Title: Generalized LTB spacetime and dust collapse in polymerized spherical symmetric models

Speakers: Hongguang Liu

Series: Quantum Gravity

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Abstract: Recently, models with different properties have been proposed for polymerized dust collapse and regular black holes. To fully understand their properties and differences, we provide a systematic procedure to construct effective polymerized spherically symmetric models encoding holonomy corrections as $1+1$d field theory from effective regular cosmological dynamics or stationary effective metrics. We apply this formalism and consider models that have the following advantages: The effective dynamics can be derived from a class of extended mimetic gravity Lagrangians in 4 dimensions. The models admit a consistent Lemaitre-Tolman-Bondi (LTB) condition, by which the dynamics is completely decoupled along the radial direction in LTB coordinates, trivializing the junction condition in dust collapse. The class of effective dynamics admits a polymerized Birkhoff-like theorem, which leads to a stationary effective metric in the polymerized vacuum. The effective dynamics can reproduce known regular black hole solutions, including Bardeen and Hayward, by a suitable choice of holonomy corrections. As a concrete example, we construct an effective model compatible with the improved dynamics of loop quantum cosmology in the decoupled LTB sector. We compare it with several effective polymerized models recently introduced in the context of loop quantum gravity and gain some new insights into the presence of shocks.

Zoom link https://pitp.zoom.us/j/99966795418?pwd=Ty9mRXNML3NsUXdvcU1WUTdCaWpVZz09

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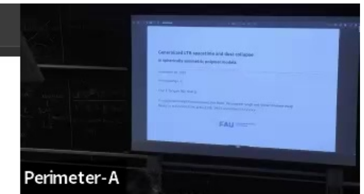


Generalized LTB spacetime and dust collapse in spherically symmetric polymer models

November 30, 2023

Hongguang Liu

FAU Erlangen Nürnberg

In collaboration with Kristina Giesel, Eric Rullit, Parampreet Singh and Stefan Andreas Weigl
Based on arXiv:2308.1094, arXiv:2308.10953, and arXiv:2312.xxxxx

FAU Friedrich-Alexander-Universität Erlangen-Nürnberg

Outline

Spherically symmetric polymer models

Covariance, polymerized LTB condition, polymerized vacuum solution, and reconstruction

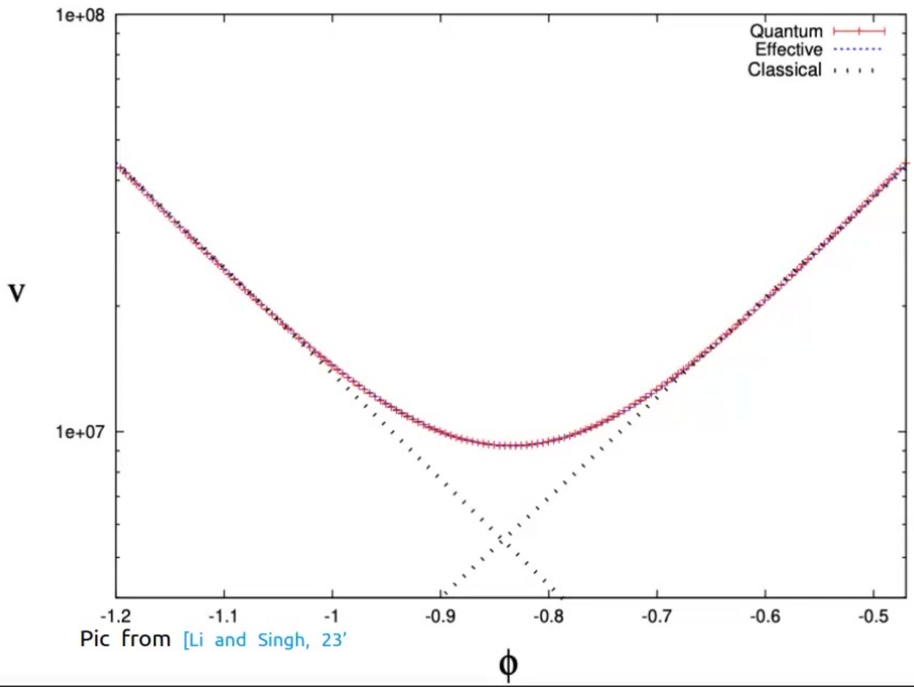
- Introduction:
  Review polymer models, classical spherically symmetric spacetime and LTB condition
- Spherically symmetric polymer models:
  - Polymerized LTB condition: decoupled dynamics along radial direction, trivializing the junction condition in dust collapse
  - Polymerized vacuum solution and Birkhoff-like theorem
  - Extended mimetic theory as the underlying covariant Lagrangian
- Examples
  - Bardeen and Hayward as the polymerized vacuum solution
  - LQC as the decoupled theory in the LTB sector: bouncing solution without a shock
- Summary and Outlook

Bouncing cosmology

Effective dynamics from loop quantum cosmology (LQC)

$$\mathcal{C} = \frac{3b^2|v|}{\kappa} + \frac{\pi_\phi^2}{2|v|} \xrightarrow[\quad b \to \frac{\sin(\alpha b)}{\alpha} \quad]{\text{Polymerization}} \mathcal{C}^\Delta = \frac{3|v|}{\kappa}\frac{\sin^2(\lambda b)}{\lambda^2} + \frac{\pi_\phi^2}{2|v|}$$

Classical — Effective dynamics



Pic from [Li and Singh, 23'

- Scalar field defines the internal time
- Effective dynamics from underlying quantum theory
- Effective dynamics captures the main quantum effects encoded in α (sharply peaked states remain sharply peaked)
- Bouncing solution with a critical density

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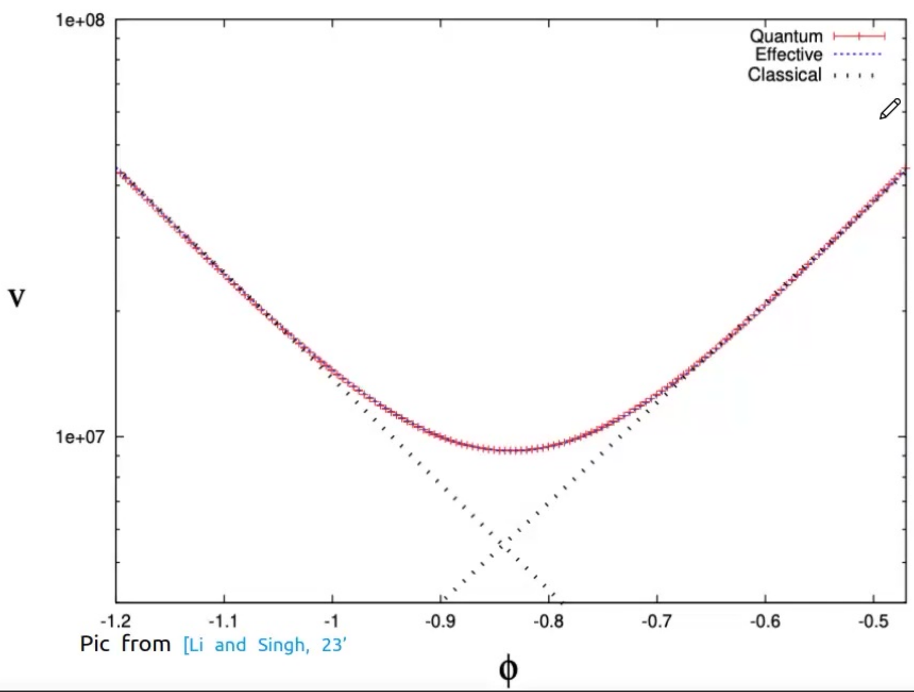

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Spherically symmetric spacetime

1+1d field theory (infinitely many d.o.f) with canonical variables E^x, E^ϕ and their conjugate momenta K_x, K_ϕ

Generalized Gullstrand–Painlevé metric: $\mathrm{d}s^2 = -N(t,x)\mathrm{d}t^2 + \frac{E^\phi(t,x)^2}{|E^x(t,x)|}(\mathrm{d}x + N^x(t,x)\mathrm{d}t)^2 + |E^x(t,x)|\mathrm{d}\Omega^2$

Temporal gauge fixing: $N=1, T=t$ with non-rotational dust

Brown and Kuchar 95'
Husain and Pawłowski 11',12',13'
Giesel and Thiemann 07',15'

$$S_{ND} = -\int \mathrm{d}^4x\,\sqrt{|\det(g)|}\frac{\rho}{2}\,(g^{\mu\nu}\partial_\mu T\partial_\nu T + 1)$$

Physical Hamiltonian: $\mathbf{H}_0 = \int \mathrm{d}x\, C + N^x C_x\,,\qquad C = -\rho\sqrt{|\det(g)|}$

$$C(x) = \frac{1}{2G}\frac{E^\phi}{\sqrt{E^x}}\left[-E^x\left(\frac{4K_xK_\phi}{E^\phi} + \frac{K_\phi^2}{E^x}\right) + \left(\frac{E^{x\prime}}{2E^\phi}\right)^2 - 1 + 2\frac{E^x}{E^\phi}\left(\frac{E^{x\prime}}{2E^\phi}\right)'\right](x)\,,\quad C_x(x) = \frac{1}{G}\left(E^\phi K_\phi{}' - K_x E^{x\prime}\right)(x)$$

EoMs: Hamilton equation:

$$\frac{dO}{dt} = \{O, \mathbf{H}_0\}$$

Conserved quantity: scalar density C conserved on diffeo invariant solutions $C_x = 0$

Lemaître–Tolman-Bondi (LTB) condition and dust collapse

Gauge fixing of C_x:

from Generalized Gullstrand–Painlevé metric: $-dt^2 + \frac{(E^\phi)^2}{E^x}(dx + N^x dt)^2 + E^x d\Omega^2, \quad E^x = R^2$

Areal gauge: $E^x = r^2$

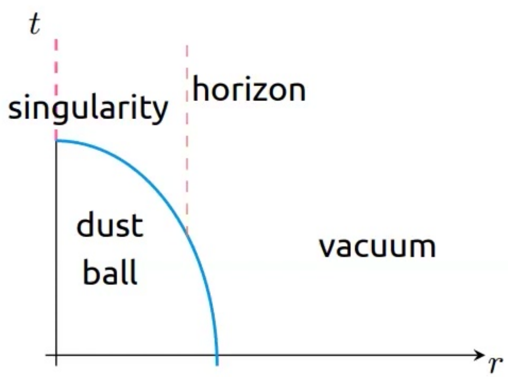
$$-dt^2 + \frac{(E^\phi)^2}{r^2}(dr + N^x dt)^2 + r^2 d\Omega^2$$

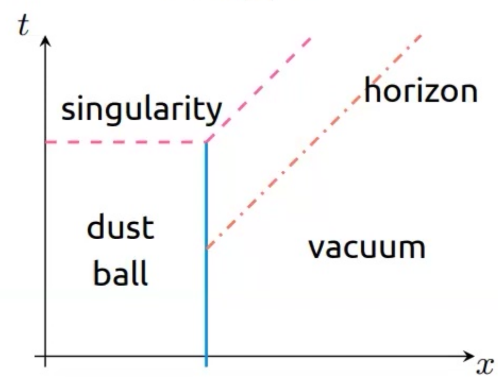
LTB condition

$$(E^x)' - 2\sqrt{1+\mathcal{E}(x)}E^\phi$$

LTB metric

$$-dt^2 + \frac{(R')^2}{1+\mathcal{E}(x)}dx^2 + R^2 d\Omega^2$$





Cosmology $R = xa(t)$

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Lemaître–Tolman-Bondi (LTB) condition and dust collapse

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$$-dt^2 + \frac{(E^\phi)^2}{r^2}(dr + N^x dt)^2 + r^2 d\Omega^2$$

$N^x = \sqrt{\frac{2M}{r}}$

LTB condition

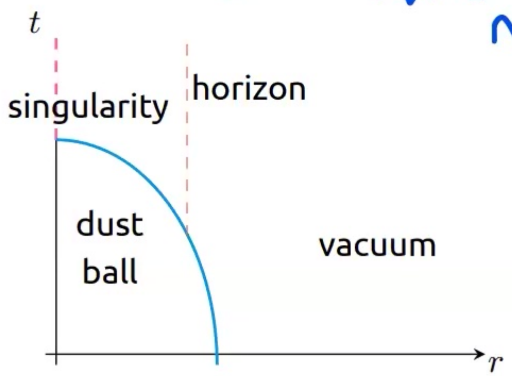
$$(E^x)' - 2\sqrt{1+\mathcal{E}(x)}E^\phi \quad + C_x$$

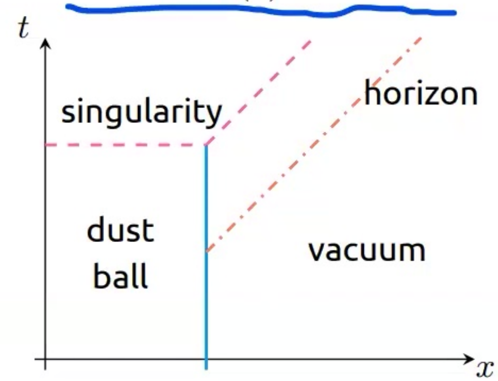
LTB metric

$$-dt^2 + \frac{(R')^2}{1+\mathcal{E}(x)}dx^2 + R^2 d\Omega^2$$

$R(t,x)$

$R(x-t)$





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Lemaître–Tolman-Bondi (LTB) condition and dust collapse

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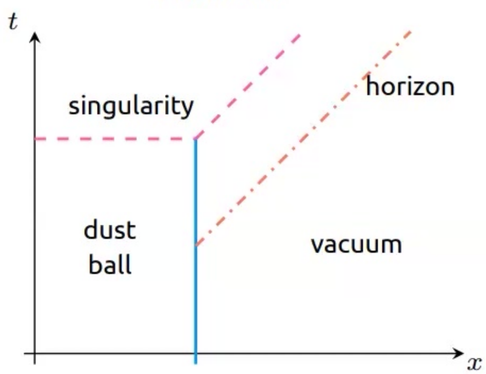

1+1d field theory $\Longrightarrow$ infinitely many decoupled cosmological theory (QM)

Effective dynamics of LQC can be used!

- dust worldline: $x = const$ lines
- decoupled system along x: trivial junction condition

$$C(x) = -\frac{\partial_x \tilde{C}}{\sqrt{1+\mathcal{E}(x)}}, \quad \tilde{C}(x) = \frac{1}{2G}\sqrt{E^x}(K_\phi)^2(x)$$

$\tilde{C}$ conserved quantity

- EoM: Friedmann equation at each x

$$\frac{\dot{R}^2}{R^2}(x) = \left(\frac{\kappa\rho}{6} + \frac{\mathcal{E}}{R^2}\right)(x), \quad \rho := \frac{4\pi\tilde{C}}{3R^3}$$

- general solution in marginally bound case $\mathcal{E} = 0$

$$R = \sqrt{E^x} = \left[\frac{9}{4}\sqrt{2GM(x)}(\beta(x)-t)^2\right]^{\frac{1}{3}}$$

$M(x) := \tilde{C}(x)$: homogeneous/inhomogeneous dust collapse

- Vacuum solution and Birkhoff theorem

$$C(x) = 0 \implies M(x) = \tilde{C}(x) = m = const$$

$$R = \left[\frac{3}{2}\sqrt{m}(x-t)\right]^{\frac{2}{3}}$$

  - Stationary with killing vector field $\partial_t = -\partial_x$.
  - Uniquely determined by integration constant m.

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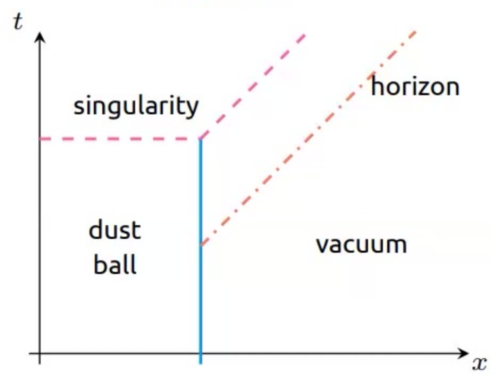

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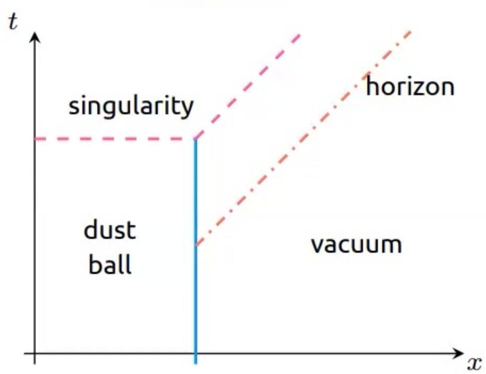

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Spherically symmetric polymer theory

Ashtekar, Bodendorfer, Bojowald, Boehmer, Brahma, Campiglia, Corichi, Chi Gambini, Li, HL, Ma, Mele, Mena Marugá, Modesto, Münch, Navascués, Nou, Olmedo, Pullin, Rastgoo, Rovelli, Saini, Singh, Speziale, Pranzetti, Perez, Wang, Wilson-Ewing ...

Effective Hamiltonian $C(x) \to C^{\Delta}(x)$

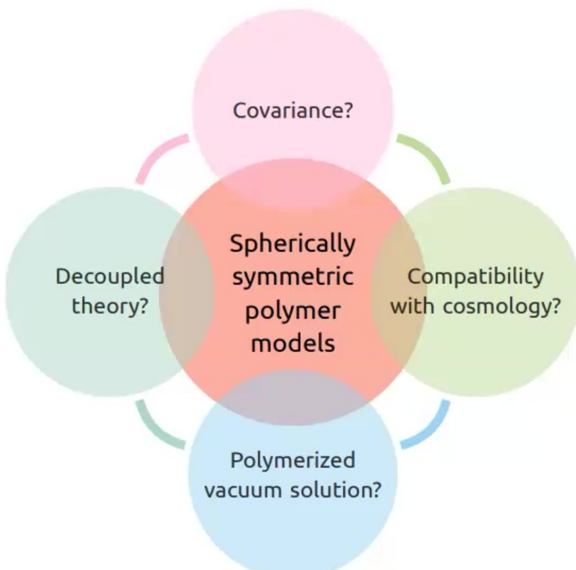
- Polymerization: $\left(\tilde{K}_x, K_\phi\right) \to f\left(\tilde{K}_x, K_\phi, E^x\right)$, $\tilde{K}_x = \frac{K_x}{E^\phi}$ density weight 0, E^x can encode $\bar{\mu}$ Giesel, HL 23'
- Inverse triad corrections: $\frac{1}{\sqrt{E^x}} \to \frac{h_1(E^x)}{\sqrt{E^x}}, \sqrt{E^x} \to h_2(E^x)\sqrt{E^x}$

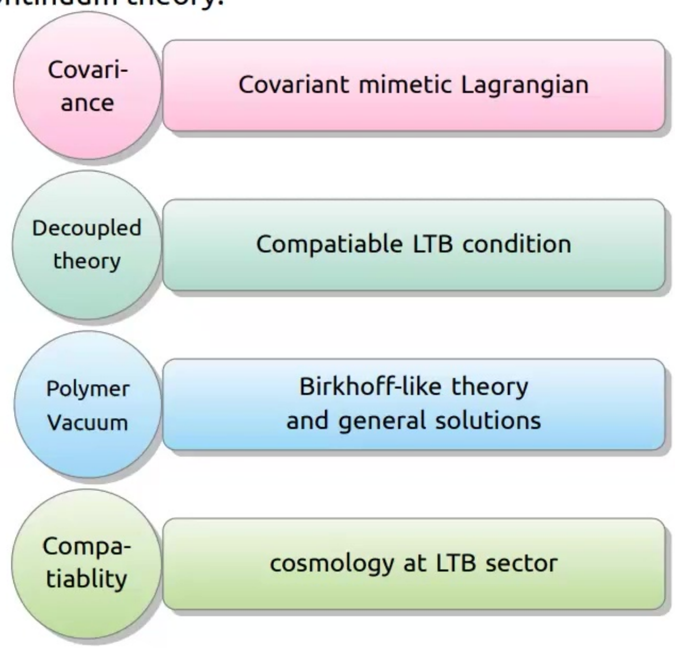
$\bar{\mu}$-scheme:

$\bar{\mu}_x K_x = \sqrt{E^x}\widetilde{K_x}$,

$\bar{\mu}_\phi K_\phi = b := \frac{K_\phi}{\sqrt{E^x}}$

No polymerization of C_x: keep spatial diffeomorphism as a continuum theory.





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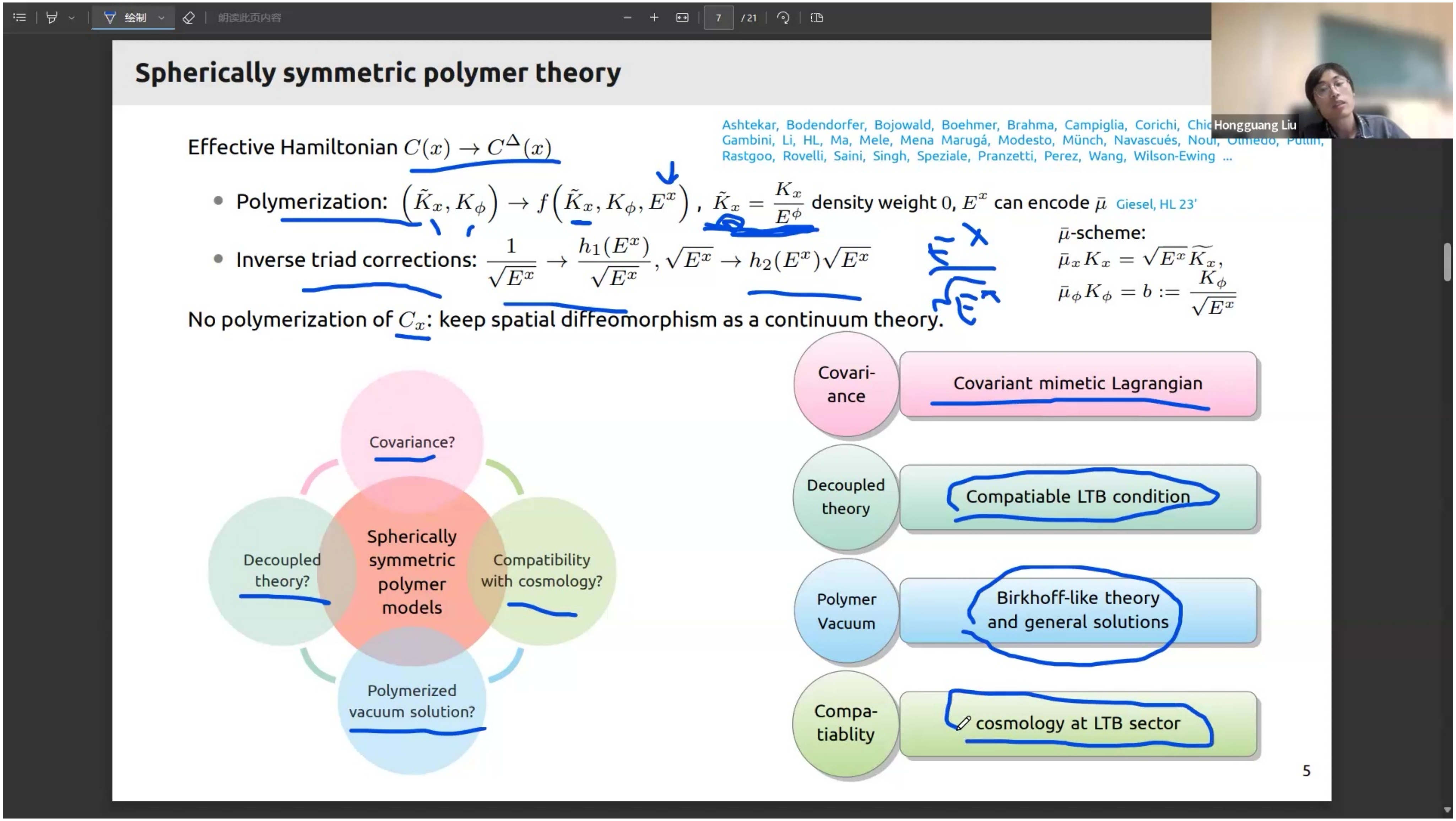

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Spherically symmetric polymer theory
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Covariance?
Decoupled theory?
Spherically symmetric polymer models
Compatibility with cosmology?
Polymerized vacuum solution?
Covari-ance
Covariant mimetic Lagrangian
Decoupled theory
Compatiable LTB condition
Polymer Vacuum
Birkhoff-like theory and general solutions
Compa-tiablity
cosmology at LTB sector
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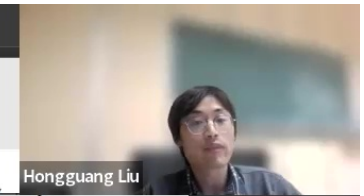
Polymerized LTB condition



Polymerized LTB condition $g_\Delta(\tilde{K}_x, K_\phi, E^x)$: $(E^x)' = g_\Delta(1+\mathcal{E}(x))E^\phi$, $(K_\phi)' = g_\Delta(1+\mathcal{E}(x))K_x \Longleftrightarrow$ $C_x = 0$

Decouples the EoMs:
4 EoMs of
$\{K_x, E^x, K_\phi, E^\phi\}$ $\overset{\text{LTB}}{\Longrightarrow}$

$$\partial_t K_\phi = \frac{g^2(2h_2 + 4E^x\partial_{E^x}h_2 - h_1) - h_1}{2\sqrt{E^x}} + \mathcal{F}_{K_\phi}(K_\phi, E^x, \partial_x K_\phi, \partial_x E^x)$$

$$\partial_t E^x = \mathcal{F}_{E^x}(K_\phi, E^x, \partial_x K_\phi, \partial_x E^x)$$

Compatible LTB condition exists if and only if **no polymerization of** K_x

$$g_\Delta = g_\Delta(E^x), \qquad 1 - \frac{2E^x\partial_{E^x}g_\Delta}{g_\Delta} = \frac{-4E^x\partial_{E^x}f^{(2)}(K_\phi, E^x) + \partial_{K_\phi}f^{(1)}(K_\phi, E^x)}{2f^{(2)}(K_\phi, E^x)} = \mathrm{Con}_f$$

with polymerized Hamiltonian

$$C^\Delta(x) = \frac{E^\phi}{2G\sqrt{E^x}}\left[-E^x\left(\frac{4K_x f^{(2)}(K_\phi, E^x)}{E^\phi} + \frac{f^{(1)}(K_\phi, E^x)}{E^x}\right) + h_1\left(\left(\frac{E^{x\prime}}{2E^\phi}\right)^2 - 1\right) + 2\frac{E^x}{E^\phi}h_2\left(\frac{E^{x\prime}}{2E^\phi}\right)'\right](x)$$

Classical limit: $f^{(1)} \to K_\phi^2, f^{(2)} \to K_\phi, h_1 \to 1, h_2 \to 1$

LTB coordinates

$$ds^2 = -dt^2 + \frac{(R')^2}{g_\Delta^2(1+\mathcal{E}(x))}dx^2 + R^2 d\Omega^2$$

Generalize early attempt in Bojowald, Harada, Tibrewala, 08', Bojowald, Reyes, Tibrewala, 09'

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Polymerized vacuum solution

Flat Minkowski spacetime is always a vacuum solution – Let's do not take it into account

Existence of vacuum solution $C^{\Delta} = 0 \Longleftrightarrow$ Conservation of the scalar Hamiltonian C^{Δ}

$$\{\mathbf{H}_0, C^{\Delta}\} \sim C_x$$

Only possible when there is no polymerization of K_x, and

$$\frac{h_1(E^x) - 2E^x \partial_{E^x} h_2(E^x)}{h_2(E^x)} = \frac{-4E^x \partial_{E^x} f^{(2)}(K_\phi, E^x) + \partial_{K_\phi} f^{(1)}(K_\phi, E^x)}{2 f^{(2)}(K_\phi, E^x)} = \text{Con}_f$$

Compatible LTB condition exists with $\dfrac{2E^x \partial_{E^x} g_\Delta}{g_\Delta} = 1 - \dfrac{h_1 - 2E^x \partial_{E^x} h_2}{h_2}$

Hamiltonian can be rewrite as $\partial_{K_\phi} F := 2 f^{(2)}$

$$C^{\Delta} = -\frac{\partial_x \widetilde{C}^{\Delta}}{\sqrt{1+\mathcal{E}(x)}} \quad \text{with} \quad \widetilde{C}^{\Delta} = \frac{\sqrt{E^x}}{2G g_\Delta}\left(F + h_2\left(g_\Delta^2(1+\mathcal{E}(x)) - 1\right)\right)$$

$\tilde{C}^{\Delta}(x)$ conserved quantity
Decoupled dynamics generated by $\tilde{C}^{\Delta}$ with $\{K_\phi, E^x\} = 2G g_\Delta$

- Modified Friedmann eq: $\dfrac{\dot{R}}{R} = \mathcal{F}(M(x), R)$, integration leads to $R = \mathcal{R}_{M(x)}(t - \beta(x))$
- Unique stationary solution for given integration constant $M(x) = m = const$ at vacuum.
- Schwarzschild coordinates:

$$\mathrm{d}s^2 = -(1-\mathcal{G}(r)^2)\mathrm{d}t^2 + \frac{1}{g_\Delta^2\,(1-\mathcal{G}(r)^2)}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2 \text{ with } \mathcal{G} := -\frac{R'}{g_\Delta} = \frac{\mathcal{F}(m,r)r}{g_\Delta(r^2)}$$

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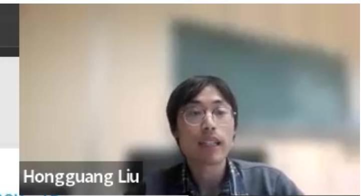
Extended mimetic gravity: underlying covariant Lagrangian



Chamseddine, Mukhanov, 16'
Takahashi, Kobayashi, 17'
Ben Achour, Noui, HL, 17'
Langlois, Mancarella, Noui, Vernizzi 18'
Han, HL, 22'

$$S[g_{\mu\nu},\phi,\lambda]=\frac{1}{8\pi G}\int_{\mathscr{M}_4}\mathrm{d}^4x\,\sqrt{-g}\,\left[\frac{1}{2}\mathcal{R}^{(4)}+L_\phi(\chi_1,\chi_2)+\frac{\lambda}{2}(\nabla_\mu\varphi\nabla^\mu\varphi+1)\right]$$

$$\chi_1=\Box\varphi,\;\;\chi_2=\varphi_{\mu\nu}\varphi^{\mu\nu},\;\;\varphi_\mu=\nabla_\mu\varphi,\;\;\varphi_{\mu\nu}=\nabla_\mu\nabla_\nu\varphi$$

- L_ϕ contains the higher derivative couplings. $L_\phi\to 0$ recovers classical GR with non-rotating dust
- Extra gauge symmetry s.t. the theory propagates only 2 + 1 d.o.f.
- EoM: Modified Einstein Eq: $G^\Delta_{\mu\nu}:=G_{\mu\nu}-T^\varphi_{\mu\nu}=-\lambda\varphi_\mu\varphi_\nu$
- scalar field φ is a natural observer (clock field) that defines the internal time $\tau=\varphi$: temporal gauge fixing with $N=1$
- Foliation with constant φ slices: $\chi_1=K,\chi_2=-K_{ij}K^{ij}$. The Hamiltonian analysis gives the effective physical Hamiltonian C^Δ with $\bar{\mu}$**-scheme polymerization** (holonomy corrections)
  L_ϕ can be reconstructed from given Hamiltonian with $\bar{\mu}$-scheme polymerization, e.g. Ben Achour, Noui, HL, 17', Han, HL, 22'
- temporal gauge fixing solves λ with $\sqrt{\gamma}\lambda=C^\Delta$. λ plays the role of dust energy density in the polymerized theory.
- Polymerized vacuum solution with $\lambda=0=G^\Delta_{\mu\nu}$, not necessarily Ricci flat

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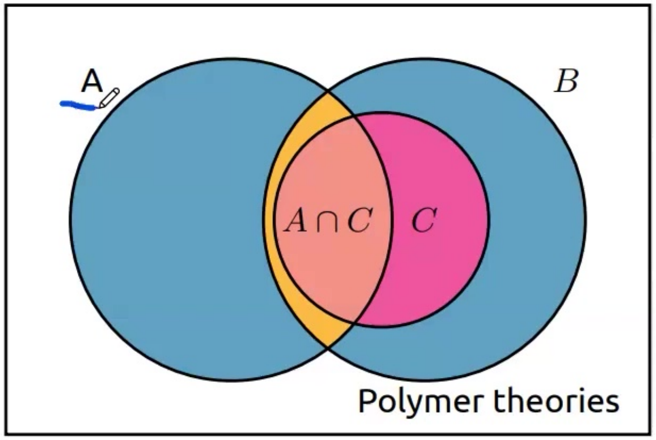
Brief summary



A: Covariant Mimetic Gravity Lagrangian
B: Polymerized LTB condition
C: Polymerized vacuum solution

- $A - B$: Mimetic theory contains $\bar{\mu}$-polymerization of K_x, e.g. Han, HL, 22'
- $B - A$: Inverse triad corrections + polymerization of K_ϕ
  - Polymerized LTB condition $g_\Delta(E^x)$
- C: Inverse triad corrections + polymerization of K_ϕ with constraint
  - Polymerized LTB condition $g_\Delta(E^x)$ and polymerized vacuum solution:
  - polymerized vacuum solution gives the most general form of Schwarzschild like metric
- $A \cap B$: $\bar{\mu}$-polymerization of K_ϕ
  - Covariant Mimetic Lagrangian
  - Polymerized LTB condition g_Δ
- $A \cap C$: $\bar{\mu}$-polymerization of K_ϕ with constraint
  - Covariant Mimetic Lagrangian
  - Classical LTB condition $g_\Delta = 1$ and polymerized vacuum solution
  - polymerized vacuum solution gives the Bardeen-like metric

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Polymer theory in $A \cap C$

Polymerization of K_ϕ with $\bar{\mu}$-scheme + Polymerized vacuum: (Can be derived from extended mimetic)

$$C^\Delta = -\frac{1}{2G} E^\phi \sqrt{E^x}\left[4\widetilde{K}_x f^{(2)}(b) + f^{(1)}(b) + ...\right]$$

with $4f^{(2)}(b) - f^{(1)\prime}(b) = 2bf^{(2)\prime}(b)$, $b = \frac{K_\phi}{\sqrt{E^x}}$ $\bar{\mu}$-scheme

Compatible LTB condition is classical LTB condition $g_\Delta = 1$ $\partial_b F = 2f^{(2)}$, $v = (E^x)^{\frac{3}{2}} = R^3$

$$C^\Delta = -\frac{\partial_x \widetilde{C}^\Delta}{\sqrt{1+\mathcal{E}(x)}} \quad \text{with} \quad \widetilde{C}^\Delta = \frac{v}{2G}\left(F(b) + \frac{\mathcal{E}(x)}{v^{\frac{2}{3}}}\right)$$

- $\tilde{C}^\Delta$ conserved quantity. Decoupled dynamics generated by $\tilde{C}^\Delta$ with $\{b, v\} = 3G$
- Modified Friedmann equation: $\frac{\dot{R}}{R} = \frac{1}{2}F'\left((F)^{-1}\left(\frac{2GM(x)}{R^3} - \frac{\mathcal{E}(x)}{R^2}\right)\right) \Longrightarrow$ reconstruction
- Integration: $t - \beta(x) = \int_{R_0}^{R} \frac{dr}{f^{(2)}\left((F)^{-1}\left(\frac{2GM(x)}{r^3} - \frac{\mathcal{E}(x)}{r^2}\right)\right)}$
- Birkhoff-like theorem at polymerized vaccum $M(x) = m = const$ in marginally bound case $\mathcal{E} = 0$
- Polymerized vacuum solution in Schwarzschild coordinates: $\mathcal{G}(r) = \frac{r}{2}F'\left((F)^{-1}\left(\frac{r_s}{r^3}\right)\right)$
- Reconstruction from $G(r)$: $(F)^{-1}\left(\frac{r_s}{r^3}\right) = \int d\left(\frac{r_s}{r^3}\right)\frac{r}{2\mathcal{G}(r)}$

Bardeen-like solution: $\mathrm{d}s^2 = -(1-\mathcal{G}(r)^2)\mathrm{d}t^2 + \frac{1}{(1-\mathcal{G}(r)^2)}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$

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Polymer theory in $A \cap C$

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Examples:Bardeen and Hayward metric

Bardeen-like solution from extend Mimetic gravity: $\mathrm{d}s^2 = -(1-\mathcal{G}(r)^2)\mathrm{d}t^2 + \frac{1}{(1-\mathcal{G}(r)^2)}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$

Schwarzschild: $\mathcal{G}(r)^2 = \frac{r_s}{r} \implies \frac{r}{2\mathcal{G}(r)} = \frac{1}{2}\sqrt{\frac{r^3}{r_s}}$ ✓

- Integration gives $F(b) = b^2$ with marginally bound solution $R(t,x) = (2GM(x))^{\frac{1}{3}}\left(\frac{3}{2}(\beta(x)-t\right)^{\frac{2}{3}}$

Bardeen BH: $\mathcal{G}(r)^2 = \frac{r_s r^2}{(r^2+\alpha^{\frac{4}{3}}m^{\frac{2}{3}})^{\frac{3}{2}}} \implies \frac{r}{2\mathcal{G}(r)} = \frac{1}{2}\left(\alpha^{\frac{4}{3}} + \left(\frac{r^3}{r_s}\right)^{\frac{2}{3}}\right)^{\frac{3}{4}}$ ✓ Bardeen, 68

- Integration gives $F^{-1} = {}_2F_1\left(-\frac{3}{2}, -\frac{3}{4}, -\frac{1}{2}, -\left(\alpha^2\frac{r^3}{r_s}\right)^{\frac{2}{3}}\right)$ with marginally bound solution:

$$R(t,x) = (2GM(x))^{\frac{1}{3}}\sqrt{\eta^{\frac{4}{3}} - \alpha^{\frac{4}{3}}}, \quad t-\beta(x) = \frac{2}{3}\eta + \alpha\tan^{-1}\left(\eta^{\frac{1}{3}}\alpha^{-\frac{1}{3}}\right) + \alpha\,\mathrm{Re}\tanh^{-1}\left(\eta^{\frac{1}{3}}\alpha^{-\frac{1}{3}}\right)$$

Hayward BH: $\frac{mr^2}{r^3+\alpha^2 m} \implies \frac{r}{2\mathcal{G}(r)} = \frac{1}{2}\sqrt{\frac{r^3}{r_s}+\alpha^2}$ ✓ Hayward, 05

- Integration gives $F^{-1} = \frac{2\eta\alpha + \sinh(2\alpha\eta)}{4\alpha}, \quad \alpha\eta = \sinh^{-1}\sqrt{\frac{\alpha^2 r_s}{r^3}}$
- Marginally bound solution: $R(t,x) = \frac{2GM(x)\alpha^2}{\sinh^2\alpha\eta}$, $t-\beta(x) = \frac{2}{3}\alpha(-\alpha\eta + \coth\alpha\eta)$

Examples: LQC as decoupled theory

Polymerization on the decoupled sector: $F(b) = \frac{\sin^2(\alpha b)}{\alpha^2}$

$$C^\Delta = -\frac{\partial_x \widetilde{C}^\Delta}{\sqrt{1+\mathcal{E}(x)}} \quad \text{with} \quad \widetilde{C}^\Delta = \frac{v}{2G}\left(\frac{\sin^2(\alpha b)}{\alpha^2} + \frac{\mathcal{E}(x)}{v^{\frac{3}{2}}}\right)$$

Polymerization in spherically symmetric spacetime: recovers Hamiltonian in [Tibrewala, 12

$$C^\Delta = -\frac{E^\phi\sqrt{E^x}}{2G}\left[\frac{3}{\alpha^2}\sin^2\left(\frac{\alpha K_\phi}{\sqrt{E^x}}\right) + \frac{(2E^x K_x - E^\phi K_\phi)}{\alpha\sqrt{E^x}E^\phi}\sin\left(\frac{2\alpha K_\phi}{\sqrt{E^x}}\right) + R_3\right]$$

Effective EoMs in Aerial gauge $E^x = r^2$: recovers EoMs in [Husain, Kelly, Santacruz, Wilson-Ewing 21',22'

$$\partial_t E^\phi = -r^2\partial_r\left(\frac{E^\phi}{r}\right)\frac{\sin\left(\frac{2\alpha K_\phi}{r}\right)}{2\alpha}, \quad \partial_t K_\phi = -\frac{1}{2r\alpha^2}\partial_r\left(r^3\sin^2\left(\frac{\alpha K_\phi}{r}\right)\right) - \frac{1}{2r} + \frac{r}{2{E^\phi}^2}$$

Covariant Lagrangian and polymerized junction condition:

- Israel junction condition: Solution to the Einstein equation in the weak sense: allow discontinuities
- Modified Einstein Eq: Modified junction condition
- We can derive polymerized junction condition in corresponding Mimetic theory for arbitrary coordinates
- Gluing along geodesics is still allowed, modifications arise when there is no trivial surface-energy tensor

Standard LQC as decoupled theory

Effective equations of motion $\widetilde{C}^{\Delta} = \frac{v}{2G}\left(\frac{\sin^2(\alpha b)}{\alpha^2} + \frac{\mathcal{E}(x)}{v^{\frac{2}{3}}}\right)$

$$\frac{\dot{R}^2}{R^2}(x) = \left(\frac{\kappa\rho}{6} + \frac{\mathcal{E}}{R^2}\right)\left(1 - \alpha^2\left(\frac{\kappa\rho}{6} + \frac{\mathcal{E}}{R^2}\right)\right)(x)$$

Modified LQC Friedmann equation on each x

General solution in marginally bound case $\mathcal{E} = 0$

$$R(x,t) = \left(2GM(x)\left(\frac{9}{4}(\tilde{\beta}(x) - t)^2 + \alpha^2\right)\right)^{\frac{1}{3}}$$

$$\begin{aligned} \mathrm{d}s^2 &= -\mathrm{d}\tau^2 + (\partial_x R)^2\,\mathrm{d}x^2 + R^2\mathrm{d}\Omega^2 \\ &= -\mathrm{d}\tau^2 + (\mathrm{d}r - \partial_t R\mathrm{d}t)^2 + r^2\mathrm{d}\Omega^2 \end{aligned}$$

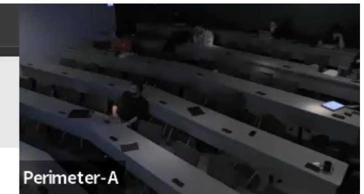
- General mass function $M(x) = \tilde{C}^{\Delta}(x)$: inhomogeneous dust collapse allowed
- Vacuum solution: $M(x) = m$, $\beta(x) = x$
- classical solution: $\alpha \to 0$
- Metric degenerate at shell-crossing $\partial_x R = 0$

Null expansion $\theta_{\pm} = \frac{\sqrt{2}}{R}\left(\dot{R} \pm 1\right)$ horizon at $\dot{R} = \pm 1$, critical mass $M_c = \frac{8\alpha}{3\sqrt{3}G}$

- When $M(x) > M_c$, inner horizons $\tilde{\beta}(x) - t = \pm h_+(x)$ and outer horizons $\tilde{\beta}(x) - t = \pm h_-(x)$ exist.
- Trapped: $[h_-, h_+]$, anti-trapped: $[-h_+, -h_-]$, untrapped otherwise.

Oppenheimer-Snyder (OS) dust collapse x_s position of dust shell

$$M(x) = 2G\left[x^3 E_0 \Theta(x_s - x) + x_s^3 E_0 \Theta(x - x_s)\right], \qquad \tilde{\beta}(x) = (x - x_s)\,\Theta(x - x_s)$$



Standard LQC as decoupled theory: Polymerized vacuum

similar solution in [Fazzini, Rovelli, Soltani, 23'

Vacuum solution: $R(x,\tau) = R(z) = \left(r_s\left(\frac{9}{4}z^2 + \alpha^2\right)\right)^{\frac{1}{3}}$, $z := x - \tau$ symmetric function in z

- Bouncing solution with minimal radius $R_0 = (r_s\alpha^2)^{\frac{1}{3}}$ at $z = 0$
- Dust worldlines $x = const$ pass bouncing surface $z = 0$ at different time τ and do not intersect
- Metric degenerate at $z = 0$ in LTB coordinates: $\mathrm{d}s^2 = -\mathrm{d}\tau^2 + (\partial_z R)^2\,\mathrm{d}x^2 + R^2\mathrm{d}\Omega^2$
- $E^\phi = 2RR'$ thus signed volume $E^\phi\sqrt{E^x}$ changes sign after the bounce
- Bounded curvature invariants: no shell crossing singularity

$$\mathcal{R} = -\frac{96\alpha^2}{(4\alpha^2+9z^2)^2}, \quad \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma} = \frac{576\left(160\alpha^4 - 96\alpha^2z^2 + 27z^4\right)}{(4\alpha^2+9z^2)^4}$$

- Polymerized vacuum: Ricci scalar $\mathcal{R} \neq 0$

Solution in Schwarzschild coordinate:

Next-to-leading order expansion of Hayward metric

$$\mathrm{d}s^2 = -A(r)\mathrm{d}t^2 + \frac{1}{A(r)}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2, \quad A(r) = 1 - \frac{r_s}{r}\left(1 - \frac{\alpha^2 r_s}{r^3}\right)$$

Coordinate transformation only well-defined for $z > 0$ or $z < 0$ s.t. $r > R_0$
Jacobian of coordinate transformation for $z > 0$ and $z < 0$ has different signs

- Solution can be obtained directly from solving Mimetic gravity Einstein equations in Schwarzschild coordinate.
- The lower-bound $r \geq R_0$ is encoded in the mimetic condition where $(\partial_r\phi)^2 = 1 - \frac{\alpha^2 r_s}{r^3} \geq 0$

Standard LQC as decoupled theory: Polymerized vacuum

Vacuum solution: $R(x,\tau) = R(z) = \left(r_s\left(\frac{9}{4}z^2 + \alpha^2\right)\right)^{\frac{1}{3}}, \; z := x - \tau$ symmetric function in z

- Bouncing solution with minimal radius $R_0 = (r_s\alpha^2)^{\frac{1}{3}}$ at $z = 0$
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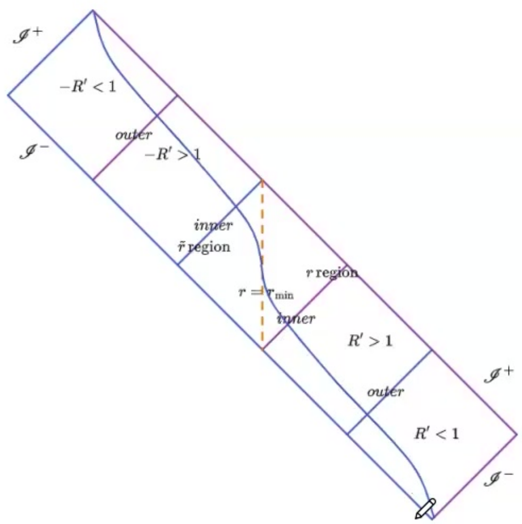

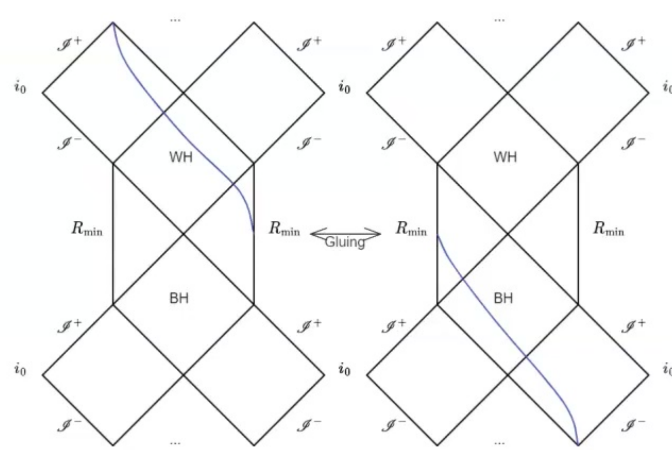

Similar result in [Munch, 21' with junction condition

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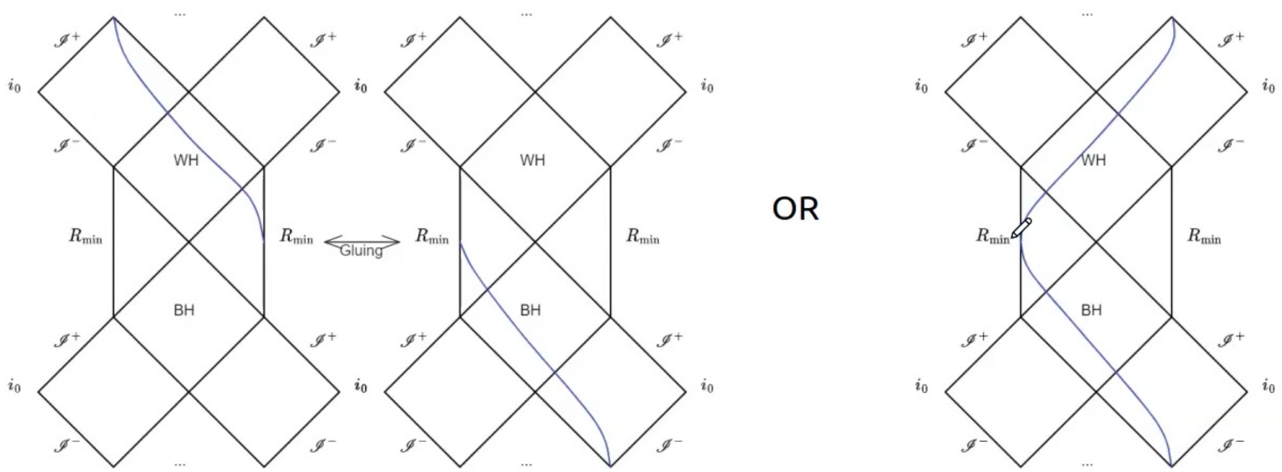


model using Isreal junction condition with LQC

[Lewandowski, Ma, Yang,Zhang, 22'

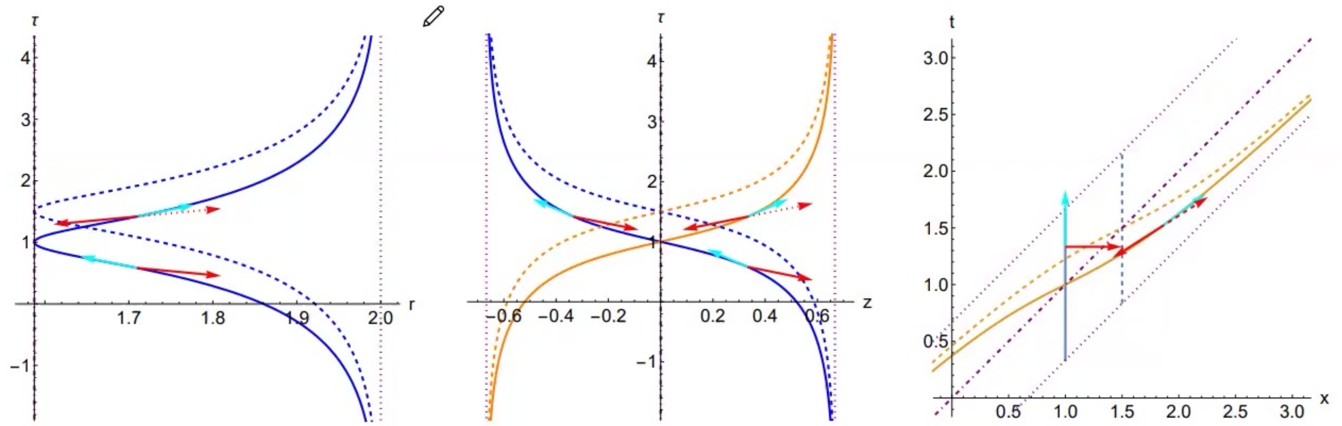
[Han, Rovelli, Soltani, 23'

[Fazzini, Rovelli, Soltani, 23'

Standard LQC as decoupled theory: Polymerized vacuum

Vacuum solution: $R(x,\tau) = R(z) = \left(r_s \left(\frac{9}{4} z^2 + \alpha^2 \right) \right)^{\frac{1}{3}}, \quad z := x - \tau$ symmetric function in z

- $(t,x) \to (\tau, z)$ coordinates: $\mathrm{d}s^2 = -(1-(R'(z))^2)\mathrm{d}\tau^2 + \frac{R'(z)^2}{1-(R'(z))^2}\mathrm{d}z^2 + R(z)^2\mathrm{d}\Omega^2$
- (τ, z) to Schwarzschild (τ, r): 2-to-1 correspondence as $r = R(z)$ symmetric in z



(a) $x = 1$ and $x = 3/2$ curves plot in (τ, r) coordinates. They will intersect each other. Blue and orange line coincide in this case.

(b) $x = 1$ and $x = 3/2$ curves plot in (τ, z) coordinates. Blue lines will not intersect with blue lines but will intersect with orange lines.

(c) $x = 1$ and $x = 3/2$ curves plot in (t, x) coordinates. Blue lines will not intersect with blue lines but will intersect with orange lines.

- Intersection of worldlines implies non-continuity of the mimetic field φ (clock field)
- Result from junction condition consistents with inital value problem in decoupled coordinates

Summary

Spherically symmetric polymer models

Covariance, polymerized LTB condition, polymerized vacuum solution, and reconstruction

- We perform a general analysis to spherically symmetric polymer models
- A subclass of the model corresponds to extended mimetic gravity and admits polymerized LTB condition and polymerized vacuum solution:
  - Completely decoupled cosmological dynamics in LTB sector:
    - (inhomogeneous) dust collapse as decoupled ODE
    - consistent reduced phase space quantization of cosmology in LTB sector and reconstruction
  - Birkhoff-like theorem with general Schwarzschild-like solution
  - Allow reconstruction from general Schwarzschild-like solution or cosmological dynamics in LTB sector
- Examples
  - Reconstruction from Bardeen-like metrics:
    Bardeen and Hayward metric
  - LQC as decoupled model: new insights into spacetime strcuture and the presence of shocks

Standard LQC as decoupled theory: Polymerized vacuum

Vacuum solution: $R(x,\tau) = R(z) = \left(r_s\left(\frac{9}{4}z^2 + \alpha^2\right)\right)^{\frac{1}{3}}, \quad z := x - \tau$ symmetric function in z

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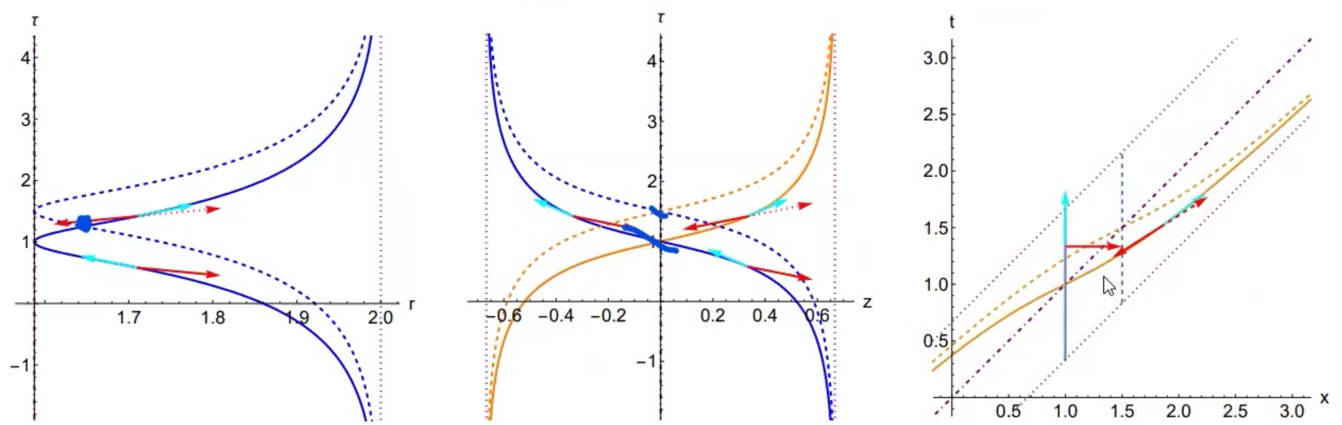

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Outlook

- Within the model
  - Relation with stationary solutions in Kantowski-Saches model of stationary BH interior, e.g. AOS model
  - Axially symmetric model and its solution, generalized Newman-Janis algorithm?
  - Generalized Vaidya solution $\Longrightarrow$ BH evaporation:
    - Possible to solve directly in the mimetic Lagrangian with a null dust energy-momentum tensor in the generalized Vaidya coordinate.
    - Send m directly to $m(r)$ do not solve the modified Einstein equation
    - Conformal scalar field similar to the mimetic CGHS.
  - 4d mimetic Lagrangian gives new insight into perturbations
    - Cosmological perturbation
    - BH perturbation and quasi-normal modes
  - Gravitational wave memory effect using corresponding extended mimetic Lagrangian
  - Connection with other regular BH and dust collapse spacetimes
- Beyond the model
  - Covariant Lagrangian beyond mimetic class, e.g. in general degenerate higher-order scalar-tensor theories
  - Polymerization of C_x: deparametrization of x needed

...

Thank you

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