Title: Gyroscopes orbiting gargantuan black holes - VIRTUAL

Speakers: Lisa Drummond

Series: Strong Gravity

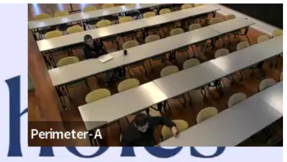
Date: November 30, 2023 - 1:00 PM

URL: https://pirsa.org/23110083

Abstract: Extreme mass-ratio binary black hole systems, known as EMRIs, are expected to radiate low-frequency gravitational waves detectable by planned space-based Laser Interferometer Space Antenna (LISA). We hope to use these systems to probe black hole spacetimes in exquisite detail and make precision measurements of supermassive black hole properties. Accurate models using general relativistic perturbation theory will allow us to unlock the potential of these unique systems. Such models must include post-geodesic corrections, which account for forces driving the smaller black hole away from a geodesic trajectory. When a spinning body orbits a black hole, its spin couples to the curvature of the background spacetime, introducing post-geodesic correction called the spin-curvature force. In this talk, I will present our calculation of EMRI waveforms that include both spin-curvature forces and the leading backreaction due to gravitational radiation. We use a near-identity transformation to eliminate dependence on the orbital phases, allowing for very fast computation of completely generic worldlines of spinning bodies; such efficiency is crucial for LISA data analysis. Finally, I will discuss what aspects still need to be included in future calculations so that we can use EMRIs for a new era of precision gravitational-wave astronomy, addressing outstanding puzzles in astrophysics, cosmology and fundamental theoretical physics.

Zoom link https://pitp.zoom.us/j/91917788358?pwd=MWp5OUhxbkRmZDFxWWE4cHR0VlBTUT09

Gyroscopes orbiting gargantuan black h

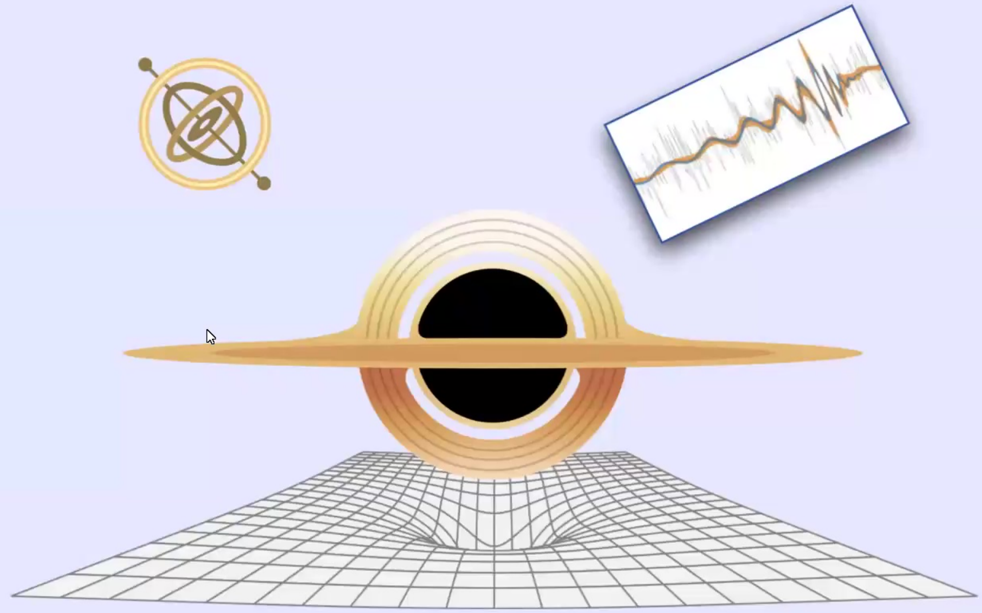


Spinning secondary** effects in **extreme mass-ratio inspirals

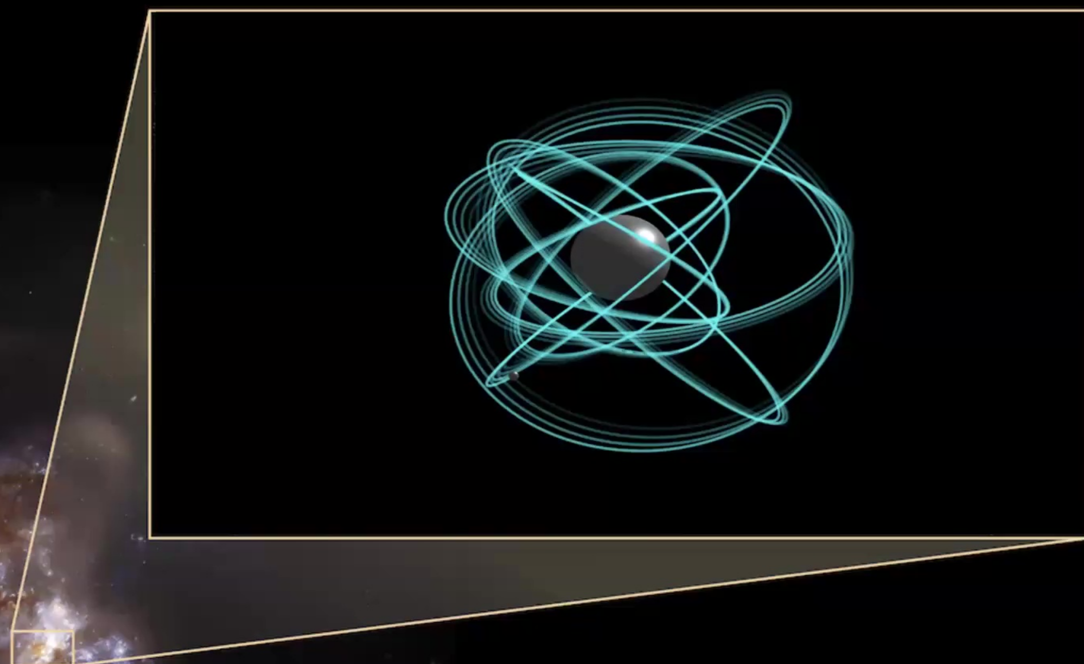
Lisa V. Drummond

MIT Physics PhD Candidate

Strong Gravity Seminar, Perimeter Institute

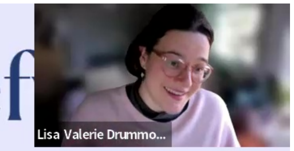


What are **extreme mass-ratio inspirals (EMRIs)***?*



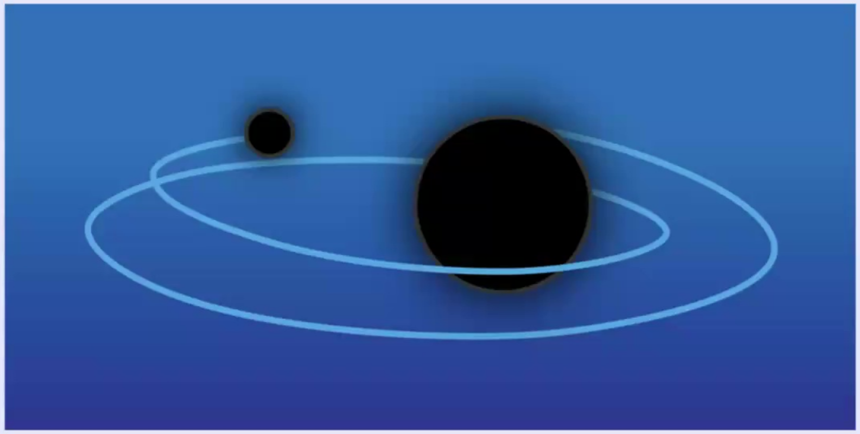
Stellar mass black hole in a ***strong-gravity orbit*** *around a supermassive black hole*

2



Why are **extreme mass-ratio inspirals** usef

Geodesy for black holes



[Image: APS/Alan Stonebraker, 2020]

Large mass ratio means the secondary body makes **thousands to millions** of orbits before merging. **Gravitational waves** from these systems enable **precision measurement** of properties of the larger black hole

Infer black hole mass, spin: $\delta M/M$ and $\delta a \sim 10^{-4} - 10^{-2}$

Infer orbit's geometry: $\delta e_0 \sim 10^{-3} - 10^{-2}$

Infer distance to the binary: $\delta D/D \sim 0.03 - 0.1$

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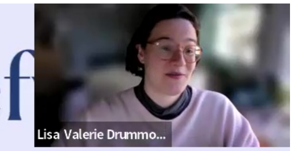
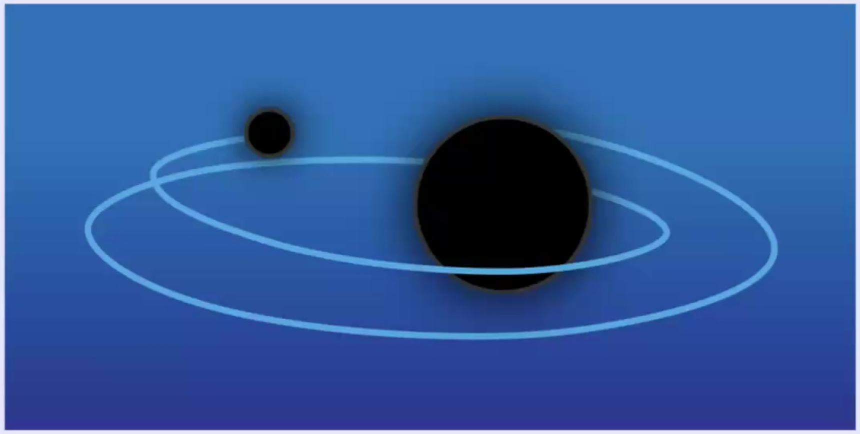
[Barack & Cutler PRD **69**, 082005 (2004)]

Why are **extreme mass-ratio inspirals** usef



Geodesy for black holes



[Image: APS/Alan Stonebraker, 2020]

Large mass ratio means the secondary body makes **thousands to millions** of orbits before merging. **Gravitational waves** from these systems enable **precision measurement** of properties of the larger black hole

With these precise measurements we can:

- ★ Test **theories of gravity**
- ★ Learn about **supermassive black hole** formation
- ★ Constrain the **Hubble constant**

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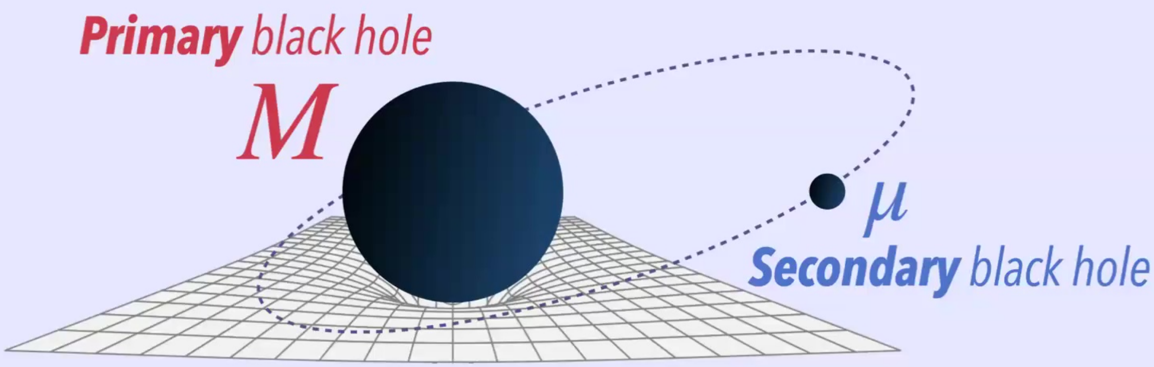
[Barack & Cutler PRD **69**, 082005 (2004)]

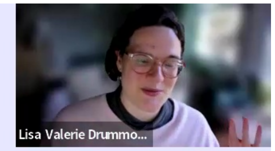
Why are **extreme mass-ratio inspirals** useful?

1. EMRIs are of ***astrophysical importance*** → *source for future gravitational-wave detectors*
2. ***Clean limit of the relativistic two-body problem*** → *potential to shed light on the more general two-body problem*

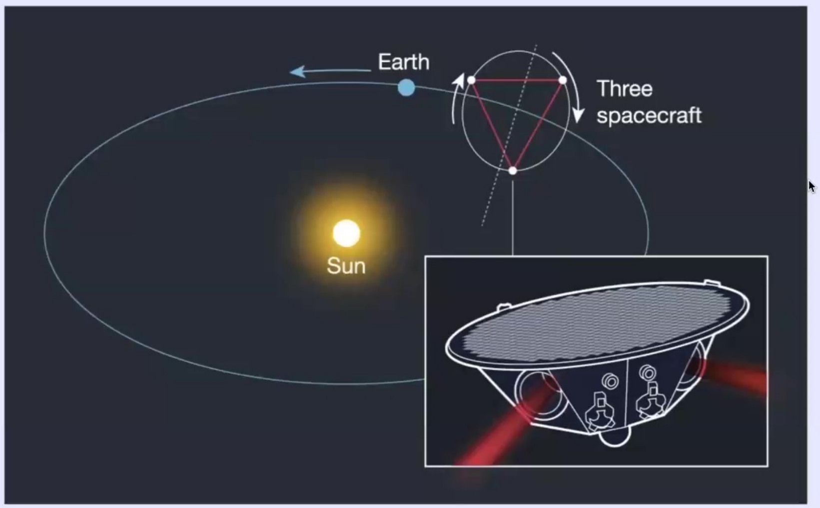


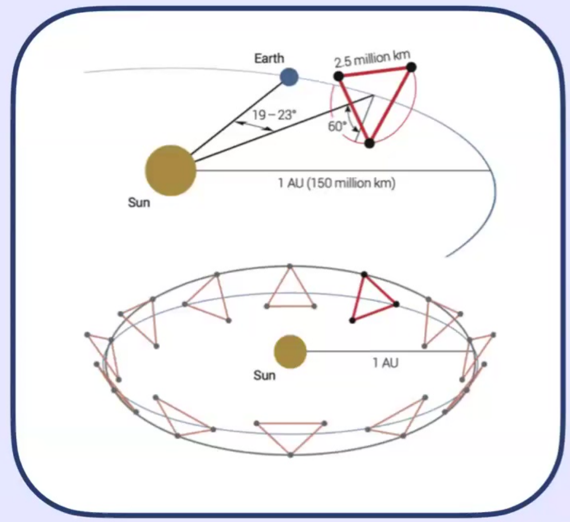
$$\epsilon = \frac{\mu}{M} \sim \text{very small}$$

How will we detect **EMRIs**?



Seismic noise limits the sensitivity of ground-based detectors such as LIGO and VIRGO to frequencies above one Hz. Therefore, we need to send a gravitational wave detector **into space**! This is **LISA** (Laser Interferometer Space Antenna).





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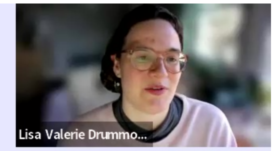
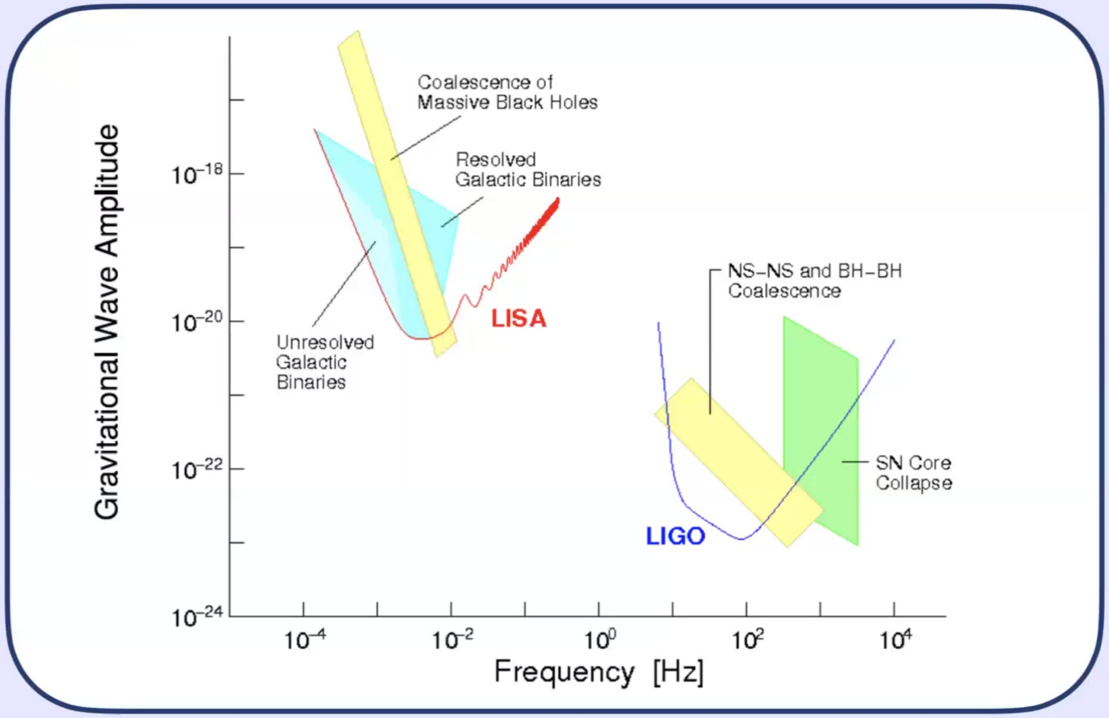


How will we detect **EMRIs**?

Gravitational waves from **EMRIs** → need to observe **millihertz** signals.

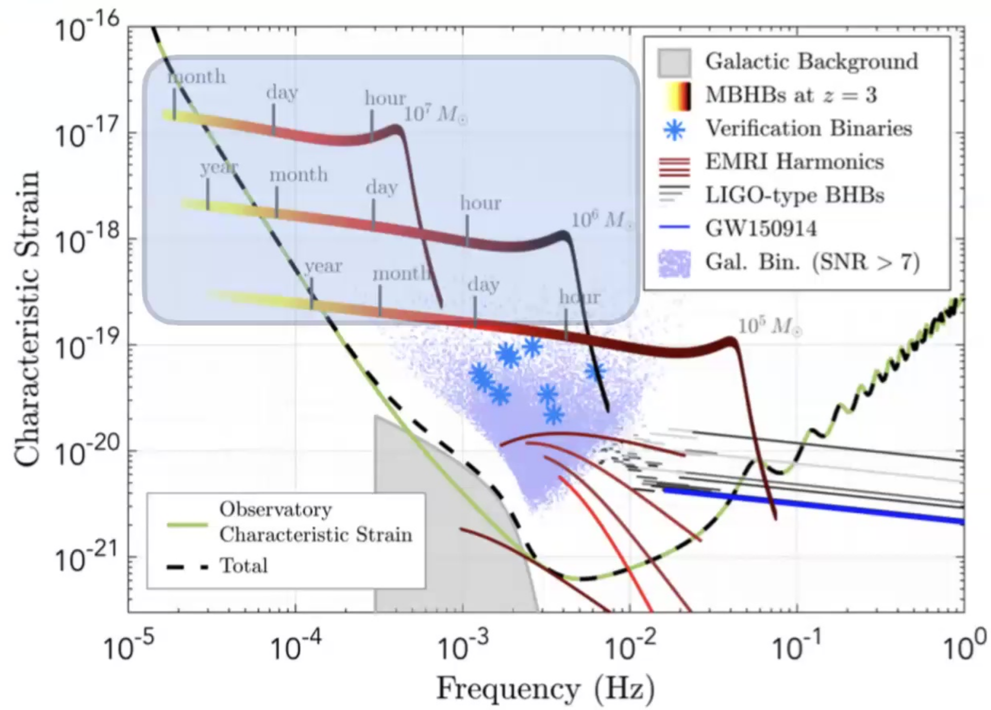
Extended systems (LISA) ⟷ ***Compact systems (LIGO)***



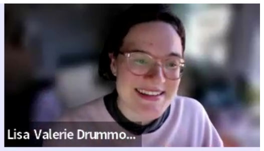
What can we detect with **LISA**?

Massive black hole binaries

~ 10



[Littenberg et al., 2020]



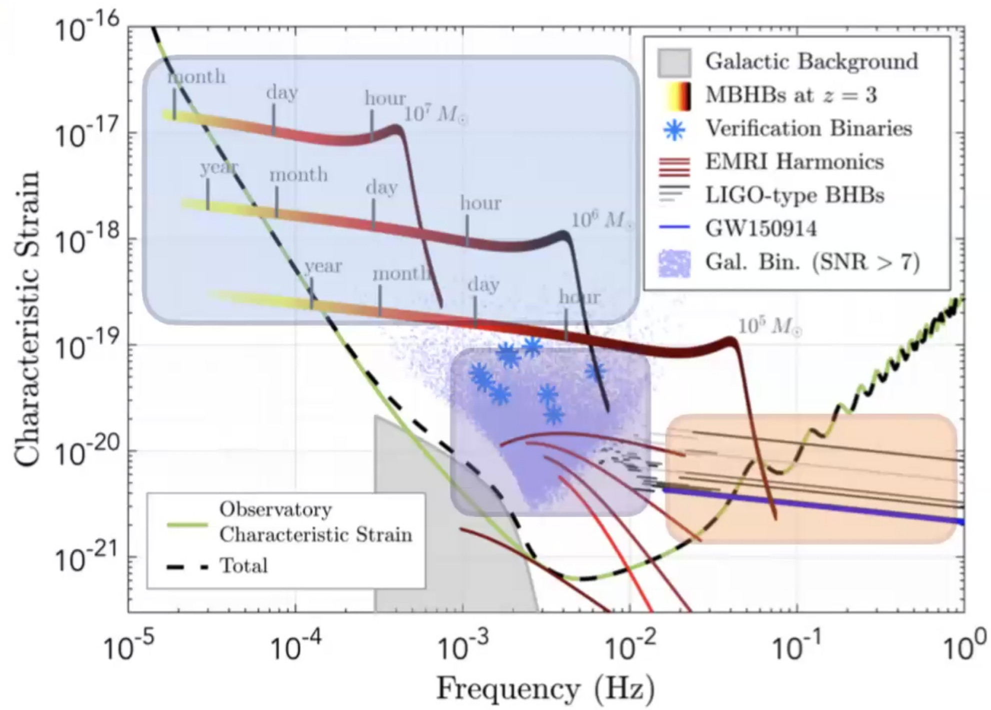
What can we detect with **LISA**?

Massive black hole binaries

~ 10

Resolved galactic binaries

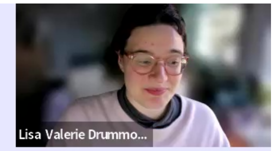
$\sim 10^3 - 10^4$



LIGO-type black hole binaries

$\sim 1 - 10$

[Littenberg et al., 2020]

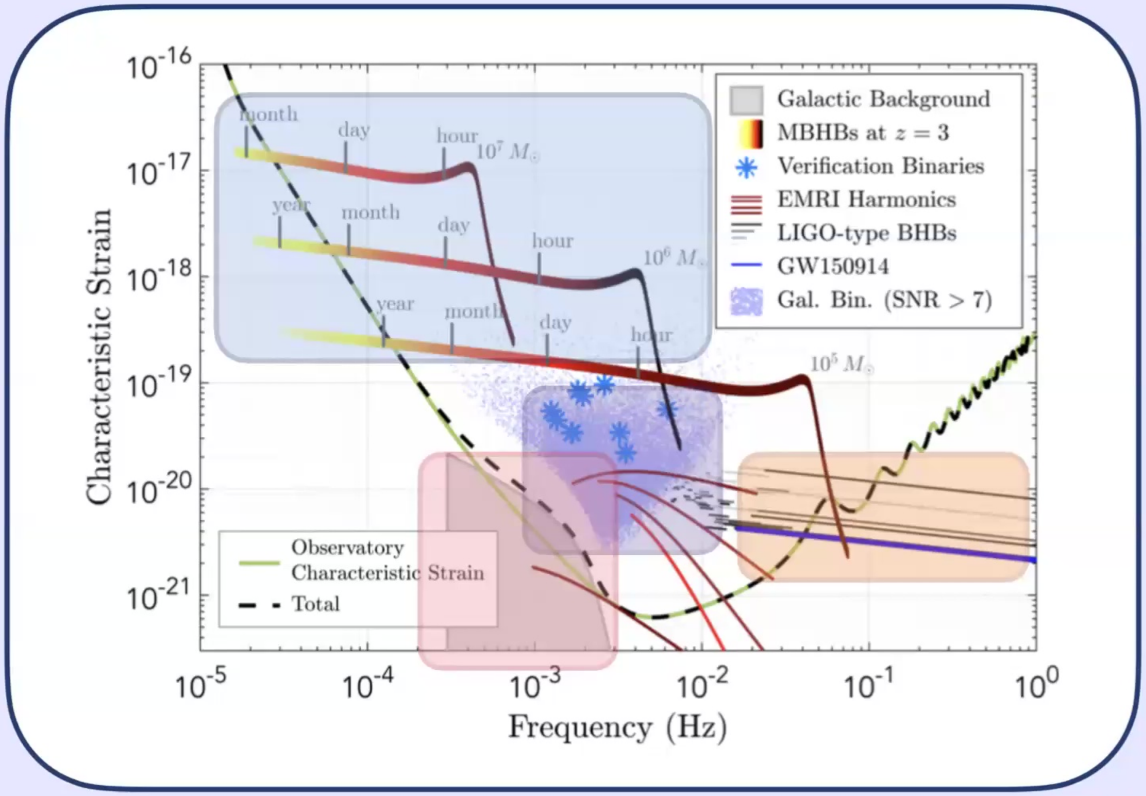


What can we detect with LISA?

Massive black hole binaries
~ 10

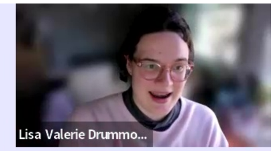
Resolved galactic binaries
$\sim 10^3$—10^4

Unresolved galactic binaries



LIGO-type black hole binaries
~ 1—10

[Littenberg et al., 2020]



What can we detect with LISA?

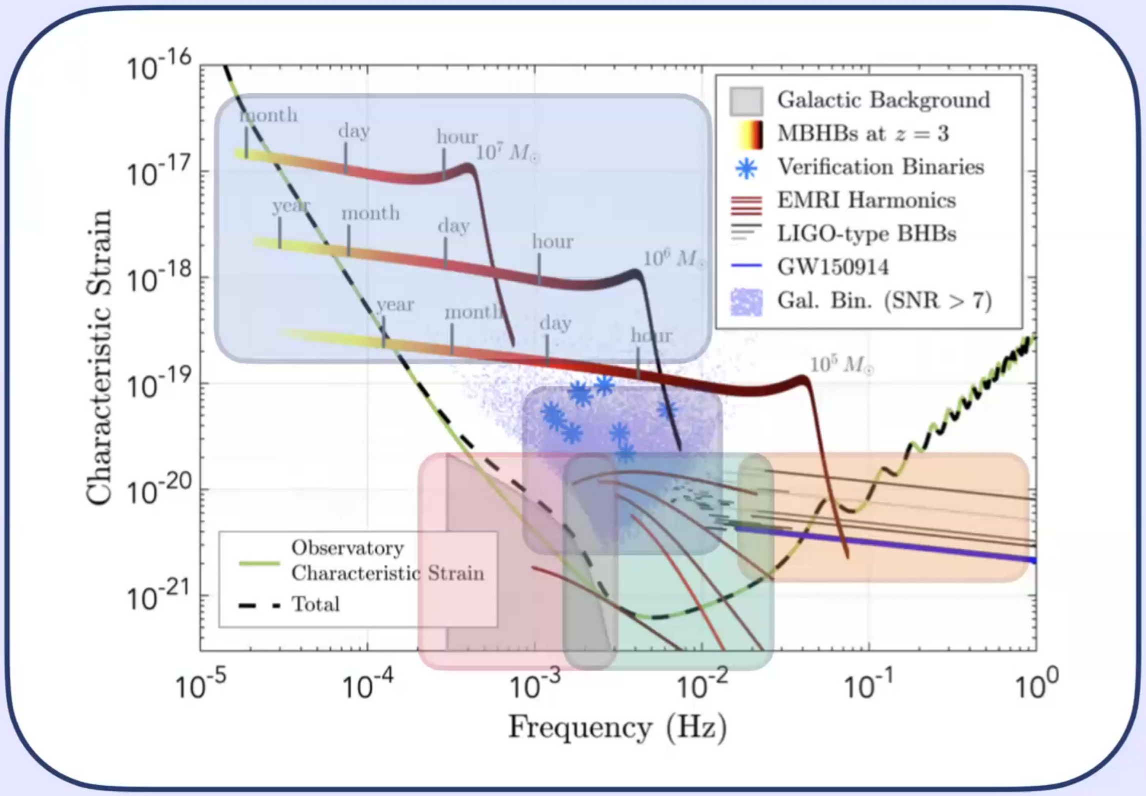
Massive black hole binaries

~ 10

Resolved galactic binaries

$\sim 10^3$—10^4

Unresolved galactic binaries



LIGO-type black hole binaries

~ 1—10

Extreme mass-ratio inspirals

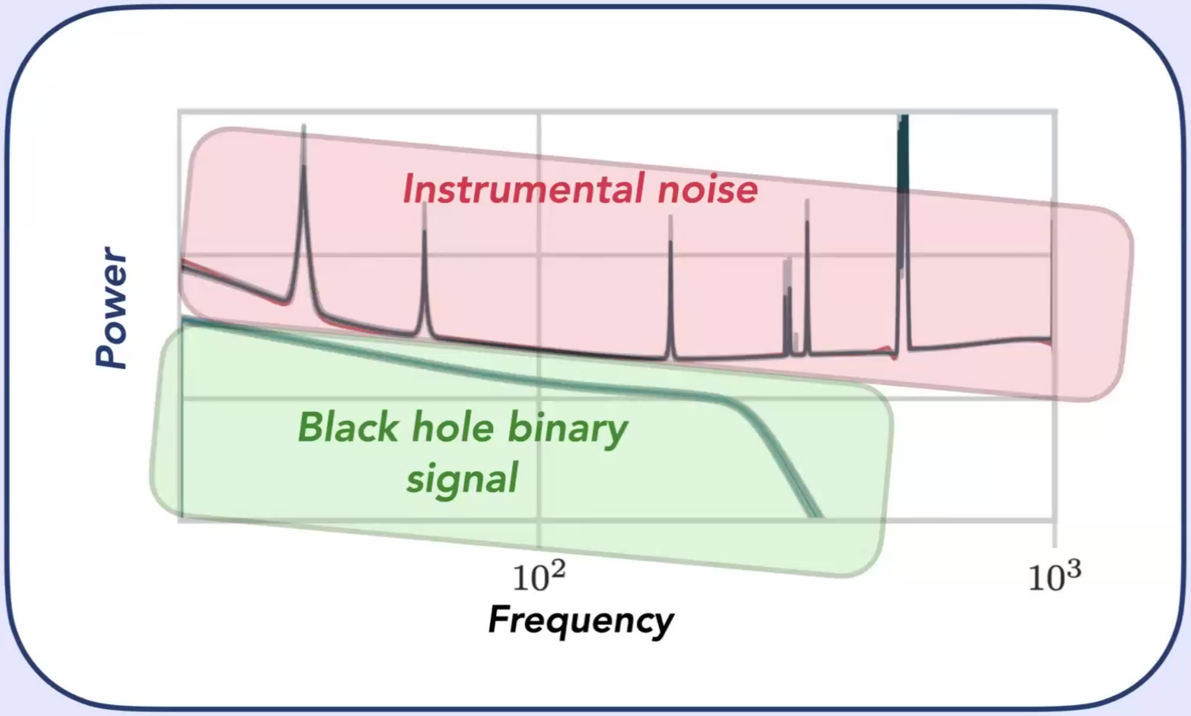
$\sim 1 - 10^3$

[Littenberg et al., 2020]

Why is it hard to detect **EMRIs**?

First consider a LIGO signal

Usually see **one source** at a time!



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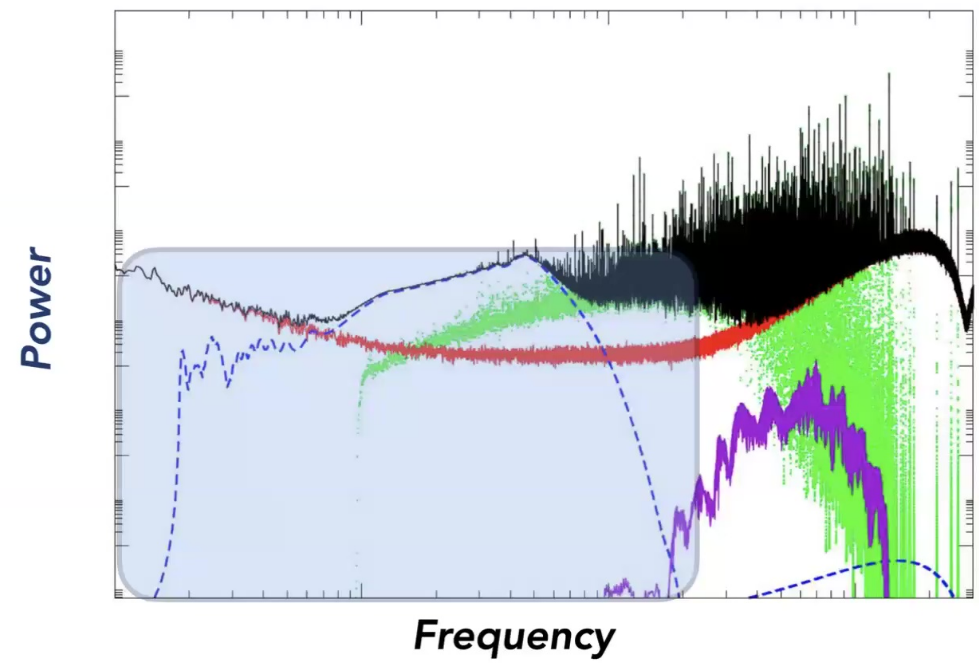
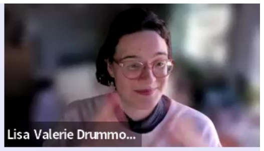
Why is it hard to detect **EMRIs**?

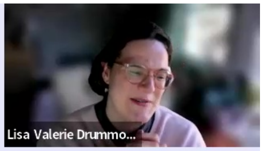
Massive black hole binary inspiral

Merging supermassive black hole binaries are likely to have very high signal to noise ratios; overwhelming the signal



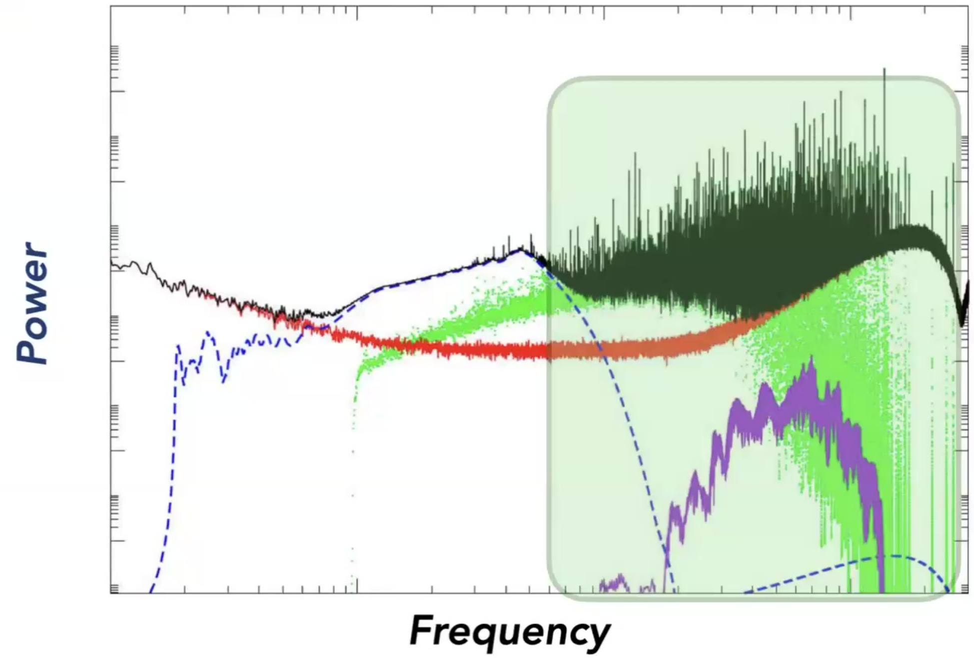


Why is it hard to detect **EMRIs?**



27 million galactic white dwarf binaries

LISA data will be strongly coloured by GW signals from white dwarf binaries in our galaxy which create confusion noise at frequencies below a few millihertz



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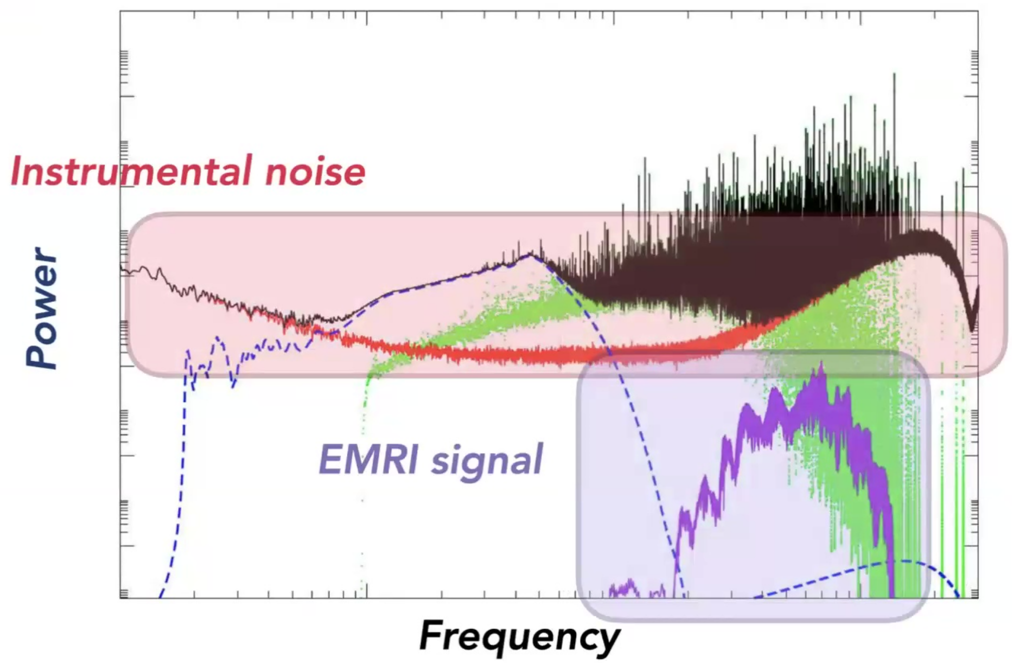
Why is it hard to detect **EMRIs**?

The **EMRI signal** is buried underneath the **instrumental noise** and **other GW sources**!



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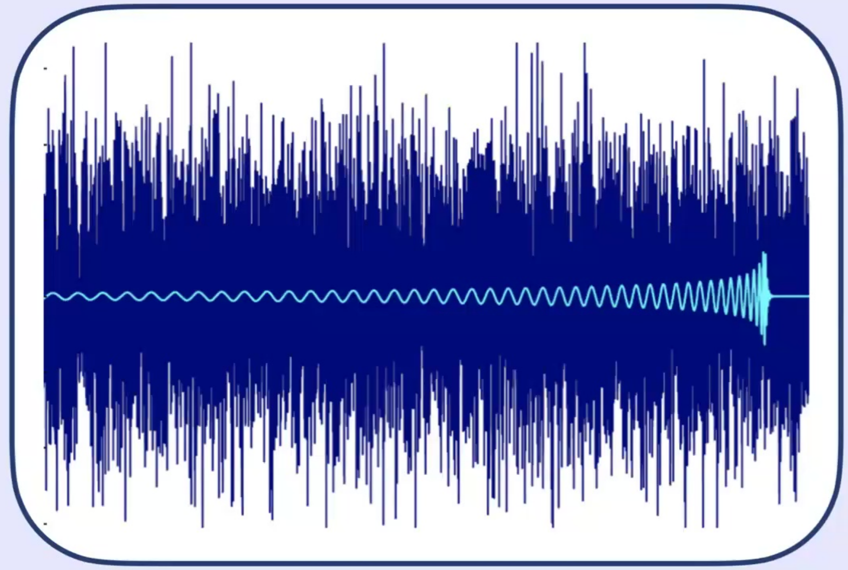
Why is it hard to detect **EMRIs?**

★ EMRI signals will be **an order of magnitude** below LISA's instrumental noise and **orders of magnitude** below the gravitational wave foreground

★ But: signals are **very long-lived** which allows the signal-to-noise-ratio (SNR) to be built up over time



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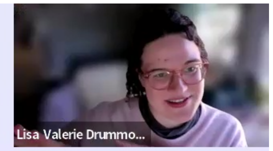
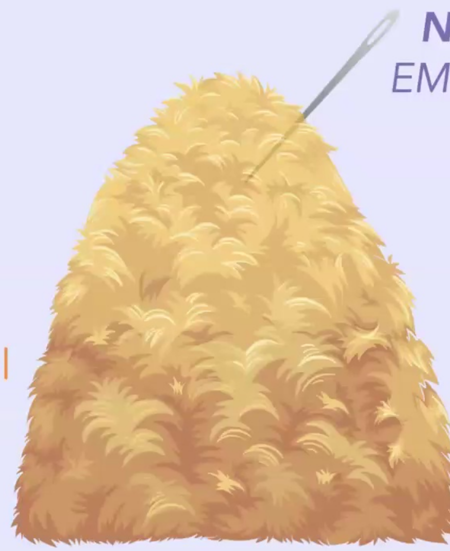
Hunting for EMRIs

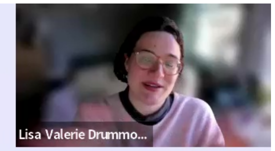
Needle** in a relativistic **haystack



Needle:
EMRI signal



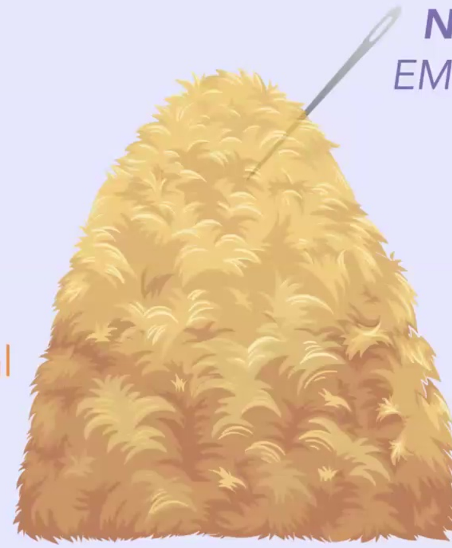
Haystack:
Instrumental noise
and other gravitational
wave sources



Hunting for EMRIs

Needle** in a relativistic **haystack

Haystack:
Instrumental noise
and other gravitational
wave sources



Needle:
EMRI signal

Use perturbation theory to develop **precise, tractable** models that can stay in phase over the **large number of orbits** of the inspiral. Therefore, can build up enough **signal-to-noise** that the EMRI can be detectable

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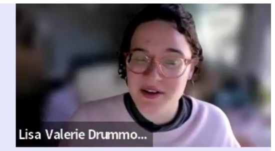
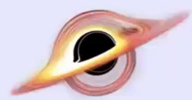
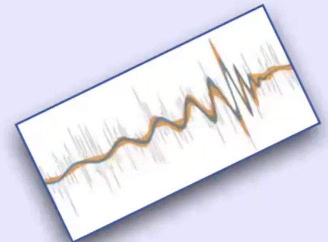
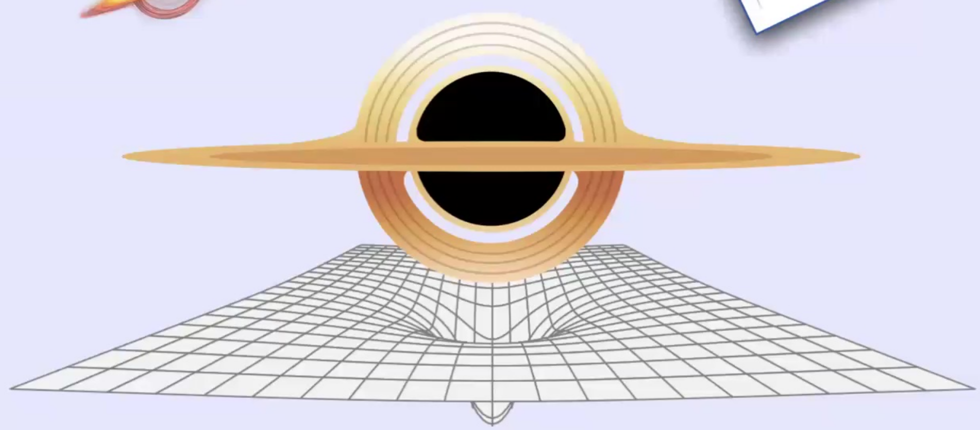
Hunting for EMRIs



Needle** in a relativistic **haystack







The plan is for LISA to launch in the 2030s; we need to have models for EMRI waveforms fully developed by then. This is a ***considerable challenge***!

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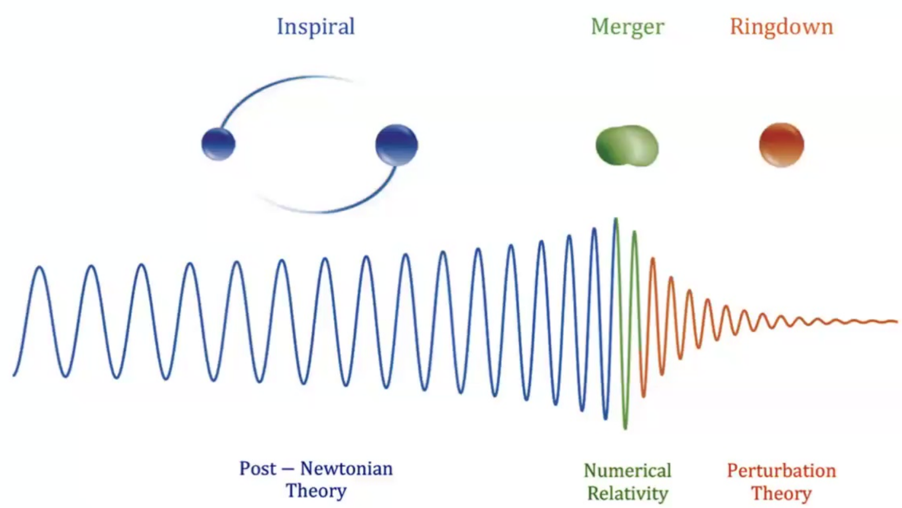
Why use perturbation theory?

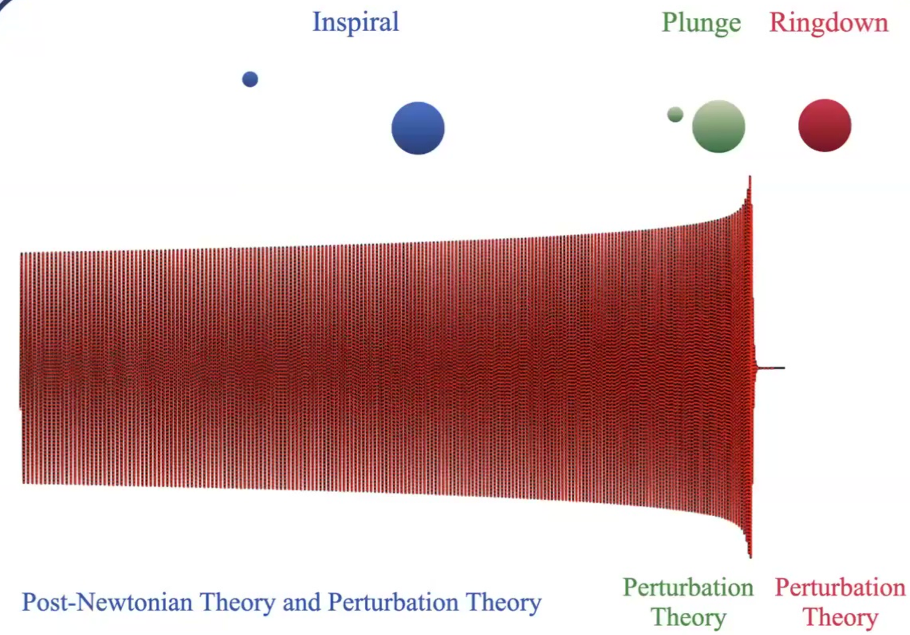
Hundreds of thousands of cycles and **disparate length scales**

Comparable-mass-ratio waveform

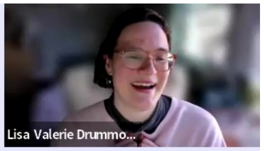


Schematic from: *Antelis & Moreno, 2017, arXiv: 1610.03567*

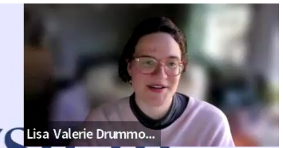
EMRI waveform

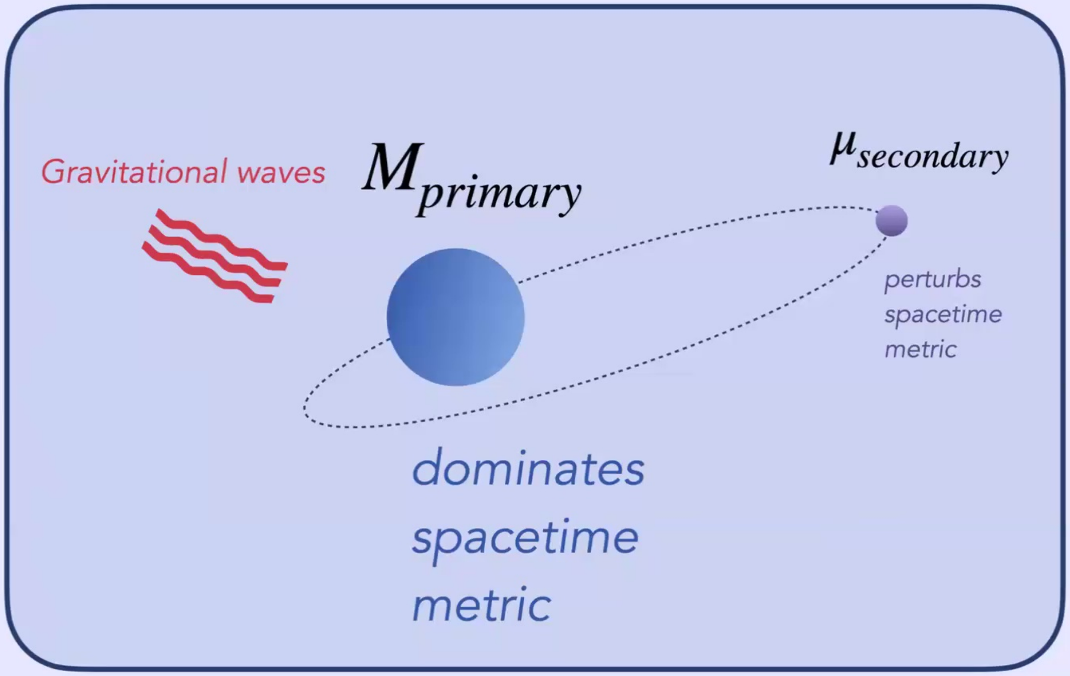


EMRI waveform from: *Zenginoğlu and Khanna, 2011, arXiv: 1108.1816*



EMRIs: Relativity's **very own** perturbative system





Use general relativistic perturbation theory with $\mu_{secondary}/M_{primary}$ as the small parameter

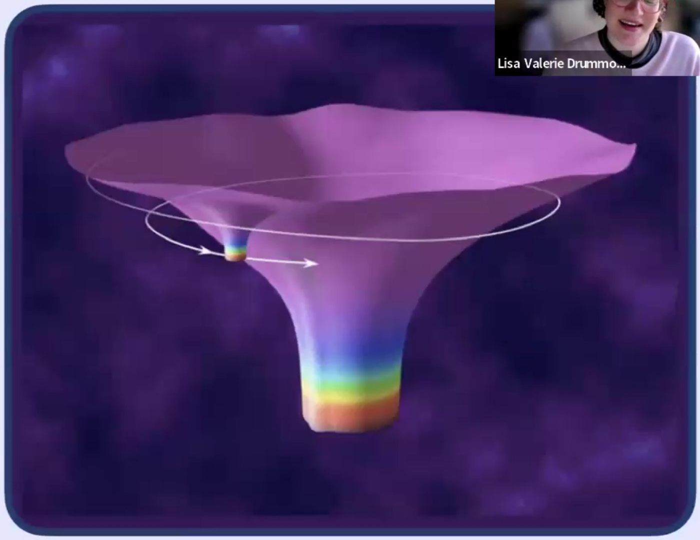
Black hole perturbation theory:

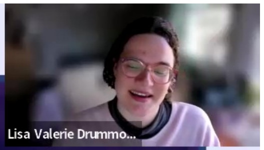
$$0^{th} + 1^{st} + 2^{nd} + \ldots$$

order corrections to the background spacetime of the larger body

What about the *gravity* due to the small black hole?

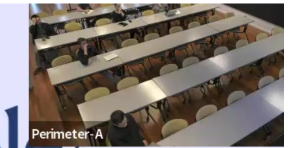
What does the geodesic picture omit?





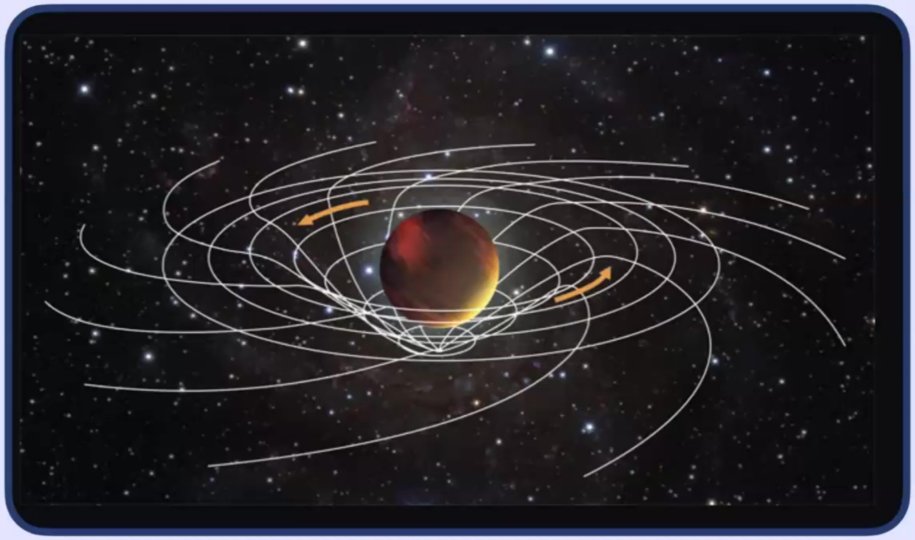
Assumes that the small body **responds to gravity** but **doesn't generate gravity** (doesn't bend spacetime)

Small body generates its own gravity → gravitational self-force



What about the *spin* of the small black hole?

What does the geodesic picture omit?



In general, **all astrophysical black holes** will have some spin, including the **smaller "secondary" black hole!**

Small body has spin → **spinning-secondary** effects, including **spin-curvature force**

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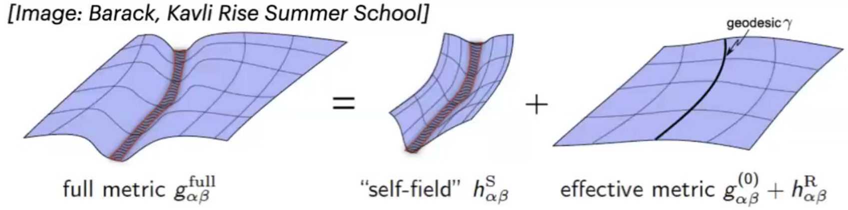
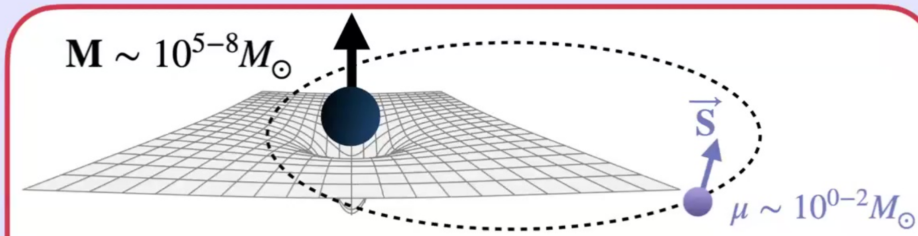
Post-geodesic corrections

[Image: Barack, Kavli Rise Summer School]



Gravitational-self force: *How the small body dimples spacetime, and how that backreacts on its motion*



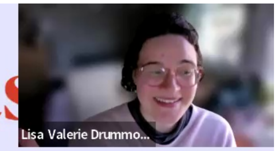
Spin-curvature coupling: *How the small body's spin couples to curvature, and how that backreacts on its motion*

Accurate waveform models require ***two types of corrections***:

1. ***gravitational self-force,*** and
2. ***spin-curvature force***

We must include both of these effects in **EMRI models** for LISA data analysis

Secondary spin contributes to the GW phas

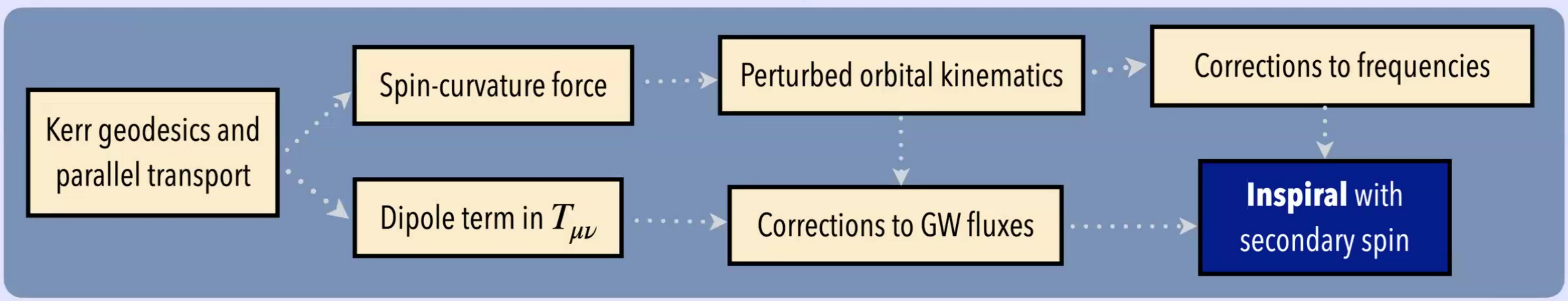


GW phase:

$$\Phi = \underbrace{\varphi_0 \varepsilon^{-1}}_{\text{adiabatic: } f^\alpha_{\text{ad}}} + \underbrace{\varphi_1 \varepsilon^{0}}_{\text{post-1-adiabatic: } \langle f^\alpha_{\text{SCF}} \rangle + f^\alpha_{\text{oscil}} + \langle f^{(1)\alpha}_{\text{cons}} \rangle + \langle f^{(2)\alpha}_{\text{diss}} \rangle + \langle f^\alpha_{\text{dipole,diss}} \rangle}$$

Gravitational self force

Secondary spin effects



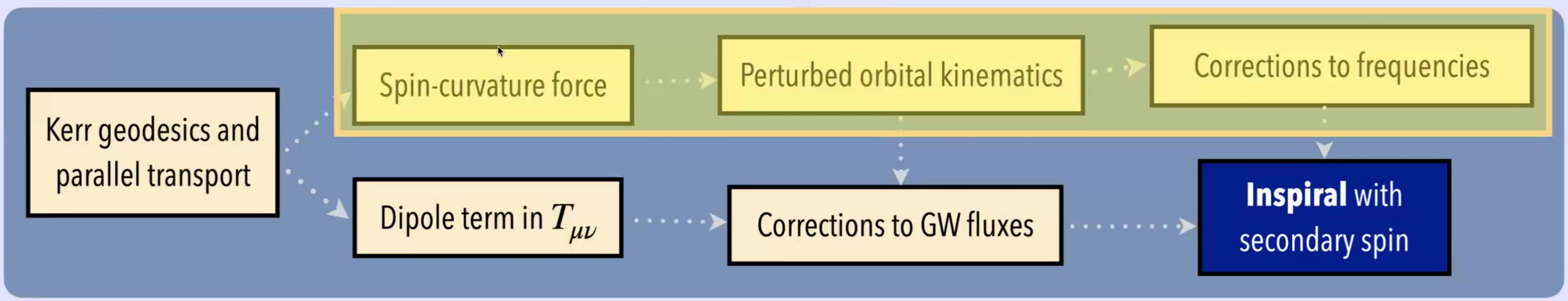
Secondary spin contributes to the GW phas

GW phase:

$$\Phi = \underbrace{\varphi_0 \varepsilon^{-1}}_{\text{adiabatic: } f^\alpha_{\text{ad}}} + \underbrace{\varphi_1 \varepsilon^{0}}_{\text{post-1-adiabatic: } \langle f^\alpha_{\text{SCF}}\rangle +}$$

$$f^\alpha_{\text{oscil}} + \langle f^{(1)\alpha}_{\text{cons}}\rangle + \langle f^{(2)\alpha}_{\text{diss}}\rangle + \langle f^\alpha_{\text{dipole,diss}}\rangle$$

Secondary spin effects



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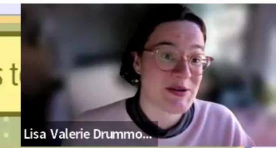
Flowchart** for spinning secondary contribution to **GW phase

Kerr geodesics and parallel transport

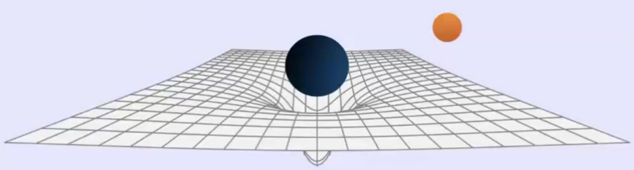
Spin-curvature force → Perturbed orbital kinematics → Corrections t

Dipole term in $T_{\mu\nu}$ → Corrections to GW fluxes → **Inspiral** with secondary spin



Kinematics of an orbiting small body

Non-spinning body: Geodesic equations



$$\ddot{x}^\alpha = f^\alpha_{geo}\,, \quad f^\alpha_{geo} \equiv -\Gamma^\alpha_{\beta\gamma}\frac{dx^\beta}{d\tau}\frac{dx^\gamma}{d\tau}$$

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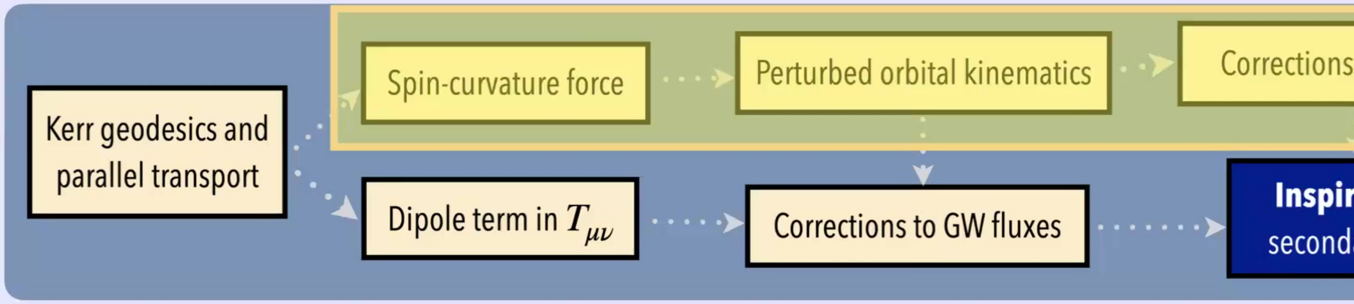
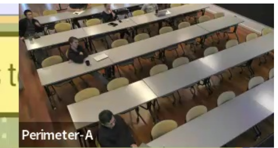
Flowchart** for spinning secondary contribution to **GW phase





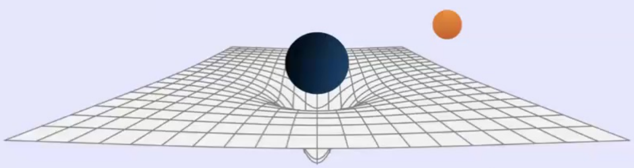
Kinematics of an orbiting small body

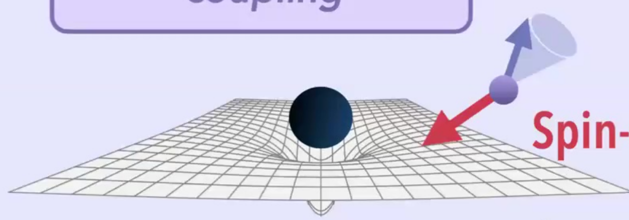
★ *Coupling between curvature and small-body spin leads to **spin-curvature force***

★ *Pushes the motion of the small body **away from** the geodesic orbit and causes small body's **spin to precess***

Non-spinning body: Geodesic equations

Spinning body: Spin-curvature coupling





$$\ddot{x}^{\alpha} = f^{\alpha}_{geo}\,, \quad f^{\alpha}_{geo} \equiv -\Gamma^{\alpha}_{\beta\gamma}\frac{dx^{\beta}}{d\tau}\frac{dx^{\gamma}}{d\tau}$$

$$\ddot{x}^{\alpha} = f^{\alpha}_{geo} + f^{\alpha}_{SCF}\,, \quad f^{\alpha}_{SCF} \equiv -\frac{1}{2\mu}R^{\alpha}{}_{\nu\lambda\sigma}u^{\nu}S^{\lambda\sigma}$$

M. Mathisson,1937; A. Papapetrou, 1951; W. G. Dixon, 1970

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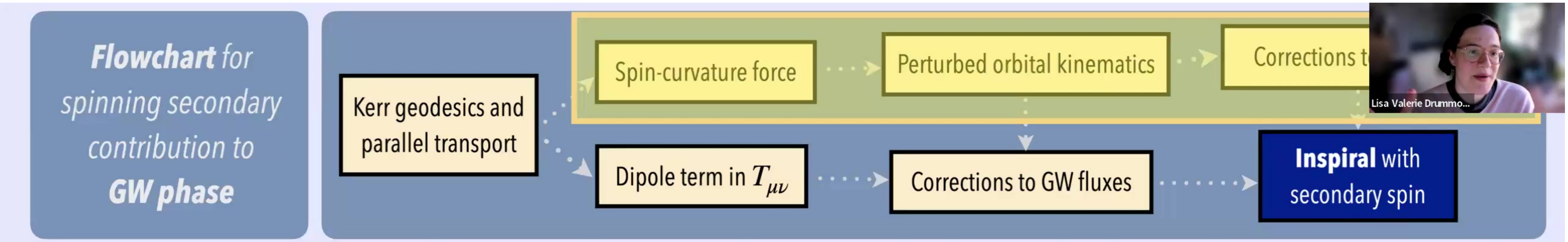


Mathisson-Papapetrou-Dixon equations

Equations describing the motion of a ***spinning test body*** *in curved spacetime*

$$\frac{Dp^\alpha}{d\tau} = -\frac{1}{2}R^\alpha_{\ \beta\gamma\delta}u^\beta S^{\gamma\delta} := f^\alpha_S/\mu \qquad (1)$$

Spin-curvature force

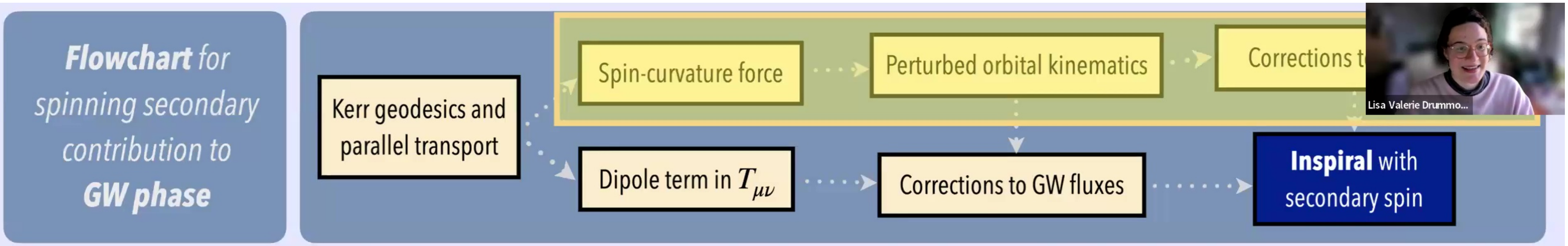
$$\frac{DS^{\alpha\beta}}{d\tau} = p^\alpha u^\beta - p^\beta u^\alpha \qquad (2)$$

$$p_\mu S^{\mu\nu} = 0 \qquad (3)$$

→ Tulczyjew-Dixon spin-supplementary condition

$S^{\mu\nu}$ *is the spin tensor of the secondary*

$S^\mu = -\frac{1}{2}\epsilon^{\mu\nu}{}_{\alpha\beta}p_\nu S^{\alpha\beta}$ *is the spin vector of the secondary*



Mathisson-Papapetrou-Dixon equations

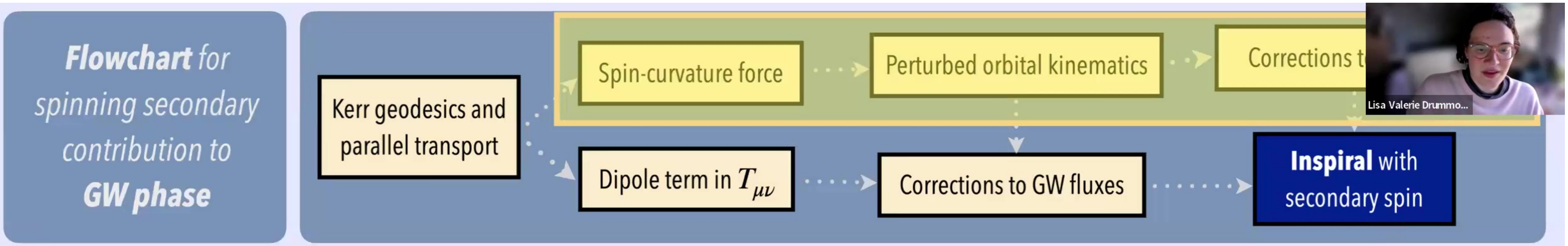
…to leading-order in spin

$$\frac{Du^\alpha}{d\tau} = -\frac{1}{2\mu} R^\alpha_{\ \beta\gamma\delta} u^\beta S^{\gamma\delta}$$

Motion of the small body

$$\frac{DS^\mu}{d\tau} = 0$$

Evolution of spin vector



Kinematics of an orbiting small body

Spin of small body orbiting a black hole **modifies orbital frequencies Ω_r, Ω_θ and Ω_ϕ.**

Aim of our analysis: Find corrections to Ω_r, Ω_θ and Ω_ϕ as functions of completely general orbital geometry (p, e, I)

1. Parameterization
in terms of orbital geometry (p, e, I) →

$$r = \frac{p}{1 + e\cos\left(\hat{\chi}_r + \chi_r^S\right)} + \delta r_S$$

L. V. Drummond & S. A. Hughes, arXiv:2201.13334, arXiv:2201.13335

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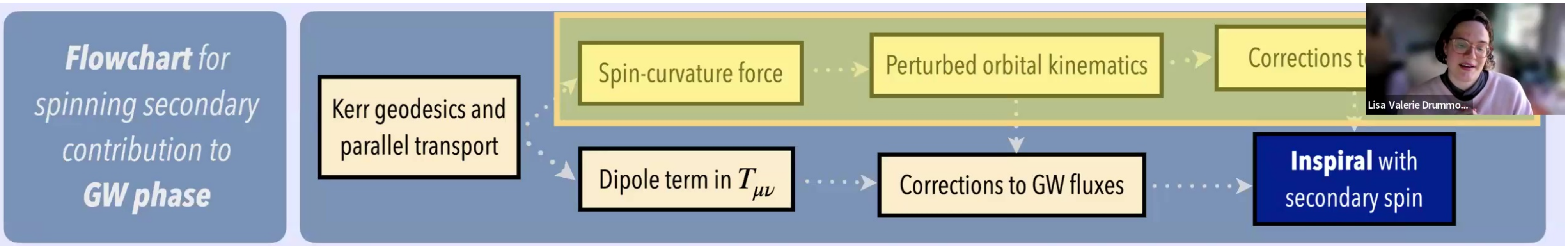

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Why? Provides physical insight into completely general orbits, which are astrophysically realistic for EMRIs

L. V. Drummond *& S. A. Hughes,* ***arXiv:2201.13334,*** ***arXiv:2201.13335***

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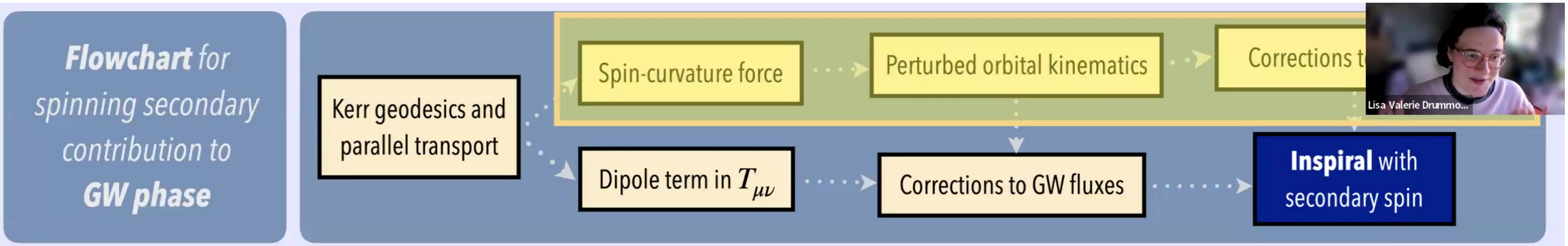

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1. Parameterization *in terms of orbital geometry (p, e, I)* → **Why?** Provides physical insight into completely general orbits, which are astrophysically realistic for EMRIs

2. Frequency-domain *approach: Orbital quantities as Fourier expansions* →

$$f[r, \theta, S] = \sum_{j=-1}^{1} \sum_{k,n=-\infty}^{\infty} f_{jkn} e^{-(ij\Upsilon_s + ik\Upsilon_\theta + in\Upsilon_r)\lambda}$$

***L. V. Drummond** & S. A. Hughes, arXiv:2201.13334, arXiv:2201.13335*

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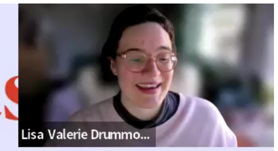
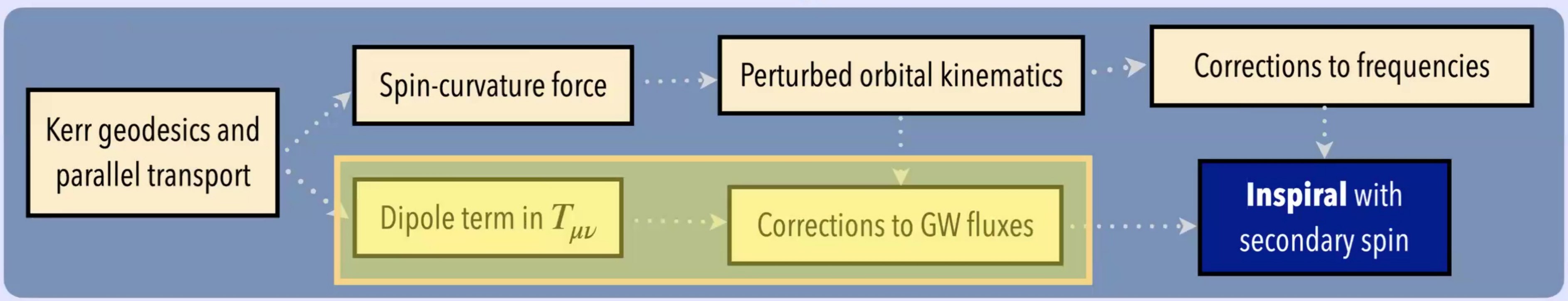
Secondary spin contributes to the GW phas



GW phase:

$$\Phi = \underbrace{\varphi_0 \varepsilon^{-1}}_{\text{adiabatic: } f^{\alpha}_{\text{ad}}} + \underbrace{\varphi_1 \varepsilon^{0}}_{\text{post-1-adiabatic: } \langle f^{\alpha}_{\text{SCF}} \rangle + f^{\alpha}_{\text{oscil}} + \langle f^{(1)\alpha}_{\text{cons}} \rangle + \langle f^{(2)\alpha}_{\text{diss}} \rangle + \langle f^{\alpha}_{\text{dipole,diss}} \rangle}$$

Secondary spin effects



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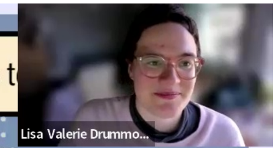
Flowchart for *spinning secondary contribution to* ***GW phase***

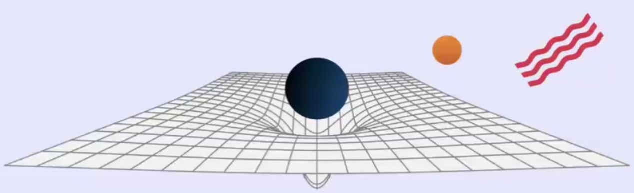
Kerr geodesics and parallel transport → Spin-curvature force → Perturbed orbital kinematics → Corrections t… → **Inspiral** with secondary spin

Kerr geodesics and parallel transport → Dipole term in $T_{\mu\nu}$ → Corrections to GW fluxes → **Inspiral** with secondary spin



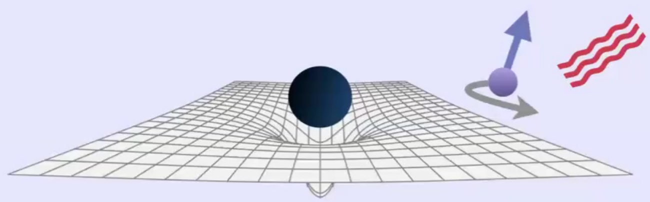
Radiation due to an orbiting small body

Non-spinning body: Point-particle GW fluxes



$$T^{\mu\nu}_{geo} = \int d\tau \left(\frac{\mu u^{\mu}_{geo} u^{\nu}_{geo}}{\sqrt{-g}} \delta^4 \left(x^\rho - z^\rho_{geo}(\tau) \right) \right)$$

Spinning body: Spinning-particle GW fluxes

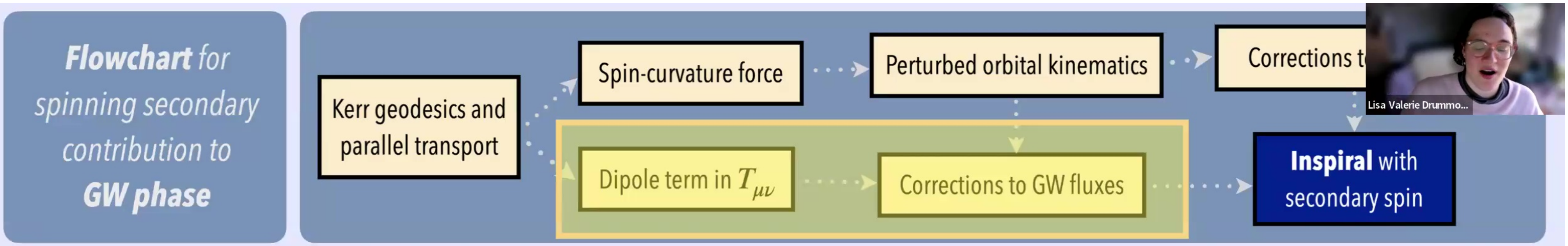


$$T^{\mu\nu}_{spin} = \int d\tau \left(\frac{p^{(\mu} u^{\nu)}}{\sqrt{-g}} \delta^4 \left(x^\rho - z^\rho(\tau) \right) - \nabla_\alpha \left(\frac{S^{\alpha(\mu} u^{\nu)}}{\sqrt{-g}} \delta^3 \left(x^\rho - z^\rho(\tau) \right) \right) \right)$$

Compute GW radiation using the **Teukolsky equation**

$${}_{-2}\mathcal{O}\ {}_{-2}\Psi = 4\pi\Sigma\mathcal{T}$$

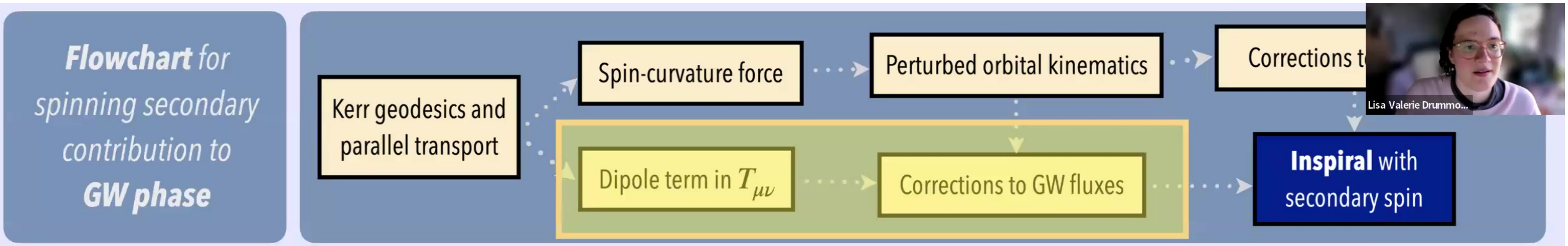
The source term $\mathcal{T}$ in the **Teukolsky equation** can be found from the **stress-energy tensor** $T^{\mu\nu}$ describing the small body



We used the framework I developed in my previous work (**arXiv:2201.13334** and **arXiv:2201.13335**) to compute the **gravitational wave fluxes** for an EMRI system with **secondary spin** for completely general orbital configurations **for the first time**.

*Skoupý, Lukes-Gerakopoulos, **L. V. Drummond** & Hughes, 2023, **arXiv:2303.16798***

This is **essential** for astrophysically realistic systems detectable by LISA



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$$\langle \dot{J}_z \rangle = -q \left(\langle \mathcal{F}^{J_z\mathcal{I}^+} \rangle_S + \langle \mathcal{F}^{J_z\mathcal{H}^+} \rangle_S \right)$$

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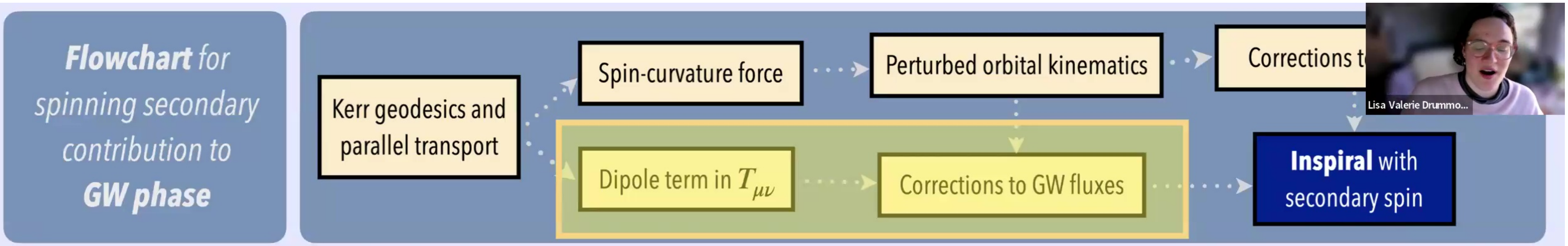

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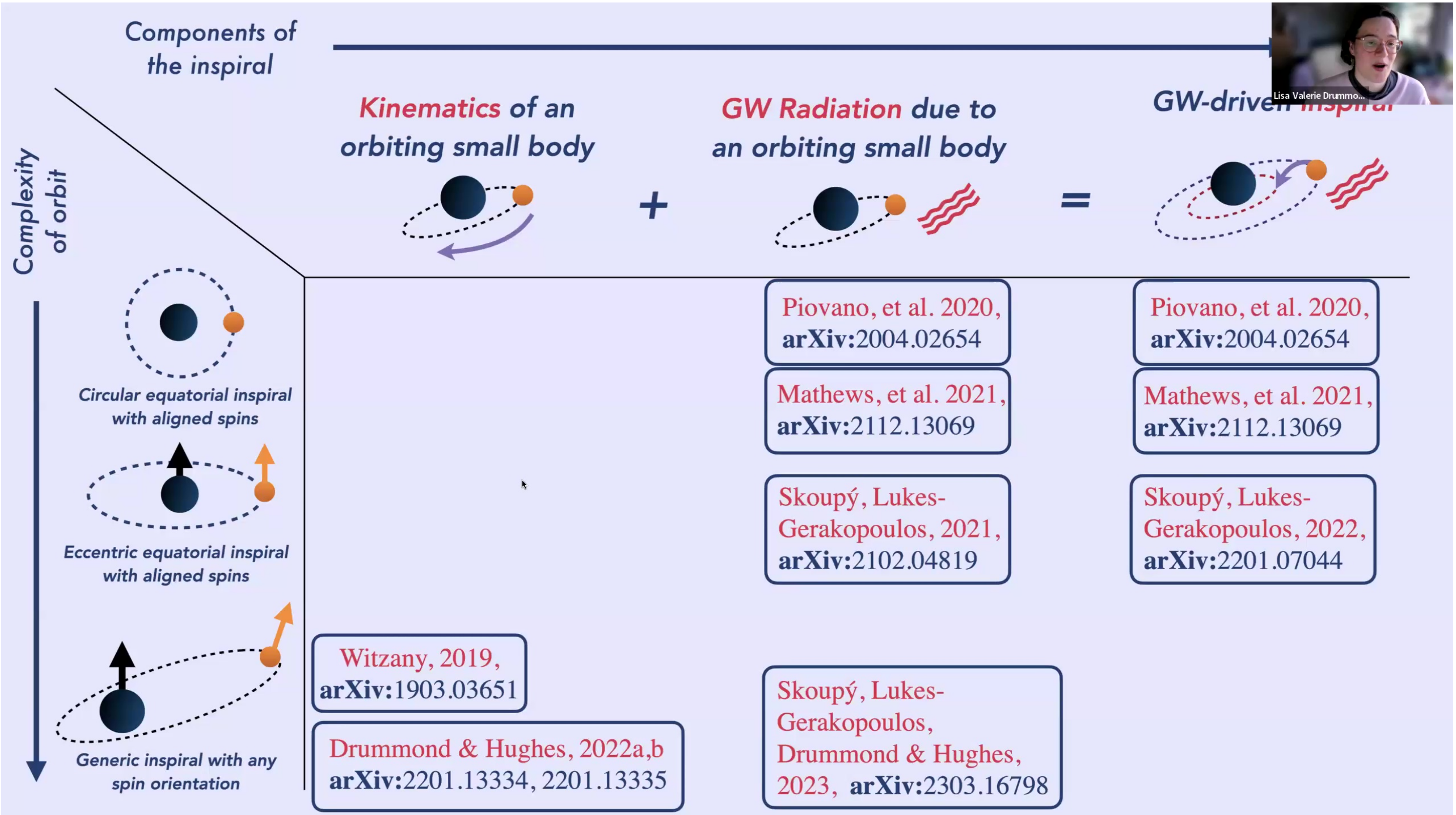
Components of the inspiral
Kinematics of an orbiting small body
+
GW Radiation due to an orbiting small body
=
GW-drive
Lisa Valerie Drummo...
Complexity of orbit
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Eccentric equatorial inspiral with aligned spins
Generic inspiral with any spin orientation
Piovano, et al. 2020, arXiv:2004.02654
Mathews, et al. 2021, arXiv:2112.13069
Skoupý, Lukes-Gerakopoulos, 2021, arXiv:2102.04819
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Skoupý, Lukes-Gerakopoulos, 2022, arXiv:2201.07044
Witzany, 2019, arXiv:1903.03651
Drummond & Hughes, 2022a,b arXiv:2201.13334, 2201.13335
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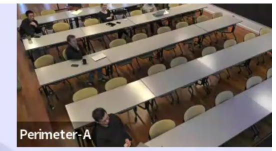
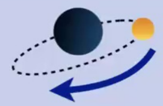


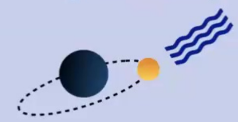
How do we build a ***generic*** inspiral?

Kinematics *of an orbiting small body*



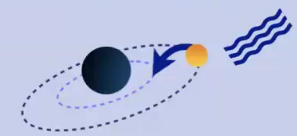
+

GW ***radiation*** *due to an orbiting small body*



=

GW-driven ***inspiral***



Spinning-body orbital kinematics

Drummond & Hughes, 2022a and 2022b

Spinning-particle GW fluxes

Skoupý, Lukes-Gerakopoulos, Drummond & Hughes, 2023

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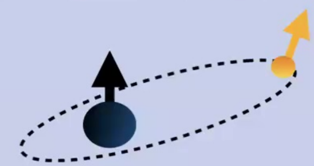
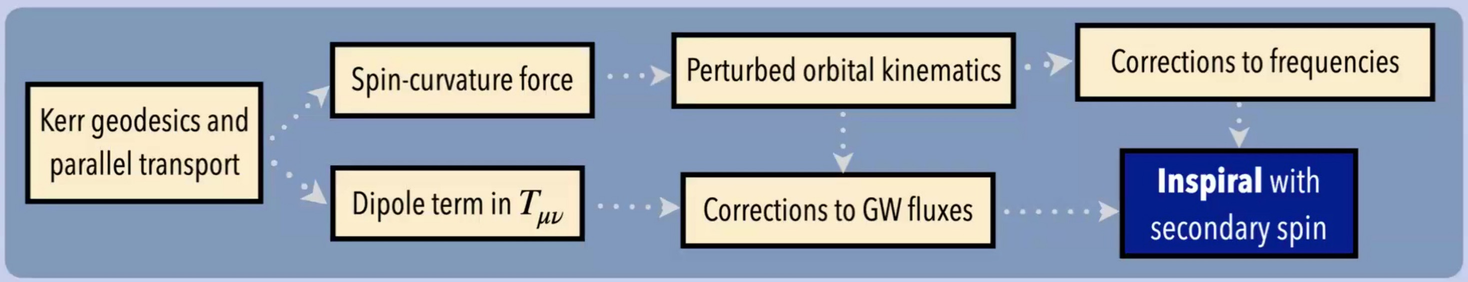


Work in progress; close, but challenges remain!



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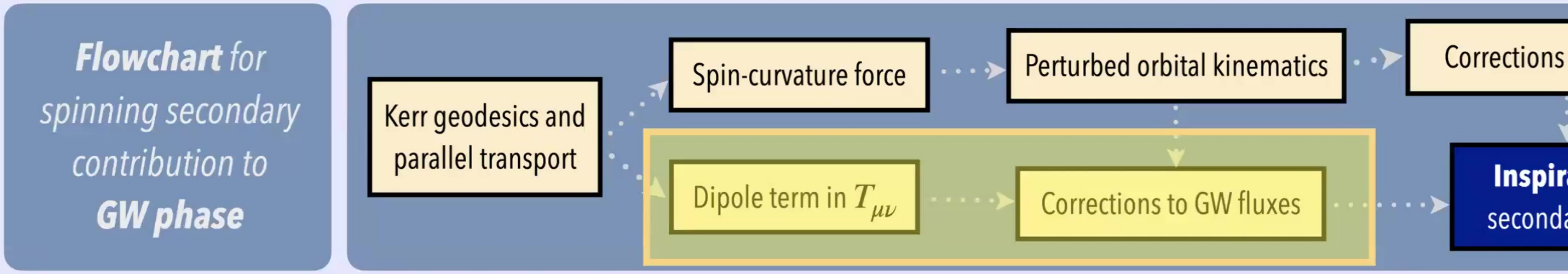

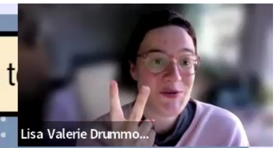

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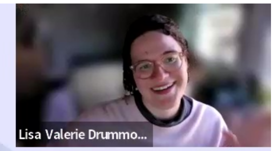
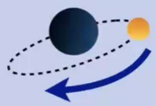
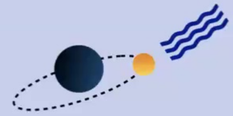


Kinematics *of an orbiting small body*



+

GW ***radiation*** *due to an orbiting small body*



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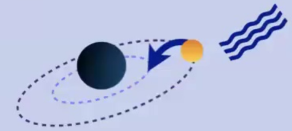
GW-driven ***inspiral***



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Skoupý, Lukes-Gerakopoulos, Drummond & Hughes, 2023

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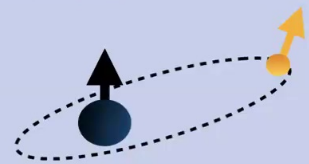
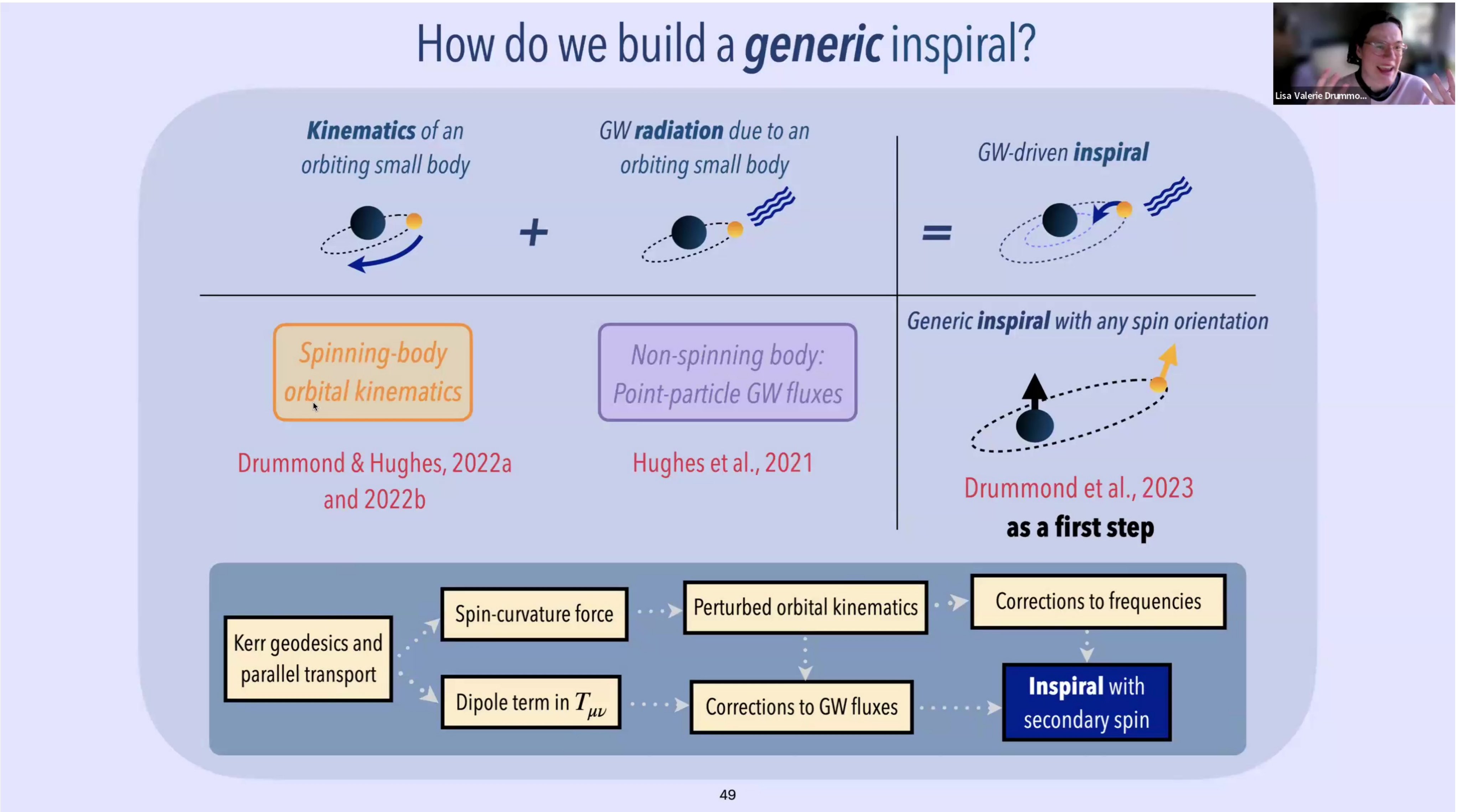


Work in progress; close, but challenges remain!

- Kerr geodesics and parallel transport
- Spin-curvature force
- Dipole term in $T_{\mu\nu}$
- Perturbed orbital kinematics
- Corrections to GW fluxes
- Corrections to frequencies
- **Inspiral** with secondary spin

48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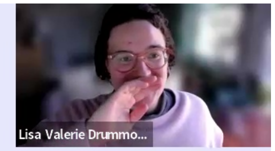
49
Lisa Valerie Drummo...



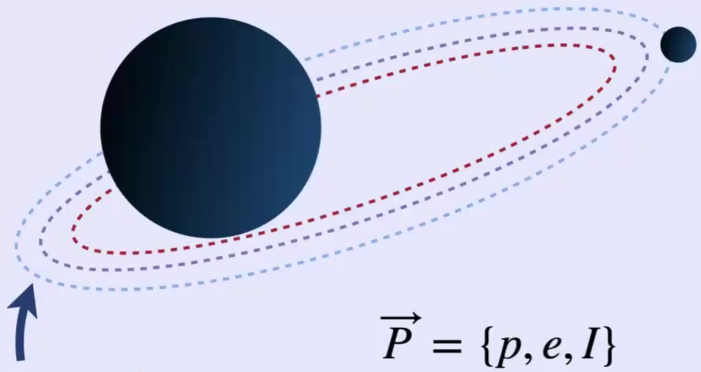
How do we build a ***generic*** waveform?

Osculating geodesics + near-identity transformations

Why **osculating geodesics**?

Flexible and **modular**, can straightforwardly combine different **relativistic** contributions to the motion, as well as **environmental** effects

Stitch together *a sequence of* ***osculating orbits*** *to construct an* ***inspiral***

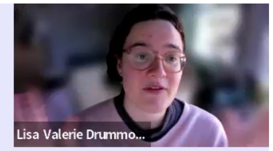


Sequence of osculating orbits

$$\vec{P} = \{p, e, I\}$$

$$\vec{q} = \{q_r, q_z, q_s\}$$

Drummond *et al.*, 2023, **arXiv:2310.08438**



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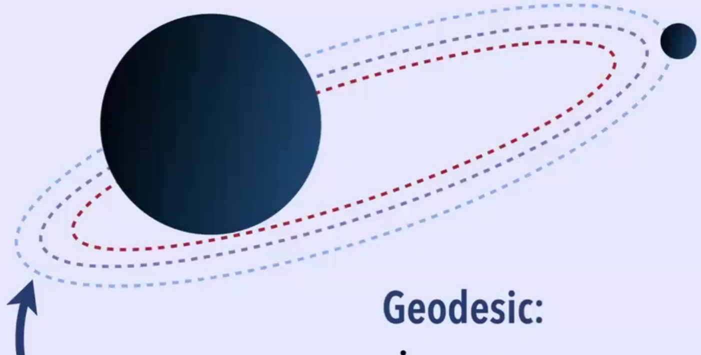


Sequence of osculating orbits

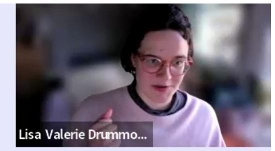
Geodesic:

$$\dot{P}_i = 0$$

$$\dot{q}_i = \Upsilon_i^{(0)}(\vec{P})$$

Drummond *et al.*, 2023, **arXiv:2310.08438**

How do we build a ***generic*** waveform?



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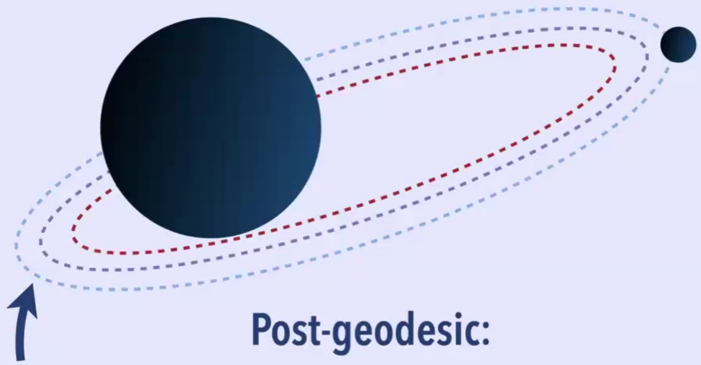


Sequence of osculating orbits

Post-geodesic:

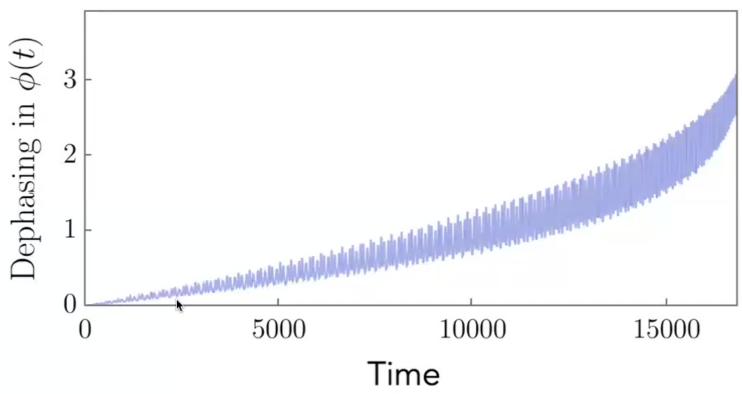
$$\dot{P}_i = F_i(\overrightarrow{P}(t), \vec{q}(t))$$

$$\dot{q}_i = \Upsilon_i^{(0)}(\overrightarrow{P}(t)) + f_i(\overrightarrow{P}(t), \vec{q}(t))$$

Drummond *et al.*, 2023, **arXiv:2310.08438**

How do we build a ***generic*** waveform?

Osculating geodesics + **near-identity transformations**

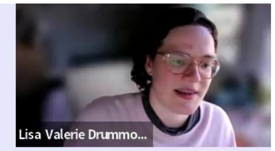


Dephasing *between the spinning- and non-spinning-body trajectory. Many* ***short timescale oscillations****; only need the averaged behavior on long timescales*

Why **near-identity transformations**?

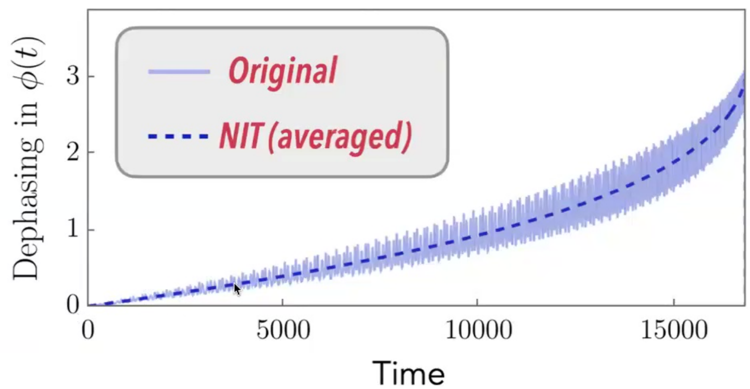
Naturally **interfaces with** the osculating geodesic approach. **Speeds up** the evaluation of the trajectory by 2–5 **orders of magnitude**.

Drummond *et al.*, 2023, **arXiv:2310.08438**



How do we build a *generic* waveform?

Osculating geodesics + near-identity transformations



*Use a **NIT (Near-Identity Transformation)** to isolate the long timescale evolution*

$$P_i \to \tilde{P}_i \qquad \dot{P}_i = \tilde{F}_i(\tilde{P}_i)$$

$$q_i \to \tilde{q}_i \qquad \dot{q}_i = \Upsilon_i^{(0)}(\vec{P}) + \tilde{f}_i(\tilde{P}_i)$$

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Drummond *et al.*, 2023, **arXiv:2310.08438**

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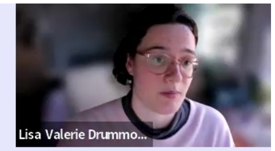
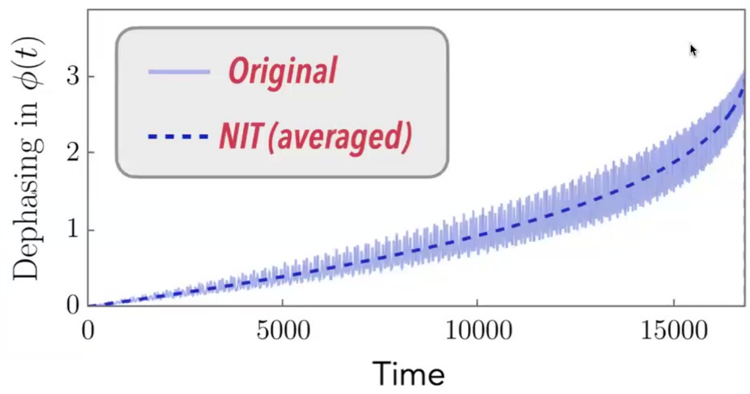


Osculating geodesics + near-identity transformations



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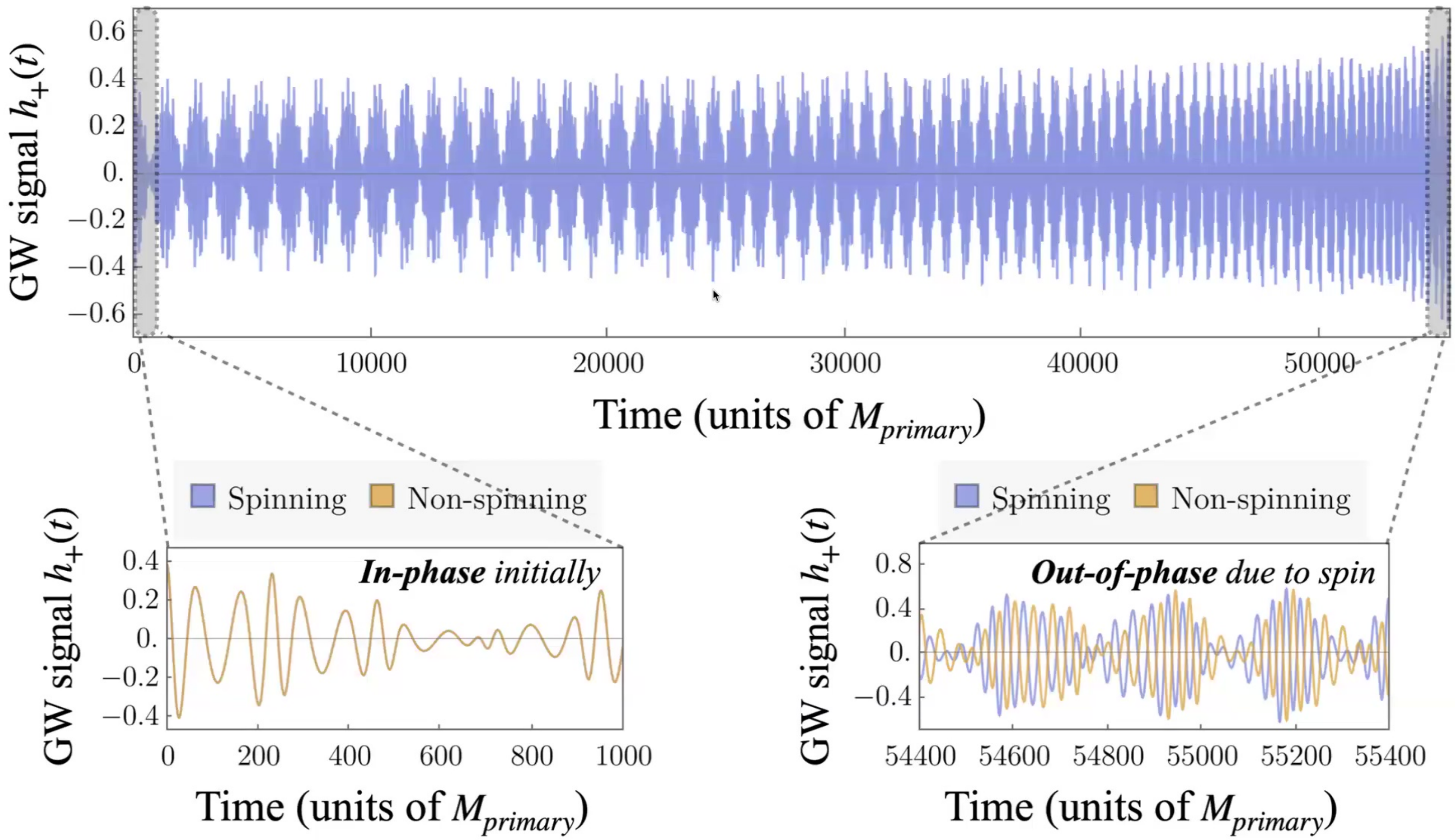

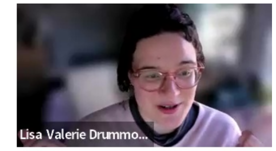

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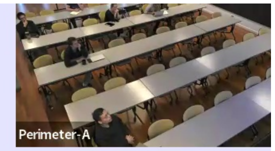

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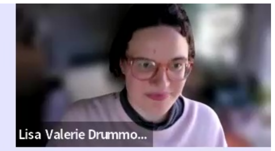
Open questions and *future work*



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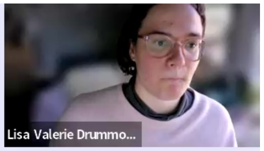

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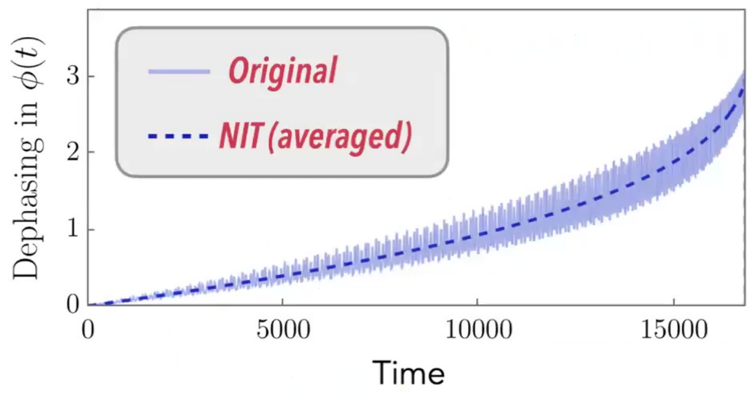

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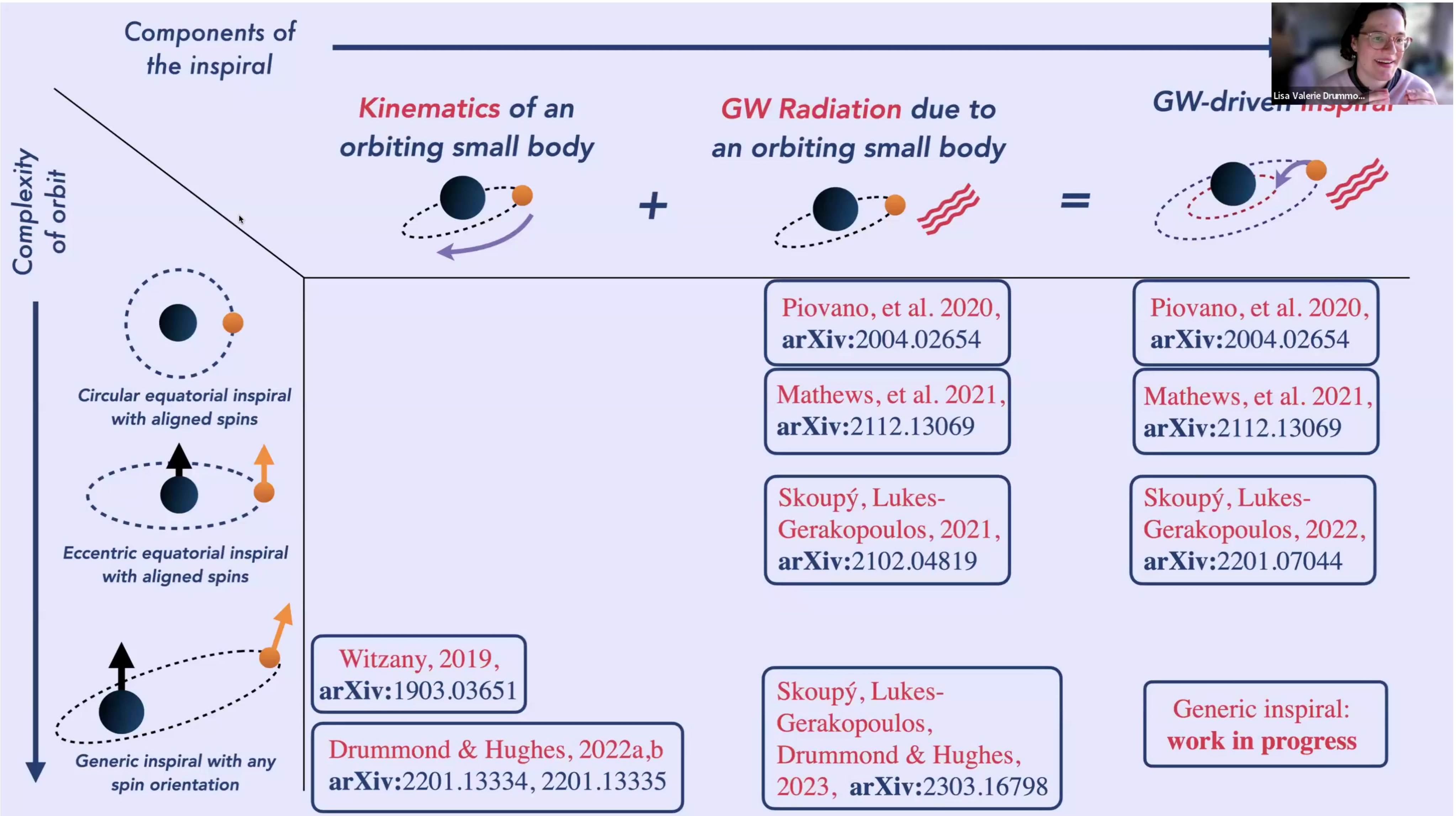
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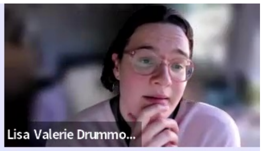
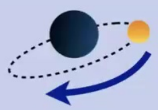


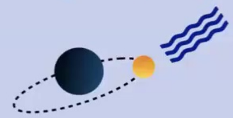
How do we build a ***generic*** inspiral?

__Kinematics__ of an orbiting small body



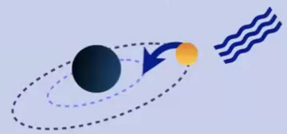
\+

GW __radiation__ due to an orbiting small body



=

GW-driven __inspiral__



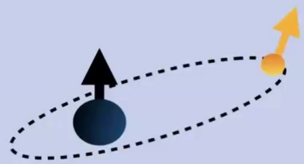
Spinning-body orbital kinematics

Drummond & Hughes, 2022a and 2022b

Non-spinning body: Point-particle GW fluxes

Hughes et al., 2021

Generic __inspiral__ with any spin orientation



Drummond et al., 2023

as a first step

Kerr geodesics and parallel transport → Spin-curvature force → Perturbed orbital kinematics → Corrections to frequencies → **Inspiral** with secondary spin

Kerr geodesics and parallel transport → Dipole term in $T_{\mu\nu}$ → Corrections to GW fluxes → **Inspiral** with secondary spin

Perturbed orbital kinematics → Corrections to GW fluxes

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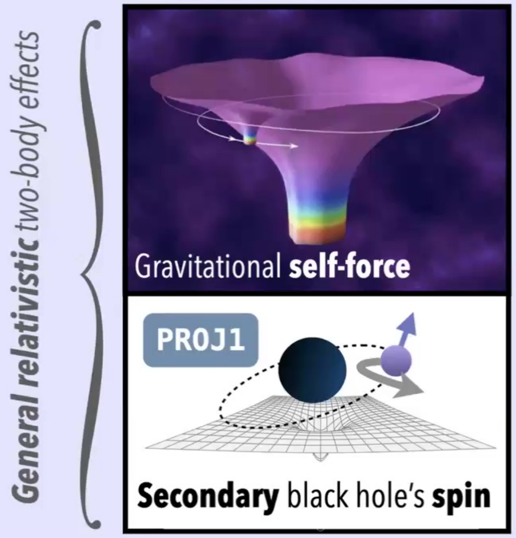
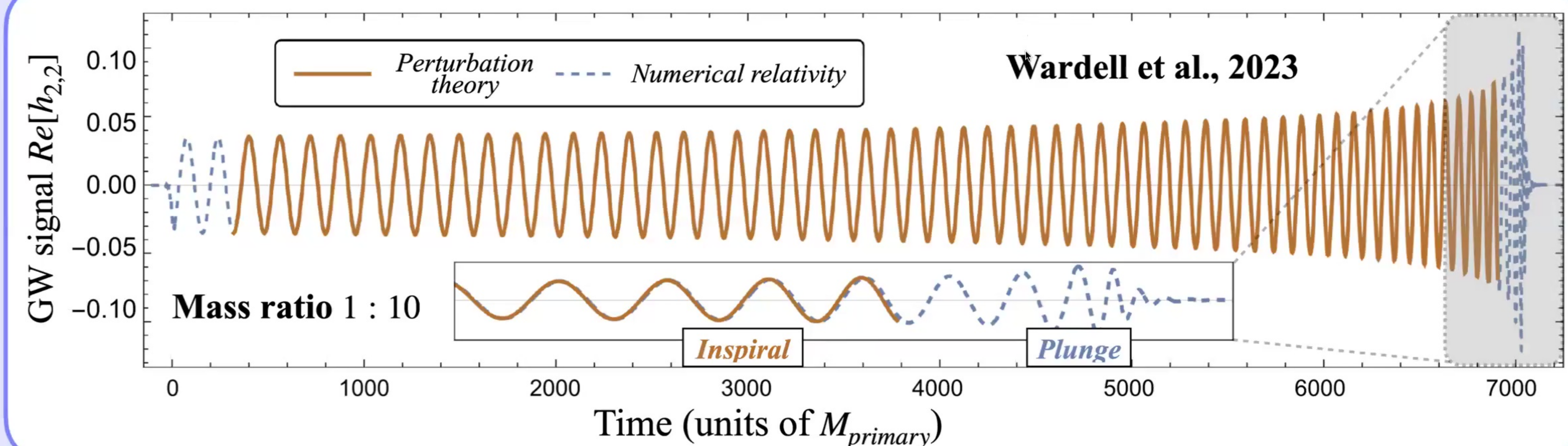
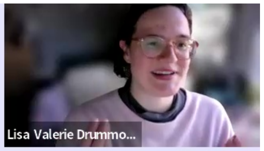
Avenue 1: Perturbation theory is ***surprisingly effecti*** beyond the regime of its formal validity and can be combined with numerical relativity to make powerful surrogate models

Therefore, progress in perturbation theory can improve **LIGO** source models as well, even before LISA launches!



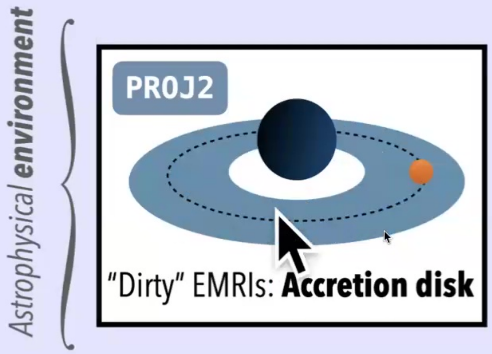




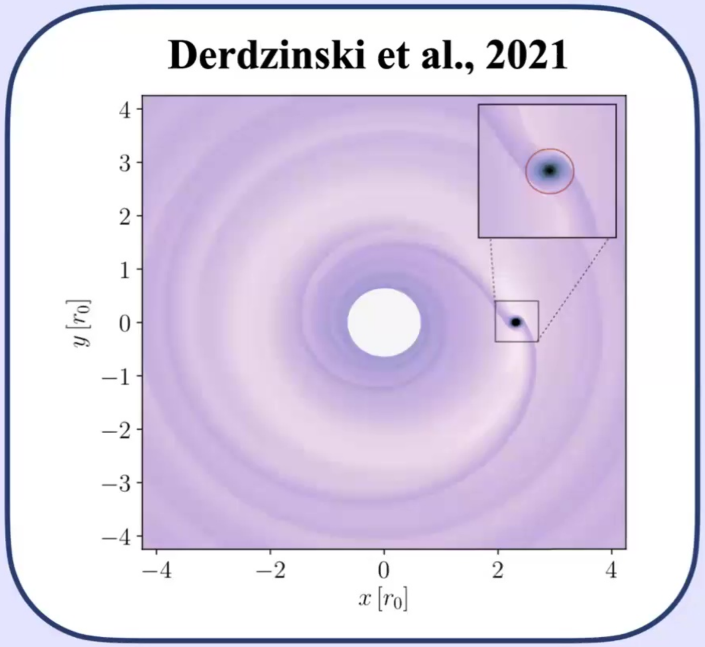
Open questions and *future work*

★ **Avenue 1:** Enhance intermediate mass ratio inspiral (relevant to LIGO) models using perturbation theory

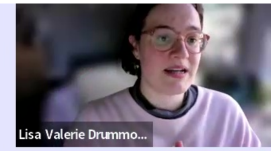
★ **Avenue 2:** Include realistic effects from the astrophysical environment surrounding the black hole into EMRI models, including the **accretion disk**



Avenue 2: We need to include realistic effects from the astrophysical environment surrounding the black hole into EMRI models, including the **accretion disk**



- Combine gas-embedded EMRI accretion disk simulations with **GW-driven** orbital dynamics
- Incorporate gas-drag effects into EMRI waveform models



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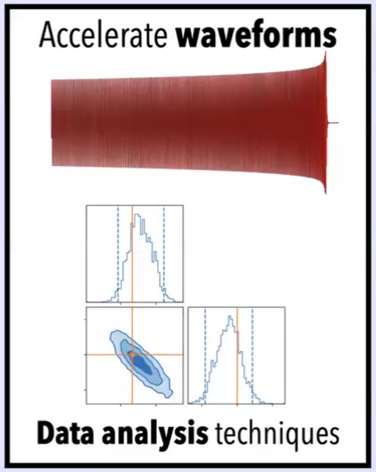
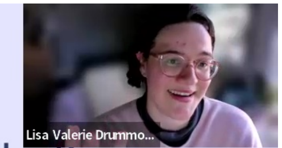
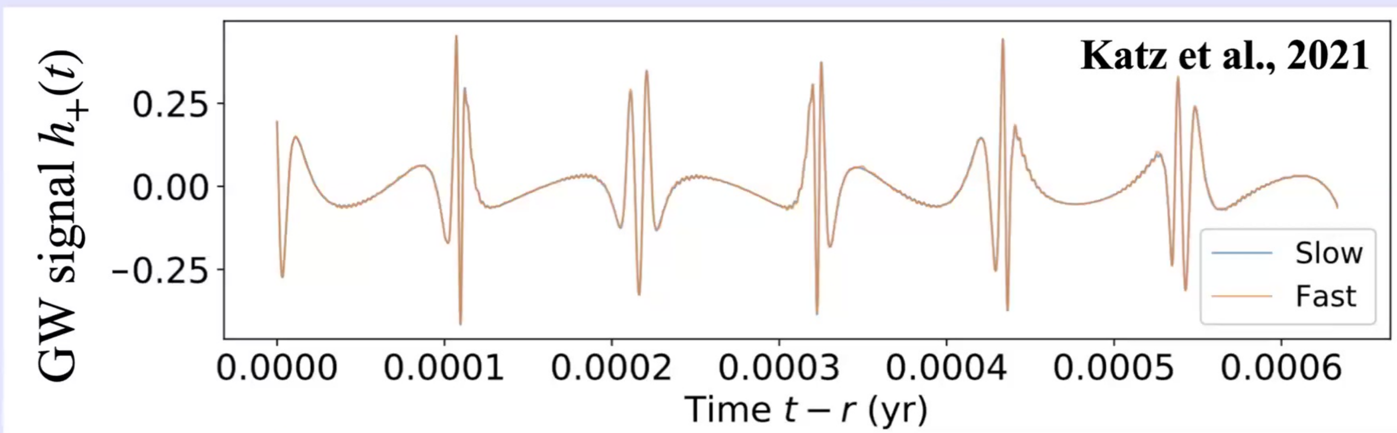
★ **Avenue 3:** Accelerate the computation of the enhanced waveform mode and incorporate all of these developments into the **Black Hole Perturbation Toolkit**

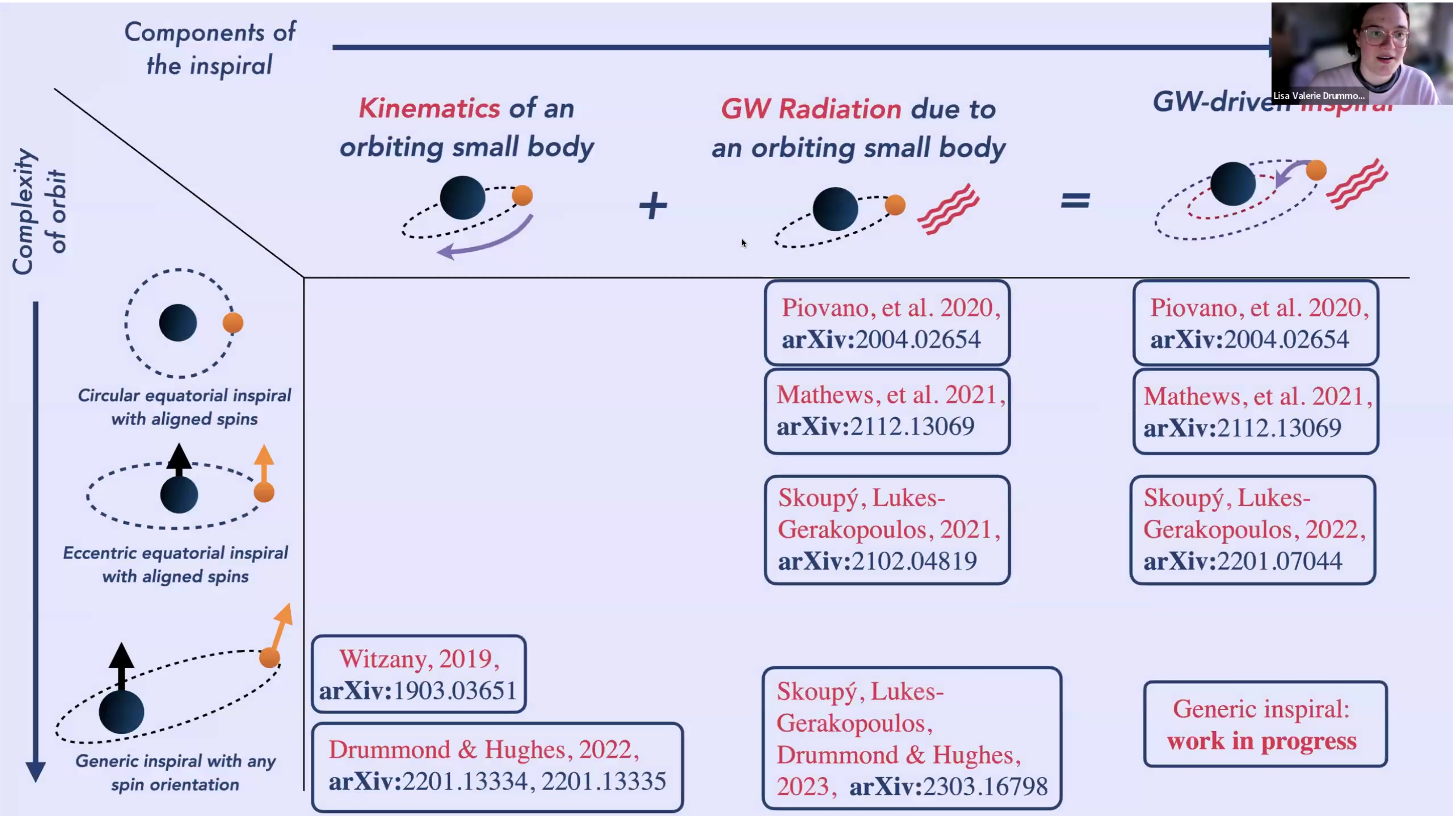




Avenue 3:

1. Consolidate the various effects on the EMRI waveform including spinning-secondary and accretion disk effects.
2. Accelerate the computation of this enhanced waveform model using GPU acceleration and neural network interpolation via the Fast EMRI Waveforms package (arXiv:2104.04582), enabling inference over a **high-dimensional** parameter space and **precise measurements** of black hole properties.

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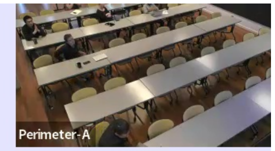

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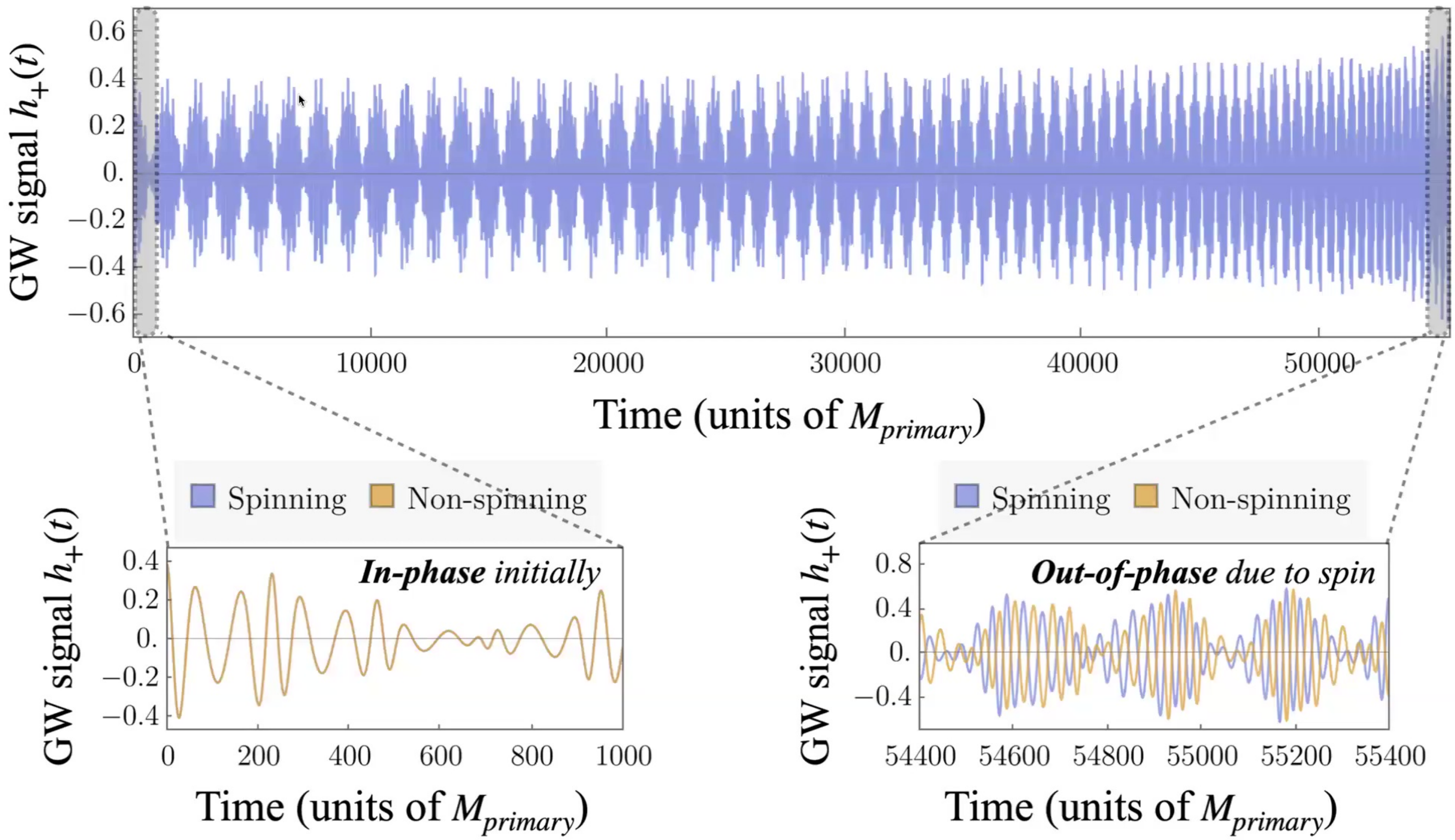

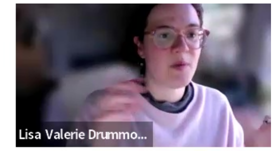

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