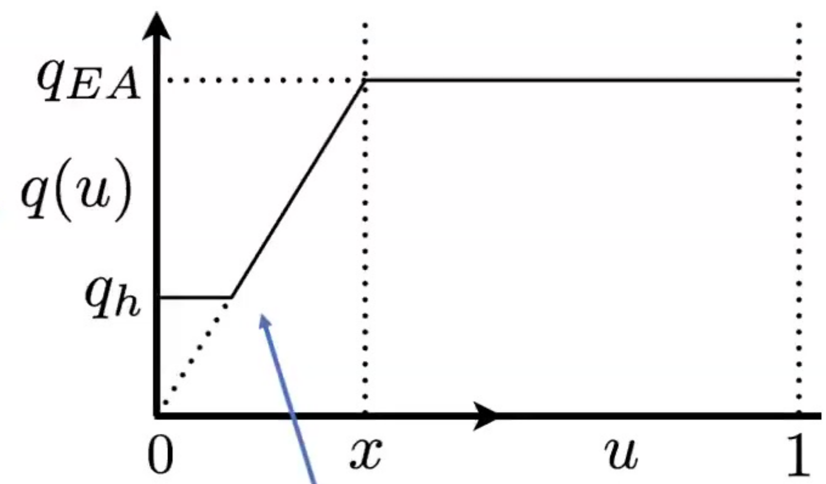

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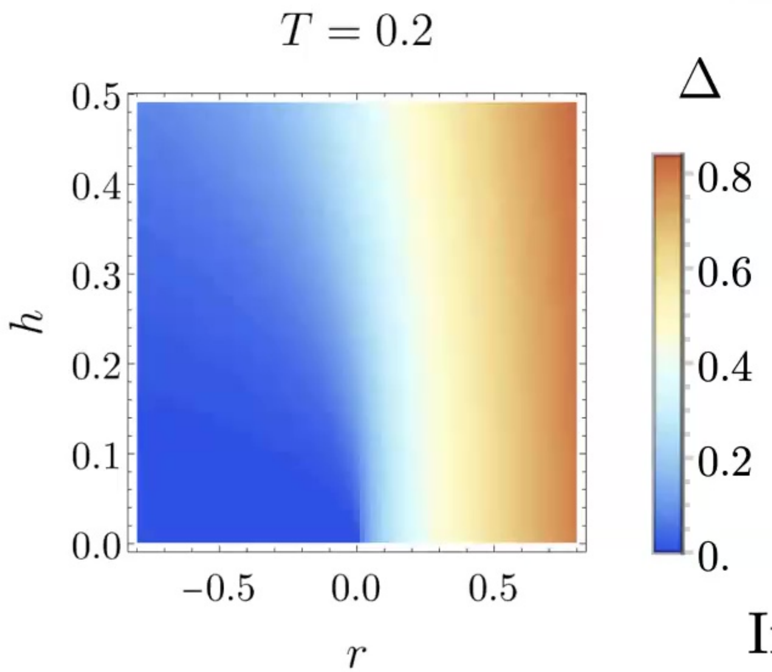

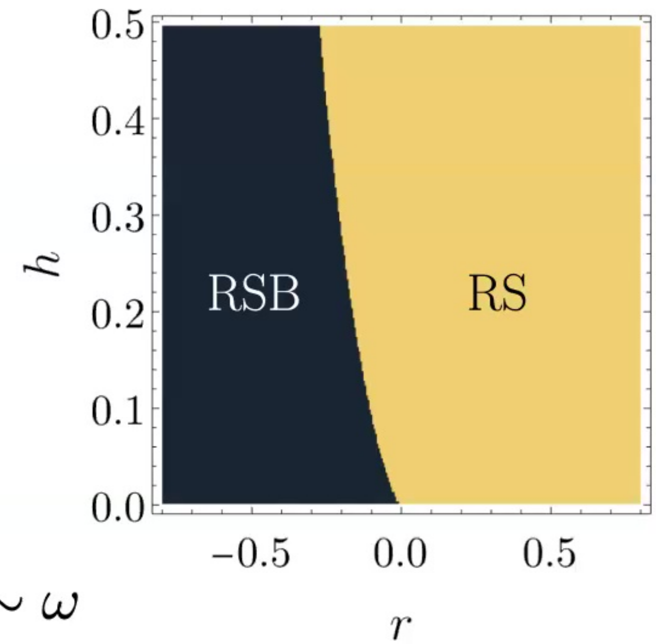

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p=3 – rotor model

Quantum p>2-rotor model in a field

The model:

$$Z[J_{i_1\ldots i_p}] = \int \mathcal{D}\sigma_i(\tau)\ \exp\left[-\int_0^\beta d\tau \left(\frac{1}{2g}\dot{\sigma}_i(\tau)\dot{\sigma}_i(\tau) + \sum_{i_1<\ldots<i_p} J_{i_1\ldots i_p}\sigma_{i_1}(\tau)\ldots\sigma_{i_p}(\tau) - h\sum_i \sigma_i(\tau)\right)\right]$$

Spherical constraint $$\sum_{i=1}^{N} \sigma_i(\tau)\,\sigma_i(\tau) = N$$

Distribution of couplings $$P(J_{i_1\ldots i_p}) \propto \exp\left[-\frac{N^{p-1}}{p!}\frac{J^2_{i_1\ldots i_p}}{J^2}\right]$$

Use replica trick

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Spherical constraint $\sum_{i=1}^{N} \sigma_i(\tau)\,\sigma_i(\tau) = N$

Distribution of couplings $P(J_{i_1\ldots i_p}) \propto \exp\left[-\frac{N^{p-1}}{p!}\frac{J^2_{i_1\ldots i_p}}{J^2}\right]$

Self-consistent equations

$$1 = \frac{1}{\beta}\sum_{\omega_n} Q_{aa}(\omega_n),$$

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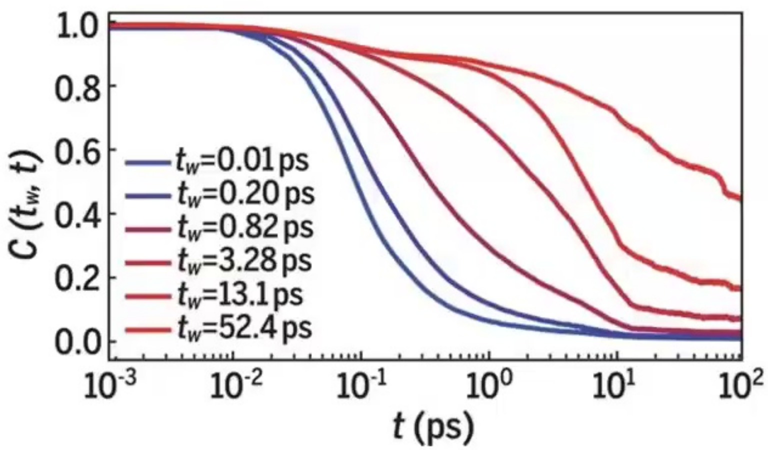
$$gQ^{-1}_{ab}(\omega_n) = \delta_{ab}(\omega_n^2 + \overline{\lambda}) - \Sigma_{ab}(\omega_n) - \beta h^2 g\delta_{\omega_n,0},$$

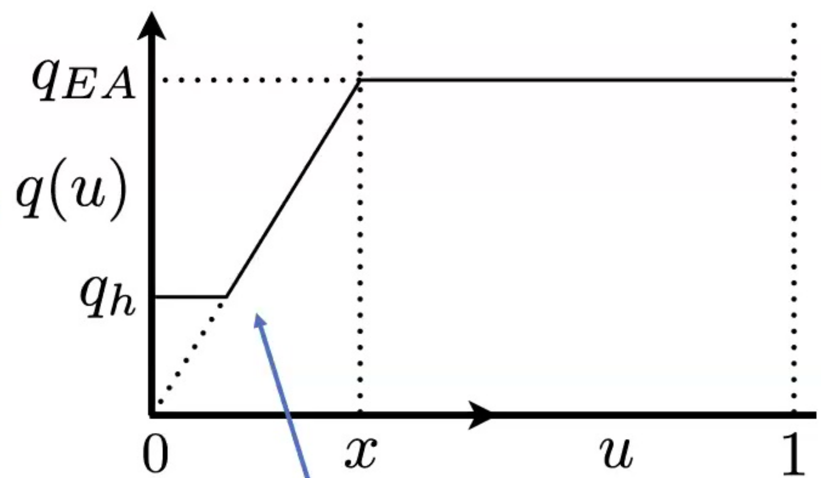
Quantum Ising spin glass in a field

$h \neq 0$

$$Q_{ab}(i\nu_n) = \begin{cases} Q_r(i\nu_n) + \beta q_{EA}\delta_{\nu_n,0}, \\ \beta q_{ab}\delta_{\nu_n,0}, \end{cases}$$



Structure of the order parameter



Due to replica symmetry breaking

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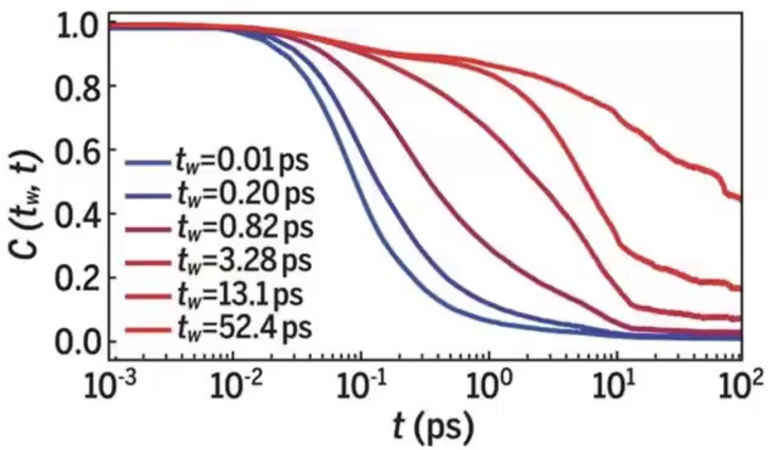
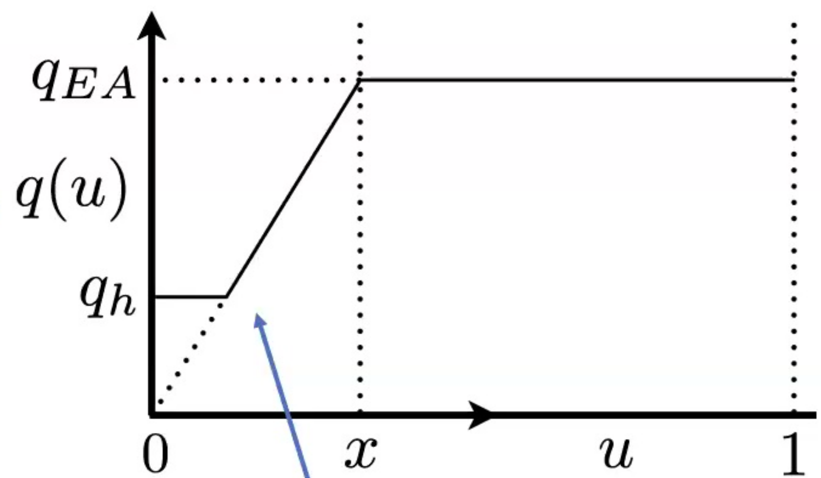


Long time

???

Structure of the order parameter



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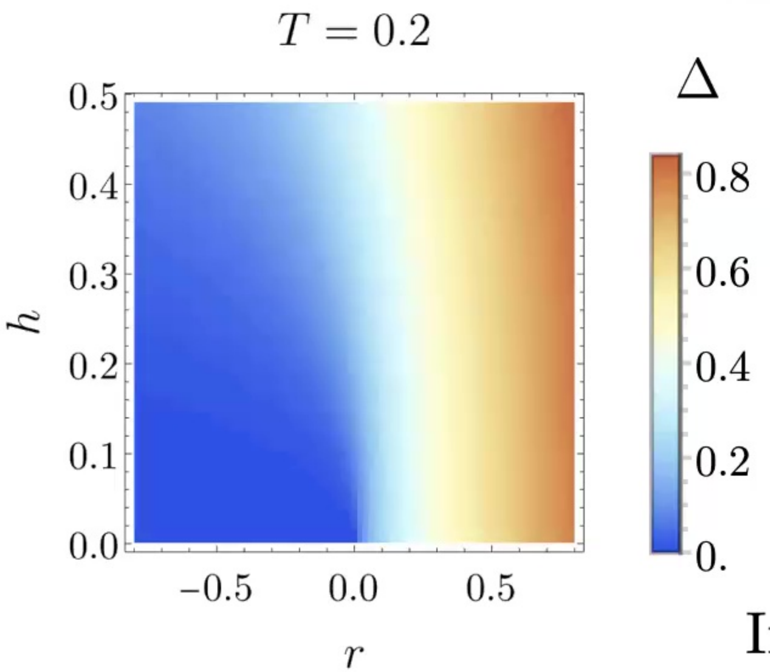
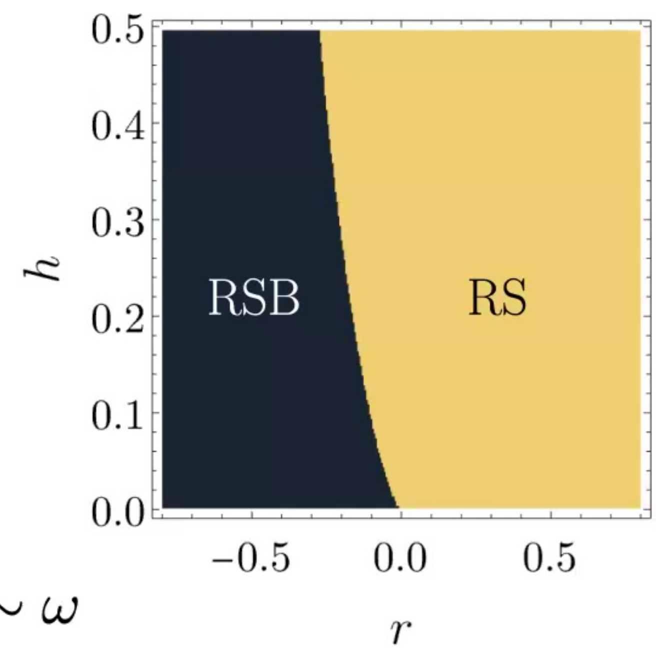
Quantum Ising spin glass in a field



Phase diagram



$$\mathrm{Im}\, Q_r(\omega) \sim \omega$$

Landau theory. r-parameter that tunes across the SG phase transition

N. Read, S. Sachdev, and J. Ye, Phys. Rev. B 52, 384 (1995)

Quantum p>2-rotor model in a field

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Replica-symmetric solution

Take replica symmetric ansatz:

$$Q_{ab}(\omega_n) = \beta q \delta_{\omega_n,0} + Q_r(\omega_n)\delta_{ab}$$

Equations in real time:

$$\rho(\omega) = \frac{g}{\pi}\frac{\Sigma_r''(\omega)}{(\Sigma_r''(\omega))^2 + (-\omega^2 + \overline{\lambda} - \Sigma_r'(\omega))^2},$$

$$\Sigma_r''(\omega) = 3\pi g J^2 \left(q\rho(\omega) + \frac{1}{2}\int_0^\omega d\omega_1 \rho(\omega_1)\rho(\omega - \omega_1) \right),$$

$$\Sigma_r'(\omega) = 2\int_0^{+\infty} \frac{d\nu}{\pi} \frac{\nu\Sigma_r''(\nu) - \omega\Sigma_r''(\omega)}{\nu^2 - \omega^2},$$

$$\varrho = \frac{3}{2} g J^2 q^2,$$

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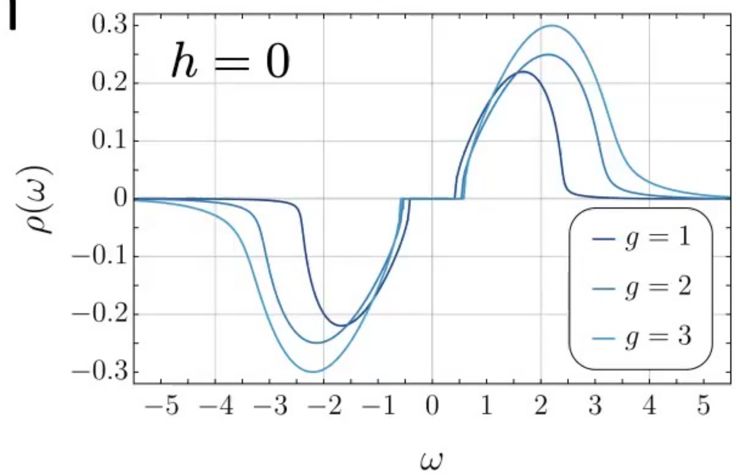
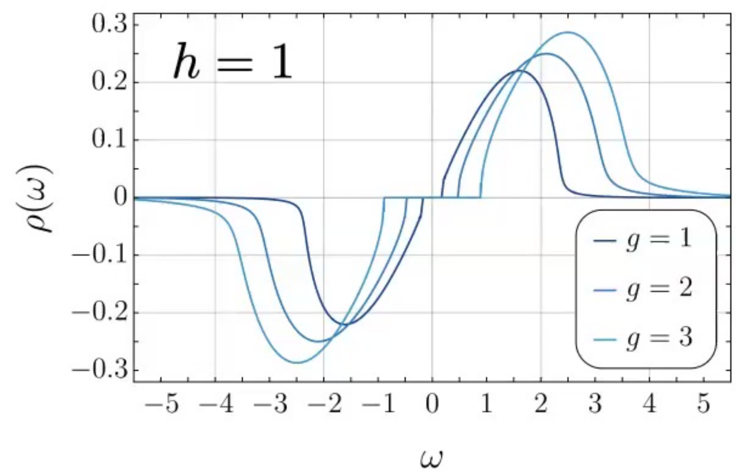
$$1 = q + \int_0^{+\infty} d\omega \rho(\omega),$$





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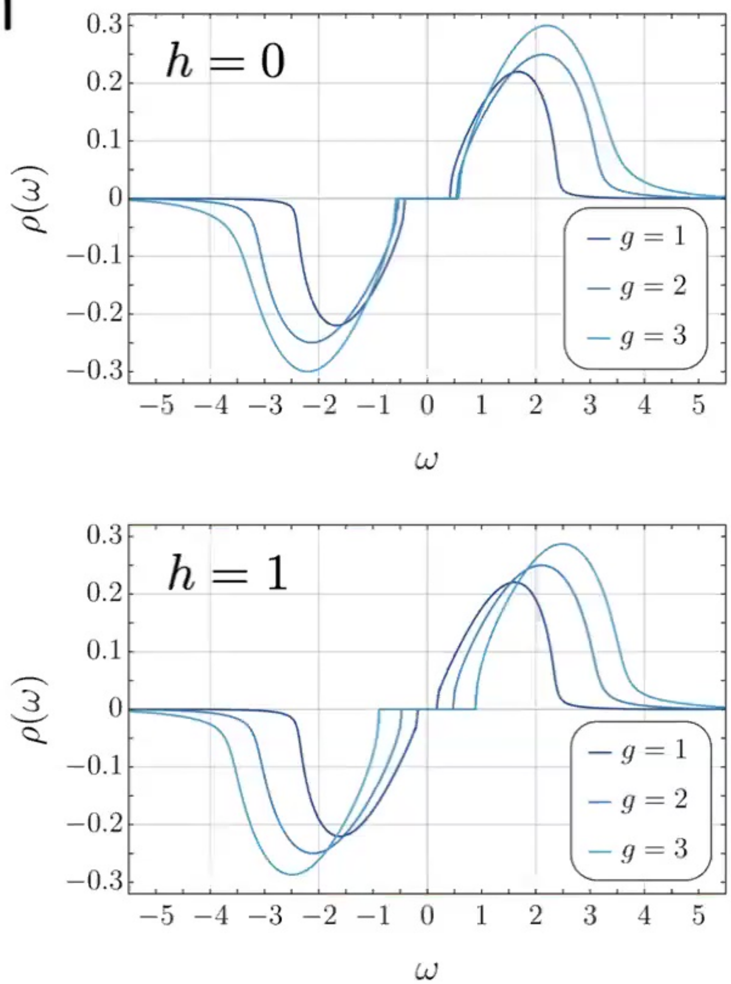
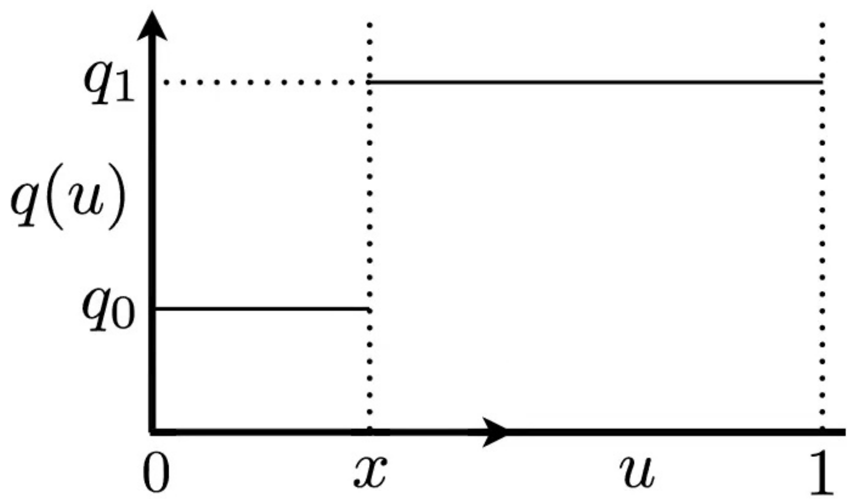
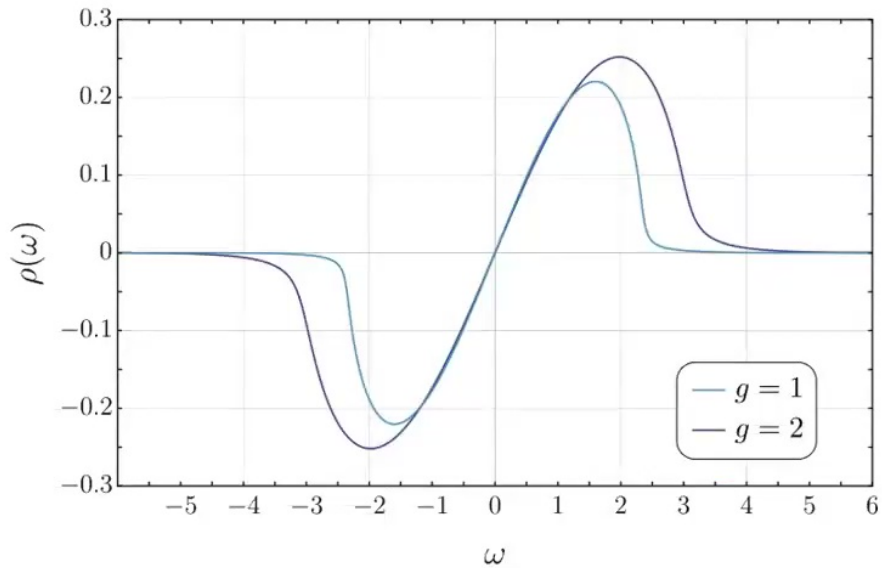


Replica symmetry breaking solution

Structure of the order parameter



Spectral function



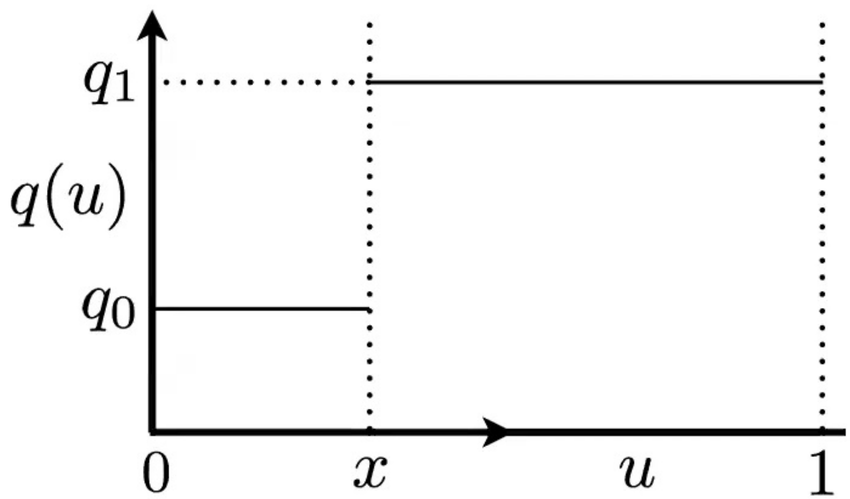
$h \neq 0$

$q_0 \sim h^2$

gapless in the RSB phase!

Replica symmetry breaking solution

Structure of the order parameter



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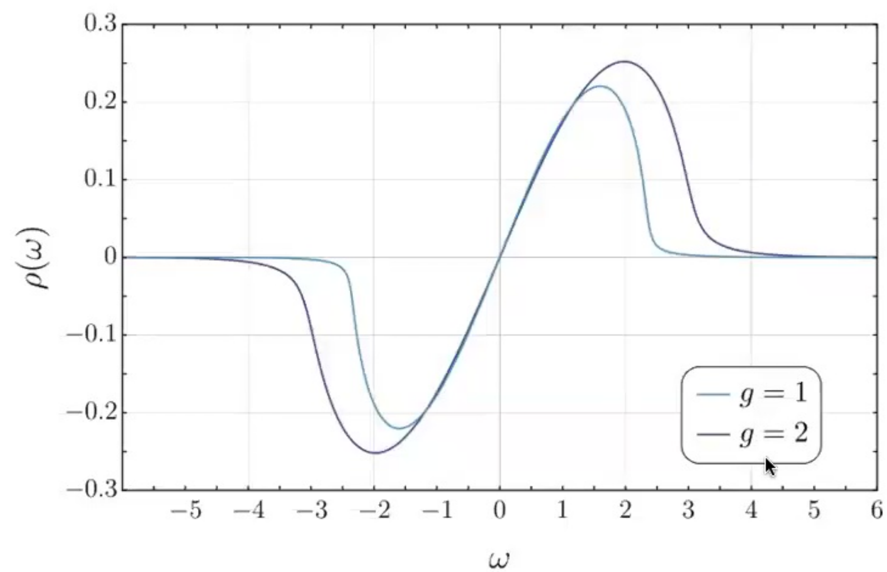
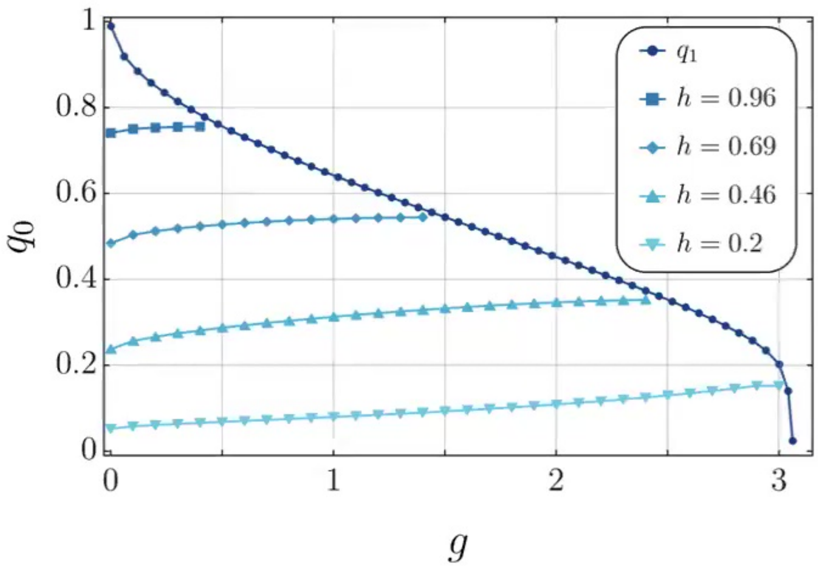
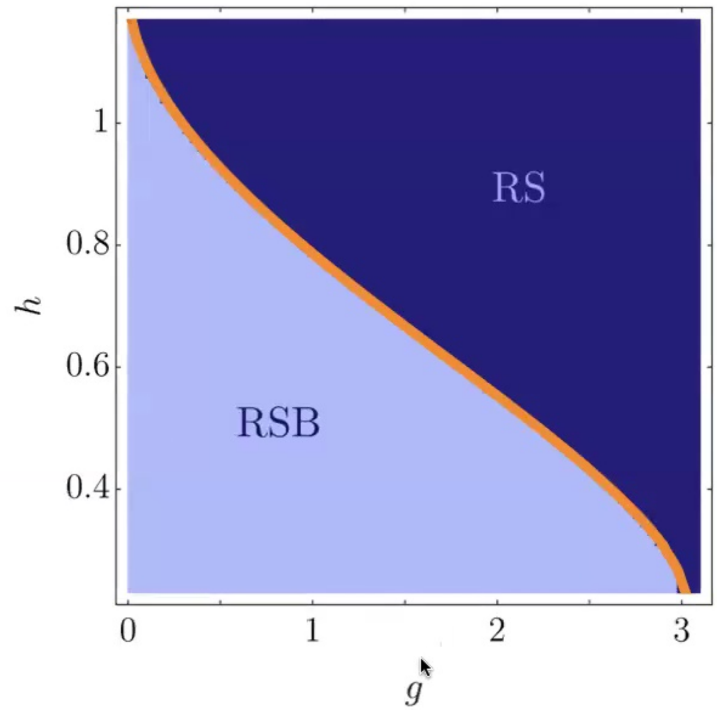


gapless in the RSB phase!

Phase diagram

RSB solution occupies a large region of the phase diagram





Conclusion

- The quantum Ising spin glass in an external longitudinal field has a spin glass phase
- Inside the spin glass phase, the spectral functions are gapless
- p-rotor model in a field exhibit RSB phase and occupies a large region of the phase diagram

Future directions

1. Study out-of-equilibrium dynamics of spin glasses (quantum and classical) to find relation to order parameter structure

2. Find parameter regimes and study specific models for experimental setups

3. Other glassy models: explore dynamics and relevance to quantum simulators

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Thank you!