

Title: Twisted Holography Mini-Course - Lecture 20231130

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Collection: Twisted Holography Mini-Course

Date: November 30, 2023 - 9:00 AM

URL: <https://pirsa.org/23110042>

$$\underbrace{X, Y, b, c}_{\phi^n}$$

$$X(z)Y(u) \sim -\frac{h}{z-u} \delta_{\tau}^c \delta_{\tau}^a$$

\Leftarrow

$$O(\phi) = \int_{(u,\tau)(u',\tau')} \mathcal{T} \phi^{a_1} \dots \phi^{a_n}$$

$$b \rightarrow [c, b] + [c, \tau]$$

$$X \rightarrow [c, X]$$

$$S \mathcal{T} X^n Y^m$$

$$Z(u; z) = X(z) + uY(z)$$

$$\mathcal{T} Z(u; z)^m = \sum_m u^m S \mathcal{T} X^{n-m} Y^m$$

$$Z(u; z) Z(u'; z') \sim h \frac{u-u'}{z-z'}$$

$$\Sigma K[X, Y] \quad O(Y, \tau)$$

$\mathcal{T} X^n$ IS CLOSED! $\Rightarrow \mathcal{T} Z(z, u)^n$ IS CLOSED!

$$\underbrace{x, y, b, c}_{\phi^n}$$

$$X(z)Y(u) \sim -\frac{\hbar}{z-u} \delta_{\tau}^i \delta_{\tau}^j$$

\rightleftharpoons

$$O(d) = \int_{(b, \tau) \times (c, \tau)} \mathcal{T}_2 \partial \phi^{a_1} \dots \partial \phi^{a_n}$$

$$b \rightarrow [c, b] + [c, \tau]$$

$$X \rightarrow [c, X]$$

$$S \mathcal{T}_2 X^m Y^m$$

$$Z(u; z) = X(z) + uY(z)$$

$$\mathcal{T}_2 Z(u; z)^m = \sum_m u^m S \mathcal{T}_2 X^{m-m} Y^m$$

$$\Sigma \mathcal{K}[x, \tau] \quad \mathcal{O}(y, \tau)$$

$$Z(u; z) Z(u'; z') \sim \hbar \frac{u-u'}{z-z'}$$

$\mathcal{T}_2 X^n$ IS CLOSED! $\Rightarrow \mathcal{T}_2 Z(z, u)^n$ IS CLOSED!

$$\mathcal{T}_2 b \cdot Z(u; z)^n$$

$$\mathcal{T}_2 b X^m \rightarrow \mathcal{T}_2 [x, \tau] X^m = 0$$

$$c[x, \tau]$$

$$\hbar N \mathcal{T}_2 \partial X X^m + \mathcal{E} \mathcal{T}_2 \dots$$

$$O(d) = \int_{(a,b) \times (c,d)} \tau \partial \phi^{a_1} \dots \partial \phi^{a_n}$$

$$b \rightarrow [c, b] + [c, \tau]$$

$$X \rightarrow [c, X]$$

$$S \int_{\mathbb{R}} X^n Y^m \quad Z(u; z) = X(z) + uY(z)$$

$$\int_{\mathbb{R}} Z(u; z)^m = \sum_m u^m \int_{\mathbb{R}} X^{n-m} Y^m$$

$$Z(u, z) Z(u', z') \sim k \frac{u-u'}{z-z'}$$

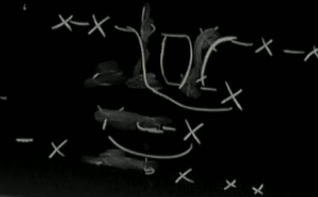
$\int_{\mathbb{R}} X^n$ IS CLOSED! $\Rightarrow \int_{\mathbb{R}} Z(z, u)^m$ IS CLOSED!

$$\int_{\mathbb{R}} b \circ Z(u, z)^m$$

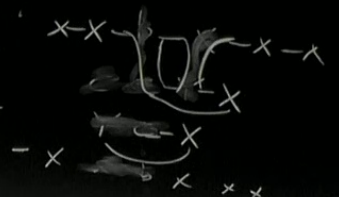
$$+ \int_{\mathbb{R}} b \circ \tau \int_{\mathbb{R}} Z^n$$

$$\int_{\mathbb{R}} b X^m \rightarrow \tau [X, Y] X^m = 0$$

$$\rightarrow \sum k \tau \cdot \tau$$



$T_2 X^n$ IS CLOSED! $\Rightarrow T_2 Z(z, u)^n$ IS CLOSED!
 $B_m \partial \left[T_2 b Z(u)^n \right]$
 $T_2 b Z(u)^n \rightarrow T_2 b Z^2$
 $T_2 b X^n \rightarrow T_2 c(x, y)$
 $T_2 c(x, y) X^n = 0$
 $\partial Y = C_1 \partial X + D_1 C_2 \partial Y$



$T_2 \partial \phi \phi \phi \phi$
 $C_1 \left[T_2 \partial c Z^n(u) \right] \leftarrow T_2 \partial c X^n$
 $T_2 \partial X X^n = \frac{1}{n} \partial T_2 X^{n+1}$
 $T_2 \partial Y X^n \rightarrow T_2 [D_1 c, Y] X^n$
 $D_m \left[T_2 \partial Y X^n + ? T_2 \partial X Y X^n \right]$

$T_2 X Y X Y$
 $C_1(z, \theta_1, \theta_2) = c + \theta_1 X + \theta_2 Y + \theta_3 \theta_2$
 $d \cdot \theta \rightarrow da \quad z \rightarrow dz$
 $Q_0 C_1 = [C_1, C_1]$
 $Q_0 dC_1 = [C_1, dC_1]$
 $T_2 dC_1^n$
 $B_{PST} \text{ CLOSED} \Rightarrow \text{TREE} + \text{PLANE}$

$\Rightarrow \mathbb{T} Z(z, u)^n$ IS CLOSED!
 $B_m \supset \mathbb{T} b Z(u)^n$
 $\mathbb{T} b X^m \rightarrow \mathbb{T}(x, y) X^m = 0$
 $\mathbb{T} b Z^m$
 $c(x, y)$
 $\rightarrow \mathbb{T} \& \mathbb{T} \cdot \mathbb{T} \cdot$

$C_1 \supset \mathbb{T} \partial c Z^n(u) \leftarrow \mathbb{T} \partial c X^m$

$\mathbb{T} \partial X X^m = \frac{1}{m} \mathbb{T} X^{m+1}$

$\mathbb{T} \partial Y X^m \rightarrow \mathbb{T}(c, y) X^m$

$D_m \supset \mathbb{T} \partial Y X^m + ? \mathbb{T} \partial X Y X^m$
 $glu[z, 0, 0]$

$\mathbb{T} X Y X Y$

$C(z, \theta_1, \theta_2) = c + \theta_1 X + \theta_2 Y + \theta_1 \theta_2$

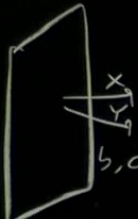
$d \cdot \theta \rightarrow d\theta \quad z \rightarrow dz$

$Q_0 C = [C, C]$

$Q_0 dC = [C, dC]$

$\mathbb{T} dC^n$

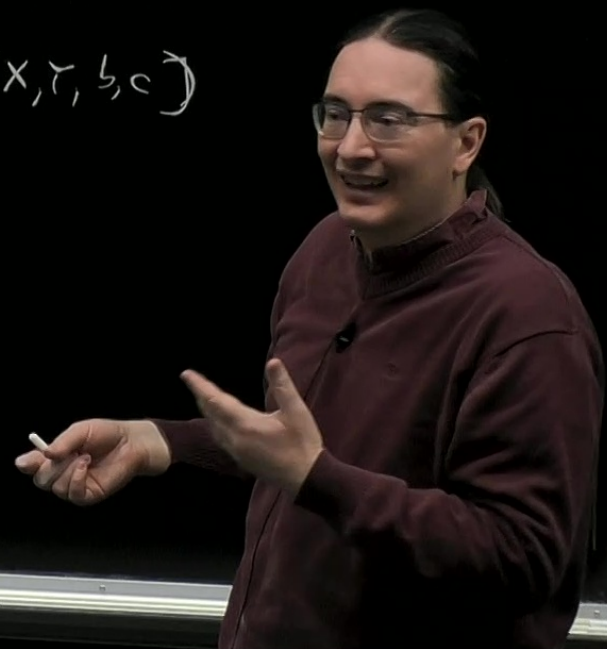
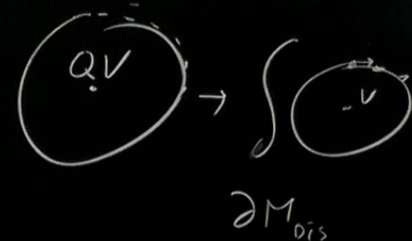
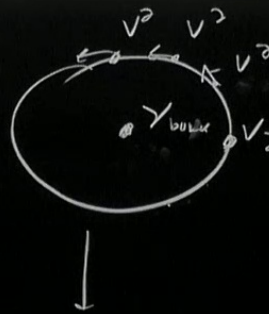
$B P S Y$ CLOSED \Rightarrow TREE + PLAMM



b, c : GAUGE
GHOSTS

$$N \times D1 \subset \mathbb{C} \times \{0,0\} \subset \mathbb{C} \times \mathbb{C}^2$$

$$S_{\text{BRANE}} = \int -\frac{1}{4} \Sigma X \bar{\Sigma} \Psi + \int \bar{L} \bar{\partial} c + \sum \text{DERIVATIVES OF BULK FIELD} \times \mathbb{R} \mathbb{O}_i(x, \tau, b, c)$$



$$\underbrace{x, y, b, c}_{\phi^n}$$

$$X(z)Y(u) \sim \frac{h}{z-u} \delta_{\epsilon}^i \delta_{\epsilon}^j$$

$$\mathcal{O}(d) = \int_{\mathcal{D}} \mathcal{T}_z \partial \phi^{a_1} \dots \partial^n \phi^{a_n}$$

$$Q_0 \mathcal{O}(d) = \mathcal{O}(Qd)$$

$$b \rightarrow [c, b] + [x, \tau]$$

$$x \rightarrow [c, x]$$

$$\partial c^i \rightarrow [c, \partial c^i]$$

$$\Sigma K[x, \tau] \quad \mathcal{O}(y, \tau)$$

$$S \mathcal{T}_z X^m Y^n$$

$$Z(u; z) = X(z) + uY(z)$$

$$A_m \partial^n \int_{\mathcal{D}} Z(u; z)^m = \sum_m u^m S \mathcal{T}_z X^{n-m} Y^m$$

$$Z(u; z)$$

$$\int_{\mathcal{D}} X^n \text{ is}$$

$$\sim h \frac{u-u'}{z-z'}$$

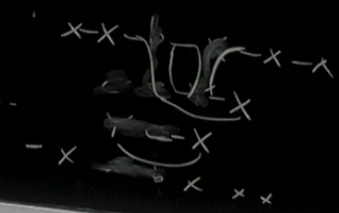
$$J_{PSTF} = bcc \quad cxy$$

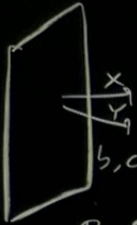
$$B_m \partial^n \int_{\mathcal{D}} b \partial z$$

$$\int_{\mathcal{D}} Z(z, u)^n$$

IS CLOSED!

$$\int_{\mathcal{D}} b X^n \rightarrow \int_{\mathcal{D}} [x, y] X^n = 0$$

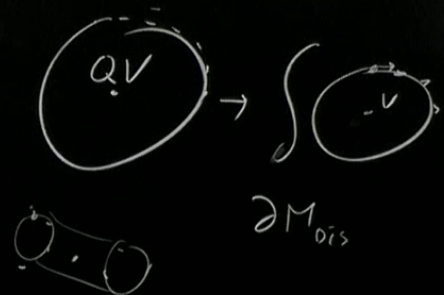
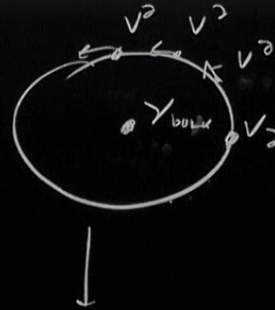


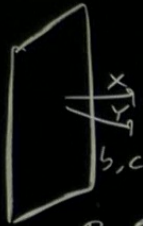


b, c : GAUGE
GHOSTS

$$N \times D1 \subset \mathbb{C} \times \{0,0\} \subset \mathbb{C} \times \mathbb{C}^2$$

$$S_{\text{BRANE}} = \int \frac{1}{4} \Sigma X^2 Y + \mathcal{L} b \delta c + \sum \text{DERIVATIVES OF BULK FIELD} \times \mathbb{C} \cdot (X, Y, b, c)$$



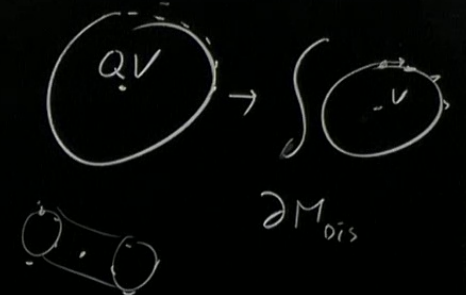
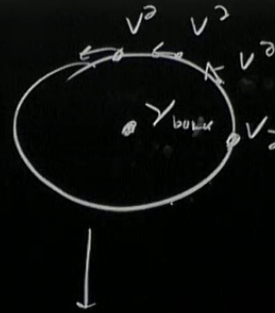


$$N \times D1 \subset \mathbb{C} \times \{0,0\} \subset \mathbb{C} \times \mathbb{C}^2$$

$$S_{\text{BRANE}} = \int \frac{1}{4} \Sigma X \bar{Y} + \epsilon \bar{b} \delta c + \sum_{\text{DERIVATIVES OF BULK FIELD}} * \mathbb{Z} \mathbb{O}_i(x, \tau, b, c) + N \cdot \text{BULK FIELD}$$

ADD

$$\text{INFIM} \tau \in \mathbb{M} \subset \mathbb{R} = \epsilon_i \beta_i \rightarrow \langle \tau, \mathbb{Z} \mathbb{O}_i \rangle$$

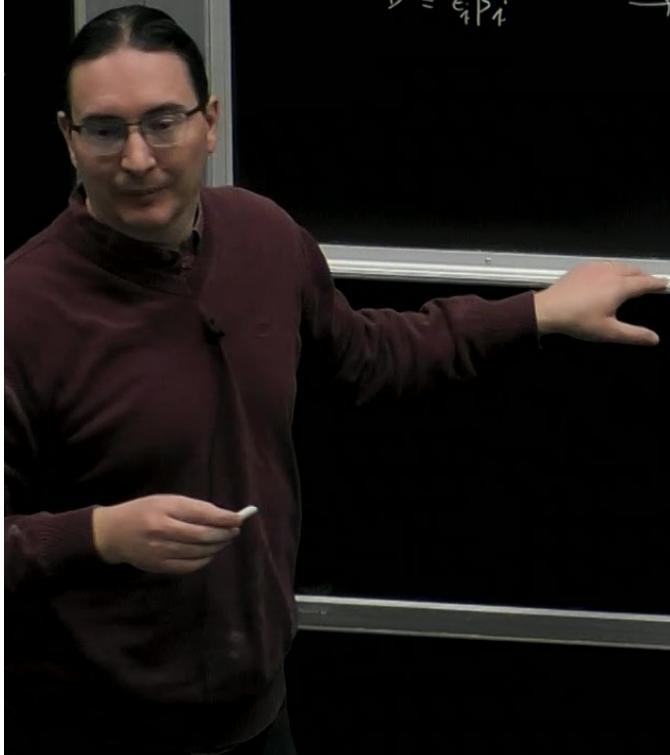


$$S_{\text{BRANE}} = \int \frac{1}{4} \Sigma X \bar{\Sigma} \Psi + \int \bar{\Sigma} \bar{\Sigma} \bar{\Psi} C + \sum \text{DERIVATIVES OF BULK FIELD} \times \frac{1}{2} \mathcal{O}_i(X, \bar{\Sigma}, \bar{\Psi}, C) + N \cdot \text{BULK FIELD}$$



ADD INFINITESIMAL $\beta = \epsilon_i \beta_i \rightarrow \langle \bar{\Sigma}, \bar{\Psi}, \mathcal{O}_i \rangle$

$B = \epsilon_i \beta_i \rightarrow$ INSERT $\epsilon_i \int \beta_i \mathcal{O}_i$



$\frac{\mathbb{C} \times \mathbb{C}^2}{z, a, b}$

$$\beta = \beta^{(2)} \otimes \delta^{(2)}(z-w) d\bar{z}$$

$$\Rightarrow \int_{\mathbb{R}^2} p(x, y) (w)$$

$$\partial \beta^{(2)} = 0$$

$$\beta^{(2)} = \partial^{(1)} p(a, b) = \partial_b p \partial_a - \partial_a p \partial_b$$

$$\beta^{(1)} = p(a, b) \Rightarrow \int_{\mathbb{R}^2} b p(x, y)$$

$$\beta = \partial \beta^{(1)} \otimes \delta^{(2)}(z-w) d\bar{z} \partial_z + \beta^{(2)} \otimes \partial_z \delta^{(2)}(z-w) d\bar{z}$$

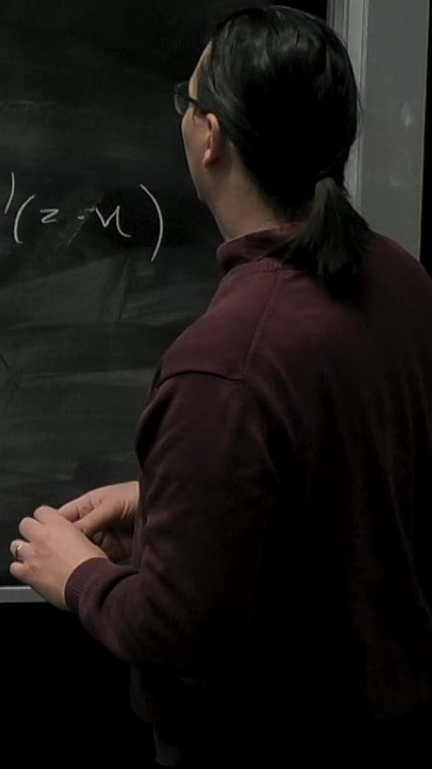
$$\beta^{(1)} = p(a, b) \rightarrow \int_{\mathbb{R}^2} p(x, y)$$

$$\beta^{(2)} = \partial p(a, b) \rightarrow C_1$$

$$\begin{array}{c}
 O_1(w) \quad O_2(w') \\
 \left. \begin{array}{l} \\ \\ \end{array} \right\} f \quad O_2 \\
 (O_2(w) \quad O_1(w'))_{REG} = O_2(w') O_1(w) - \frac{f}{u'-w} \dots (w) \dots
 \end{array}$$

$$B = \beta_1^{\alpha_2} \delta^{\alpha_1}(z-w) d\bar{z} + \beta_2^{\alpha_2} f(z)$$

$$\begin{array}{c}
 \bar{\partial} \beta_1 + \{ \beta_2, \beta_1 \} = 0 \\
 \bar{\partial} \beta_1 + \{ \beta_2^{\alpha_2}, \beta_1^{\alpha_1} \} f(w) = \delta^{\alpha_1}(z-w)
 \end{array}$$



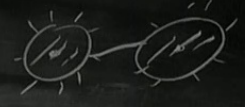
CAUTION
 Do not touch the surface of the chalkboard.
 Do not use the chalk for writing on the board.

$O_1(w)$ $O_2(w')$

$f O_2$

$$(O_2(w) O_1(w'))_{REG} = O_2(w') O_1(w) - \frac{h}{u' - w} T \dots (w) \dots$$

$$B = \beta_1^c \int \delta^{(2)}(z-w) d\bar{z} + \beta_2^c f(z)$$



$$\partial \beta + \{ \beta, \beta \} = 0 \quad \partial \beta_1 + \{ \beta_2, \beta_1 \} = 0$$

$$\beta_1 = \beta_1^c \int \delta^{(2)}(z-w) + \{ f(w) \beta_2^c, \beta_1 \} \frac{1}{z-w}$$

$O_1(w)$ $O_2(w')$

$f O_2$

$$(O_2(w) O_1(w'))_{REG} = O_2(w') O_1(w) - \frac{h}{u' - w} T \dots (w)$$

$$B = \beta_1^c \int \delta^{(2)}(z-w) d\bar{z} + \beta_2^c f(z)$$

$$\bar{\partial} B + \{ \beta_2, \beta_1 \} = 0$$

$$\beta_1 = \beta_1^c \int \delta^{(2)}(z-w) + \{ f(w) \beta_2^c, \beta_1 \} \frac{1}{z-w}$$



CAUTION
DO NOT TOUCH THE BOARD SURFACE.
DO NOT TOUCH THE BOARD SURFACE.
DO NOT TOUCH THE BOARD SURFACE.

$N \times D1 \subset \mathbb{C} \times \mathbb{C}^2$

$S_{\text{GRAVE}} = \int \frac{1}{4} \Sigma X \Sigma Y + \text{E} b \delta c + \sum \text{DETERMINANTS OF BULK FIELD} * \mathbb{E} O_i(x, y, b, c) + N \cdot \text{BULK FIELD}$

ADD IMMUTABLE $\beta = \epsilon_i \beta_i \rightarrow \langle \tau_i, z_i \rangle$

$B = \epsilon_i \beta_i \rightarrow \text{INSERT } \epsilon_i \int \delta_{i, \text{FIELD}} Q_i(u)$

$\beta = \int \beta_i^{a_i} + S^{(i)}(z_i, u) dz \Rightarrow \text{ST}_P(x, y)(u)$

$\partial \beta^{a_i} = 0 \quad \beta^{a_i} = \partial^{a_i} p(a, b) = \partial_b p \partial_a - \partial_a p \partial_b$
 $\beta^{a_i} = p(a, b) \Rightarrow \text{ST}_b p(x, y)$

$\beta = \partial \beta_i^{a_i} \circ S^{(i)}(z_i, u) dz + \beta_i^{a_i} \circ \partial_z S^{(i)}(z_i, u) dz$

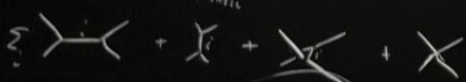
$\beta^{a_i} = p(a, b) \rightarrow \text{ST}_b p$

$\beta^{a_i} = \partial p(a, b) \rightarrow C_i$

$\int \partial_x \phi \partial^x \phi + \phi^3 \quad \int \partial_x \tilde{\phi} \partial_x \tilde{\phi} + \tilde{\phi}^3$
 $\phi = \tilde{\phi} + \epsilon \hat{\phi}^2$

$\frac{1}{2} (\tilde{\phi}, Q \tilde{\phi}) + \frac{1}{3} (\phi, \phi, \phi) + \frac{1}{24} (\phi, \phi, \phi, \phi)$
 $\phi \rightarrow \varphi_1(\phi') + \varphi_2(\phi', \phi'') + \dots$

$\Delta \text{diagram} = \int \text{diagram} \Leftrightarrow S^T S = 11$



$a + \int \frac{f(u)}{z-u} \partial_b p$

$a + \int \frac{\partial_b p}{z-u} \quad b + \int \frac{\partial_a p}{z-u}$

$\partial a_i = \int \text{ST}_P S^{(i)}(z_i, u), a_i$

