Title: Bosonic quantum sensing and communication in the presence of loss and noise - VIRTUAL

Speakers: Haowei Shi

Series: Perimeter Institute Quantum Discussions

Date: November 27, 2023 - 3:30 PM

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Abstract: Squeezing has proven to be a powerful tool for suppressing noise in bosonic quantum sensing and communication. However, it is fragile and the resulting quantum advantage is extremely vulnerable to loss and noise. In this seminar, I will first overview the method of formulating loss and noise and thereby characterizing the practical quantum advantages. Then I will present our recent progress on entanglement-assisted protocols using two-mode squeezed-vacuum states, which are robust to loss and noise. I will demonstrate the quantum advantages in three scenarios: dark matter search, absorption spectroscopy, and telecommunication. Notably, we derived the ultimate precision limit of noise sensing and dark matter search. As a result, we found the two-mode squeezed vacuum is the optimal quantum source for dark matter search at the limit of strong squeezing. This optimality extends to entanglement-assisted communication. In each of the presented scenarios, entanglement-assisted protocols yield quantum advantages of orders of magnitude over classical protocols.

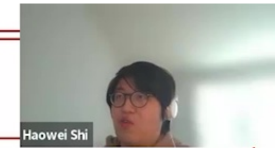
Zoom link https://pitp.zoom.us/j/94873478582?pwd=c1dxNEVtMGx0ZU4vZjRvTU5OakZoUT09

BOSONIC QUANTUM SENSING AND COMMUNICATION IN THE PRESENCE OF LOSS AND NOISE

Presenter: Haowei Shi

Collaborators: Zaijun Chen, Scott E. Fraser, Saikat Guha, Min-Hsiu Hsieh, Stefano Pirandola, Jeffery H. Shapiro, Mengjie Yu, Zheshen Zhang, Quntao Zhuang, and more

Nov 27 @ PI



CONTENTS

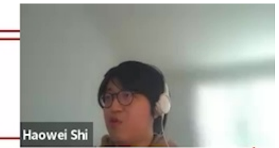
- Method
  - “Quantum advantage”
  - Formulation of loss and noise

- Noise Sensing and Dark Matter Search

  Shi, H., & Zhuang, Q. (2023). *npj Quantum Information*, *9*, 27.
- Absorption Spectroscopy

  Shi, H., Zhang, Z., Pirandola, S., & Zhuang, Q. (2020). *Physical review letters*, *125*(18), 180502.
- Entanglement-Assisted Communication

  Shi, H., Zhang, Z., & Zhuang, Q. (2020). *Physical Review Applied*, *13*(3), 034029.



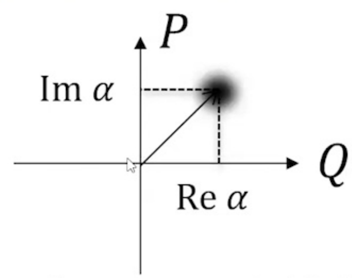
BOSONIC FORMULATION: QUANTUM OPTICS

- Second harmonic oscillator

$$\hat{H} = \frac{1}{2}\int (\epsilon_0 \boldsymbol{E}^2 + \mu_0 \boldsymbol{H}^2) d\boldsymbol{r} = \sum_k \hbar\omega_k \left(\frac{1}{2} + \hat{a}_k^\dagger \hat{a}_k\right)$$

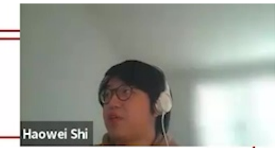
  - Axion dark matter Weinberg, S. (1978). A new light boson?. *Physical Review Letters*, *40*(4), 223.

- Annihilation operator $\hat{a} = \hat{Q} + i\hat{P}$
  - $[\hat{Q}, \hat{P}] = i/2$



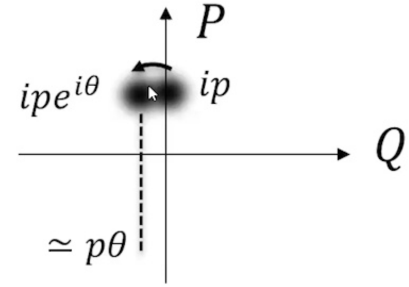
Coherent state $|\alpha\rangle\langle\alpha|$

- Boson ↔ SU(2) spin ensemble: Holstein-Primakoff mapping
  Klein, A., & Marshalek, E. R. (1991). *Reviews of modern physics*, *63*(2), 375.

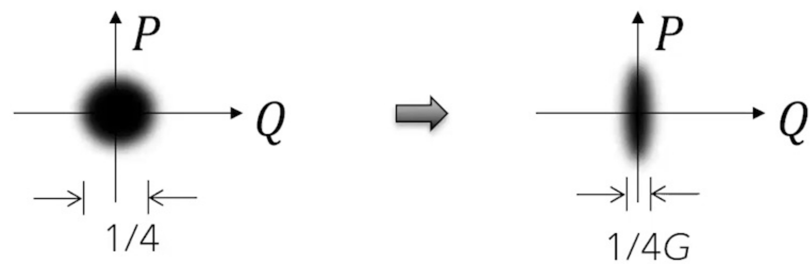


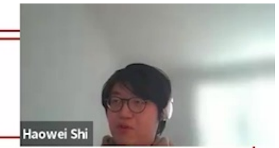
DISPLACEMENT SENSING

- Displacement $\mathcal{D}_{q+ip}$: $\hat{Q} \to \hat{Q} + q$, $\hat{P} \to \hat{P} + p$
  - Connection to phase sensing
    $|ipe^{i\theta}\rangle\langle ipe^{i\theta}| \simeq \mathcal{D}_{-p\theta}(|ip\rangle\langle ip|), \theta \to 0$



- Squeezing $\mathcal{S}$: $\hat{Q} \to \frac{\hat{Q}}{\sqrt{G}}$, $\hat{P} \to \sqrt{G}\hat{P}$





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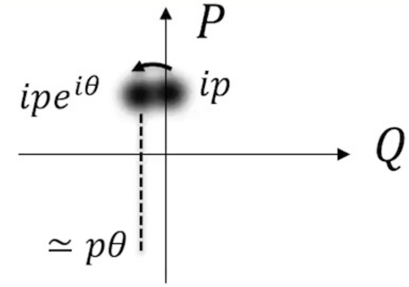
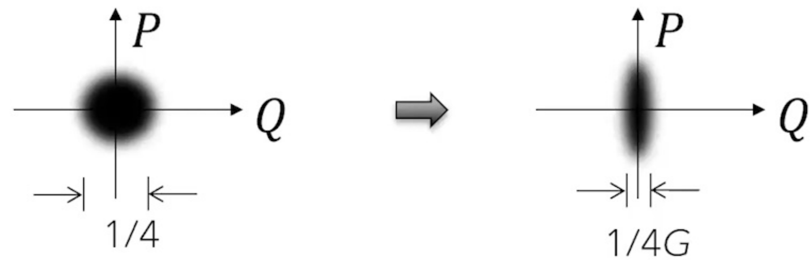
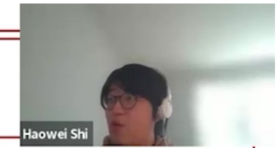
- Squeezing $\mathcal{S}$: $\hat{Q} \to \frac{\hat{Q}}{\sqrt{G}}, \hat{P} \to \sqrt{G}\hat{P}$
- Squeezing-enhanced sensing
  - Squeezed source: $\mathcal{S}(|0\rangle\langle 0|)$
  - Probing: $\mathcal{D}_q \circ \mathcal{S}(|0\rangle\langle 0|)$
  - Homodyne (measure $\hat{Q}$)
- Quantum advantage: noise $1/4G \to 0$ for $G \to \infty$
  - Vulnerable to loss and noise





Classical

Displaced squeezed state



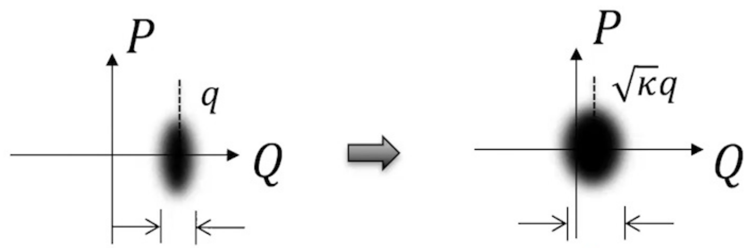
FORMULATING LOSS AND NOISE: BOSONIC LOSS CHANNEL

- Cavity loss, diffraction, scattering…
- Input-output relation: $\hat{a}_{out} = \sqrt{\kappa}\hat{a}_{in} + \sqrt{1-\kappa}\hat{e}$

Background dark count $n_B := \langle \hat{a}^\dagger_{out}\hat{a}_{out}\rangle|_{\langle \hat{a}^\dagger_{in}\hat{a}_{in}\rangle=0} = (1-\kappa)\langle \hat{e}^\dagger\hat{e}\rangle$

$\to \hat{X}_{out} = \sqrt{\kappa}\hat{X}_{in} + \sqrt{1-\kappa}\hat{X}_e$

Additive quadrature noise $= (1-\kappa)\langle \hat{X}_e^2\rangle = \frac{1}{4}[(1-\kappa)+2n_{\mathrm{B}}]$

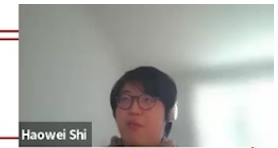


$\frac{1}{4G}$

Pure state

$\frac{\kappa}{4G} + \frac{1}{4}[(1-\kappa)+2n_B] > 0$ for $G \to \infty$

Thermal mixture



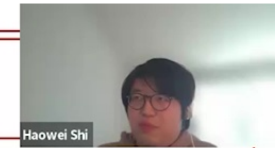
NOISE (RANDOM DISPLACEMENT) SENSING

- Axion dark matter:
  - Random-phase displacement: $\alpha = |\alpha| e^{i\theta},\ \theta \sim U[0,2\pi)$
  - Random wave vector direction: $|\alpha| \sim \mathcal{N}(0, \sigma^2)$

  $\rightarrow$ Gaussian thermal noise, σ^2 unknown

- 1. How does squeezing help noise sensing?

- 2. Is there a loss/noise-robust sensing protocol?
  - Single-mode squeezing noise: $\frac{\kappa}{4G} + \frac{1}{4}[(1-\kappa) + 2n_B] > 0$ for $G \rightarrow \infty$

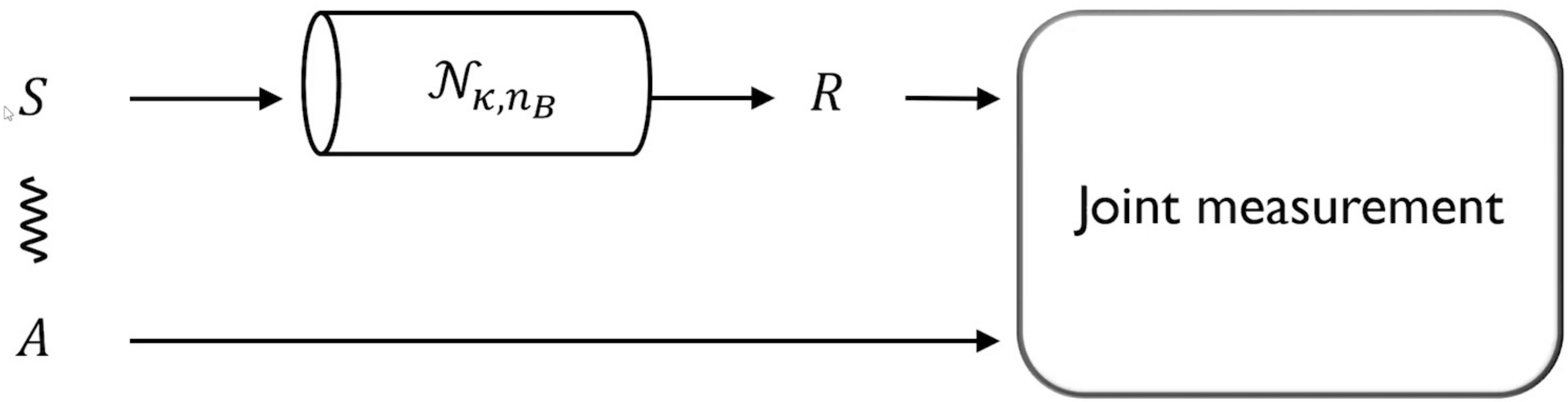


CONTENTS

- Method
- Noise Sensing and Dark Matter Search
  - Ultimate precision limit
  - Quantum-optimum: two-mode squeezed-vacuum nulling receiver
  - Cavity dark matter search
- Absorption Spectroscopy
- Entanglement-Assisted Communication

ENTANGLEMENT-ASSISTED NOISE SENSING

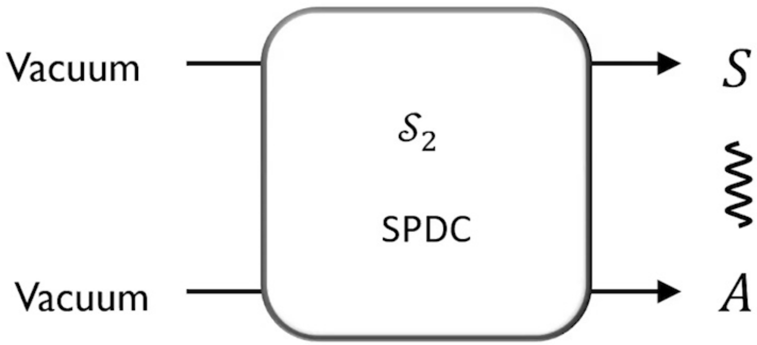
- $\mathcal{N}_{\kappa,n_B}$: general covariant Gaussian channel
  - Loss, $\kappa < 1$; AWGN, $\kappa = 1$; amplifier, $\kappa > 1$



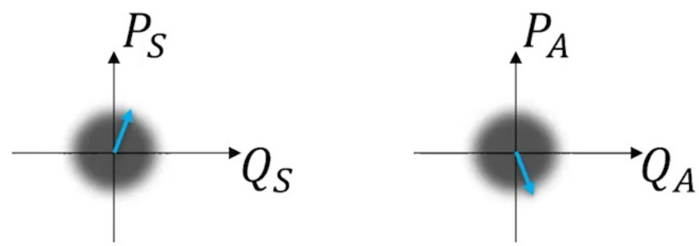
- Task: estimate n_B, minimize the mean square error (MSE)

EXAMPLE OF BOSONIC ENTANGLEMENT

- Two-mode squeezed vacuum (TMSV): Gaussian EPR state



$$|\mathrm{TMSV}\rangle = \sum_{n=0}^{\infty} \frac{N_S^n}{(1+N_S)^{n+1}} |n\rangle_S |n\rangle_A$$



$$\left\langle \left(\hat{Q}_S - \hat{Q}_A\right)^2 \right\rangle = \left\langle \left(\hat{P}_S + \hat{P}_A\right)^2 \right\rangle = \frac{1}{4G}$$

- Physical, with energy constrained:

$$\langle \hat{a}_S^\dagger \hat{a}_S \rangle = N_S = \frac{(G-1)^2}{4G}$$

ULTIMATE PRECISION LIMIT OF NOISE SENSING

- Classical Fisher information (CFI)

$$I(\hat{\rho}_{in}, \{\Pi\}) := \mathrm{MMSE}^{-1}(\{\Pi \mathcal{N}_{\kappa,n_B}(\hat{\rho}_{in})\Pi\})$$

Input quantum state POVM

- Quantum Fisher information (QFI)

$$\mathcal{J}(\hat{\rho}_{in}) := \max_{\{\Pi\}} I(\hat{\rho}_{in}, \{\Pi\})$$

- Upper bound

$$\mathcal{J}_{\mathrm{UB}} \geq \max_{\hat{\rho}_{in}} \mathcal{J}(\hat{\rho}_{in})$$

  - Our results: unitary extension bound $\mathcal{J}_{\mathrm{UB-UE}}$ and teleportation simulation bound $\mathcal{J}_{\mathrm{UB-TP}}$

Pirandola, S., & Lupo, C. (2017). PRL, 118(10), 100502.

$$\mathcal{J}_{\mathrm{UB}} = \max\{\mathcal{J}_{\mathrm{UB-UE}}, \mathcal{J}_{\mathrm{UB-TP}}\}$$

DERIVATION OF UNITARY EXTENSION BOUND

$$\mathcal{J}(\hat{\rho}_{in}) = -4\partial^2_{n_B'} F\left[\mathcal{N}_{\kappa,n_B}(\hat{\rho}_{in}), \mathcal{N}_{\kappa,n_B'}(\hat{\rho}_{in})\right]\Big|_{n_B'=n_B}$$

Braunstein, S. L., & Caves, C. M. (1994). PRL, 72(22), 3439.

› Challenge: Evaluating fidelity $F[\hat{\rho}, \hat{\sigma}] = \mathrm{Tr}\sqrt{\sqrt{\hat{\rho}}\hat{\sigma}\sqrt{\hat{\rho}}}$ for arbitrary M-mode states

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› Challenge: Evaluating fidelity $F[\hat{\rho}, \hat{\sigma}] = \mathrm{Tr}\sqrt{\sqrt{\hat{\rho}}\hat{\sigma}\sqrt{\hat{\rho}}}$ for arbitrary M-mode states

› Ulmann's theorem:

$$F\left[\hat{\rho}(n_{\mathrm{B}}), \hat{\rho}(n_{\mathrm{B}}')\right] \geq \left| \langle \psi(n_{\mathrm{B}}) | \psi(n_{\mathrm{B}}') \rangle \right|$$

where $|\psi\rangle$ is the purification of $\hat{\rho}$.

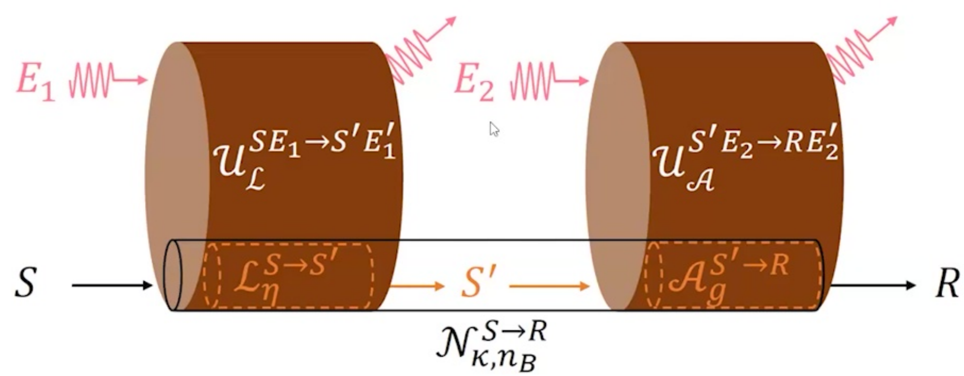
› Purification of output state: Unitary extension (Stinespring representation)

DERIVATION OF UNITARY EXTENSION BOUND

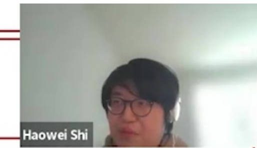
- Decomposition into quantum-limited channels (pure amplifier and pure loss)

$$\mathcal{N}_{\kappa,n_{\mathrm{B}}} = \mathcal{A}_{g(n_{\mathrm{B}})} \circ \mathcal{L}_{\eta(n_{\mathrm{B}})}$$

  - Kraus operators simple
- Unitary extension



- The overall output quantum state $|\psi\rangle_{E'RA}$ is always pure and easy to calculate.



UNITARY EXTENSION BOUND

- Energy constrained: $\langle \hat{a}_S^\dagger \hat{a}_S \rangle \le N_S$.

$$\mathcal{J}_{\mathrm{UB,UE}} = \underbrace{\boxed{\frac{1}{n_B(n_B+1)}}}_{\mathcal{J}_{\mathrm{VL}}} + \frac{\kappa N_S(2n_B - \kappa + 1)}{n_B(n_B+1)^2(n_B - \kappa + 1)} \overset{\substack{\kappa \to 1 \\ n_B \to 0}}{\simeq} \frac{2N_S + 1}{n_B} \simeq \mathcal{J}_{\mathrm{TMSV}}$$

- Additivity

  To estimate a global parameter θ for compound channel $\bigotimes_{\{\ell=1\}}^{K} \mathcal{N}_{\kappa_\ell, n_{\mathrm{B},\ell}(\theta)}$:

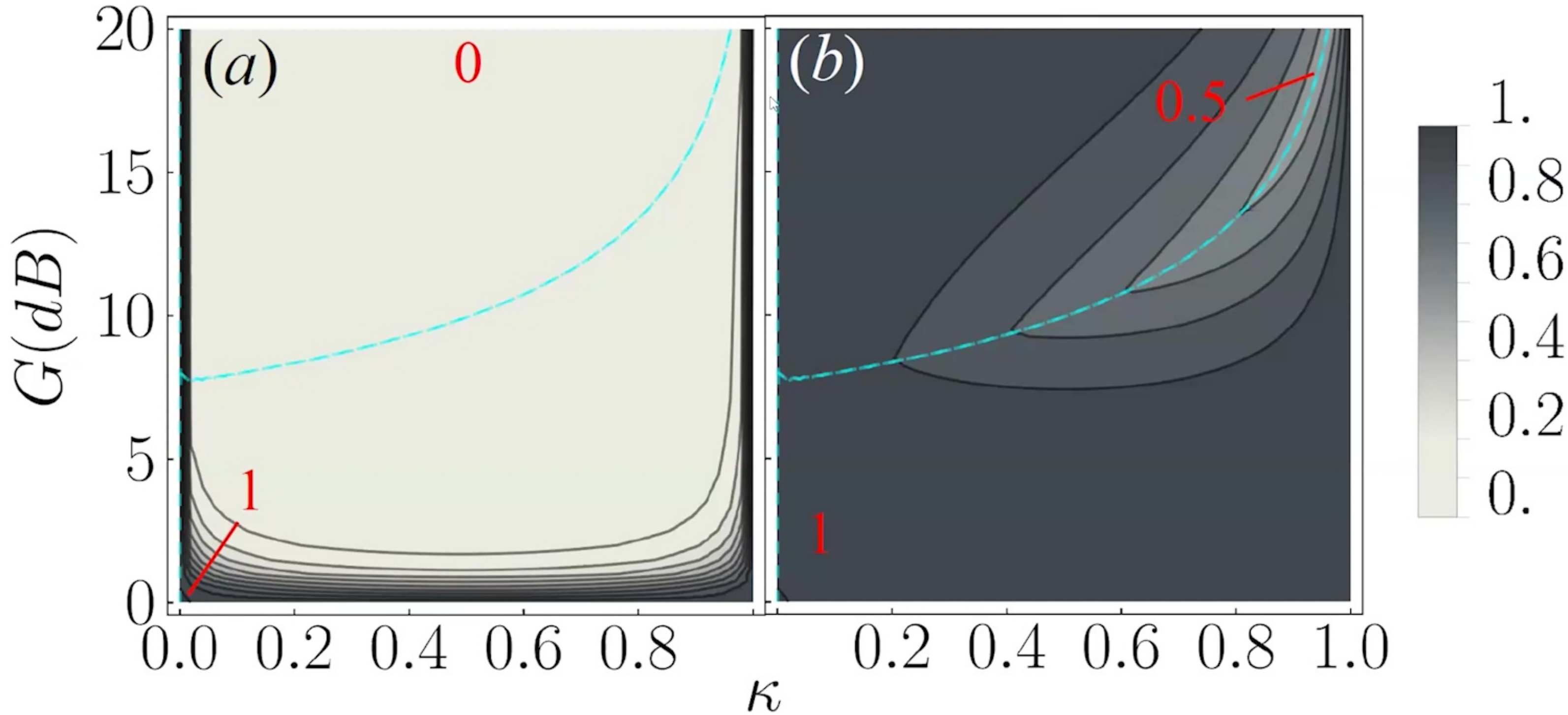
$$\mathcal{J}_\theta^{\mathrm{UB,UE}} = \sum_{\ell=1}^{K} \left[\partial_\theta n_{\mathrm{B},\ell}\right]^2 \mathcal{J}_{\mathrm{UB,UE}}(N_{\mathrm{S},\ell}, \kappa_\ell, n_{\mathrm{B},\ell})$$

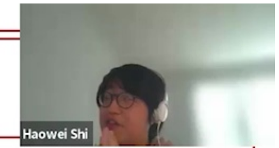
ANSATZ OF OPTIMAL INPUT STATE

$\mathcal{J}(\hat{\rho}_{in})/\mathcal{J}_{\mathrm{UB}}$

cyan: $\mathcal{J}_{\mathrm{UB-TP}} = \mathcal{J}_{\mathrm{UB-UE}}$

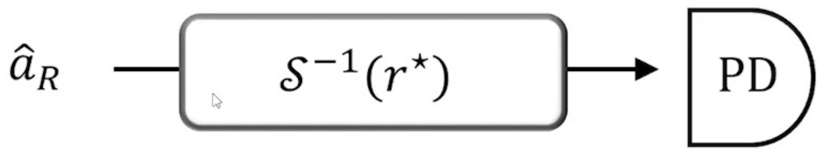
$\hat{\rho}_{in}$: Single-mode squeezed state Two-mode squeezed state

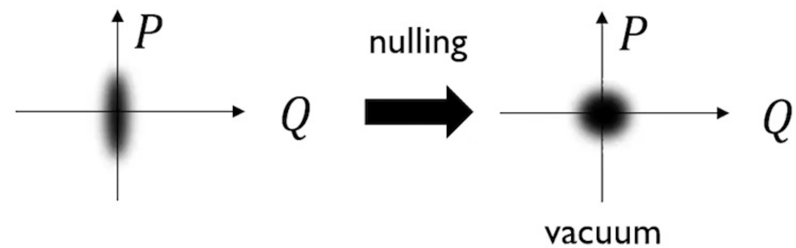




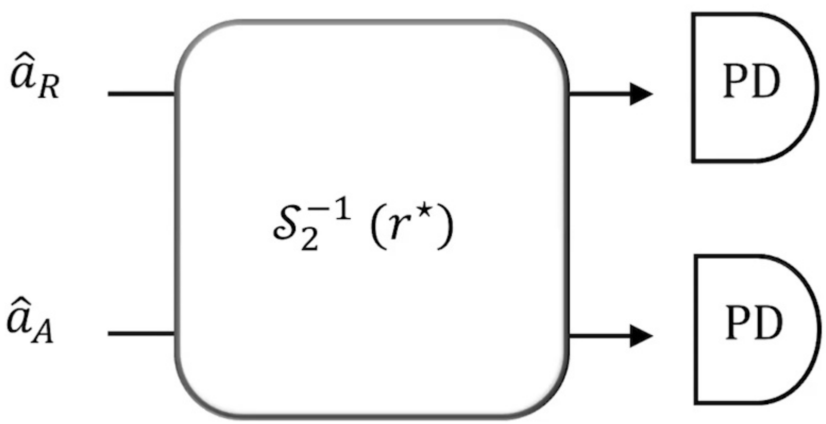
ANSATZ OF OPTIMAL MEASUREMENT: NULLING RECEIVER

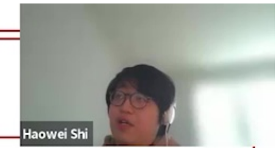
For single-mode squeezed vacuum (SV):





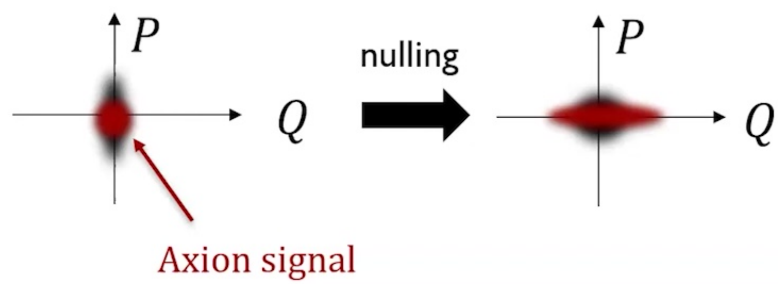
For two-mode squeezed vacuum (TMSV):





ADVANTAGE OF NULLING RECEIVER AND ENTANGLEMENT

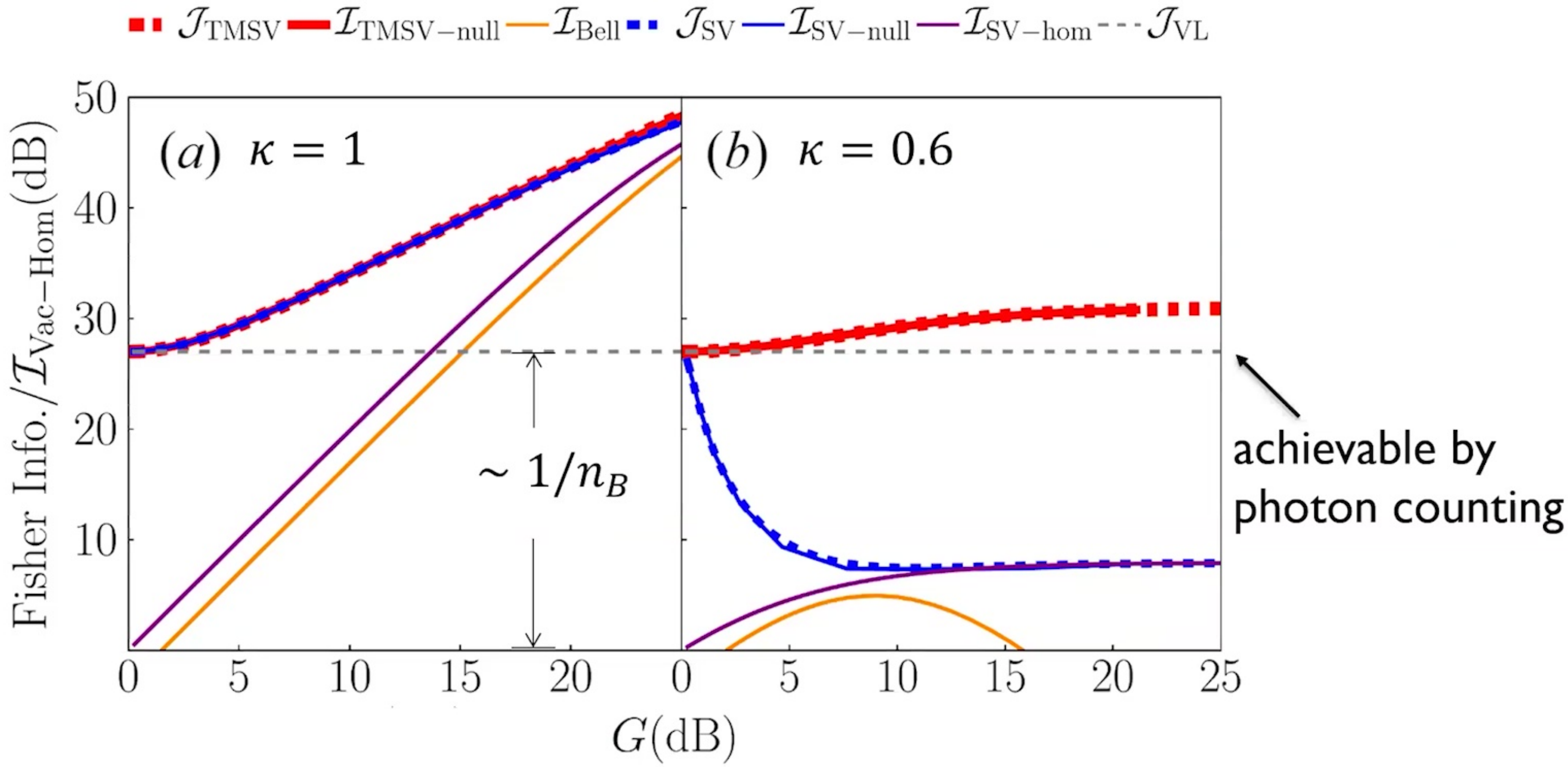
- Consider weak noise signal $n_B \to 0$
- Conventional homodyne: measure quadrature $\hat{Q}$, Var $\propto 1/G$
- Nulling receiver: null the return to vacuum and photon count, Var $\simeq n_B(1+n_B) \to 0$ at $G \to 1$
  - Single-mode squeezed source:



  - Impossible to null to vacuum in the presence of loss
- Entanglement (TMSV)
  - Loss tolerant: always possible to return vacuum for at least one mode

OPTIMALITY OF NULLING RECEIVER

- Cooled environment: $n_B = 10^{-3}$



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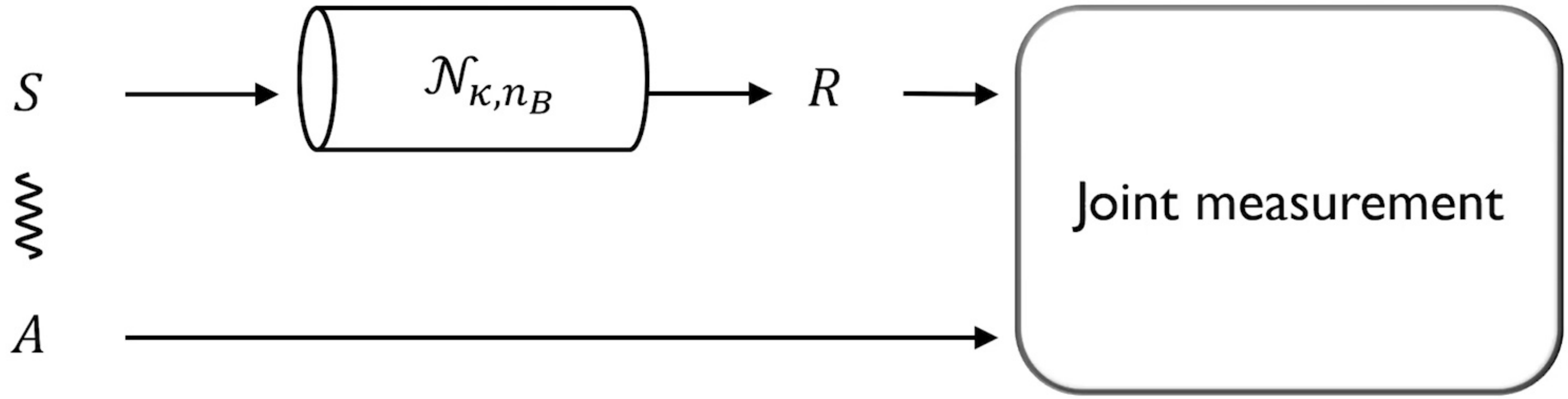

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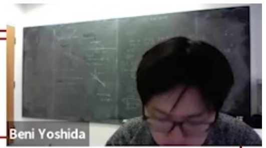

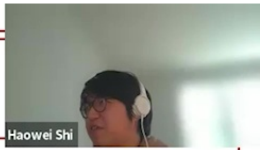

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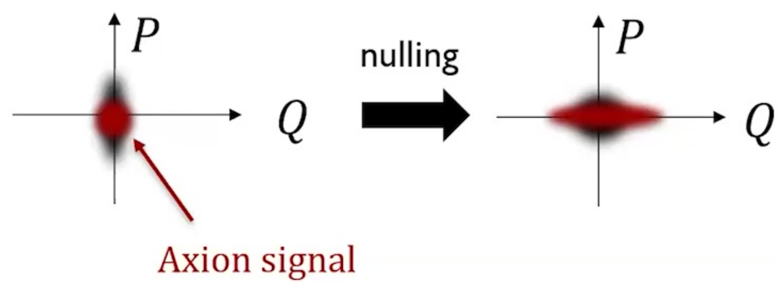

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OPTIMALITY OF NULLING RECEIVER

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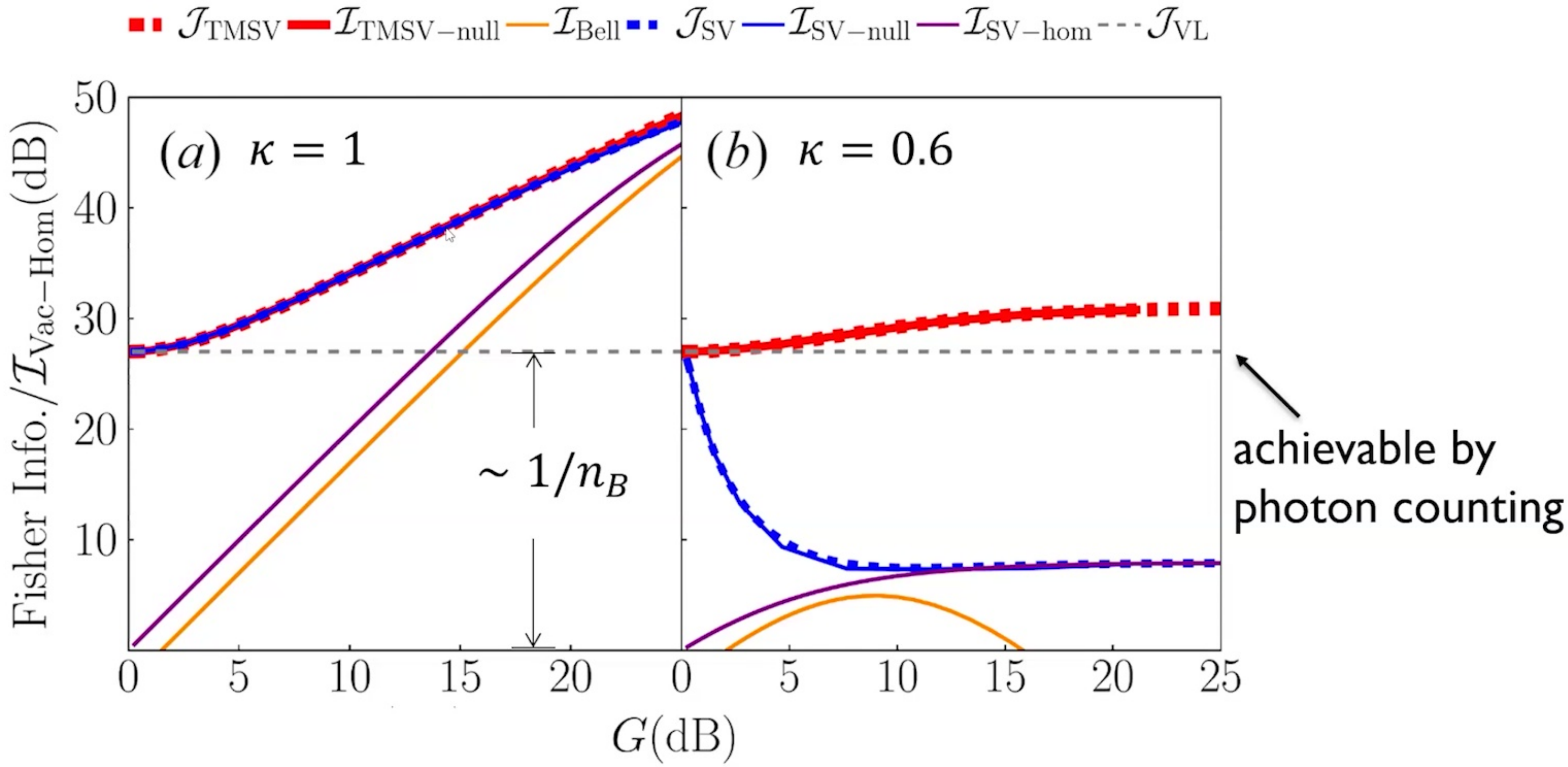
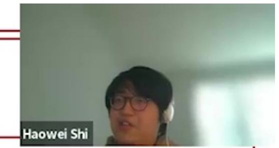
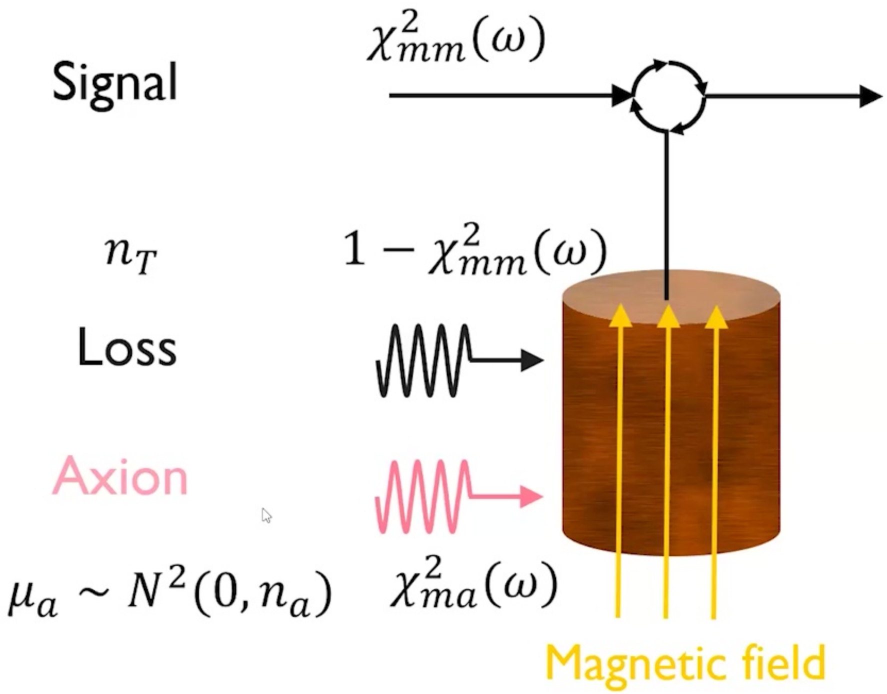


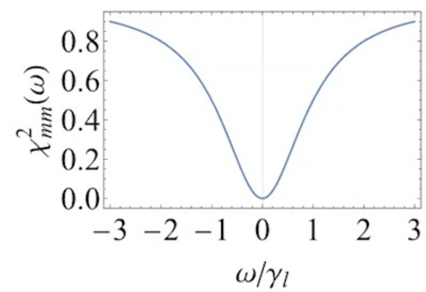


MICROWAVE COUPLING



$$\boxed{\hat{a}_{out} = \chi_{mm}\hat{a}_{in} + \sqrt{1-\chi_{mm}^2}\,\hat{a}_l + \chi_{ma}\mu_a}$$

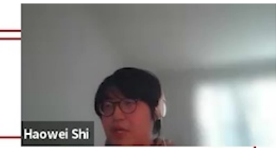
Lorentzian transmissivity:



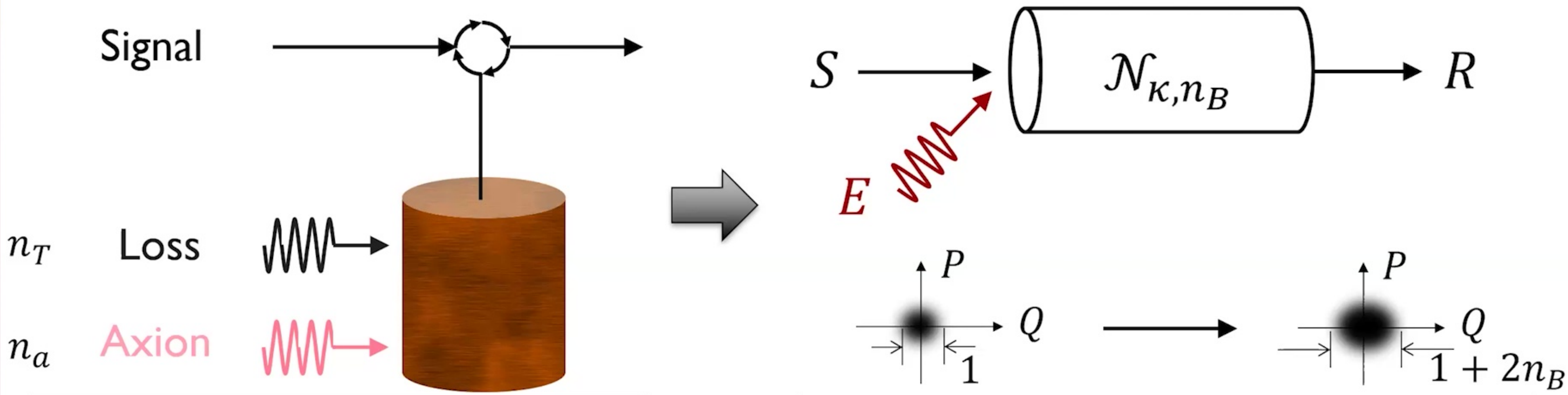
$$n_a \sim \frac{\rho_a V}{m_a} \gg 1$$

$$\chi_{ma} n_a \ll 1$$

Brady, Anthony J., et al.
PRX Quantum 3.3 (2022): 030333.



MICROWAVE COUPLING AS BOSONIC CHANNEL



$$\hat{a}_{out} = \chi_{mm}\hat{a}_{in} + \sqrt{1-\chi_{mm}^2}\,\hat{a}_l + \sqrt{1-\chi_{mm}^2}\,\frac{\chi_{ma}\mu_a}{\sqrt{1-\chi_{mm}^2}}$$

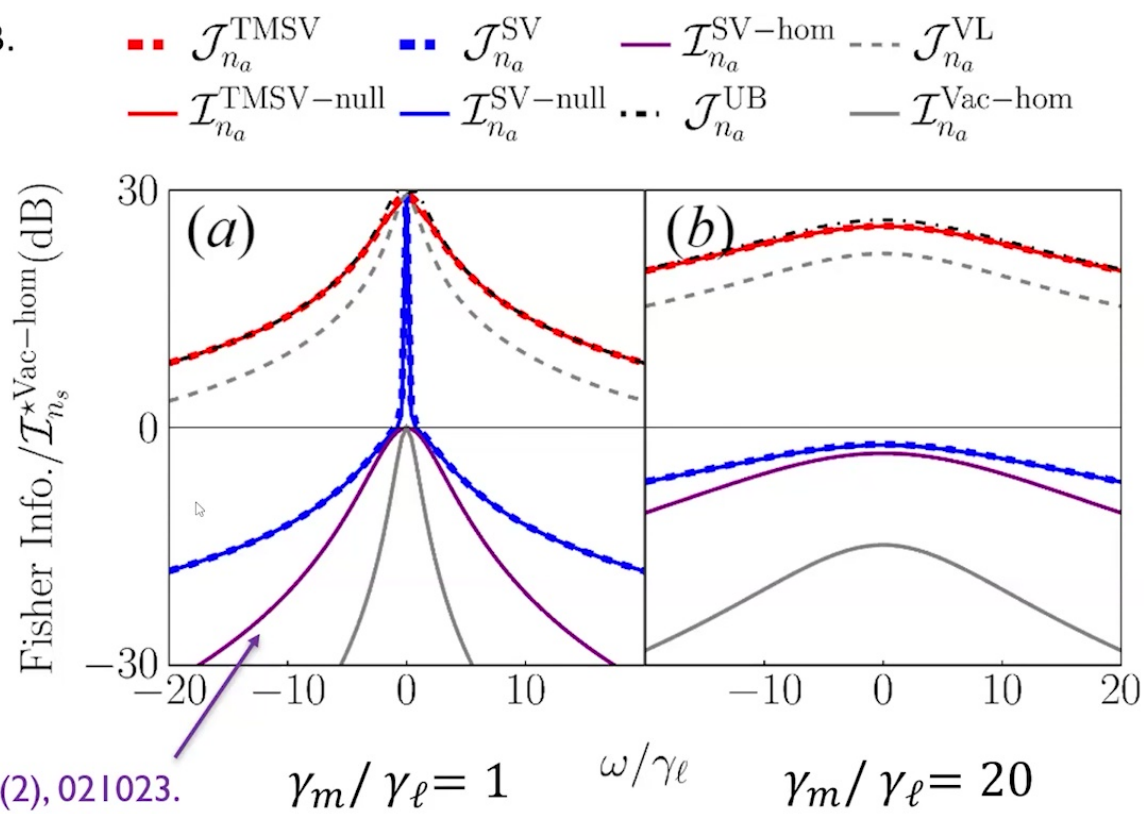
$$\hat{a}_R = \sqrt{\kappa}\hat{a}_S + \sqrt{1-\kappa}\,\hat{a}_E$$

Dark count: $n_B = (1-\kappa)n_E$

$$n_B(\omega) = [1-\chi_{mm}^2(\omega)]n_T + \chi_{ma}^2(\omega)n_a$$

FISHER INFO SPECTRUM OF CAVITY MICROWAVE COUPLING

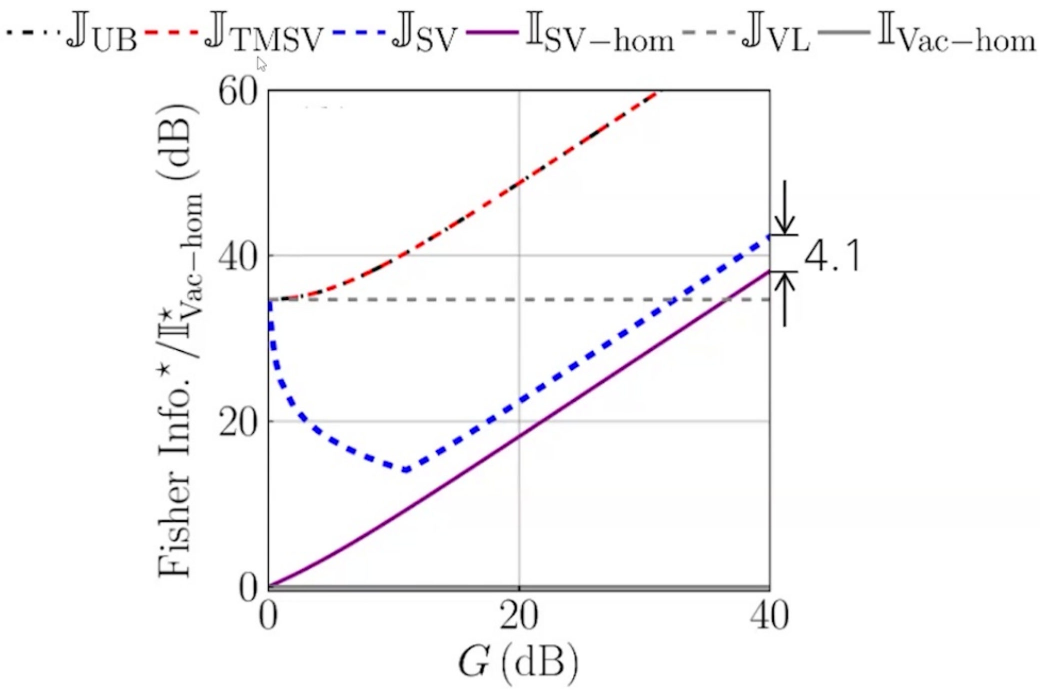
- Cooled cavity T=61mK. G=10dB.
  - $n_T < 10^{-3}$ at ω_c=10GHz.
- Coupling rates: $\gamma_m, \gamma_\ell \gg \gamma_a$
- Nulling receiver optimal
- Quantum sources prefer far detuning (transparency) → Fisher spectrum widened



Malnou, M., et al. (2019) Phys. Rev. X, 9(2), 021023.

BROADBAND: TOTAL FISHER INFORMATION (SCAN RATE)

- Cooled cavity T=61mK.

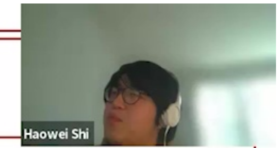


γ_m/γ_ℓ optimized

Scan rate

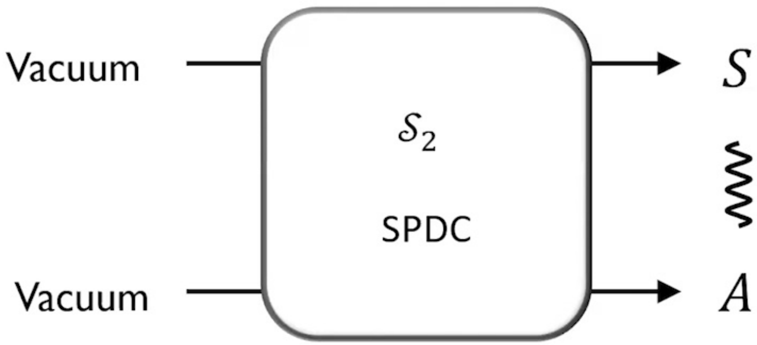
$$R \propto \int_{-\infty}^{\infty} \mathcal{I}(\omega)\,d\omega$$

Malnou, M., et al. (2019) Phys. Rev. X, 9(2), 021023.

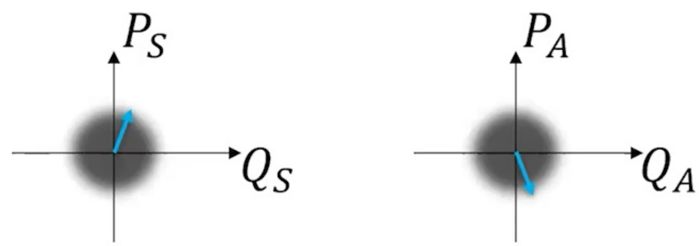


EXAMPLE OF BOSONIC ENTANGLEMENT

- Two-mode squeezed vacuum (TMSV): Gaussian EPR state



$$|\mathrm{TMSV}\rangle = \sum_{n=0}^{\infty} \frac{N_S^n}{(1+N_S)^{n+1}} |n\rangle_S |n\rangle_A$$



$$\langle (\hat{Q}_S - \hat{Q}_A)^2 \rangle = \langle (\hat{P}_S + \hat{P}_A)^2 \rangle = \frac{1}{4G}$$

- Physical, with energy constrained:

$$\langle \hat{a}_S^\dagger \hat{a}_S \rangle = N_S = \frac{(G-1)^2}{4G}$$

DERIVATION OF UNITARY EXTENSION BOUND

$$\mathcal{J}(\hat{\rho}_{in}) = -4\partial^2_{n_B'} F\left[\mathcal{N}_{\kappa,n_B}(\hat{\rho}_{in}), \mathcal{N}_{\kappa,n_B'}(\hat{\rho}_{in})\right]\Big|_{n_B'=n_B}$$

Braunstein, S. L., & Caves, C. M. (1994). PRL, 72(22), 3439.

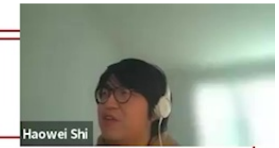
- Challenge: Evaluating fidelity $F[\hat{\rho},\hat{\sigma}] = \mathrm{Tr}\sqrt{\sqrt{\hat{\rho}}\hat{\sigma}\sqrt{\hat{\rho}}}$ for arbitrary M-mode states

- Ulmann's theorem:

$$F\left[\hat{\rho}(n_{\mathrm{B}}), \hat{\rho}(n_{\mathrm{B}}')\right] \geq |\langle\psi(n_{\mathrm{B}})|\psi(n_{\mathrm{B}}')\rangle|$$

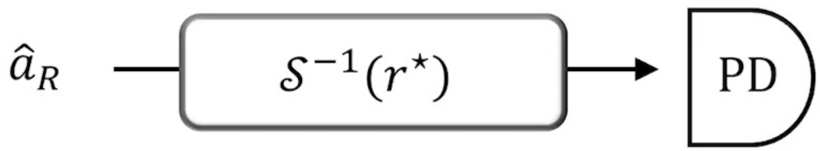
  where $|\psi\rangle$ is the purification of $\hat{\rho}$.

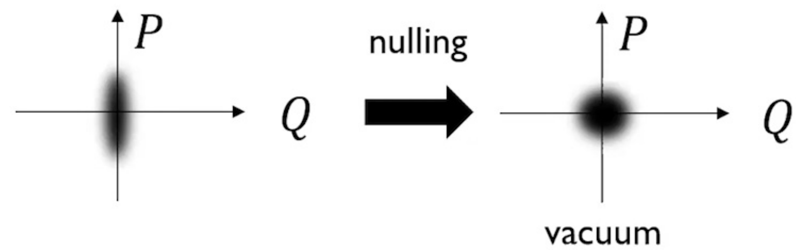
- Purification of output state: Unitary extension (Stinespring representation)



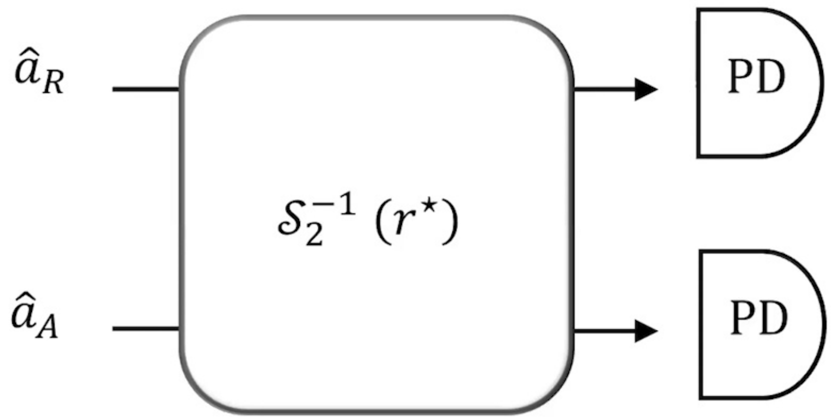
ANSATZ OF OPTIMAL MEASUREMENT: NULLING RECEIVER

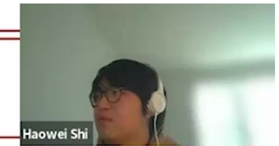
For single-mode squeezed vacuum (SV):



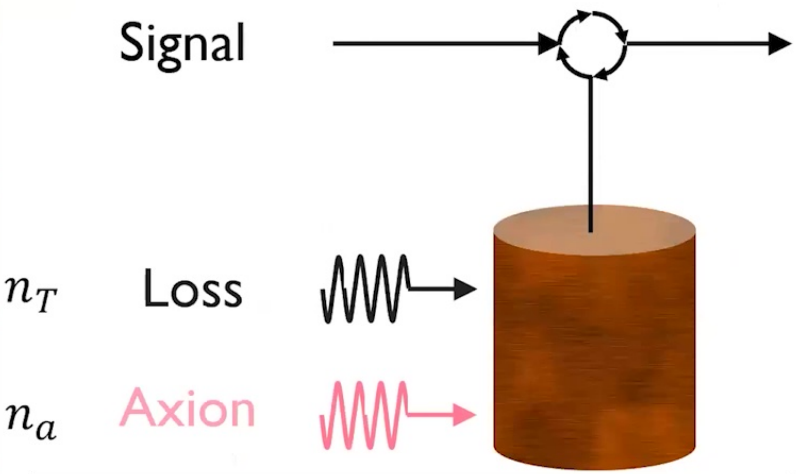


For two-mode squeezed vacuum (TMSV):

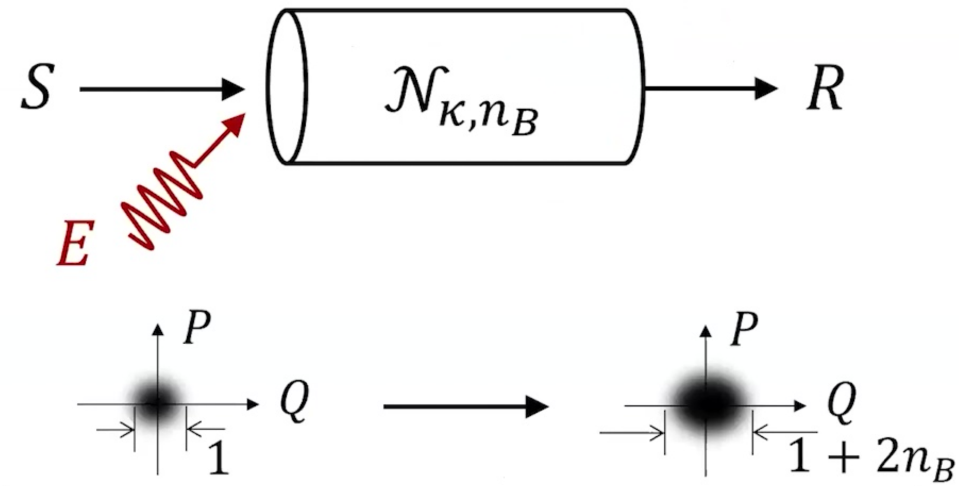




MICROWAVE COUPLING AS BOSONIC CHANNEL



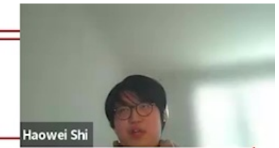
$$\hat{a}_{out} = \chi_{mm}\hat{a}_{in} + \sqrt{1-\chi_{mm}^2}\,\hat{a}_l + \sqrt{1-\chi_{mm}^2}\,\frac{\chi_{ma}\mu_a}{\sqrt{1-\chi_{mm}^2}}$$



$$\hat{a}_R = \sqrt{\kappa}\hat{a}_S + \sqrt{1-\kappa}\,\hat{a}_E$$

Dark count: $n_B = (1-\kappa)n_E$

$$n_B(\omega) = [1-\chi_{mm}^2(\omega)]n_T + \chi_{ma}^2(\omega)n_a$$



SUMMARY

- Optimality of two-mode squeezed vacuum (TMSV) in dark matter search
- Optimality of nulling receiver
- Quantum advantage of nulling receiver over vacuum homodyne measurement
  - Infinite advantage $\sim \frac{1}{n_B} \to \infty$ at weak noise limit $n_B \to 0$, even with weak squeezing $G \to 1$
  - Microwave photon counting?
- Entanglement benefit: TMSV robust to loss and noise
- Open question: General axion model and non-Gaussian random-phase displacement?

CONTENTS

Haowei Shi

ABSORPTION SPECTROSCOPY

- Fingerprint:
  Transmissivity spectrum $\kappa_h(\omega)$
  $\rightarrow$ Molecule class h

- Input-output relation:
  $\hat{a}(\omega) \rightarrow \sqrt{\kappa}(\omega)\hat{a}(\omega) + \sqrt{1-\kappa}\hat{e}(\omega)$

- Task: Distinguish different pattern $\kappa(\omega)$, minimize the error rate

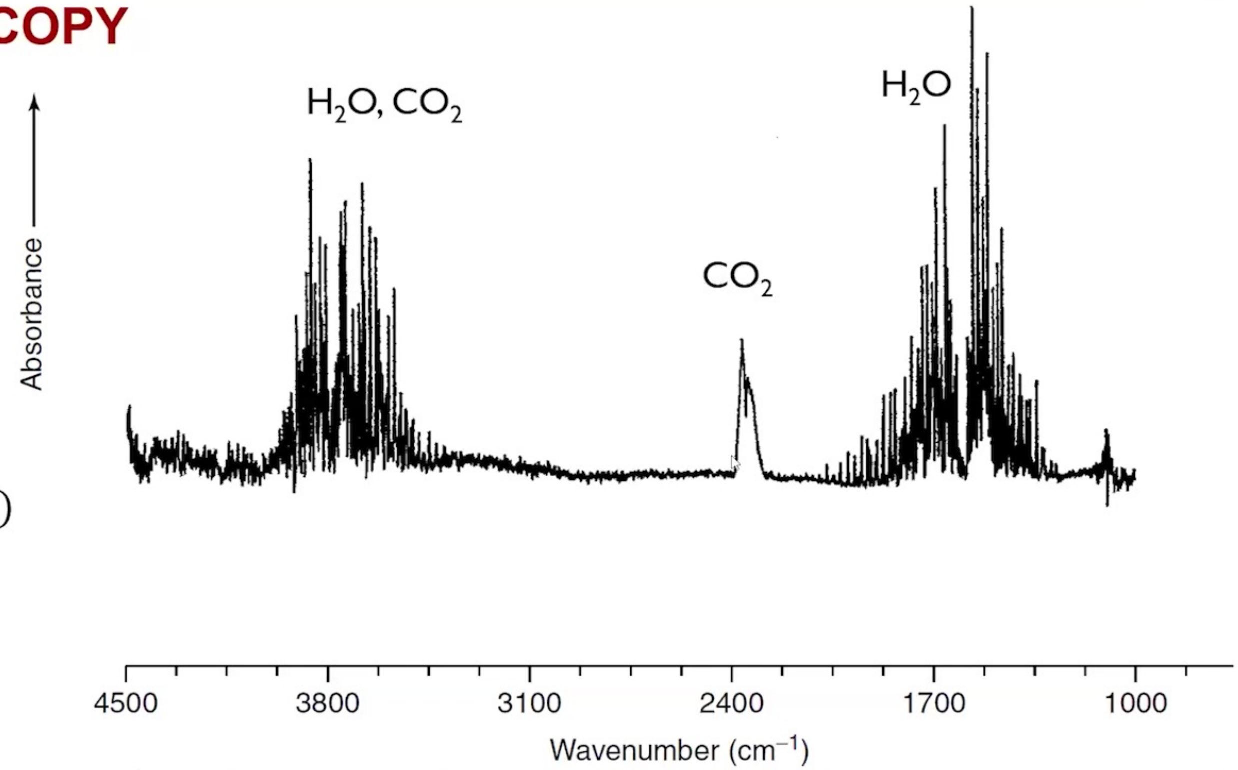
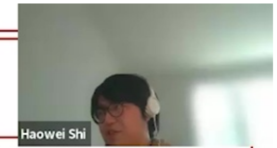


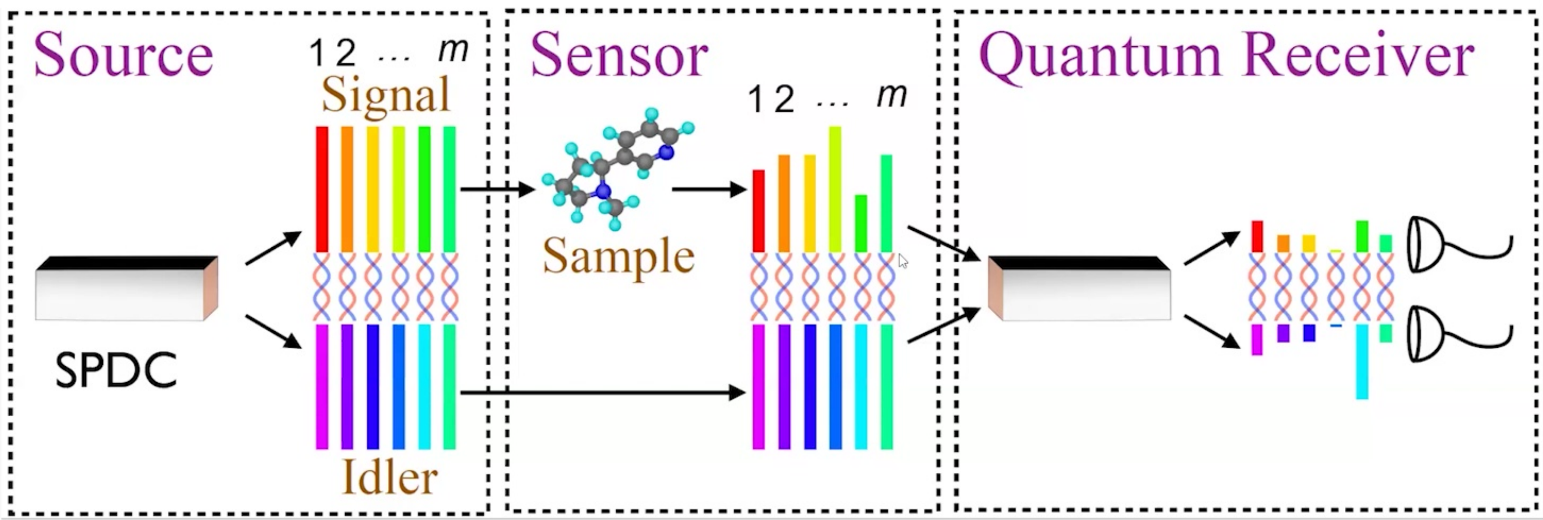
Figure Infrared spectrum of atmospheric contributions (e.g. CO_2 and H_2O).
From Stuart, B., *Modern Infrared Spectroscopy*, ACOL Series, Wiley, Chichester, UK, 1996.

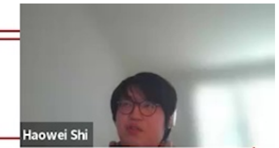


Haowei Shi

ENTANGLEMENT-ASSISTED (EA) SPECTROMETER SETUP

- m-frequency-bin probe
- M independent and identically distributed *(i.i.d.)* probe copies





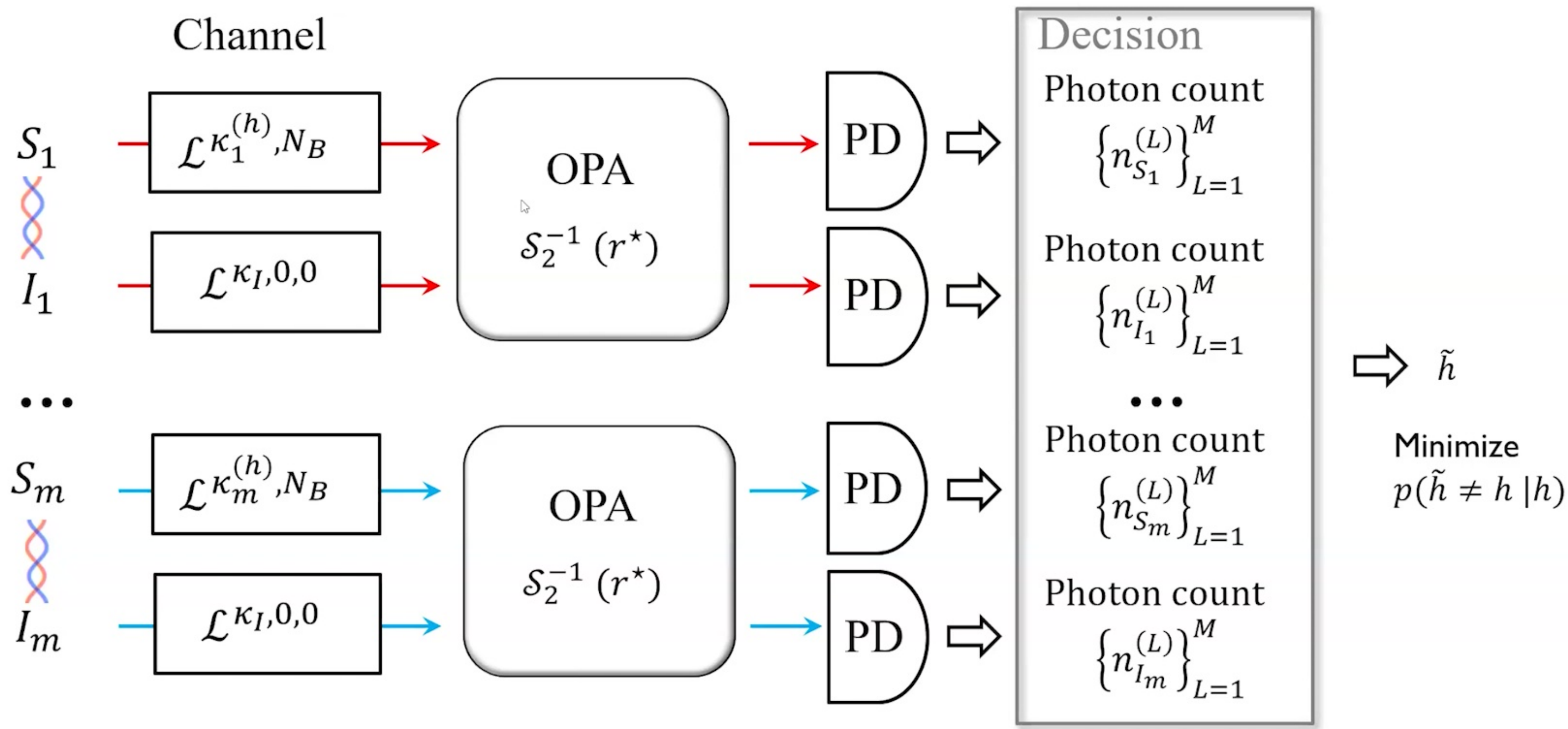
ULTIMATE CLASSICAL BENCHMARK

- "Classical" source: a statistical ensemble of coherent states

- Lower bound on *any* such classical source and *any* receiver
  - Error rate lower bounded by fidelity
  - Concavity of fidelity: ensemble → coherent state
  - Fidelity formula of coherent state is well-known:

    C. Weedbrook, S. Pirandola, R. García-Patrón, N. J. Cerf, T. C. Ralph, J. H. Shapiro, and S. Lloyd, Gaussian quantum information, Rev. Mod. Phys. 84, 621 (2012).

OPTICAL PARAMETRIC AMPLIFIER (OPA) NULLING RECEIVER



ABSORPTION DETECTION AND PEAK POSITIONING

(a) Absorption detection.

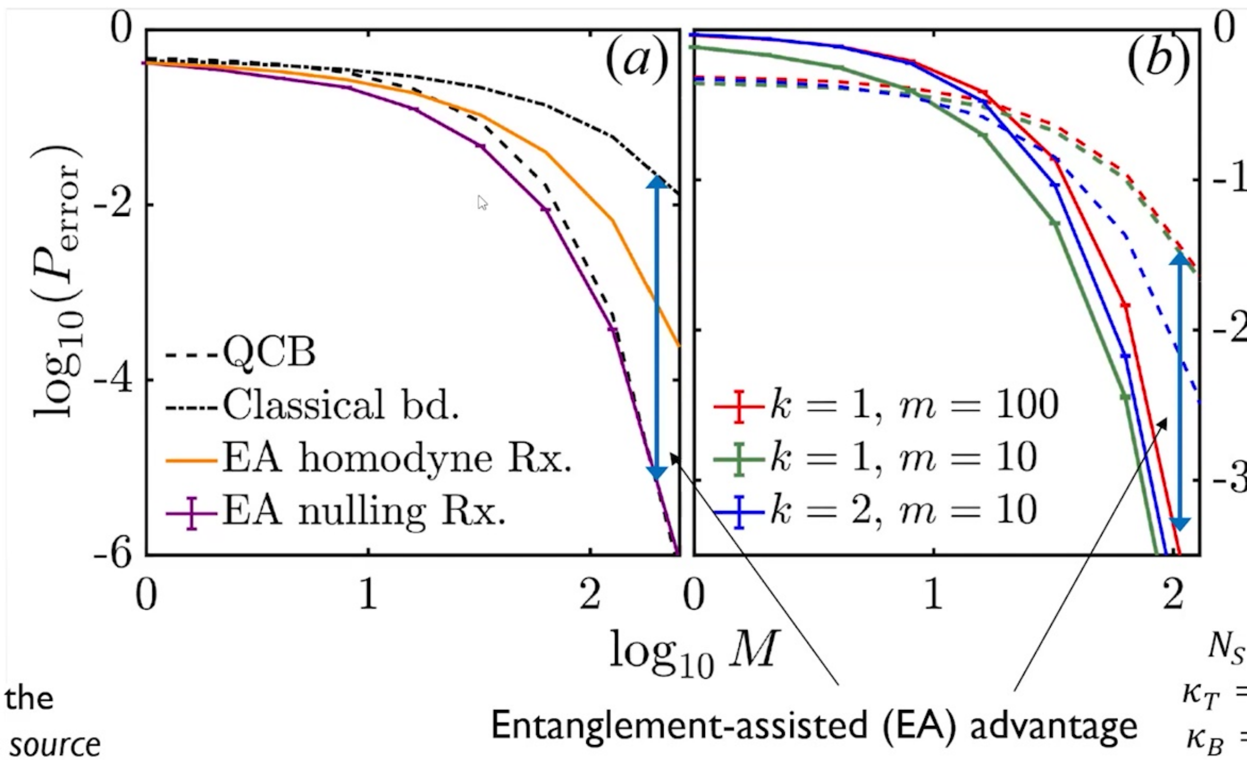
(b) k-peak positioning among m bins.

Solid lines:

Nulling receiver;

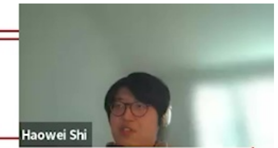
Dot-dashed lines:

ultimate classical lower bounds.



$N_S = 1$, $\kappa_T = 0.75$, $\kappa_B = 0.95$.

QCB: quantum Chernoff bound, the optimal error rate *given the source* over *any receiver* for large M.



ROBUSTNESS AGAINST LOSS AND NOISE

$\kappa_I = 1, N_B = 0.$

Row by row:

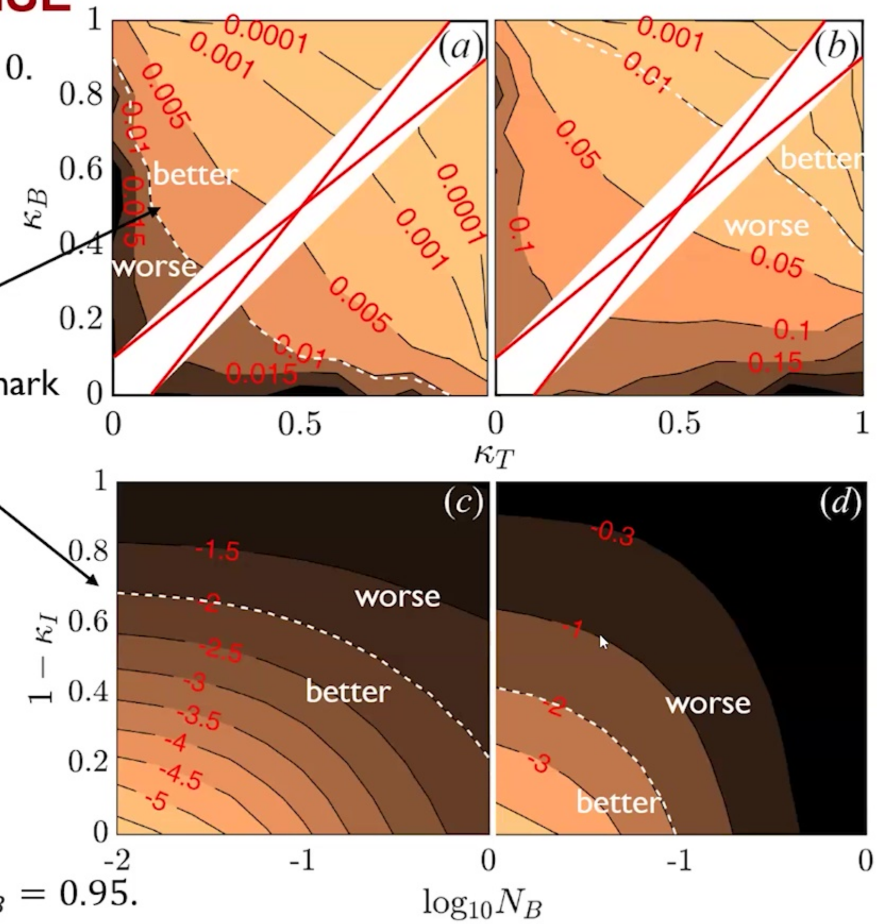
- (a)(b) Robustness against sample absorption;
- (c)(d) Robustness against noise and idler loss.

Column by column:

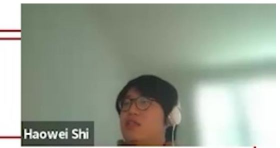
- (a)(c): Absorption detection (m=1);
- (b)(d): single-peak positioning (m=100, k=1).

- Classical benchmarks fixed to 0.01 (white-dashed). Source power N_S=1.

Classical benchmark

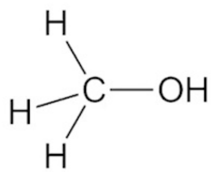


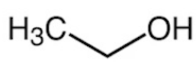
$\kappa_T = 0.75, \kappa_B = 0.95.$

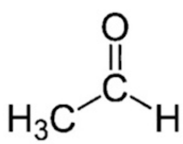


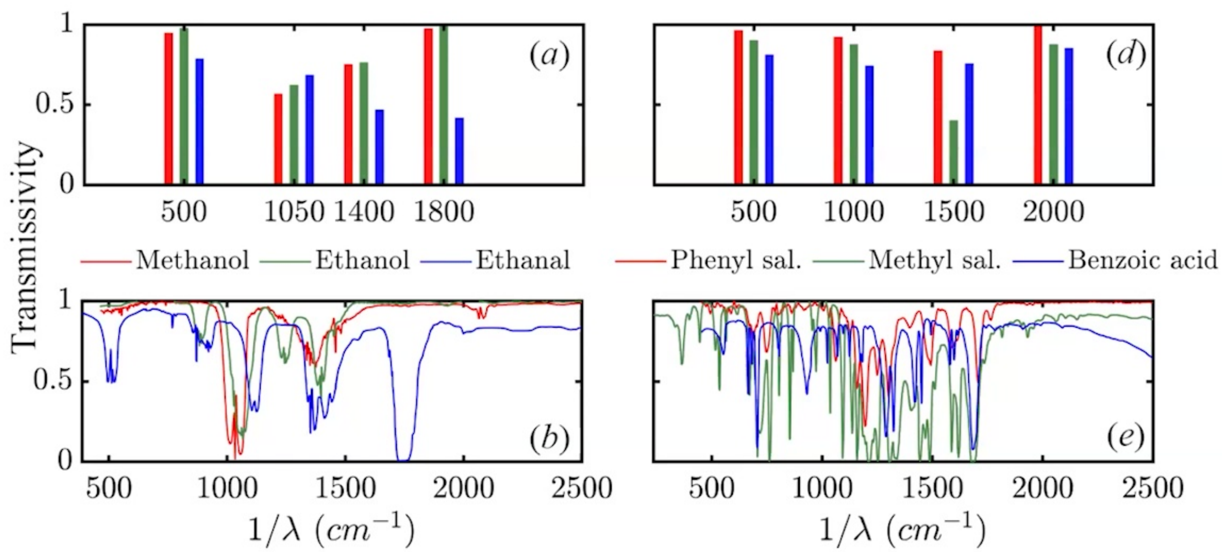
PATTERN CLASSIFICATION: REAL MOLECULES

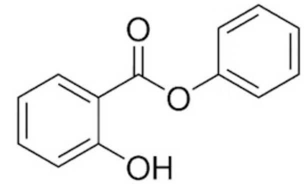
- Figure (a): alcohol scenario {h_1=methanol, h_2= ethanol, h_3= ethanal};
  Figure (d): drug scenario {h_1= phenyl salicylate, h_2= methyl salicylate, h_3= benzoic acid}.
- (a)(d): m=4 discrete spectra, sampled on the real FTIR spectra (b)(e)†.

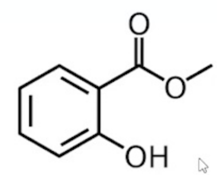


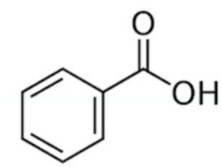




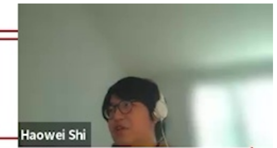






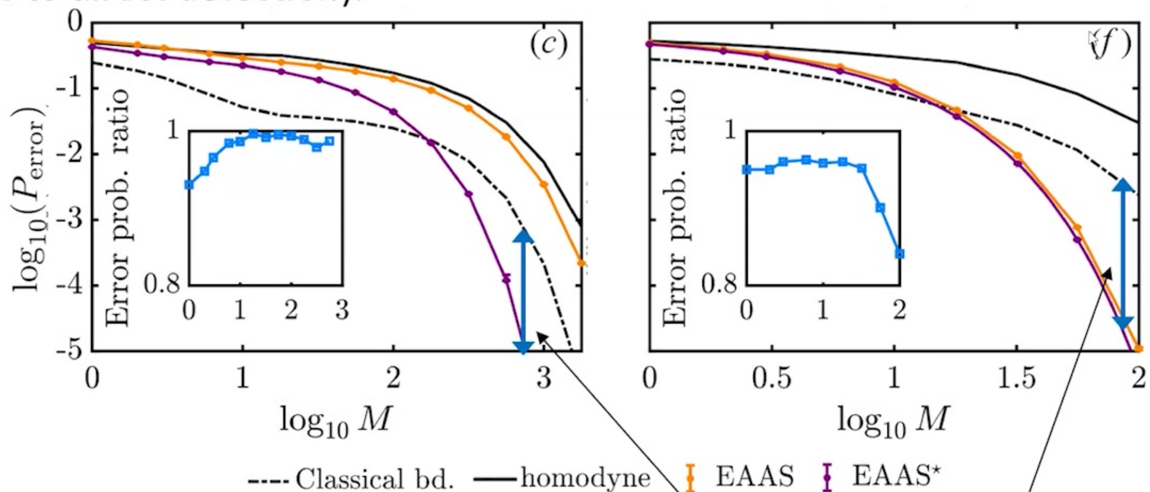


† Coblentz Society, Inc. ''Evaluated Infrared Reference Spectra'' in NIST Chemistry WebBook, NIST Standard Reference Database Number 69, Eds. P.J. Linstrom and W.G. Mallard, National Institute of Standards and Technology, Gaithersburg MD, 20899.



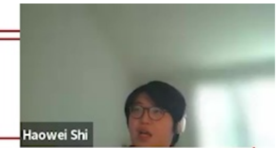
PATTERN CLASSIFICATION: REAL MOLECULES

- Error rate of classification of 3 molecules within (c) alcohol set; (f) drug set.
- N_S=1, G=1 (the nulling receiver reduces to *direct detection*).



Entanglement-assisted (EA) advantage

- EAAS: uniform *energy* distribution over *m* bins;
  EAAS*: numerically optimized *energy* distribution.
- Insets of (c) and (f): error probability ratio of gain-optimized case over G=1 case.

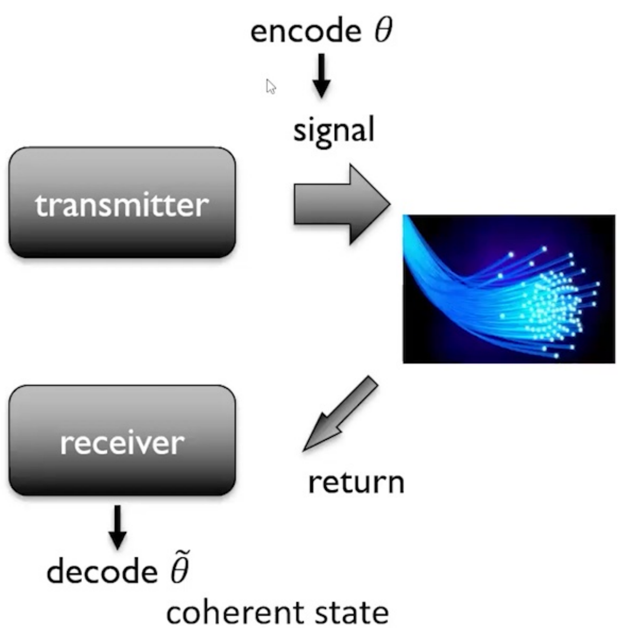


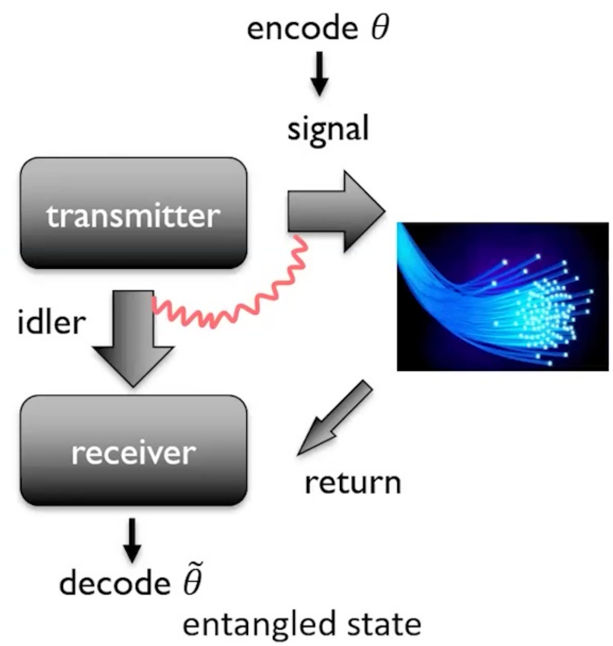
SUMMARY

- OPA nulling receiver: Provable entanglement-assisted (EA) advantages over the ultimate classical limit
- Robust against loss and noise
- All components off-the-shelf. Available for near-term experiment.
- Optimum receiver for general pattern? Ultimate EA limits?

CONTENTS

ENTANGLEMENT-ASSISTED COMMUNICATION





QUANTUM ADVANTAGE IN THE PRESENCE OF NOISE

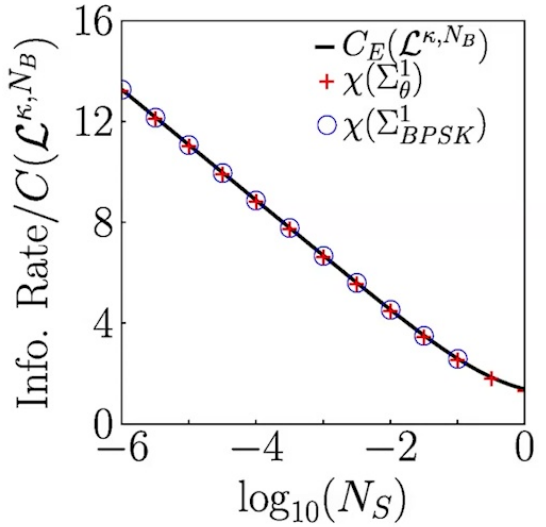
- Lossy and noisy: $\kappa \ll 1, N_B \gg 1$
- Ultimate limits:

  Entanglement-assisted capacity C_E, unassisted capacity C

  - Infinite quantum advantage $\sim \log\frac{1}{N_S}$ at $N_S \to 0$
  - Achieved by BPSK two-mode squeezed vacuum

    χ: Holevo information (given input state ensemble)

$\kappa = 0.1, N_B = 10$

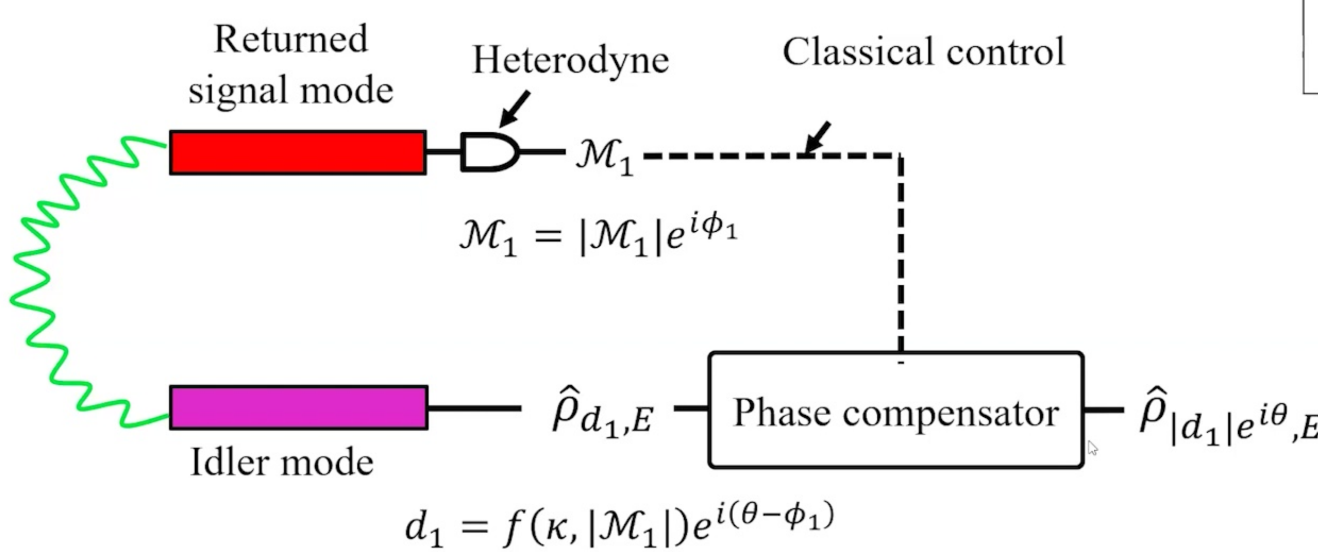


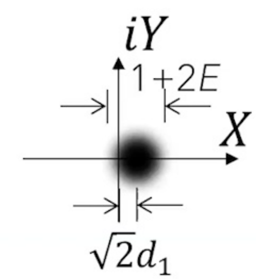
RECEIVER: CORRELATION TO DISPLACEMENT (C→D) CONVERSION

Shi, H., Zhang, B., Shapiro, J. H., Zhang, Z., & Zhuang, Q. (2023). *arXiv preprint arXiv:2309.12629*.

- 1. Heterodyne the signal; 2. Phase-correct the idler accordingly
- Output state: almost a coherent state
  - Green machine   Guha, S. (2011). Physical review letters, 106(24), 240502.

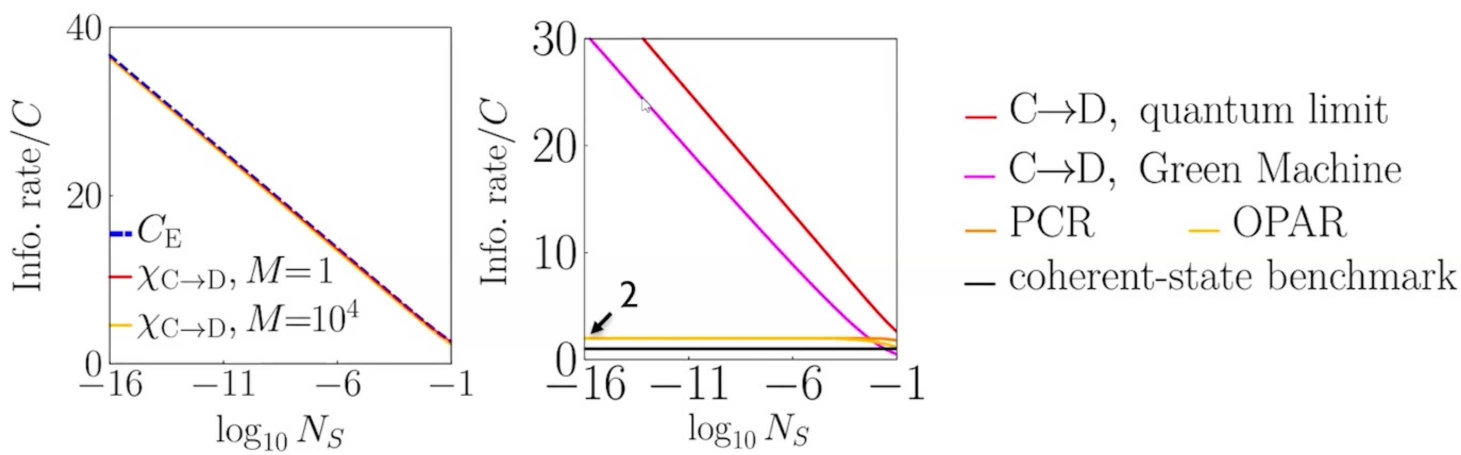
$\hat{\rho}_{d,E}$: displaced thermal state with mean $\sqrt{2}d$ and variance 1/2+E.
$E \simeq N_S \ll 1$

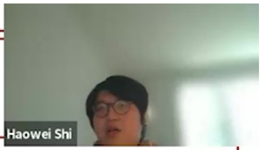




QUANTUM OPTIMALITY IN LOSS-TOLERANT COMMUNICATION

- Lossy and noisy: $\kappa = 10^{-3}, N_B = 10^4$
- At $N_S \to 0$, C→D conversion achieves the entanglement-assisted capacity C_E.
- Green machine : capacity-achieving in the scaling of N_S



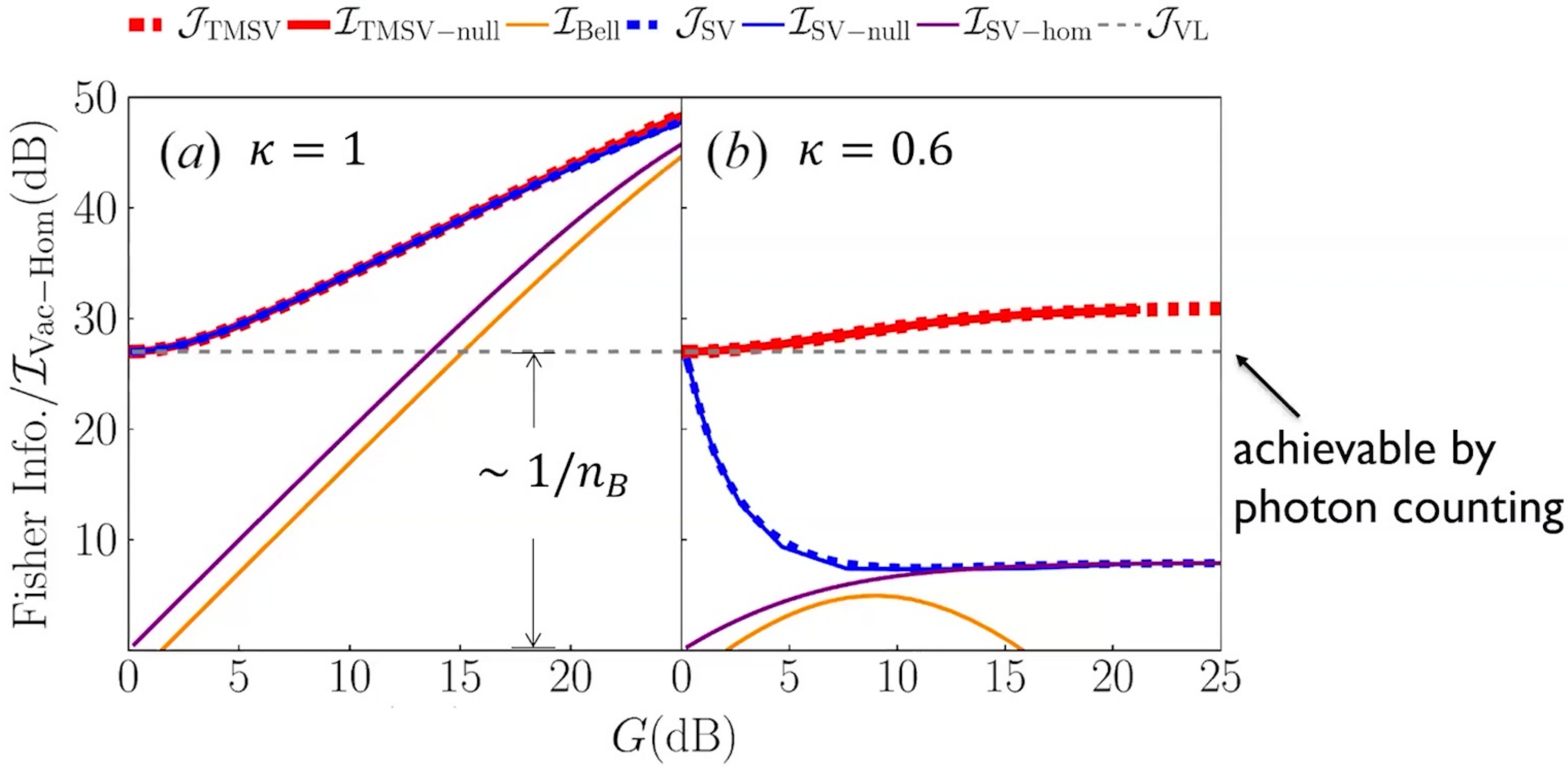


OPEN QUESTIONS

- Generalization of the ultimate precision limit
  - Non-Gaussian axion models, e.g. random-phase displacement
  - Cavity optomechanical systems
  - Spin systems
- General pattern classification with entanglement assistance
  - Phase sensing, phase noise
- Entanglement-assisted communication under non-Gaussian noises

OPTIMALITY OF NULLING RECEIVER

- Cooled environment: $n_B = 10^{-3}$



UNITARY EXTENSION BOUND

- Energy constrained: $\langle \hat{a}_S^\dagger \hat{a}_S \rangle \le N_S$.

$$\kappa \to 1$$
$$n_B \to 0$$

$$\mathcal{J}_{\mathrm{UB,UE}} = \underbrace{\boxed{\frac{1}{n_B(n_B+1)}}}_{\mathcal{J}_{\mathrm{VL}}} + \frac{\kappa N_S(2n_B - \kappa + 1)}{n_B(n_B+1)^2(n_B - \kappa + 1)} \simeq \frac{2N_S + 1}{n_B} \simeq \mathcal{J}_{\mathrm{TMSV}}$$

- Additivity

  To estimate a global parameter θ for compound channel $\bigotimes_{\{\ell=1\}}^{K} \mathcal{N}_{\kappa_\ell, n_{\mathrm{B},\ell}(\theta)}$:

$$\mathcal{J}_\theta^{\mathrm{UB,UE}} = \sum_{\ell=1}^{K} \left[\partial_\theta n_{\mathrm{B},\ell}\right]^2 \mathcal{J}_{\mathrm{UB,UE}}(N_{\mathrm{S},\ell}, \kappa_\ell, n_{\mathrm{B},\ell})$$

ULTIMATE PRECISION LIMIT OF NOISE SENSING

- Classical Fisher information (CFI)

$$I(\hat{\rho}_{in}, \{\Pi\}) := \mathrm{MMSE}^{-1}(\{\Pi \mathcal{N}_{\kappa,n_B}(\hat{\rho}_{in})\Pi\})$$

Input quantum state POVM

- Quantum Fisher information (QFI)

$$\mathcal{J}(\hat{\rho}_{in}) := \max_{\{\Pi\}} I(\hat{\rho}_{in}, \{\Pi\})$$

- Upper bound

$$\mathcal{J}_{\mathrm{UB}} \geq \max_{\hat{\rho}_{in}} \mathcal{J}(\hat{\rho}_{in})$$

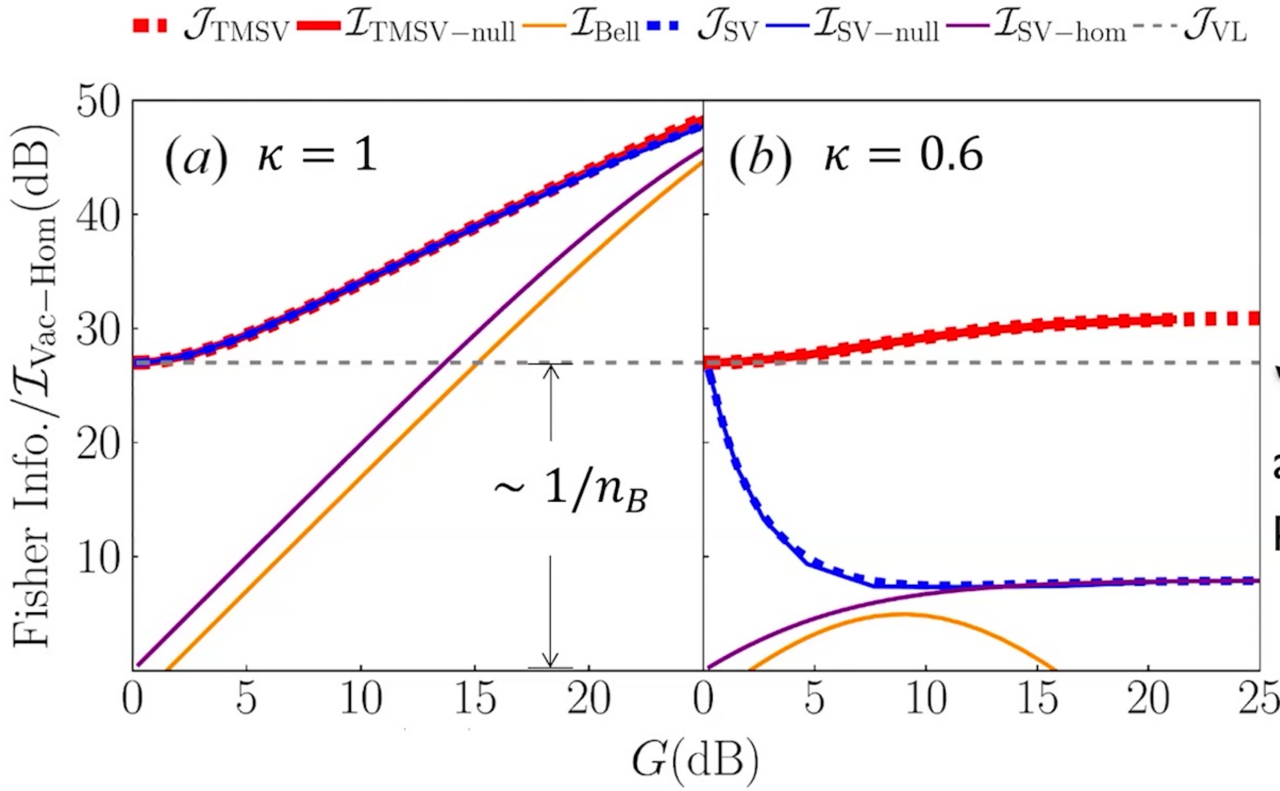
  - Our results: unitary extension bound $\mathcal{J}_{\mathrm{UB-UE}}$ and teleportation simulation bound $\mathcal{J}_{\mathrm{UB-TP}}$

Pirandola, S., & Lupo, C. (2017). PRL, 118(10), 100502.

$$\mathcal{J}_{\mathrm{UB}} = \max\{\mathcal{J}_{\mathrm{UB-UE}}, \mathcal{J}_{\mathrm{UB-TP}}\}$$

OPTIMALITY OF NULLING RECEIVER

- Cooled environment: $n_B = 10^{-3}$



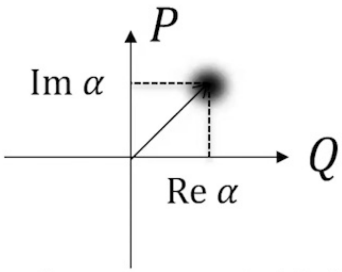
achievable by photon counting

BOSONIC FORMULATION: QUANTUM OPTICS

- Second harmonic oscillator

$$\hat{H} = \frac{1}{2}\int (\epsilon_0 \boldsymbol{E}^2 + \mu_0 \boldsymbol{H}^2) d\boldsymbol{r} = \sum_k \hbar\omega_k (\frac{1}{2} + \hat{a}_k^\dagger \hat{a}_k)$$

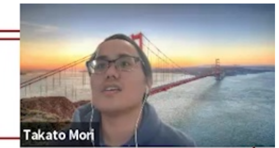
  - Axion dark matter Weinberg, S. (1978). A new light boson?. *Physical Review Letters*, *40*(4), 223.
- Annihilation operator $\hat{a} = \hat{Q} + i\hat{P}$
  - $[\hat{Q}, \hat{P}] = i/2$



Coherent state $|\alpha\rangle\langle\alpha|$

- Boson $\leftrightarrow$ SU(2) spin ensemble: Holstein-Primakoff mapping

  Klein, A., & Marshalek, E. R. (1991). *Reviews of modern physics*, *63*(2), 375.



FORMULATING LOSS AND NOISE: BOSONIC LOSS CHANNEL

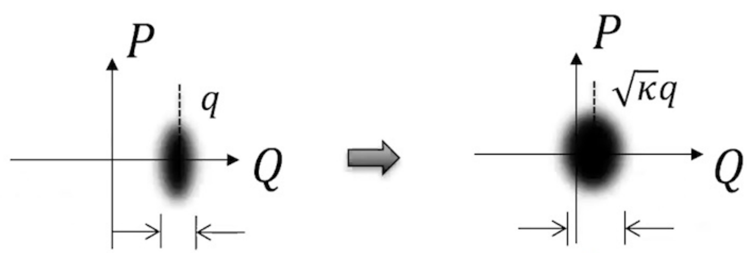
- Cavity loss, diffraction, scattering…

- Input-output relation: $\hat{a}_{out} = \sqrt{\kappa}\hat{a}_{in} + \sqrt{1-\kappa}\hat{e}$

  Background dark count $n_B := \langle \hat{a}_{out}^\dagger \hat{a}_{out} \rangle \big|_{\langle \hat{a}_{in}^\dagger \hat{a}_{in} \rangle = 0} = (1-\kappa)\langle \hat{e}^\dagger \hat{e} \rangle$

  $\rightarrow \hat{X}_{out} = \sqrt{\kappa}\hat{X}_{in} + \sqrt{1-\kappa}\hat{X}_e$

  Additive quadrature noise $= (1-\kappa)\langle \hat{X}_e^2 \rangle = \frac{1}{4}[(1-\kappa) + 2n_{\mathrm{B}}]$

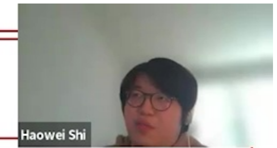


$\frac{1}{4G}$

Pure state

$\frac{\kappa}{4G} + \frac{1}{4}[(1-\kappa) + 2n_B] > 0$ for $G \to \infty$

Thermal mixture



ULTIMATE PRECISION LIMIT OF NOISE SENSING

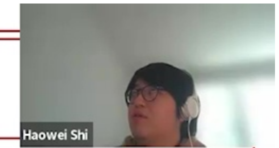
- Classical Fisher information (CFI)

$$I(\hat{\rho}_{in}, \{\Pi\}) := \mathrm{MMSE}^{-1}\left(\left\{\Pi \mathcal{N}_{\kappa,n_B}(\hat{\rho}_{in})\Pi\right\}\right)$$

Input quantum state POVM

- Quantum Fisher information (QFI)

$$\mathcal{J}(\hat{\rho}_{in}) := \max_{\{\Pi\}} I(\hat{\rho}_{in}, \{\Pi\})$$



DERIVATION OF UNITARY EXTENSION BOUND

$$\mathcal{J}(\hat{\rho}_{in}) = -4\partial^2_{n_B'} F\left[\mathcal{N}_{\kappa,n_B}(\hat{\rho}_{in}), \mathcal{N}_{\kappa,n_B'}(\hat{\rho}_{in})\right]\Big|_{n_B'=n_B}$$

Braunstein, S. L., & Caves, C. M. (1994). PRL, 72(22), 3439.

› Challenge: Evaluating fidelity $F[\hat{\rho}, \hat{\sigma}] = \mathrm{Tr}\sqrt{\sqrt{\hat{\rho}}\hat{\sigma}\sqrt{\hat{\rho}}}$ for arbitrary M-mode states