Title: Non-abelian symmetries can increase entanglement and induce critical dynamics

Speakers: Shayan Majidy

Series: Perimeter Institute Quantum Discussions

Date: November 29, 2023 - 11:00 AM

URL: https://pirsa.org/23110082

Abstract: Measuring the temperature of your coffee should not change the amount of coffee in your cup. This holds because the operators representing the coffee's energy and volume commute. The intuitive assumption that conserved quantities, also known as charges, commute, underpins basic physics derivations, like that of the thermal state's form and Onsager coefficients. Yet, operators' failure to commute plays a key role in quantum theory, e.g. underlying uncertainty relations. Lifting this assumption has spawned a growing subfield of quantum many-body physics [1].

How can one argue that charges' noncommutation caused a result? To isolate the effects of charges' noncommutation, we created analogous models that differ in whether their charges commute and discovered more entanglement in the noncommuting-charge model [2]. We further introduce noncommuting charges (an SU(2) symmetry) into monitored quantum circuits, circuits with unitary evolutions and mid-circuit projective measurements. Numerically, we find that the SU(2)-symmetric model has a critical phase in place of the area-law phase typically found in these circuits [3]. I will focus on the results from Ref 2 and 3. Time permitting, I'll briefly explain how one can use Lie Algebra theory to build the Hamiltonians necessary for testing the predictions of noncommuting charge physics [4].

[1] Majidy et al. "Noncommuting conserved charges in quantum thermodynamics and beyond." Nat Rev Phys (2023)
[2] Majidy et al. "Non-Abelian symmetry can increase entanglement entropy." PRB (2023)
[3] Majidy et al. "Critical phase and spin sharpening in SU(2)-symmetric monitored quantum circuits." PRB (2023)
[4] Yunger Halpern and Majidy "How to build Hamiltonians that transport noncommuting charges in quantum thermodynamics" npj QI (2022)

Zoom link https://pitp.zoom.us/j/97193579200?pwd=MkdmbWo1S2lUcUZtUFpORk5VbnFBdz09

Noncommuting charges can increase entanglement and induce critical dynamics

Shayan Majidy

Based on:

S Majidy, W Braasch, A Lasek, T Updahyaya, A Kalev, N Yunger Halpern *Nat Rev Phys* (2023)

S Majidy, A Lasek, D Huse, N Yunger Halpern *PRB* (2023)

S Majidy, U Agrawal, S Gopalakrishnan, AC Potter, R Vasseur, N Yunger Halpern *PRB* (2023)

N Yunger Halpern and **S Majidy** *npj QI* (2022)

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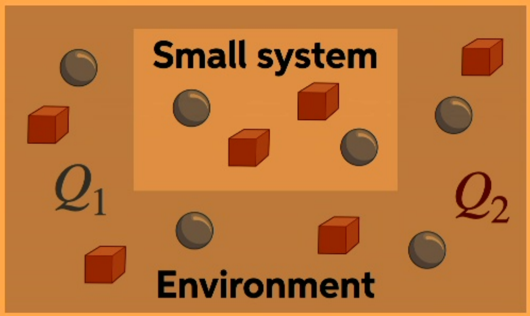
Vanier Canada Graduate scholarships

Noncommuting charges: what they are and why people care

What are they?

- Systems exchanging quantities (energy, particles, etc.)
- Called charges Q_α if conserved globally



Noncommuting charges: what they are and why people care

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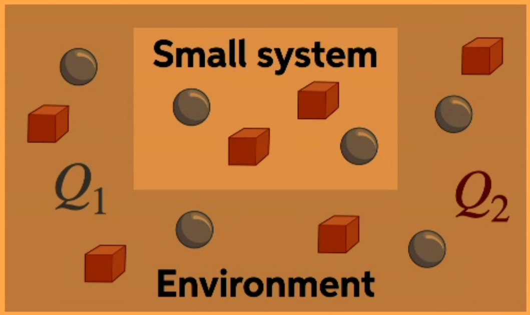

Meaningful physical difference[1]

[1] Majidy, et. al (2023)

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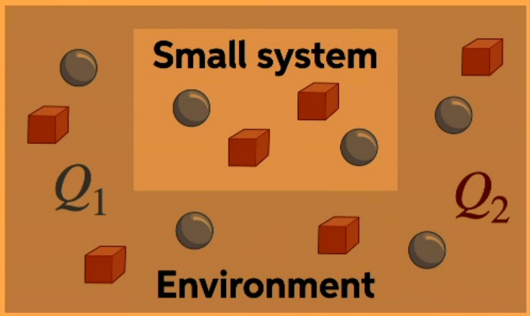
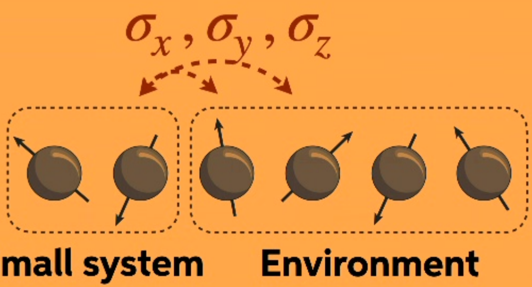


Meaningful physical difference[1]

- Onsager coefficients: Reduce entropy-production rates[2]
- Eigenstate thermalization hypothesis (ETH): Anomalous deviation from thermal state[3]

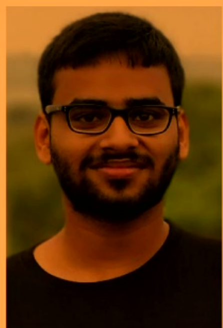
Illustrative example

- 1D spin chain + Heisenberg Hamiltonian
- Experimental testing has begun with trapped-ion systems[4-5]



[1] Majidy, et. al (2023) [2] Manzano, Parrondo, Landi (2022) [3] Murthy et. al. (2023) [4] Kranzl et. al. (2023), [5] Yunger Halpern, Majidy (2022)

Results: How $[Q_1, Q_2] \neq 0$ affects entanglement



Utkarsh Agrawal

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Increases average entanglement

- Models to isolate the effects of charges' noncommutation

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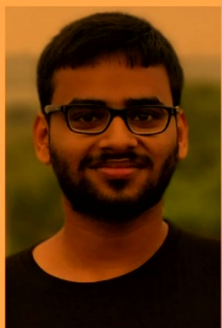

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Increases average entanglement

- Models to isolate the effects of charges' noncommutation
- Compare models' entanglement using Page curves
- S Majidy, A Lasek, D Huse, N Yunger Halpern *PRB* (2023)

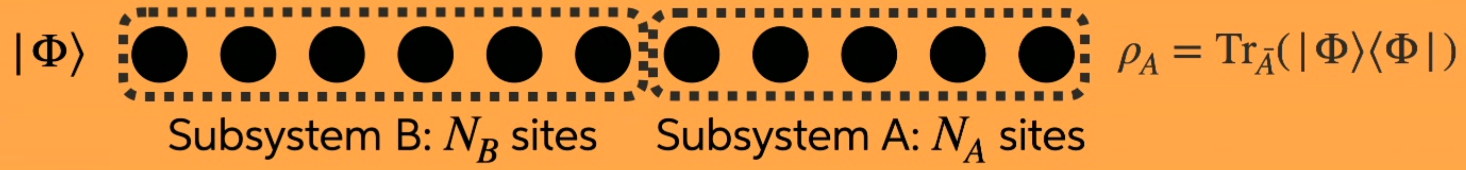
Induces a critical phase, i.e. long-range entanglement

- Introduce noncommuting charges into monitored circuits.
- Found a critical phase

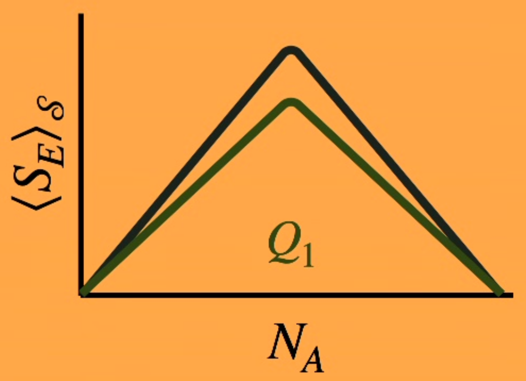
Page-curve background

How will we quantify entanglement? With Page curves. [6]

- Setup: N d-dimensional sites in a random pure state $|\Phi\rangle$ drawn from the Hilbert space $\mathcal{S}$



- Entanglement entropy: $S_E := -\mathrm{Tr}(\rho_A \log \rho_A)$.
- Average over $|\Phi\rangle$ from $\mathcal{S}$: $\langle S_E \rangle_{\mathcal{S}}$
- Plot $\langle S_E \rangle_{\mathcal{S}}$ against N_A: Page curve
- Fixed charge (fixing $\mathcal{S}$) lowers Page curve[7]



[6] Page (1993), [7] Altland, Huse, Micklitz (2022)

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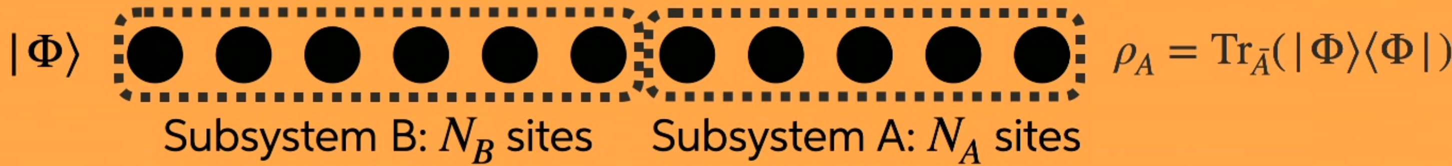

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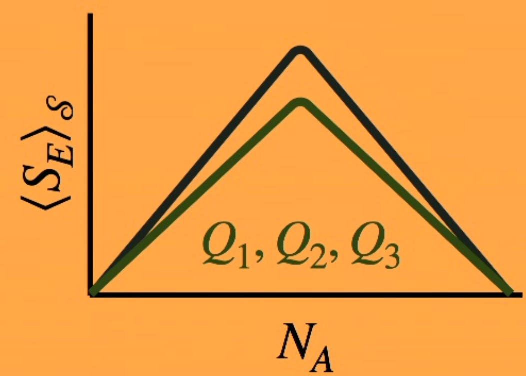
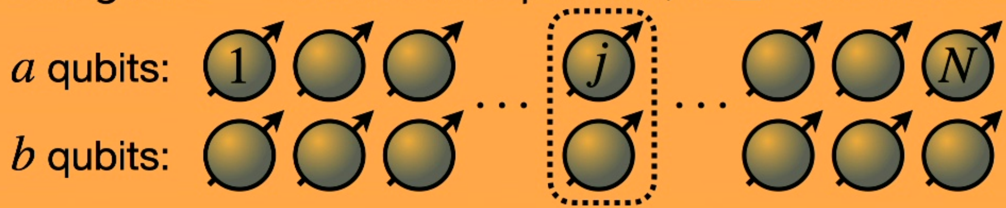


[6] Page (1993), [7] Altland, Huse, Micklitz (2022)

How can you compare commuting and noncommuting charges?

Analogous models

- NC charges: $\mathfrak{su}(2)$, $c = 3$ conserved charges.
- C charges: $3 + 1$ restrictions per site, $d \geq 4$ dim. sites



- $Q_\alpha^{\text{tot}} := \sum_{j=1}^{N} Q_\alpha^{(j)}$ or $C_\alpha^{\text{tot}} := \sum_{j=1}^{N} C_\alpha^{(j)}$, where $Q_\alpha^{(j)}$ and $C_\alpha^{(j)}$ are local operator on the jth site.
- $Q_1 = \sigma_x^{(a)} \otimes \mathbb{I}^{(b)}$, $Q_2 = \sigma_y^{(a)} \otimes \mathbb{I}^{(b)}$, and $Q_3 = \sigma_z^{(a)} \otimes \mathbb{I}^{(b)}$
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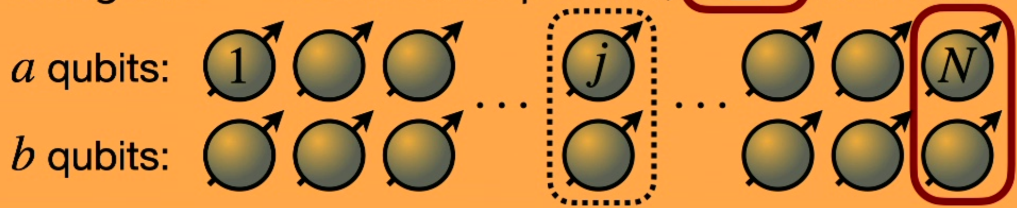

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- For unequal indices $\alpha, \beta, \gamma \in \{1,2,3\}$:

$$Q_\alpha Q_\beta = i\epsilon_{\alpha\beta\gamma} Q_\gamma \quad \text{and} \quad C_\alpha C_\beta = -C_\gamma .$$

  Multiplying two distinct charges equals the third times a constant. For equal indices: $Q_\alpha Q_\alpha = C_\alpha C_\alpha = \mathbb{I}$

1. **Same set-up:** Same N, d, and c.

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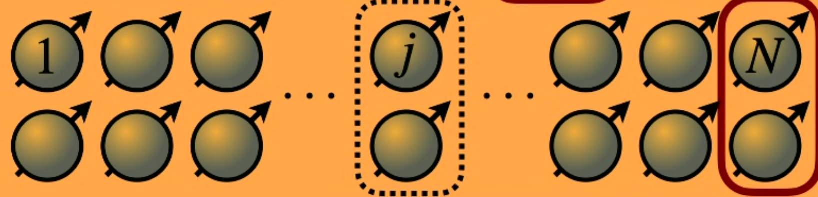

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3. **Same spectrum:** Each NC charge (Q_α^{tot}) has the same spectrum as its analogous commuting charge (C_α^{tot}).

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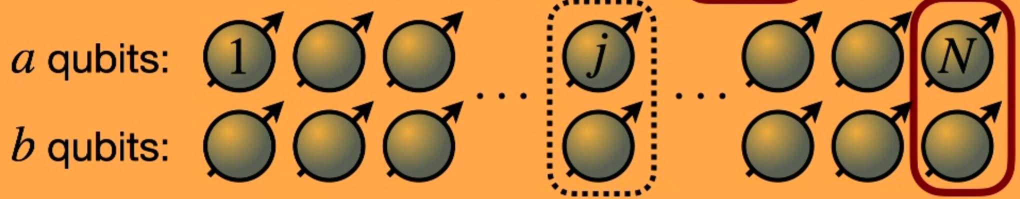

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4. **Same probability of outcomes**: For the constrained subspaces $\mathcal{N}$ and $\mathcal{C}$, the probability of outcomes are equal $[p_\alpha^{\mathcal{N}}(\gamma) = p_\alpha^{\mathcal{C}}(\gamma)]$.
5. **Similar algebras:** The commuting charges generate an algebra that "*resembles*" the noncommuting-charge algebra.

Numerical comparison with microcanonical subspaces

Microcanonical (simultaneous-eigenspace) comparison

- NC model has one such subspace: $\gamma = (0,0,0)$.
- Thus, commuting model $\gamma = (0,0,0)$.
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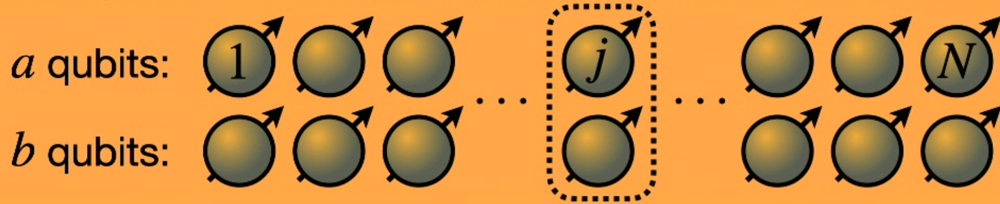

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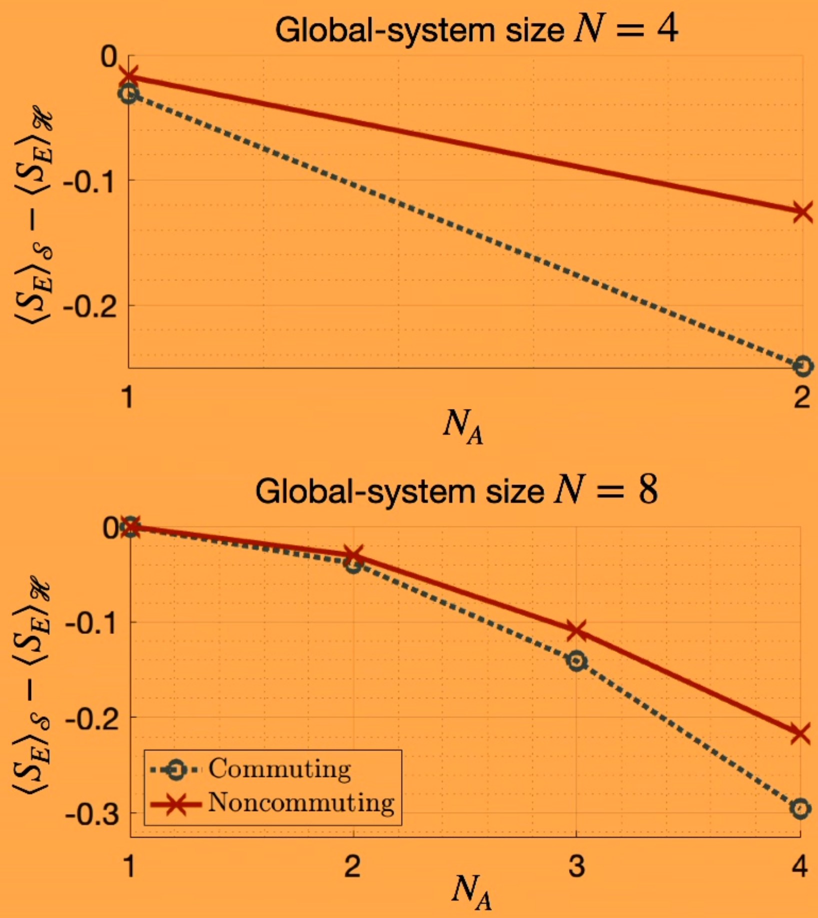
Numerical comparison

- Haar-random states $\propto \Sigma_\ell c_\ell |\psi_\ell\rangle$, $\{|\psi_\ell\rangle\}$ basis for the subspace, c_ℓ drawn from complex normal distribution.
- $\langle S_E \rangle_{\mathcal{H}}$ is the Page curve for unrestricted space.
- $\langle S_E \rangle_{\mathcal{S}}$ is the Page curve for spaces $\mathcal{S} = \{\mathcal{N}_0, \mathcal{C}_0\}$.



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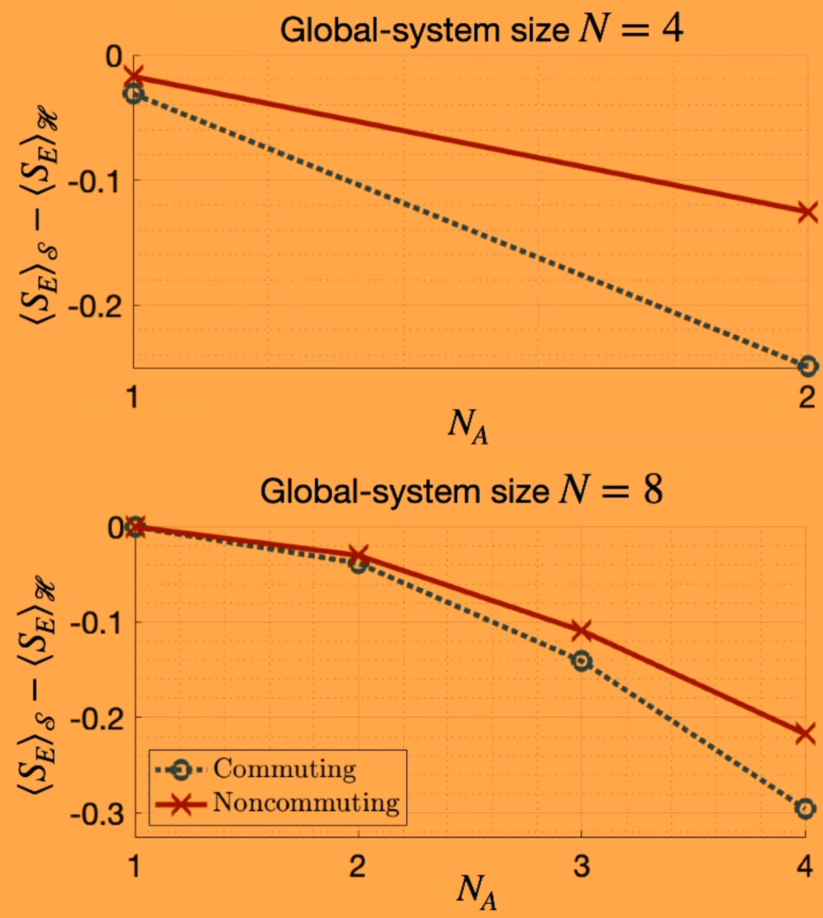
- For all N_A, the noncommuting Page curve is higher

Explanation?

- NC model's (i) dimension is equal or greater in this comparison and (ii) minimally entangled basis is more entangled.



Analytic comparison with microcanonical subspaces

Average over states, *then* calculate the entropy: $S(\langle\rho_A\rangle_S)$. If $|N_B - N_A| \gg 1$, $\langle S(\rho_A)\rangle_{\mathcal{S}} \approx S(\langle\rho_A\rangle_S)$ [6].

Exact expression

- $$S(\langle\rho_A\rangle_S) = -\sum_{\{A\}} \frac{D_A}{D} \log\left(\frac{D_A}{d_A D}\right)$$
  where D = Dim. of the global system's Hilbert space, $\{A\}$ denotes a charge configuration on subsystem A, D_A = Dim. of the global system's Hilbert space for $\{A\}$, d_A = Dim. of subsystem A's Hilbert space for $\{A\}$
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[6] Page (1993),

Analytic comparison with microcanonical subspaces

Closed form approximation

- The term outside the log is well-approximated as a Gaussian (a product of quadnomials). We make this approximation and then Taylor-expand around it's maximum:

$$\approx \frac{4(2N)^{\frac{3}{2}}}{(N_A N_B)^{\frac{3}{2}}\sqrt{\pi}} s_A^2 \exp(\frac{-2Ns_A^2}{N_A N_B})\left[1 + \frac{1}{s_A} - \frac{2Ns_A}{N_A N_B} + \frac{9}{4N} - \frac{17N}{4N_A N_B} + \frac{6s_A^2}{N_A^2} + \frac{6s_A^2}{N_B^2} + \frac{4s_A^2}{N_A N_B} + \frac{1}{4s_A^2} - \frac{4s_A^4}{3N_A^3} - \frac{4s_A^4}{3N_B^3} + \mathcal{O}\left(N^{-3/2}\right)\right].$$

- The log term we expand with the Stirling expansion and Taylor-approximate

$$\log(\frac{d_B}{D(2s_A+1)}) = -N_A \log(2) + \frac{3}{2}\log(\frac{N}{N_B}) - \frac{2s_A^2}{N_B} - \frac{2s_A}{N_B} - \frac{4s_A^4}{3N_B^3} + \frac{4s_A^2}{N_B^2} + \frac{9}{4N} - \frac{9}{4N_B} + \mathcal{O}\left(N^{-3/2}\right).$$

- Evaluate a closed form approximation

Results

- Both subspaces' Page curves have the leading to $O(N^0)$, term I call L. The NC/C Page curves are:

$$L + \frac{3N_A}{4N^2} \pm \frac{N_A^2}{2N^2 N_B} + O(N^{-3/2})$$

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- For all N_A, the noncommuting Page curve is higher. Decreases $\sim 1/N$

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Part 1: Summary & Outlook

Summary

- Isolate the effects of charges' noncommutation using analogous models.
- Noncommuting-charge model has more entanglement on average.

Outlook

- Puzzle: Does charges' noncommutation hinder or enhance thermalization?

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- Noncommuting-charge model has more entanglement on average.

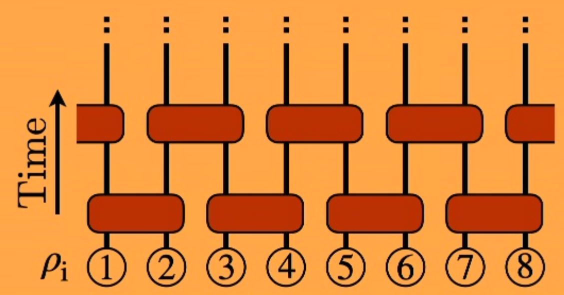
Outlook

- Puzzle: Does charges' noncommutation hinder or enhance thermalization?
- Experimental observation: Precession necessary to measure this difference has been achieved[16-17]
- Effects of charges' noncommutation on: Chaos, trot errors, decoherence, hydrodynamics, etc.

Monitored quantum circuits background

A toolbox for studying entanglement dynamics[9-11]

- L qubits in an initial state ρ_i
- Random unitaries (entangling)



[9] Potter, Vasseur (2021), [10] Fisher, Khemani, Nahum, Vijay (2022), [11] Skinner (2023)

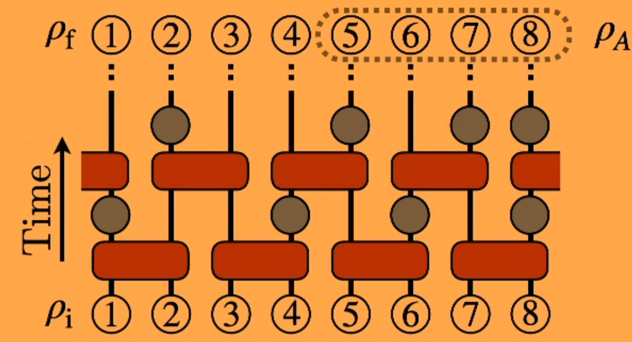
Monitored quantum circuits background

A toolbox for studying entanglement dynamics[9-11]

- L qubits in an initial state ρ_i
- Random unitaries (entangling)
- Projective measurements done with probability p (disentangling)
- Three types of randomness: unitaries, measurements locations and outcomes

Phase transitions

- Late-time bipartite entanglement entropy, $S := S(\rho_A)$
- An entanglement phase transition occurs at $p = p_c$[12-14].



$p < p_c$ $p = p_c$ $p > p_c$

[9] Potter, Vasseur (2021), [10] Fisher, Khemani, Nahum, Vijay (2022), [11] Skinner (2023)
[12] Skinner, Ruhman, Nahum (2019), [13] Li, Chen, Fisher (2018), [14] Gullans and Huse (2020)

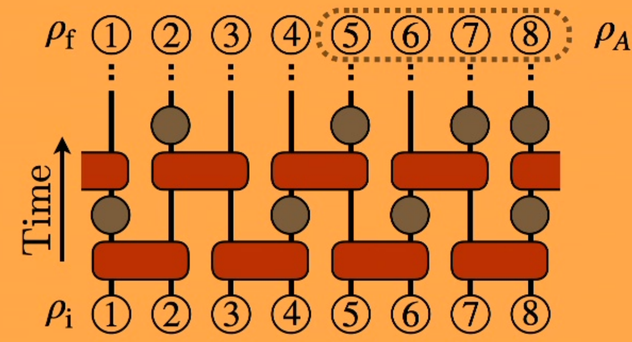
Monitored quantum circuits background

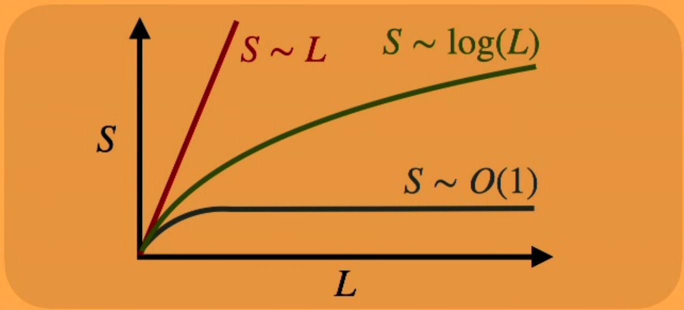
A toolbox for studying entanglement dynamics[9-11]

- L qubits in an initial state ρ_i
- Random unitaries (entangling)
- Projective measurements done with probability p (disentangling)
- Three types of randomness: unitaries, measurements locations and outcomes

Phase transitions

- Late-time bipartite entanglement entropy, $S := S(\rho_A)$
- An entanglement phase transition occurs at $p = p_c$[12-14].
- If ρ_i is a maximally mixed state, it evolves to a pure state ρ_f at a time scale τ_P, called the purification time
- Entanglement transition ↔ purification phase transition[22]





Volume-law/ mixed phase	Critical dynamics	Area-law/ pure phase
$p < p_\mathrm{c}$	$p = p_\mathrm{c}$	$p > p_\mathrm{c}$
$S \sim L$	$S \sim \log(L)$	$S \sim O(1)$
$\tau_\mathrm{P} \sim e^L$	$\tau_\mathrm{P} \sim L$	$\tau_\mathrm{P} \sim O(1)$

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Monitored quantum circuits background

***Second phase transition when you add a charge*[15]**

- $S_z^{\text{tot}} = \sum_{j=1}^{L} \sigma_z^{(j)}$, m is the eigenvalue of S_z^{tot} (proportional to the number of qubits in the state $|1\rangle$)
- Gates and measurement conserve m.
- Charge-sharpening transition
  - $p > p_{\text{cs}}$: One can learn the global charge's value from local measurements efficiently
  - $p < p_{\text{cs}}$: One cannot

What if the charges are noncommuting?

- Can you learn the charge?
- Do the entanglement dynamics change?

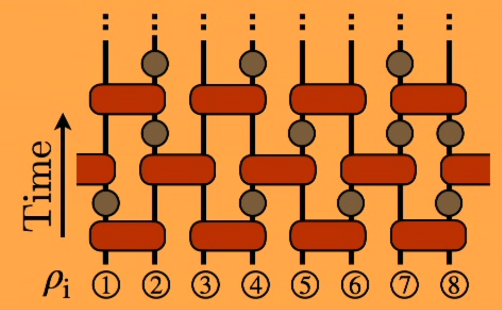
[15] Agrawal, Zabalo, Chen, Wilson, Potter, Pixley, Gopalakrishnan, Vasseur (2022)

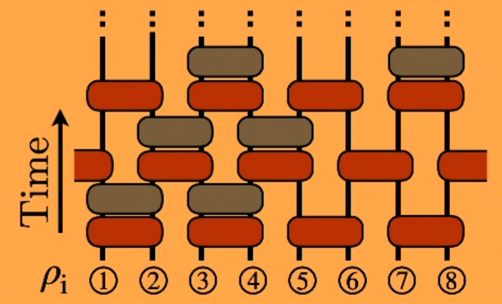
SU(2)-symmetric monitored quantum circuits

Introduce noncommuting charges

- $S_\alpha^{\mathrm{tot}} = \sum_{j=1}^{L} \sigma_\alpha^{(j)}$ are the total spin components
- Find unitaries, U, and projections operators P
  $[U, S_\alpha^{\mathrm{tot}}] = 0 = [P, S_\alpha^{\mathrm{tot}}]$ for all α
- $U = \cos(\phi)\mathbb{I} - i\sin(\phi)\mathrm{SWAP}$
- $|\mathrm{s}_0\rangle$ = two-qubit singlet state.
- $|\mathrm{t}_{m'}\rangle$ = two-qubit eigenvalue-m triplet state.
- $P_{s_0} = |\mathrm{s}_0\rangle\langle \mathrm{s}_0|$ and $P_{t_m} = \sum_m |\mathrm{t}_m\rangle\langle \mathrm{t}_m|$

Symmetry-free circuit





Critical-phase evidence 1 - Scale-invariance

Characteristic feature of critical phases: Scale invariance: $f(L) \rightarrow f(\lambda L) = \lambda^{\nu} f(L)$.

E.g., power laws have this feature: $\tau_{\mathrm{p}} = aL^{b}$.

[14] Gullans, Huse (2020)

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***Measuring the purification time*[14]**

- $|s, m, \lambda_0\rangle$ and $|s, m, \lambda_1\rangle$ are orthogonal states from the same (s, m) sector.
- Entangle our system with an ancilla as follows: $|\tilde{\psi}_i\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A |s, m, \lambda_0\rangle + |1\rangle_A |s, m, \lambda_1\rangle\right)$
- Measure the entanglement entropy of the ancilla, S_A.

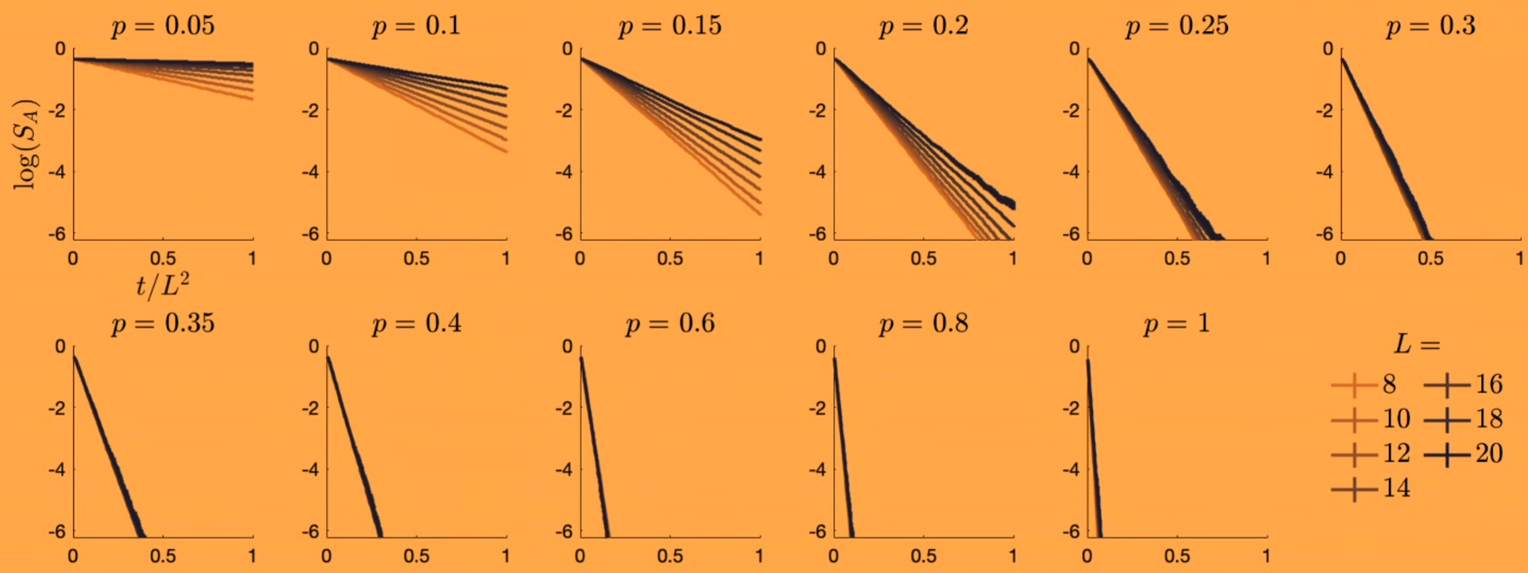
Plots

- The purification time is a decay rate, defined: $\log(S_A) \sim -t/\tau_{\mathrm{P}}$
- We fit the data to estimate $\tau_{\mathrm{P}} \sim f(L)$
- We plot $\log(S_A)$ against $-t/f(L)$ for many values of L

[14] Gullans, Huse (2020)

Critical-phase evidence 1 - Scale-invariance

Numerical results



Critical-phase evidence 2 - Large mutual information

- $I^{(2)}_{1,L/2}$ mutual information between antipodal pairs of sites

Critical-phase evidence 3 - Log entanglement growth

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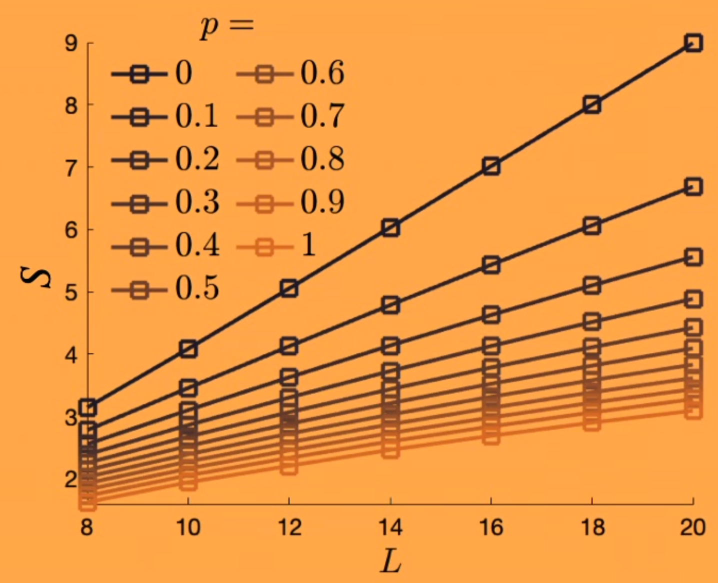
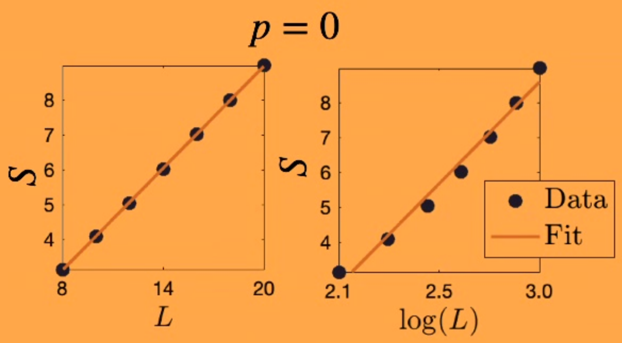
Critical phase: no area-law

- There is no area law.
- In the volume-law phase, every qubit is entangled with all others. Our measurements only create entanglement with one other.
- Two-site measurement-only dynamics can have volume-law and area-law phases[15].





[15] Ippoliti et al (2021)

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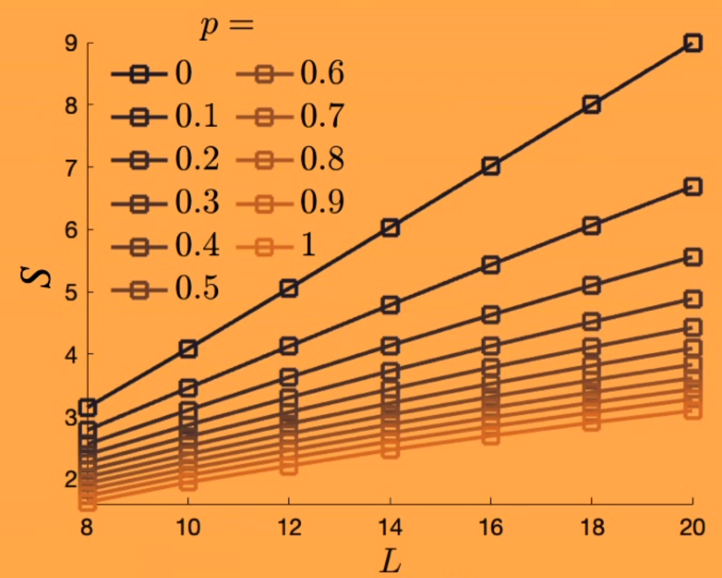
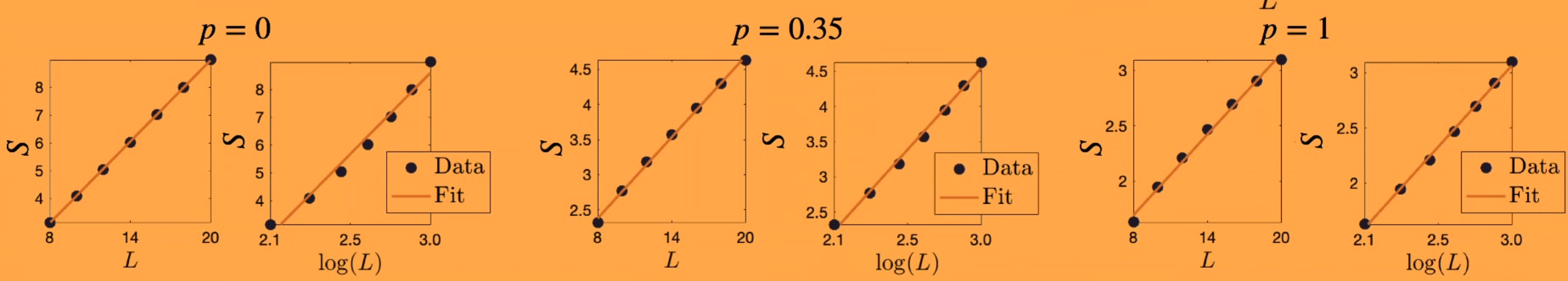
- $p < p_c$: Expected volume-law, with $S \sim L$.





[15] Ippoliti et al (2021)

Critical-phase's evidence recap

Symmetry-free and U(1) cases

Volume-law/ mixed phase	Critical dynamics	Area-law/ pure phase
$p < p_c$	$p = p_c$	$p > p_c$
$S \sim L$	$S \sim \log(L)$	$S \sim O(1)$
$\tau_P \sim e^L$	$\tau_P \sim L$	$\tau_P \sim O(1)$

SU(2) case

Volume-law/ mixed phase		
$p < p_c$	$p = p_c$	$p > p_c$
$S \sim L$		
$\tau_P \sim e^L$		$\tau_P \sim L^2$

- The volume-law phase is expected ($S \sim L, \tau_P \sim e^L$)
- Purification numerics demonstrate $\tau_P \sim L^2$
- Entanglement numerics clearly show no area-law and are consistent with $S \sim \log(L)$

Summary & Outlook

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- Introduced noncommuting charges into monitored quantum circuits
- Found a critical phase
- Discovered a “spin-sharpening” phase transition

Outlook

- Understand the critical phase analytically.

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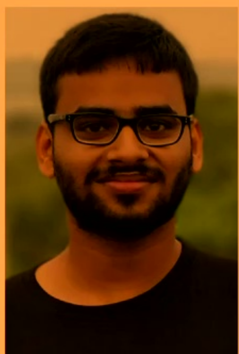
- Introduced noncommuting charges into monitored quantum circuits
- Found a critical phase
- Discovered a "spin-sharpening" phase transition

Outlook

- Understand the critical phase analytically.
- Again, more entanglement. What does this mean, if anything, for thermalization?
- Leverage spin-sharpening to help with the post-selection problem
- Relation to the entanglement necessary for quantum computing models[18]

[18] Rudolph, Virmani (2023)

Thanks for listening



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- **Majidy**, Agrawal, Gopalakrishnan, Potter, Vasseur, Yunger Halpern *"Critical phase and spin sharpening in SU(2)-symmetric monitored quantum circuits.* Phys. Rev. B 108, 054307 (2023)
- **Majidy**, Braasch Jr., Lasek, Upadhyaya, Kalev, Yunger Halpern "Noncommuting conserved charges in quantum thermodynamics and beyond" Accepted by Nat. Rev. Phys. (2023)
- **Majidy**, Lasek, Huse, Yunger Halpern "Non-Abelian symmetry can increase entanglement entropy." Phys. Rev. B 107 045102 (2023)
- Yunger Halpern and **Majidy** "How to build Hamiltonians that transport noncommuting charges in quantum thermodynamics" npj Quantum Information (2022)