Title: Quantum Prison Break and the Infinitesimal Limit of Quantum Teleportation

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Series: Quantum Foundations

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Abstract: We will present a teleportation protocol which uses the properties of superoscillatory wavefunctions. Then we will briefly introduce a different protocol, the port based teleportation. We will use it to study teleportation over infinitesimally small distances, where the vacuum of a quantum field serves as the source of entanglement. We find that the resulting motion is equivalent to a quantum teleportation-induced Brownian motion. Purifying the interactions, from measurements to unitary operations, leads to motion described by Schrodinger's equation. Therefore, we find that the notions of teleportation and continuous quantum and classical motion are unified in this sense.

Zoom link https://pitp.zoom.us/j/93504138534?pwd=VUFPSXpRd2g0TVNxVTNlY3Rxa1ZjUT09

Motion as the Infinitesimal Limit of Quantum Teleportation

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Teleportation = entanglement resource + measurement + classical. communication (+ unitary corrections)

Alice | **Bob**

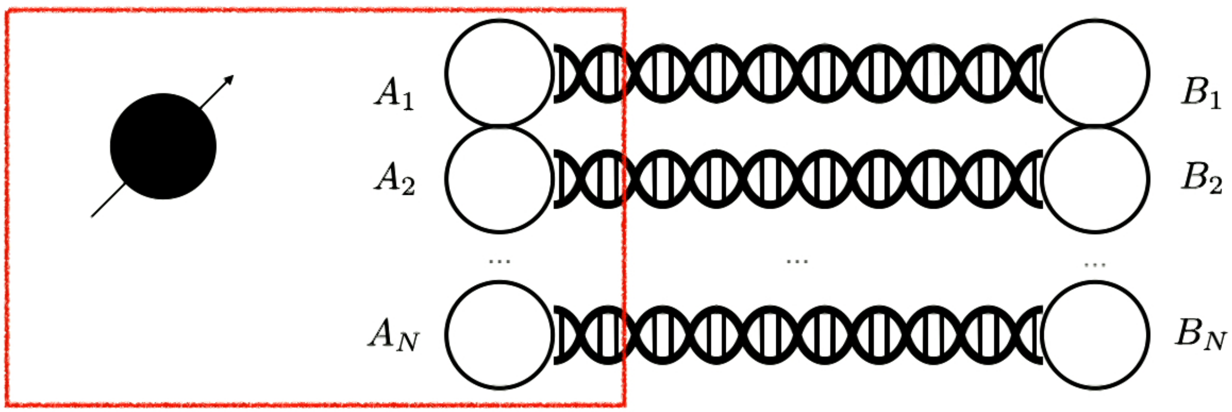
unknown state

$$|\psi\rangle$$

N shared Bell pairs, e.g.

$$\otimes_i \left|\phi^+\right\rangle_{A_i B_i} = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

“Lossy” measurement:
one of N states



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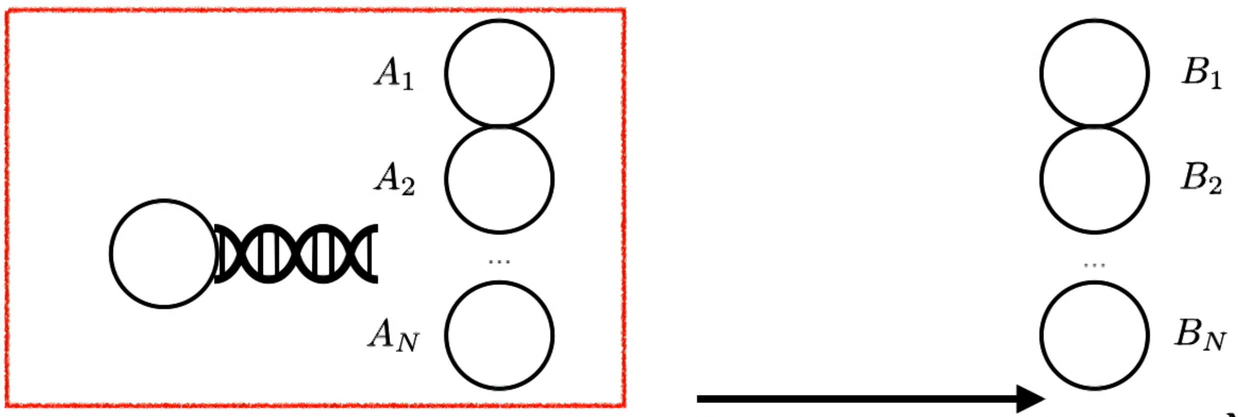
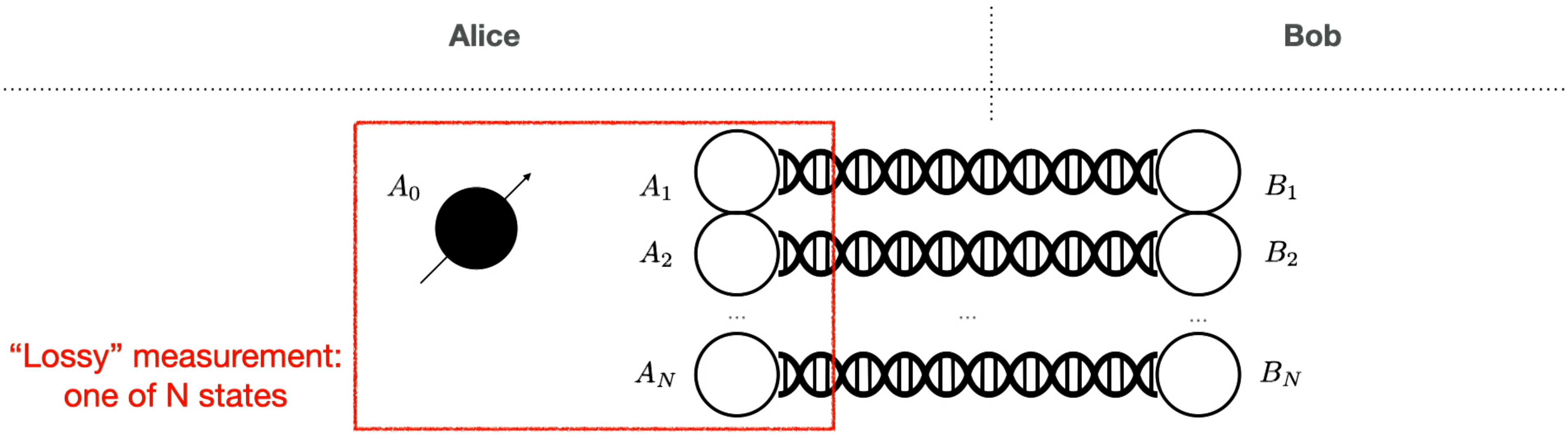


Classical communication (2^N bits) ->which i?

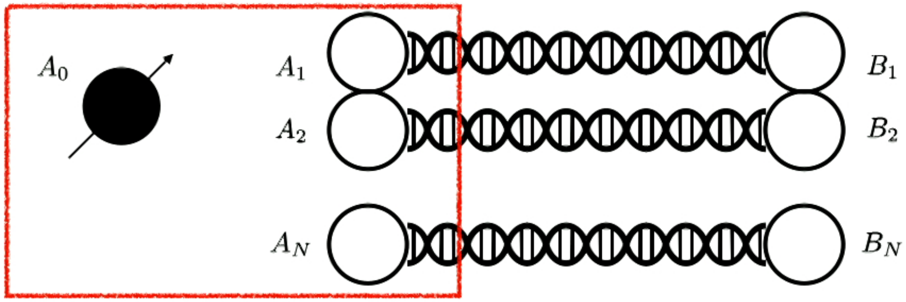
Measurement details



POVM operators: Projectors onto entangled states: $\left|\phi^+\right\rangle_{A_0 A_i} = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$

$$\Pi_i = \left|\phi^+\right\rangle_{A_0 A_i}\left\langle\phi^+\right|$$

Measurement details



The projectors are normalized: $P_i = \rho^{-1/2}\Pi_i\rho^{-1/2}$

'Normalization' operator: $\rho = \sum_i \Pi_i$ -> Pretty Good Measurement

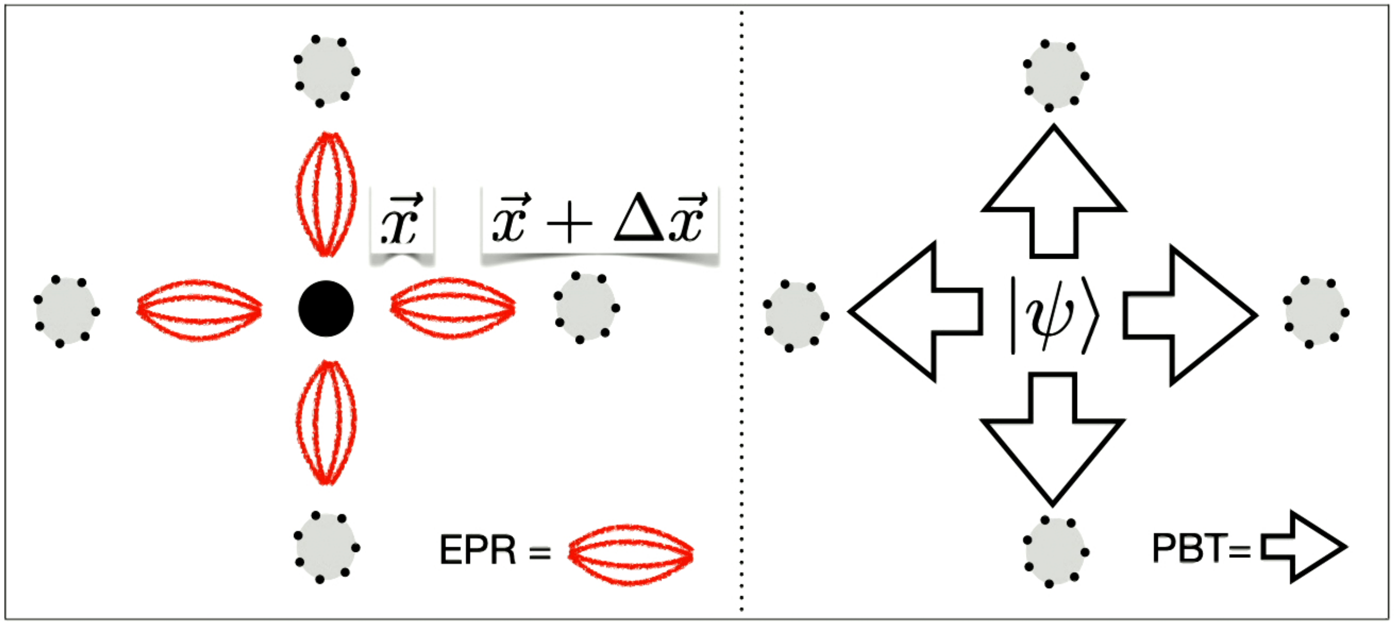
These projectors don't sum up to one, so to each we add 1/N $\Delta = \mathbf{1} - \sum_i \Pi_i$

"Lossy measurement"

Probabilistic protocol: loss leads to sometimes fail.
Deterministic protocol: loss leads to less than perfect fidelity.

Infinitesimal Limit of Quantum Teleportation

Infinitesimal Limit of Quantum Teleportation



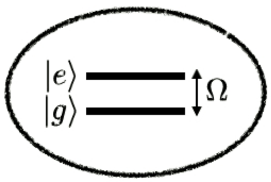
Entanglement Resource in the Quantum Field

- The vacuum of a free scalar quantum field is an entangled state.
- Can be quickly seen in the Klein-Gordon eq. $$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + \frac{m^2c^2}{\hbar^2}\right)\psi = 0$$
- Generically, the entanglement entropy diverges at short distances, and falls off quickly as the distance between two points increases.
- We model this entanglement as N shared Bell pairs at very short distances.

Measurements

- Can be modelled e.g. by the interactions of an Unruh-DeWitt detector with a scalar quantum field.

- UDW detector:



$$H_0^{(D)} = \Omega\, \hat{\sigma}_z$$

- Scalar quantum field:

$$H_0^{(F)} = \int d^3k\, \omega_{\mathbf{k}}\, \hat{a}_{\mathbf{k}}^{\dagger} \hat{a}_{\mathbf{k}}$$

- Interaction Hamiltonian: $H_{int} = G\, \hat{\sigma}_x(\tau) \otimes \hat{\phi}(t(\tau), \mathbf{x}(\tau))$

- We are looking at the infinitesimal limit: brief interaction

$$U = \mathbf{1} + \frac{i\Delta t}{\hbar} g\, \hat{X} \otimes \int dk \left(\hat{a}_k e^{ik^\mu x_\mu} + \hat{a}_k^\dagger e^{-ik^\mu x_\mu} \right)$$

- The contains interactions with all modes of the field -> measurements

The resulting Brownian motion

- Assuming only finite entanglement resource in the quantum field: N Bell pairs
- Naturally-induced port-based entanglement via interactions with the quantum field, probability of fail:

$$p_f = \sqrt{\frac{d}{N-1}}c_d + o(\frac{1}{\sqrt{N}})$$

d Size of the teleported state, for qubits =2

N Number of entangled pairs

PBT, EPR pairs as the entanglement resource

- The resulting motion is a massive Brownian motion, with mass related to the probability of success:

$$\frac{1}{p_s} - 1 = \frac{m^2 l^2}{2D}$$

l UV cutoff length

D Dimension of space

The resulting Brownian motion

- Assuming N is very large: success probability of PB teleportation is close to 1.
- In this limit, we have a diffusion equation:

$$\dot{p}\left(\vec{x}, t\right) = \alpha^2 \nabla^2 p\left(\vec{x}, t\right)$$

- Where the diffusion constant arising from infinitesimal PB teleportation is:

$$\alpha^2 \propto \sqrt{p_f}$$

- i.e.

$$\alpha^2 \propto \frac{d^a}{N^b}$$

Purification of Brownian motion

Quantum random walks

- How do we go beyond the classical Brownian motion?
- Recall that we modelled the port-based teleportation with measurements.
- -> purify the measurements into unitary interactions.
- Resulting motion is due to a quantum random walk.

Purification of Brownian motion

- The resulting matching of the diffusion constant arising from PB teleportation, to the one in Schrödinger's equation:

$$\hbar \propto \frac{d^a}{N^b}$$

- N is a property of the quantum field's vacuum, while d is a property of the system being teleported.

$$\hbar \propto \frac{1}{N^b}$$

- Reduced Planck constant as a measure of amount of entanglement in the vacuum.

Discussion & Outlook

- Motion due to continuous differential equations unified with teleportation
    - via the infinitesimal limit of port-based teleportation
- $\hbar$'s connection to the amount of entanglement comes from the Born rule, not from details of interactions between matter and quantum field
- Backreaction in general relativity - entanglement in the vacuum affects motion. Motion affects entanglement - quantifiable by calculations of recycled entanglement resource in PBT (*)
- Relation to Nelson's derivation of the Schrodinger equation - teleportation as the source of diffusion

(*) M. Studzinski, M. Mozrzymas, and P. Kopszak, Journal of Physics A: Mathematical and Theoretical 55, 375302 (2022)