Title: Dynamics from Dispersion: a versatile tool

Speakers: Makinde Ogunnaike

Series: Quantum Matter

Date: November 28, 2023 - 11:00 AM

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Abstract: Driven by rapid advancements in quantum simulation capabilities across diverse physical platforms, open quantum systems are now of great interest, with special focus on thermalization processes of interacting many-body systems. Various techniques have been used to study operator spreading, to characterize entanglement dynamics, and even to identify exotic phases enabled by dynamical symmetries.

This talk will present a novel perspective on dynamical quantum systems that is capable of reproducing many previous results under a single intuitive framework and enables new results in symmetry-constrained systems. This is accomplished via a mapping between the dynamics averaged over Brownian random time evolution and the low-energy spectrum of a Lindblad superoperator, which acts as an effective Hamiltonian in a doubled Hilbert space. Doing so, we identify emergent hydrodynamics governing charge transport in open quantum systems with various symmetries, constraints, and ranges of interactions. By explicitly constructing dispersive excited states of this effective Hamiltonian using a single mode approximation, we provide a comprehensive understanding of diffusive, subdiffusive, and superdiffusive relaxation in many-body systems with conserved multipole moments and variable interaction ranges. Our approach further allows us to identify exotic Krylov-space-resolved diffusive relaxation despite the presence of dipole conservation, which we verify numerically. Therefore, we provide a simple, general, and versatile framework to qualitatively understand the dynamics of conserved operators under random unitary time evolution, and by extension, thermalizing quantum systems.

O. Ogunnaike, J. Feldmeier, J.Y. Lee, "Unifying Emergent Hydrodynamics and Lindbladian Low-Energy Spectra across Symmetries, Constraints, and Long-Range Interactions," arXiv:2304.13028 (accepted to PRL)

Zoom link https://pitp.zoom.us/j/94890217558?pwd=dTg4Mm9xOTBJUzNCeHFDaTdoNlFSQT09

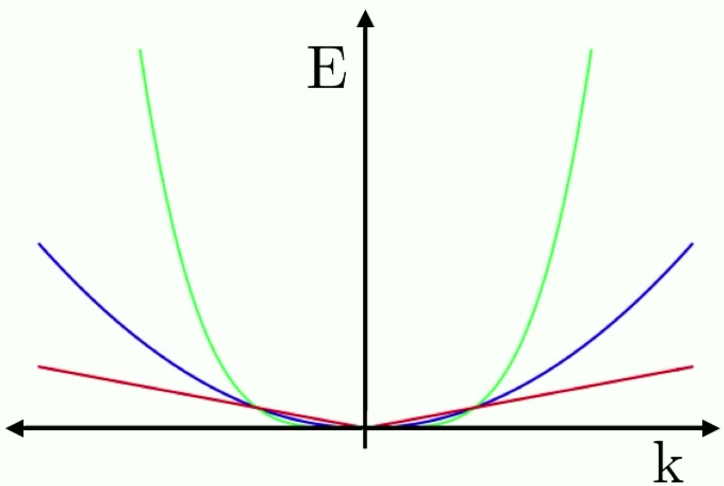
Dynamics from Dispersion

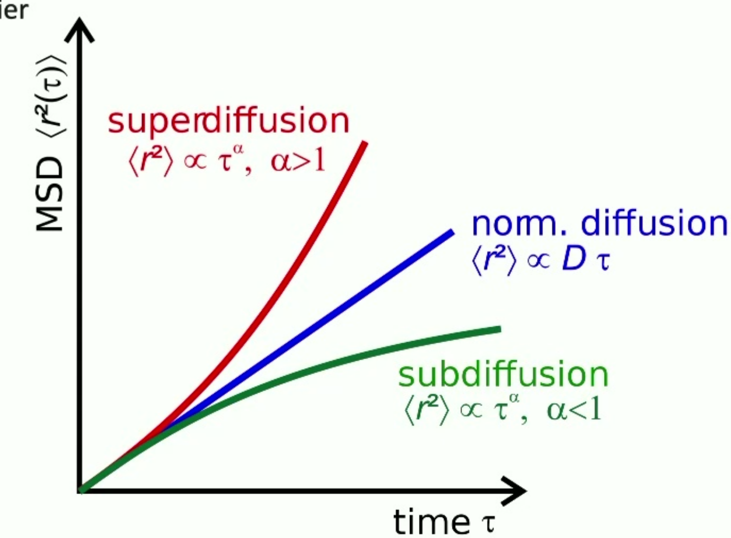
A versatile tool

Olumakinde Ogunnaike
MIT

arXiv:2304.13028, O Ogunnaike, JY Lee, J Feldmeier
Accepted to PRL

+ Work in Progress with JY Lee





Wikipedia: Anomalous Diffusion

Perimeter Institute
11/28/23

Motivation

Thermalization from Random Unitary Circuits

Decoherence/Thermalization comes from tracing out interactions with the environment

$$\hat{\rho}_{s,e} \equiv |\psi\rangle\langle\psi| = \hat{\rho}_s \otimes \hat{\rho}_e$$

$$\hat{\rho}_s(t) = Tr_e[U_{s,e}(t)\hat{\rho}_{s,e}(0)U^{\dagger}_{s,e}(t)]$$

H-J, Nicholas. *arXiv: Quantum Physics* (2018): n. pag.

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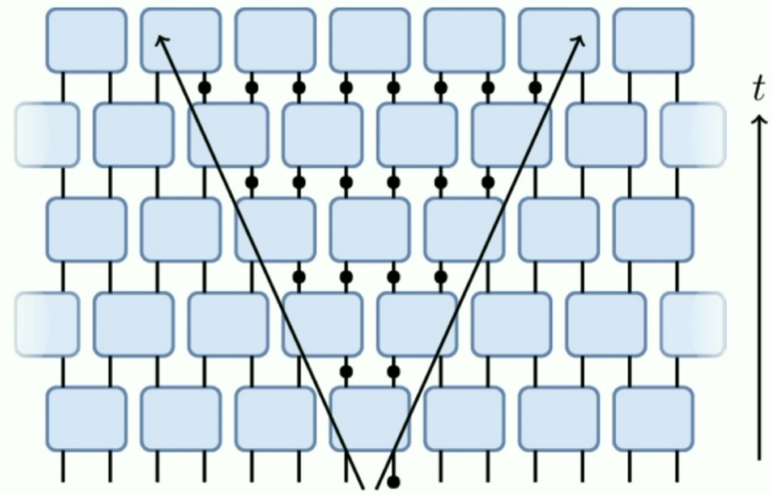
Environmental interactions make evolution look "random"

$$\hat{\rho}_s(t) = \mathbb{E}_{U_s}[U_s(t)\hat{\rho}_s(0)U^{\dagger}_s(t)]$$

Random Unitary Circuits approximate local, thermalizing systems



$$O_x|\psi\rangle = \bigotimes_{i<x}|\phi_0\rangle_i \otimes O_x|\phi_0\rangle_x \otimes \bigotimes_{i>x}|\phi_0\rangle$$

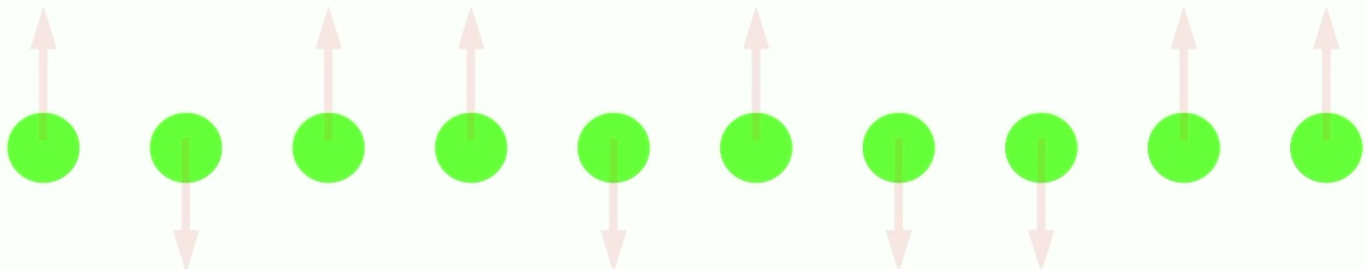
H-J, Nicholas. *arXiv: Quantum Physics* (2018): n. pag.

An Example

Spin - 1/2 Chain

$$H_0 = -J \sum_{\langle x, x' \rangle} (\vec{\sigma}_x \cdot \vec{\sigma}_{x'}) - h \sum_x \sigma_x^z$$

$$\Delta h_x = \sum_x (B_{x,t})$$



$$|\psi_0\rangle = \bigotimes_x |\uparrow\rangle_x$$

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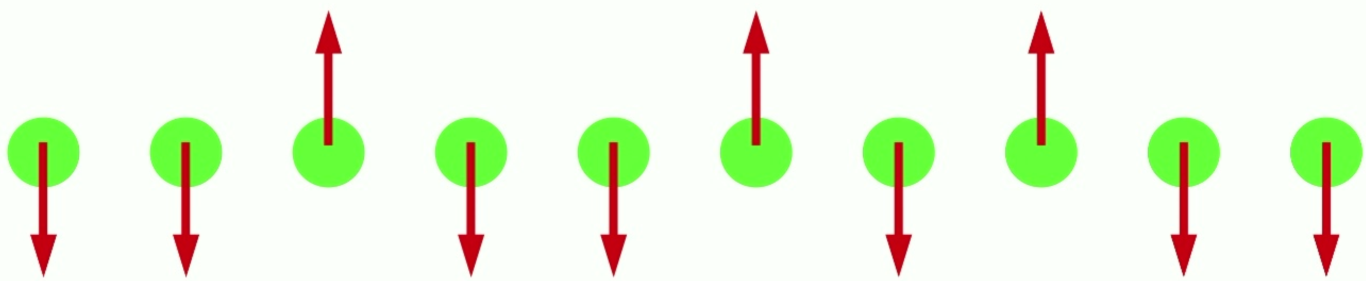

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An Example

Spin - 1/2 Chain

$$H_0 = -J \sum_{\langle x,x' \rangle} (\vec{\sigma}_x \cdot \vec{\sigma}_{x'}) - h \sum_x \sigma_x^z$$

$$\Delta J_x = \sum_{\langle x,x' \rangle} \Gamma_{x,x'} \qquad \Delta h_x = \sum_x (B_{x,t})$$

$$|\psi_0\rangle = \bigotimes_x |\uparrow\rangle_x$$

$$\hat{\rho} = \sum_\alpha p_\alpha |\psi_\alpha\rangle\langle\psi_\alpha|$$

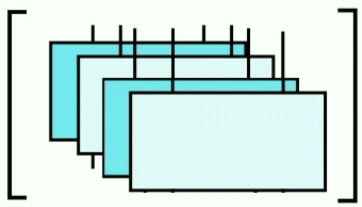
Haar-Averaging

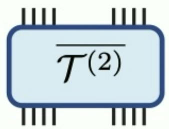
The only way?

The Haar-measure is an extremely powerful tool

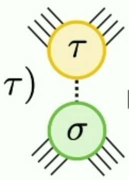
Random Circuits become Tensor Networks

Tensor Networks become d+1 statistical ensembles

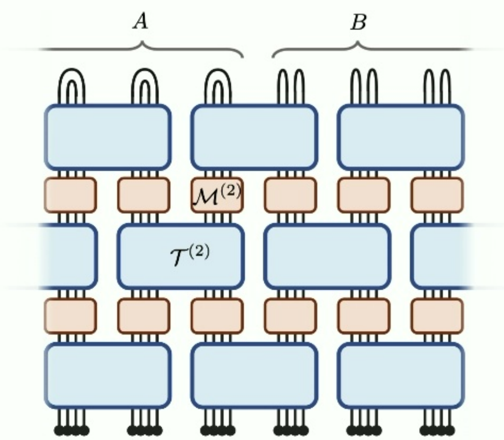
$\mathbb{E}$ 

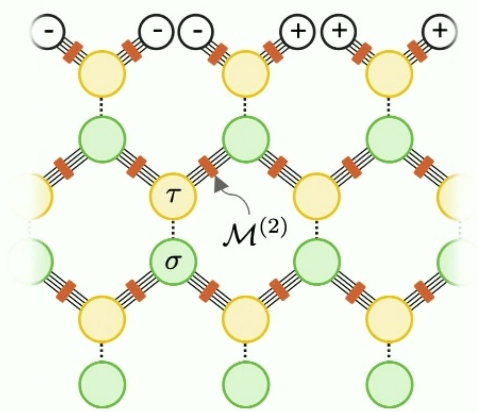


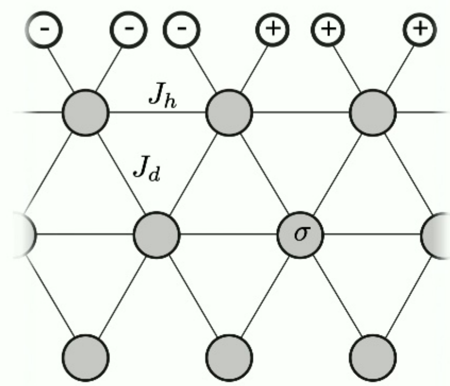
$$= \sum_{\sigma,\tau=\pm 1} w_g^{(2)}(\sigma,\tau)$$



$$\mu_{AM}^{(2)} = \mathcal{Z}_{AM}^{(2)} = \sum_{\{\sigma\}} \prod_{\sigma_1,\sigma_2,\sigma_3} \bar{w}(\sigma_1,\sigma_2,\sigma_3)$$



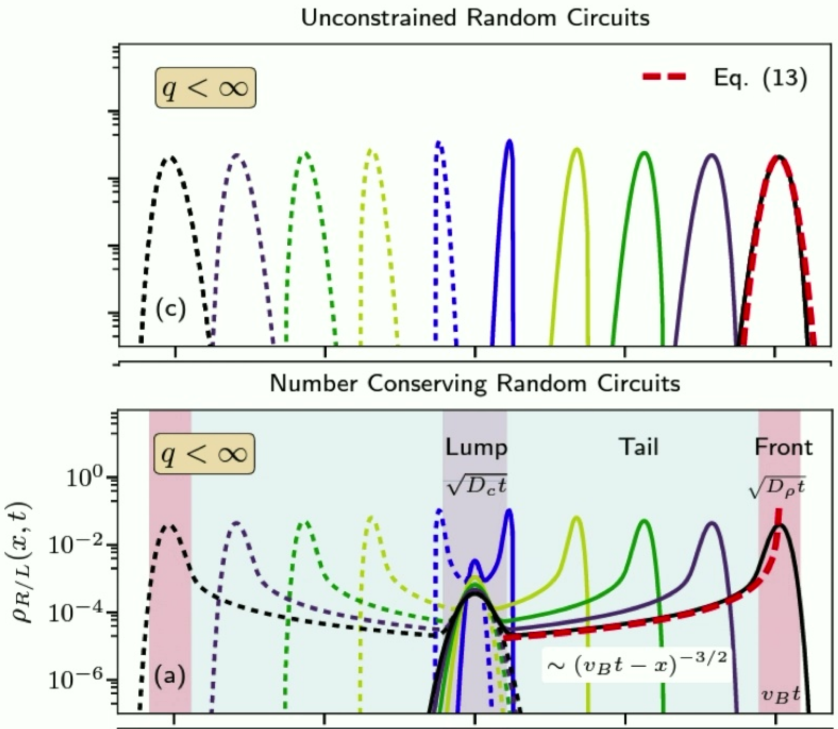


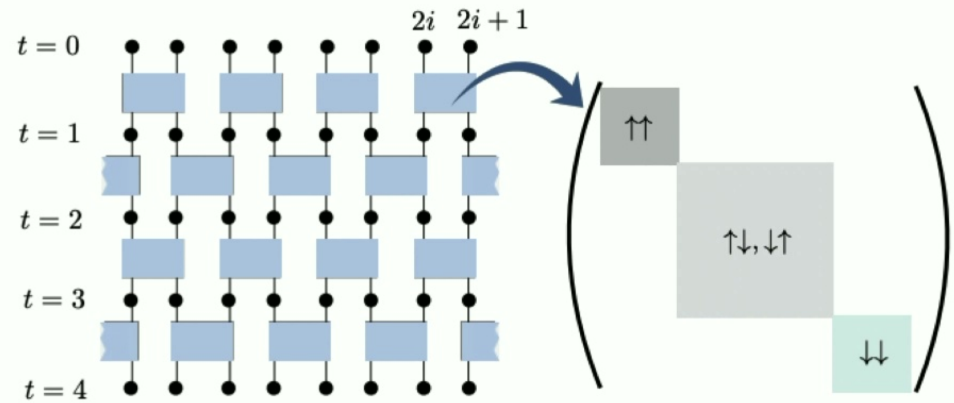


Y. Bao, S. Choi, and E. Altman, Phys. Rev. B **101**, (2020)

Motivation

Late-time decoherence dynamics with constraints





Local Conservation of $\sigma^z_{2i} + \sigma^z_{2i+1}$ causes diffusive dynamics of σ^z_i operators

$$\langle \sigma^z_i(0)\sigma^z_i(t)\rangle \sim \frac{1}{\sqrt{t}}$$

V. Khemani, A. Vishwanath, and D. A. Huse, Phys. Rev. X **8**, 031057 (2018)

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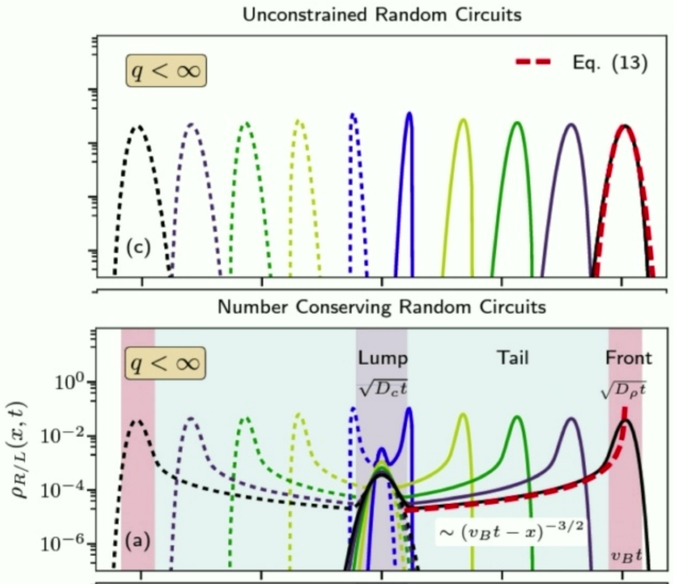

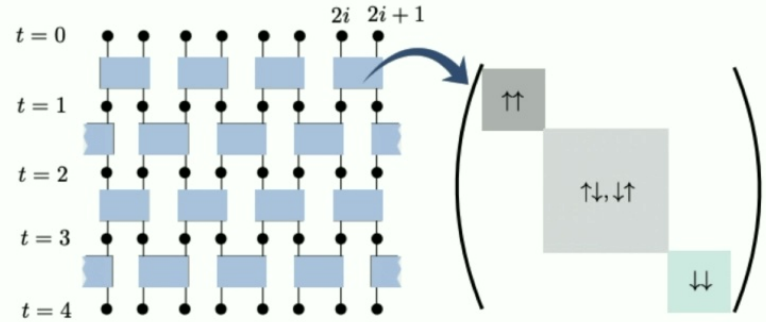

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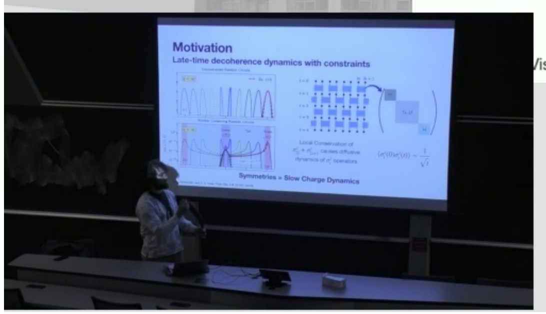
Symmetries = Slow Charge Dynamics



Vishwanath, and D. A. Huse, Phys. Rev. X **8**, 031057 (2018)

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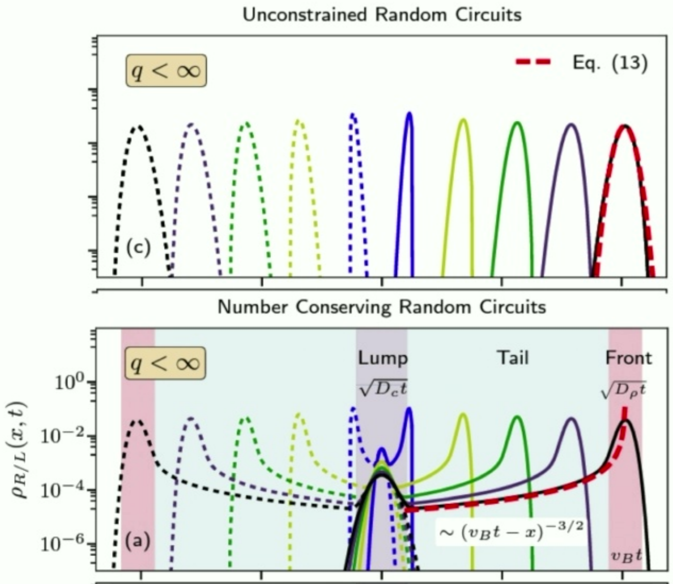

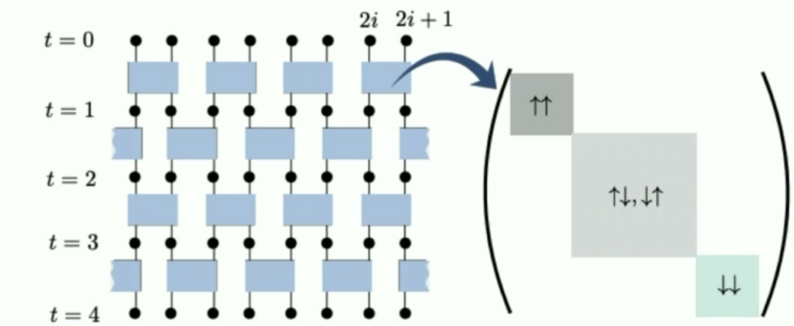

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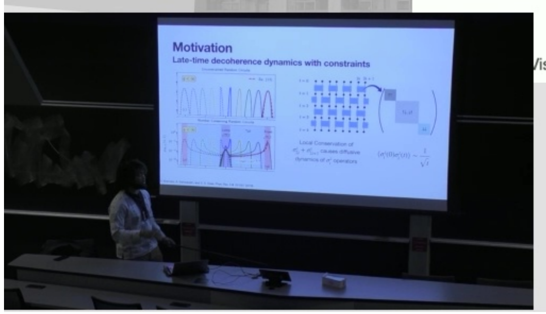

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Results

Many phenomena - one framework

A new perspective on thermalizing and decoherence phenomena

$$\hat{\rho}_s(t+\Delta t) = \mathbb{E}_{U_s}[U_s(\Delta t)\hat{\rho}_s(t)U_s^\dagger(\Delta t)]$$

$$\|\rho(t+\Delta t)\rangle\rangle = e^{-\overline{H}_{eff}\Delta t}\|\rho(t)\rangle\rangle$$

Examples

- Charge Conservation

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Duality between low-energy dispersion and late-time dynamics

$$\overline{H}_{eff}\|O_k\rangle\rangle = E_k\|O_k\rangle\rangle \qquad C_t = \mathbb{E}\langle O_x(0)O_x(t)\rangle_\rho$$

$$E_k \sim k^n \Rightarrow C_t \underset{t\to\infty}{\sim} t^{-d/n}$$

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- Charge Conservation

$E_k \sim k^2$ → **Diffusion** $C_t \underset{t\to\infty}{\sim} t^{-d/2}$

- Multipole Conservation

$E_k \sim k^{2(m+1)}$ → **Sub-diffusion**

- Constrained Dynamics

$E_k \sim k^{2(m-p+1)}$ → **(Sub)-diffusion**

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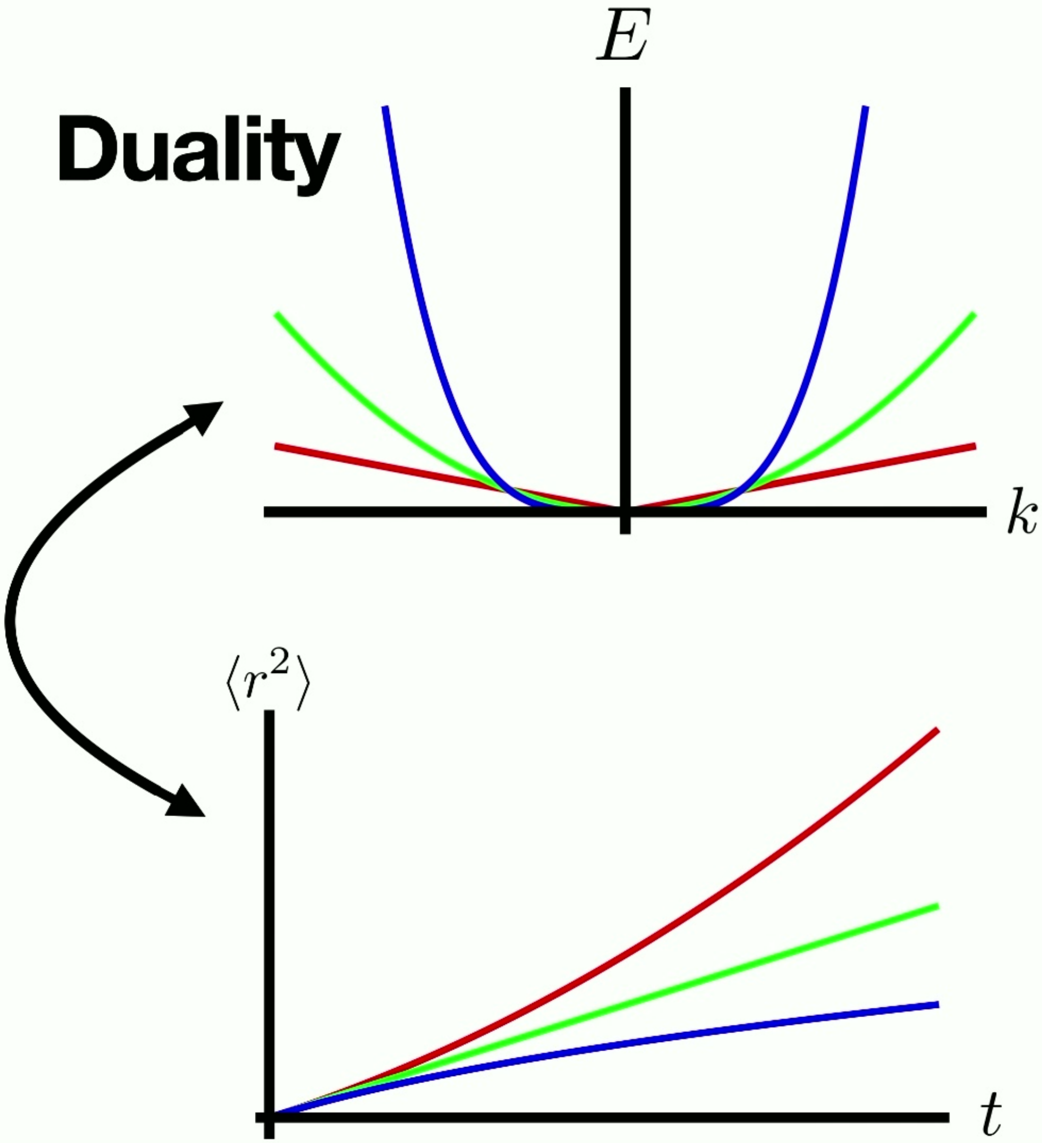
$E_k \sim k^{2(m-p+1)}$ ➡ **(Sub)-diffusion**

- Long-Range Interactions $\sim \frac{1}{r^\alpha}$

$E_k \sim k^{f(\alpha)}$ ➡ **Phases**

Duality



Examples

- Charge Conservation
- Multipole Conservation
- Constrained Dynamics
- Long-Range Interactions $\sim \frac{1}{r^\alpha}$

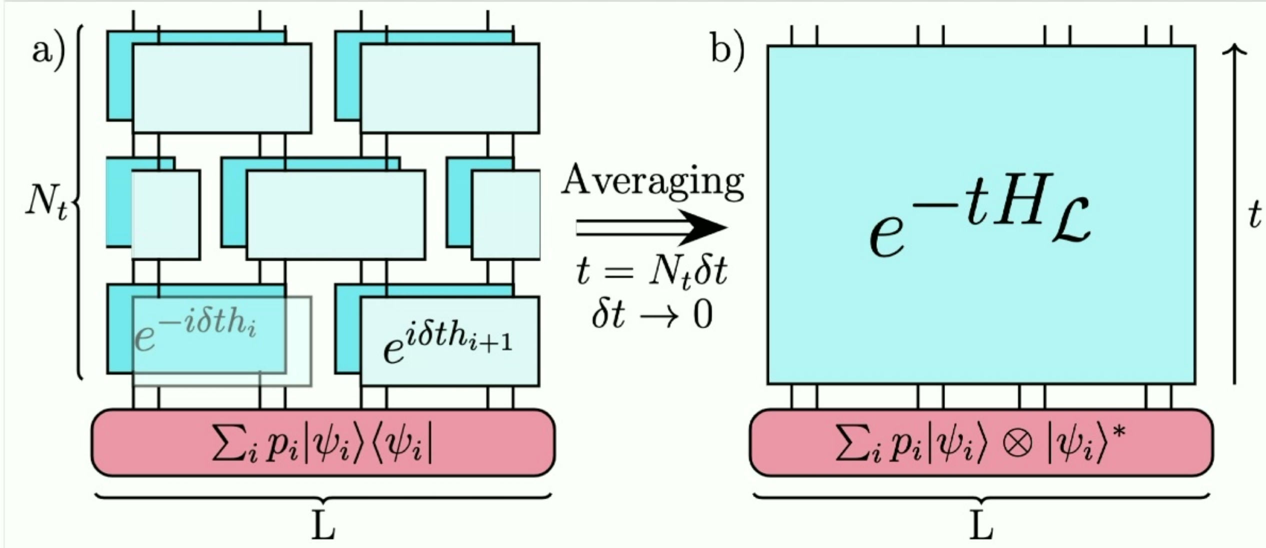
Framework

Brownian evolution to Effective Hamiltonian

$$O(t+\delta t) \equiv e^{iH_t\delta t} O(t) e^{-iH_t\delta t}$$

$$H_t = \sum_{\alpha=(i,\lambda)} h_\alpha dB_{\alpha,t}$$

$$\mathbb{E}[dB_{\alpha,t}] = 0 \qquad \mathbb{E}[dB^2_{\alpha,t}] = \frac{1}{\delta t}$$



O. Ogunnaike , J. Feldmeier, and J Y Lee, arXiv:2304.13028 (PRL forthcoming) (2023)

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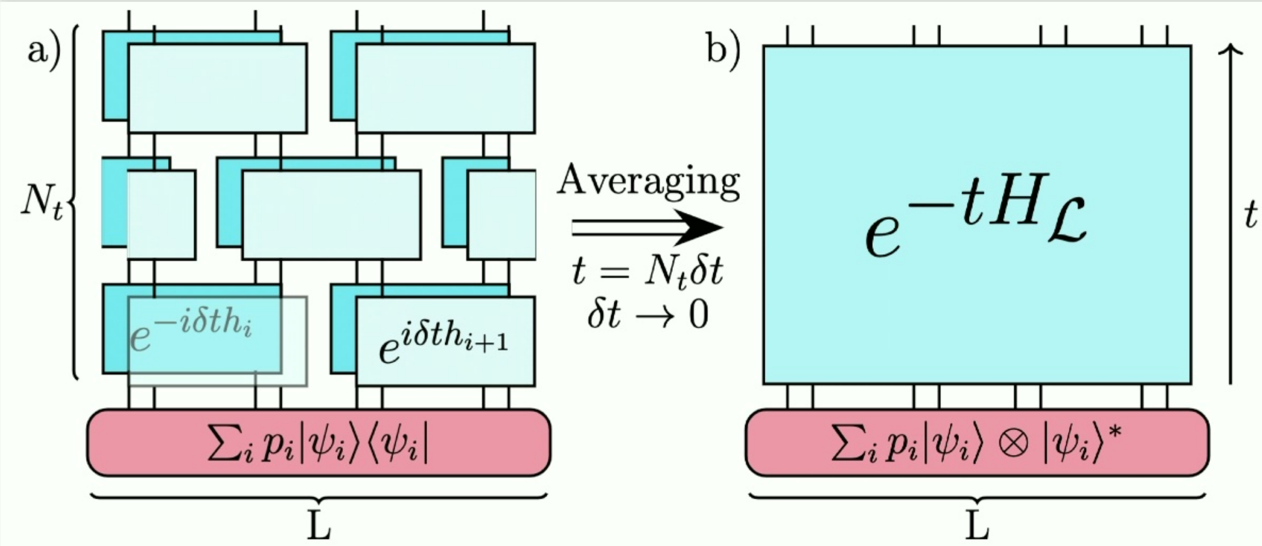

Lindbladian Evolution

$$\mathcal{L}[O] \equiv -\mathbb{E}[\partial_t O]$$

$$\mathcal{L}[O] = \frac{1}{2} \sum_{\alpha=(i,\lambda)} (h_\alpha^2 O - 2h_\alpha O h_\alpha + O h_\alpha^2)$$

Choi Isomorphism

$$O \mapsto \|O\rangle\rangle \equiv \sum_n |n\rangle \otimes (O|n\rangle)$$

Imaginary Schrödinger Evolution

$$\partial_t \|O\rangle\rangle = -H_{\mathcal{L}} \|O\rangle\rangle$$

$$H_{\mathcal{L}} = \sum_{\alpha=(i,\lambda)} |h_\alpha^T \otimes \mathbb{I} - \mathbb{I} \otimes h_\alpha|^2$$

O. Ogunnaike , J. Feldmeier, and J Y Lee, arXiv:2304.13028 (PRL forthcoming) (2023)

Framework

Ground States & Commutant Algebra

Positive Definite Spectrum

$$H_{\mathcal{L}} = \sum_{\alpha=(x,\lambda)} |h_\alpha^T \otimes \mathbb{I} - \mathbb{I} \otimes h_\alpha|^2$$

$$H_{\mathcal{L}} = \sum_{x,\lambda} \mathcal{O}_{x,\lambda}^\dagger \mathcal{O}_{x,\lambda}$$

Framework

Ground States & Commutant Algebra

Positive Definite Spectrum

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$$H_{\mathcal{L}} = \sum_{x,\lambda} \mathcal{O}_{x,\lambda}^\dagger \mathcal{O}_{x,\lambda}$$

Frustration-free Ground State

$$\mathcal{O}_{x,\lambda} ||\mathbb{I}\rangle\rangle \longleftrightarrow [h_\alpha, \mathbb{I}] = 0$$

$$\mathcal{O}_{x,\lambda} ||\mathbb{I}\rangle\rangle = 0 \longrightarrow H_{\mathcal{L}} ||\mathbb{I}\rangle\rangle = 0$$

Bond Algebra - interactions

$$\mathcal{A} = \langle\langle \{h_\alpha\} \rangle\rangle$$

Commutant Algebra - $\mathcal{C}$ - ground states

$$\hat{O} \in \mathcal{C} \longrightarrow [\hat{O}, \hat{h}_\alpha] = 0$$

$$\mathcal{O}_{x,\lambda} ||\hat{O}\rangle\rangle = 0$$

$$||\hat{O}\rangle\rangle = [\hat{O}^T \otimes \mathbb{I} + \mathbb{I} \otimes \hat{O}] ||\mathbb{I}\rangle\rangle$$

Ground State Degeneracy

$$d_0 = \dim(\mathcal{C})$$

Symmetries and Excitations

Approximate (Strong) Symmetries

Strong Symmetry: U(1)

$$[h_{x,\lambda}, \sum_x \rho_x] = 0$$

$$[\mathcal{O}_{x,\lambda}, \left(\sum_x \rho_x\right) \otimes \mathbb{I}] = [\mathcal{O}_{x,\lambda}, \mathbb{I} \otimes \left(\sum_x \rho_x\right)] = 0$$

Weak Symmetry: U(1)

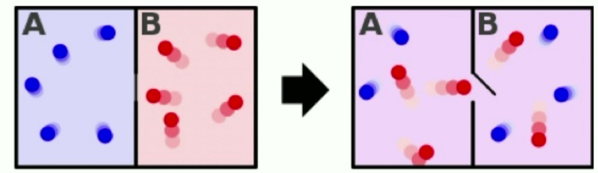
$$[\hat{\rho}(t), \sum_x \rho_x] = 0$$

$$[H_{\mathcal{L}}, \left(\sum_x \rho_x\right) \otimes \mathbb{I} - \mathbb{I} \otimes \left(\sum_x \rho_x\right)] = 0$$

Symmetries and Excitations

Approximate (Strong) Symmetries

$\rho_{eq} \sim e^{-\beta H}$ $\rho_{eq} \sim e^{-\beta(H-\mu N)}$



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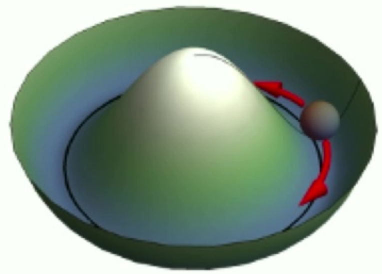
Enlarged Symmetry

$$G = [U(1) \times U(1)] \rtimes \mathbb{Z}_2^{\mathbb{H}}$$



Weak Symmetry: U(1)

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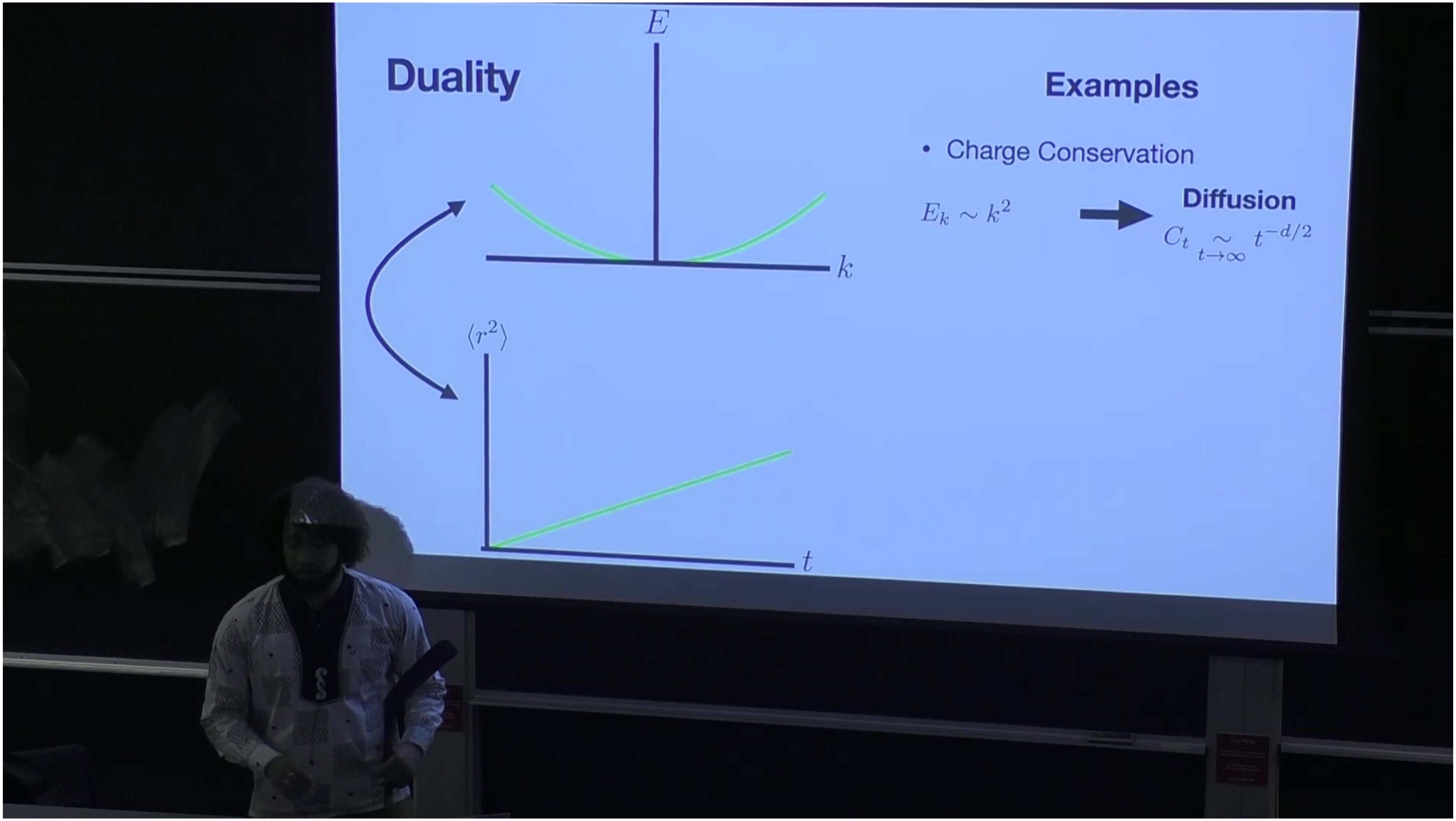
Enlarged Symmetry

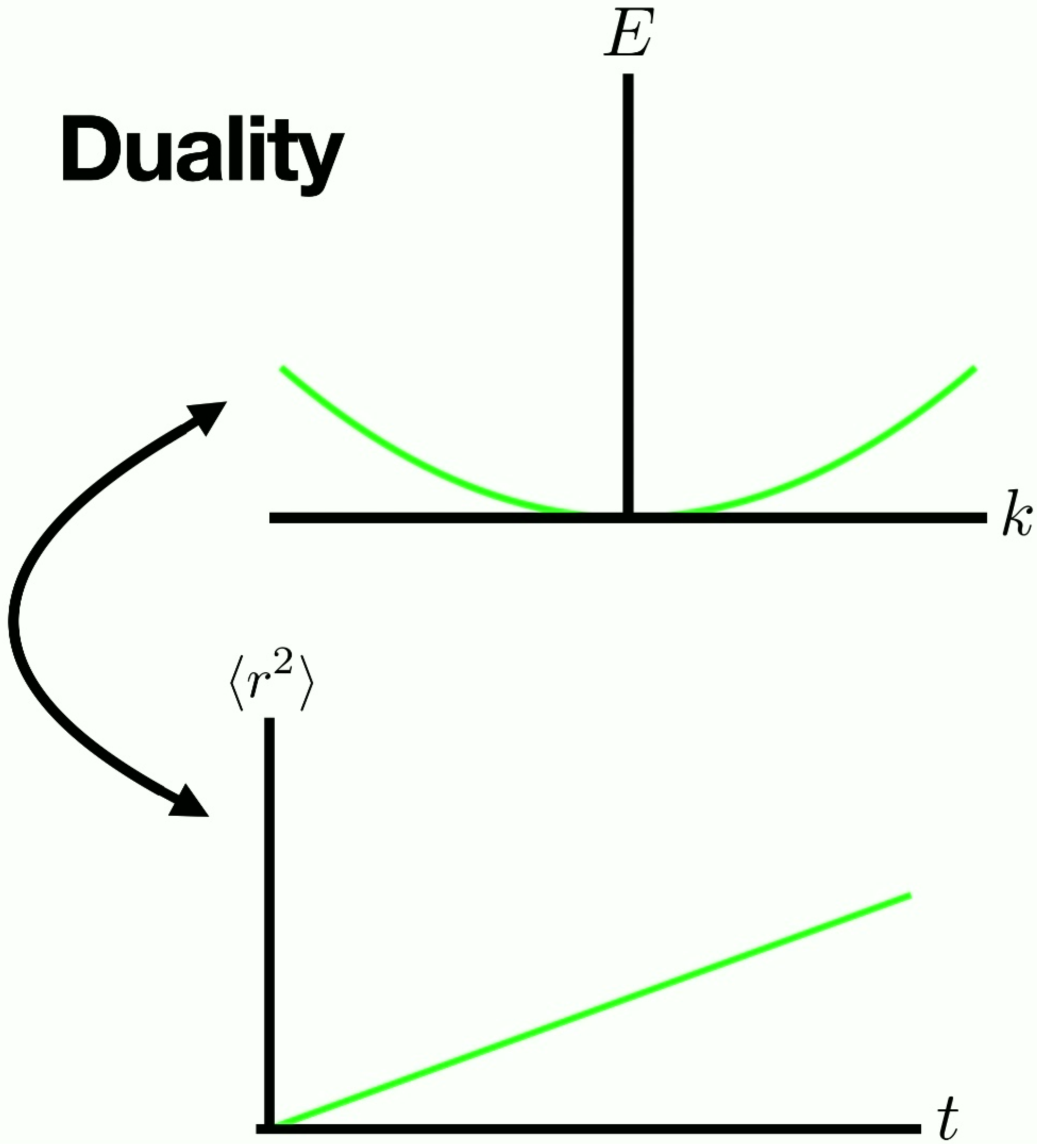
$$G = U(1) \rtimes \mathbb{Z}_2^{\mathbb{H}}$$

$$H_{\mathcal{L}} \| \sum_x \rho_x \rangle\rangle = 0 \quad \Rightarrow \quad [h_{x,\lambda}, \rho_k = \sum_x e^{ik\cdot x} \rho_x] \sim k^n \rho_k \quad \Rightarrow \quad H_{\mathcal{L}} \| \rho_k \rangle\rangle \sim k^{2n} \| \rho_k \rangle\rangle$$

Wikipedia: Grand canonical ensemble


Duality
E
k
$\langle r^2 \rangle$
t
Examples
• Charge Conservation
$E_k \sim k^2$
Diffusion
$C_t \underset{t \to \infty}{\sim} t^{-d/2}$

Duality



Examples

- Charge Conservation

$E_k \sim k^2$ → **Diffusion** $C_t \underset{t\to\infty}{\sim} t^{-d/2}$

Charge Fluctuations
Single-Mode, Double Hilbert Space

$$H_{\mathcal{L}} = \sum_{x,\lambda} \mathcal{O}^{\dagger}_{x,\lambda}\mathcal{O}_{x,\lambda} \qquad [\mathcal{O}_{x,\lambda}, \sum_{x} \rho_x] = 0$$

Uniform charge acts as ground state and decomposes into charge sectors

$$H_{\mathcal{L}}\|\mathbb{I}\rangle\rangle = 0 \qquad H_{\mathcal{L}}\|P_m\rangle\rangle = 0$$

Diagonal U(1) x U(1) density modulation

$$\rho_k \sim \sum_x e^{ik\cdot x}(\rho_x \otimes \mathbb{I} + \mathbb{I} \otimes \rho_x) \qquad \|m_k\rangle\rangle = \frac{1}{\mathcal{N}_k}\rho_k\|P_m\rangle\rangle$$

$$E_k \sim \langle\langle m_k\|H_{\mathcal{L}}\|m_k\rangle\rangle$$

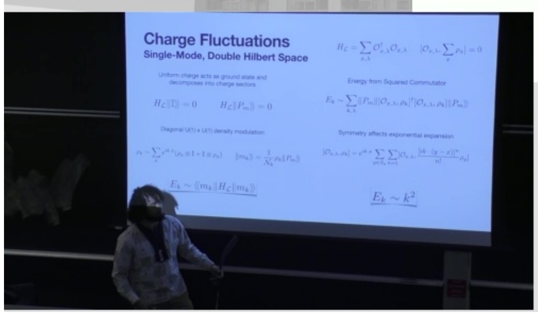
Energy from Squared Commutator

$$E_k \sim \sum_{k,\lambda}\langle\langle P_m\|[\mathcal{O}_{x,\lambda},\rho_k]^{\dagger}[\mathcal{O}_{x,\lambda},\rho_k]\|P_m\rangle\rangle$$

Symmetry affects exponential expansion

$$[\mathcal{O}_{x,\lambda},\rho_k] = e^{ik\cdot x}\sum_{y\in S_x}\sum_{n=1}[\mathcal{O}_{x,\lambda}, \frac{[ik\cdot(y-x)]^n}{n!}\rho_y]$$

$$E_k \sim k^2$$



Justifying Variational Estimates

Feynman-Bijl Formula

$$|\psi_k\rangle = \frac{1}{L^{d/2}}\rho_k|\phi_0\rangle = \frac{1}{L^{d/2}}\sum_x e^{ik\cdot x}\rho_x|\phi_0\rangle$$

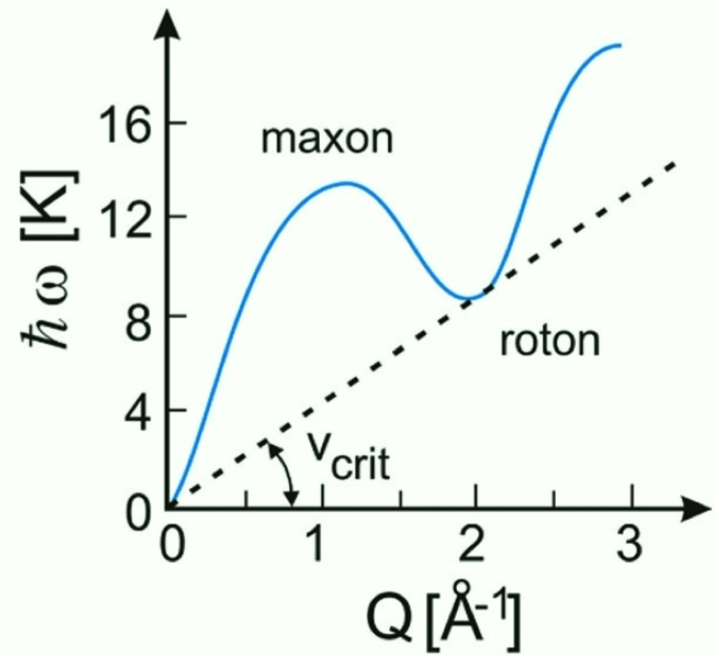
$$E_k \approx \frac{\langle\psi_k|H|\psi_k\rangle}{\langle\psi_k|\psi_k\rangle} = \frac{f(k)}{s(k)}$$

$$\langle\psi_k|\psi_{k'}\rangle \sim \delta_{k,k'}$$

$$f(k) = \frac{1}{2L^d}\langle\phi_0|[\rho_k^\dagger,[H,\rho_k]]|\phi_0\rangle$$

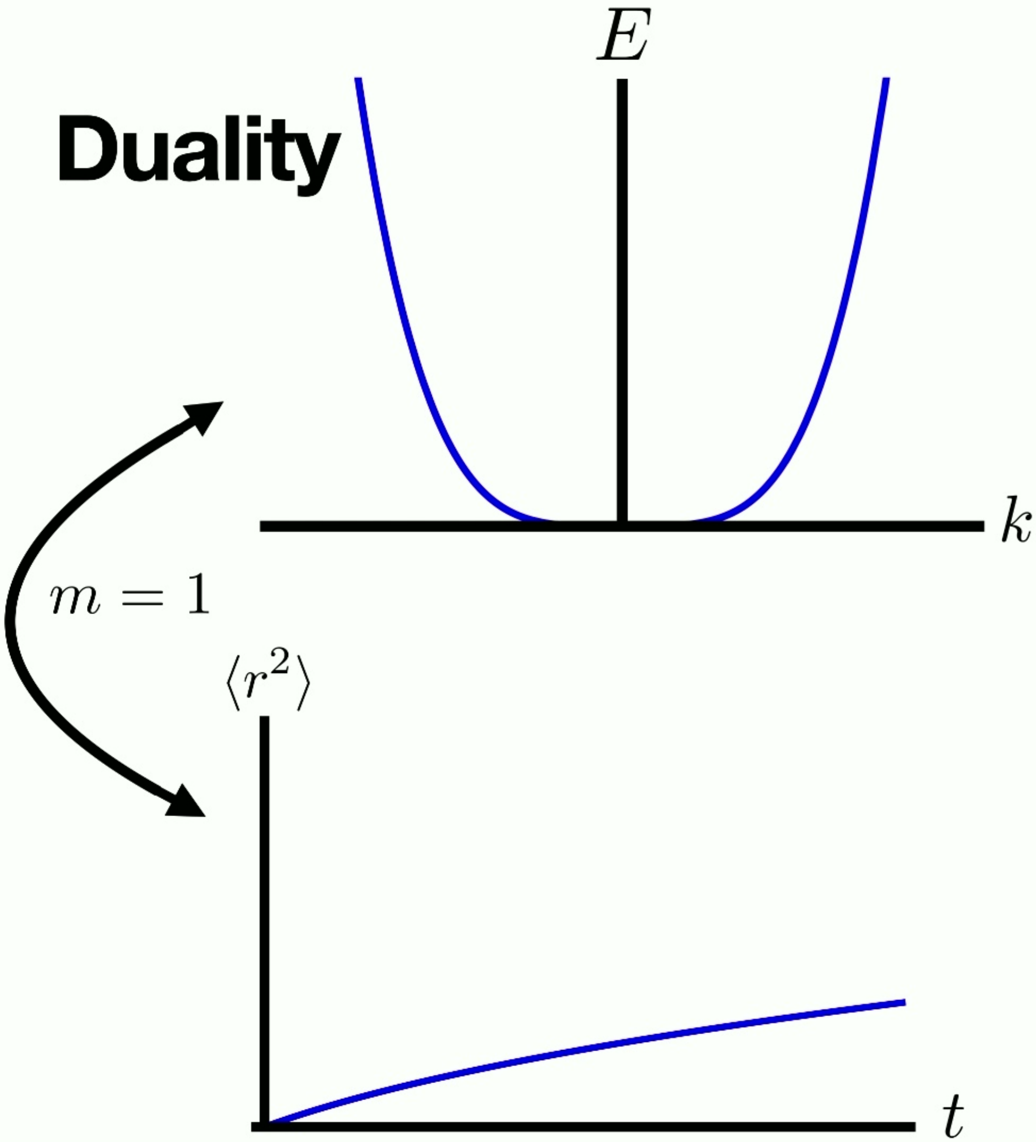
$$s(k) = \langle\psi_k|\psi_k\rangle = \frac{1}{L^d}\langle\phi_0|\rho_k^\dagger\rho_k|\phi_0\rangle$$

Superfluid He-4: $E_k = \frac{f(k)}{s(k)} \sim \frac{k^2}{k} = k$



Toennies, J.P. (2022).

Duality



Examples

- Multipole Conservation

$E_k \sim k^{2(m+1)}$ → **Sub-diffusion** $C_t \underset{t\to\infty}{\sim} t^{-d/2(m+1)}$

Multipole Conservation

$$[\mathcal{O}_{x,\lambda}, Q^{(m)}] = [\mathcal{O}_{x,\lambda}, \sum_x x^m \rho_x] = 0$$

More terms removed from exponential expansion

$$[\mathcal{O}_{x,\lambda}, \rho_k] = e^{ik\cdot x} \sum_{y\in S_x} \sum_{n=m+1} [\mathcal{O}_{x,\lambda}, \frac{[ik\cdot(y-x)]^n}{n!}\rho_y]$$

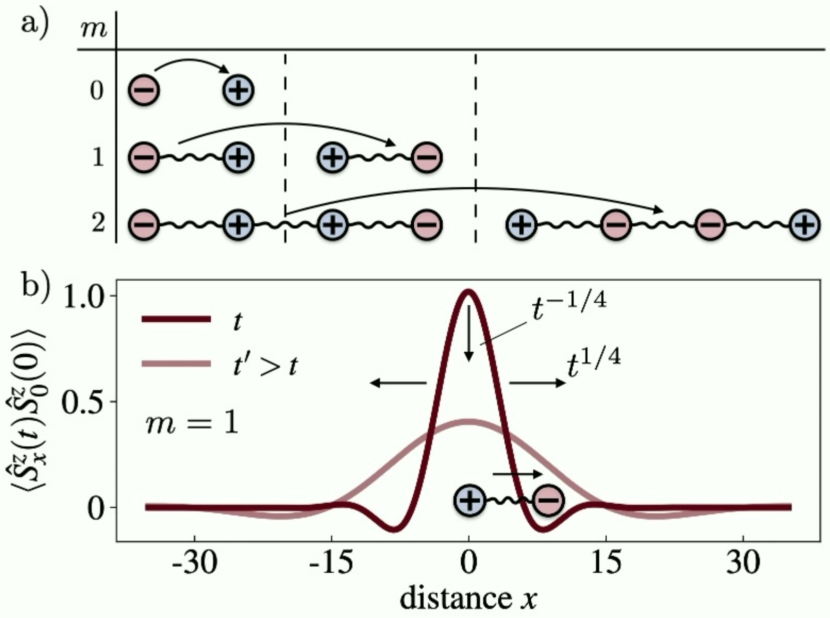
Higher power dispersion

$$\langle\langle m_k \| H_{\mathcal{L}} \| m_k \rangle\rangle = \frac{1}{\mathcal{N}_k} \sum_{k,\lambda} \langle\langle P_m \| [\mathcal{O}_{x,\lambda}, \rho_k]^\dagger [\mathcal{O}_{x,\lambda}, \rho_k] \| P_m \rangle\rangle \sim k^{2(m+1)}$$

Assuming:

$$lim_{k\to 0} \mathcal{N}_k = \text{Const.}$$

Dipole conserving system:



J. Feldmeier, P. Sala, G. De Tomasi, F. Pollmann, and M. Knap, Phys. Rev. Lett. **125**, 245303 – (2020)

Hilbert Space Fragmentation

More than symmetry sees

Krylov Subspaces — dynamically connected sectors

$$\mathcal{K}_i^{(\mathcal{S})} \equiv \underset{n\to\infty}{\text{span}}\{H^n|\psi_i^{(\mathcal{S})}\rangle\}$$

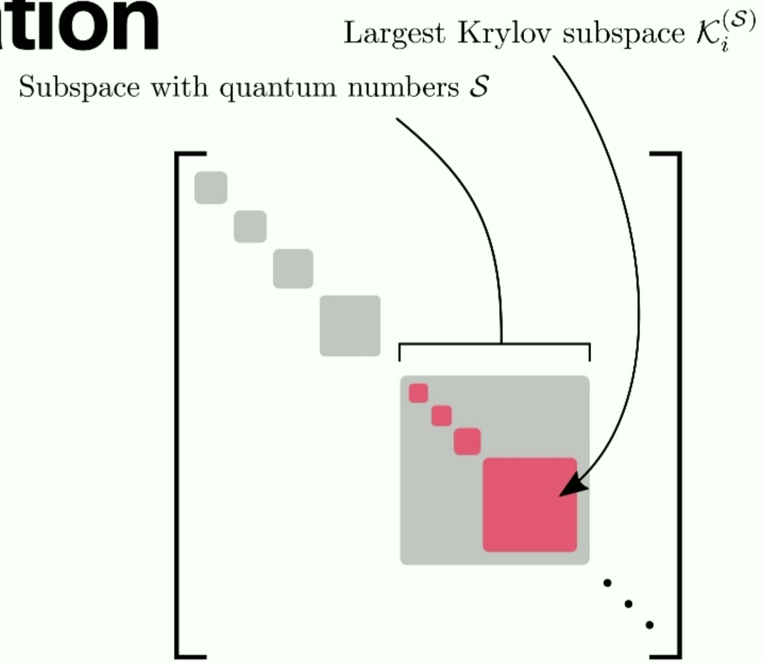
$$\widehat{W} = \bigotimes_{i=1}^{D^{(\mathcal{S})}} \widehat{W}^{(\mathcal{K}_i^{(\mathcal{S})})}$$

Ergodicity Breaking

QMBS - Weak Ergodicity Breaking

Weak HSF - $\frac{D_{max}}{D_{\mathcal{S}}} = \frac{\dim(\mathcal{K}_i^{(\mathcal{S})})}{\dim(\mathcal{S})} \underset{L\to\infty}{\longrightarrow} \text{Const.}$

Strong HSF - $\frac{D_{max}}{D_{\mathcal{S}}} = \frac{\dim(\mathcal{K}_i^{(\mathcal{S})})}{\dim(\mathcal{S})} \underset{L\to\infty}{\longrightarrow} 0$

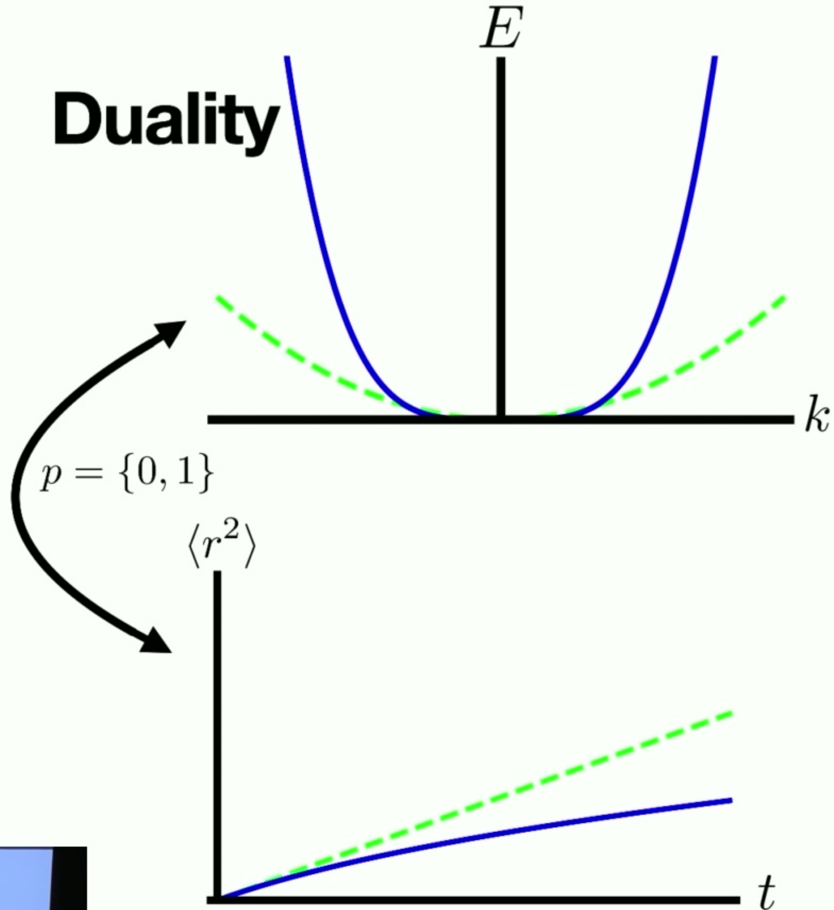


Momentum and multipole moments do not commute

$$T_{\mathbf{a}}Q^{(m)}T_{\mathbf{a}}^{-1} \neq Q^{(m)}$$

S. Moudgalya, A. Prem, D. H. Use, A. Chan, arXiv:2009.11863 - (2021)

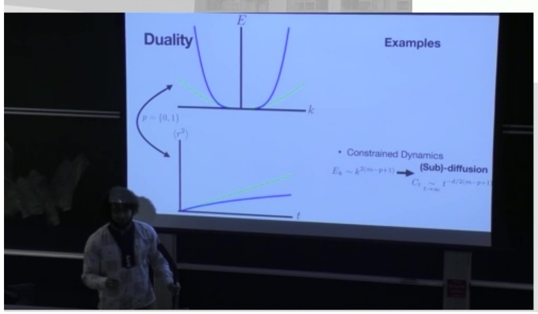
Duality



Examples

- Constrained Dynamics

$E_k \sim k^{2(m-p+1)}$ ➡ **(Sub)-diffusion** $C_t \underset{t\to\infty}{\sim} t^{-d/2(m-p+1)}$



Constrained Dynamics

$$\|\mathcal{K}_k\rangle\rangle = \frac{1}{\mathcal{N}_k^{\mathcal{K}}}\rho_k\|\mathcal{K}\rangle\rangle$$

$$lim_{k\to 0}\mathcal{N}_k^{\mathcal{K}} = 0$$

E.g. Projection into Krylov Subspace

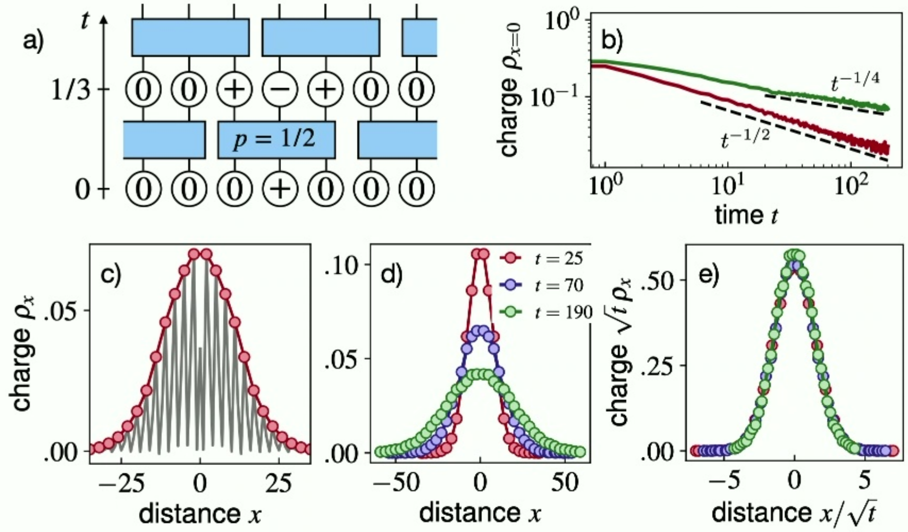
Hilbert Space Fragmentation leads to new ground states: Krylov-resolved dynamics

$$H_{\mathcal{L}}\|\mathcal{K}\rangle\rangle = 0$$

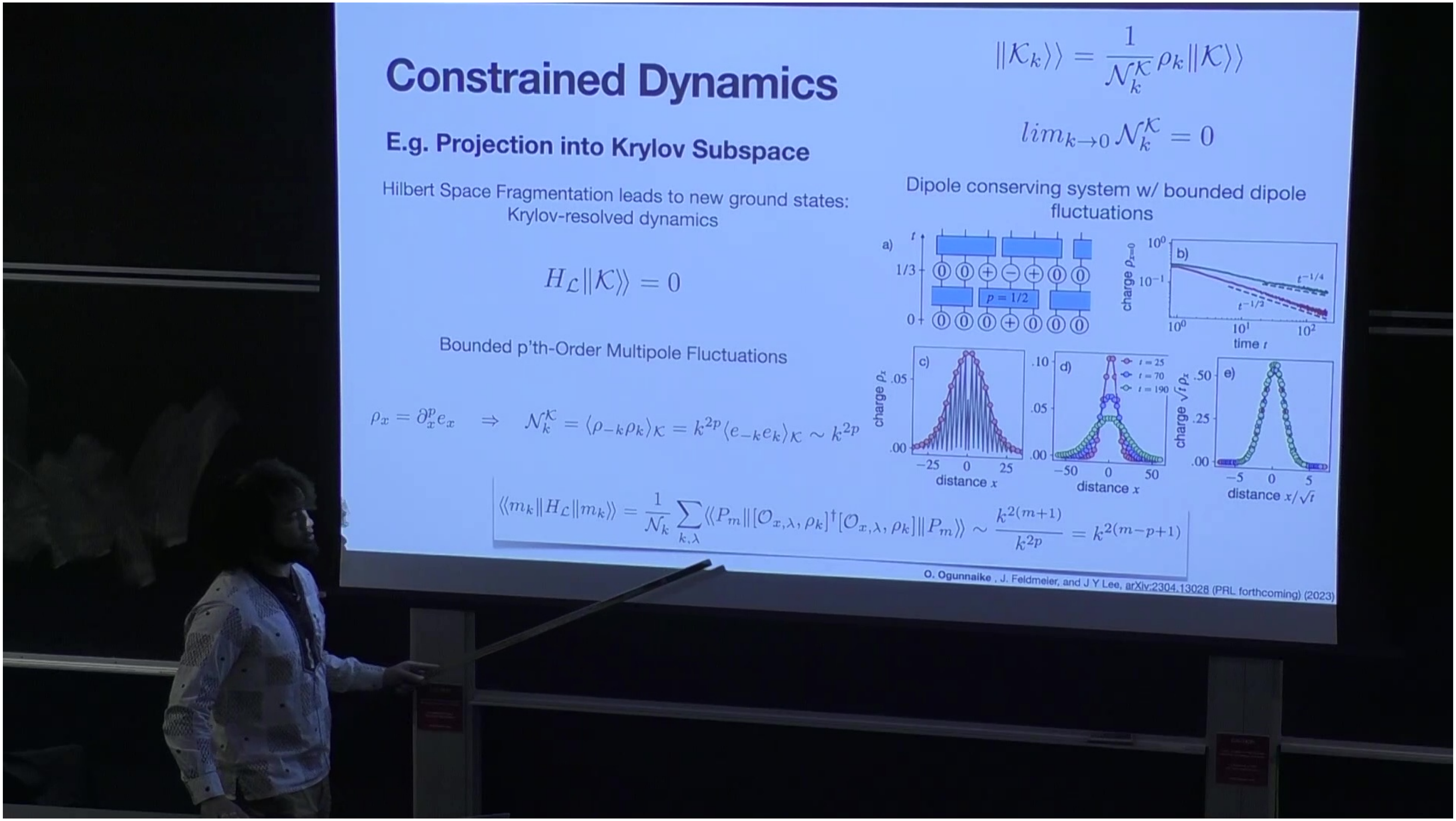
Bounded p'th-Order Multipole Fluctuations

$$\rho_x = \partial_x^p e_x \quad \Rightarrow \quad \mathcal{N}_k^{\mathcal{K}} = \langle\rho_{-k}\rho_k\rangle_{\mathcal{K}} = k^{2p}\langle e_{-k}e_k\rangle_{\mathcal{K}} \sim k^{2p}$$

Dipole conserving system w/ bounded dipole fluctuations



O. Ogunnaike , J. Feldmeier, and J Y Lee, arXiv:2304.13028 (PRL forthcoming) (2023)



Constrained Dynamics
E.g. Projection into Krylov Subspace
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$\|\mathcal{K}_k\rangle\rangle = \frac{1}{\mathcal{N}_k^{\mathcal{K}}}\rho_k\|\mathcal{K}\rangle\rangle$
$lim_{k\to 0}\mathcal{N}_k^{\mathcal{K}} = 0$
Dipole conserving system w/ bounded dipole fluctuations
a)
t
1/3
0
$p = 1/2$
b)
charge $\rho_{x=0}$
10^0
10^{-1}
$t^{-1/4}$
$t^{-1/2}$
time t
c)
charge ρ_x
distance x
d)
$t = 25$
$t = 70$
$t = 190$
distance x
e)
charge $\sqrt{t}\,\rho_x$
distance $x/\sqrt{t}$
$\langle\langle m_k \| H_{\mathcal{L}} \| m_k \rangle\rangle = \frac{1}{\mathcal{N}_k}\sum_{k,\lambda}\langle\langle P_m \| [\mathcal{O}_{x,\lambda}, \rho_k]^\dagger [\mathcal{O}_{x,\lambda}, \rho_k] \| P_m \rangle\rangle \sim \frac{k^{2(m+1)}}{k^{2p}} = k^{2(m-p+1)}$
O. Ogunnaike, J. Feldmeier, and J Y Lee, arXiv:2304.13028 (PRL forthcoming) (2023)

Long-Range Interactions

$$h_{x,x'} \equiv \frac{S_x^+ S_{x'}^- + h.c.}{|x'-x|^\alpha}$$

Commutator Becomes Distance-Dependent

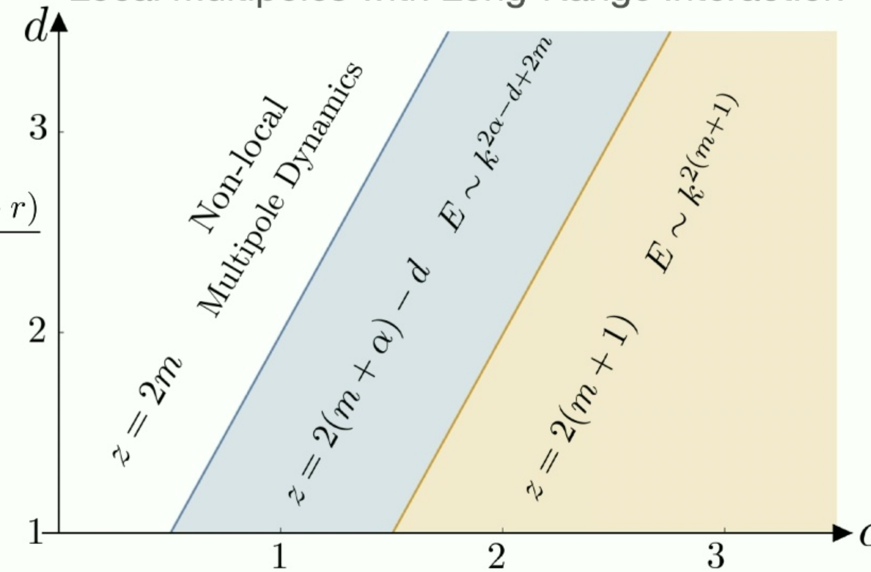
$$[\mathcal{O}_{x,x'}, \rho_k] = e^{ik\cdot x}\frac{1-e^{ik\cdot(x'-x)}}{|x'-x|^\alpha}[\tilde{\mathcal{O}}_{x,x'}, \rho_k]$$

α-Dependent Dispersion

$$\langle\langle m_k \| H_{\mathcal{L}} \| m_k \rangle\rangle = \frac{1}{\mathcal{N}_k}\sum_{k,\lambda}\langle\langle P_m \| [\mathcal{O}_{x,\lambda}, \rho_k]^\dagger [\mathcal{O}_{x,\lambda}, \rho_k] \| P_m \rangle\rangle \sim \int d^d r \frac{1-\cos(k\cdot r)}{r^{2\alpha}}$$

$$\int_{r>1} d^d r \frac{1-\cos(k\cdot r)}{r^{2\alpha}} \underset{k\to 0}{\propto} \begin{cases} C_0(\alpha, L) \sim L^{d-2\alpha} & (\alpha \le \frac{d}{2}) \\ C_1(\alpha)|k|^{2\alpha-d} + C_2(\alpha)k^2 & (\alpha > \frac{d}{2}) \end{cases}$$

Local Multipoles with Long-Range Interaction



$$h^{(m)}_{x,x'} \equiv \frac{(q_x^{(m)})^\dagger q_{x'}^{(m)} + h.c.}{|x'-x|^\alpha}$$

O. Ogunnaike , J. Feldmeier, and J Y Lee, arXiv:2304.13028 (PRL forthcoming) (2023)

Long-Range Interactions

$$h_{x,x'} \equiv \frac{S_x^+ S_{x'}^- + h.c.}{|x' - x|^\alpha}$$

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$$\langle\langle m_k \| H_{\mathcal{L}} \| m_k \rangle\rangle = \frac{1}{\mathcal{N}_k} \sum_{k,\lambda} \langle\langle P_m \| [\mathcal{O}_{x,\lambda}, \rho_k]^\dagger [\mathcal{O}_{x,\lambda}, \rho_k] \| P_m \rangle\rangle \sim \int d^d r \frac{1 - \cos(k\cdot r)}{r^{2\alpha}}$$

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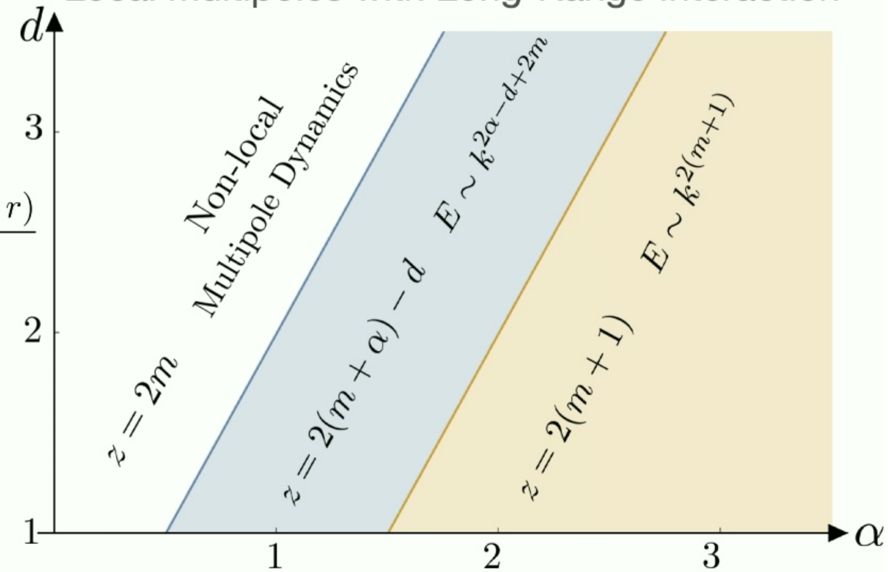
$$E_k \sim \begin{cases} k^{2m} & (\alpha \leq \frac{d}{2}) \\ |k|^{2(m+\alpha)-d} & (\frac{d}{2} \leq \alpha \leq \frac{d}{2} + 1) \\ k^{2(m+1)} & (\alpha \geq \frac{d}{2} + 1) \end{cases}$$

Local Multipoles with Long-Range Interaction



$$h^{(m)}_{x,x'} \equiv \frac{(q_x^{(m)})^\dagger q_{x'}^{(m)} + h.c.}{|x' - x|^\alpha}$$

O. Ogunnaike , J. Feldmeier, and J Y Lee, arXiv:2304.13028 (PRL forthcoming) (2023)

Summary - Just the beginning

Examples from super- to sub- diffusion

- Multipole Symmetries -
- Krylov-constrained dynamics -
- Long-range interactions -
- Non-abelian symmetries -
- Any local hilbert space dim.

Summary - Just the beginning

Examples from super- to sub- diffusion

- Multipole Symmetries - $[h_{x,\lambda}, Q^{(m)}] = 0$ ➡ $E_k \sim k^{2(m+1)}$
- Krylov-constrained dynamics - $\lim_{k \to 0} \mathcal{N}_k^{\mathcal{K}} = k^{2p} \langle e_{-k} e_k \rangle_{\mathcal{K}} \sim k^{2p}$ ➡ $E_k \sim k^{2(m-p+1)}$
- Long-range interactions - $h_{x,x'}^{(m)} \equiv \frac{(q_x^{(m)})^\dagger q_{x'}^{(m)} + h.c.}{|x'-x|^\alpha}$ ➡ $E_k \sim \begin{cases} k^{2m} & (\alpha \leq \frac{d}{2}) \\ |k|^{2(m+\alpha)-d} & (\frac{d}{2} < \alpha \leq \frac{d}{2}+1) \\ k^{(2m+1)} & (\alpha > \frac{d}{2}+1) \end{cases}$
- Non-abelian symmetries - $[\rho_i^y, \rho_j^z] = i\rho_i^x \delta_{ij}$ ➡ $\|\rho_k^x\rangle\rangle = (Q^y \otimes \mathbb{I} - \mathbb{I} \otimes Q^y)\|\rho_k^z\rangle\rangle$
- Any local hilbert space dim.

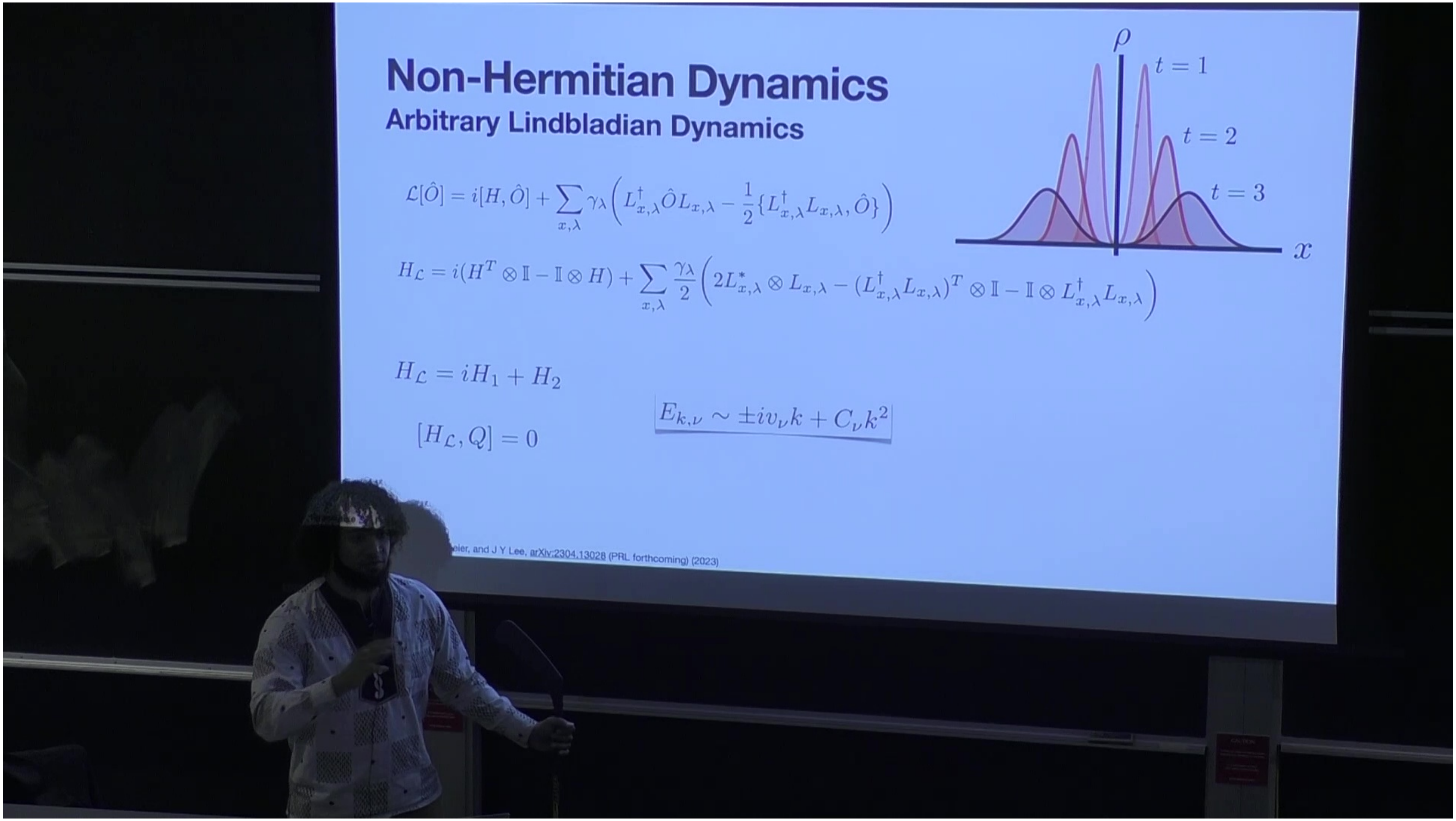
Non-Hermitian Dynamics

Arbitrary Lindbladian Dynamics

$$\mathcal{L}[\hat{O}] = i[H, \hat{O}] + \sum_{x,\lambda} \gamma_\lambda \left(L^\dagger_{x,\lambda} \hat{O} L_{x,\lambda} - \frac{1}{2}\{L^\dagger_{x,\lambda} L_{x,\lambda}, \hat{O}\} \right)$$

$$H_\mathcal{L} = i(H^T \otimes \mathbb{I} - \mathbb{I} \otimes H) + \sum_{x,\lambda} \frac{\gamma_\lambda}{2} \left(2L^*_{x,\lambda} \otimes L_{x,\lambda} - (L^\dagger_{x,\lambda} L_{x,\lambda})^T \otimes \mathbb{I} - \mathbb{I} \otimes L^\dagger_{x,\lambda} L_{x,\lambda} \right)$$

O. Ogunnaike , J. Feldmeier, and J Y Lee, arXiv:2304.13028 (PRL forthcoming) (2023)


Non-Hermitian Dynamics
Arbitrary Lindbladian Dynamics
$\mathcal{L}[\hat{O}] = i[H, \hat{O}] + \sum_{x,\lambda} \gamma_\lambda \left(L^\dagger_{x,\lambda} \hat{O} L_{x,\lambda} - \frac{1}{2}\{L^\dagger_{x,\lambda} L_{x,\lambda}, \hat{O}\} \right)$
$H_\mathcal{L} = i(H^T \otimes \mathbb{I} - \mathbb{I} \otimes H) + \sum_{x,\lambda} \frac{\gamma_\lambda}{2} \left(2L^*_{x,\lambda} \otimes L_{x,\lambda} - (L^\dagger_{x,\lambda} L_{x,\lambda})^T \otimes \mathbb{I} - \mathbb{I} \otimes L^\dagger_{x,\lambda} L_{x,\lambda} \right)$
$H_\mathcal{L} = iH_1 + H_2$
$[H_\mathcal{L}, Q] = 0$
$E_{k,\nu} \sim \pm i v_\nu k + C_\nu k^2$
ρ
$t = 1$
$t = 2$
$t = 3$
x
nier, and J Y Lee, arXiv:2304.13028 (PRL forthcoming) (2023)

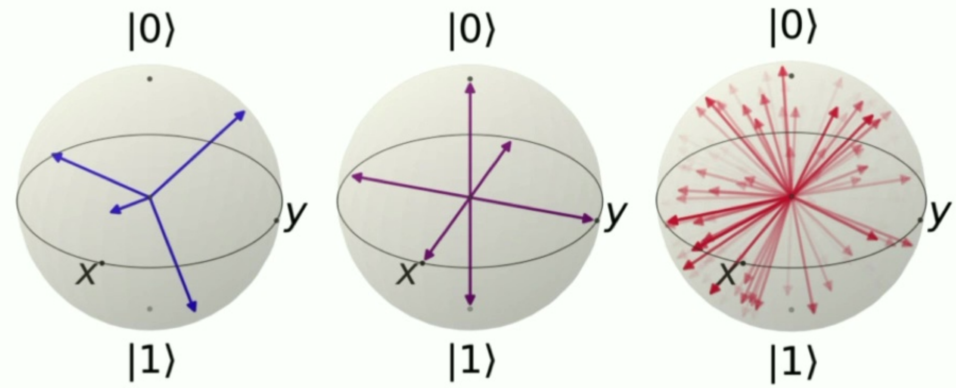
Future Directions

K-designs

Multiple system copies

$$H_{\mathcal{L}}^{(k)} = \frac{1}{2} \sum_{\alpha=(x,\lambda)} \left(\sum_{n=1}^{k} [(h_{x,\lambda})^T \otimes \mathbb{I} - \mathbb{I} \otimes h_{x,\lambda}] \right)^2$$

$$H_{\mathcal{L}}^{(k)} = \frac{1}{2} \sum_{x,\lambda} \mathcal{O}_{x,\lambda}^{(k)\dagger} \mathcal{O}_{x,\lambda}^{(k)}$$



(a) *2-design* (b) *3-design* (c) *approximate design*

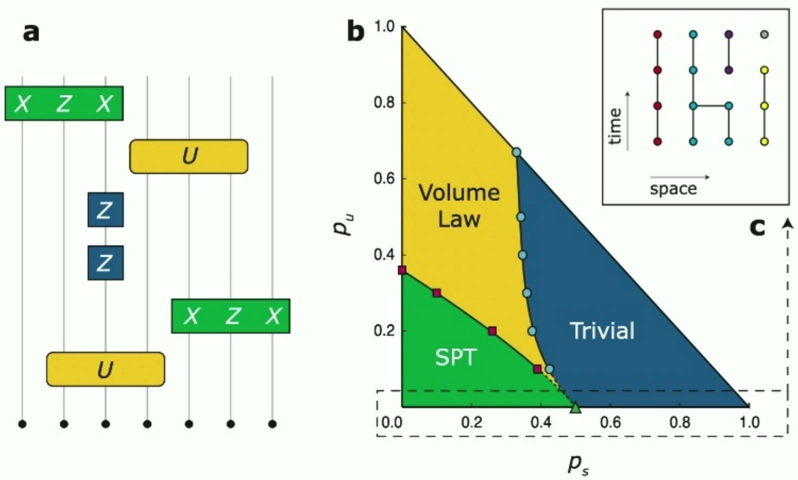
S. N. Hearth, M. O. Flynn, A. Chandran, and C. R. Laumann, arXiv:2306.01035, (2023)

L. Versini, K. Alaa El-Din, F. Mintert, R. Muckherjee, arXiv:2009.11863 - (2023)

Future Directions

Measurement

New ground state/steady state orders parametrized by measurement rates

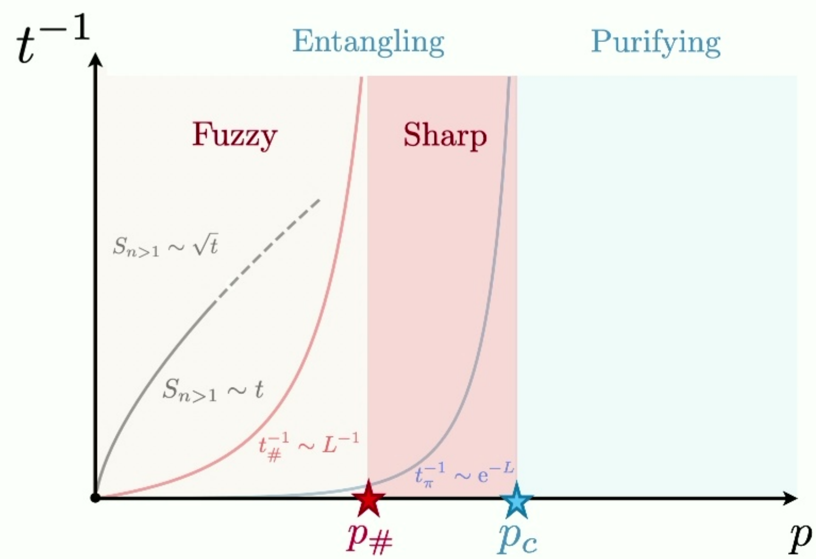


A. Lavasani, Y. Alavirad, M Barkeshi, Nature Physics 17, 342-347 (2021)

$$H' = H_{\mathcal{L}} + H_{\mathcal{M}}$$

$$H_{\mathcal{M}} = \sum_p \Gamma_p \left[\mathbb{I}^{\otimes k} - \sum_{x,m} (\mathcal{P}_{m,x}^{\otimes k}) \otimes (\mathcal{P}_{m,x}^{\otimes k}) \right]$$

Charge-sharpening transition



U Agrawal, A. Zabalo, K Chen, J. H. Wilson, A. C. Potter, J. H. Pixley, S. Gopalakrishnan, R. Vasseur, arXiv:2107.10279 (2022)

Future Directions

- Non-Hermitian classification and characterization (Phys. Rev. X **13**, 031019)

$$\mathcal{P} H_{\mathcal{L}} \mathcal{P}^{-1} = -H_{\mathcal{L}} \qquad \mathcal{C}_{\pm} H_{\mathcal{L}} \mathcal{C}_{\pm}^{-1} = \pm H_{\mathcal{L}} \qquad \mathcal{Q}_{\pm}^2 = 1 \quad \mathcal{T}_{+}^2 = \pm 1 \quad \mathcal{C}_{+}^2 = \pm 1$$

$$\mathcal{Q}_{\pm} H_{\mathcal{L}} \mathcal{Q}_{\pm}^{-1} = \pm H_{\mathcal{L}} \qquad \mathcal{T}_{\pm} H_{\mathcal{L}} \mathcal{T}_{\pm}^{-1} = \pm H_{\mathcal{L}} \qquad \mathcal{P}^2 = 1 \quad \mathcal{T}_{-}^2 = \pm 1 \quad \mathcal{C}_{-}^2 = \pm 1$$

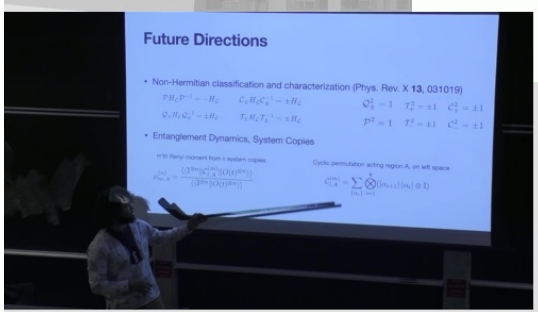
- Entanglement Dynamics, System Copies

m'th Renyi moment from n system copies:

$$\mu_{m,A}^{(n)} = \frac{\langle\langle \mathbb{I}^{\otimes n} \| \mathcal{C}_{l,A}^{(m)} \| O(t)^{\otimes n} \rangle\rangle}{\langle\langle \mathbb{I}^{\otimes n} \| O(t)^{\otimes n} \rangle\rangle}$$

Cyclic permutation acting region A, on left space

$$\mathcal{C}_{l,A}^{(m)} = \sum_{\{\alpha_i\}} \bigotimes_{i=1}^{k} (|\alpha_{i+1}\rangle\langle\alpha_i| \otimes \mathbb{I})$$



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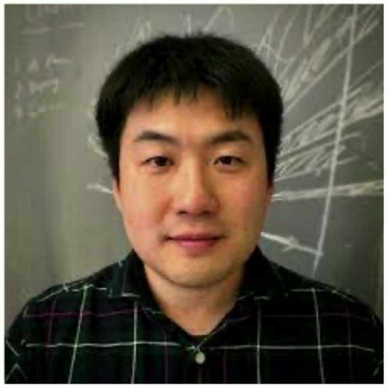
Jong Yeon Lee, KITP -> UIUC

Johannes Feldmeier, Harvard



Soonwon Choi, MIT