Abstract: We use a result of Hawking and Gilkey to define a Euclidean path integral of gravity and matter which has the special property of being independent of the choice of basis in the space of fields. This property allows the path integral to describe also physical regimes that do not admit position bases. These physical regimes are pre-geometric in the sense that they do not admit a mathematical representation of the physical degrees of freedom in terms of fields that live on a spacetime. In regimes in which a spacetime representation does emerge, the geometric properties of the emergent spacetime, such as its dimension and volume, depend on the balance of fermionic pressure and bosonic and gravitational pull. That balance depends, at any given energy scale, on the number of bosonic and fermionic species that contribute, which in turn depends on their masses. This yields an explicit mechanism by which the effective spacetime dimension can depend on the energy scale.

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Zoom link https://pitp.zoom.us/j/91471624951?pwd=RDBYTXc2MGs5bkFKUytYR0huOHYwdz09
Model for Emergence of Spacetime from Fluctuations

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- accepted to PRL
- arXiv: 2303.01519
RESULTS

- Dimension & Volume of Emergent ST depend on Gravity, Boson Pull, Fermion Push
- Energy Dependent Spacetime Dim.
- Consistency OK
HAWKING - GILKEY FORMULA

\[ \Delta \text{ OPERATOR ON RIEM. MFLOPS} \]

\[ N(\lambda < \Lambda) = \frac{1}{6\pi^2} \int d^4x \sqrt{g} \left( \frac{\Lambda^2}{2} + \frac{\Delta}{6} R + O(R^2) \right) \]

\[ S_j = \lambda \cdot N \quad \mu = \frac{6\pi}{\Lambda} \]

\[ = \mu \cdot \text{Tr}(\mathbf{M}) \]
HAWKING - GILKEY FORMULA

$\Delta$ OPERATOR ON RIEM. MFLOS

$\langle \lambda, \langle \lambda \rangle \rangle = \frac{1}{64\pi^3} \int d^4x \sqrt{g} \left( \frac{\Lambda^2}{\ell^2} + \frac{\Delta}{6} R + O(R^4) \right)$

$S_j = \mu N \nabla \cdot \frac{\Delta}{32\pi^2} + \text{Tr}(\Pi)$

$\mu = \frac{6\pi}{\ell^2}$

$\frac{N}{\lambda} = \frac{\Lambda^2}{32\pi^2} + S \lambda O(R^n)$
FREE BOSONS

\[ S_B = \sum_{n=1}^{N_B} \frac{1}{2} \text{Tr} \left( (\Delta + m^2) \psi_1^\dagger \psi_1 \right) \]

\[ = \frac{1}{2} \sum_{1=1}^{N_B} \sum_{n=1}^{N} \lambda_n (\phi_n^\dagger)^2 \]

FREE FERMIONS

\[ S_F = \sum_{n=1}^{N_F} \sum_{n=1}^{N} \sqrt{\lambda_n} \bar{\theta}_n \theta_n \]
\[ S = S_g + S_b + S_f \]

\[ Z = \sum_{N=N_0}^{N_{\text{max}}} \int \ldots \int \int \ldots \int \frac{e^{-\beta S}}{N! e^{\beta S}} \left( \ldots \right) \]

\[ Z \sim C e^{k c s} \left( \ldots e^{-\beta P} \right) \]

\[ B_{\text{max}} = \frac{2M_e - N_0}{2\mu} \]
Weyl scaling $f$:

\[ f(\lambda) \sim \lambda^{d/2-1} \]

\[ d_{\text{eff}} = 2 + N_5 - N_\rho \]
\[ \langle N \rangle \]

VOLUME \[ \langle v \rangle \]

\[ \log \langle N \rangle \sim (N_f - N_s) \]

\[ N_f - N_s > 2 \]

\[ \log \langle v \rangle \sim \frac{1}{T} \]

\[ N_s > N_f - 2 \]
\[ \frac{N}{V} = \frac{A^2}{32\pi^2} + \int \alpha(x) \]

\[ \frac{\langle N \rangle}{\langle V \rangle} = \text{const} \]
\[ \frac{N}{V} = \frac{A^2}{3 \pi \nu^2} + \int \alpha(x) \]

\[ \frac{\langle N \rangle}{\langle V \rangle} = \text{constant} + \tau \]