

Title: Mathematics, Physics, and Machine Learning

Speakers: Sergei Gukov

Series: Quantum Fields and Strings

Date: November 27, 2023 - 11:00 AM

URL: <https://pirsa.org/23110087>

Abstract: Abstract TBA

Zoom link <https://pitp.zoom.us/j/93671537911?pwd=UjFJK3B5Y1ZQbU9ucm9yK3dNVmNSZz09>

Data Science

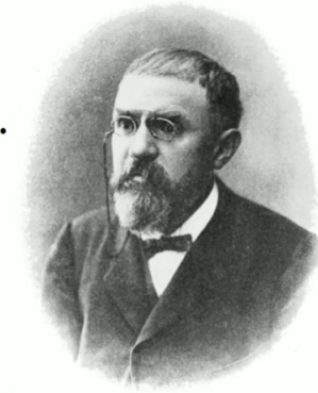


Physics

Math



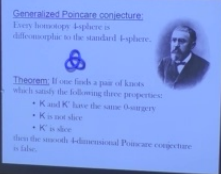
Generalized Poincare conjecture:
Every homotopy 4-sphere is
diffeomorphic to the standard 4-sphere.



Theorem: If one finds a pair of knots
which satisfy the following three properties:

- K and K' have the same 0-surgery
- K is not slice
- K' is slice

then the smooth 4-dimensional Poincare conjecture
is false.

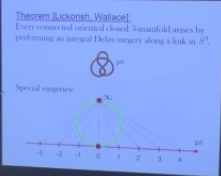
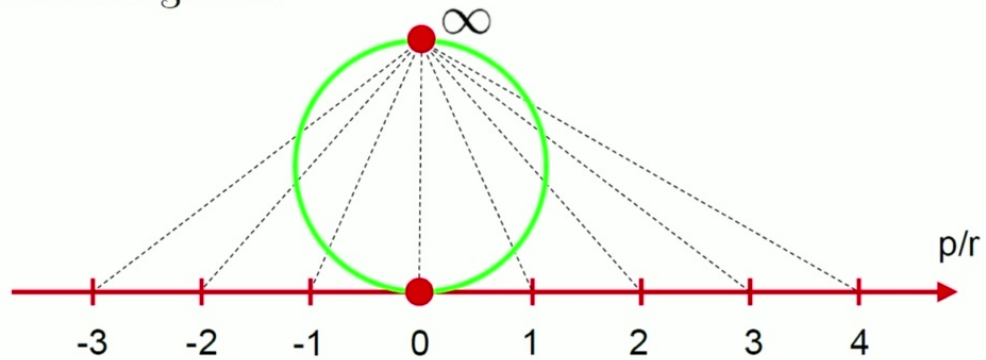


Theorem [Lickorish, Wallace]:

Every connected oriented closed 3-manifold arises by performing an integral Dehn surgery along a link in S^3 .



Special surgeries:



FIBERED KNOTS AND POTENTIAL COUNTEREXAMPLES TO THE PROPERTY 2R AND SLICE-RIBBON CONJECTURES

ROBERT E. GOMPF, MARTIN SCHARLEMANN, AND ABIGAIL THOMPSON

48 crossings

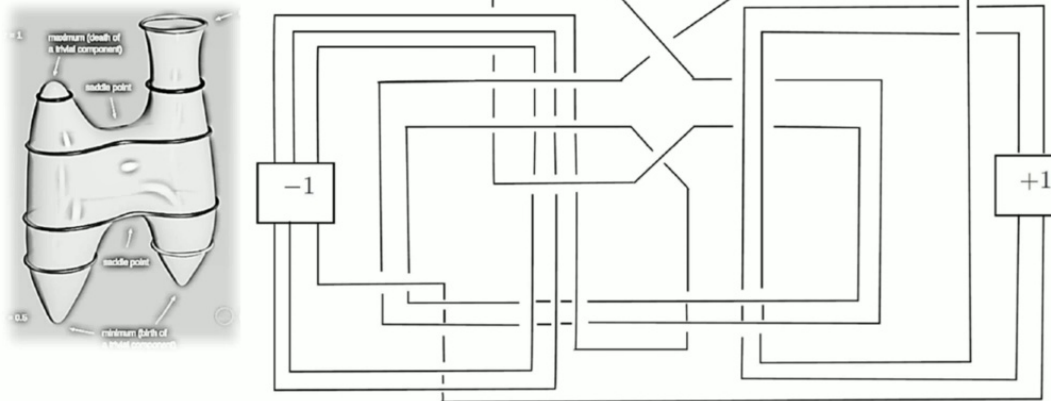


FIGURE 2. A slice knot that might not be ribbon

computation of
"quantum" invariants



1,388,705 knots



potential counterexamples
to SPC4 (ruled out)



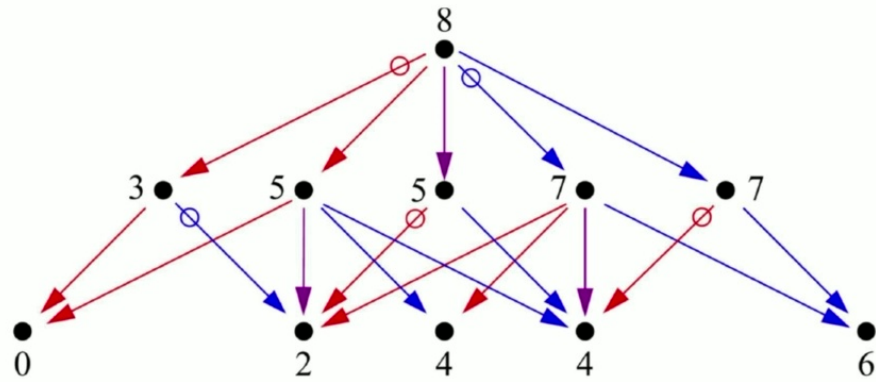
165 knots



potential counterexample
to slice-ribbon conj.

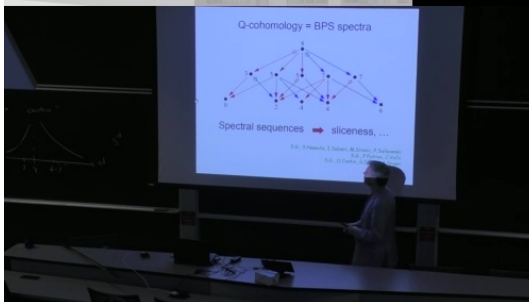


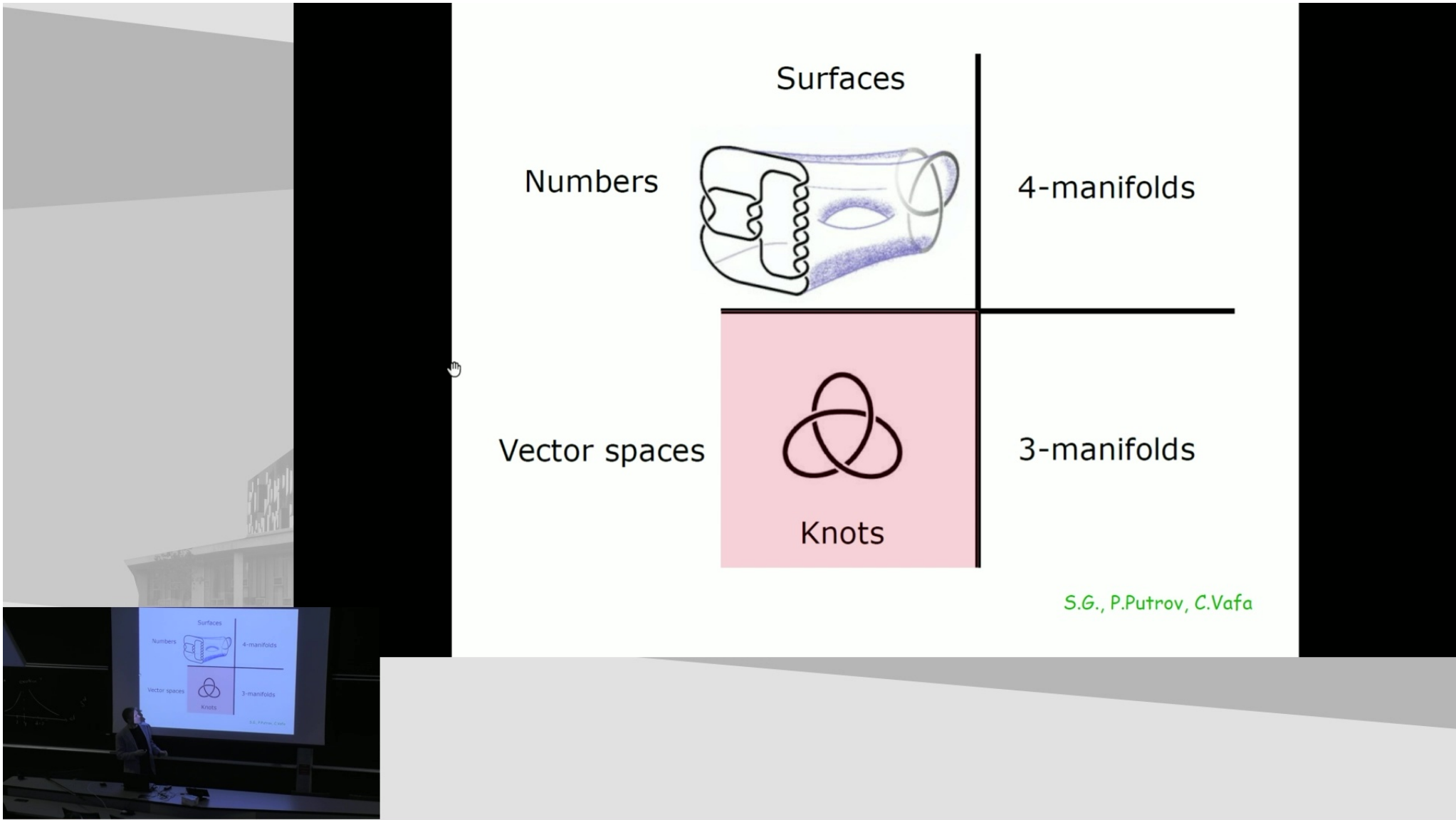
Q-cohomology = BPS spectra



Spectral sequences \rightarrow sliceness, ...

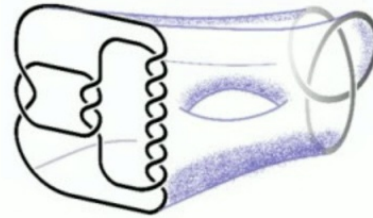
S.G., S.Nawata, I.Saberi, M.Stosic, P.Sulkowski
S.G., P.Putrov, C.Vafa
S.G., O.Costin, G.Dunne, A.Gruen
:





Surfaces

Numbers



4-manifolds

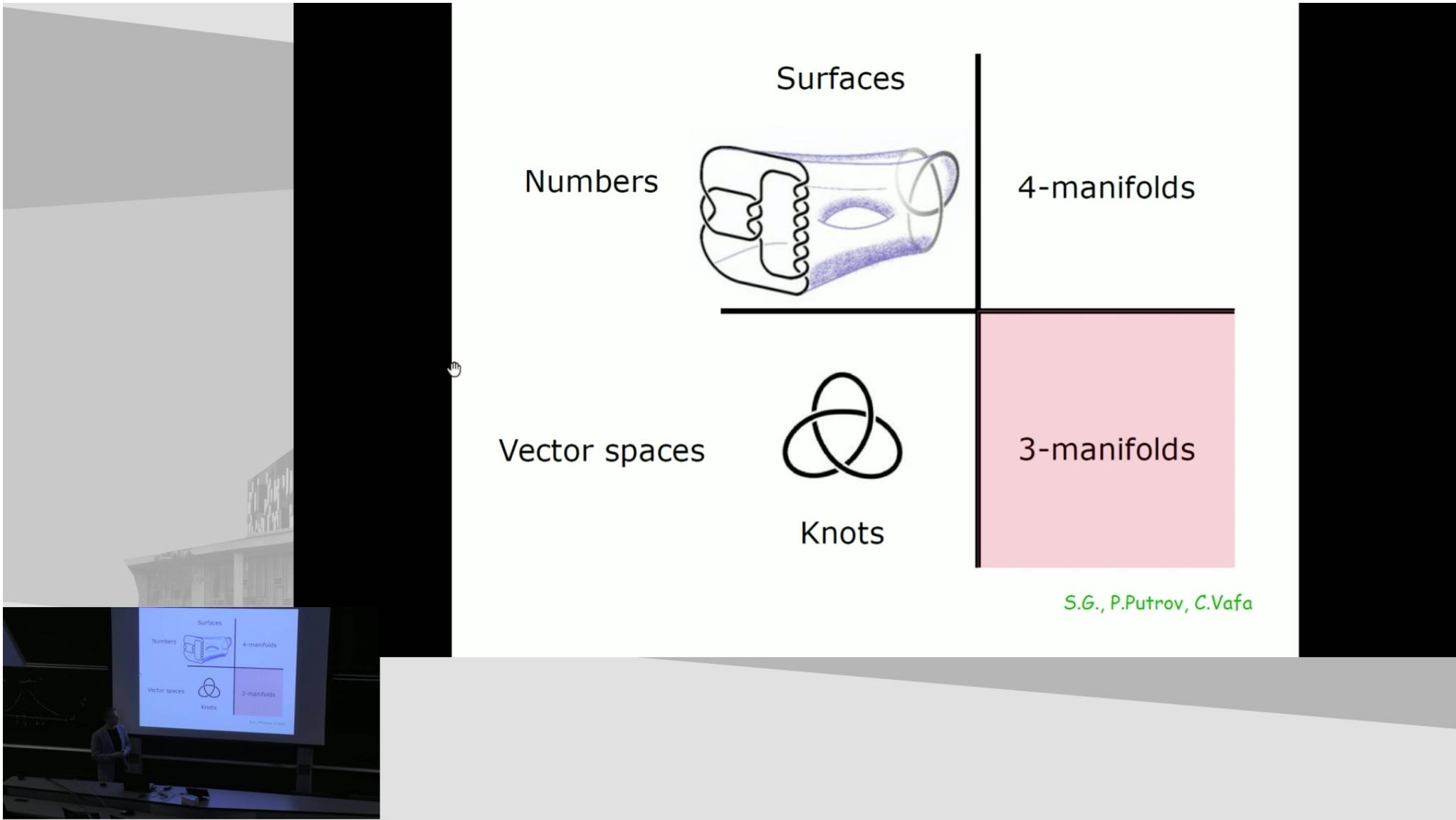
Vector spaces



Knots

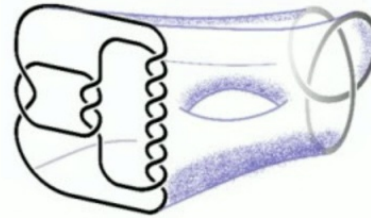
3-manifolds

S.G., P.Putrov, C.Vafa



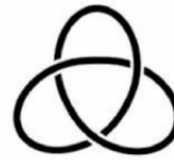
Surfaces

Numbers



4-manifolds

Vector spaces



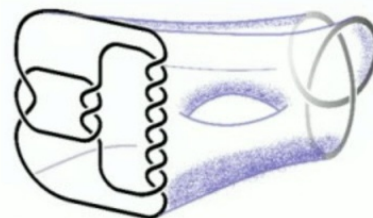
3-manifolds

Knots

S.G., P.Putrov, C.Vafa

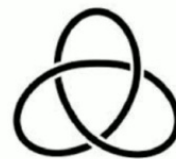
Surfaces

Numbers



4-manifolds

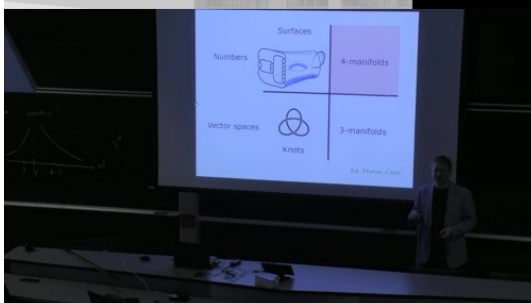
Vector spaces



3-manifolds

Knots

S.G., P.Putrov, C.Vafa



Definition: (6-n)-dimensional theory

$$T[M_n, \dots] := \text{6d (0,2) theory on } M_n$$

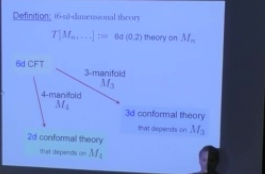
6d CFT

3-manifold
 M_3

4-manifold
 M_4

3d conformal theory
that depends on M_3

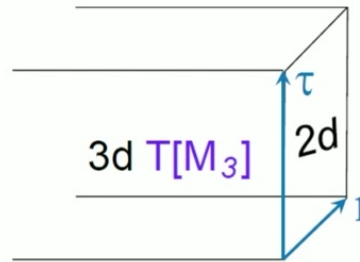
2d conformal theory
that depends on M_4



Even though in most cases we don't know 3d $\mathcal{N}=2$ theory and can not determine its BPS spectrum, we can explicitly compute the graded trace

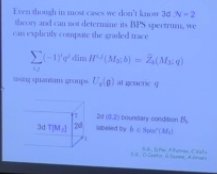
$$\sum_{i,j} (-1)^i q^j \dim H^{i,j}(M_3; \mathfrak{b}) = \widehat{Z}_{\mathfrak{b}}(M_3; q)$$

using quantum groups $U_q(\mathfrak{g})$ at generic q



2d (0,2) boundary condition $\mathcal{B}_{\mathfrak{b}}$
labeled by $\mathfrak{b} \in \text{Spin}^c(M_3)$

S.G., D.Pei, P.Putrov, C.Vafa
S.G., O.Costin, G.Dunne, A.Gruen



Question: $T[T^2] \stackrel{?}{=} 4d \mathcal{N}=4$ super-Yang-Mills

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Question: $T[T^2] \stackrel{?}{=} 4d \mathcal{N}=4$ super-Yang-Mills

Answer: No

6d: 2-form symmetry $C \longrightarrow$ on T^2 :

$$G = U(1) : C = U(1)$$

$$G = SU(2) : C = \mathbb{Z}_2$$

2-form

1-form 1-form

0-form

D.Gaiotto, A.Kapustin, N.Seiberg, B.Willet

Question: $T[T^2] \stackrel{?}{=} 4d \mathcal{N}=4$ super-Yang-Mills

Answer: No

6d: 2-form symmetry $C \longrightarrow$ on T^2 :

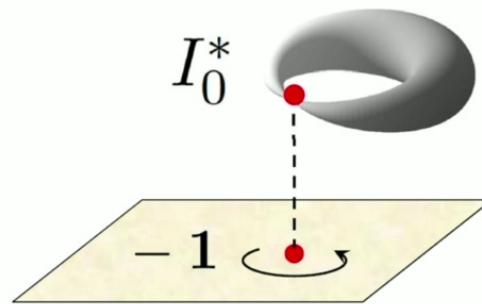
$$G = U(1) : C = U(1)$$

$$G = SU(2) : C = \mathbb{Z}_2$$

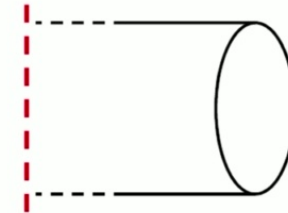
D.Gaiotto, A.Kapustin, N.Seiberg, B.Willet

Coulomb branches

4d $\mathcal{N}=4$ $G = SU(2)$



$T[T^2]$

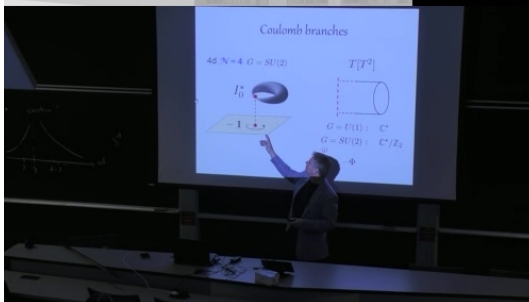


$$G = U(1) : \mathbb{C}^*$$

$$G = SU(2) : \mathbb{C}^*/\mathbb{Z}_2$$

\cup

$$\Phi \rightarrow -\Phi$$



n=3: $T[T^3]$

$$\mathcal{M}_{U(1)} = \mathbb{C}^* \times \mathbb{C}^* \times \mathbb{C}^* \times \mathbb{C}$$

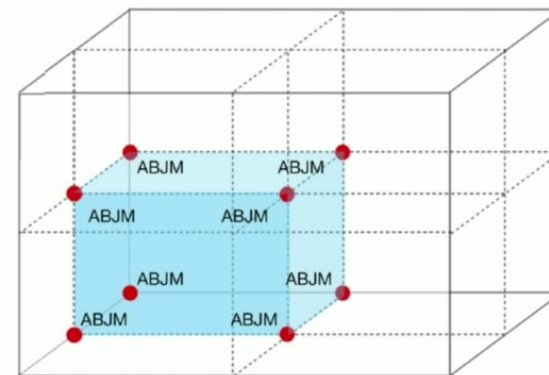
$$\mathcal{M}_{SU(2)} = \frac{\mathbb{C}^* \times \mathbb{C}^* \times \mathbb{C}^* \times \mathbb{C}}{\mathbb{Z}_2}$$

6d 3d

$B \rightarrow A, \varphi_1, \varphi_2, \dots$ dual to $\varphi_3 \in S^1$

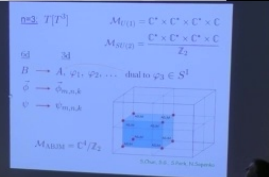
$\vec{\phi} \rightarrow \vec{\phi}_{m,n,k}$

$\psi \rightarrow \psi_{m,n,k}$

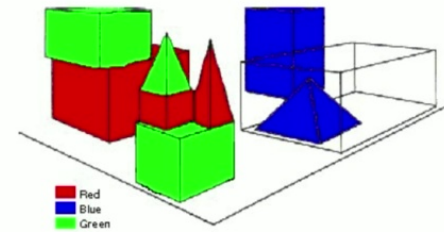


$$\mathcal{M}_{\text{ABJM}} = \mathbb{C}^4 / \mathbb{Z}_2$$

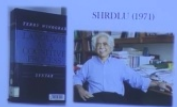
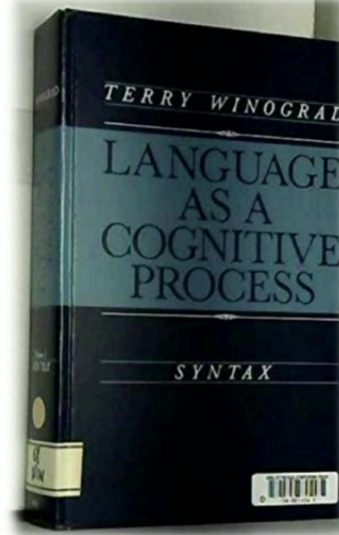
S.Chun, S.G., S.Park, N.Sopenko

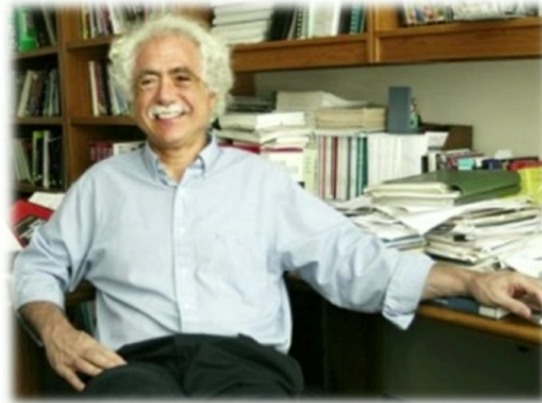


Person: Pick up a big red block.
Computer: OK.
Person: Grasp the pyramid.
Computer: I DON'T UNDERSTAND WHICH PYRAMID YOU MEAN.
Person (changing their mind): Find a block which is taller than the one you are holding and put it into the box.
Computer: BY "IT", I ASSUME YOU MEAN THE BLOCK WHICH IS TALLER THAN THE ONE I AM HOLDING.
Computer: OK.
Person: What does the box contain?
Computer: THE BLUE PYRAMID AND THE BLUE BLOCK.
Person: What is the pyramid supported by?
Computer: THE BOX.
Person: How many blocks are not in the box?
Computer: FOUR OF THEM.
Person: Is at least one of them narrower than the one which i told you to pick up?
Computer: YES, THE RED CUBE.
Person: Is it supported?
Computer: YES, BY THE TABLE.
Person: Can the table pick up blocks?
Computer: NO.
Person: Can a pyramid be supported by a block?
Computer: YES.
Person: Can a pyramid support a pyramid?
Computer: I DON'T KNOW.
Person: Stack up two pyramids.
Computer: I CAN'T.
Person: Will you please stack up both of the red blocks and either a green cube or a pyramid?
Computer: OK.
Person: Which cube is sitting on the table?
Computer: THE LARGE GREEN ONE WHICH SUPPORTS THE RED PYRAMID.



SHRDLU (1971)





1997





Winograd schemas:

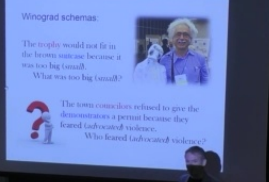
The **trophy** would not fit in the brown **suitcase** because it was too big (*small*).

What was too big (*small*)?



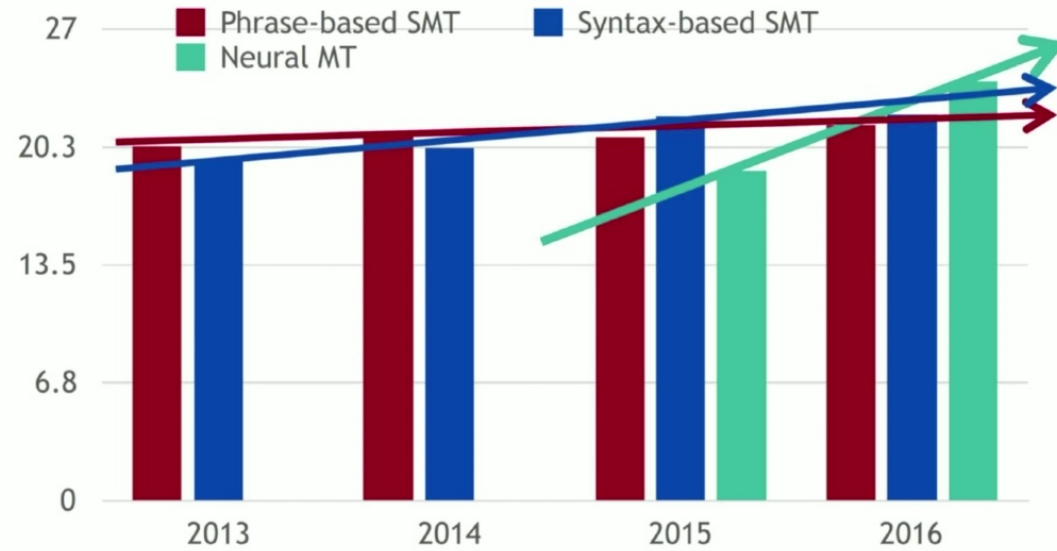
The town **councilors** refused to give the **demonstrators** a permit because they feared (*advocated*) violence.

Who feared (*advocated*) violence?

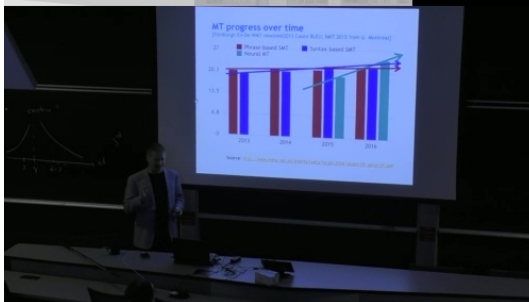


MT progress over time

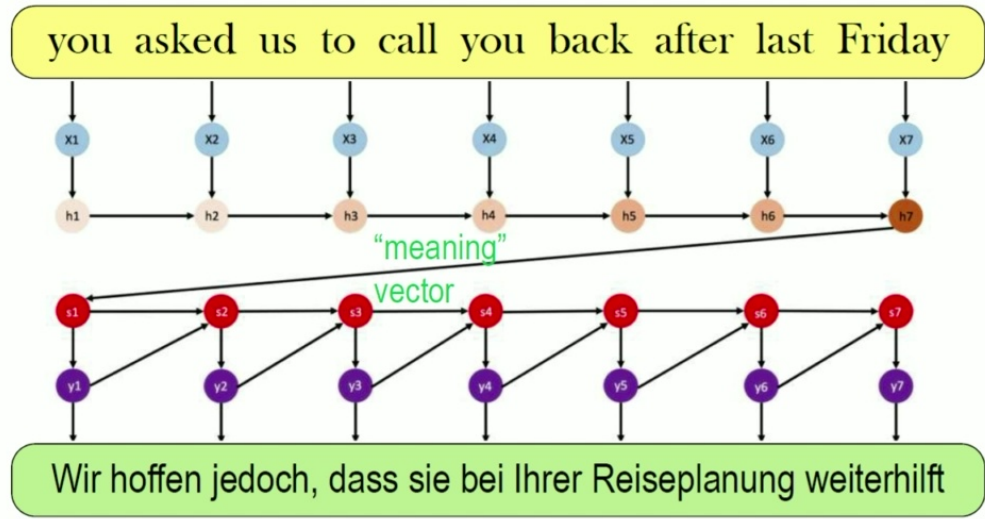
[Edinburgh En-De WMT newstest2013 Cased BLEU; NMT 2015 from U. Montréal]



Source: http://www.meta-net.eu/events/meta-forum-2016/slides/09_sennrich.pdf

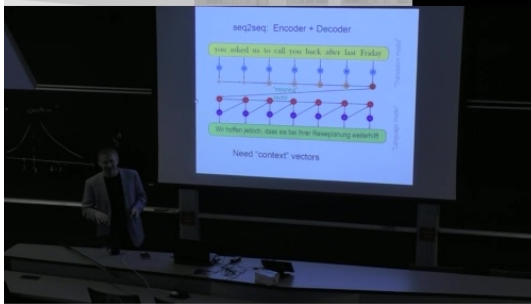


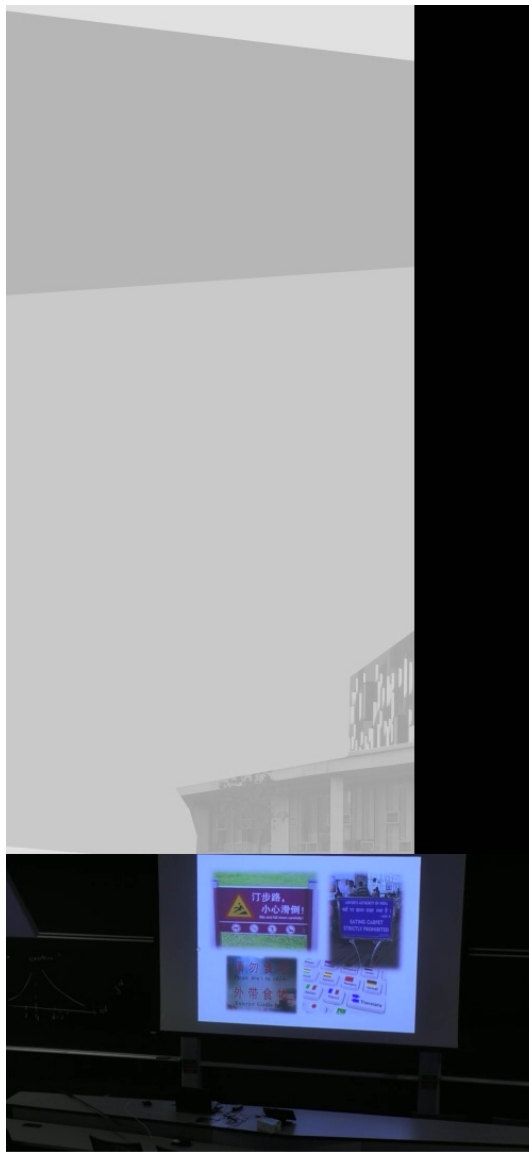
seq2seq: Encoder + Decoder



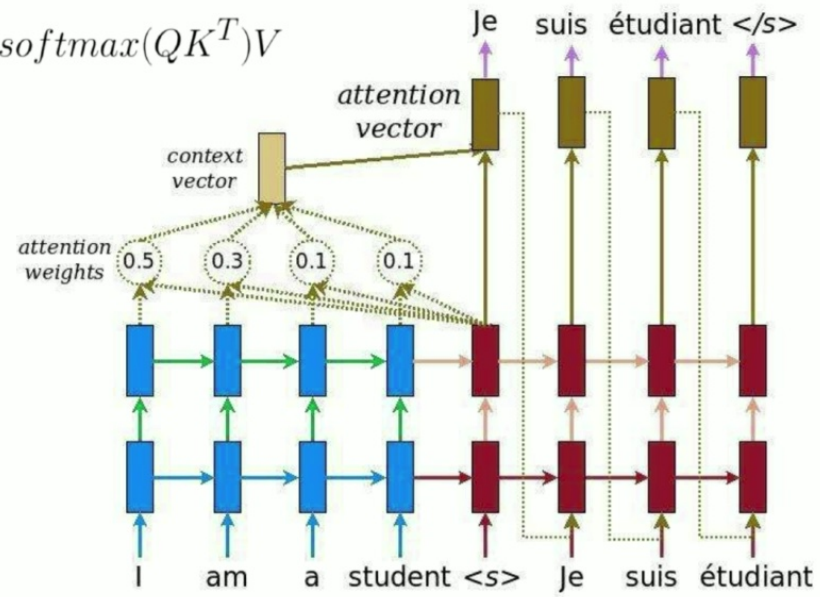
"Translation model"
"Language model"

Need "context" vectors





$$\text{softmax}(QK^T)V$$

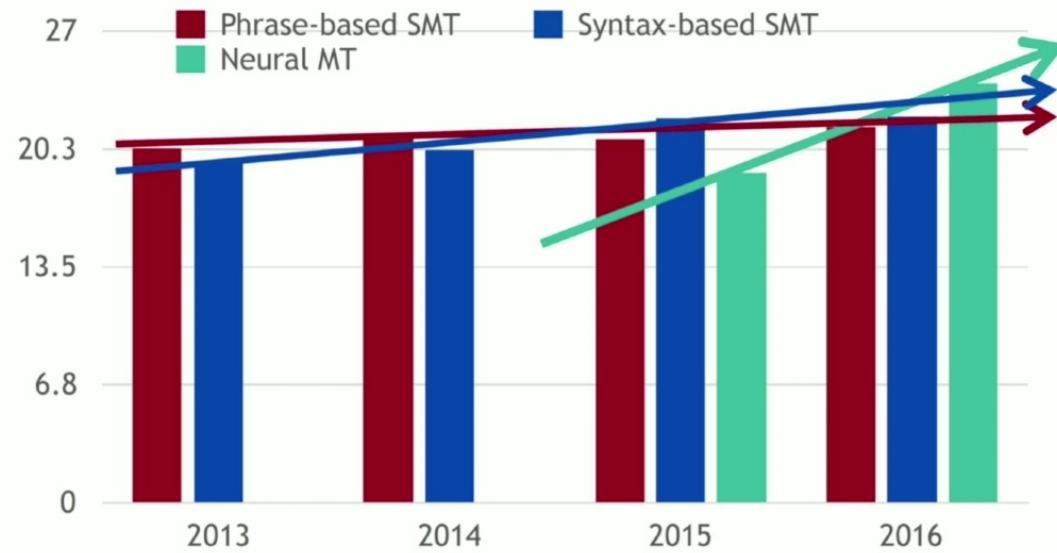


Source: <https://github.com/tensorflow/nmt>

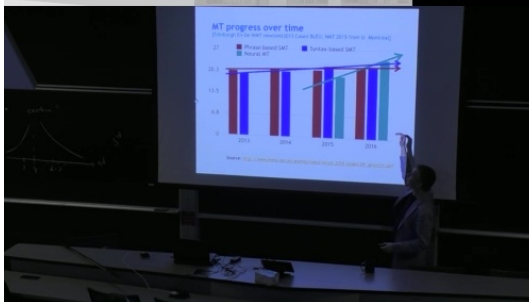


MT progress over time

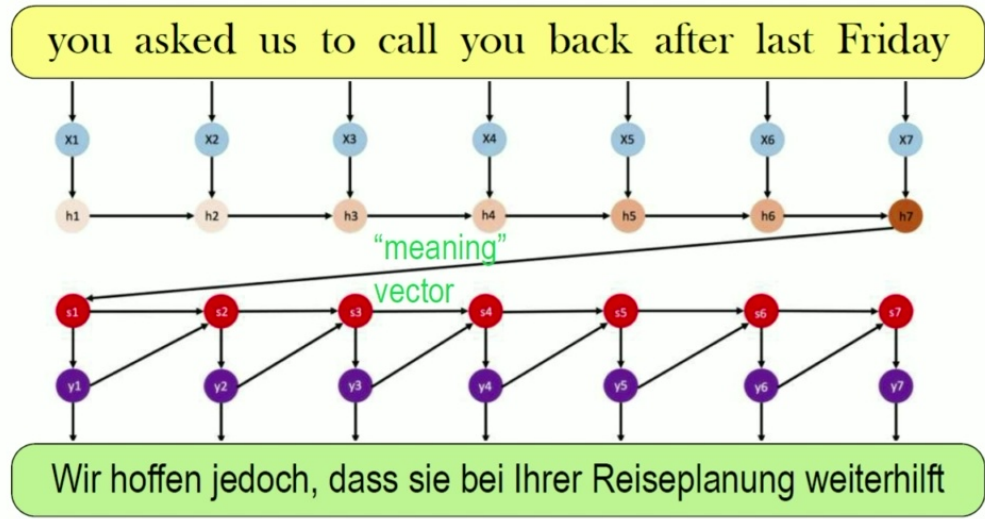
[Edinburgh En-De WMT newstest2013 Cased BLEU; NMT 2015 from U. Montréal]



Source: http://www.meta-net.eu/events/meta-forum-2016/slides/09_sennrich.pdf

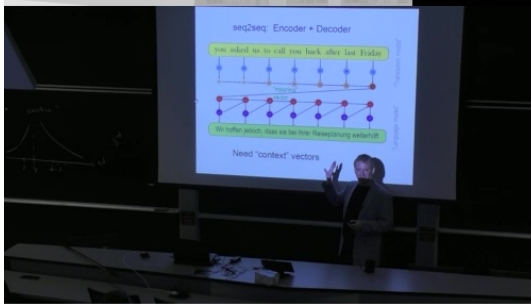


seq2seq: Encoder + Decoder

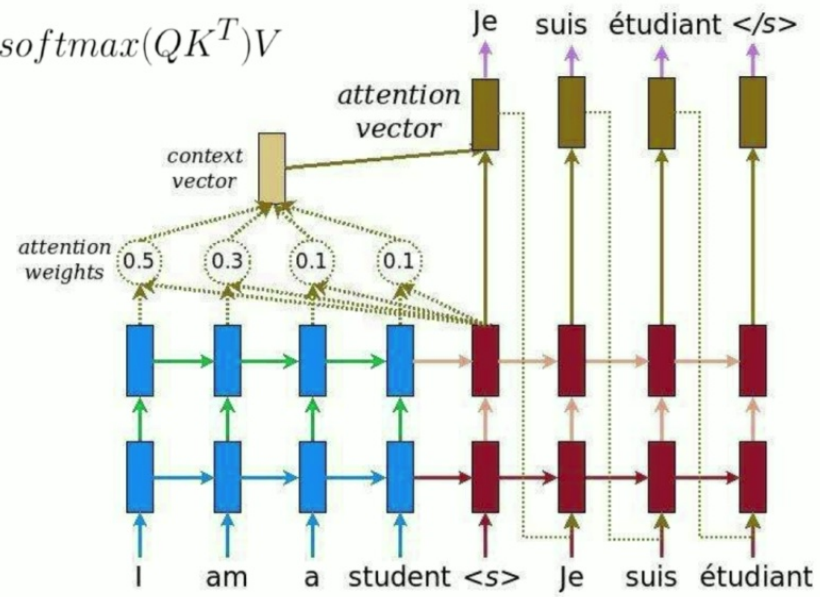


"Translation model"
"Language model"

Need "context" vectors

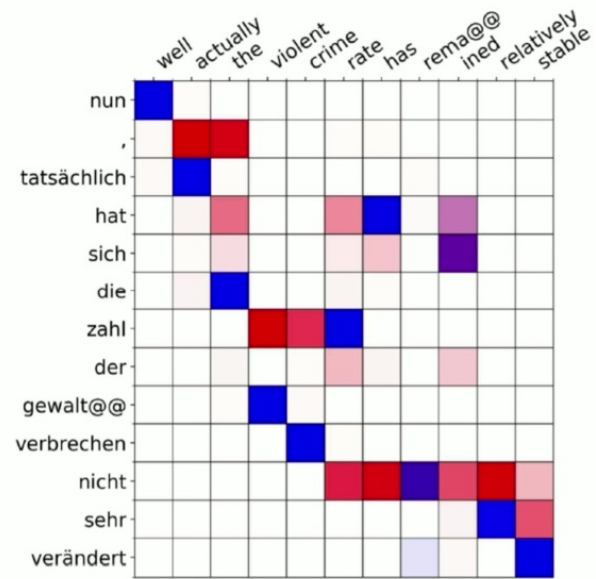


$$\text{softmax}(QK^T)V$$

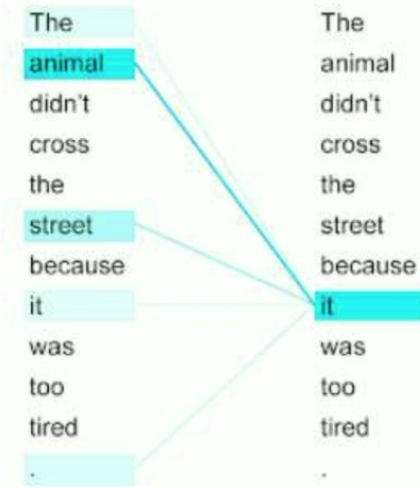


Source: <https://github.com/tensorflow/nmt>





variational attention (blue)
vs prior alignment (red)



self-attention



Attention Is All You Need

Ashish Vaswani*
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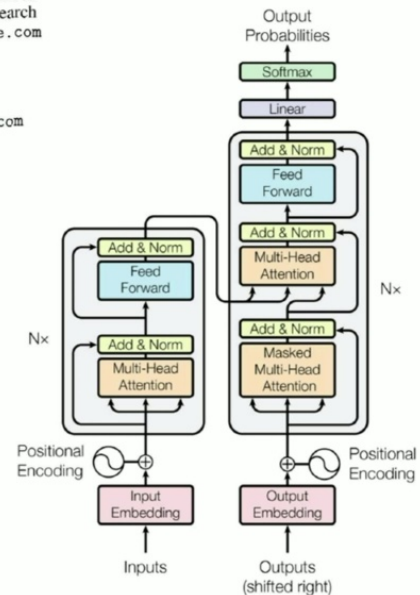
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Abstract

The dominant sequence transduction models are based on complex recurrent or convolutional neural networks that include an encoder and a decoder. The best performing models also connect the encoder and decoder through an attention mechanism. We propose a new simple network architecture, the Transformer, based solely on attention mechanisms, dispensing with recurrence and convolutions entirely. Experiments on two machine translation tasks show these models to be superior in quality while being more parallelizable and requiring significantly less time to train. Our model achieves 28.4 BLEU on the WMT 2014 English-to-German translation task, improving over the existing best results, including ensembles, by over 2 BLEU. On the WMT 2014 English-to-French translation task, our model establishes a new single-model state-of-the-art BLEU score of 41.8 after training for 3.5 days on eight GPUs, a small fraction of the training costs of the best models from the literature. We show that the Transformer generalizes well to other tasks by applying it successfully to English constituency parsing both with large and limited training data.

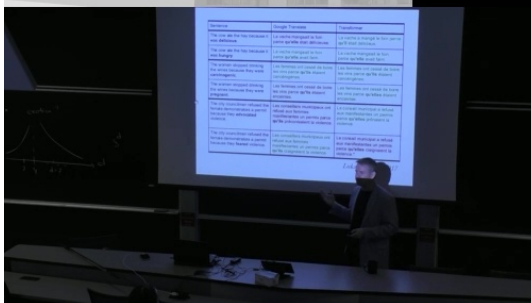


arXiv:1706.03762



Sentence	Google Translate	Transformer
The cow ate the hay because it was delicious .	La vache mangeait le foin parce qu'elle était délicieuse.	La vache a mangé le foin parce qu'il était délicieux.
The cow ate the hay because it was hungry .	La vache mangeait le foin parce qu'elle avait faim.	La vache mangeait le foin parce qu'elle avait faim.
The women stopped drinking the wines because they were carcinogenic .	Les femmes ont cessé de boire les vins parce qu'ils étaient cancérogènes.	Les femmes ont cessé de boire les vins parce qu'ils étaient cancérogènes.
The women stopped drinking the wines because they were pregnant .	Les femmes ont cessé de boire les vins parce qu'ils étaient enceintes.	Les femmes ont cessé de boire les vins parce qu'elles étaient enceintes.
The city councilmen refused the female demonstrators a permit because they advocated violence.	Les conseillers municipaux ont refusé aux femmes manifestantes un permis parce qu'ils préconisaient la violence.	Le conseil municipal a refusé aux manifestantes un permis parce qu'elles prônaient la violence.
The city councilmen refused the female demonstrators a permit because they feared violence.	Les conseillers municipaux ont refusé aux femmes manifestantes un permis parce qu'ils craignaient la violence	Le conseil municipal a refusé aux manifestantes un permis parce qu'elles craignaient la violence.*

Lukasz Kaiser, 2017



“The Transformer” are a Japanese [[hardcore punk]] band.

--Early years--

The band was formed in 1968, during the height of Japanese music history. Among the legendary [[Japanese people|Japanese]] composers of [[Japanese lyrics]], they prominently exemplified Motohiro Oda's especially tasty lyrics and psychedelic intention. Michio was a longtime member of the every Sunday night band PSM. His alluring was of such importance as being the man who ignored the already successful image and that he municipal makeup whose parents were - the band was called

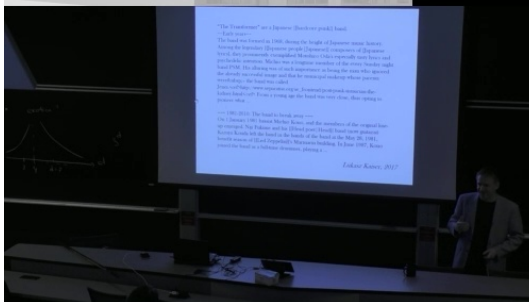
Jenei.<ref>http://www.separatist.org/se_frontend/post-punk-musician-the-kidney.html</ref> From a young age the band was very close, thus opting to pioneer what ...

:

--- 1981-2010: The band to break away ---

On 1 January 1981 bassist Michio Kono, and the members of the original line-up emerged. Niji Fukune and his [[Head poet|Head]] band (now guitarist) Kazuya Kouda left the band in the hands of the band at the May 28, 1981, benefit season of [[Led Zeppelin]]'s Marmarin building. In June 1987, Kono joined the band as a full-time drummer, playing a ...

Lukasz Kaiser, 2017



arXiv:2001.04451v2 [cs.LG] 18 Feb 2020

Published as a conference paper at ICLR 2020

REFORMER: THE EFFICIENT TRANSFORMER

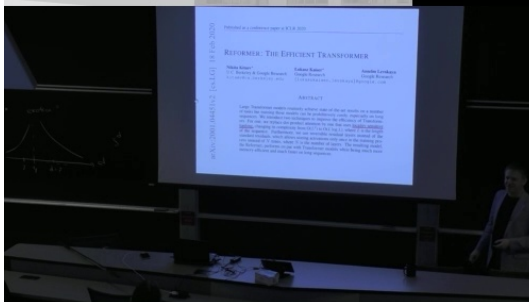
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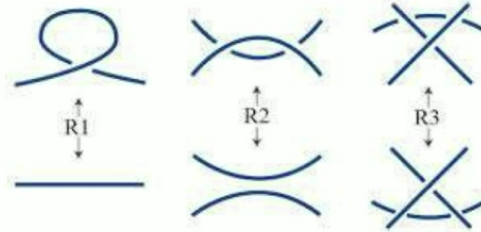
ABSTRACT

Large Transformer models routinely achieve state-of-the-art results on a number of tasks but training these models can be prohibitively costly, especially on long sequences. We introduce two techniques to improve the efficiency of Transformers. For one, we replace dot-product attention by one that uses locality-sensitive hashing, changing its complexity from $O(L^2)$ to $O(L \log L)$, where L is the length of the sequence. Furthermore, we use reversible residual layers instead of the standard residuals, which allows storing activations only once in the training process instead of N times, where N is the number of layers. The resulting model, the Reformer, performs on par with Transformer models while being much more memory-efficient and much faster on long sequences.



- Is it knotted?

S.G., J.Halverson, F.Ruehle, P.Sulkowski



- Is it ribbon? Is it slice?

S.G., J.Halverson, C.Manolescu, F.Ruehle

(SPC4, slice-ribbon, ...)

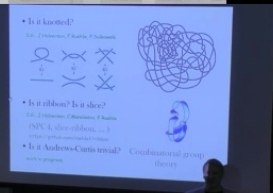
<https://github.com/ruehle/ribbon>

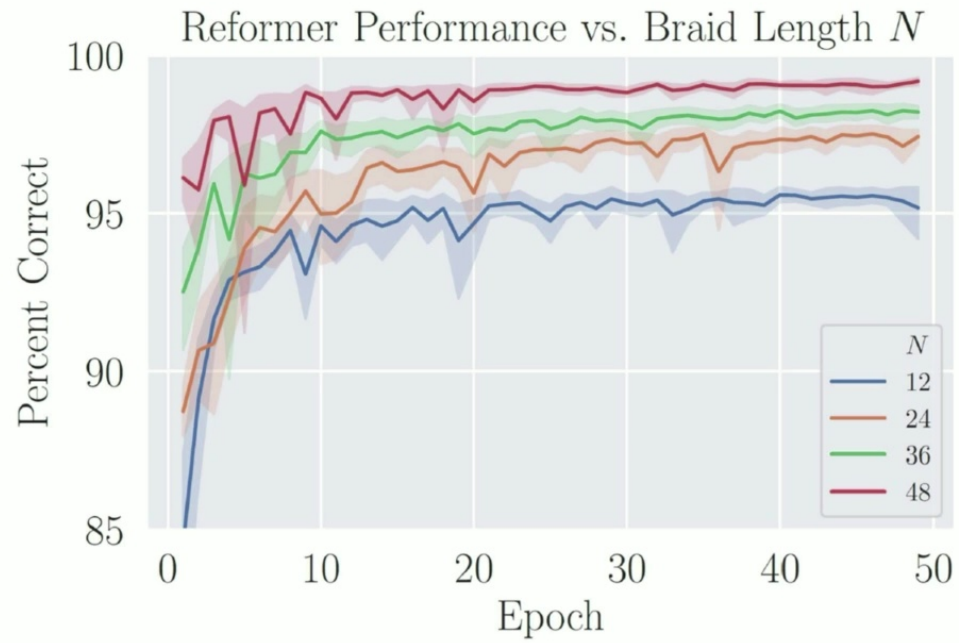


- Is it Andrews-Curtis trivial?

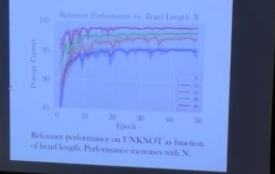
work in progress

Combinatorial group theory



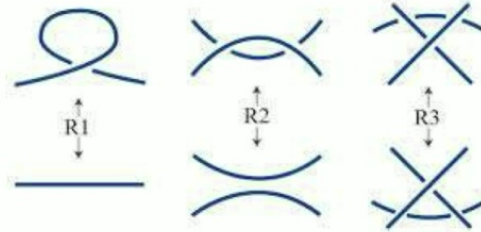


Reformer performance on UNKNOT as function of braid length. Performance increases with N .



- Is it knotted?

S.G., J.Halverson, F.Ruehle, P.Sulkowski



- Is it ribbon? Is it slice?

S.G., J.Halverson, C.Manolescu, F.Ruehle

(SPC4, slice-ribbon, ...)

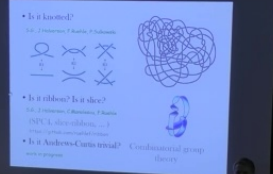
<https://github.com/ruehle/ribbon>

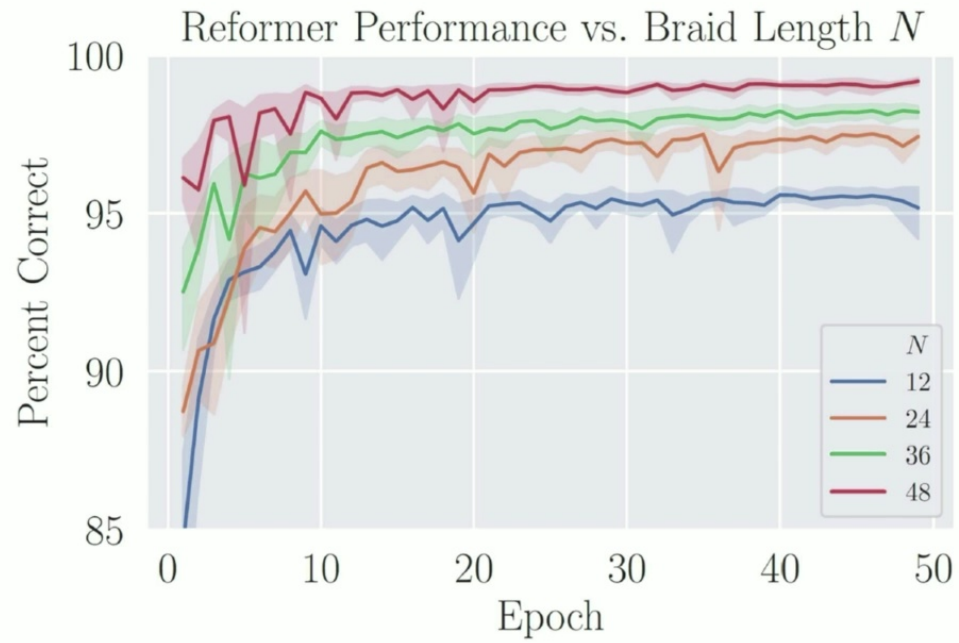


- Is it Andrews-Curtis trivial?

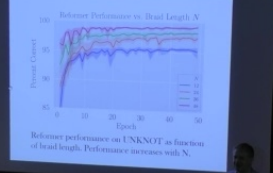
work in progress

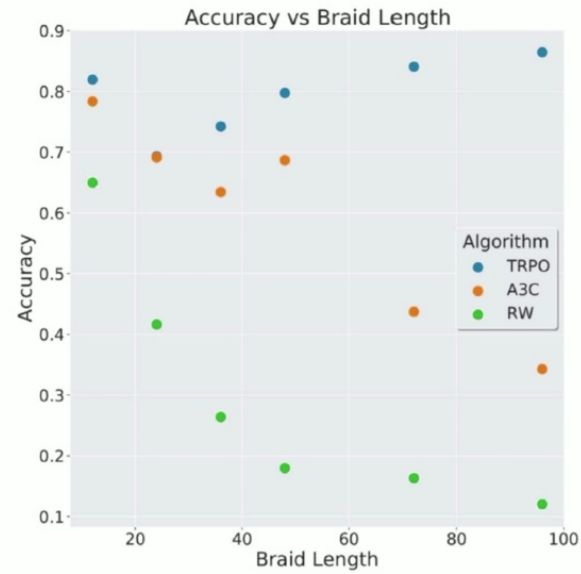
Combinatorial group theory





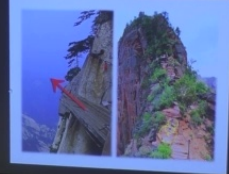
Reformer performance on UNKNOT as function of braid length. Performance increases with N .





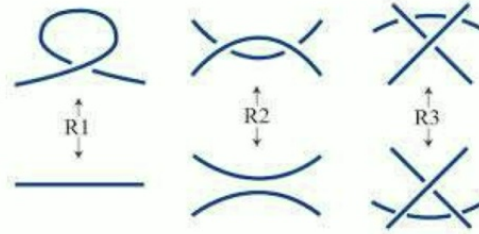
Fraction of unknots whose braid words could be reduced to the empty braid word as a function of initial braid word length.





- Is it knotted?

S.G., J.Halverson, F.Ruehle, P.Sulkowski



Hard unknots



- Is it ribbon? Is it slice?

S.G., J.Halverson, C.Manolescu, F.Ruehle

(SPC4, slice-ribbon, ...)

<https://github.com/ruehle/ribbon>

Hard ribbon knots

- Is it Andrews-Curtis trivial?

Hard AC presentations

work in progress

