

Title: The recent gravitational wave observation by pulsar timing arrays and primordial black holes: the importance of non-gaussianities -

VIRTUAL

Speakers: Antonio Iovino

Series: Particle Physics

Date: November 07, 2023 - 1:00 PM

URL: <https://pirsa.org/23110050>

Abstract: The recent data releases by multiple pulsar timing array (PTA) experiments show evidence for Hellings-Downs angular correlations indicating that the observed stochastic common spectrum can be interpreted as a stochastic gravitational wave background. We study whether the signal may originate from gravitational waves induced by high-amplitude primordial curvature perturbations. Such large perturbations may be accompanied by the generation of a sizable primordial black hole (PBH) abundance. We discuss in which scenarios the inclusion of non-Gaussianities in the computation of the abundance can lead to a signal compatible with the PTA experiments without overproducing PBHs.

Zoom link <https://pitp.zoom.us/j/95261778825?pwd=QndRd0xQVFpNVzk0VXpRUkNqR1JXZz09>

The recent gravitational wave observation by
pulsar timing arrays and primordial black holes
the importance of non-gaussianities.

Antonio Junior Iovino



SAPIENZA
UNIVERSITÀ DI ROMA



Primordial Black Holes: outline presentation

REVIEW: A. M. Green and B. J. Kavanagh.— arXiv:2007.10722

PART 1) ABUNDANCE of PBH: The role of NGs

arXiv:2211.01728 (Published on PRD)

G.Ferrante, G.Franciolini, A.J.I., A.Urbano

PART 2) GRAVITATIONAL WAVES: PBH as possible explanation of PTA

arXiv:2306.17149 (To appear on PRL)

G.Franciolini, A.J.I., H. Veermae, V. Vaskonen

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Black Holes: Astro vs Primordial

Astro BH: forms from the gravitational collapse of a star. We know they exist.

$$M > \mathcal{O}(1) M_{\odot}$$

PBH: forms in the Early universe through a gravitational collapse of large over densities in the primordial density contrast field δ .

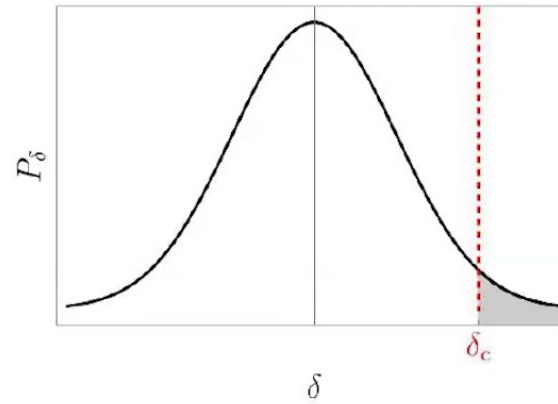
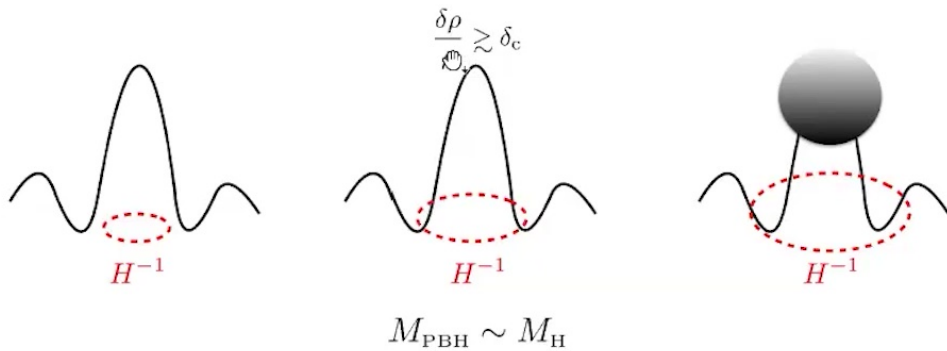
They can still be around as long as they do not evaporate within the age of the universe

$$M > 10^{-18} M_{\odot}$$

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Abundance of PBHs

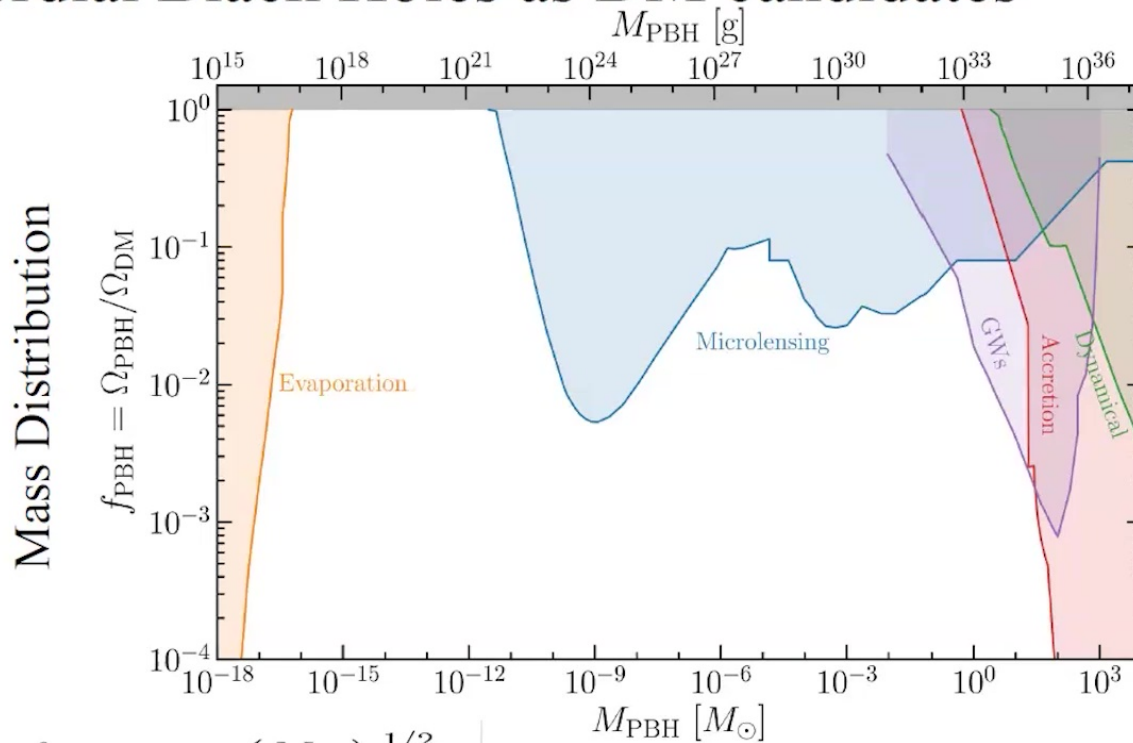


$$\beta = \int_{\delta_c}^{\infty} \mathcal{K}(\delta - \delta_c)^\gamma P_\delta(\delta) d\delta$$

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Primordial Black Holes as DM candidates



$$\Omega_{\text{PBH}} = \int d \log M_H \left(\frac{M_{\text{eq}}}{M_H} \right)^{1/2} \beta$$

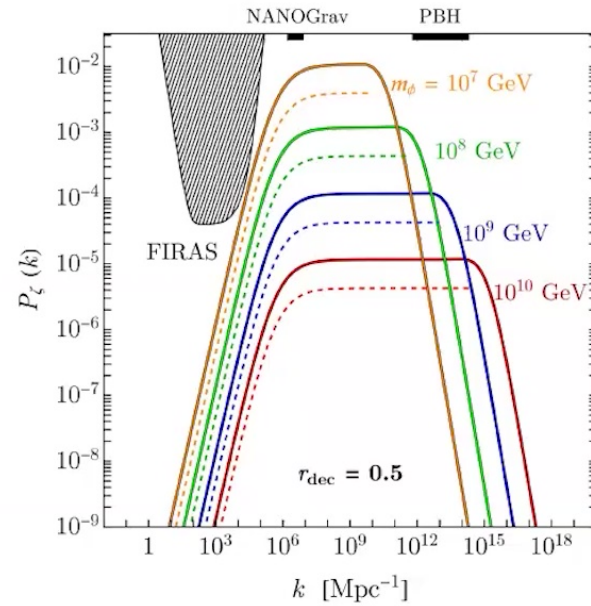
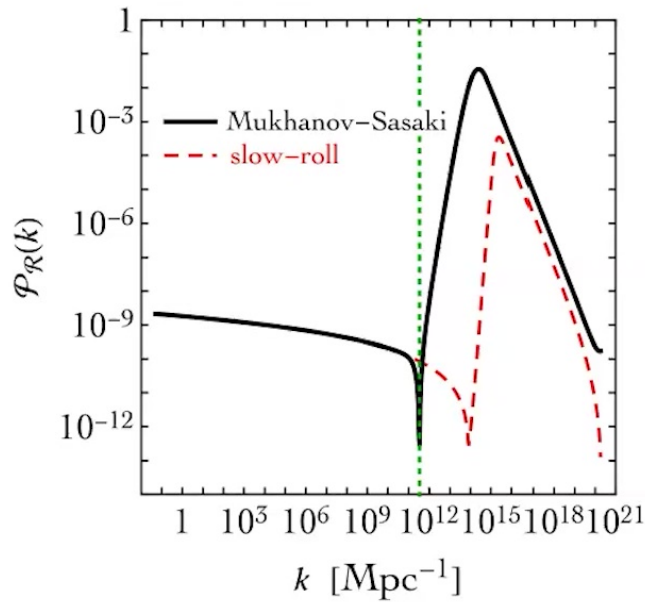
PBH as Dark Matter:

- USR models
- Hybrid inflation
- Curvaton field
- exotic formation mechanisms (bubble collisions and so on)
- And etc etc...



Primordial Black Holes as DM candidates

What we can compute during inflation is the curvature perturbation field ζ (or R).



PBH as Dark Matter:

- USR models
- Hybrid inflation
- Curvaton field
- exotic formation mechanisms (bubble collisions and so on)
- And etc etc...

$$M_H \simeq 17M_\odot \left(\frac{g_\star}{10.75} \right)^{-1/6} \left(\frac{k/\kappa}{10^6 \text{Mpc}^{-1}} \right)^{-2}$$



Abundance of PBHs: The role of Non-Gaussianities (NG).

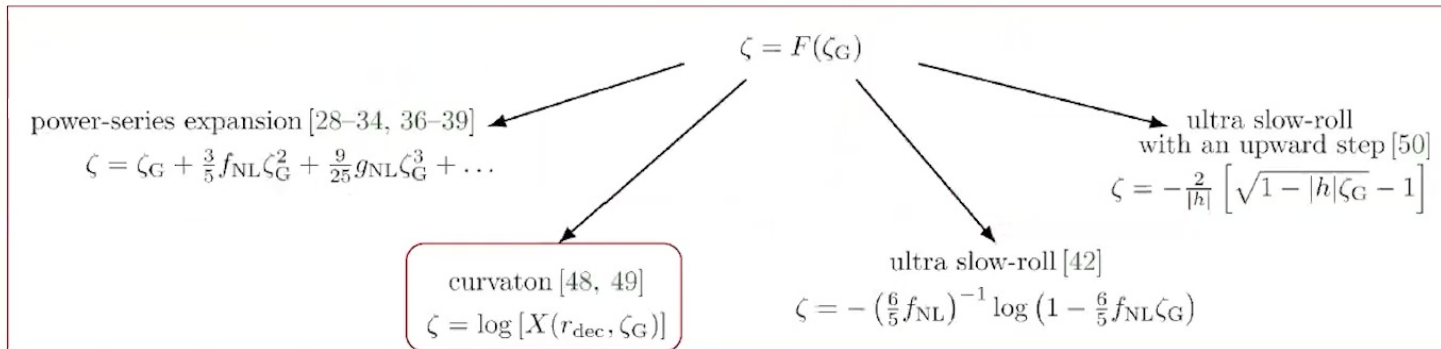
An exact formalism for the computation of PBHs mass fraction abundance NGs in the curvature perturbation field ζ :

NON-LINEARITIES (NL)

$$\delta(\vec{x}, t) = -\frac{2}{3} \Phi \left(\frac{1}{aH} \right)^2 e^{-2\zeta(\vec{x})} \left[\nabla^2 \zeta(\vec{x}) + \frac{1}{2} \partial_i \zeta(\vec{x}) \partial_i \zeta(\vec{x}) \right]$$

T. Harada, C. M. Yoo, T. Nakama and Y. Koga, - arXiv:1503.03934

PRIMORDIAL NG IN $\zeta = F(\zeta_G)$



$$r_{\text{dec}} \equiv \frac{3\rho_\phi}{3\rho_\phi + 4\rho_\gamma} \Big|_{\text{curvaton decay}}$$

7



Abundance of PBHs: The role of Non-Gaussianities (NG).

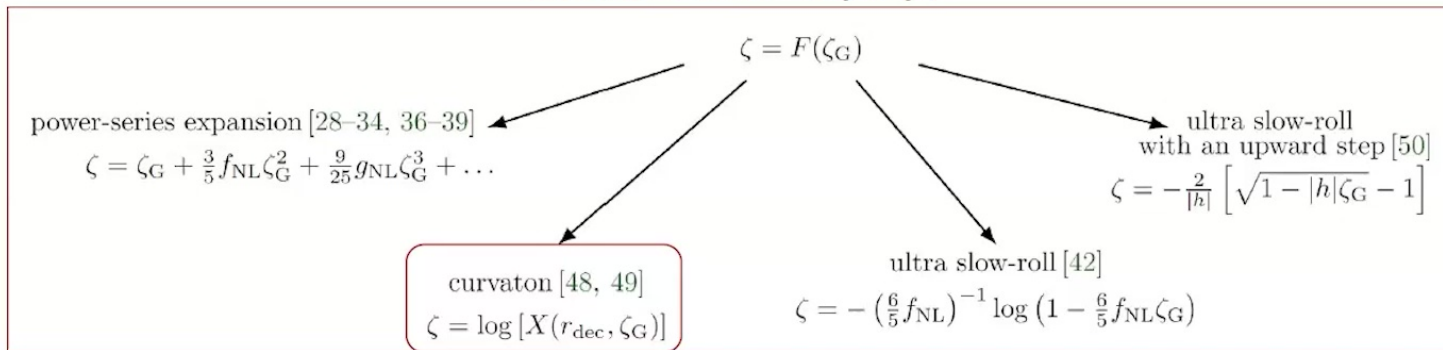
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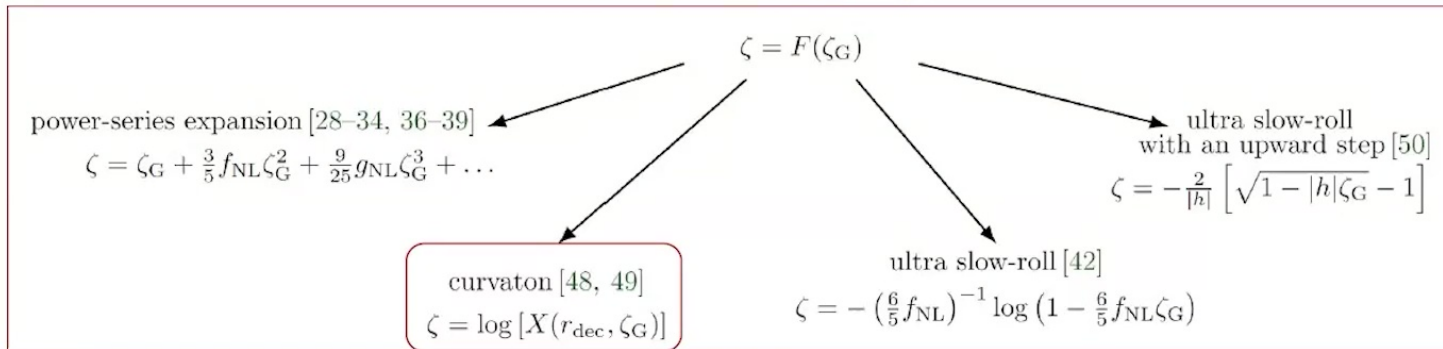
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PRIMORDIAL NG IN $\zeta = F(\zeta_G)$



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Mathematical formulation

By integrating δ over the radial coordinate r we get the compaction function C

$$\mathcal{C}(r) = -2\Phi r \zeta'(r) \left[1 + \frac{r}{2} \zeta'(r) \right] = \mathcal{C}_1(r) - \frac{1}{4\Phi} \mathcal{C}_1(r)^2, \quad \mathcal{C}_1(r) := -2\Phi r \zeta'(r)$$

In the presence of NG C_l takes the form

$$\mathcal{C}_1(r) = -2\Phi r \zeta'_G(r) \frac{dF}{d\zeta_G} = \mathcal{C}_G(r) \frac{dF}{d\zeta_G}, \quad \text{with } \mathcal{C}_G(r) := -2\Phi r \zeta'_G(r)$$

From the two-dimensional joint PDF of ζ_G and C_G , called P_G

Later on confirmed
also by
A.Gow et al
atXiv:2211.08348



NG PBH mass fraction adopting threshold statistics on the compaction function

$$\beta_{\text{NG}} = \int_{\mathcal{D}} \mathcal{K}(\mathcal{C} - \mathcal{C}_{\text{th}})^\gamma P_G(\mathcal{C}_G, \zeta_G) d\mathcal{C}_G d\zeta_G, \quad (56)$$

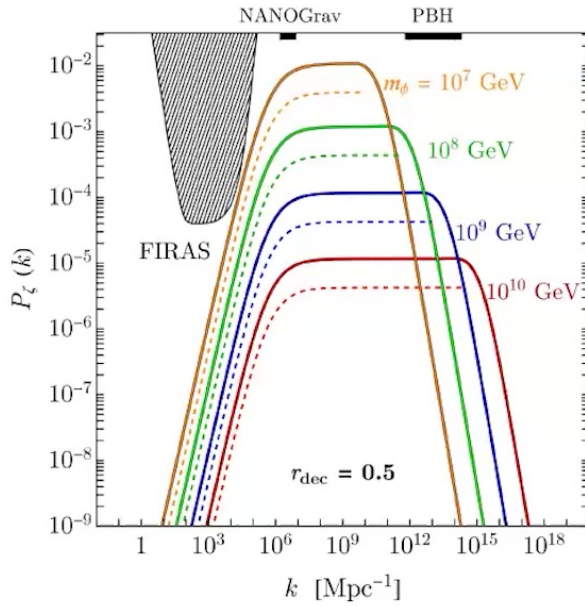
$$P_G(\mathcal{C}_G, \zeta_G) = \frac{1}{(2\pi)\sigma_c\sigma_r\sqrt{1-\gamma_{cr}^2}} \exp\left(-\frac{\zeta_G^2}{2\sigma_r^2}\right) \exp\left[-\frac{1}{2(1-\gamma_{cr}^2)} \left(\frac{\mathcal{C}_G}{\sigma_c} - \frac{\gamma_{cr}\zeta_G}{\sigma_r}\right)^2\right], \quad (57)$$

$$\mathcal{D} = \{\mathcal{C}_G, \zeta_G \in \mathbb{R} : \mathcal{C}(\mathcal{C}_G, \zeta_G) > \mathcal{C}_{\text{th}} \wedge \mathcal{C}_1(\mathcal{C}_G, \zeta_G) < 2\Phi\}, \quad (58)$$

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Mathematical formulation



$$\langle \mathcal{C}_G \mathcal{C}_G \rangle = \sigma_c^2 = \frac{4\Phi^2}{9} \int_0^\infty \frac{dk}{k} (kr_m)^4 W^2(k, r_m) T^2(k, r_m) P_\zeta(k),$$

$$\langle \mathcal{C}_G \zeta_G \rangle = \sigma_{cr}^2 = \frac{2\Phi}{3} \int_0^\infty \frac{dk}{k} (kr_m)^2 W(k, r_m) W_s(k, r_m) T^2(k, r_m) P_\zeta(k),$$

$$\langle \zeta_G \zeta_G \rangle = \sigma_r^2 = \int_0^\infty \frac{dk}{k} W_s^2(k, r_m) T^2(k, r_m) P_\zeta(k),$$

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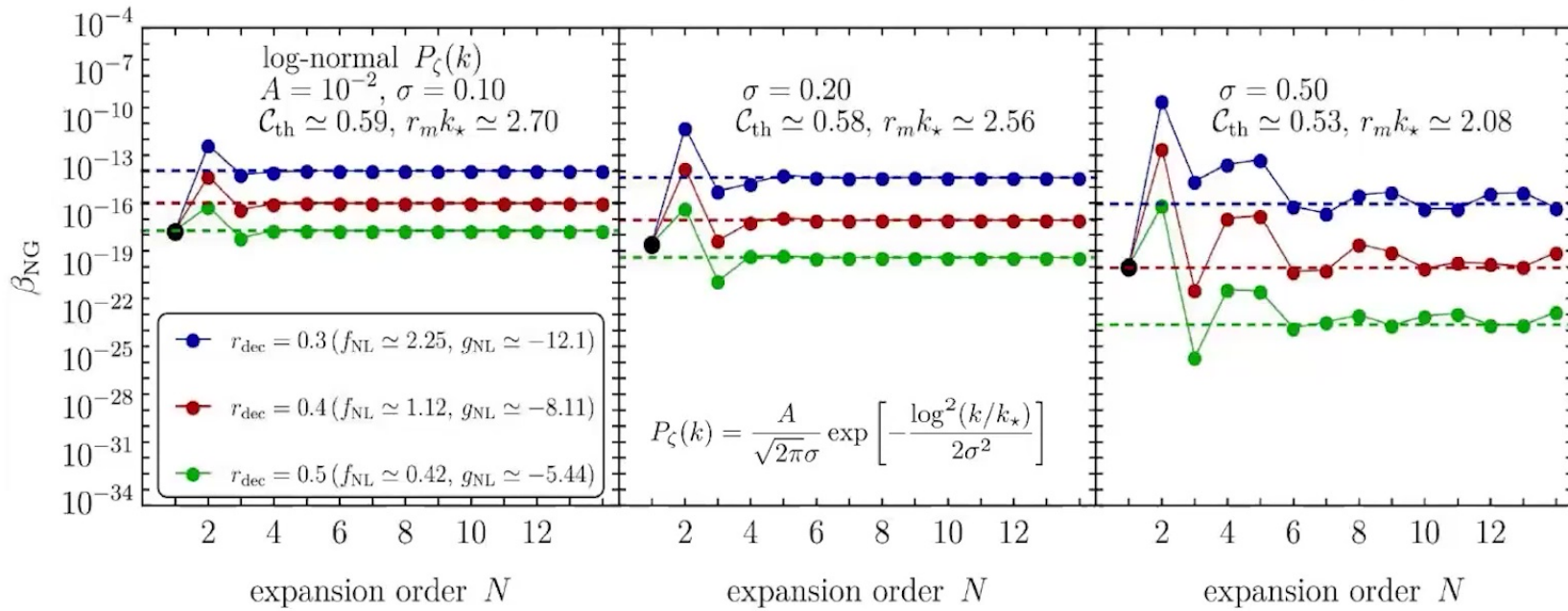


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Application to the curvaton model

Failure of the perturbative approach (Narrow)

- - - - $\zeta = \log[X(r_{\text{dec}}, \zeta_G)]$
 • $\zeta_N = \sum_{n=1}^N c_n(r_{\text{dec}}) \zeta_G^n$

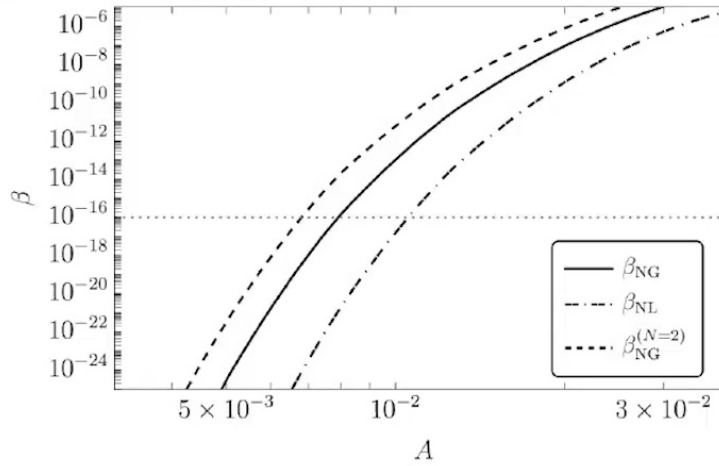
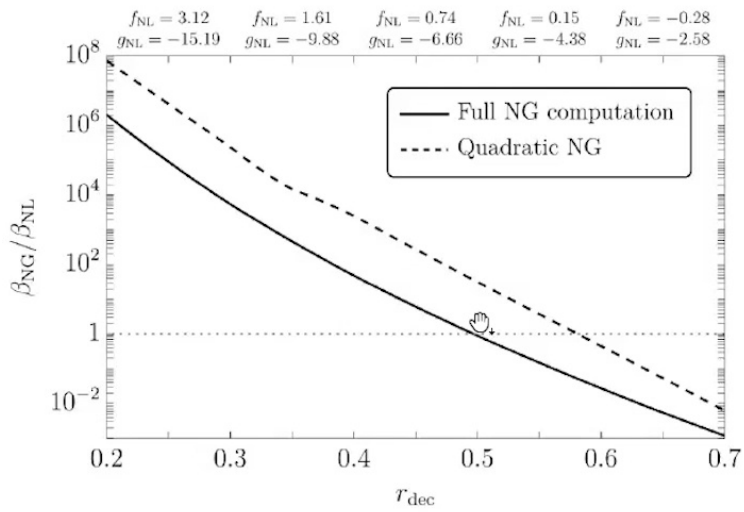


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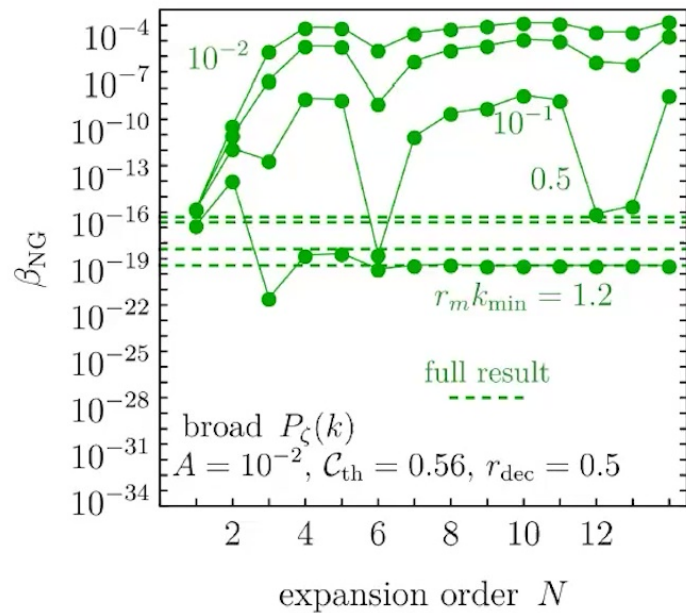
Application to the curvaton model

Quadratic app. overestimates the abundance



Application to the curvaton model

Failure of the perturbative approach (Broad)



$$P_\zeta(k) = A \Theta(k - k_{min}) \Theta(k_{max} - k)$$

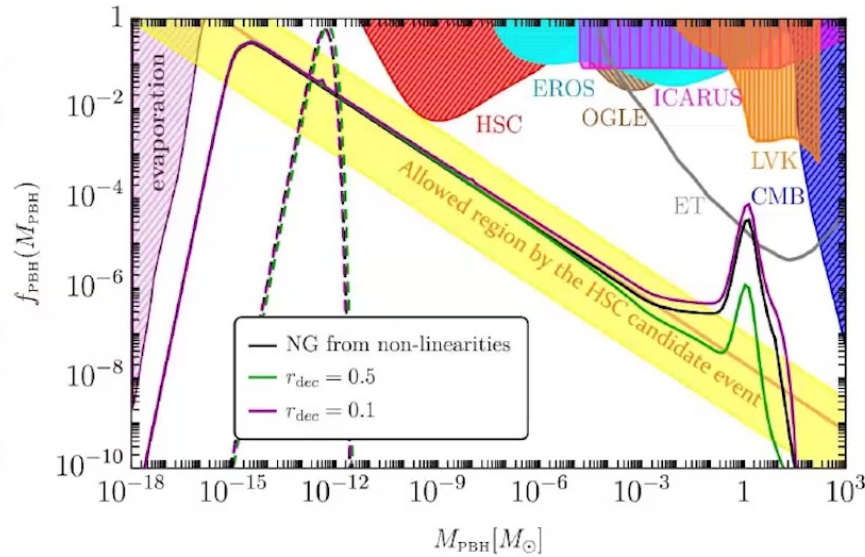
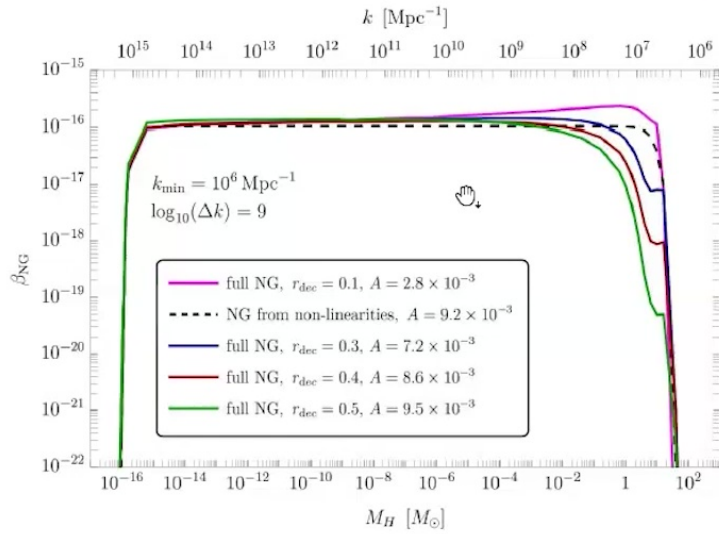
For a broad Power spectrum the power-series expansion is simply wrong and one is forced to use the full result NG.



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Application to the curvaton model (2)

Breaking of M_H -Independence



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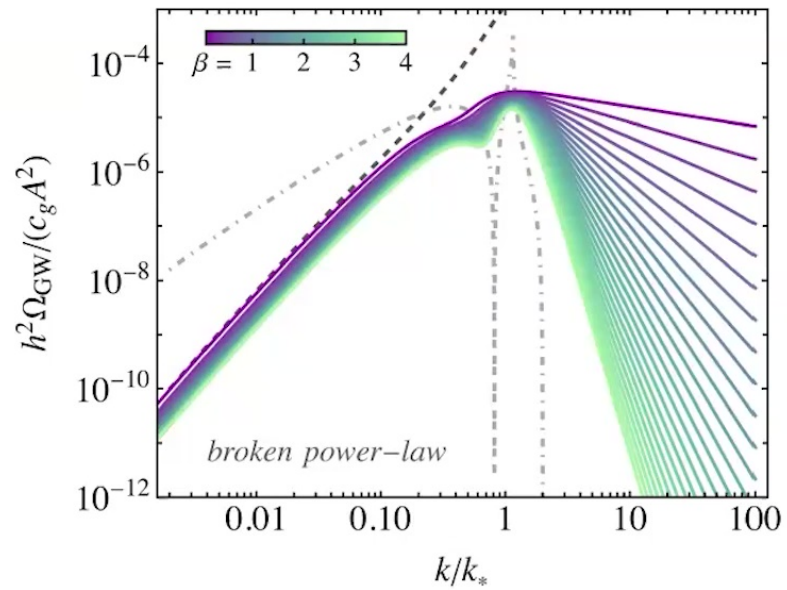
We need another observable: The induced Gravitational waves

PBH and SGWB

SGWB are produced by a second-order effect when scalar perturbations re-enter the horizon.

$$h^2 \Omega_{\text{GW}}(k) = \frac{h^2 \Omega_r}{24} \left(\frac{g_*}{g_*^0} \right) \left(\frac{g_{*s}}{g_{*s}^0} \right)^{-\frac{4}{3}} \mathcal{P}_h(k)$$

$$\mathcal{P}_h(k) \propto \mathcal{P}_\zeta^2(k)$$



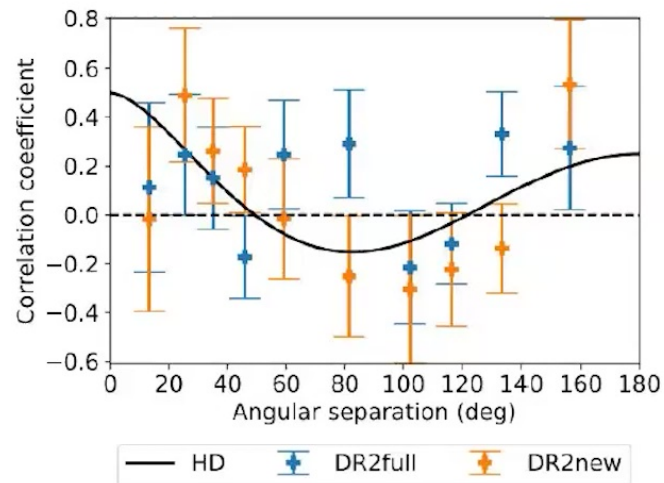
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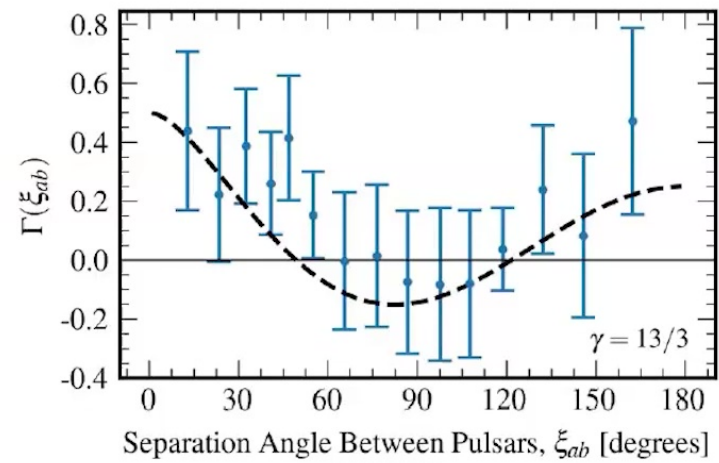
PBH and SGWB

Several PTA collaborations show that the correlations follow the Hellings–Downs pattern expected for a stochastic gravitational-wave background.

EPTA – arXiv:2306.16214



NANOGrav – arXiv:2306.16213
arXiv:2306.16219



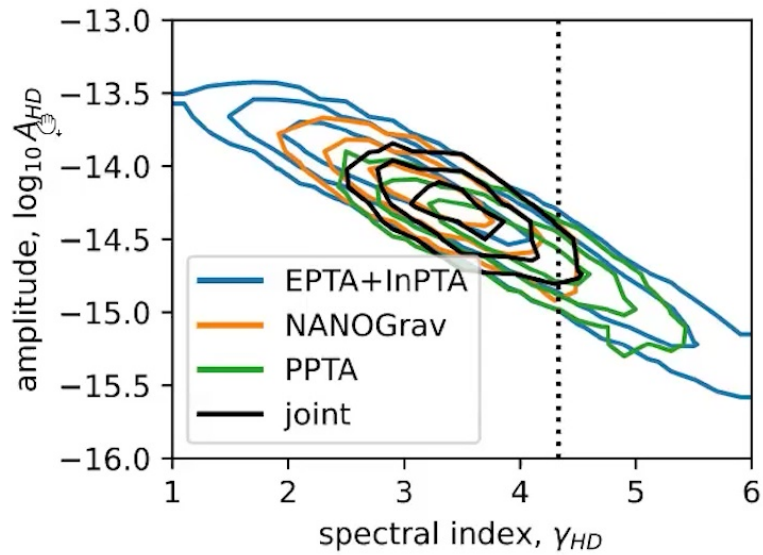
16



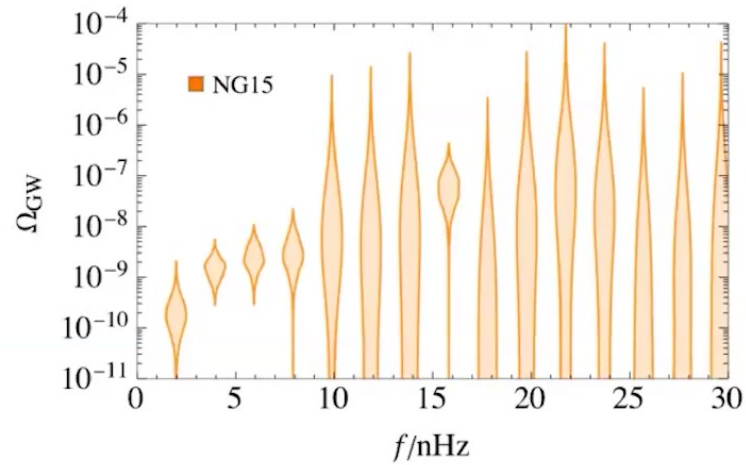
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IPTA – arXiv:2309.00693



NANOGrav – arXiv:2306.16213
arXiv:2306.16219



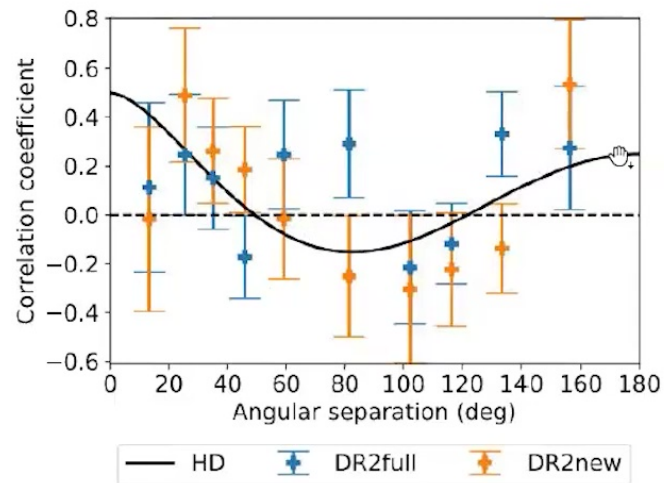
17



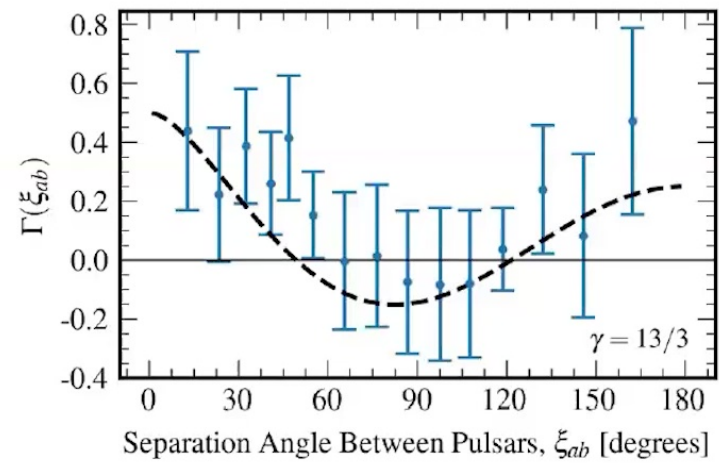
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PBH and SGWB

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Log-likelihood analysis

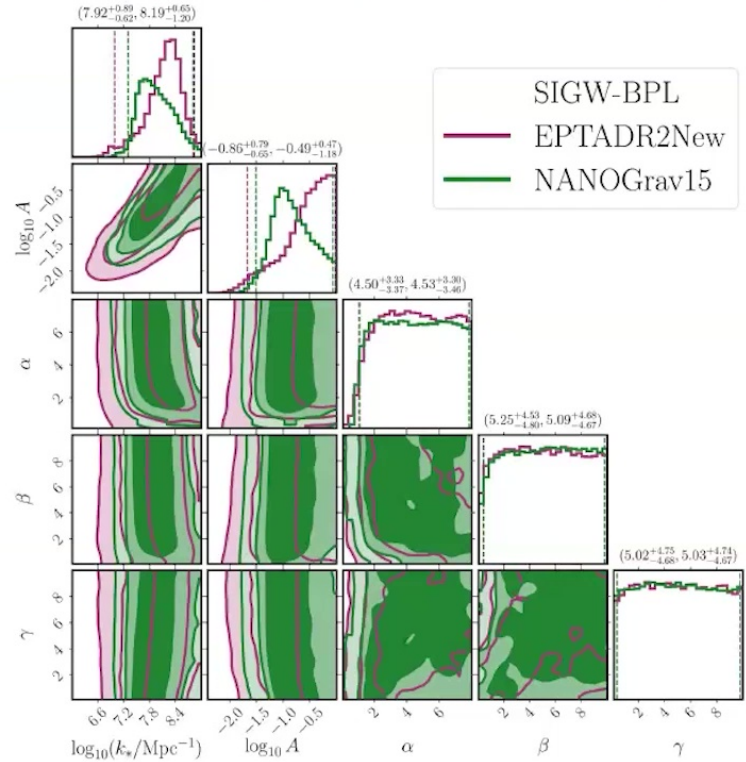
Fitting the posterior distributions

$$\mathcal{P}_\zeta^{\text{BPL}}(k) = A \frac{(\alpha + \beta \ln^2 k)^{\gamma}}{\left(\beta (k/k_*)^{-\alpha/\gamma} + \alpha (k/k_*)^{\beta/\gamma}\right)^{\gamma}}$$

$$\mathcal{P}_\zeta^{\text{LN}}(k) = \frac{A}{\sqrt{2\pi}\Delta} \exp\left(-\frac{1}{2\Delta^2} \ln^2(k/k_*)\right)$$

The causality tail is not good:

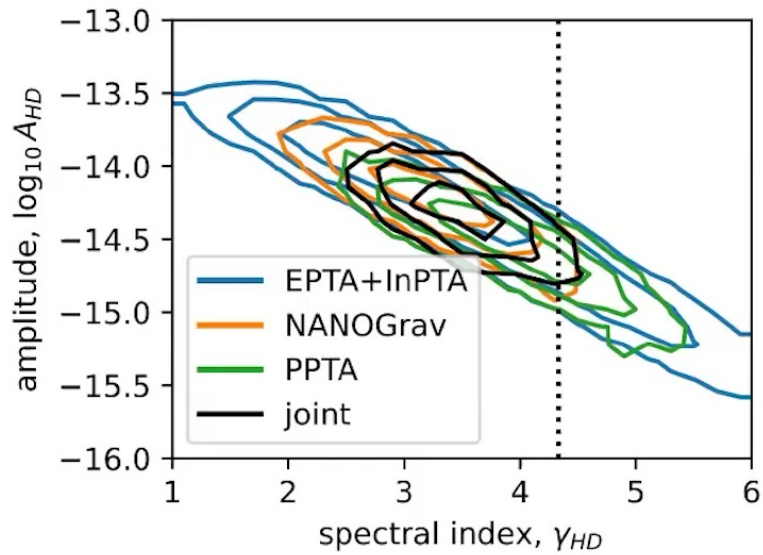
$$\Omega_{\text{GW}}(k \ll k_*) \propto k^3 (1 + \tilde{A} \ln^2(k/\tilde{k}))$$



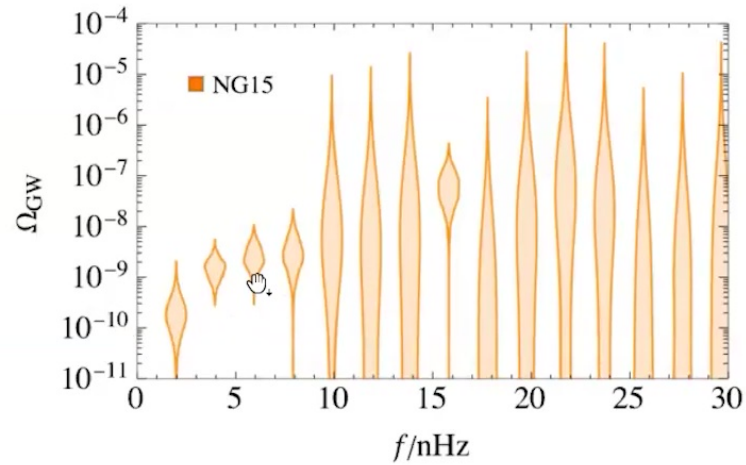
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PBH and SGWB

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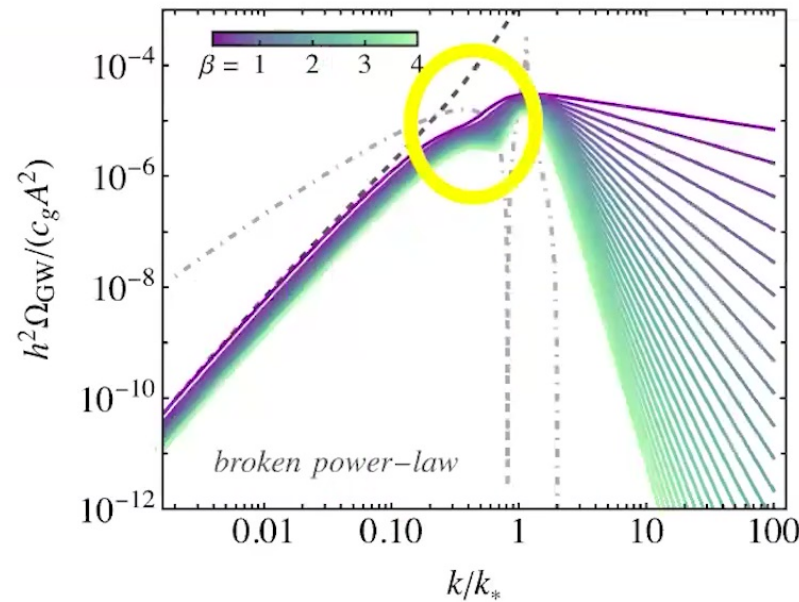
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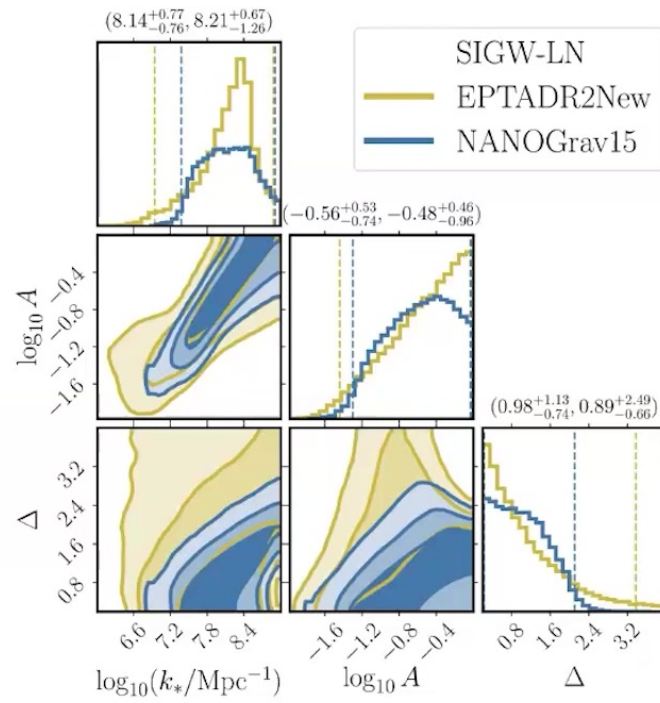
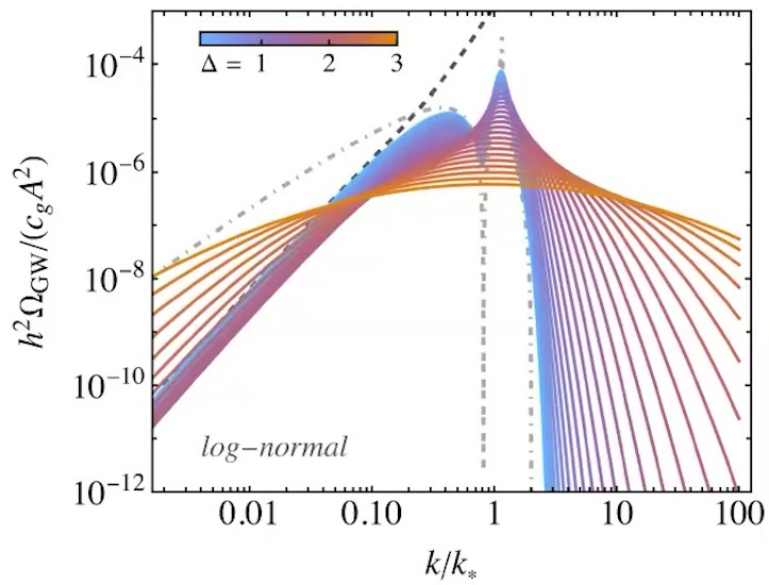
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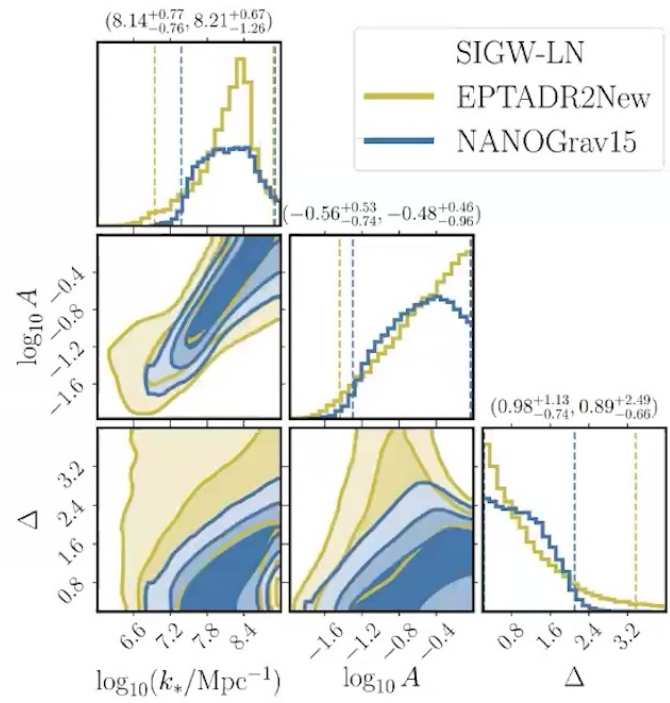
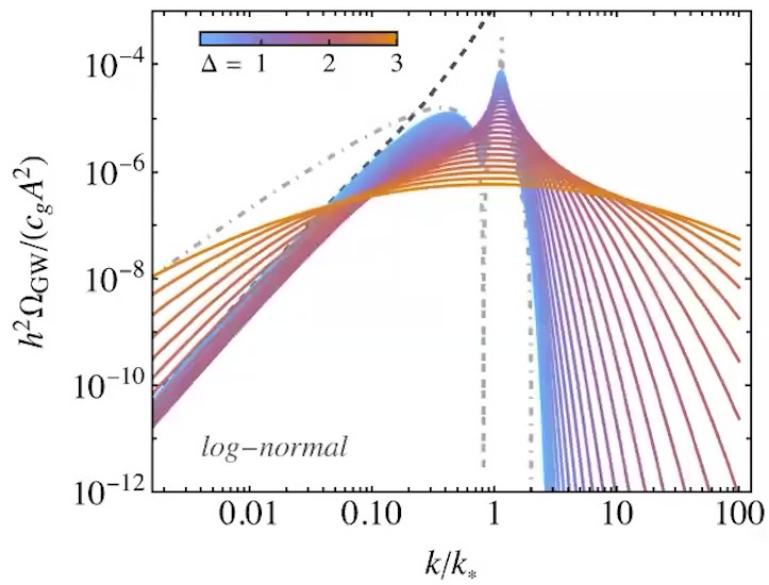
The casualty tail is not good:

$$\Omega_{\text{GW}}(k \ll k_*) \propto k^3 (1 + \tilde{A} \ln^2(k/\tilde{k}))$$



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Improvement respect to NANOGrav analysis.

NANOGrav collaboration
arXiv:2306.16219

Power spectrum <> Abundance <> GWs

- Non-Gaussianities in the abundance.
- Dependency of the PBH formation parameters on the PS shape.
- QCD impact on threshold.

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NGs in the abundance: Cases under consideration

NON-LINEARITIES (NL)

$$\delta(\vec{x}, t) = -\frac{2}{3}\Phi\left(\frac{1}{aH}\right)^2 e^{-2\zeta(\vec{x})} \left[\nabla^2 \zeta(\vec{x}) + \frac{1}{2} \partial_i \zeta(\vec{x}) \partial_i \zeta(\vec{x}) \right]$$

$$\delta(\vec{x}, t) = -\frac{4}{9a^2 H^2} \nabla^2 \zeta(\vec{x})$$

PRIMORDIAL NG IN $\zeta=F(\zeta_G)$

$$\zeta = \log [X(r_{\text{dec}}, \zeta_G)]$$

curvaton case

$$\zeta = -\frac{2}{\beta} \log \left(1 - \frac{\beta}{2} \zeta_G \right)$$

Inflection-point (USR) case

$$\zeta = \zeta_G + \frac{3}{5} f_{\text{NL}} \zeta_G^2$$

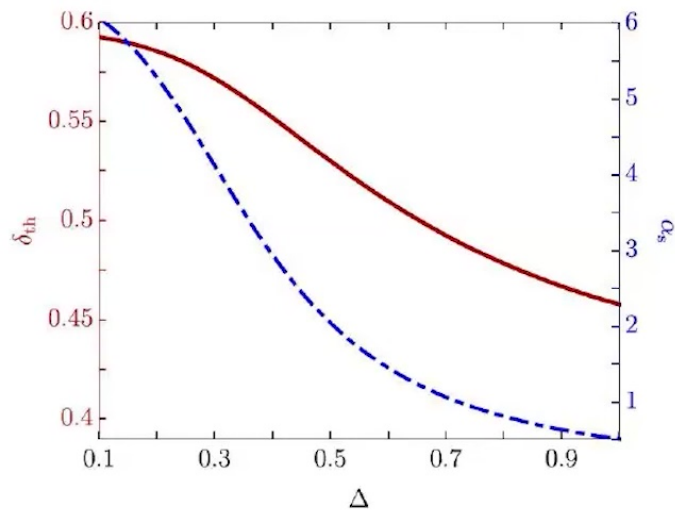
Quadratic approx.

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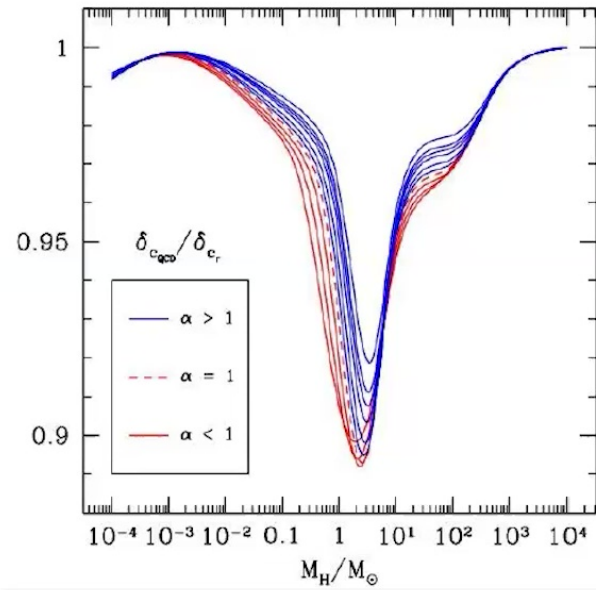
Abundance of PBHs: Shape dependencies

I. Musco, V. De Luca, G. Franciolini, A. Riotto. – arXiv:2011.03014



QCD phase transitions

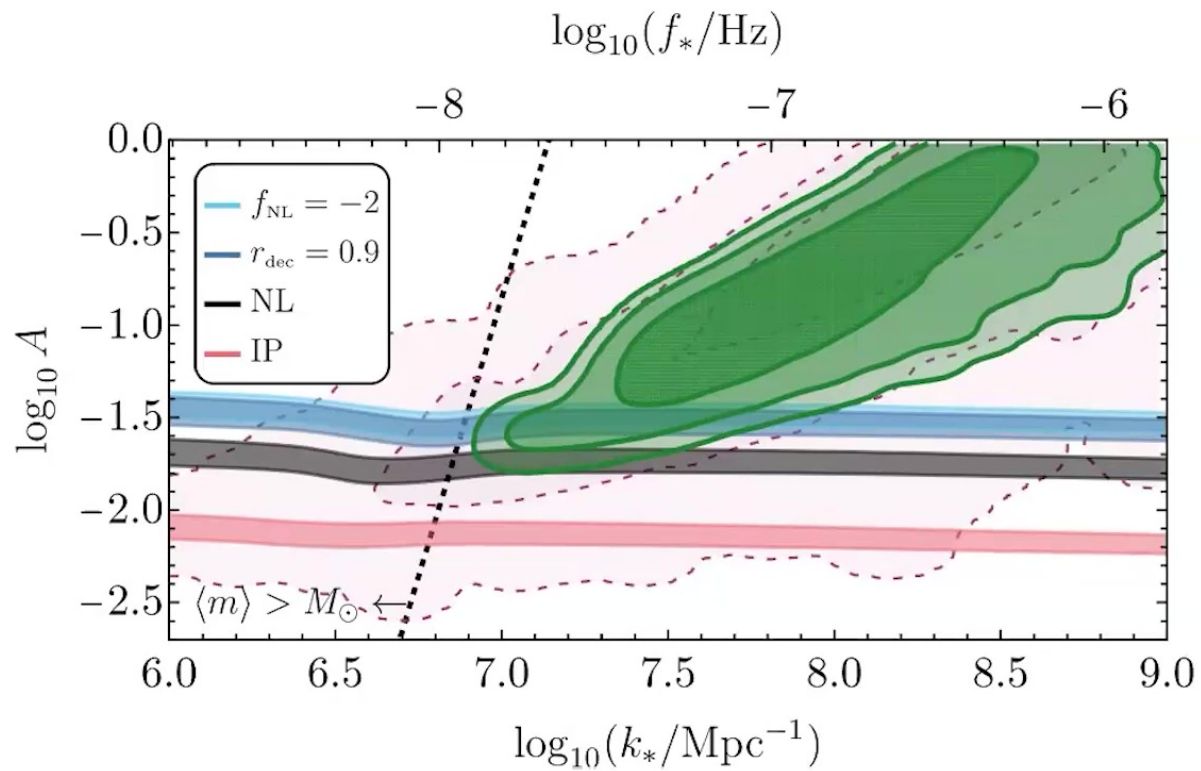
I. Musco, K. Jedamzik, S. Young. – arXiv:2303.07980



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Tension between NANOGrav and PBHs



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Conclusions

- *Fundamental to take into account both kind of NGs in the computation for the abundance.*
- *Negative NGs to alleviate the tension between PTA and PBH overproduction.*

A potential issue

Threshold values maybe are not correct? Different super-horizon threshold conditions may lead to an overestimation of the abundance, due to non-linear effects not included in the linear transfer function. *Next step: we need a new prescription.*

V. De Luca, A. Kehagias, A. Riotto – arXiv:2307.13633

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