Title: Kazhdan-Lusztig Equivalence and Kac-Moody Localization

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Series: Mathematical Physics

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Abstract: We will begin by reviewing the work of Kazhdan and Lusztig, who established an equivalence between certain affine Lie algebra representations and quantum group representations. It can be thought of as a logarithmic version of the CS-WZW correspondence. In a joint work with Lin Chen, we used factorization algebras to establish an extended version of this equivalence. We will explain the main structure of the proof, and draw some connections with Kazhdan and Lusztig's original proof. The key idea is that of Kac-Moody localization, a parametrized version of factorization homology. Time permitting, we will also explain how this idea plays a role in the recently announced proof (by Gaitsgory et al.) of the de Rham geometric Langlands conjecture.

Zoom link https://pitp.zoom.us/j/98932564942?pwd=WWdMLzI5WktkZnMrQmplU0J3Mk43dz09

$\mathcal{Z}$ 4d SYM k-twisted
(k positive)

$\mathcal{Z}$(1 manifold) : 2-cats

1-cat

$\mathcal{Z}(X) = \mathrm{DMod}_k(\mathrm{Bun}_G)$

$\mathcal{Z}(S^1) = \mathrm{DMod}_k(LG)\text{-mod}$

$S^1 \downarrow \mathring{D}_x$

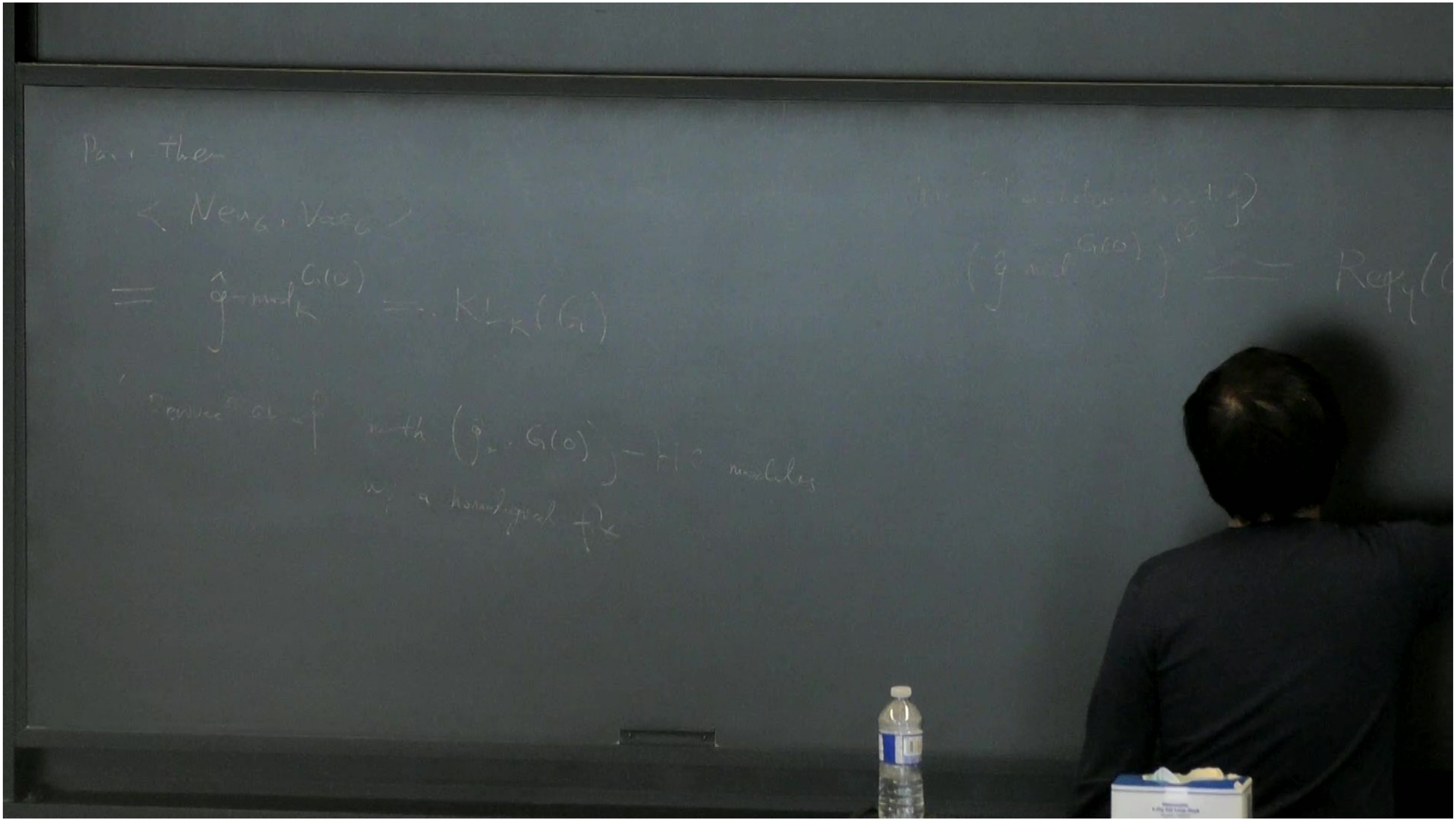
$\mathcal{Z}(S^1)$ roughly chiral.

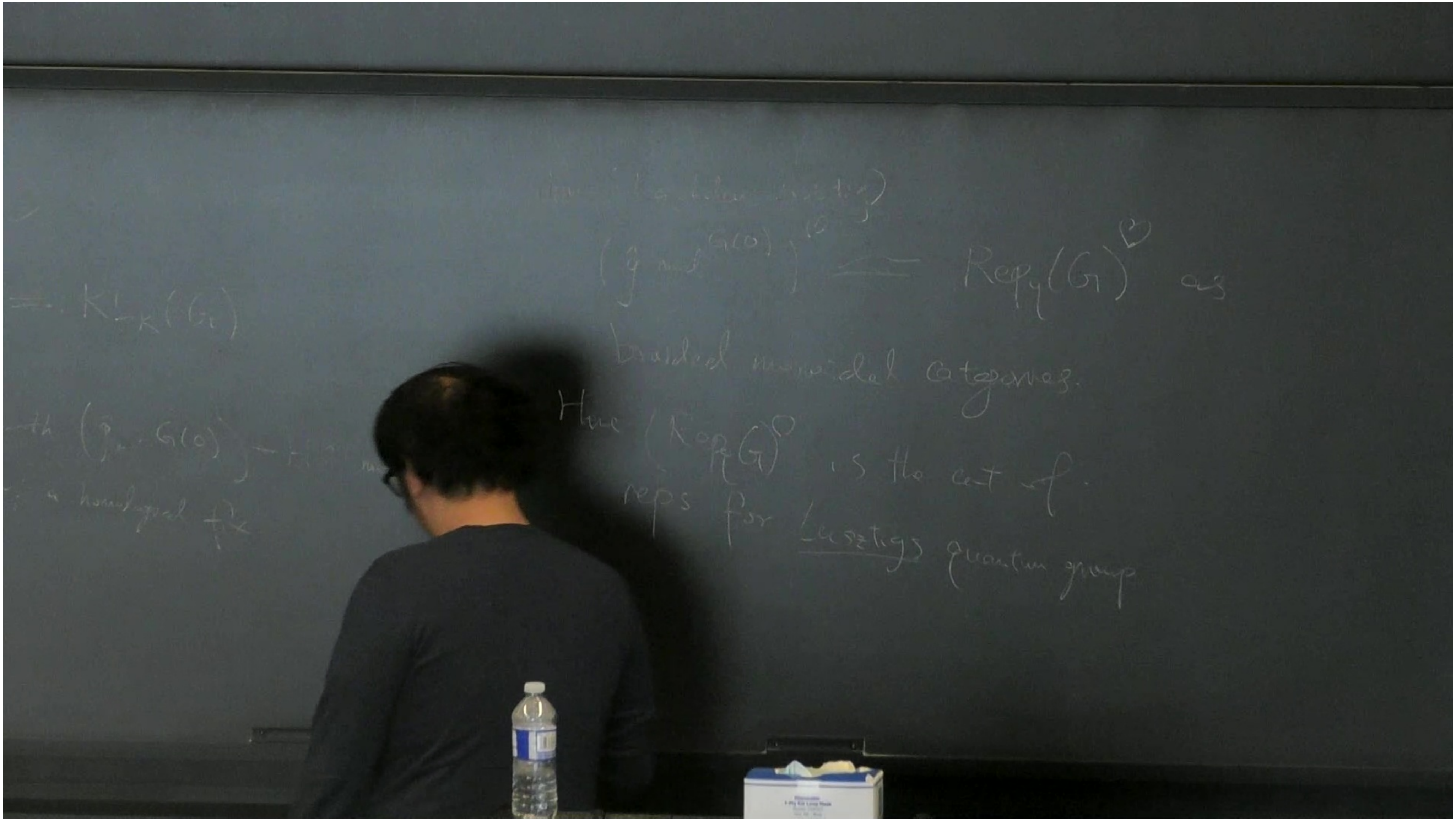
Facts: $\mathcal{Z}(S^1) \hookrightarrow \mathrm{DMod}_k(\mathrm{Gr})\text{-Fact/Mod}$

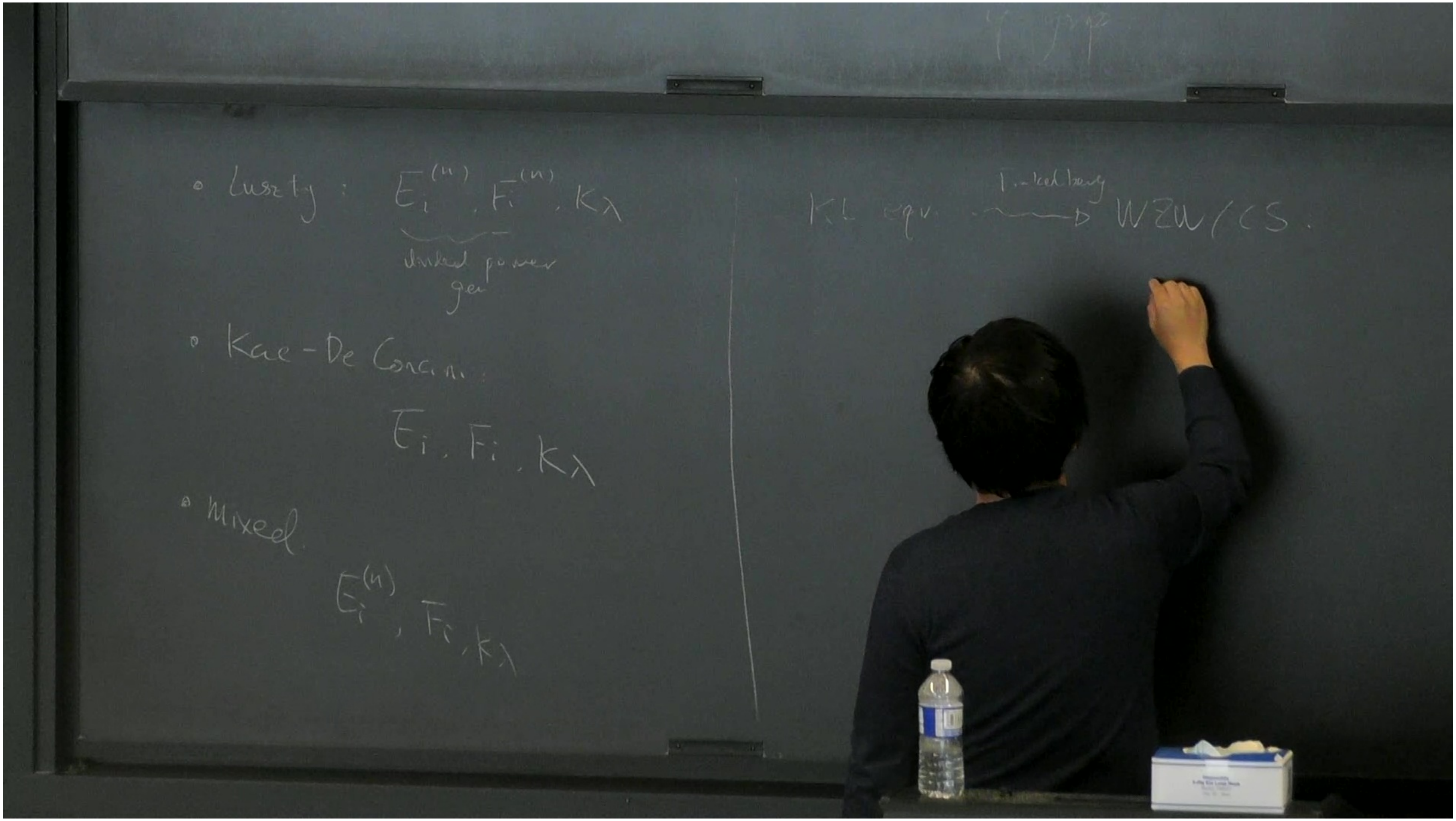
vac of $\mathcal{Z}(S^1)$

Boundary Conditions. $\mathcal{B}(S^1) \in \mathcal{Z}(S^1)$

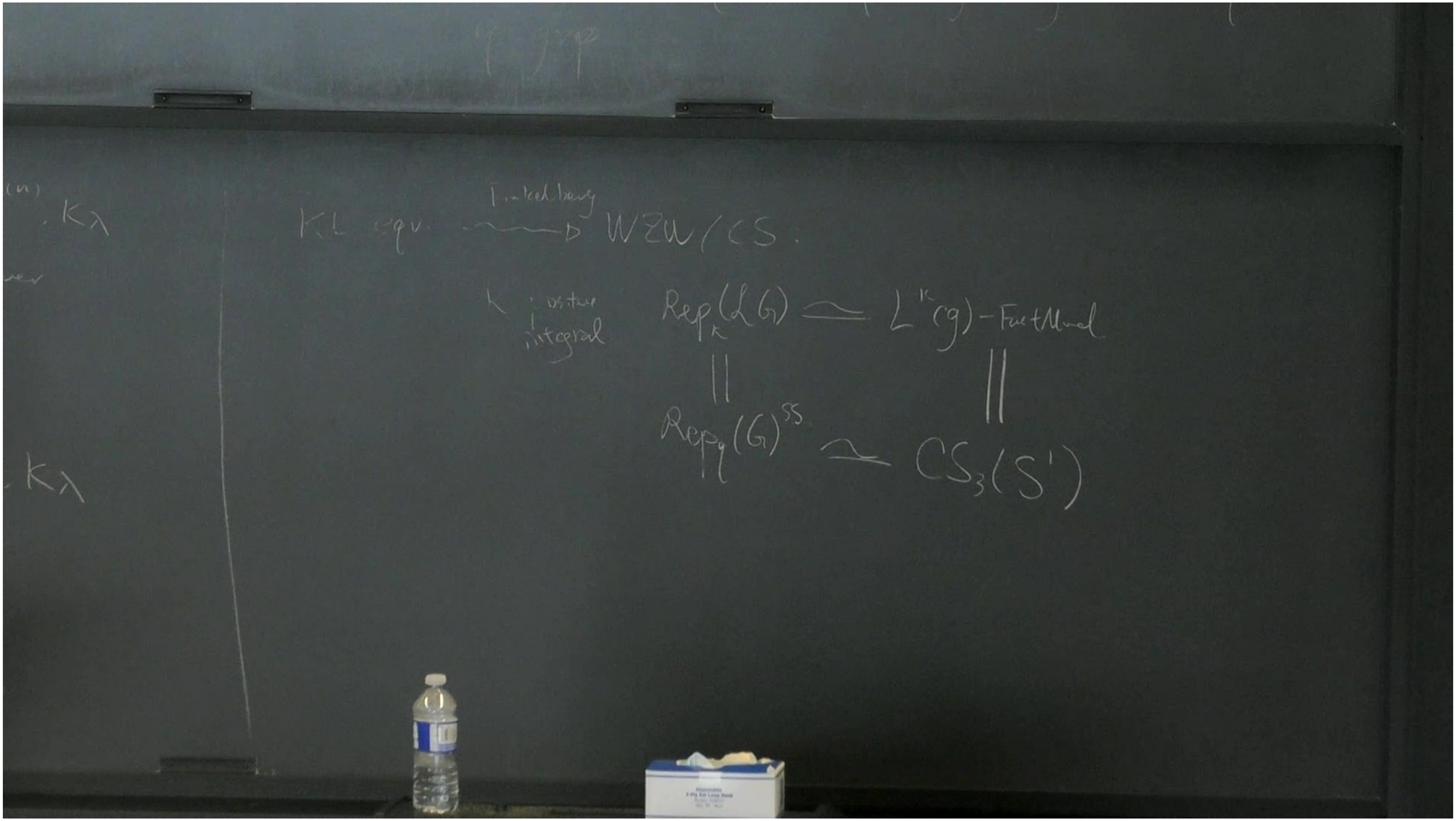
- Vac $\rightsquigarrow \mathrm{DMod}_k(\mathrm{Gr}_G)$
- Neumann $\rightsquigarrow \hat{\mathfrak{g}}\text{-mod}_k$
- Gauge-breaking $H \subseteq G \rightsquigarrow \mathrm{DMod}_k(LG/LH)$


Pair them
$\langle \mathrm{Nm}_G, \mathrm{Vec}_G \rangle$
$= \hat{\mathfrak{g}}\text{-mod}_\kappa^{G(O)} = KL_\kappa(G)$
with $(\hat{\mathfrak{g}}_\kappa, G(O))$-HC modules
w/ a homological fix
$(\hat{\mathfrak{g}}\text{-mod}^{G(O)})$
Rep_q


braided monoidal categories.
Here
is the cat of
reps for Lusztig's quantum group



• Lusztig: $E_i^{(n)}, F_i^{(n)}, K_\lambda$
divided power gen
• Kac-De Concini:
E_i, F_i, K_λ
• Mixed.
$E_i^{(n)}, F_i, K_\lambda$
KL equiv. $\xrightarrow{\text{Finkelberg}}$ WZW/CS:


KL eqv. ⇝ WZW/CS.
k : positive integral
Rep_k(LG) ≃ L^k(g)-FactMod
Rep_q(G)^ss ≃ CS_3(S^1)
Kλ
Kλ

Given parabolic $P \subseteq G$

$\exists$ domain wall Par_P between Z_G and Z_M (M Levi)

For now, $P = B$, $M = T$.

$$\mathrm{Par}_B(S^1) \in Z(G)(S^1)^\vee \otimes Z(T)(S^1)$$
$$\|$$
$$\mathrm{DMod}_\kappa(\mathcal{L}G/\mathcal{L}N)$$

l Parp between Z_G and Z_M (M Levi)

$M = T$.

$Z(G)(S^1)^\vee \otimes Z(T)(S^1)$

N)

$\langle Neu_G, Par_B \rangle (S^1)$

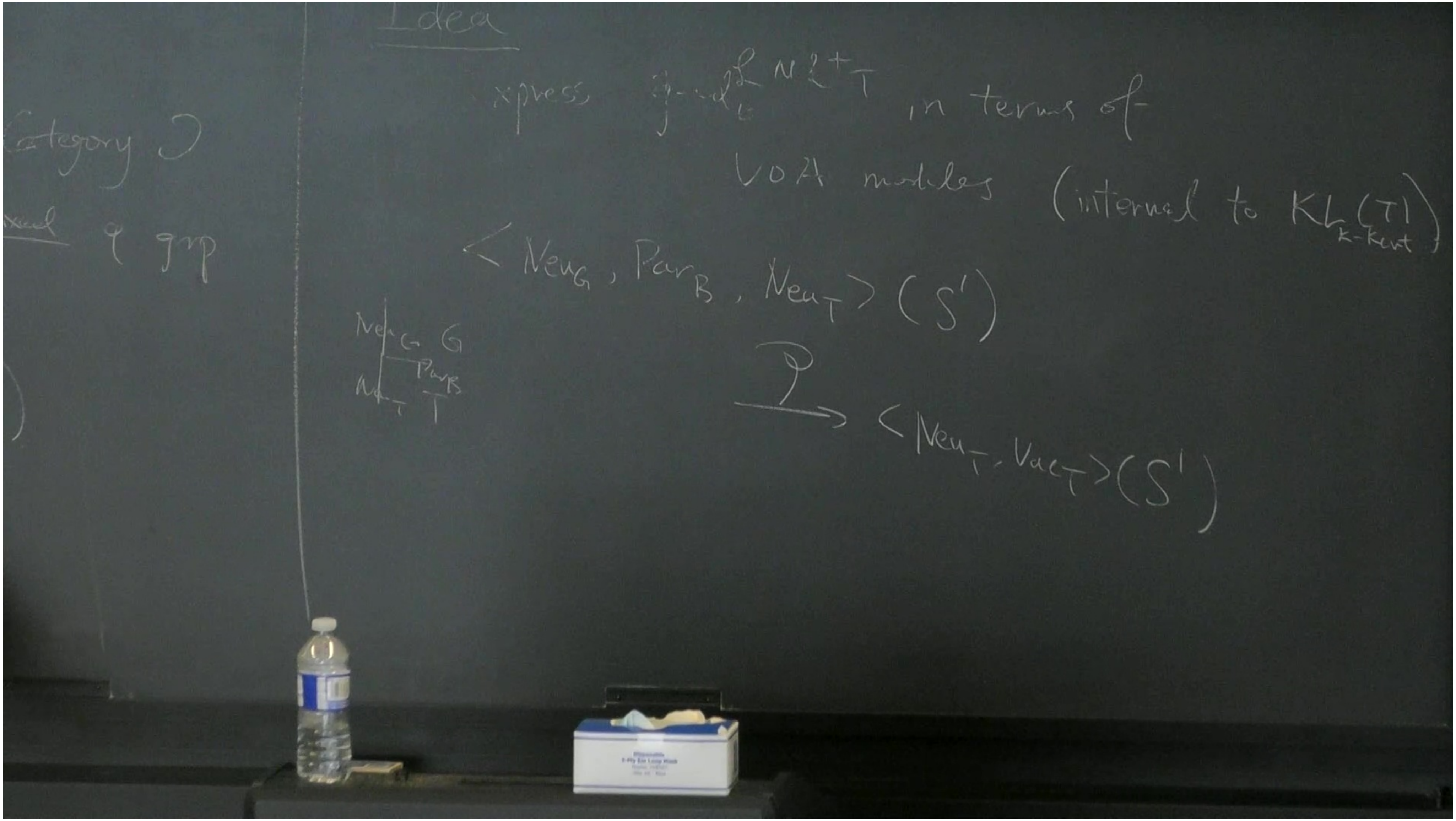
$= \mathfrak{g}\text{-mod}_K^{LN} \in$

$DMod_{K\text{-}Kont}(LT)$

$\langle Neu_G, Par_B, Vac_T \rangle (S^1)$

$= \boxed{\mathfrak{g}\text{-mod}_K^{LN L^+ T}}$


Thm (joint w/ Lin Chen)
$\hat{\mathfrak{g}}\text{-mod}_k^{LN\mathcal{L}^+T} \simeq$ BGG Category $\mathcal{O}$
for mixed q grp
(Fact: $\hat{\mathfrak{g}}\text{-mod}_k^{LN\mathcal{L}^+T} \simeq \hat{\mathfrak{g}}\text{-mod}_k^{I}$)
I Iwahori
not t-exact


Idea
in terms of
VOA modules (internal to
<Neu_G, Par_B, Neu_T>(S')
<Neu_T, Vac_T>(S')
Category ?
q grp

Neu_G G

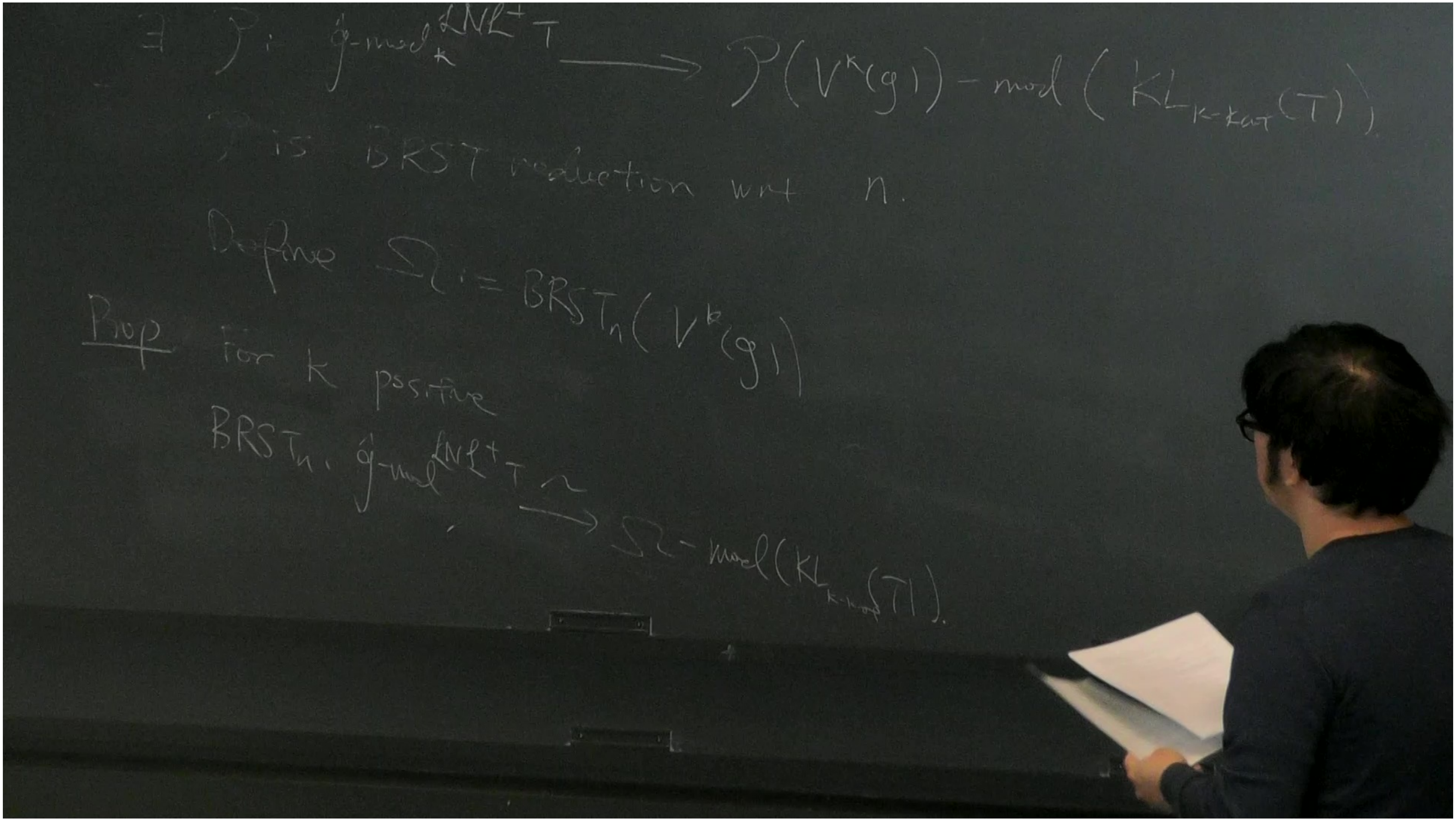
Par_B

Neu_T T

$\langle \mathrm{Neu}_G, \mathrm{Par}_B, \mathrm{Neu}_T \rangle (S^1)$

$$\xrightarrow{\ \mathcal{P}\ } \langle \mathrm{Neu}_T, \mathrm{Vac}_T \rangle (S^1)$$

Moreover, any $\mathcal{P}(\mathcal{L})$ is a module over the VOA object $\mathcal{P}(\mathcal{L}_{triv})$


$\exists\ \mathcal{P} : \hat{\mathfrak{g}}\text{-mod}_k^{LNL^+T} \longrightarrow \mathcal{P}(V^k(\mathfrak{g}))\text{-mod}(KL_{k-k_{crit}}(T))$.
$\mathcal{P}$ is BRST reduction wrt $\mathfrak{n}$.
Define $\Omega := BRST_{\mathfrak{n}}(V^k(\mathfrak{g}))$
Prop For k positive
$BRST_{\mathfrak{n}} : \hat{\mathfrak{g}}\text{-mod}^{LNL^+T} \xrightarrow{\sim} \Omega\text{-mod}(KL_{k-k_{crit}}(T))$.

$\longrightarrow \mathcal{P}(V^k(\mathfrak{g}))\text{-mod}(KL_{k\text{-}Kat}(T))$

...ction wrt n.

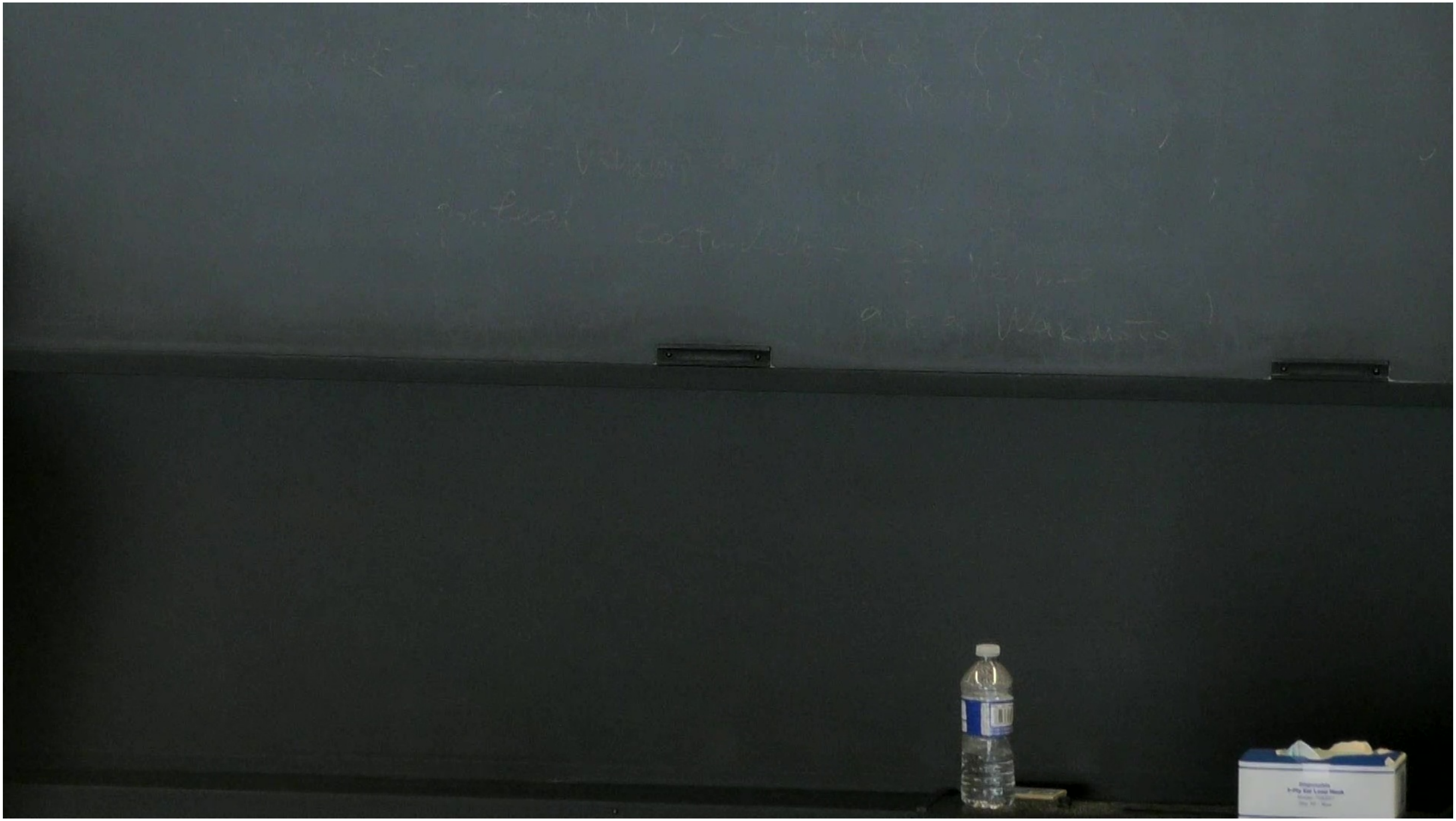
Further:

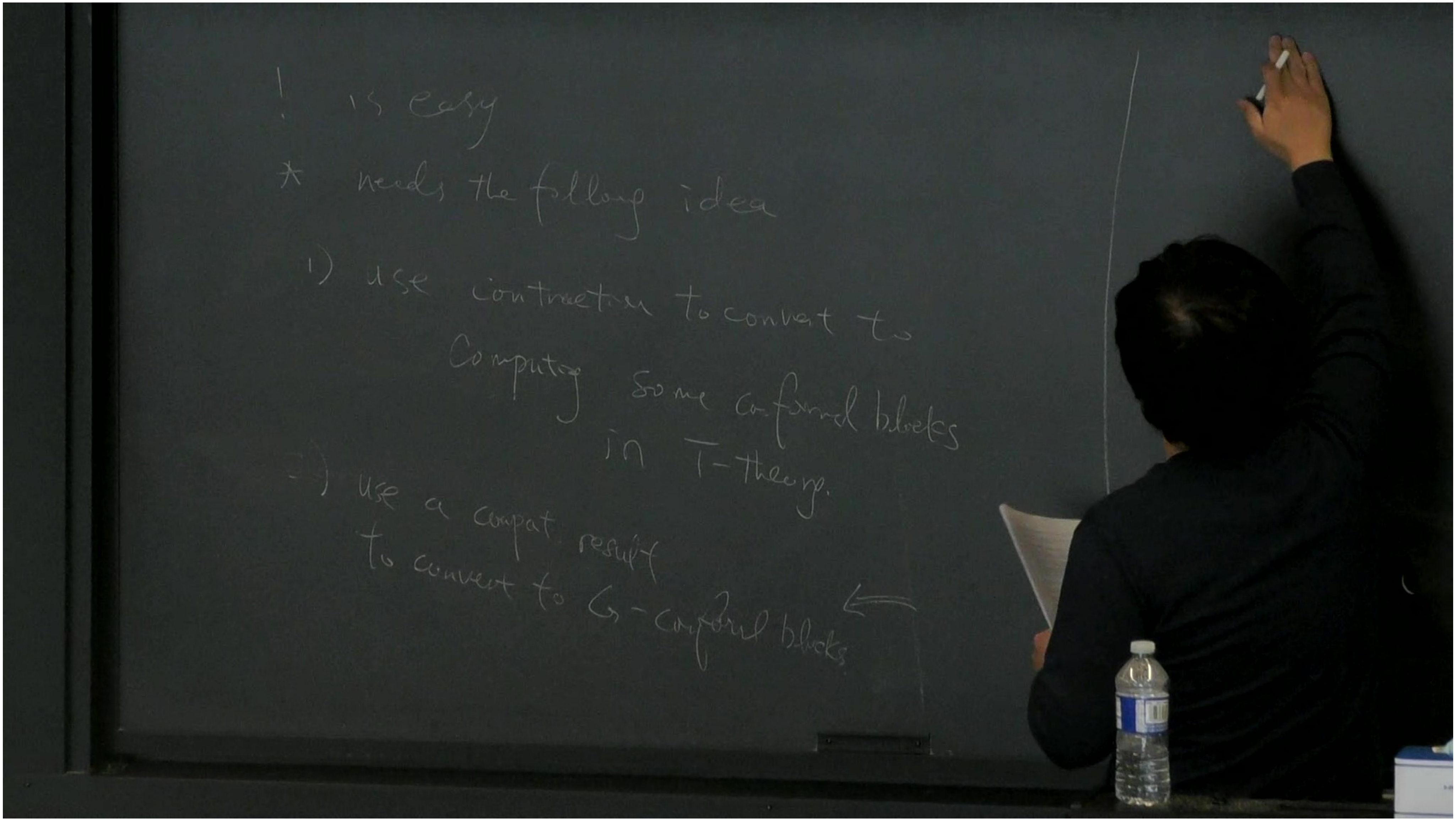
Ω is topological

i.e. an $\mathbb{E}_2$-algebra object

$T_n(V^k(\mathfrak{g}))$

$\Omega\text{-mod}(KL_{k\ldots}(T))$.




! is easy
* needs the following idea
1) use contraction to convert to
Computing some conformal blocks
in T-theory.
2) use a compact result
to convert to G-conformal blocks

In general:

$$\exists : \mathcal{B}(\mathring{\mathbb{D}}_x) \xrightarrow{\mathrm{Loc}} \mathrm{DMod}_\kappa(\mathrm{Bun}_G^{\infty\text{-level}, x})$$

Moreover:

$$\mathcal{B}' \longrightarrow \mathcal{B}$$

$$\mathrm{Hom}(\mathcal{B}'(\mathring{\mathbb{D}}_x), \mathcal{B}(\mathring{\mathbb{D}}_x)) \longrightarrow \mathrm{Hom}(\mathcal{B}'(\mathring{\mathbb{D}}_x), \mathrm{DMod}_\kappa(\mathrm{Bun}_G^{\infty\text{-level}, x}))$$

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<u>Example:</u>

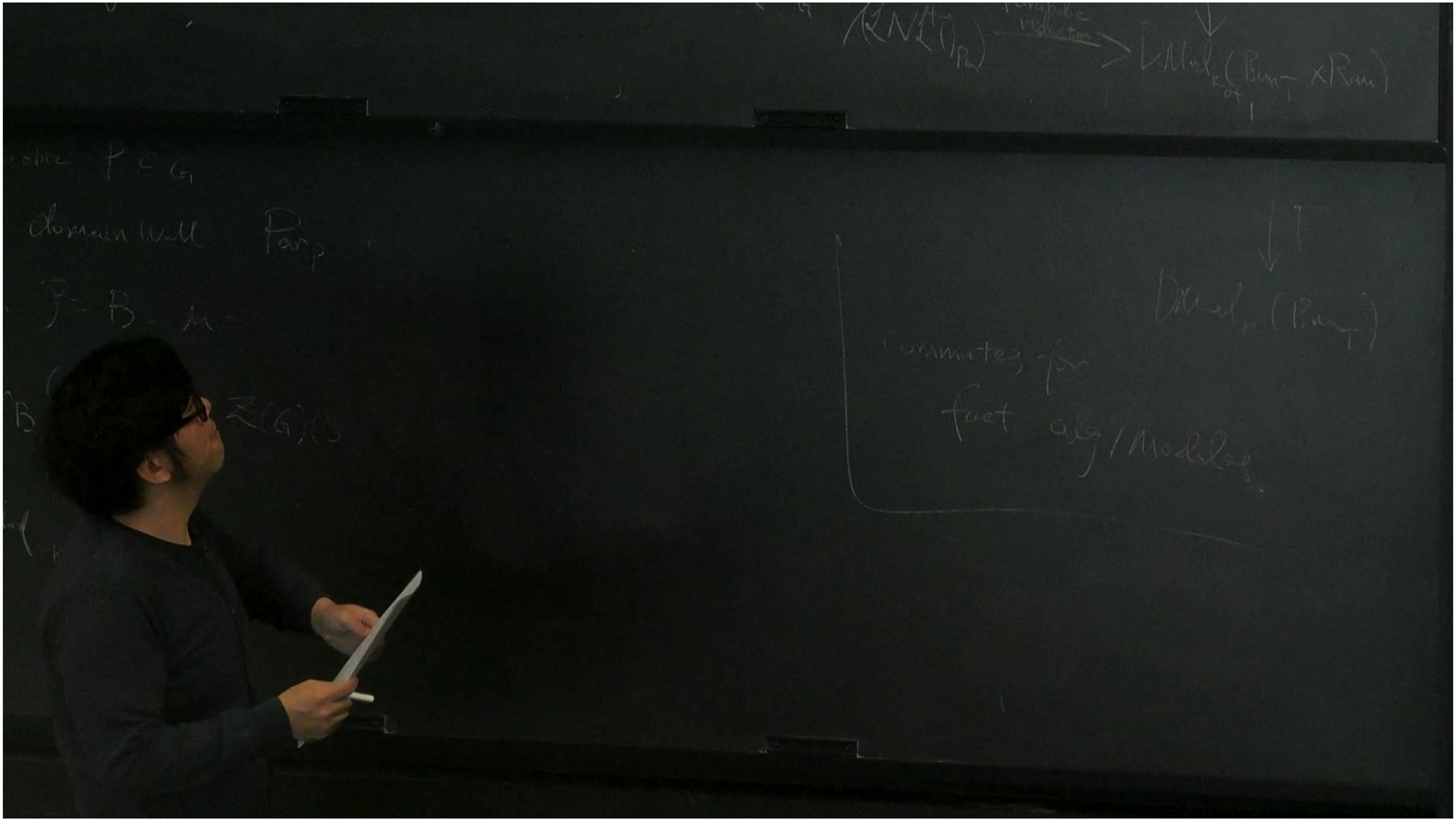
$\mathcal{B} = \mathrm{Neum}$, $\mathcal{B}' = \mathrm{Vac}_G$

$$KL_\kappa(G) \xrightarrow{\text{Loc}} \mathrm{DMod}_\kappa(\mathrm{Bun}_G)$$

$$\hat{\mathfrak{g}}\text{-mod}_\kappa^{\mathcal{L}N\mathcal{L}^+T} \xrightarrow{\text{Loc}} \text{D-mod}_\kappa\left(\text{Bun}_G^{\infty\text{-level}} / \mathcal{L}N\mathcal{L}^+T\right)$$

Moreover ∃ multi-pt version

$\text{KL}_\kappa(G)$


domain wall
Commutes for
fact alg/Modules