Title: Generalized angular momentum via Wald-Zoupas

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Abstract: In the last years, asymptotic symmetries have regained a lot of interest, and various extensions of the well known BMS group have been considered in the literature. Many charges associated to the diffeomorphisms of the sphere (superboosts and superrotations) have been proposed, but it has not been clear if these charges can be derived from a symplectic potential that is covariant and stationary, i.e satisfying the Wald-Zoupas usual requirements. In this talk I will consider a new asymptotic symmetry group, which is a one dimensional extension of the generalized-BMS group, and construct a stationary symplectic potential, covariant with respect to these symmetries, by adding corner terms to the usual Einstein-Hilbert symplectic potential. Then, we will recover the charges introduced by Compère, Fiorucci and Ruzziconi for superboosts and superrotations. In order to ensure covariance, we will need to introduce an edge mode which has already appeared in the literature, the supertranslation field. I will also explain that its introduction as a corner term can lead us to construct a local (asymptotic) notion of energy for the gravitational waves, providing a physical interpretation of the new charges.

Zoom link https://pitp.zoom.us/j/97926664729?pwd=VzV2VmQ4eVlzcFdaZkNBNnpqRkMvUT09

Generalized angular momentum via Wald Zoupas

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Motivations

Asymptotic symmetries in General Relativity Bondi,van der Burg,Metzner, Sachs

BMS symmetries, preserve the normal and the unphysical metric up to a conformal factor. We obtain the Lorentz transformations (rotations + boosts) and all the supertranslations.

Soft theorems and asymptotic symmetries Strominger, Pasterski, Campligia, Laddha

The subleading soft theorems imply more symmetries. Local conformal transformations, extended BMS. Barnich, Troessart

Corner proposal Chandrasekaran, Ciambelli, Compère, Fiorucci, Freidel, Geiller, Leigh, Oliveri, Pranzetti, Ruzziconi, Speranza, Speziale, Wieland, Zwickel

We know that the degenerate directions of the pre-symplectic form are gauge symmetries. The renormalized symplectic form at $\mathscr{I}$ does not exhibit degenerate directions for the diffeomorphisms of the sphere. It suggests an extension of the *BMS* group.

$\Rightarrow$ Proposal : Promoting the non-vanishing charges to operators

Plan of the talk

We will

- Review the Wald-Zoupas procedure, its suitable properties, and its limitations.
- Review BMS, Ashtekar-Streubel flux and Dray-Streubel charges.
- Construct a new covariant and stationary symplectic potential and new charges, that will measure a local (and asymptotic) notion of energy with the help of an edge mode, the supertranslation field. No central extension.
- Explain the problem of covariance with generalized BMS.
- Try to solve the covariance problem by introduce a symmetry new group, preserving the round sphere.
- Find a covariant and stationary symplectic potential at $\mathscr{I}$. We will give the Noether charges for a vanishing supertranslation field. This procedure removes many ambiguities in the construction of the charges. Central extension is cross section independent.

What are good asymptotic charges ? Wald,Zoupas,99'

Symplectic structures

The symplectic potential θ and the symplectic form $\omega = \delta\theta$ encode the symplectic structure $\delta L = E(g)\delta g + d\theta(g, \delta g, \chi)$.

⚠ θ is ambiguous up to exact terms in field space and spacetime

We want Wald,Zoupas,99'; Grant,Prabhu,Shezad,21'; Odak, ARB, Speziale,22'

- A symplectic potential $\bar{\theta} = \theta_*^{EH} + \delta l - d\vartheta$
- Covariance, i.e $(\delta_\xi - \mathscr{L}_\xi)\bar{\theta} = 0$
- Stationarity requirement $\Theta(g, \delta g) = 0$ in non-radiative spacetime

Charges Harlow,Wu,20'and Freidel,Geiller,Pranzetti,20'

- à la Noether $\bar{q}_\xi = \kappa_*^{EH} + i_\xi l - I_\xi \vartheta$
- à la Wald-Zoupas $-I_\xi \bar{\omega} + di_\xi \bar{\theta} = d\delta\bar{q}_\xi$ + one reference solution

Covariance $\Rightarrow d\bar{q}_\xi = I_\xi \bar{\theta}$

The unphysical metric is background

Asymptotic flat spacetime

- We have $\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ with $\Omega = 0$ at $\mathscr{I}$. Null infinity has topology $\mathbb{R} \times S^2$ and is complete.
- We have that $n_\mu = \partial_\mu \Omega$ ans is null through Einstein's equations.

BMS symmetries

We are looking for the diffeomorphisms satisfying
$\delta \hat{g}_{\mu\nu}|_{\mathscr{I}} = 0 = \mathscr{L}_\xi \hat{g}_{\mu\nu} - 2\Omega^{-1} n_\alpha \xi^\alpha \hat{g}_{\mu\nu}$

Ashtekar and Streubel flux

We have $\bar{\theta}^{AS} = -\frac{1}{2} \hat{N}_{AB} \delta C^{AB} \varepsilon_I$, covariant and stationary

Dray-Streubel charges

$\bar{Q}^{DS}_\xi = \int_S (4\tau \hat{M} + 2Y^A P_A) \varepsilon_S$

Introduction of the edge mode Donnelly,Freidel,16',Rovelli,13'

Supertranslation field Compère,Fiorucci,Ruzziconi,18'

This supertranslation field $u_0(x^A)$ plays the role of a reference frame for an asymptotic observer at $\mathscr{I}$. We start from the requirement $(\delta_\xi - \mathscr{L}_\xi)(u - u_0) = -\dot{\tau}(u - u_0)$ implying $\delta_{\tau,Y} u_0 = Y^A \partial_A u_0 - T - \dot{\tau} u_0$

Covariant shear

The shear transforms inhomogenously. Consider $\hat{C}_{AB} = C_{AB} - (u - u_0)\rho_{<AB>} - 2D_{<A}\partial_{B>} u_0$.
We can check that $(\delta_\xi - \mathscr{L}_\xi)\hat{C}_{AB} = -\dot{\tau}\hat{C}_{AB}$

Boundary condition

We will set the boundary condition that in the far future $\lim_{u\to+\infty} \hat{C}_{AB} = 0$
Define u_0 as $(C_{AB} - u\rho_{<AB>}) \underset{u\to+\infty}{=} 2D_{<A}\partial_{B>} u_0 - u_0\rho_{<AB>}$

New symplectic potential and new charges

Corner ambiguity

We use the corner ambiguity to add the corner term
$\vartheta^{new} = -\frac{1}{2}\hat{C}_{AB}\delta(D^A\partial^B u_0 + u_0\rho^{AB})\varepsilon_S$

New symplectic potential

$\bar{\theta}^{new} = \bar{\theta}^{AS}_* + d\vartheta^{new} = -\frac{1}{2}\hat{N}_{AB}\delta\hat{C}^{AB}$ It is covariant and stationary.

New charges see also Moreschi,88

$Q^{new}_S = \int_S 4\tau[\hat{M} - \frac{1}{8}\tau(u_0)(D^A D^B + \rho^{AB})\hat{C}_{AB}]_S$
$+ 2Y^A[P_A + \frac{1}{4}\partial_A u_0(D^B D^C + \rho^{BC})\hat{C}_{BC}]\varepsilon_S$ so $\rightarrow$ DS + f($\hat{C}_{AB}$)

Flux of supertranslation

Local energy flux $\Delta Q^{new}_T = -\frac{1}{2}T\hat{N}_{AB}\hat{N}^{AB}\varepsilon_{\mathscr{I}} \leq 0$

Symplectic form and bracket

No local symplectic form density

The corner term ϑ^{new} is defined at $\mathscr{I}$ only. The symplectic form is obtained by integrating on a Cauchy surface intersection $\mathscr{I}$
$\Omega_\Sigma = \int_\Sigma \omega^{EH} + d\vartheta^{new}$

Barnich-Troessart bracket

$\{Q_\xi, Q_\chi\}^{BT}|_S = \delta_\chi Q_\xi - i_\xi I_\chi \bar{\theta}|_S$ generates symmetry if $\bar{\theta} = 0$, i.e $\hat{N}_{AB} = 0$

No central extension Barnich,Troessart,11'

- $(\delta_\xi - \mathscr{L}_\xi)\bar{\theta} = 0 \Rightarrow \{Q_\xi, Q_\chi\}^{BT}|_{S_2} - \{Q_\xi, Q_\chi\}^{BT}|_{S_1} = Q_{[[\xi,\chi]]}|_{S_2} - Q_{[[\xi,\chi]]}|_{S_1} \Rightarrow$ central extension independent on S.
- As $\lim_{u \to +\infty} \hat{C}_{AB} = 0$, we have that
$\{Q_\xi^{new}, Q_\chi^{new}\}^{BT}|_{S_{+\infty}} = \{Q_\xi^{BT}, Q_\chi^{new}\}^{BT}|_{S_{+\infty}} = \delta_\chi Q_\xi^{new/BT}|_{S_{+\infty}}$
- Therefore, on any cross section S : $\{Q_\xi^{new}, Q_\chi^{new}\}|_S = Q_{[[\xi,\chi]]}^{new}|_S$

Metric is not background anymore

BMSW group CFP,18', Freidel,Oliveri,Pranzetti,Speziale,21'

Diffeomorphisms preserving $\delta n_\mu =, \delta n^\mu = \delta k_n = 0$, but $\delta q_{AB} \neq 0$.
Contains gBMS and BMS as subgroup.
$BMSW = (Diff(S)^2 \ltimes \mathbb{R}^W_S) \ltimes \mathbb{R}^T_S$

Renormalization Freidel,Oliveri,Pranzetti,Speziale,21'

We have to renormalize the EH symplectic potential by a corner term
$\vartheta^{div} = (\frac{r^2}{2}\delta\sqrt{q} - \frac{r}{4}C^{AB}\delta q_{AB})\varepsilon_S - r\vartheta^A \varepsilon_{AB} du \wedge dx^B$

Non covariance of the pullback

We write $\theta^R = \theta^{EH} - d\vartheta^R|_*$.
We can prove that
$(\delta_\xi - \mathscr{L}_\xi)\theta^R = \partial_A W F^A(q_{BC}, C_{BC}, \delta q_{BC})\varepsilon_{\mathscr{I}} + D_A X^A \varepsilon_{\mathscr{I}} + dc$
Therefore $\bar{\theta} = \theta^R + \delta b - d\beta$ cannot be covariant for any b and β.

New background structure

Symmetry vector field preserving the round sphere

- Through a conformal transformation $\Omega \to \omega\Omega$ we have that $R \to \frac{1}{\omega^2}(R - 2\Delta \ln \omega)$.
- At the infinitesimal level we want $\delta_\xi R = Y^A \partial_A R + 2\dot{\tau} R + \frac{1}{2} R \dot{\tau} = KR$ whit constant $K \Rightarrow l = 0$ and $l = 1$ spherical harmonics of $\dot{\tau}$
- Lie bracket closes if we impose $\dot{\tau} = \frac{1}{2} D_A Y^A$ and keep conformal isometries (BMS) OR if we remove the $l = 1$ harmonics of $\dot{\tau}$ (BRS)

BRS group

- $BRS = (Diff(S)^2 \ltimes \mathbb{R}_S) \ltimes \mathbb{R}_S^T$
- Infinitesimal generator :$\xi = T(x^A)\partial_u + uW\partial_u + Y^A(x^B)\partial_A$
- The algebra $\xi_3 = [\xi_1, \xi_2]$ is
  $T_3 = Y_1^A \partial_A T_2 - W_2 T_1 - (1 \leftrightarrow 2)$
  $W_3 = 0$
  $Y_3^A = Y_1^A \partial_A Y_2^A - (1 \leftrightarrow 2)$

New background structure 2

Diffeomorphisms BRS

- Therefore, we have the supertranslations, the diffeomorphisms of the sphere, and the constant rescalings.
- We can decompose the diffeomorphisms of the sphere between the superrotations, satisfying $D_A Y^A = 0$ and the superboosts, generalizing the notion of boosts, which have a non vanishing divergence.

Unimodular BRS

- Unimodular BRS : In addition of imposing a round sphere, we impose that the determinant of the metric is fixed.
- $\Rightarrow$ We keep the superrotations but we loose the superboosts, and even the boosts.
- The vectors fields generating the Lie algebra are $\xi = T(x^A)\partial_u + Y^A(x^B)\partial_A$ with $D_A Y^A = 0$

Hierarchy of the groups

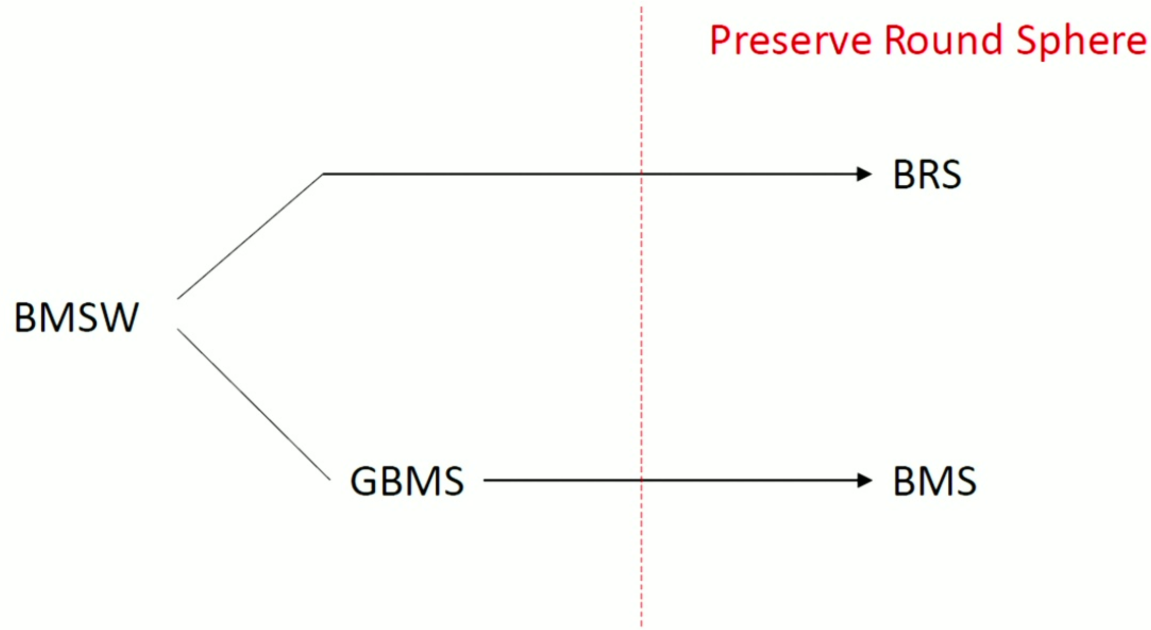

Figure 1: The RBS algebra is a one-dimensional extension of the generalized-BMS group. However, we have removed the vertical part of the superboosts, so we cannot recover the generalized BMS group by restricting to supertranslations and diffeomorphsims of the sphere.

Covariant and stationary symplectic potential

Finding a stationary symplectic potential

- We have $\theta^R = -\frac{1}{2}N_{AB}\delta C^{AB}\varepsilon_{\mathscr{I}} + X_{AB}\delta q^{AB}\varepsilon_{\mathscr{I}}$
- As $\mathscr{I}$ is expansion-free shear-free, then $d(u\varepsilon_S) = \varepsilon_{\mathscr{I}}$
- $\theta' = \theta^R - d(uX_{AB}\delta q^{AB}\varepsilon_S) = -\frac{1}{2}N_{AB}\delta C^{AB}\varepsilon_{\mathscr{I}} - u\partial_u X_{AB}\delta q^{AB}\varepsilon_{\mathscr{I}}$

Restoring covariance

- We can find a covariant symplectic potential by adding corner terms depending on the supertranslation field u_0.
- $\bar{\theta} = \theta^R + \delta b - d\vartheta = -\frac{1}{2}N_{AB}\delta\hat{C}^{AB}\varepsilon_{\mathscr{I}}$
  $+\varepsilon_{\mathscr{I}}(u-u_0)(D^{<A}\partial_u\hat{U}^{B>} + D^{<A}u_0\partial_u^2\hat{U}^{B>} + \frac{R}{4}N^{AB})\delta q_{AB}$
  $+(u-u_0)(-2D_A\partial_u\hat{U}^A - 2D_A u_0\partial_u^2\hat{U}^A - \frac{1}{4}N_{AB}N^{AB})\delta\varepsilon_{\mathscr{I}}$
  $+(u-u_0)D_A(\frac{1}{2}q^{BC}\delta q_{BC}\partial_u\hat{U}^A)\varepsilon_{\mathscr{I}} + (u-u_0)D_A u_0\partial_u^2\hat{U}^A\delta\varepsilon_{\mathscr{I}}$
  $+\beta N_{AB}\hat{C}^{AB}\delta\varepsilon_{\mathscr{I}}$
- Only one ambiguity β Elashash,Nichols,21'

Charges

Improved Noether charges

- We take a frame where $u_0 = 0$.
- $\bar{Q}_{T,W,Y^A} = \int_S 4T(M - \frac{1}{4}D_A D_B C^{AB})\varepsilon_S$
  $+ 2W\beta C_{AB}C^{AB}\varepsilon_S$
  $+2Y^A(P_A + \frac{1}{2}(\beta - \frac{1}{16})\partial_A(C_{BC}C^{BC}) - u\partial_A M - u\frac{1}{2}D^B D_{[A}D^C C_{B]C})$
- We recover the Dray Streubel angular momentum and boost charges by taking $\beta = \frac{1}{16}$
- For $u_0 = 0$, same super-rotation and super-boost charge as Compère-Fiorucci-Ruzziconi, which is DS + soft term. CFR18'

Different supermomentum

- Even if $u_0 = 0$, we do not have $\delta u_0 = 0$. We need to change C_{AB} into $\hat{C}_{AB}$ in the symplectic potential to ensure covariance.
- We can apply the IPP method and recover the CFR charges, but no covariance.

Bracket and ambiguity in the charge

Time independent central extensions

As we have $(\delta_\xi - \mathscr{L}_\xi)\bar{\theta} = 0$, as for our charge in BMS, we have
$\{\bar{Q}_\xi, \bar{Q}_\chi\}^{BT}|_{S_2} - \{\bar{Q}_\xi, \bar{Q}_\chi\}^{BT}|_{S_1} = \bar{Q}_{[[\xi,\chi]]}|_{S_2} - \bar{Q}_{[[\xi,\chi]]}|_{S_1}$
If $\bar{\theta}$ vanishes at S_1 and S_2 (no news), we have that
$\{\bar{Q}_\xi, \bar{Q}_\chi\}^{BT}|_{S_2} - \{\bar{Q}_\xi, \bar{Q}_\chi\}^{BT}|_{S_1} = \delta_\chi \bar{Q}_\xi|_{S_2} - \delta_\chi \bar{Q}_\xi|_{S_1}$

Ambiguity in the charge

- We can always add terms $X\varepsilon_S$ to the charge such that $d(X\varepsilon_S) = 0$
- Ideally, we would like no central extension at all. In BMS, we got rid of it by imposing a boundary condition at $u \to +\infty$ in order to recover the DS charges. Here, it is unclear if we can do the same by playing with the ambiguity. Indeed, the Dray-Streubel charges vanish in flat spacetime because $Y^A\partial_A$ is a conformal Killing field. It is not true for a superrotation or a superboost.

Thank You