Title: Black holes in cosmological settings: thermodynamics, semi-classical models, and applications - VIRTUAL

Speakers: Filip Simovic

Series: Cosmology & Gravitation

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Abstract: Black hole thermodynamics plays a central role in the theoretical landscape, acting as both synthesizer and sieve for concepts in field theory, quantum gravity, information theory, and more. While its formulation and applications are well understood in asymptotically flat and anti-de Sitter spaces, significant difficulties arise when generalizing to cosmological settings. In this talk, I will discuss how the thermodynamic properties of black holes can be understood in such scenarios through the Euclidean path integral. I will also discuss black holes in a more generalized, semi-classical setting, and suggest a consistent framework for describing a wide variety of dynamical black hole models and ultracompact objects, with consequences for horizon formation, back-reaction, and the growth of astrophysical black holes.

Zoom link https://pitp.zoom.us/j/97647270696?pwd=bXhFYWYyYUlrTEUyVXhtOUNDakhhQT09

Black holes in cosmological settings:

Thermodynamics, semi-classical models, and applications.

Fil Simovic

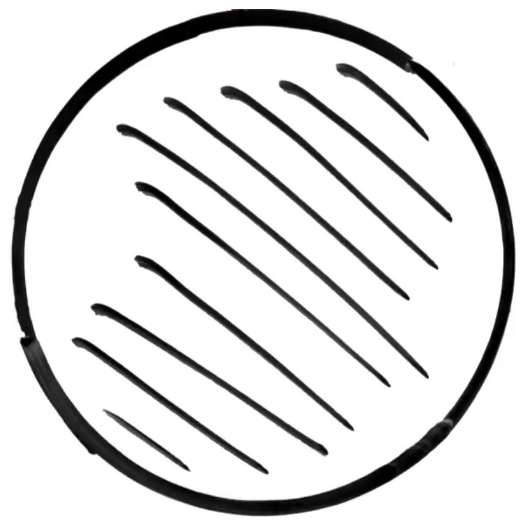
Nov. 8th, Perimeter Institute for Theoretical Physics, Waterloo, Canada

MACQUARIE University
Research Centre for Astronomy, Astrophysics & Astrophotonics

Outline

Part I

- Black hole thermodynamics
- Phase transitions and holography
- Generalizing to de Sitter

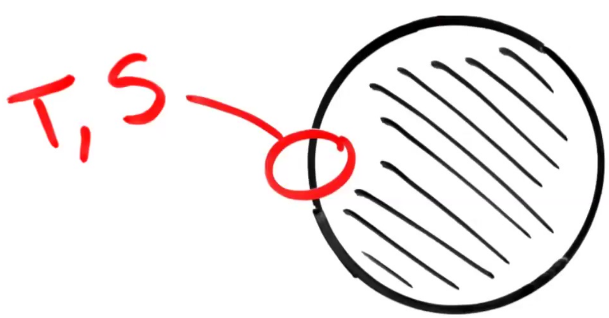


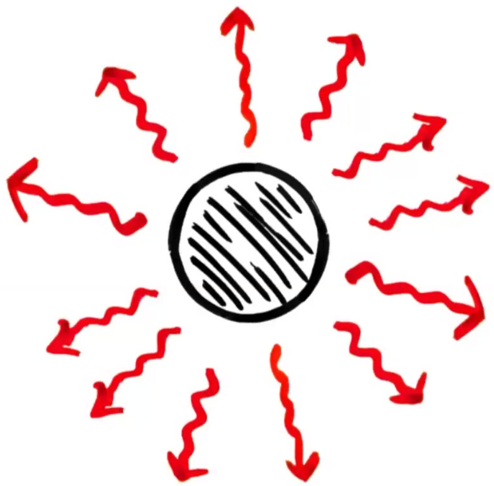
Part II

- Semi-classical black holes
- Viability of dynamical models
- Observational signatures?

Why black holes?

- Black holes are one of the few objects in the universe where curved-space QFT and quantum gravity effects manifest.
- They appear to have thermodynamic properties:





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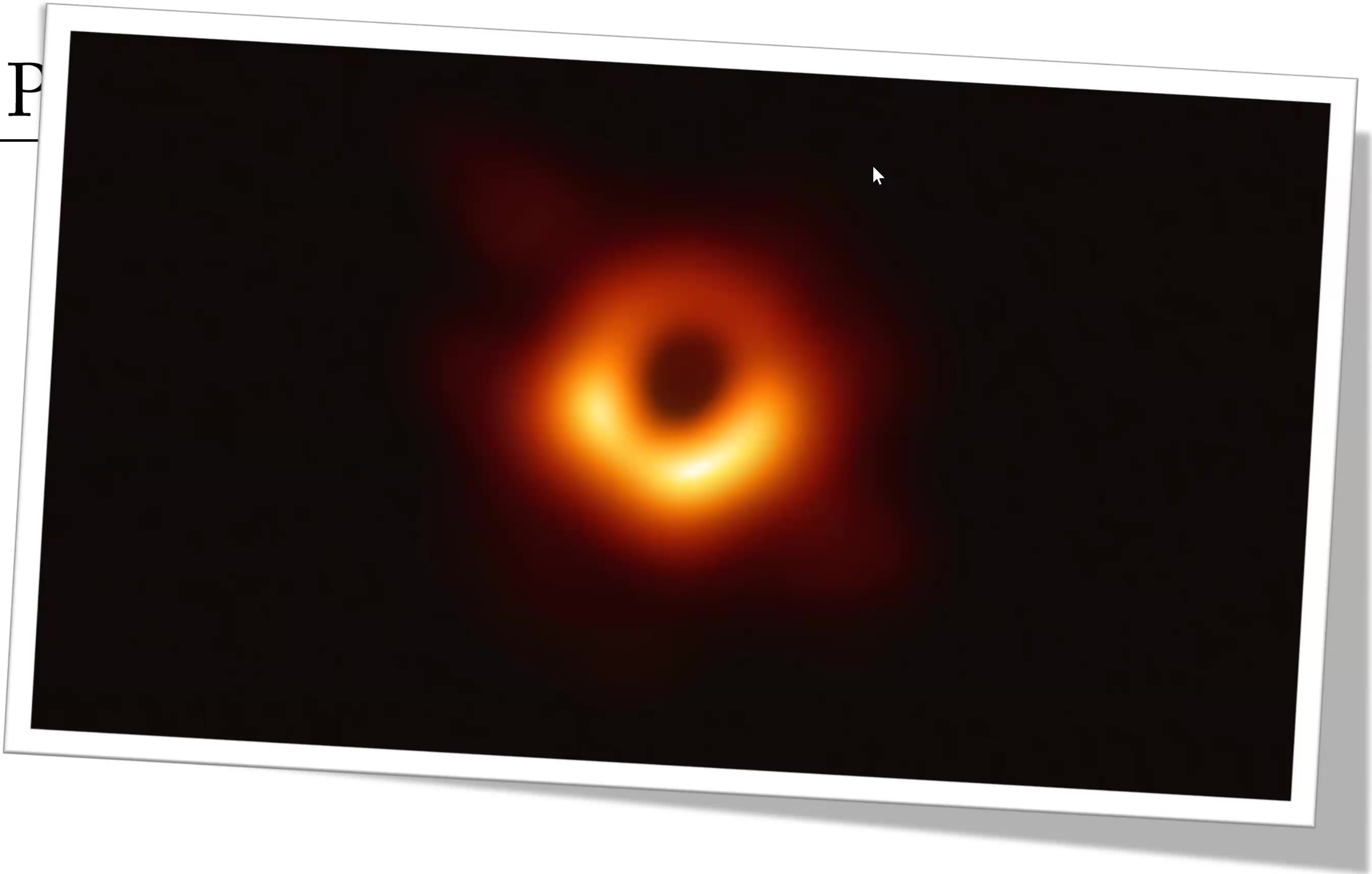


Laws of black hole mechanics

- Black holes can be assigned entropy, temperature, etc. and obey the laws of thermodynamics:

Bardeen, Carter, Hawking 1973
Bekenstein 1974

$$\delta M = \frac{\kappa}{8\pi}\,\delta A + \Omega\,\delta J + \ldots \qquad \kappa \sim T_H \ , \ A \sim S_{BH}$$

- If solution is not vacuum or there is matter:

$$\cdots + \frac{1}{2}\int_\Sigma \xi^\rho \epsilon_\rho T^{\mu\nu}\delta g_{\mu\nu} + \int_\Sigma \delta\left(T^{\mu\nu}\xi_\mu\epsilon_\nu\right)$$

Laws of black hole mechanics

- Can be understood in a covariant form:

Lee, Wald 1993
Iyer, Wald 1994
Wald, Zoupas 2000

$$\delta H_\xi = \int_\Sigma \Omega(\phi, \delta\phi, \mathcal{L}_\xi \phi)$$

$$\int_{\mathcal{B}} \delta Q_\xi = \int_\infty \delta Q_\xi - i_\xi \theta$$

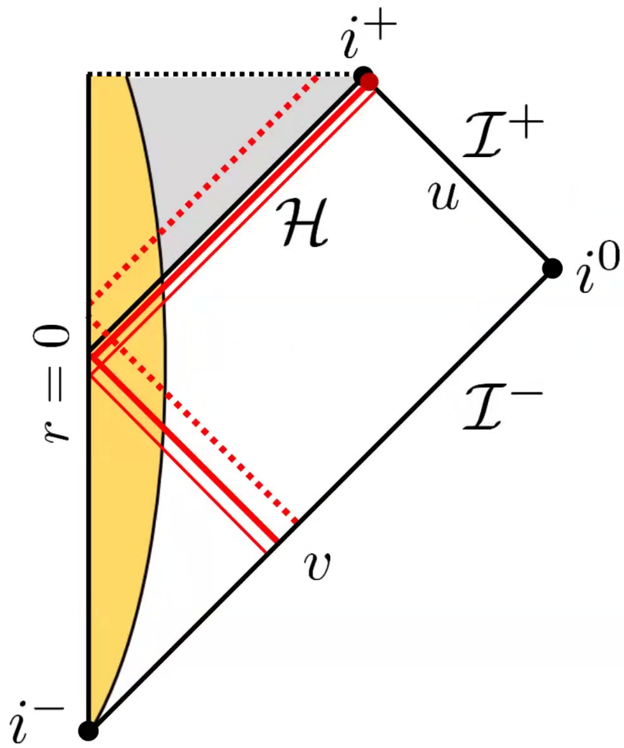
$$T_H \delta S = \delta \mathcal{E} - \Omega_H \, \delta J$$

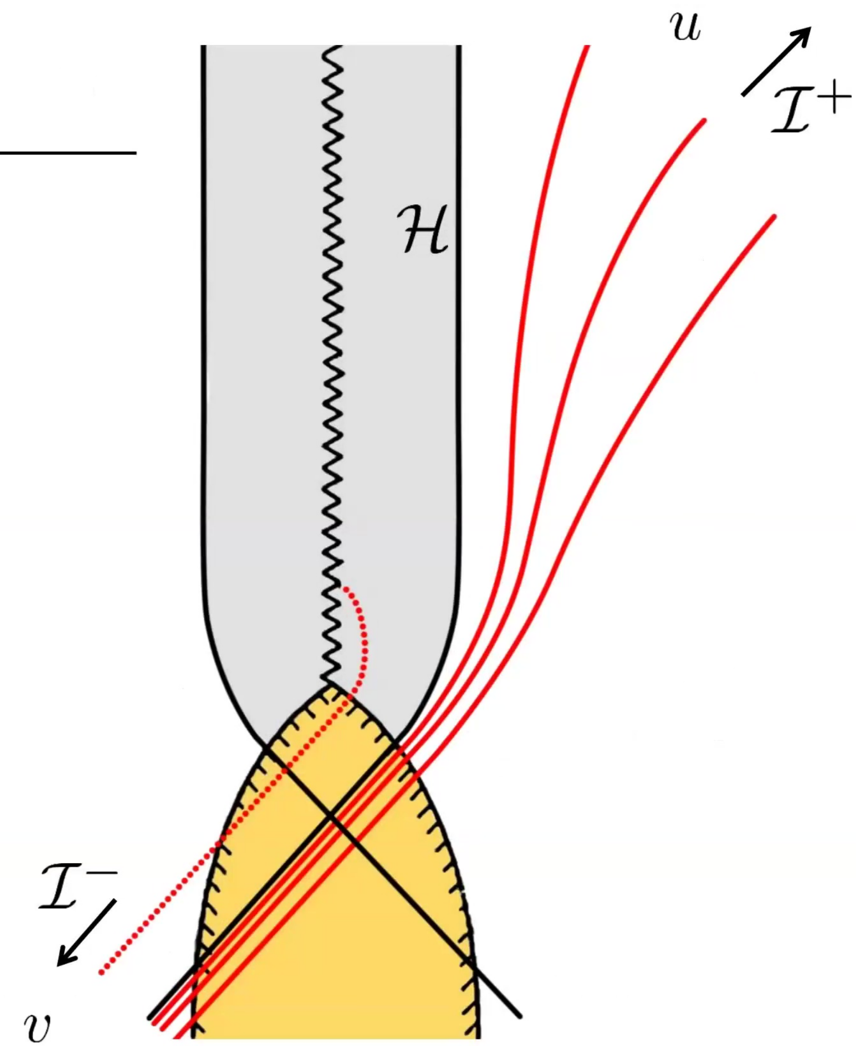
Entropy $\leftrightarrow$ Diffeos.

$$Q_\xi \sim -\frac{1}{16\pi} \epsilon_{ab} \nabla^a \xi^b$$

- Can be interpreted as a physical process or global state comparison.

Hawking Radiation





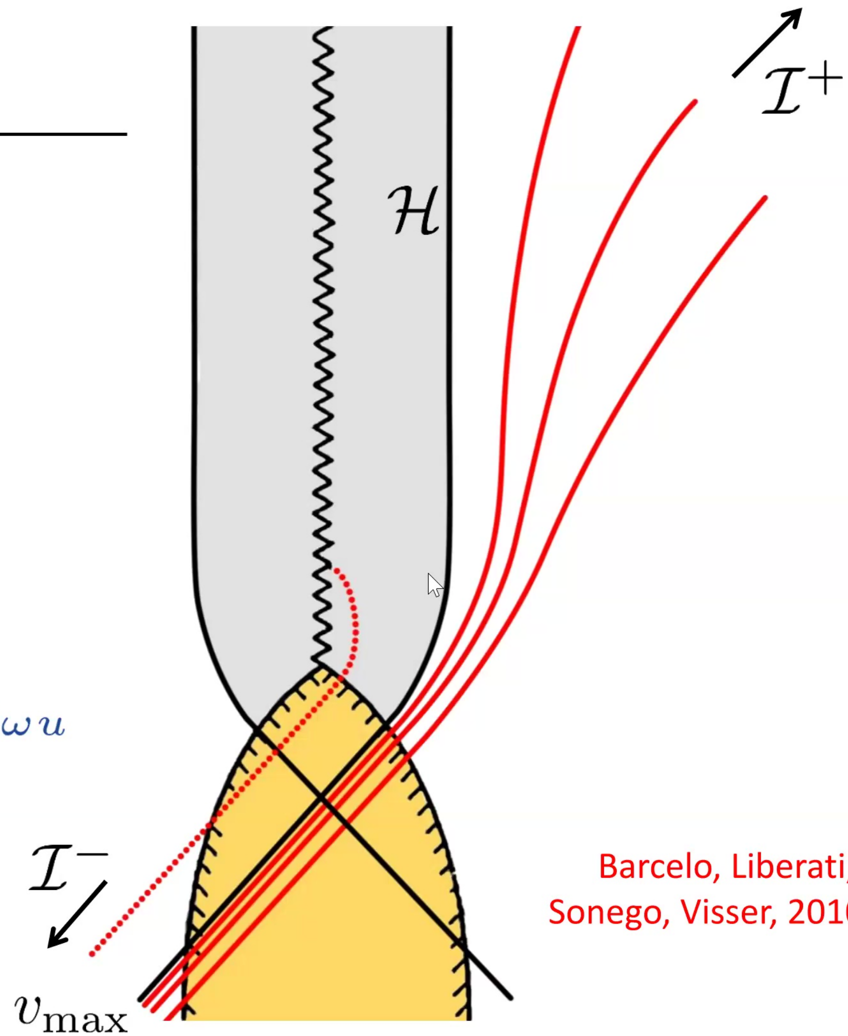
Hawking Radiation

- The evaporation process is ordinarily thought to be a feature particular to event horizons.

- Really all we need is a particular relation between null generators:

$$\beta_{\omega'\omega} = \frac{1}{2\pi}\sqrt{\frac{\omega}{\omega'}}\int du\, e^{-i\omega' v(u)} e^{-i\omega u}$$

$$v(u) \approx v_0 - C e^{-\kappa u}$$



Barcelo, Liberati, Sonego, Visser, 2010

Anti-de Sitter space ($\Lambda < 0$)

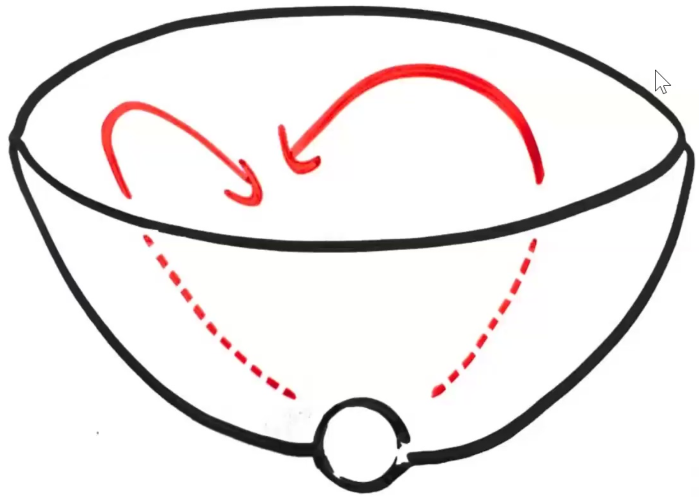
- Possesses nice boundary structure suitable for holography.

- Periodic (closed) orbits.

$$L = \frac{1}{2}\eta_{AB}\dot{X}^A\dot{X}^B - \lambda(\tau)(X_A X^A + a^2)$$

$$\implies \ddot{X}^A \pm \frac{1}{a^2}X^A = 0 \quad \text{and} \quad \ddot{X}^A = 0$$

(timelike) (null)



- A natural length scale l_{AdS} separating two distinct regions.

Variable Λ

- When Λ is treated as a constant, no notion of volume/pressure:

$$\delta M(S,\Lambda,Q) = T\,\delta S + \Omega\,\delta J + \left(\frac{\partial M}{\partial \Lambda}\right) d\Lambda$$

- Allowing Λ to vary:

$$\delta M = T\,\delta S + \Omega\,\delta J + \Theta\,\delta\Lambda$$

Kastor, Ray, Traschen 2009

- Integrating the Killing potential then leads to a volume term:

$$\Theta = \int_{\partial\Sigma_h} d\Sigma_{ab}\,\omega^{ab} - \int_{\partial\Sigma_\infty} d\Sigma_{ab}\left(\omega^{ab} - \omega^{ab}_{AdS}\right) = \boxed{-V_{BH}}$$

Variable Λ

- The first law is restored:

$$\delta M = T\,\delta S + \boxed{V\,\delta P} \quad \Longrightarrow \quad P = -\frac{\Lambda}{8\pi}$$

- LHS must correspond to a variation of enthalpy, not energy:

$$\begin{aligned} \delta H = \delta(E + PV) &= \delta E + P\delta V + V\delta P \\ &= T\delta S - P\delta V + P\delta V + V\delta P \end{aligned}$$

$$\delta H = T\delta S + V\delta P$$

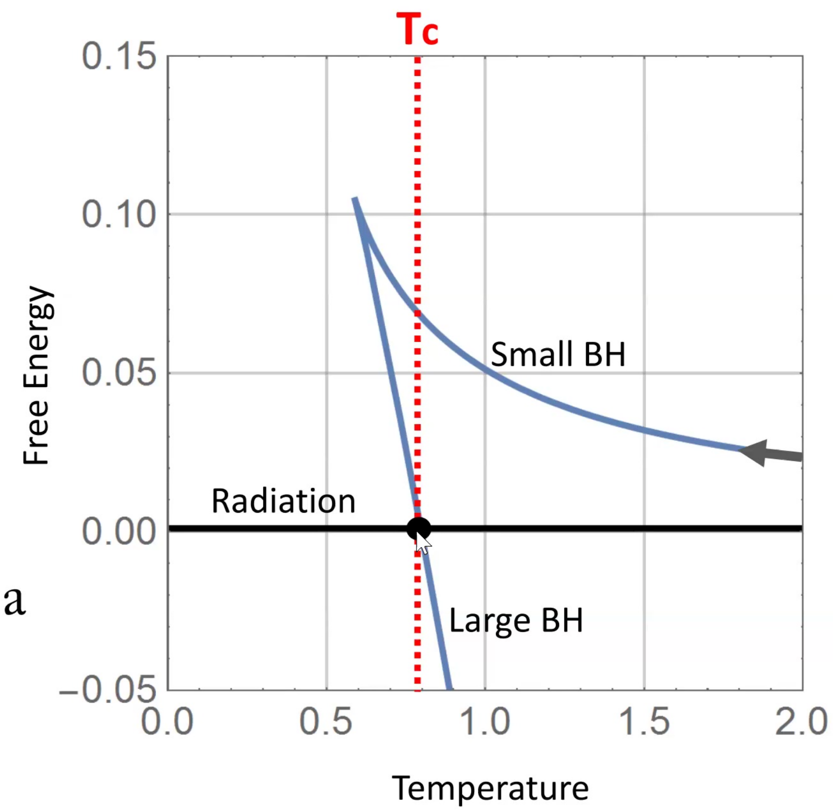
The Hawking-Page transition

- Consider an asymptotically AdS ($\Lambda < 0$) black hole spacetime.

- The free energy is:

$$G = M_{ADM} - T_H S_{BH}$$

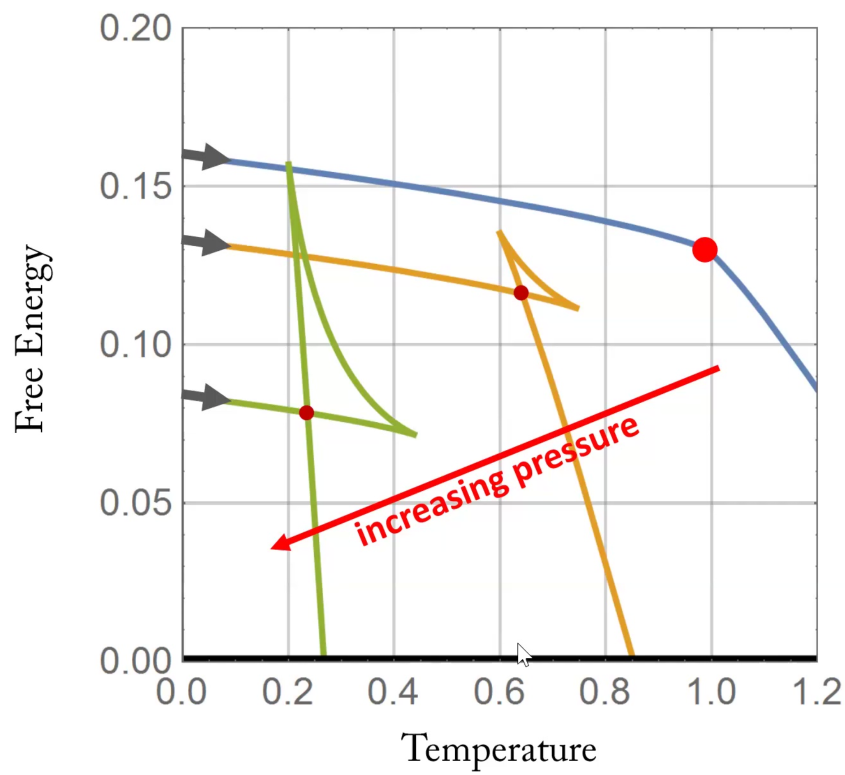
- When the temperature increases past Tc, a phase transition occurs.

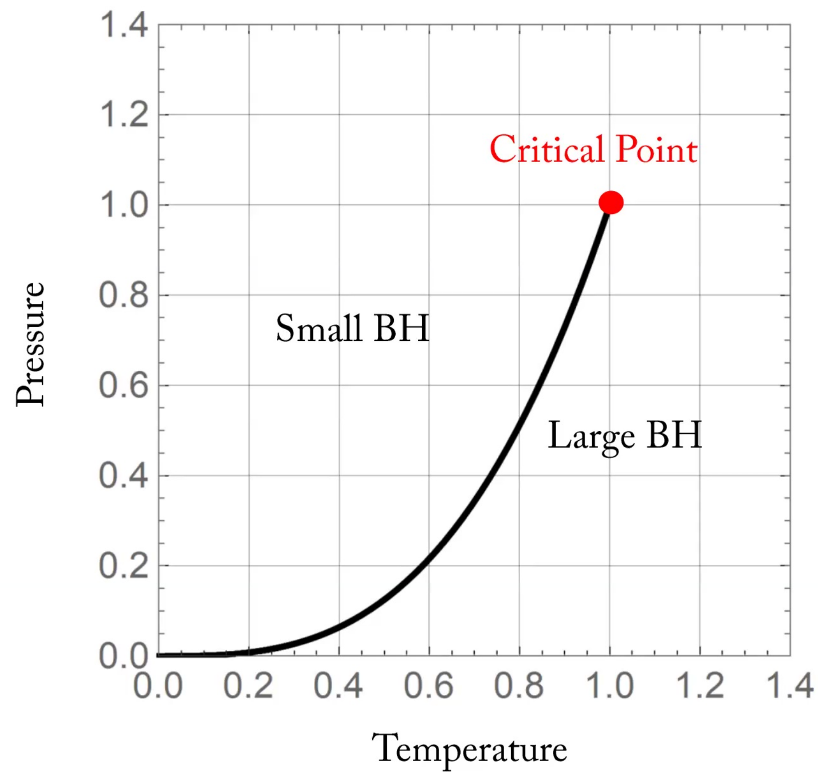


Hawking, Page 1983

Reissner-Nordstrom-AdS

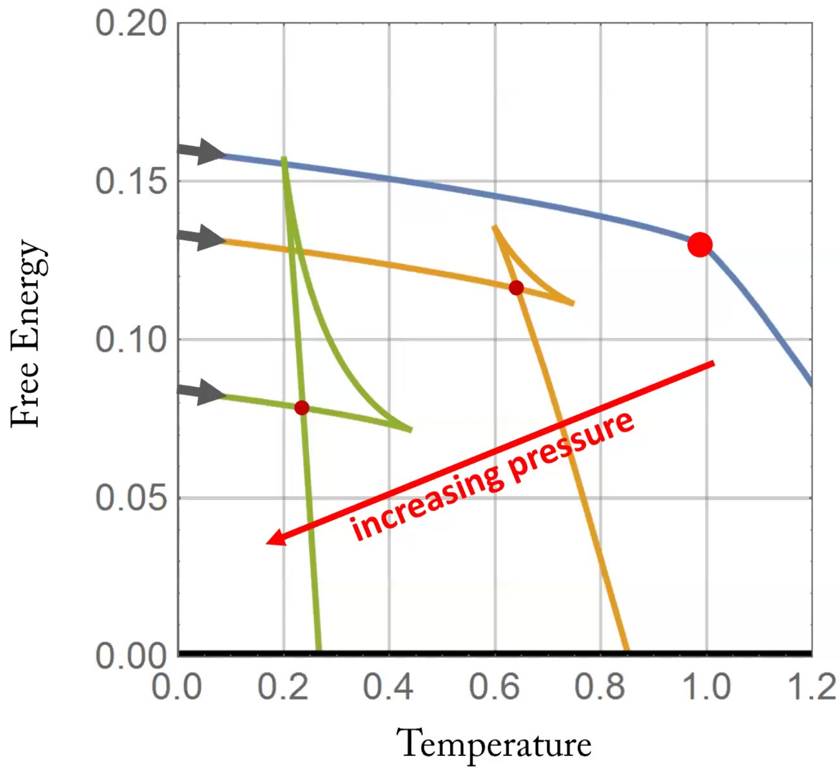
Kubiznak, Mann, Teo 2016

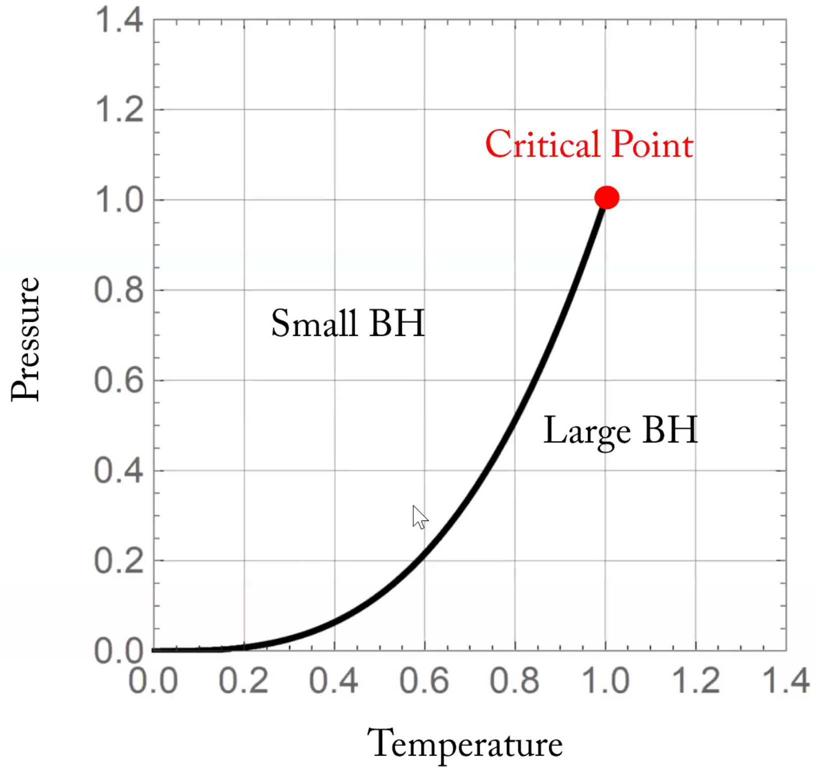




Reissner-Nordstrom-AdS

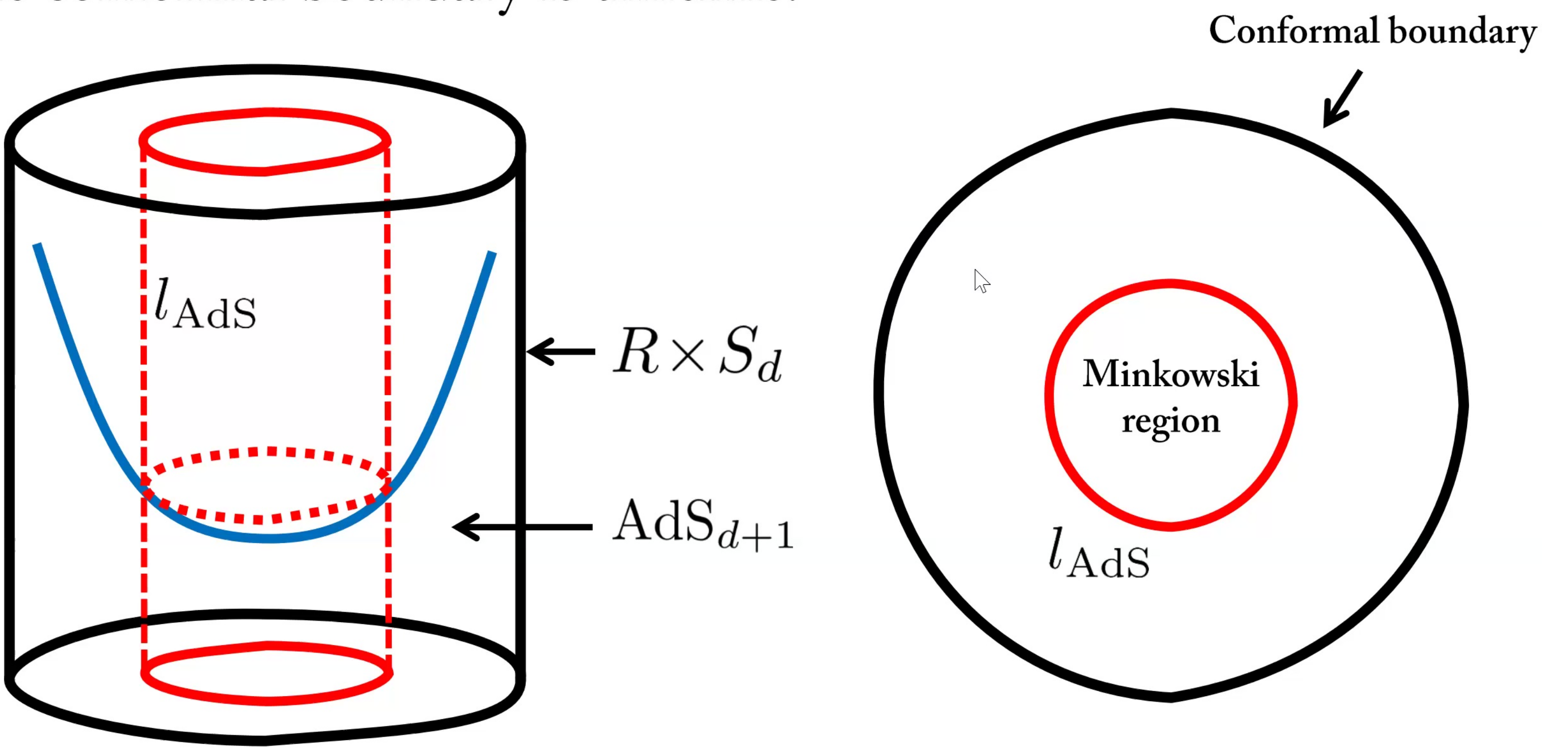
Kubiznak, Mann, Teo 2016





Anti-de Sitter Space

- The conformal boundary is timelike:



The dual description

- In AdS/CFT, we identify the stress tensors as functionals of the boundary metric:

$$T^{\mu\nu} = \frac{2}{\sqrt{-\gamma}} \frac{\delta S_{\text{grav}}}{\delta \gamma_{\mu\nu}} \quad \longleftrightarrow \quad \langle T^{\mu\nu} \rangle = \frac{2}{\sqrt{-\gamma}} \frac{\delta S_{\text{CFT}}}{\delta \gamma_{\mu\nu}}$$

- The partition function of the gravity theory and CFT are the same:

$$\mathcal{Z}_{\text{AdS}}[g, \phi_0] = \mathcal{Z}_{\text{CFT}}[h, \phi_0]$$

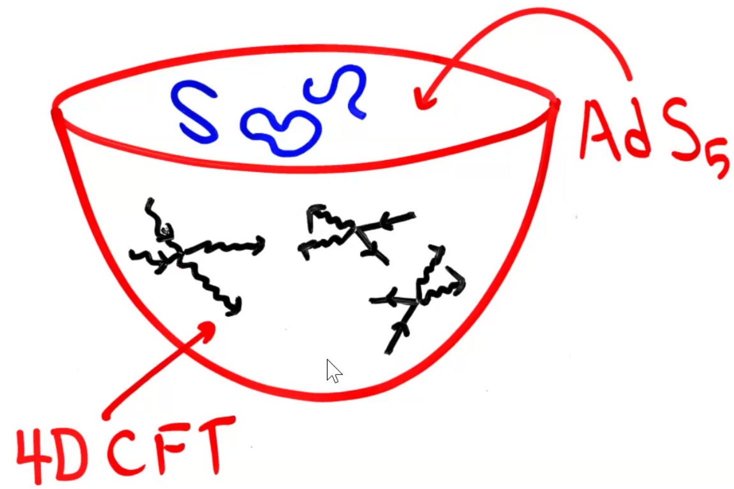
- Bulk theory is typically some variety of SUGRA at large N.

The dual description

- AdS/CFT allows one to probe strongly coupled systems through a weakly coupled gravitational system.

- BH transition in AdS is dual to a deconfinement transition in SYM.

$$T_{\text{QCD}} \longleftrightarrow M_{BH}$$



- Bulk phase structure can describe an enormous variety of systems!

The dual description

- Corrections to the bulk have implications for the boundary theory:

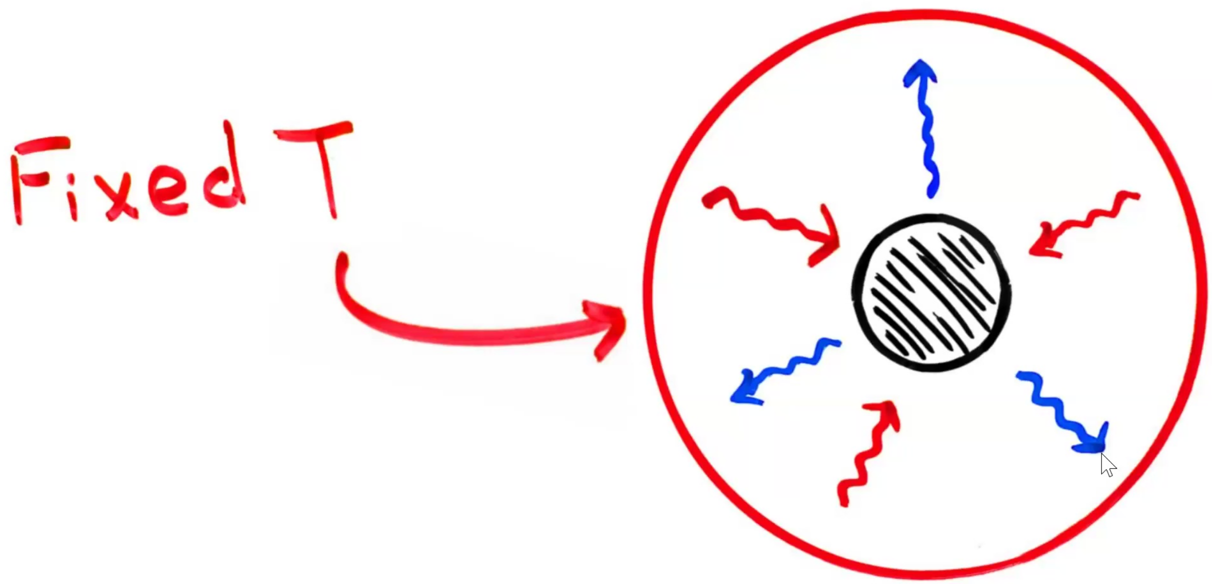
$$\int \sqrt{-g}\,(R - 2\Lambda) \;\rightarrow\; \int \sqrt{-g}\,(R - 2\Lambda + \lambda G_{GB})$$

- Viscosity bound in QFTs: $\frac{\eta}{s} \geq \frac{1}{4\pi}$ Kovtun, Son, Starinets 2005

- Corrections are obtained: $\frac{\eta}{s} \geq \frac{1}{4\pi}\left(1 + \lambda \frac{4\Lambda}{3}\right)$

What about de Sitter?

- In asymptotically flat and de Sitter spacetimes, black holes evaporate and there is no thermal equilibrium.

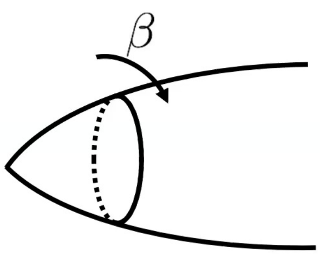


What about de Sitter?

- Take a semi-classical path integral approach:

Gibbons, Hawking 1977

$$\mathcal{Z} = \mathrm{Tr}\, e^{-\beta H} = \oint \mathcal{D}[g]\, e^{-I_E/\hbar} \approx \sum_{g_{cl}} e^{-I_E[g_{cl}]/\hbar}$$



- Data is fixed at a finite boundary in the spacetime:

$$I_{\mathrm{E}} = -\frac{1}{16\pi}\int_{\mathcal{M}} d^4x\sqrt{g}\,(R-2\Lambda) + \frac{1}{8\pi}\int_{\partial\mathcal{M}} d^3x\sqrt{k}\,K - I_0$$

$$\langle E\rangle = -\frac{\partial \log \mathcal{Z}}{\partial \beta}\,, \quad S = \beta\left(\log \mathcal{Z} - \beta\frac{\partial \log \mathcal{Z}}{\partial \beta}\right)\,, \quad F = -\beta^{-1}\log \mathcal{Z}$$

What about de Sitter?

- Consider a class of regular black holes:

Soranidis, Simovic: 2309.09439

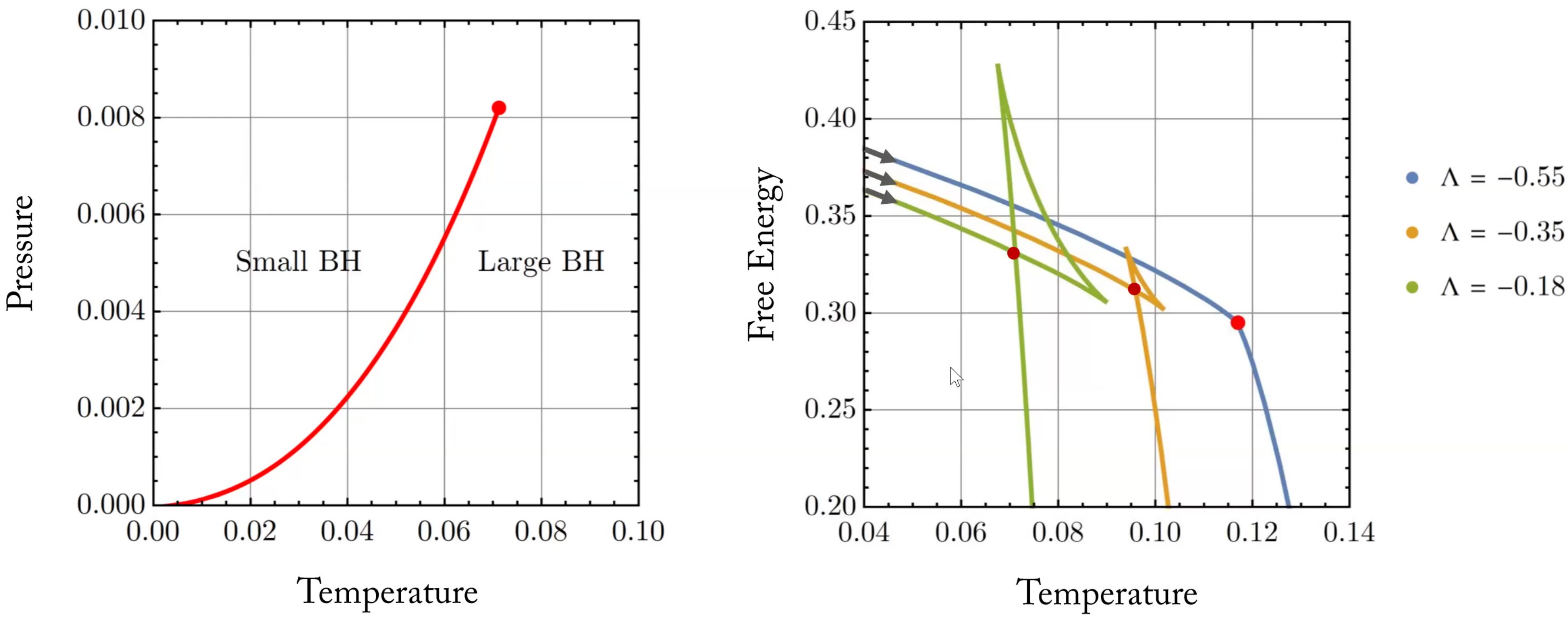
$$I_E = -\frac{1}{16\pi}\int_{\mathcal{M}} d^4x\sqrt{g}\Big(R - 2\Lambda + 4\mathcal{L}(\mathcal{F},\mathcal{G})\Big)$$

$$+\frac{1}{8\pi}\int_{\partial\mathcal{M}} d^3x\sqrt{k}\;K \;-\frac{1}{16\pi}\int_{\partial\mathcal{M}} d^3x\sqrt{k}\;\left(\frac{\partial\mathcal{L}}{\partial\mathcal{F}}\right)F^{\mu\nu}n_\nu A_\mu$$

where $\mathcal{L}(\mathcal{F}) = \frac{4\mu}{\alpha}\frac{(\alpha\mathcal{F})^{\frac{\mu+3}{4}}}{(1+(\alpha\mathcal{F})^{\mu/4})^2}$, $\mathcal{F} = F_{\mu\nu}F^{\mu\nu}$, $Q_m = \frac{1}{4\pi}\int_{\mathcal{S}} F$

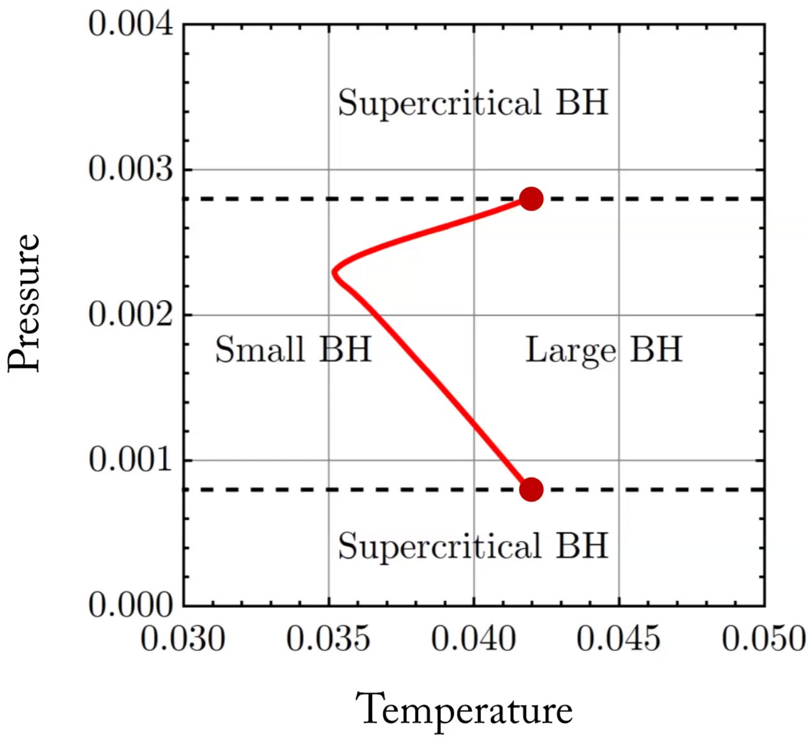
What about de Sitter?

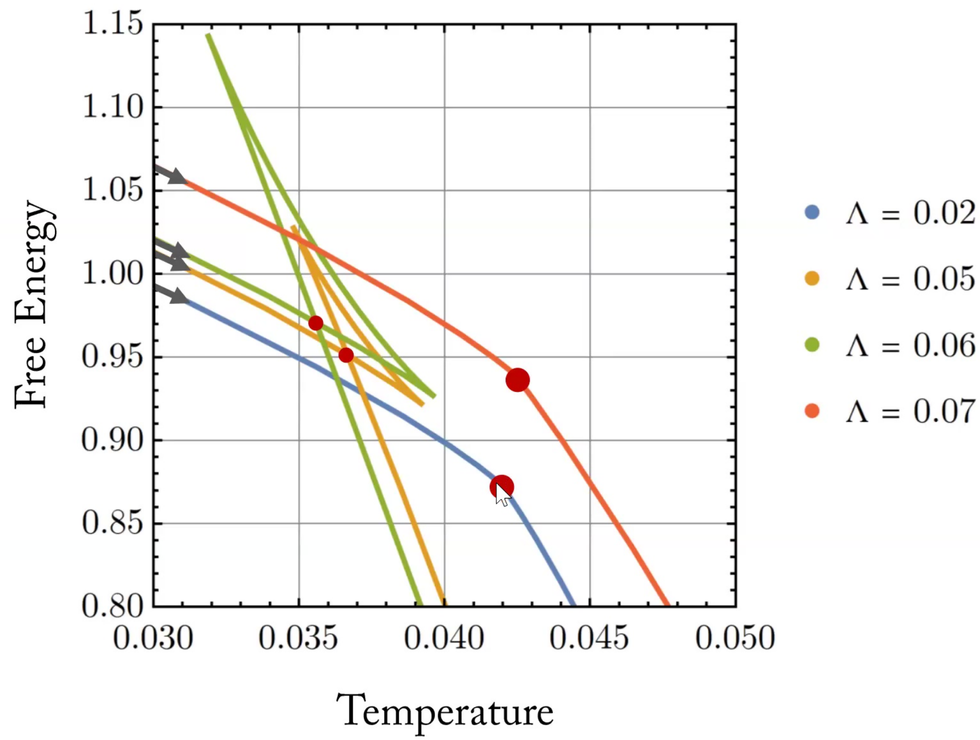
Soranidis, Simovic: 2309.09439



What about de Sitter?

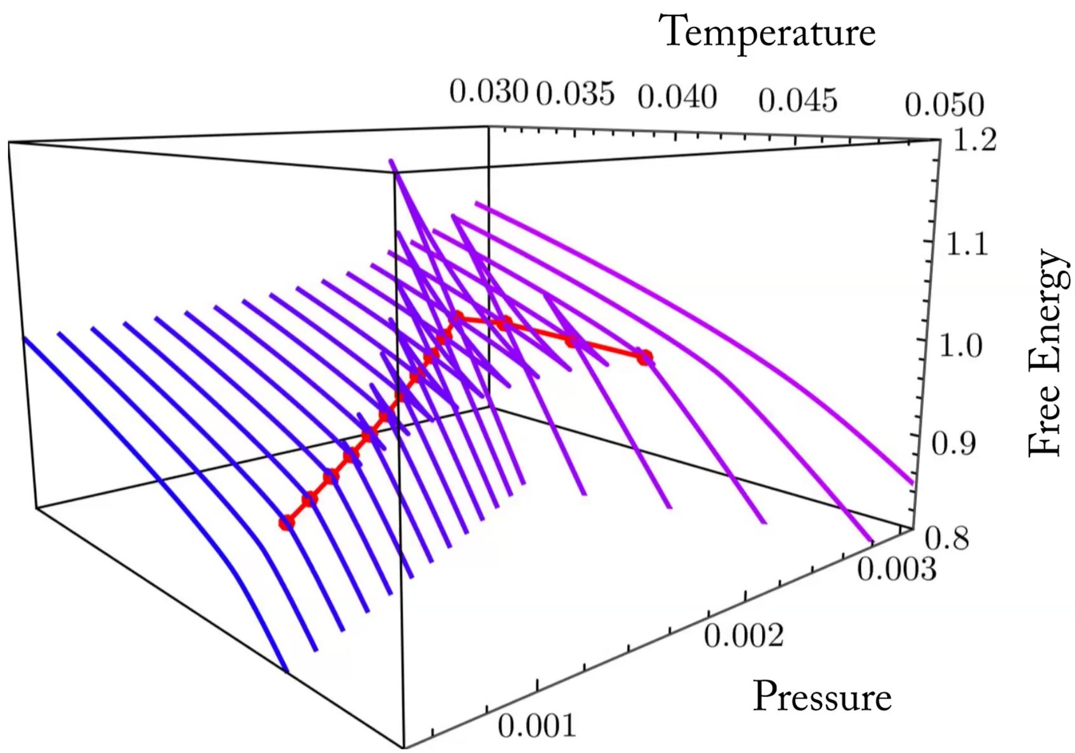
Soranidis, Simovic: 2309.09439





What about de Sitter?

- Coexistence line is compact, terminating at second-order critical points with cosmological horizon.
- New critical behaviour may emerge where the end points meet.
- Some features of non-Fermi metals, FCCs, etc. are realized.



Soranidis, Simovic: 2309.09439

What about de Sitter?

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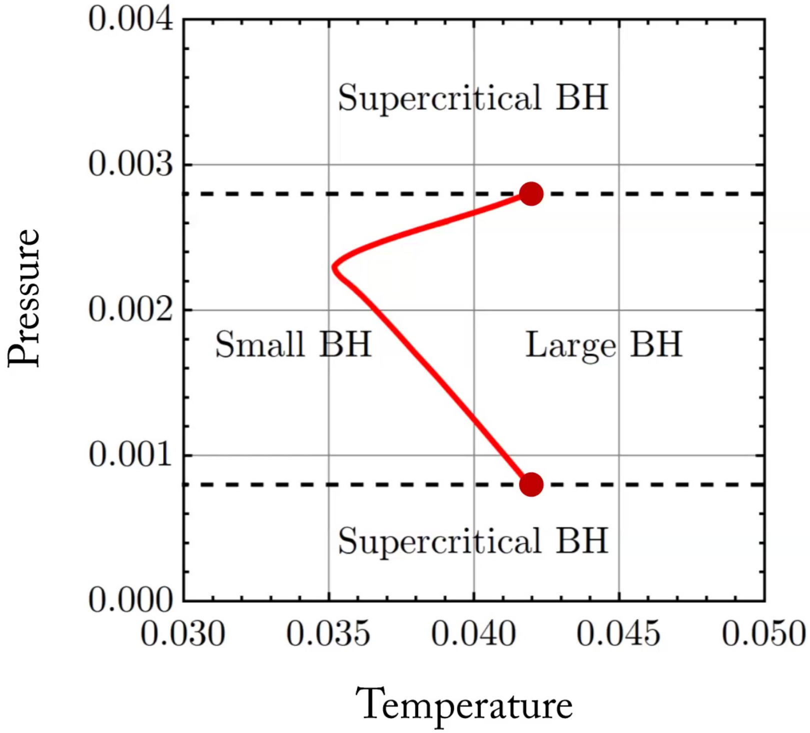
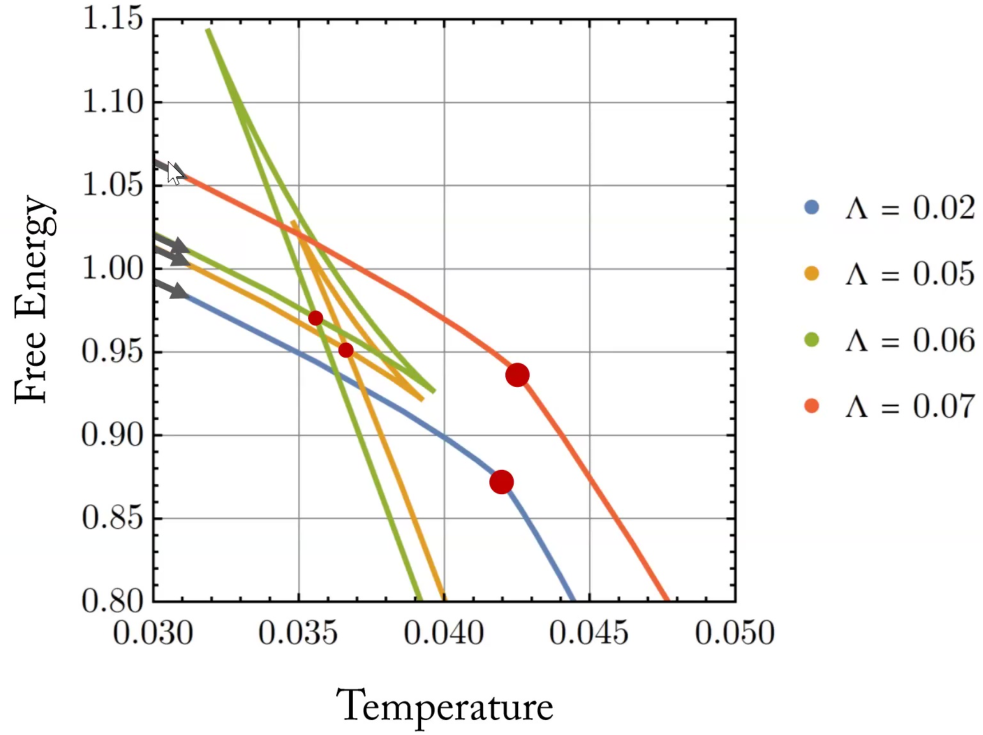




Studied examples

- Scalar fields: $$\mathcal{L}_\phi = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{12}\phi^2 R - V(\phi)$$

  Fusco, Mann, Simovic 2021

- 4D Gauss-Bonnet: $$\mathcal{L}_{GB} = \psi\mathcal{G} - 2G^{ab}\nabla_a\psi\nabla_b\psi - \frac{1}{4}(\nabla\psi)^4 - (\nabla\psi)^2\Box\psi$$

  Marks, Mann, Simovic 2021

- Exotic black holes: $$\mathcal{L} = \frac{1}{16\pi}\sum_k \frac{\hat{\alpha}_k}{2^k}\delta^{a_1 b_1 \ldots a_k b_k}_{c_1 d_1 \ldots c_k d_k} R_{a_1 b_1}{}^{c_1 d_1} \ldots R_{a_k b_k}{}^{c_k d_k}$$

  Hull, Simovic 2022

- Regular black holes: $$\mathcal{L}_{A^\mu} = \frac{4\mu}{\alpha}(\alpha F_{\mu\nu}F^{\mu\nu})^{\frac{\mu+3}{4}}\left(1 + (\alpha F_{\mu\nu}F^{\mu\nu})^{\frac{\mu}{4}}\right)^{-2}$$

  Soranidis, Simovic 2023

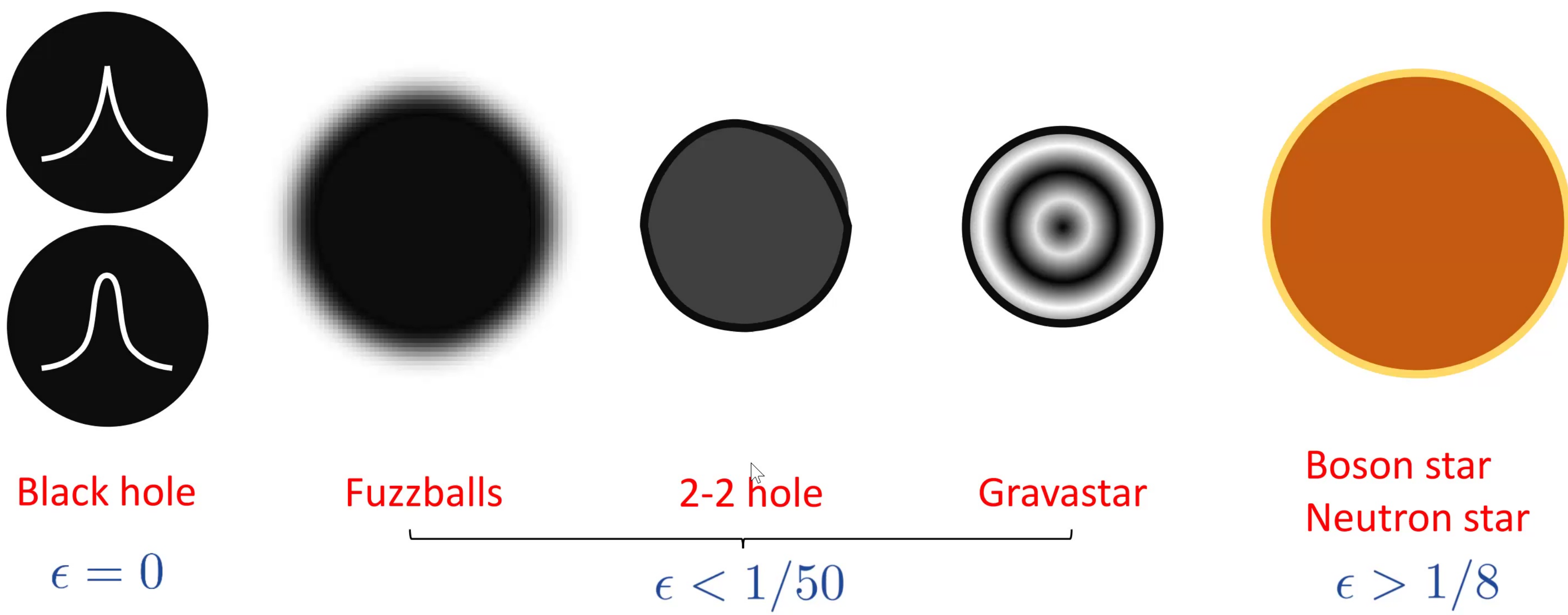
Incredible diversity of bulk phenomena in dS!

The CMB

- CMB acts as a thermal bath at temperature: $T = 2.726 \text{ K}$
- A BH in thermal equilibrium needs: $r_g \approx 67\mu\text{m}$, $M = 0.0075 = M_\oplus$
- As Universe expands, CMB redshifts more and more.
- Critical points are usually $\sim l_{(\text{A})\text{dS}}$, but HP occurs even when cosmological constant is negligible.

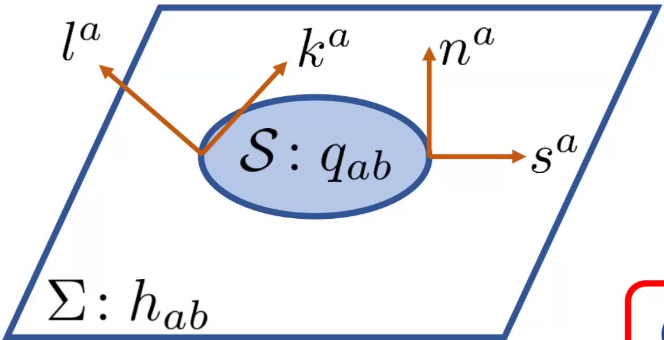
The black hole zoo

- Wide variety of models with varying compactness: $r_0 = 2M(1+\epsilon)$

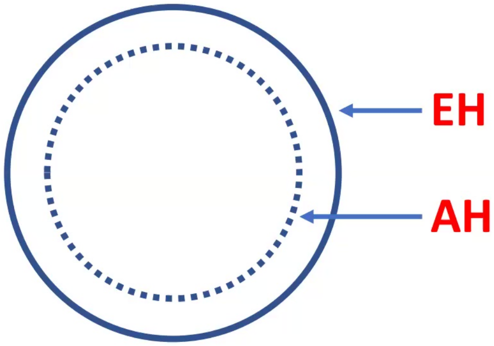


Physical black holes

- An operationalist viewpoint might require finite formation time of the apparent horizon:



$$K_{ab} = h^c{}_a h^d{}_b \nabla_c n_d$$
$$k_{ab} = q^c{}_a q^d{}_b \nabla_c s_d$$



$$\Theta_l = K + k - s^a s^b H_{ab} = 0 \quad \rightarrow \quad r_{AH}$$

- The equivalence principle also suggests regularity at the apparent horizon as a natural feature of such objects.

Physical black holes

- Assume classical spacetime is sourced by the expectation value of some appropriately renormalized energy-momentum tensor:

$$G_{\mu\nu} = 8\pi \left\langle T_{\mu\nu} \right\rangle_{\omega}$$

- Assume a general metric with Misner-Sharp mass $C(t, r)$.

$$ds^2 = -e^{2h(t,r)} f(t,r) dt^2 + \frac{dr^2}{f(t,r)} + r^2 d\Omega_2^2 \ , \quad f(t,r) = 1 - \frac{C(t,r)}{r}$$

- From these assumptions one can construct a semiclassical framework which encompasses many known black hole models.

Mann, Murk, Terno: 2112.06515

Physical black holes

- Introduce rescaled RSET components and resulting EFEs:

$$\tau_t = e^{-2h} T_{tt} \qquad \partial_r C = 8\pi r^2 \tau_t / f$$
$$\tau^r = T^{rr} \quad \longleftrightarrow \quad \partial_t C = 8\pi r^2 e^h \tau_t{}^r$$
$$\tau_t^r = e^{-h} T_t^r \qquad \partial_r h = 4\pi r \left(\tau_t + \tau^r\right) / f^2$$

- Finiteness of curvature invariants constrains the allowed scaling:

$$\mathrm{T} \equiv \frac{\tau^r - \tau_t}{f}, \quad \mathfrak{T} \equiv \frac{\left(\tau^r\right)^2 + \left(\tau_t\right)^2 - 2\left(\tau_t^r\right)^2}{f^2}$$

$$\boxed{\tau \sim f^k}$$

Physical black holes

- Properties of the metric and RSET are given in a near-horizon expansion $(x \equiv r - r_g)$. Regularity implies two types of solutions:

$$k = 0$$

$$C = r_g - 4\sqrt{\pi} r_g^{3/2} \Upsilon \sqrt{x} + \mathcal{O}(x)$$

$$h = -\frac{1}{2} \ln\left(\frac{x}{\xi}\right) + \mathcal{O}(\sqrt{x})$$

$$r_g' = \pm 4\sqrt{\pi r_g \xi} \Upsilon$$

$$k = 1$$

$$C = r_g - 4\sqrt{-\pi e_2/3} r_g^{3/2} x^{3/2} + \mathcal{O}(x^2)$$

$$h = -\frac{3}{2} \ln\left(\frac{x}{\xi}\right) + \mathcal{O}(\sqrt{x})$$

$$r_g' = \pm 4\sqrt{\pi r_g \xi^3 e_2/3}$$

Physical black holes

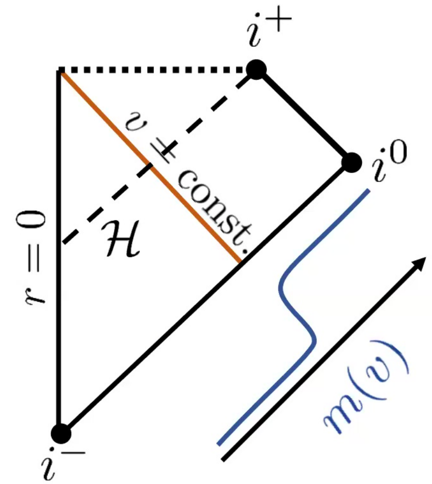
- Leads to constraints on the dynamics of physical black holes:

$$T^a{}_b = \begin{pmatrix} \Upsilon^2/f & \pm e^{-h}\Upsilon^2/f^2 \\ \mp e^h \Upsilon^2 & -\Upsilon^2/f \end{pmatrix} \quad , \qquad T^{ab}k_a k_b < 0$$

- Only two types of Vaidya metrics are valid:

$$ds^2 = -\left(1 - \frac{2M(v)}{r}\right) dv^2 + 2dvdr + r^2 d\Omega^2$$

$$ds^2 = -\left(1 - \frac{2M(u)}{r}\right) du^2 - 2dudr + r^2 d\Omega^2$$



Physical black holes

- Astrophysical black holes rotate so use Kerr-Vaidya metrics instead:

$$ds^2 = -\left(1 - \frac{2M(v)r}{\rho^2}\right)dv^2 + 2dvdr - \frac{4aM(v)r\sin^2\theta}{\rho^2}dvd\psi - 2a\sin^2\theta drd\psi$$
$$+\rho^2 d\theta^2 + \frac{\Sigma^2}{\rho^2}\sin^2\theta d\psi^2 \ , \quad (v = t + r^*)$$

Dahal, Maharana, Simovic, Terno: 2311.02981

- All four solutions are admissible!

- Exactly solvable model possible in a de Sitter background.

Dahal, Simovic, Soranidis, Terno: 2309.13894

Physical black holes

- Assume classical spacetime is sourced by the expectation value of some appropriately renormalized energy-momentum tensor:

$$G_{\mu\nu} = 8\pi \langle T_{\mu\nu} \rangle_\omega$$

- Assume a general metric with Misner-Sharp mass $C(t, r)$.

$$ds^2 = -e^{2h(t,r)} f(t,r) dt^2 + \frac{dr^2}{f(t,r)} + r^2 d\Omega_2^2 \ , \quad f(t,r) = 1 - \frac{C(t,r)}{r}$$

- From these assumptions one can construct a semiclassical framework which encompasses many known black hole models.

Mann, Murk, Terno: 2112.06515

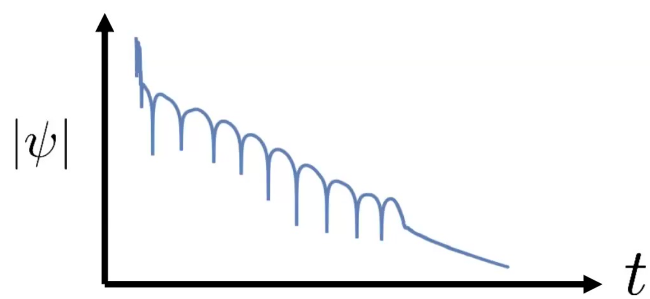
Observational signatures?

- Quasi-normal mode spectra are highly sensitive to BCs at the horizon:

$$\Box\Phi = 0\ , \quad \Phi = \sum_{l,m} \frac{1}{r}\psi(r) Y_{l,m}(\phi,\theta) e^{-i\omega t}$$

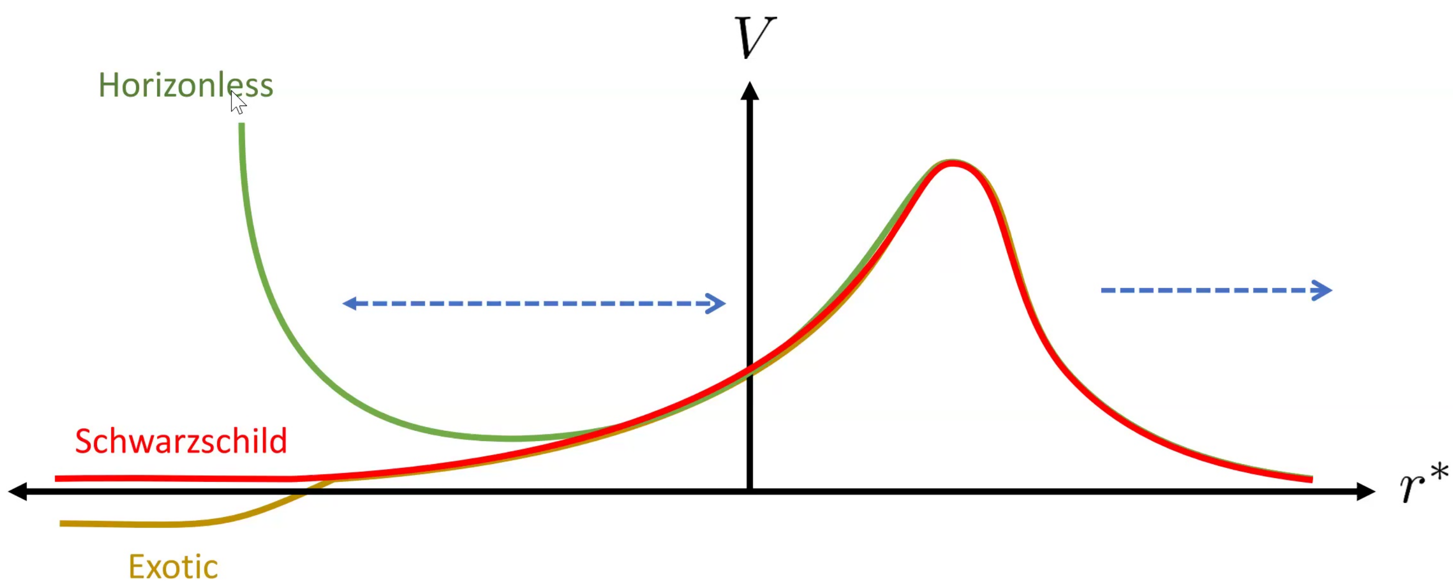
$$\frac{\partial^2\psi}{\partial r^{*2}} + \Big[\omega^2 - \underbrace{f(r)\left(\frac{l(l+1)}{r^2} + \frac{f'(r)}{r}\right)}_{V}\Big]\psi = 0 \qquad \frac{dr^*}{dr} = \frac{1}{f}$$

- Characteristic 'ringing' in GW spectrum:



Observational signatures?

- Provides an observational signature of the near-horizon characteristics of different BH models



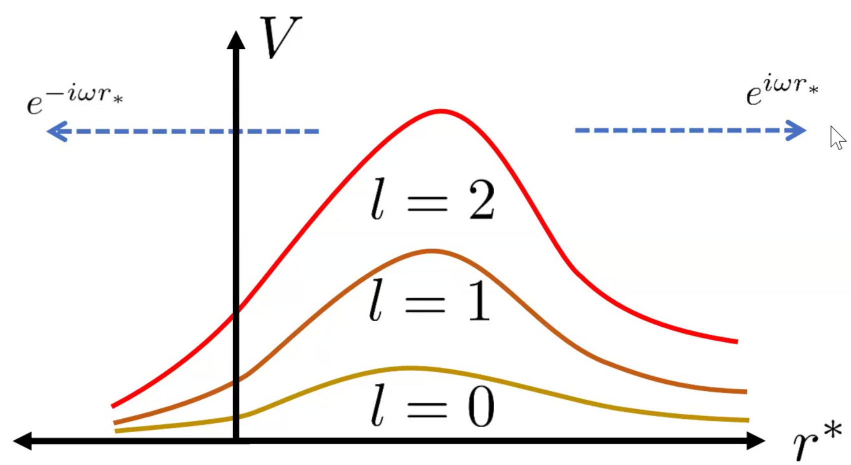
Observational signatures?

- The radial part of the wavefunction contains a potential term:

$$\frac{\partial^2 \psi}{\partial r^{*2}} + \Big[\omega^2 - \underbrace{f(r)\left(\frac{l(l+1)}{r^2} + \frac{f'(r)}{r}\right)}_{V}\Big]\psi = 0$$

- For a BH the potential vanishes at the horizon and infinity.

- QNMs are purely ingoing at H and purely outgoing at infinity.



Observational signatures?

- In our framework, the potential is constrained to the form:

$$V(r \sim r_h) = \frac{16\pi^2 e_2^2 l(l+1)}{9r_h} - \frac{32\left(l(l+1)\left(14\pi^2 e_2^2\sqrt{r_h} - 3\sqrt{3}\pi^{3/2}\sqrt{-e_2}e_{52}r_h^{5/2} + 3\pi e_2\right)\right)}{63 r_h^{5/2}} x + \mathcal{O}(x^2)$$

- Higher-order terms in RSET expansion are obtained through EFEs.
- Tortoise coordinate becomes difficult.

Summary

- Black holes continually provide fertile ground for understanding classical, semi-classical, and quantum gravity.
- Semi-classical effects may be important at the horizon scale and can significantly modify collapse and evaporation dynamics. Important to understand if there are observational signatures of such effects.
- Thermodynamic properties of classical solutions provide a rich set of tools for studying strongly coupled systems holographically and by analogy.
- Need to better understand the role of bulk dS phenomena in holography.

Thank you

Fil Simovic

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MACQUARIE University
Research Centre for Astronomy, Astrophysics & Astrophotonics