Title: Twisted Holography Mini-Course - Lecture 20231116

Speakers: Kevin Costello

Collection: Twisted Holography Mini-Course

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BRST reduction:

$$Z_i(z)\, Z_j(z') \sim \frac{\varepsilon_{ij}}{z-z'}$$

Introduce b of spin 1, c of spin 0 $\Big\}$ gl_N valued

$$b(z)\, c(z') \sim \frac{1}{z-z'}$$

$$Q_{BRST} = :\mathrm{tr}(c Z_1 Z_2): + \frac{1}{2} :bcc:$$

BRST reduction: $\oint_{|z|=1} Q_{BRST}\, dz$ acts on local operators: Take cohomology.

IIB on

$$\underset{Z_3}{\mathbb{R}^2_{\varepsilon}} \times \underbrace{\mathbb{R}^2_{-\varepsilon} \times \mathbb{C}}_{D3} \times \underset{Z_1}{\mathbb{C}} \times \underset{Z_2}{\mathbb{C}}$$

$\rightsquigarrow$ D1 in top' string on $\mathbb{C}^3$

Backreact: $SL_2(\mathbb{C})$

... background: Z_3 goes away

EOM are BPS configurations

Ω bg $\Rightarrow$ these live at origin in $\mathbb{R}^2_{\varepsilon}$

$Z_1(z), Z_2(z)$ must describe a hol. curve in $\mathbb{C}^3$

$\Rightarrow \bar{\partial} Z_i(z) = 0$

The theory on D1 = localized D3 has fields $Z_1, Z_2 \in \mathfrak{gl}_N$

Lagrangian $\int \varepsilon^{ij} \operatorname{tr} Z_i \bar{\partial} Z_j$

Z_i of spin $\frac{1}{2}$, bosonic

Also perform BRST reduction using adjoint g...

(gauging comes from $N=2$ vector multiplet)

Holographic dictionary

- Single trace elements of BRST coho. at $N \to \infty$
- States of KS on $SL_2(\mathbb{C})$

OPEs in CFT $\Leftrightarrow$ Scattering

At finite N, BRST coho is very hard to compute.

$N = \infty$, easy. old math results (Loday-Quillen-Tsygan ...)

IIB on

$$\underset{z_3}{\mathbb{R}^2_{\varepsilon}} \times \underbrace{\mathbb{R}^2_{-\varepsilon} \times \mathbb{C}}_{D3} \times \overset{z_1}{\mathbb{C}} \times \overset{z_2}{\mathbb{C}}$$

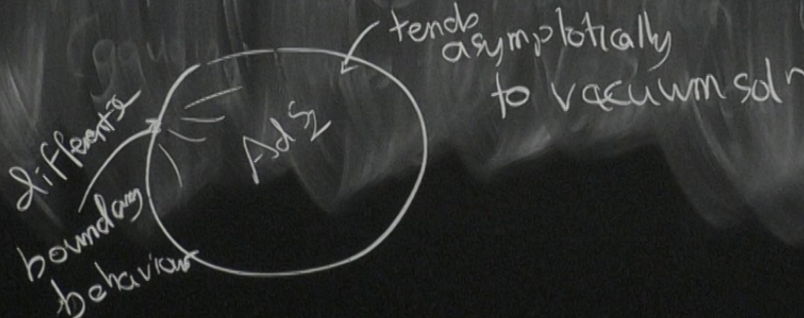
$\rightsquigarrow$ D1 in top' string on $\mathbb{C}^3$

Backreact: $SL_2(\mathbb{C})$

SUGRA states

AdS_3 says, states that correspond to local operators at $x \in \partial AdS_3$

= Sol^{n}s to EOM of gravitational theory that satisfy a b.c. everywhere except x



Want to find α_1, α_2 which satisfy b.c. except at $n=0, z=0$

Simple way to do this:

$\alpha_1 = \delta_{z=0} \frac{1}{n^k} w^l \quad l \le k$

as n has a pole at $w=\infty$

$\bar\partial \alpha_1 = 0$

$\alpha_1 = 0$ at $n=0, z \neq 0$

$\delta_{z=0}$ Spin 1

$\frac{1}{n^k}$ Spin $k/2$

w^l, $0 \le l \le k$, $SU(2)_R$ rep of spin $k/2$

Spin $1+k/2$, $SU(2)$ spin $k/2$

Fermionic States

$\mathrm{Tr}(b Z_1^r Z_2^s)$ Spin $1+\frac{r+s}{2}$

$SU(2)$ rep. spin $\frac{r+s}{2}$

Similarly, boundary operators built from μ
(or, fields w. modified boundary conditions)
give bosonic A, B towers.

$$N n^2 \frac{d\bar{w}}{(1+|w|^2)^2} \partial_z$$

describes the patch of $\overline{SL_2\mathbb{C}}$
$z \neq \infty$

Near boundary, $\overline{SL_2\mathbb{C}}$ looks like

$\mathcal{O}(1,1) \longrightarrow \mathbb{CP}^1 \times \mathbb{CP}^1$

Coordinate n on fibre, n has a pole at $z=\infty, w=\infty$

$SL_2\mathbb{C}$ Volume form is

$\mathbb{CP}^1_z = \partial AdS_3$

$\mathbb{CP}^1_w = S^3/U(1)$

$\mathbb{CP}^2 \times \mathbb{C}$ $\quad \mathbb{C}^2 \times \mathbb{C}$ $\quad \delta_{z=\omega}(\ )$

$\mathbb{CP}^1_\infty \times \mathbb{C}$