Title: The light cone bundle and its ultrarelativistic gauge symmetries

Speakers: Daniel Weiss

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The light cone bundle and its ultrarelativistic gauge symmetries

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2023

Summary

Talk based on *A microscopic analogue of the BMS group, JHEP 2023, 136* and yet unpublished results.

- Subject of the talk: Gauge groups for light cone bundle.
- Main result: New symmetry groups appear as gauge groups for this bundle.
- Result 0: Light cone bundle is a Carrollian fiber bundle.
- Result 1: Carrollian gauge groups are non-trivially represented Lorentz groups.
- Result 2: Ambiguity in those representations.
- Result 3: A BMS-like group μBMS is conformal Carrollian gauge group.
- Result 4: μBMS parametrizes ambiguity from above.

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Initial Motivation: Study of cosmological singularities

- Study of Kasner, Schwarzschild and FLRW singularities.
- Two aims:
  1. Completeness behaviour[1] of quantum fields induced by classical background? ⇒ Publication in preparation.
  2. Geometric foundations of dimensional reduction and asymptotic silence in the vicinity of cosmological singularities?

[1] cf. Hofmann and Schneider (arXiv:1504.05580, 1611.07981), Ashtekar and Schneider (arXiv:2107.08506).

Dimensional reduction and asymptotic silence

- Dimensional reduction: Evidence from various approaches to quantum gravity, that spacetime behaves effectively lower dimensional on microscopic scales. E.g.:
  - Spectral dimension in CDT (Ambjørn et al. 2005).
  - Spectral and walk dimension in Asymptotic Safety (Reuter et al. 2011).
- Asymptotic silence: Light cones collapse to lines, and nearby points become causally disconnected. E.g.:
  - Short distance Wheeler-de Witt (cf. Helfer et al. 1988).
  - Raychaudhuri equation and 2D dilaton gravity (cf. Carlip 2011).
- Further reading: Carlip 2017 (arXiv:1705.05417) and references therein.

The case of singularities:

- Evidence for dimensional reduction:
  - Heat-Kernel in Kasner spacetimes (Carlip 2017, Futamase 1984).
  - Kernel of Schrödinger ground state functional in Schwarzschild spacetimes (Hofmann et al. 2015).
  - Geodesics in Kasner spacetimes (cf. Carlip 2017, A. Harvey 1989).
- Evidence for asymptotic silence:
  - BKL conjecture (BKL 1980)
  - Particle horizons and ultralocal EFE in the vicinity of singularities:
    - General considerations (Uggla et al. 2003).
    - Gowdy spacetimes (Andersson et al. 2005).

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Questions regarding the classical geometry of singularities

Questions:

- How to characterize dimensional reduction and asymptotic silence invariantly in the vicinity of cosmological singularities from the perspective of differential geometry?
- How are dimensional reduction and asymptotic silence related to each other in this picture?

Motivation:

- Singularities as probes for gravity's strong field regime.
- Precise geometric understanding maybe transferable to other cases.

The case of singularities:

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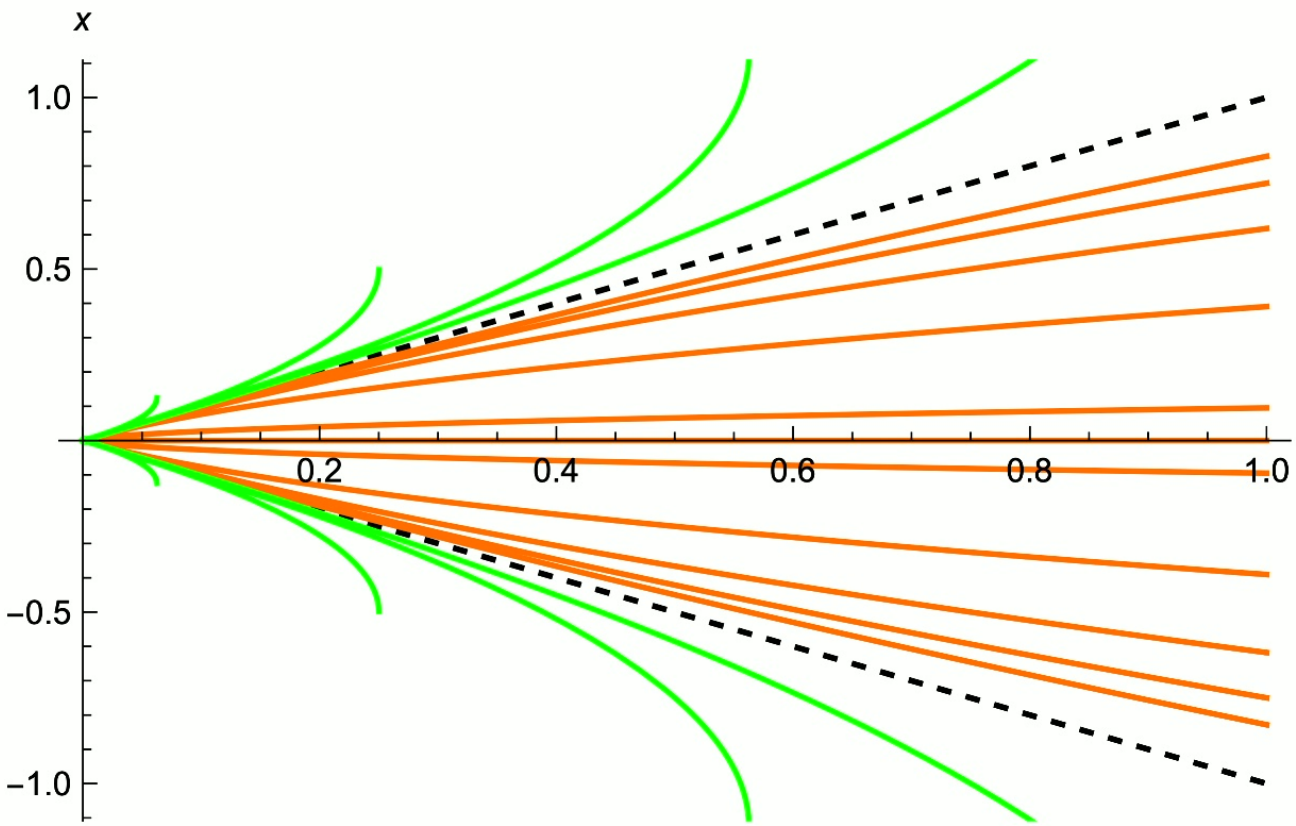
Geodesics at FRW singularities

- $ds^2 = a(\eta)^2(-d\eta^2 + dx^2 + dy^2 + dz^2)$ with $a(\eta) \to 0$ for $\eta \to 0$.
- Solve geodesic equations for $a(\eta) = a_0\eta^c$ and $c > 0$.

Geodesics at FRW singularities



Geodesics at FRW singularities

- Invariant statement:

$$\frac{dx^{\mu}_{\pm 1,\vec{P}}}{d\lambda} = A^{\mu}{}_{\nu}[\sigma, \vec{P}] \frac{dx^{\nu}_{0,\vec{P}}}{d\lambda} \tag{1}$$

  with

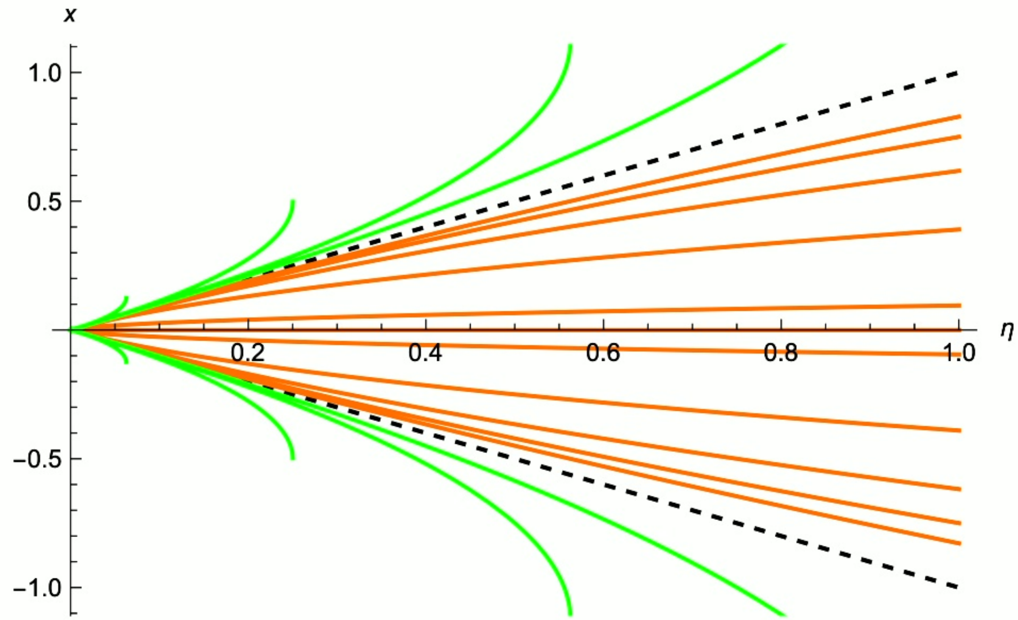
$$A^{\mu}{}_{\nu}[\sigma, \vec{P}] \to \mathbf{1} \tag{2}$$

  as $\eta \to 0$

- As a sidenote, there are some subtleties:
  - Invariant for diffeomorphisms that respect the boundary at $\eta = 0$.
  - For general diffeomorphisms: Depends on asymptotics as $\eta \to 0$.
  - $\Rightarrow$ Still some work to do ... . I'm on it!

$\Rightarrow$ Carrollian physics at singularities? (Cf. case of horizons, e.g. Donnay et al. 2020, Freidel et al. 2022.)

Geodesics at FRW singularities



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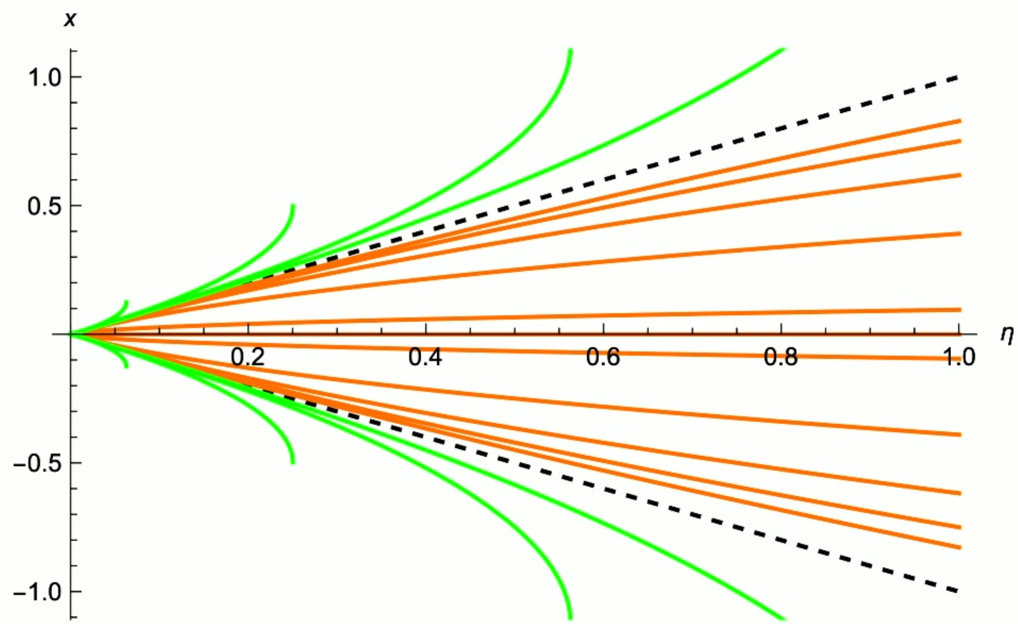


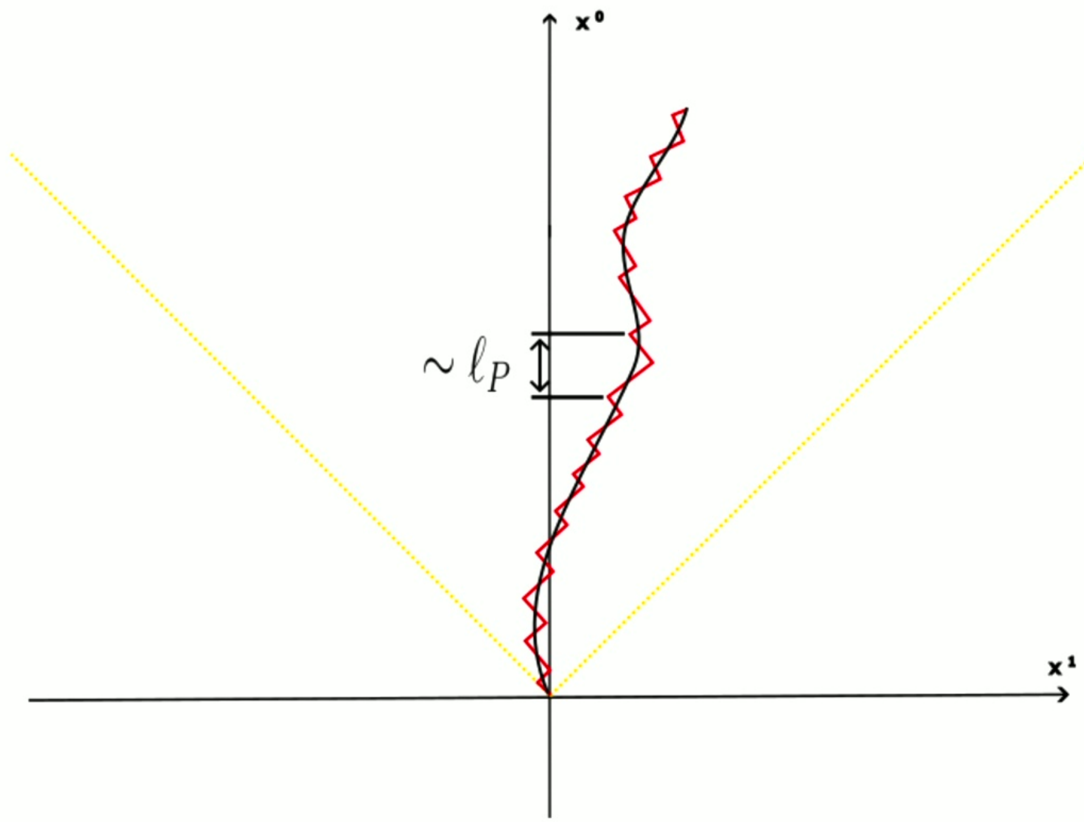
- Some other type of Carrollian limit. Not $c \to 0$ but local geometry gets deformed towards the light cone.

$\Rightarrow$ What is the Carrollian structure of the light cone bundle over the singular hypersurface?

Qualitative conclusion by analogy

- Intuition: Singularities as a probe for the strong field regime of gravity.
- Observation: For short time scales around certain singularities geometry is dominated by ultrarelativistic behaviour.
- Maybe: Asymptotic silence and dimensional reduction at singularities related to this aymptotic ultrarelativistic behaviour.
- Observation: Asymptotic silence and dimensional reduction are not only tied to singularities, but are also present in many approaches to quantum gravity.
- Assumption: Maybe, also in those cases asymptotic ultrarelativistic behaviour is present?

A Gedankenexperiment (Planckian Zitterbewegung)



cf. Feynman's checkerboard, Penrose's Zig-Zag model, ...

A question

If we have a situation, where microscopically anything moves at the speed of light in a $3+1$-dimensional pseudo-Riemannian spacetime ...

... what are then possible gauge[1] symmetries of such a theory of gravity?

[1] I mean gauge symmetries in the sense of fiber bundle theory.

A less wild question

If we have a situation, where a pseudo-Riemannian spacetime is probed just by massless classical test particles ...

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A less wild question

If we have a situation, where a pseudo-Riemannian spacetime is probed just by massless classical test particles ...

... what are the Carrollian symmetries in this situation?

The light cone bundle

- Let $(\mathscr{M}, g)$ be a $3+1$-dimensional, time- and space-orientable Lorentzian spacetime with metric signature $(-1, +1, +1, +1)$.
- If anything moves at the speed of light, all tangent vectors lie in the tangent light cones. $\Rightarrow$ Replace $T\mathscr{M}$ by the bundle of future pointing tangent light cones.
- Bundle of future pointing tangent light cones:

$$L^+\mathscr{M} = \bigsqcup_{p\in\mathscr{M}} L_p^+\mathscr{M} \subset T\mathscr{M} \tag{3}$$

with

$$L_p^+\mathscr{M} := \{v \in T_p\mathscr{M} | g(v, T) < 0 \text{ and } g(v, v) = 0\} \tag{4}$$

where T is the global time orientation vector field.

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where T is the global time orientation vector field.

- $L\mathscr{M}$ is a sub-fiberbundle of $T\mathscr{M}$.

Geometric structures on the light cone bundle

Question: Which geometric structures are induced on $L^+\mathscr{M}$ by the geometry of $(\mathscr{M}, g)$?

- Topology and smooth structure, especially $L^+_p\mathscr{M} \cong S^2 \times \mathbb{R}^+$.
- Vector space structure of $T_p\mathscr{M}$ induces cone structure on $L^+_p\mathscr{M}$:

$$\forall \alpha > 0, v \in L^+_p\mathscr{M} : \alpha \cdot v \in L^+_p\mathscr{M}. \tag{5}$$

- Metric is (degenerate) distance function on $L^+_p\mathscr{M}$:
  - $-g_p(v, w) = -g_p(w, v)$.
  - $-g_p(v, w) \geq 0$
  - $-g_p(v, w) \neq 0$ if and only if $v \neq \alpha w$ for some $\alpha > 0$

  and induces on $L^+_p\mathscr{M}$ a degenerate Riemannian metric q_p with 1-dimensional kernel.
- Kernel of q_p is generated by cone structure.

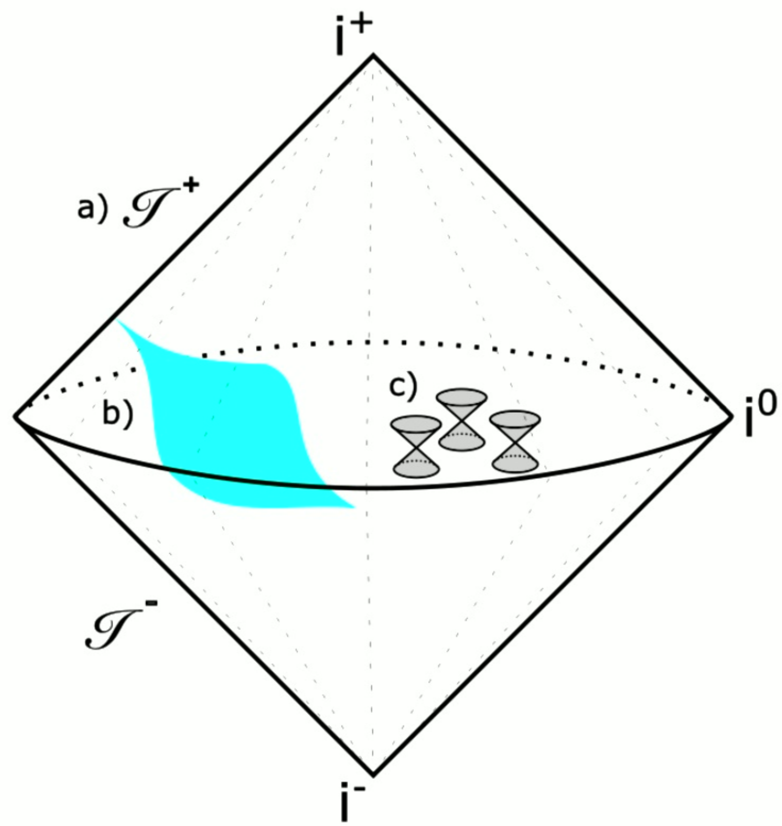
The light cone bundle as a Carrollian entity

- All together, each tangent light cone $L_p^+\mathscr{M}$ is endowed with:
  - A nowhere vanishing vector field ξ (induced by cone structure).
  - A symmetric, covariant, degenerate 2-tensor q_p with one-dimensional Kernel generated by ξ (induced by metric).
- Equals definition of a Carroll manifold (Duval et al. 2014).

$\Rightarrow$ Light cone bundle is a Carrollian fiber bundle, i.e. a fiber bundle whose fibers are Carroll manifolds.

$\Rightarrow$ It encaptures the microscopic ultrarelativistic geometry of $3+1$-dimensional pseudo-Riemannian spacetimes.

$L^+\mathscr{M}$ as an arena for the study of gravity



$L^+\mathscr{M}$ as an arena for the study of gravity

- Study of gravity on null surfaces leads often to great simplifications, the discovery of intriguing structures and interesting insights.
- Null infinity:
  - BMS, phase space of radiative modes of non-linear GR, asymptotic quantization, IR-triangle, ... (cf. e.g. Ashtekar 2015, Ashtekar 2018, Strominger 2017, ...).
- Embedded Null surfaces:
  - Phase space on embedded null surfaces (Ciambelli et al. 2023).
  - Carrollian physics on horizons (Donnay et al. 2020, Freidel et al. 2022), non smooth structures at horizons (Siino et al. 2004, Gadioux et al. 2023).
- The light cone bundle:
  - Motivation 1: Ultrarelativistic aspects of gravity via tools from gauge theory on fiber bundles.

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$L^+\mathscr{M}$ as an arena for the study of gravity

But before we are able to study connections on $L^+\mathscr{M}$ and their properties ...

... we have to analyze possible Carrollian gauge groups.

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$L^+\mathscr{M}$ as an arena for the study of gravity

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$L^+\mathscr{M}$ as an arena for the study of gravity

But before we are able to study connections on $L^+\mathscr{M}$ and their properties ...

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Coordinate systems for the tangent bundle

- Consider tangent bundle $T\mathscr{M}$ of manifold $\mathscr{M}$.
- Vector bundle structure induces a distinguished class of coordinate systems for $T\mathscr{M}$ induced by local frames. $\Rightarrow$ Gauge group is $\mathrm{GL}(\mathbb{R}, 4)$.
- If we have a pseudo-Riemannian metric, this induces a distinguished class of coordinate systems for $T\mathscr{M}$ induced (orthonormal frames). $\Rightarrow$ Gauge group (is reduced to) $\mathrm{O}(1,3)$.
- If $(\mathscr{M}, g)$ is space- and time-orientable, then the gauge group is further reduced to $\mathrm{SO}^+(1,3)$.

Strategy: Look at coordinate systems for the bundle, express geometric structures in those coordinate systems and look at transformations that preserve those structures.

Coordinate systems for tangent light cones

- How to coordinatize $L_p^+\mathscr{M}$?
- Idea: $L_p^+\mathscr{M} \cong S^2 \times \mathbb{R}^+$.
- S^2 inconvient, better Riemann sphere

$$S^2 \cong \mathbb{C}_\infty \tag{6}$$

with

$$\mathbb{C}_\infty := \mathbb{C} \cup \{\infty\}. \tag{7}$$

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- Idea: $L_p^+ \mathscr{M} \cong \mathbb{C}_\infty \times \mathbb{R}^+$.
- I.e. we identify light cones with the half line bundle $\mathbf{C}^+ := \mathbb{C}_\infty \times \mathbb{R}^+$ over the Riemann sphere.
- We will see: The identification $L_p^+ \mathscr{M} \cong \mathbf{C}^+$ has a richer structure than the projection $L_p^+ \mathscr{M} \to \mathbb{C}_\infty$.

Recap: On null vector directions

- Let (e_μ) be a vielbein frame around $p \in \mathscr{M}$.
- Stereographic projection associated with (e_μ)

$$z_p : L_p^+ \mathscr{M} \to \mathbb{C}_\infty \tag{8}$$

with

$$z_p(v^\mu e_\mu) := \frac{v^1 + iv^2}{v^0 - v^3}. \tag{9}$$

- Lorentz transformations $\Lambda \in \mathrm{SO}^+(1,3)$ act via Möbius transformations

$$z_p(\Lambda^\mu{}_\nu v^\nu e_\mu) = Z_\Lambda(z_p(v^\mu e_\mu)) \tag{10}$$

with $Z_\Lambda = (az+b)/(cz+d)$ and $ad - bc = 1$.

On the length of null vectors

- Often, the length of a null vector is not considered as a meaningful concept, since the metric is degenerate on the light cone.
- e.g. from *Spinors and spacetime* by Roger Penrose:

  The extent of a null vector cannot be characterized in an invariant way by a number, nor can null vectors of different directions be compared with respect to extent. The ratio of the extents of null vectors of the same direction is meaningful, being just the ratio of the vectors.

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- But: Things don't have to be invariant for being meaningful.
- Idea: Lorentz *covariant* length notions.

Length gauges for null vectors

- Call a smooth map
$$\chi_p : L_p^+\mathscr{M} \to \mathbb{R}^+ \tag{11}$$
with
$$\forall \alpha > 0, v \in L_p^+\mathscr{M} : \chi(\alpha v) = \alpha \chi(v) \tag{12}$$
a *length gauge* at p.
- Any such map behaves under Lorentz transformations as
$$\chi_p(\Lambda^\mu{}_\nu v^\nu e_\mu) = f_\Lambda(z) \cdot \chi_p(v^\mu e_\mu) \tag{13}$$
for $z = z_p(v^\mu)$ with $f_\Lambda : \mathrm{SO}^+(1,3) \times \mathbb{C}_\infty \to \mathbb{R}^+$ satisfying the 2-cocycle property
$$f_{\Lambda_1\Lambda_2}(z) = f_{\Lambda_1} \circ Z_{\Lambda_2}(z) \cdot f_{\Lambda_2}(z). \tag{14}$$

Length gauges for null vectors

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A simple length gauge

- A particularly simple length gauge is given by

$$\chi_p(v^\mu e_\mu) := v^0 \tag{15}$$

- In this case

$$f_\Lambda(z) = \frac{1 + Z_\Lambda(z)\overline{Z_\Lambda}(z)}{1 + z\bar{z}}|cz + d|^2 \tag{16}$$

for $Z_\Lambda(z) = (az + b)/(cz + d)$ with $ad - bc = 1$ being the Möbius transformation associated to Λ.

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for $Z_\Lambda(z) = (az + b)/(cz + d)$ with $ad - bc = 1$ being the Möbius transformation associated to Λ.

- Satisfies the covariance law

$$f_{\Lambda_1\Lambda_2}(z) = f_{\Lambda_1} \circ Z_{\Lambda_2}(z) \cdot f_{\Lambda_2}(z). \tag{17}$$

Coordinate systems for the light cone

- How to coordinatize $L^+\mathscr{M}$?
- Answer: By local trivializations which restrict on each tangent light cone to a map

$$\psi_p : L_p^+\mathscr{M} \to \mathbf{C}^+, v \in L_p^+\mathscr{M} \mapsto (z_p(v), \chi_p(v)) \tag{18}$$

with z_p stereographic projection and χ_p being a length gauge.

Coordinate expressions of the Carrollian structures

- How does the light cone metric q_p look in those trivializations?
- Answer: Pushforward of q_p along the induced map

$$\psi_p : L_p^+ \mathscr{M} \to \mathbf{C}^+ \tag{19}$$

is, in coordinates $(z, \ell) \in \mathbb{C}_\infty \times \mathbb{R}^+$, given by

$$ds^2 = 2\ell^2 \Omega_p^\chi(z)^{-2} dz d\bar{z} \tag{20}$$

- For example, for $\chi(v^\mu) := v^0$:

$$\Omega_p^\chi(z) = (1 + z\bar{z}) \tag{21}$$

Coordinate expressions of the Carrollian structures

Interpretation:

- Extrinsic perspective: Stereographic projection $\pi : L_p^+\mathscr{M} \to \mathbb{C}_\infty$ induces a conformal structure on $\mathbb{C}_\infty$ via pull back of g_p.
- Intrinsic perspective: Conformal structure is induced by complex structure on $\mathbb{C}_\infty$.
- Length gauge breaks conformal invariance by choosing a preferred metric on $\mathbf{C}^+$ and hence on $\mathbb{C}_\infty$.

Summary: What have we gained so far?

- $L_p^+\mathscr{M} \cong \mathbb{C}_\infty \times \mathbb{R}^+$ via local trivializations.
- Under this identification g_p induces a degenerate metric on $\mathbf{C}^+$ given by
$$ds^2 = 2\ell^2\Omega_p^\chi(z)^{-2}dzd\bar{z}. \tag{22}$$
- Cone structure on $\mathbb{C}_\infty \times \mathbb{R}^+$ given by
$$\forall\alpha > 0, (z,\ell) \in \mathbb{C}_\infty \times \mathbb{R}^+ : \alpha \cdot (z,\ell) = (z,\alpha\ell). \tag{23}$$
- Those two properties give Carrollian structure on $\mathbb{C}_\infty \times \mathbb{R}^+$.

Carrollian gauge groups for the light cone bundle

- Assume we have identified $L_p^+\mathscr{M} \cong \mathbb{C}_\infty \times \mathbb{R}^+$.
- Carrollian structure on $\mathbb{C}_\infty \times \mathbb{R}^+$:
  - Light cone metric q_p has coordinate expression

$$ds^2 = 2\ell^2 \Omega_p(z)^{-2} dz d\bar{z} \tag{24}$$

    in coordinates $(z, \ell) \in \mathbb{C}_\infty \times \mathbb{R}^+$.
  - Cone structure on $\mathbb{C}_\infty \times \mathbb{R}^+$ given by

$$\forall \alpha > 0, (z, \ell) \in \mathbb{C}_\infty \times \mathbb{R}^+ : \alpha \cdot (z, \ell) = (z, \alpha\ell). \tag{25}$$

- Gauge group actions should preserve those structures. Two possibilities:
  - Isometries: $\Omega_p \mapsto \Omega_p$ under gauge transformation.
  - Conformal transformations: $\Omega_p \mapsto \Omega_p'$ under gauge transformations.

The conformal Carroll gauge group

- Conformal automorphisms are homogeneous maps

$$\mathbf{C}^+ \to \mathbf{C}^+ \tag{26}$$

under which the light cone metric gets mapped to a conformally equivalent light cone metric.

- The conformal automorphism group is given by a (right) semidirect product

$$\mu\text{BMS} = \text{SO}^+(1,3) \ltimes_\kappa C^\infty(\mathbb{C}_\infty, \mathbb{R}^+) \tag{27}$$

- It acts on $\mathbf{C}^+$ via the (left) action $(\Lambda, Y) \star (z, \ell) := (Z_\Lambda(z), Y(z)\ell)$.
- Group multiplication law is $(\Lambda_1, Y_1)(\Lambda_2, Y_2) = (\Lambda_1\Lambda_2, Y_1 \circ Z_{\Lambda_2} \cdot Y_2)$.

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- Group multiplication law is $(\Lambda_1, Y_1)(\Lambda_2, Y_2) = (\Lambda_1\Lambda_2, Y_1 \circ Z_{\Lambda_2} \cdot Y_2)$.
- It maps conformal factor Ω_p under pullback on
$$\Omega_p'(z) = \Omega_p \circ Z_\Lambda(z) \cdot Y(z)^{-1} \cdot |cz + d| \tag{28}$$
for $Z_\Lambda(z) = (az + b)/(cz + d)$.

Isometric Carroll gauge groups

- Isometric automorphisms are homogeneous maps

$$\mathbf{C}^+ \to \mathbf{C}^+ \tag{29}$$

which preserve the light cone metric q_p.

- The group of those automorphisms is isomorphic $\mathrm{SO}^+(1,3)$.
- It acts on $\mathbf{C}^+$ via the (left) action $\Lambda \star (z,\ell) := (Z_\Lambda, g_\Lambda(z)\ell)$. with

$$g_\Lambda(z) = |cz+d|^2 \, \frac{\Omega_p \circ Z_\Lambda(z)}{\Omega_p(z)} \tag{30}$$

for $Z_\Lambda = (az+b)/(cz+d)$.

- g_Λ satisfies the covariance law $g_{\Lambda_1\Lambda_2}(z) = g_{\Lambda_1} \circ Z_{\Lambda_2}(z) \cdot Z_{\Lambda_2(z)}$.
- Representation of $\mathrm{SO}^+(1,3)$ depends hence on length gauge used for identification $L_p^+\mathscr{M} \cong \mathbf{C}^+$.

Lorentz subgroups of the conformal Carroll group

- μBMS has infinitely many Lorentz subgroups.
- Any map

$$f : \mathrm{SO}^+(1,3) \to C^\infty(\mathbb{C}_\infty, \mathbb{R}^+), \Lambda \mapsto f_\Lambda \tag{31}$$

satisfying the cocycle property

$$f^{\Lambda_1 \Lambda_2}(z) = f_{\Lambda_1} \circ Z_{\Lambda_2}(z) f_{\Lambda_2}(z) \tag{32}$$

defines an associated Lorentz subgroup.

Interpretation of conformal group

- Lorentz subgroups of μBMS correspond to length gauges and their associated Lorentz transformation laws.
- Infinitely many Lorentz subgroups since there are infinitely many Lorentz covariant length gauges.

$\Rightarrow$ What is the "natural" gauge group for $L^+\mathscr{M}$? Observe:

- Any Lorentz subgroup of μBMS induces a preferred length gauge with an associated Lorentz transformation law.

Interpretation of conformal group

- Lorentz subgroups of μBMS correspond to length gauges and their associated Lorentz transformation laws.
- Infinitely many Lorentz subgroups since there are infinitely many Lorentz covariant length gauges.

$\Rightarrow$ What is the "natural" gauge group for $L^+\mathscr{M}$? Observe:

- Any Lorentz subgroup of μBMS induces a preferred length gauge with an associated Lorentz transformation law.
- Personal statement: From an intrinsic perspective, I have the feeling that μBMS is more natural than any of its Lorentz subgroups.

Holographic special relativity and μBMS

Summarized:

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Emergent special relativity under symmetry breaking.

- Any length gauge induces a identification $L^+_p\mathscr{M} \cong \mathbf{C}^+$ and an associated conformal factor Ω in the metric $ds^2 = 2\ell^2\Omega_p^{-2}(z)dzd\bar{z}$.
- E.g. the length gauge $\chi(v^\mu e_\mu) := v^0$ gives $\Omega_p(z) = (1 + z\bar{z})$ as a conformal factor. Observe, that $\chi(v^\mu e_\mu) = {e_0}^\mu v^\nu \eta_{\mu\nu}$.
- Can be generalized to all timelike vectors. Let $t^\mu e_\mu$ be any timelike vector. Then this induces a length gauge

$$\chi_t(v) := g_p(t, v) \tag{34}$$

giving a conformal factor (cf. also Held et al. 1970)

$$\Omega_p^{(t)}(z) = (1 + z\bar{z})\left[t^0 Y_{0,0}(z) + t^1 Y_{1,1}(z) + t^2 Y_{1,-1}(z) + t^3 Y_{1,0}(z)\right]. \tag{35}$$

- $t \mapsto \Omega_p^{(t)}$ is a linear, Lorentz-equivariant isomorphism between the convex cone of timelike vectors and a space of conformal factors on $\mathbf{C}^+$.
- Any such conformal factor induces an associated Lorentz subgroup of μBMS.

Holographic special relativity and μBMS

Summarized:

⇒ Convex cone of timelike vectors emerges intrinsically on $\mathbf{C}^+$ as a space of conformal factors.

⇒ Any such conformal factor singles out a subgroup $\mathrm{SO}^+(1,3) \subset \mu\mathrm{BMS}$ as its Carrollian isometry group.

Interpretation:

- Extrinsic perspective: Subgroups of μBMS are *extrinsically* induced by non-null geometry. μBMS is from null-geometric perspective more natural.

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Interpretation:

- Extrinsic perspective: Subgroups of μBMS are *extrinsically* induced by non-null geometry. μBMS is from null-geometric perspective more natural.
- Intrinsic perspective: Timelike geometry emerges under some kind of conformal symmetry breaking as a space of conformal factors on $\mathbf{C}^+$.

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Sidenote:

- There is a connection to conformally and spin weighted spherical harmonics (Newman and Penrose 1966, Held et al. 1970).

Comparison with BMS analysis

- Spacetimes that are asymptotically Minkowski are at null infinity endowed with the following "universal" structure (cf. e.g. Ashtekar 2014):
  - Null infinity is topologically $S^2 \times \mathbb{R}$. ✓
  - Degenerate metric q with signature $(0, +, +)$. ✓
  - Complete null vector field n ruling $\mathcal{I}^{\pm}$ with $\text{div}(n) = \nabla_a n^a = 0$ ✓
- The Bondi-Metzner-Sachs (BMS) group is the conformal automorphism group of those structures:
  - Group of mappings with $(q, n) \mapsto (\omega^2 q, \omega^{-1} q)$. ✓
  - Mathematical structure: $\text{SO}^+(1,3) \ltimes C^\infty(\mathbb{C}_\infty, \mathbb{R})$. ✓
- Description of gravitational waves at null infinity:
  - Information on gravitational radiation encoded in connection D.

Applications of the light cone bundle and μBMS

- Direction 1: Does the analysis of gravity simplify on the light cone bundle?
  - Motivated by the apparent simplification on other null-geometric structures.
  - $\Rightarrow$ Study of gauge connections on $L_c^+\mathscr{M}$, reformulation of Einstein's equations, Corners on the light cone bundle, ...
- Direction 2: μBMS as a gauge group of a more fundamental theory of gravity?
  - Motivation: Emergence of pseudo-Riemannian geometry under some kind of symmetry breaking.
  - $\Rightarrow$ Is it possible to write down a gravitational theory for μBMS? Is it connected to GR via symmetry breaking? Are there induced corrections to GR?

Direction 3: It from Null?

- Usually: Carrollian geometry derived from pseudo-Riemannian geometry.
- Idea: Reverse the logics.
- Motivation: Small scale structure of spacetime is to a large extent fixed by microscopic ultrarelativistic aspects. E.g.:
  - Penrose and Kronheimer (1967), Minguzzi (2009): Horismos relations generate causal relations.
  - Borchers and Hegerfeldt (1972): Preservation of light cones in Minkowski spacetime implies Poincaré group.
  - Above argumentation: Breaking conformal invariance on $\mathbf{C}^+$ gives Minkowski space.

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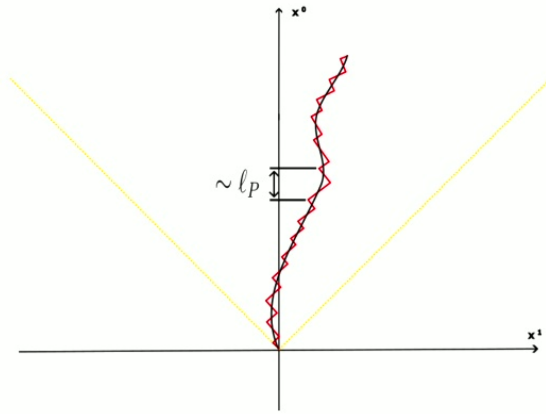
$\Rightarrow$ Defining the small scale structure of spacetime in terms of ultrarelativistic geometry?

$\Rightarrow$ Maybe a more general framework for the small scale structure of spacetime?

Direction 4: Null discretizations?



- Was suggested already in literature (e.g. Minguzzi 2009).

Personal statement

- Light cone bundle and μBMS are previously unnoticed Carrollian entities. Nobody knows their properties and capabilities.
- The idea of defining the small scale structure of spacetime via ultrarelativistic aspects could lead to generalized geometries. Maybe one can define geometries which are asymptotically lower dimensional and silent on microscopic scales by this strategy.

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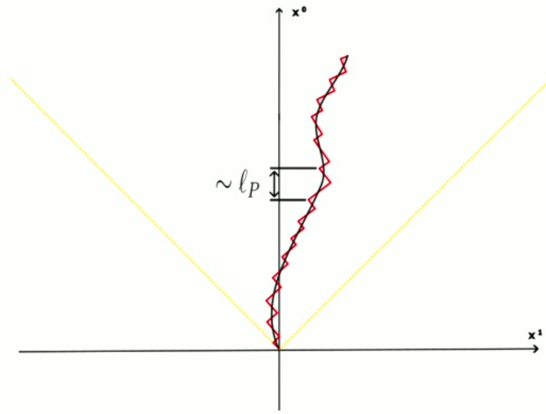

- Was suggested already in literature (e.g. Minguzzi 2009).
- But remained substantially underresearched. Exceptions: Kheyfets et al. 1988, Schaden 2015.
- Interesting consequence if this should work out: Speed of light not *maximum* velocity, but the only "velocity" of information exchange.

Thank you

Thank you very much!