Title: Cosmology from random entanglement

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Abstract: Obtaining a description of cosmology is a central open problem in holography. Studying simple models can help us gain insight on the generic properties of holographic cosmologies. In this talk I will describe the construction of entangled microstates of a pair of holographic CFTs whose dual semiclassical description includes big bang-big crunch AdS cosmologies in spaces without boundaries. The cosmology is supported by inhomogeneous heavy matter and it partially purifies the bulk entanglement of two auxiliary AdS spacetimes. In generic settings, the cosmology is an entanglement island contained in the entanglement wedge of one of the two CFTs. I will then describe the properties of the non-isometric bulk-to-boundary encoding map and comment on an explicit, state-dependent boundary representation of operators acting on the cosmology. Finally, if time allows, I will argue for a non-isometric to approximately-isometric transition of the encoding of "simple" cosmological states as a function of the bulk entanglement, with tensor network toy models of the setup as a guide.

Zoom link https://pitp.zoom.us/j/93483219872?pwd=TGpJVVlaNDVGWTN5ZHpkTHp6a2hTZz09

Cosmology from random entanglement

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November 1, 2023

Perimeter Institute

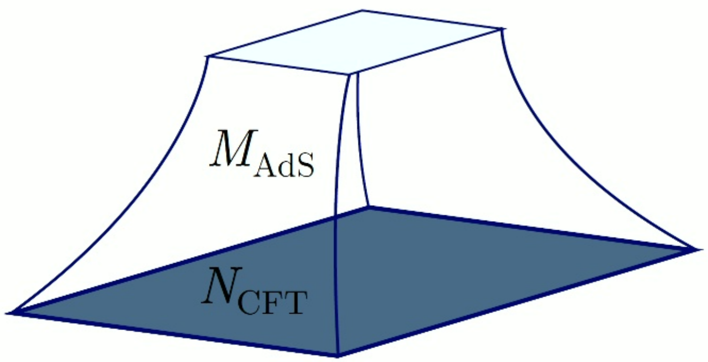
[SA, Sasieta, Swingle: 2307.14416]

- Introduction
- Cosmic PETS
- Cosmology-to-boundary map and non-isometricity

Holography and AdS/CFT correspondence

- Gravity is "holographic": $S_{BH} \propto A_{horizon}$

Quantum gravity theory in asymptotically AdS_{d+1} spacetime $\Longleftrightarrow$ d-dimensional CFT living on the asymptotic boundary



$$Z_{QG}[\phi_0] = Z_{CFT}[J]$$

$$Z_{QG}[\phi_0] \approx \mathrm{e}^{-S_E^{saddle}[\phi_0]}$$

1

Cosmology and AdS/CFT

Any quantum gravity theory aiming at describing physics in our universe must be able to describe cosmology. How can we obtain a (quantum) cosmology in holography?

- There (at least) three problems to describe a realistic cosmology:
  - AdS/CFT describes QG with $\Lambda < 0$, our universe (maybe) has $\Lambda > 0$
  - AAdS spacetimes are "asymptotically empty", FLRW spacetimes are homogeneously filled with stuff
  - FLRW spacetimes (even with $\Lambda < 0$) have no asymptotic boundary where to define the dual CFT
- The formulation of a holographic cosmological framework is a central open problem in the field

Approaches to holographic cosmology

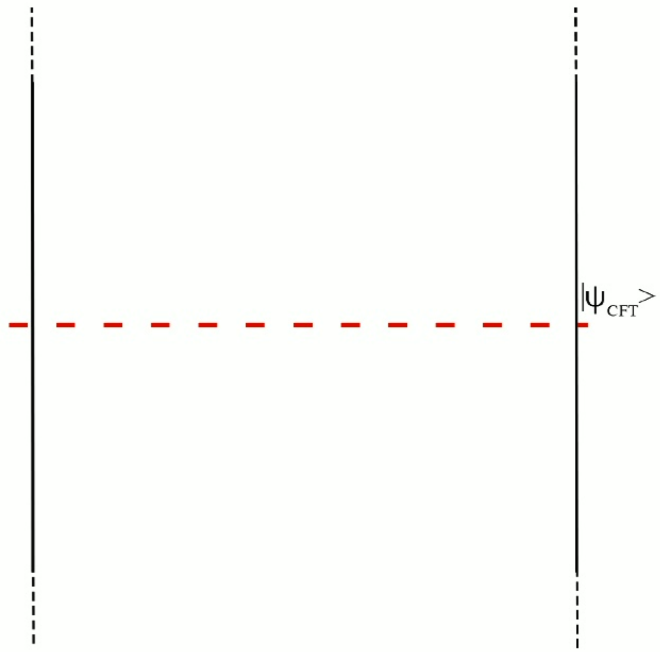
Various attempts have been made to describe cosmology holographically:

- de Sitter holography → Require a new holographic framework
  [Dong, Gorbenko, Silverstein, Torroba, Susskind,...]
- Celestial holography → Require a new holographic framework
  [Strominger, Pasterski, Puhm, Raclariu...]
- "Holographic cosmology" → Dual QFT has complex parameters
  [McFadden, Skenderis]
- Braneworld cosmologies → $\Lambda \leq 0$, 4D gravity only in limited regime
  [Karch, Randall, Sundrum, Cooper, Rozali, Swingle, Van Raamsdonk, Waddell, Wakeham, SA,...]
- Cosmology from Euclidean wormholes → $\Lambda < 0$, complicated encoding in the dual theory
  [Maldacena, Maoz, Chen, Gorbenko, SA, Simidzija, Swingle, Van Raamsdonk, Waddell]

Our approach

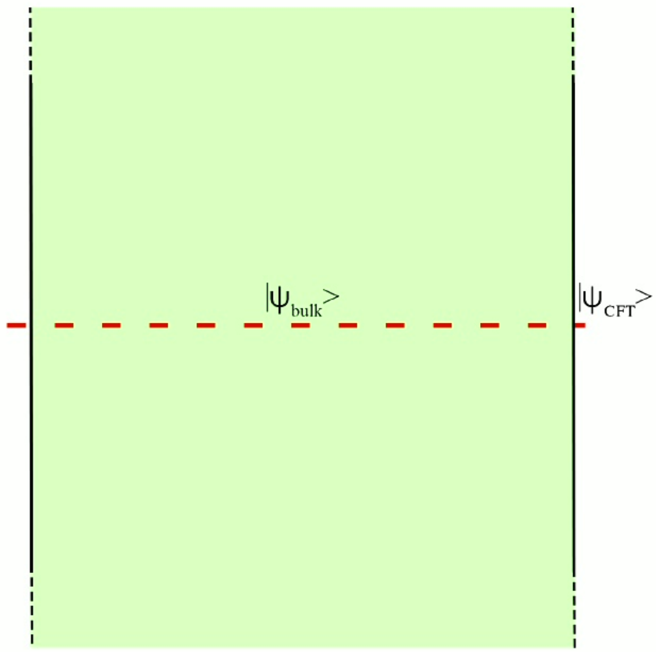
- We simplify the problem by not requiring the cosmological universe to be realistic
- We build $\Lambda < 0$ cosmologies which are:
  - Closed (compact spatial slices)
  - Inhomogeneous
  - Time-symmetric (they have a big bang and a big crunch)
- Advantages:
  - The cosmological state can be defined using a gravitational path integral in the saddle point approximation
  - There is a well-understood dual description (entangled state of two CFTs)
  - We can study in some detail the properties of the bulk-to-boundary map
- From our simple construction we could learn more general lessons about cosmology in AdS/CFT and the holographic description of disconnected spacetimes

States from Euclidean path integral



5

States from Euclidean path integral

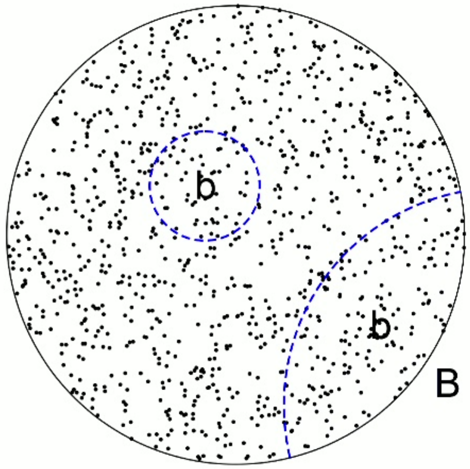


- Different CFT states = different Lorentzian geometries

5

Quantum extremal surfaces

- Entanglement entropy of subregion B is computed by area of Ryu-Takayanagi (RT) surface
- Including entanglement entropy of bulk fields, RT is replaced by QES, and "entanglement islands" can appear

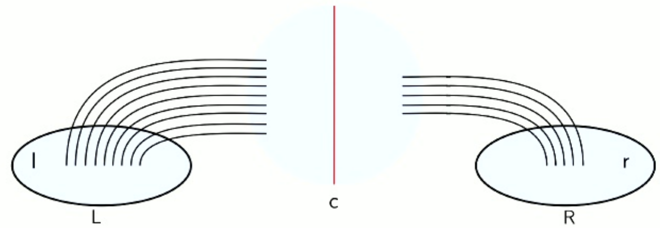


$$S(B) = \min \text{ext}_\gamma \left[\frac{\mathcal{A}(\gamma)}{4G} + S_{bulk}(b) \right]$$

- From B we can reconstruct everything in the "entanglement wedge" $W(B)$ (including the island!)

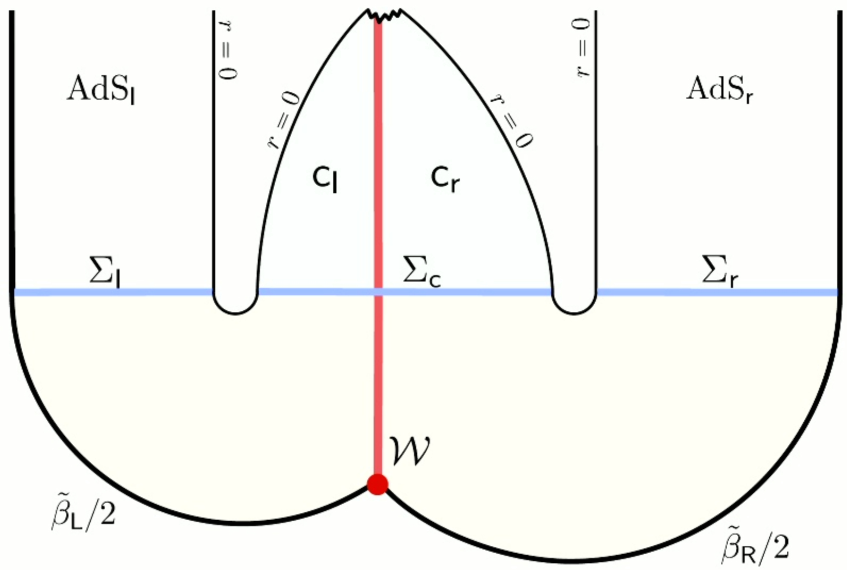
Main results

1. Cosmology entangled with two auxiliary AdS spacetimes $\Leftrightarrow$ partially entangled thermal state (PETS) of two holographic CFTs



2. If bulk entanglement is enough, the cosmology is an "entanglement island" for one of the CFTs
3. The map encoding the cosmology into the dual theory is non-isometric (but its kernel can be trivial in AdS/CFT)
4. An explicit state-dependent reconstruction for operators on the cosmology can be built
5. There is a non-isometric to nearly-isometric phase transition as the bulk entanglement grows (studied in a tensor network toy model)

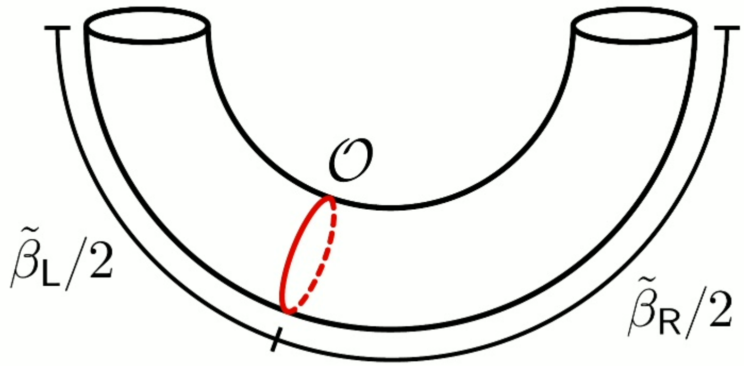
Bulk setup



$$\dot{R}^2 - f(R) + \left(\frac{4\pi G m}{(d-1)V_\Omega R^{d-2}}\right)^2 = 0$$

- $\beta_\mathsf{l}, \beta_\mathsf{r}$ are large: we are below Hawking-Page temperature
- Bulk fields are entangled between AdS$_{\mathsf{l,r}}$ and $\mathsf{c_{l,r}}$

The dual state: PETS

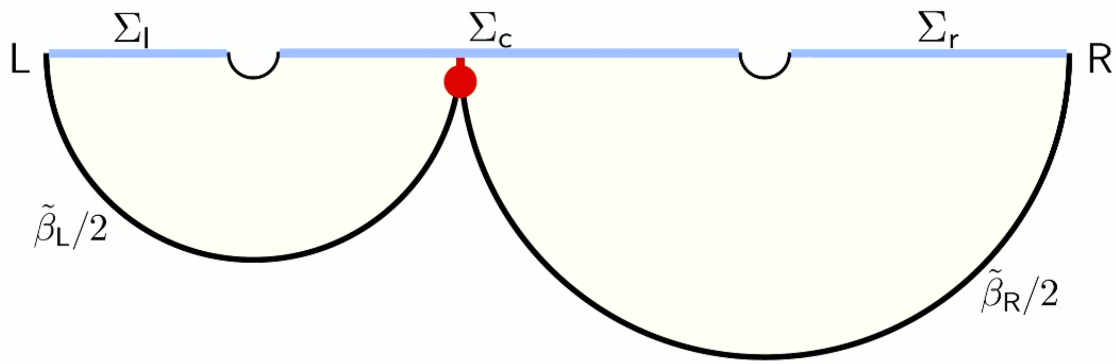


$$|\Psi_{\mathcal{O}}\rangle = |\rho_{\tilde{\beta}_{\mathsf{L}}/2}\mathcal{O}\rho_{\tilde{\beta}_{\mathsf{R}}/2}\rangle = \frac{1}{\sqrt{Z_1}}\sum_{n,m} e^{-\frac{1}{2}(\tilde{\beta}_{\mathsf{L}}E_n+\tilde{\beta}_{\mathsf{R}}E_m)}\mathcal{O}_{nm}\,|E_n\rangle_{\mathsf{L}}^{*}\otimes|E_m\rangle_{\mathsf{R}}$$

$$\mathcal{O} \sim \prod_{i=1}^{O(N)} \mathcal{O}_{\Delta}(\Omega_i)$$

- The left-right entanglement is small (order 1). The entanglement of bulk fields is also small

Large mass limit



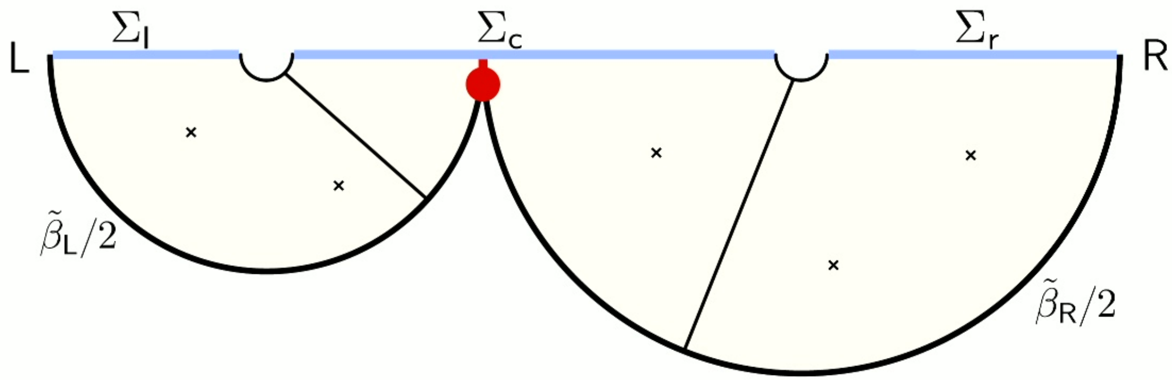
- For $m\ell \gg 1$ the bulk state approximately factorizes:

$$|\psi_0\rangle \approx |\Phi_-\rangle_{lc_l} |\Phi_+\rangle_{c_r r}$$

- Evaluation of bulk partition functions greatly simplifies in this limit

Excited states

- We can consider more general "excited states":



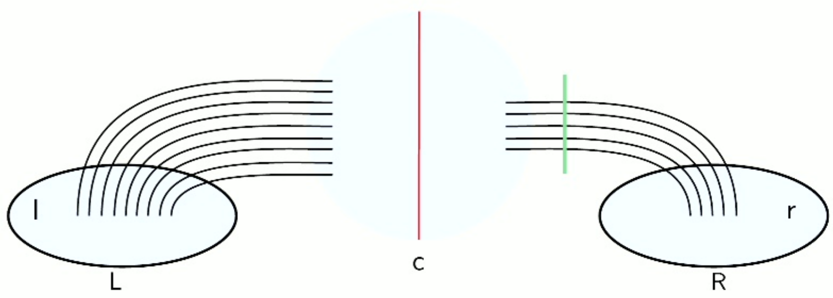
- With these insertions we can create larger entanglement between AdS regions and cosmological regions
- Bulk insertions will correspond to (non)local insertions in the dual CFT path integral

Entanglement entropy and island formula

- The cosmology could be encoded in the left or right CFTs, or in the union of the two
- The entanglement entropy of the left CFT (same as right CFT) contains this information
- For $\exp(|S(\rho_l) - S(\rho_r)|) \gg 1$ we get

$$S(\rho_L) \approx \min\{S(\rho_l), S(\rho_r)\} = \min\{S(\rho_l), S(\rho_{lc})\}$$

- The QES is the empty set, and the cosmology $c = c_l \cup c_r$ is an island for either the left or the right CFT:



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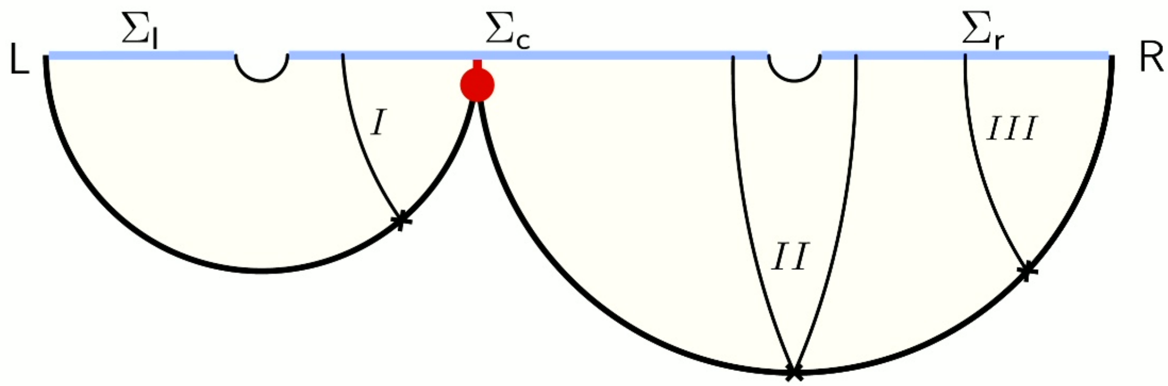
Preparing different cosmological states

- We want to study the encoding of the cosmology into the dual theory
- For simplicity, let us restrict to (not too) heavy operator insertions satisfying the geodesic approximation:



- We will focus on type I insertions, which prepare the state of a particle in the cosmology

Cosmology-to-boundary map

- Start with a reference bulk state $|\psi\rangle$ (fixed state on AdS regions and fixed cosmology-AdS entanglement)
- Consider type I insertions on top of this state:

$$|\psi_I\rangle = \frac{1}{\sqrt{z_I}} \prod_i \phi_i(y_i) |\psi\rangle$$

- The dual state is

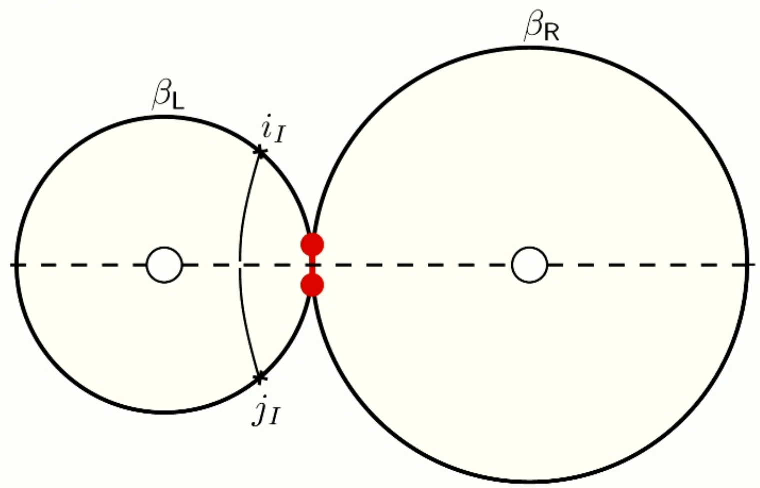
$$|\Psi_I\rangle = |\frac{1}{\sqrt{Z_I}} \mathcal{T}\{\mathcal{O}(\tilde{\beta}_{\mathrm{R}}/2)\mathcal{O}_\Psi(\mathbf{x}_\Psi)\mathbb{O}_I(\mathbf{x}_I)\}\rangle$$

- The (restricted) cosmology-to-boundary map is defined as

$$V_\psi |\psi_I\rangle = |\Psi_I\rangle$$

Non-isometricity of the cosmology-to-boundary map

- In the bulk EFT, different insertions correspond to orthogonal states (different particles in cosmology): $g_{ij} \equiv \langle \psi_{I_i} | \psi_{I_j} \rangle = \delta_{ij}$
- To understand whether V_ψ is isometric, we must study the CFT Gram matrix $G_{ij} \equiv \langle \Psi_{I_i} | \Psi_{I_j} \rangle = \langle \psi_{I_i} | V_\psi^\dagger V_\psi | \psi_{I_j} \rangle$
- We can compute CFT overlaps using the dual gravitational path integral:

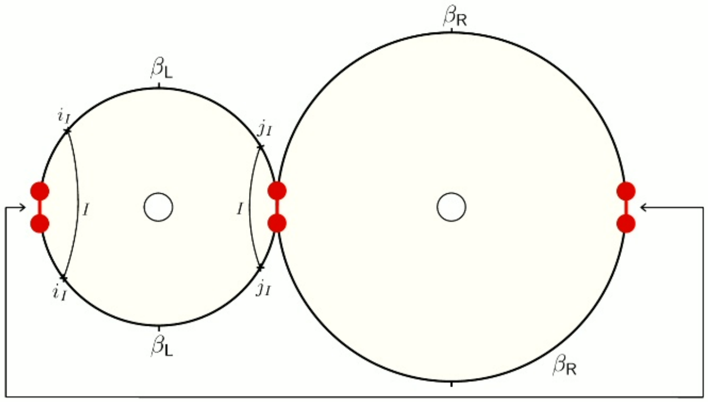


$$\overline{G_{ij}} = \delta_{ij}$$

15

Non-isometricity of the cosmology-to-boundary map

- The square of the overlaps computed with the gravitational path integral does not vanish due to Euclidean wormholes:



$$\overline{|G_{ij}|^2} \approx e^{-[S_2(\rho_r)+S_2(\rho_l)]}$$

- This is the typical size of off-diagonal overlaps (gravity computes average over ensemble of shell operators [Sasieta 2211.11794])

The map is non-isometric!

A bag-of-gold paradox?

A non-isometric map does not imply the existence of null states!

- For any set S of K bulk EFT orthogonal states, in the full CFT we expect $\text{rank}(G_{ij}) = K$ due to thermal tails: the map is non-isometric but invertible
- If we truncate to a microcanonical band (ignoring the thermal tails), a resolvent calculation yields

$$\text{rank}(G^E_{ij}) = \min\{K, e^{\mathbf{S}}\}$$

In a microcanonical band, $\ker(V_\psi) \neq \emptyset$ and some cosmological states are equivalent!

State-dependent reconstruction

- Operators acting in the cosmology can be reconstructed using a Euclidean reconstruction:

$$\Phi_{IJ} = \langle \Psi_I | \Phi | \Psi_J \rangle = \langle \Psi_I | \, V_\psi \phi(x) V_\psi^{-1} \, | \Psi_J \rangle$$
$$= \frac{1}{\sqrt{Z_I Z_J}} \mathrm{Tr}(\mathcal{T}\{\mathcal{O}(\tilde{\beta}_{\mathrm{R}}/2) \mathcal{O}_\Psi(\mathbf{x}_\Psi) \mathbb{O}_J(\mathbf{x}_J) \mathcal{O}_\phi(\mathbf{x}_\phi) \mathcal{O}^\dagger(-\tilde{\beta}_{\mathrm{R}}/2) \mathcal{O}_\Psi^\dagger(\mathbf{x}'_\Psi) \mathbb{O}_I^\dagger(\mathbf{x}'_I)\})$$

- The Euclidean time-ordering makes the reconstruction manifestly state-dependent

Conjecture. *The global bulk-to-boundary map is non-isometric iff there exists a bulk region in which the reconstruction of operators is state-dependent. If this region exists, it is causally disconnected from the global boundary.* *[Antonini, Bao, Cao, Chemissany - work in progress]*

- This is valid for old evaportating black hole interiors and any disconnected cosmological spacetime in AdS/CFT

18

Open questions

- If S_l, S_r are small, does the cosmological EFT break down?
- Cosmological states can be distinguished one-shot in the bulk but not in the boundary: bulk vs boundary experiments?
- How to consistently dress operators in the cosmology?
- Precise relationship between non-isometricity, state-dependence, and causal disconnectivity? [Antonini, Bao, Cao, Chemissany - work in progress]

Thank you! Questions?

“Cosmic PETS”, by Brian Swingle (Midjourney)