Title: Long-Range Order on Line Defects in Ising Conformal Field Theories

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Abstract: It is well-known that one-dimensional systems at finite temperature, such as the classical Ising model, cannot spontaneously break a discrete symmetry due to the proliferation of domain walls. The validity of this statement rests on a few assumptions, including the spatial locality of interactions. In a situation where a one-dimensional system exists as a defect in a critical, higher-dimensional bulk system, the coupling between defect and bulk can induce an effective long-range interaction on the defect. It is thus natural to ask if long-range order can be stabilized on a defect in a critical bulk, which amounts to asking whether domain walls on the defect are relevant or not in the renormalization group sense. I will explore this question in the context of Ising conformal field theory in two and higher dimensions in the presence of a localized symmetry-breaking field. With both perturbative techniques and numerical conformal bootstrap, I will provide evidence that indeed the defect domain wall must be relevant when 2 < d < 4. For the bootstrap calculations, it is essential to include "endpoint" primary fields of the defect, which lead to a rigorous and powerful way to input bulk data. I will additionally give tight estimates of a number of other quantities, including scaling dimensions of defect operators and the defect entropy, and I will conclude with a discussion of future directions.

Zoom link https://pitp.zoom.us/j/92671628591?pwd=WjNma3VEV2M4T011dFlLMzM2ZUJiUT09

Long-range Order on Line Defects in Ising Conformal Field Theories

Work in progress w/ Shang Liu & Max Metlitski

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November 16, 2023

Overview

1. Motivation & Setup Go to page 3

2. Pinning field defect in $2 \leq D \leq 4$

3. Bootstrap approach

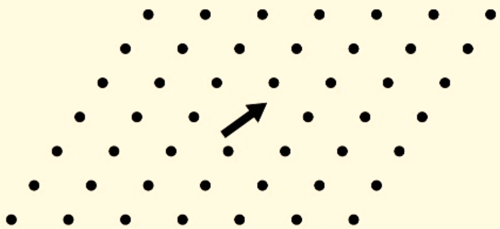
4. Conclusions

Defects and phases of matter

Defects come in many forms

$$H = H_0 + H_{defect}$$

- Kondo problem[a], impurities[b]
- Modified couplings
- Symmetry defects (background flux)[c]



[a] Kondo 1964; Wilson 1975.
[b] Sachdev, Buragohain, Vojta 2000.
[c] Thorngren, Else 2018; Barkeshli, Bonderson, Cheng, Wang 2019.

Want to understand long-distance properties of defects

Long-range order on a line?

Classical one-dimensional systems with local interactions do not typically order

- Ferromagnetic Ising: domain walls thermodynamically favored

$$\uparrow\ \uparrow\ \uparrow\ \uparrow\ \uparrow\ \uparrow \longrightarrow \uparrow\ \uparrow\ \uparrow\ |\ \downarrow\ \downarrow\ \downarrow$$

- Consider higher-dimensional critical Ising model with modified bond strength along a line

$$H = -J_* \sum_{\langle ij \rangle} s_i s_j - K \sum_{\langle ij \rangle \in L} s_i s_j$$

Is there a phase where spins spontaneously order along L?

Defects in field theory

Suppose H_0 tuned to a critical point, $D = d + 1$ spacetime dimensions

- Assume conformal field theory (CFT) description
- Want to model the combined bulk + defect system

Focus on **line defects**

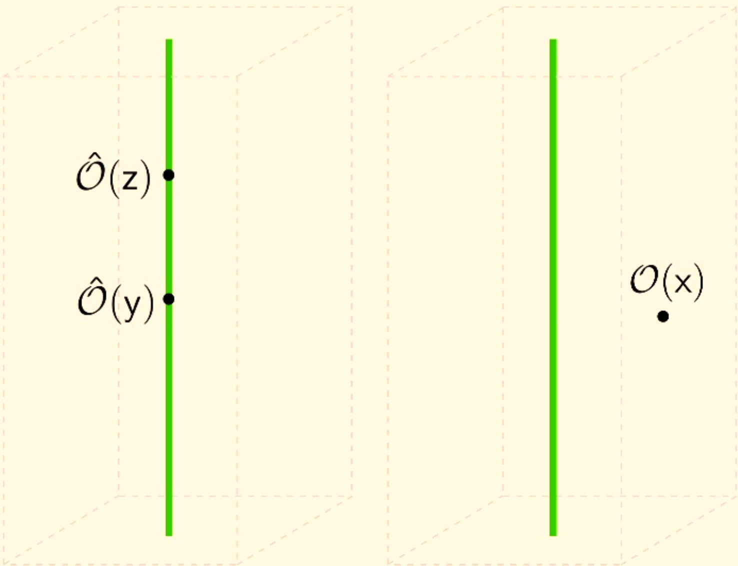
- In the IR, can flow to conformal defect preserving at most

$$SL(2, \mathbb{R}) \times SO(d)$$

i.e. defect CFT (dCFT)[a]

- Modified correlation functions, new critical exponents

$$\langle \mathcal{O}(x) \rangle = \frac{C_{\mathcal{O}}}{|x_\perp|^\Delta} \quad \langle \hat{\mathcal{O}}(y)\hat{\mathcal{O}}(z) \rangle = \frac{1}{|y - z|^{2\hat{\Delta}}}$$



[a]Billò, Gonçalves, Lauria, Meineri 2016.

Defect Renormalization Group

Simple case: perturb by relevant bulk operator, starting from no defect

$$S = S_{CFT} + h \int d\tau \, \mathcal{O}(\vec{x}, \tau) \qquad \Delta_{\mathcal{O}} < 1$$

Such RG flows constrained by monotonicity theorem[1]

Defect g-theorem

The g-function defined by

$$\log g \equiv \log Z_{defect+bulk} / Z_{bulk}$$

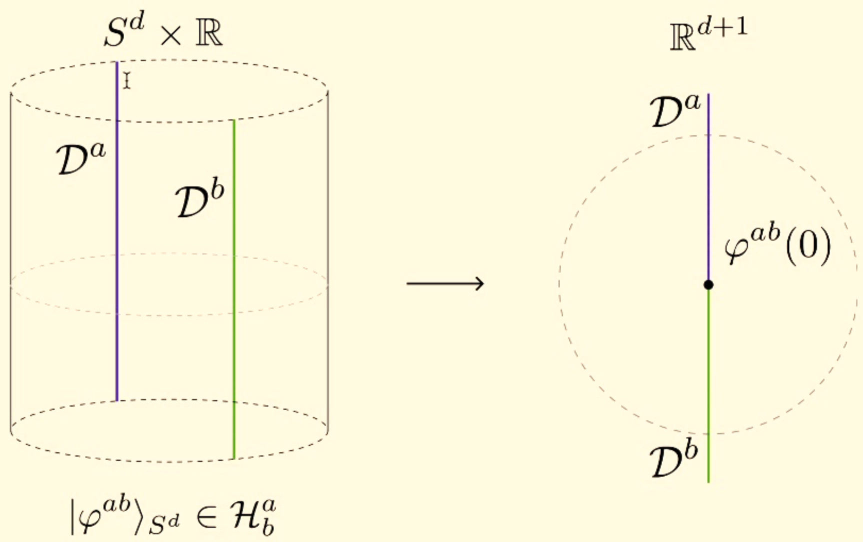
obeys $g_{UV} > g_{IR}$ at the fixed points under defect RG flow.

- In $D = 2$, follows from Affleck-Ludwig g-theorem from boundary conformal field theory
- Trivial defect has $g = 1$

[1] Zamolodchikov 1986; Casini, Landea, Torroba 2016; Friedan, Konechny 2004; Kobayashi, Nishioka, Sato, Watanabe 2019; Cuomo, Komargodski, Raviv-Moshe 2022.

Defect Hilbert spaces

Consider space as S^d, place conformal defects $\mathcal{D}^a, \mathcal{D}^b$ at antipodal points

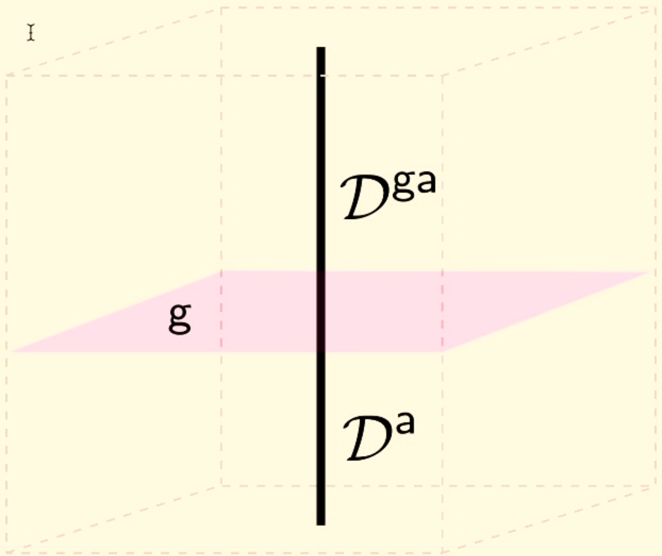


- Quantizing with different defect types gives defect Hilbert spaces $\mathcal{H}^a_b$
- φ^{ab} called *defect-changing* operators[1]

[1] Kim, Kiryu, Komatsu, Nishimura 2017.

Symmetry transformations of line defects

If we have global 0-form symmetry G, line type may transform under G[1]



[1] Kitaev, Kong 2012; Bartsch, Bullimore, Grigoletto 2023; Bhardwaj, Schafer-Nameki 2023.

Explicit and spontaneous defect symmetry breaking

Imagine a $\mathbb{Z}_2$ long-range ordered phase on a defect. As a conformal defect $\mathcal{D}^{sp\pm}$

- $\mathbb{Z}_2$ symmetric
- Expect two topological local operators corresponding to each symmetry-broken vacuum

$$I_+, I_-$$

  where $I_+ - I_-$ is an order parameter with LRO
- Can consider local and domain wall perturbations
- Operator spectrum described by

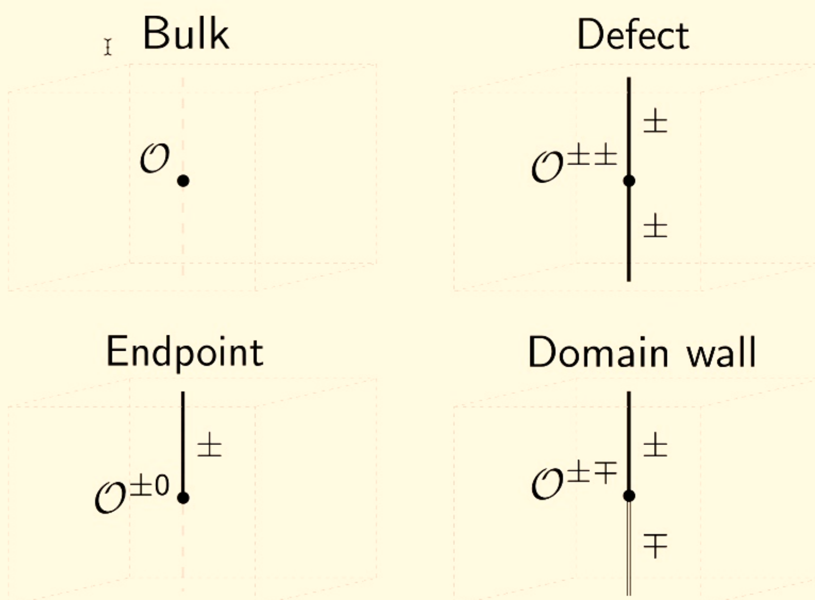
$$\mathcal{H}^{sp\pm}_{sp\pm} = \mathcal{H}^+_+ \oplus \mathcal{H}^+_- \oplus \mathcal{H}^-_+ \oplus \mathcal{H}^-_-$$

- Can separately study each $\mathcal{H}^a_b$, i.e. in the context of explicit symmetry breaking
- Sometimes domain walls forbidden in quantum problems: can be related to SPT physics[1]

[1] Liu, Shapourian, Vishwanath, M. A. Metlitski 2021; Thorngren, Vishwanath, Verresen 2021; Prembabu, Thorngren, Verresen 2022.

Pinning-field defect operators

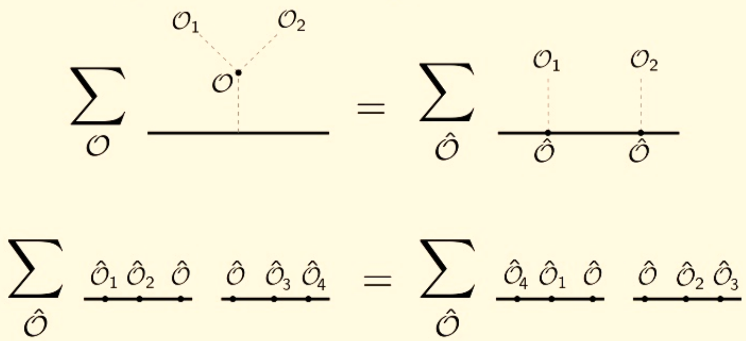
In this talk, will consider various species of operators



Goals

Want to study the pinning field defect for $2 \leq D \leq 4$

- Long history of existing literature (analytic[1], M.C.[2], bootstrap[3], fuzzy sphere[4])
- Want to advance bootstrap study of line defects
- Previous works: bulk-to-defect, defect crossing[5]



- Include defect-changing operators, do standard 4-point bootstrap

[1] Allais, Sachdev 2014; Cuomo, Komargodski, Mezei 2022, [2] Parisen Toldin, Assaad, Wessel 2017; Allais 2014, [3] Gimenez-Grau, Lauria, Liendo, Vliet 2022, [4] Hu, He, W. Zhu 2023
[5] Gaiotto, Mazac, Miguel F Paulos 2014; Padayasi, Krishnan, M. Metlitski, Gruzberg, Meineri 2022; Liendo, Rastelli, Rees 2013; Behan, Di Pietro, Lauria, Rees 2020

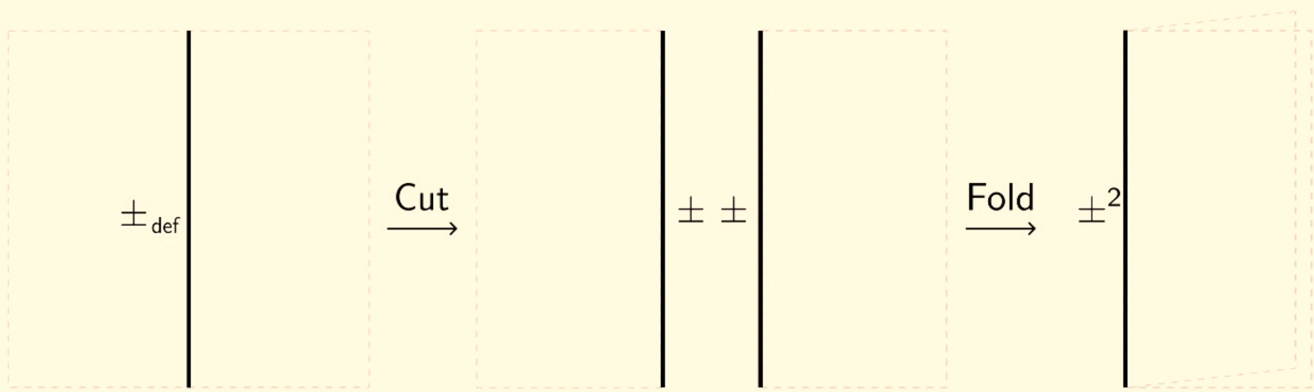
2. Pinning field defect in $2 \leq D \leq 4$

Pinning field defect in $D = 2$

All conformal defects in $D = 2$ Ising CFT known[1]

- Related to conformal boundary conditions of orbifold free boson

Pinning field defect is "separating" in $D = 2$



i.e. equivalent to cutting and imposing "fixed" conformal boundary conditions

[1] Oshikawa, Affleck 1996.

Pinning field defect in $D = 2$

Use boundary CFT

- Fixed Cardy states in Ising

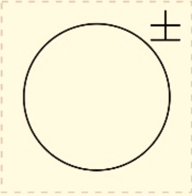
$$|\pm\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle\rangle + |\epsilon\rangle\rangle \pm 2^{1/4}|\sigma\rangle\rangle\right)$$

- Pinning field defect associated with tensor product Cardy state in Ising$^{\otimes 2}$

$$|\pm,\pm\rangle = \frac{1}{2}\left(|0\rangle\rangle + |\epsilon\rangle\rangle \pm 2^{1/4}|\sigma\rangle\rangle\right)^{\otimes 2}$$

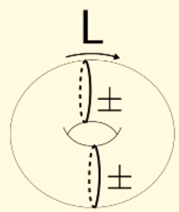
Read off defect entropy $g = \frac{1}{2}$

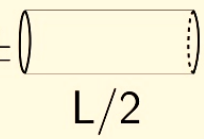
$$\tfrac{1}{2} = \langle 0,0|\pm,\pm\rangle =$$



Pinning field defect in $D = 2$

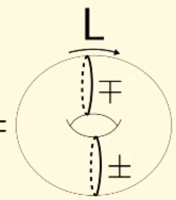
Defect partition function

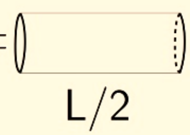
$Z_{\pm\pm}(\tau) =$ 

$= \big(\pm$ 

$\pm\big)^2 = (\chi_0(\tau))^2$

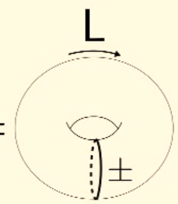
Domain wall partition function

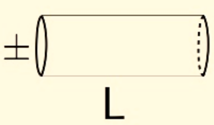
$Z_{\mp\pm}(\tau) =$ 

$= \big(\mp$ 

$\pm\big)^2 = (\chi_{1/2}(\tau))^2$

Endpoint partition function

$Z_{0\pm}(\tau) =$ 

$= \pm$ 

$\pm = \chi_0(\tau)$

Pinning field defect in $D = 2$

Operator spectra:

- Defect: Descendants of identity operator

$$\Delta_n^{\pm\pm} = 0, 2 + n \qquad n \in \mathbb{N}$$

- Domain walls: Descendants of $\Delta = 1$ primary

$$\Delta_n^{\pm,\mp} = 1 + n \qquad n \in \mathbb{N}$$

- Endpoints

$$\Delta_{n,1}^{0,\pm} = n + \frac{1}{32} \qquad n \in \mathbb{N}$$
$$\Delta_{n,2} = \frac{n}{2} + \frac{1}{32} \qquad 3 \leq n \in 2\mathbb{N} + 1$$

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Pinning field in $D = 4 - \epsilon$

Study the pinning field defect at Wilson-Fisher fixed point in $D = 4 - \epsilon$ (note $\Delta_\phi = \frac{D-2}{2}$)

$$S_{WF} = \int d^d x \left[\frac{1}{2}(\partial\phi)^2 + \frac{\lambda_0}{4!}\phi^4 \right]$$

$$S_{def} = \int d\tau \, h_0(\tau)\phi(0,\tau)$$

We can allow discontinuities in $h(\tau)$ to create endpoint, domain wall, etc.

In terms of appropriate dimensionless couplings, non-trivial fixed point appears in $D < 4$

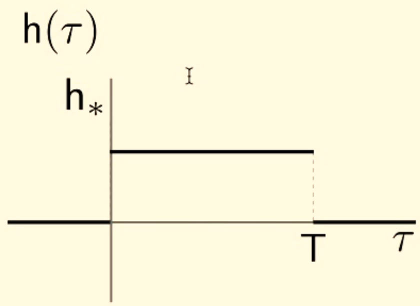
$$\frac{\lambda_*}{(4\pi)^2} = \frac{\epsilon}{3} + \frac{17}{81}\epsilon^2 + \mathcal{O}(\epsilon^3) \qquad h_*^2 = 9 + \epsilon\frac{73}{6} + \mathcal{O}(\epsilon^2)$$

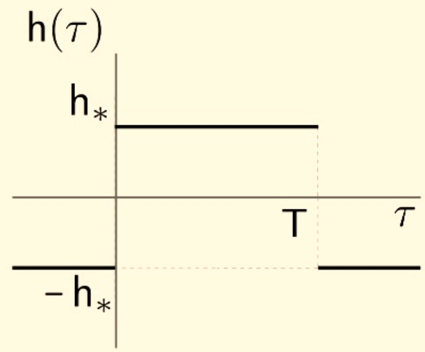
ϕ renormalizes differently along the defect

$$\Delta_{++} = 1 + \epsilon - \frac{41}{27}\epsilon^2 + \mathcal{O}(\epsilon^3) \qquad \xrightarrow{\text{Padé}} \qquad \Delta_{++}(D=3) \approx 1.55 \pm 0.14$$

Pinning field in $D = 4 - \epsilon$

Can attempt to estimate Δ_{0+} and Δ_{+-}





Both receive large corrections at $\mathcal{O}(\epsilon)$

$$\Delta_{0+} = 0.113986(1 + 1.03454\epsilon + \mathcal{O}(\epsilon^2)) \overset{d=3}{=} 0.231909$$

$$\Delta_{+-} = 0.455945(1 + 1.58261\epsilon + \mathcal{O}(\epsilon^2)) \overset{d=3}{=} 1.17753$$

At Gaussian fixed point in $d = 4$, h is exactly marginal and Δ_{+-} may be tuned continuously

Survey of defect CFT data in $d = 3$

Collect various estimates of dCFT data

	Analytic	MC	Fuzzy sphere
Δ_{++}	1.55 ±0.14[1]	1.60±0.05[2] 1.52±0.06[2] 1.4±0.03[3]	1.63 ±0.06[4]
Δ_{+-}	1.17753	?	**Unpublished**: ∼0.837[6]
Δ_{0+}	0.231909[5]	0.0935 ±0.0035[3]	**Unpublished**: ∼0.1075[6]
g	0.569783[1]	?	**Unpublished**: ∼0.6055[6]

Bootstrap?! Let's find out!

[1] Cuomo, Komargodski, Mezei 2022.
[2] Parisen Toldin, Assaad, Wessel 2017.
[3] Allais 2014.
[4] Hu, He, W. Zhu 2023.
[5] Allais, Sachdev 2014.
[6] Zou, Zhou, Gaiotto, He 2023.

3. Bootstrap approach

Symmetries and defects

For the defect problem in $D = 3$

- Consider straight line defect along τ axis
- Continuous spacetime symmetry: $SL(2, \mathbb{R}) \times SO(2)_\tau$[1]

$$\text{Generators} : D, P_\tau, K_\tau, M_{xy}$$

- Discrete spacetime symmetry[2]:

$$\mathbb{Z}_2^\tau : (\tau, x, y) \to (-\tau, x, y)$$

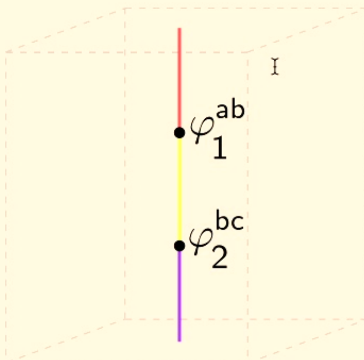
- Discrete internal symmetry: Ising $\mathbb{Z}_2$

These symmetries lead to various selection rules

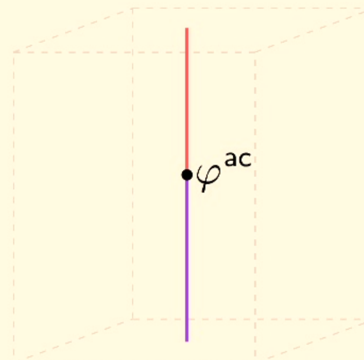
[1] Billò, Gonçalves, Lauria, Meineri 2016.
[2] Gaiotto, Mazac, Miguel F Paulos 2014.

Symmetries and defects

OPE of defect-changing operators[1]:



$$\sim \sum_{\varphi_3^{ac}} \lambda^{abc}_{\bar{\varphi}_3 \varphi_1 \varphi_2}$$



For primaries, $SL(2, \mathbb{R})$ symmetry fixes form of OPE

$$\varphi_1^{ab}(\tau) \times \varphi_2^{bc}(0) = \sum_{\varphi_3} \lambda^{abc}_{\bar{\varphi}_3 \varphi_1 \varphi_2} \tau^{\Delta_{\mathcal{O}} - \Delta_{\varphi_1} - \Delta_{\varphi_2}} \sum_{n=0}^{\infty} \tau^n \beta^{(n)}_{\bar{\varphi}_3 \varphi_1 \varphi_2} \varphi^{ac,(n)}(0)$$

[1] Runkel 2000.

Symmetries and defects

Important aside: take care of operator normalization

- When the output of OPE contains only defect operators, identity operator may appear

$$\varphi^{ab}(\tau)\varphi^{ba}(0) = \frac{\lambda^{aba}_{\varphi\varphi I} I_a}{\tau^{2\Delta}} + \ldots$$

- I^a represents infinite line defect with no local operator insertions, in general

$$g_a = \langle I_a \rangle \neq 1$$

- $SL(2,\mathbb{R})$ symmetry can cyclically permute

$$\langle \varphi^{ab}(\tau)\varphi^{ba}(0)\rangle = \langle \varphi^{ba}(\tau)\varphi^{ab}(0)\rangle \implies g_a \lambda^{aba}_{\varphi\varphi I} = g_b \lambda^{bab}_{\varphi\varphi I}$$

- Can achieve unit-normalization by choosing $\lambda^{aba}_{\varphi\varphi I} = g_a^{-1}$

Symmetries and defects

For three-point functions

$$\langle \varphi_1^{ab}(\tau_1)\varphi_2^{bc}(\tau_2)\varphi_3^{ca}(\tau_3)\rangle = \frac{\lambda^{bca}_{\varphi_1\varphi_2\varphi_3}}{\tau_{12}^{\Delta_{\varphi_1\varphi_2\varphi_3}}\tau_{13}^{\Delta_{\varphi_1\varphi_3\varphi_2}}\tau_{23}^{\Delta_{\varphi_2\varphi_3\varphi_1}}} \qquad \Delta_{\varphi_i\varphi_j\varphi_k} = \Delta_{\varphi_i} + \Delta_{\varphi_j} - \Delta_{\varphi_k}$$

There is again a constraint from cyclicity

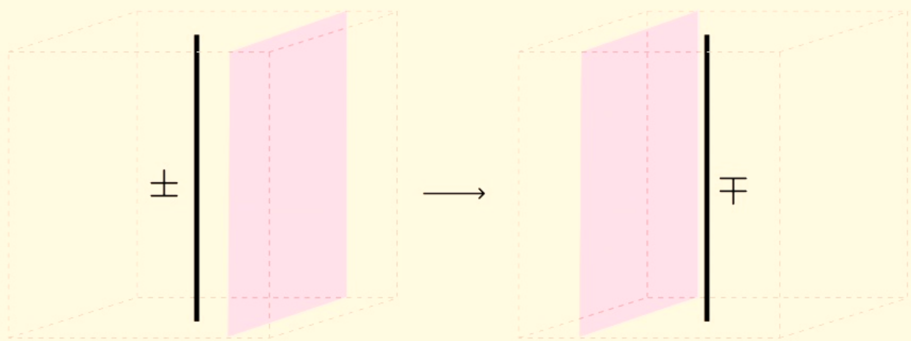
$$\lambda^{bca}_{\varphi_1\varphi_2\varphi_3} = \lambda^{cab}_{\varphi_2\varphi_3\varphi_1} = \lambda^{abc}_{\varphi_3\varphi_1\varphi_2}$$

Cross-ratio dependent piece of four-point function

$$G^{abcd}_{\varphi_i\varphi_j\varphi_k\varphi_l}(x) = \sum_{\mathcal{O}^{ca}} \frac{\lambda^{bca}_{\varphi_i\varphi_j\mathcal{O}}\lambda^{cda}_{\bar{\mathcal{O}}\varphi_k\varphi_l}}{\langle \mathcal{O}^{ca}|\mathcal{O}^{ca}\rangle} g_{\Delta_{\mathcal{O}}}^{\Delta_{\varphi_i\varphi_j},\Delta_{\varphi_k\varphi_l}}(x)$$

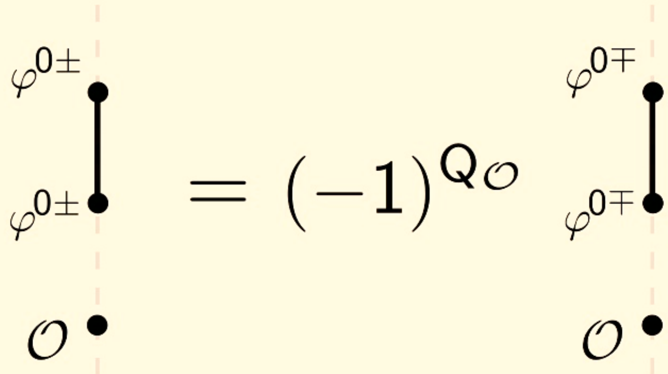
Symmetries and defects

Imposing Ising $\mathbb{Z}_2$ symmetry



Symmetries and defects

Imposing Ising $\mathbb{Z}_2$ symmetry



Selection rules

Will need more details about OPE content of bulk and endpoint operators

- Leading endpoint ($SO(2)$ singlet): $\varphi^{0\pm}$
- Leading $\mathbb{Z}_2$ even scalar: ϵ
- Leading $\mathbb{Z}_2$ odd scalar: σ
- Previous studies used just displacement operator, including makes negligible difference

$\mathbb{Z}_2^\tau$ and $SO(2)$ further constrain OPE

- Defect $SO(2)_\tau$ spin: s
- $\mathbb{Z}_2^\tau$ parity: p_τ
- Bulk $SO(3)$ spin: ℓ

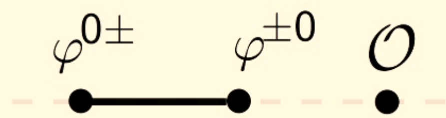
Bulk operators of $SO(3)$ spin ℓ, $s = 0$ and parity p have $p_\tau = \ell + p \mod 2$

Use as much bulk data as possible[1]

[1] Simmons-Duffin 2017; Su 2022; Wei Zhu, Han, Huffman, Hofmann, He 2023.

Selection rules

First consider endpoints fusing to bulk $SL(2,\mathbb{R})$ primary operators

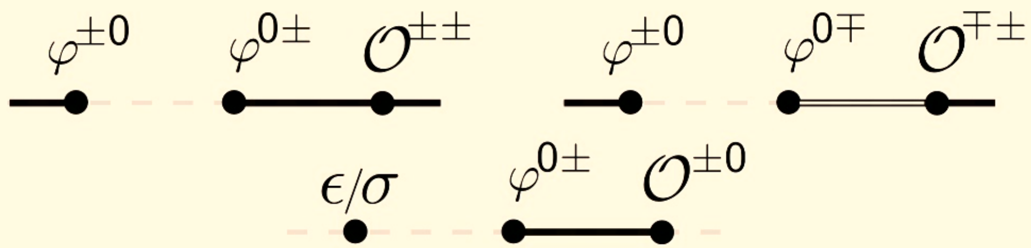


$\mathcal{O}$ must be

- $\mathbb{Z}_2$ even or odd
- $p_\tau = 0$
- $s = 0$
- $\mathcal{O}$ not necessarily a *bulk* primary
- Need to include all bulk descendants consistent with other symmetries

Selection rules

Next consider endpoints $\to$ (defect operators/domain walls) or endpoint + bulk $\to$ endpoint

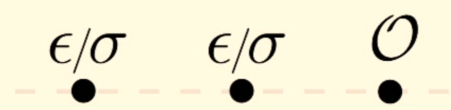


$\mathcal{O}^{\pm,\pm}, \mathcal{O}^{\pm,\mp}, \mathcal{O}^{0,\pm}$ must be

- $p_\mathcal{T} = 0$ (except for $\mathcal{O}^{0,\pm}$)
- $s = 0$

Selection rules

Finally, consider all bulk operators



<u>$\sigma \times \sigma$, $\epsilon \times \epsilon$</u>

- $\mathbb{Z}_2$ even
- $p_\tau = 0$
- $s = 0$

<u>$\sigma \times \epsilon$</u>

- $\mathbb{Z}_2$ odd
- $p_\tau = 0, 1$
- $s = 0$

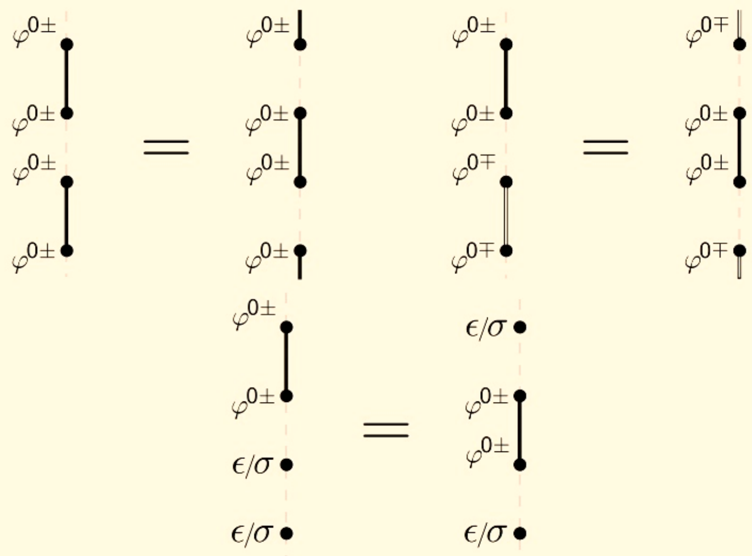
- We include known OPE coefficients: $\lambda_{\sigma\sigma\epsilon}, \lambda_{\epsilon,\epsilon,\epsilon}, \lambda_{\sigma\sigma T}, \lambda_{\epsilon\epsilon T}$
- For scalar bulk primaries, incoporate OPE coefficient of level-2 bulk descendant

$$\tilde{\mathcal{O}}^{(2)} = \mathcal{N}\left(\frac{2}{2\Delta_3 + 1}\partial_\tau^2 + \partial_i^2\right)\mathcal{O}$$

$$\lambda_{\mathcal{O}_1\mathcal{O}_2\tilde{\mathcal{O}}^{(2)}} = -\mathcal{N}\frac{2(\Delta + \Delta_{12})(\Delta - \Delta_{12})}{2\Delta + 1}\lambda_{12\mathcal{O}}$$

Crossing symmetry

Impose crossing symmetry and obtain numerical bootstrap bounds[1]

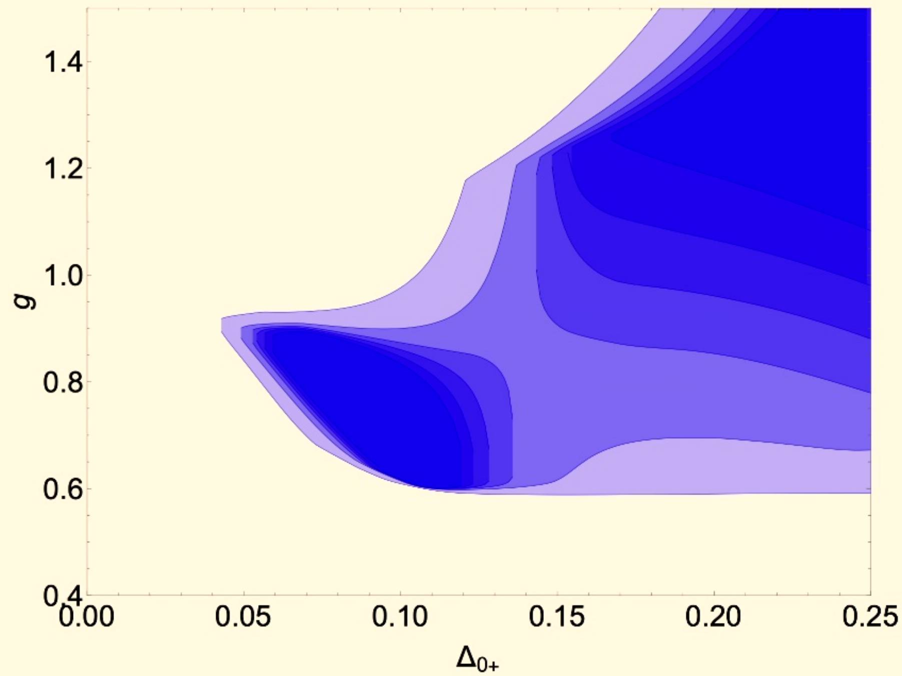


Use standard semidefinite programming techniques (SDPB[2])

[1] Rattazzi, V. S. Rychkov, Tonni, Vichi 2008.
[2] El-Showk, Miguel F. Paulos, Poland, S. Rychkov, Simmons-Duffin, Vichi 2012; Simmons-Duffin 2015.

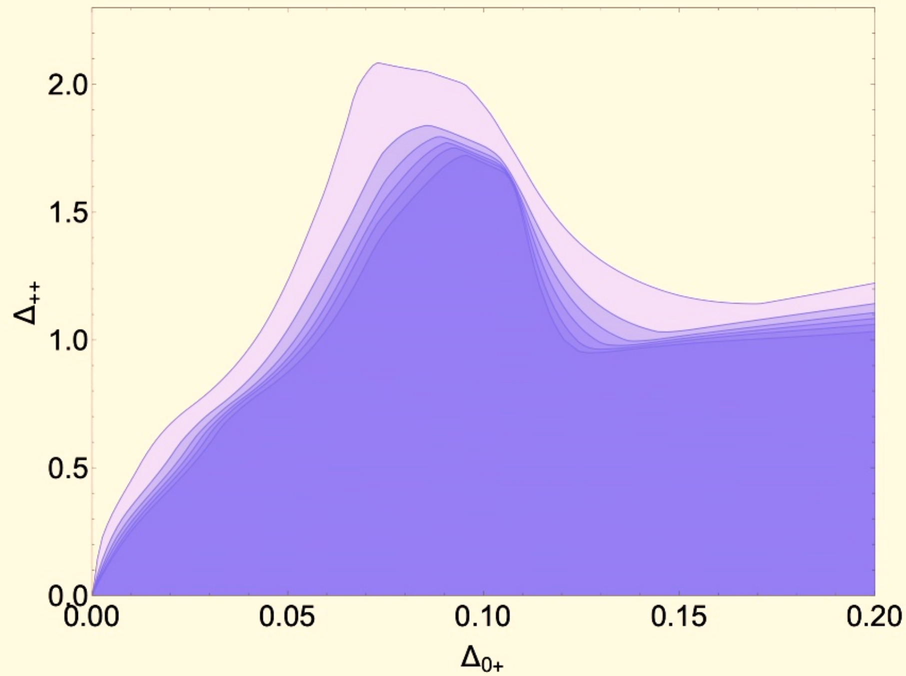
Universal Bound on defect g-function

Assume bulk 3d Ising, no assumptions about defect other than stability to local perturbations



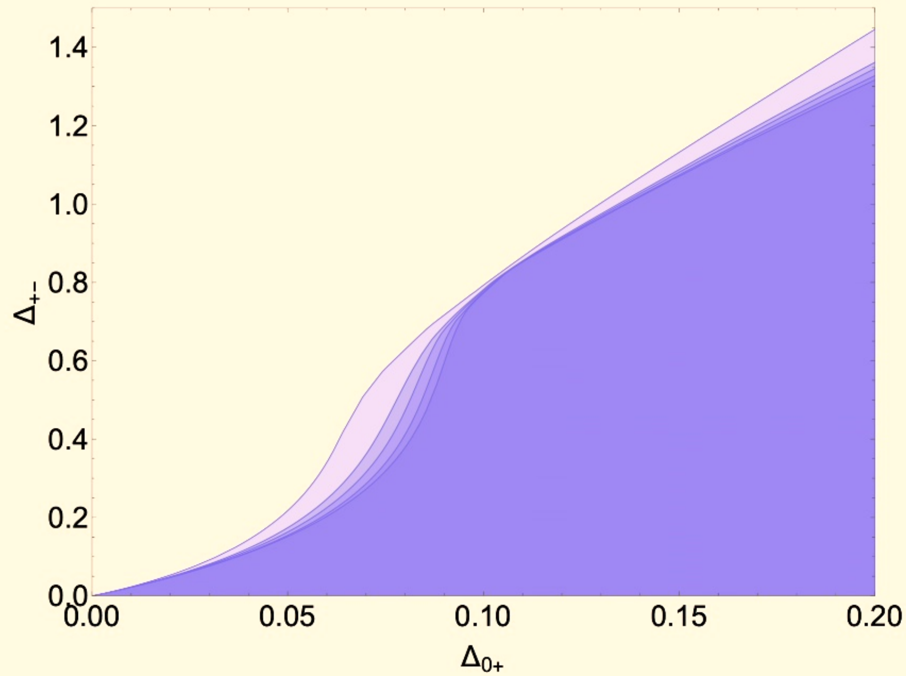
- $0.0585 \leq \Delta_{0+} \leq 0.1195$
- $0.601 \leq g \leq 0.894$

Universal bound on lightest defect operator



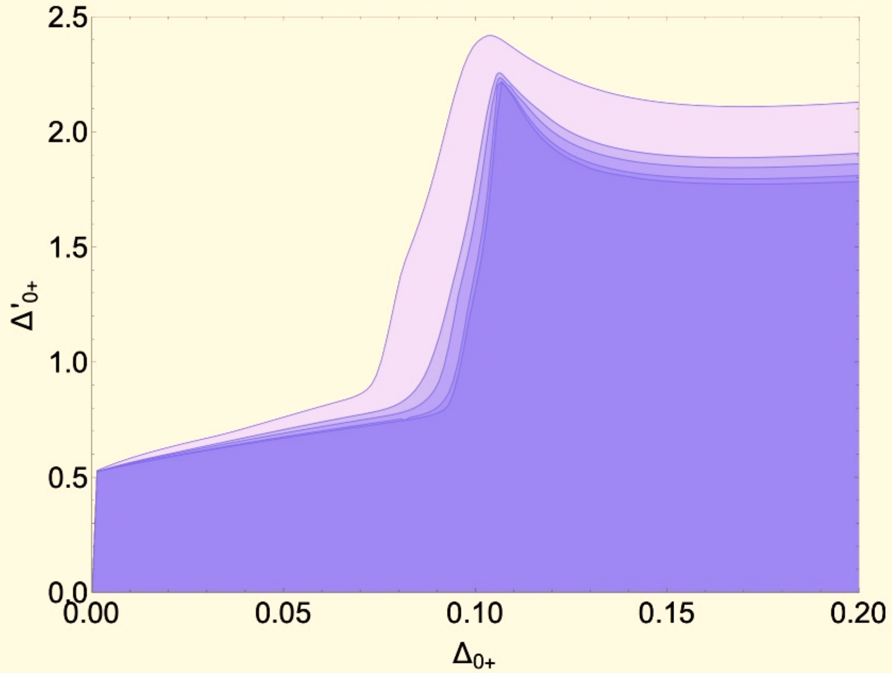
- $\Delta_{++} \leq 1.723$
- Saturates near $(\Delta_{0+}, \Delta_{++}) = (0.1074, 0.614)$!

Universal bound on lightest domain wall



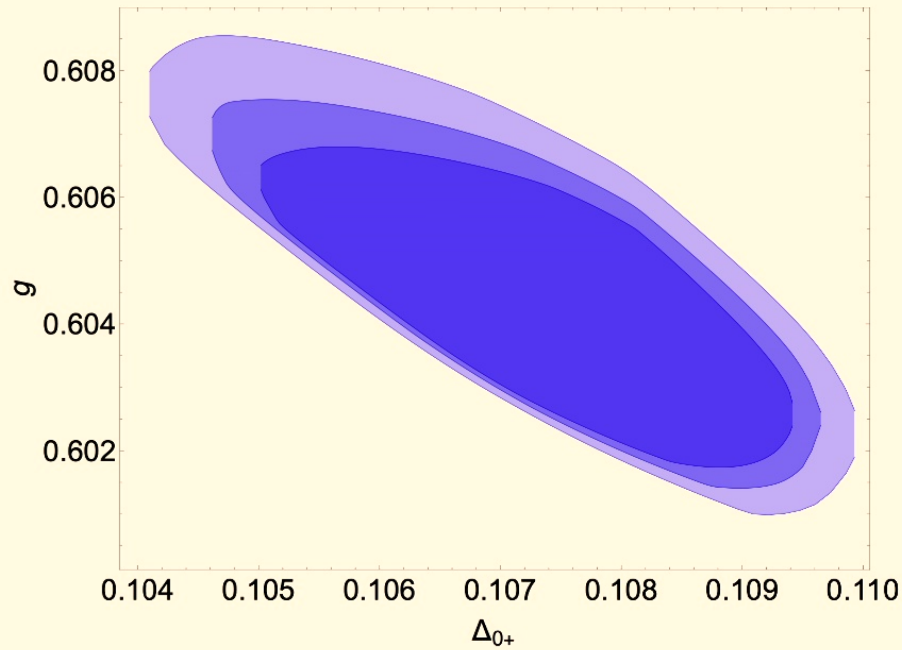
- Saturates near $(\Delta_{0+}, \Delta_{++}) = (0.1076, 0.832)$!
- Relevant if $\Delta_{0+} \leq 0.1366$!

Universal bound on subleading endpoint



More aggressive g bound

Assume gaps $\Delta_{++} \geq 1.5$, $\Delta_{+-} \geq 0.7$



- $0.105 \leq \Delta_{0+} \leq 0.10942$
- $0.6018 \leq g \leq 0.6068$

Conclusions

Summary

- We can obtain accurate defect CFT data by incorporating endpoints
- Found excellent agreement with preliminary fuzzy sphere predictions!

To-Do

- Careful spectrum analysis
- Impose gaps above Δ_{++}, Δ_{+-}

Future directions:

- Spin impurities
- Wilson lines