Title: Twisted Holography Mini-Course - Lecture 20231109

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Collection: Twisted Holography Mini-Course

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X CY3

$\mu \in \Omega^{0,1}(X, TX) = \Omega^{0,1}(X, \Lambda^2 T^* X)$

$= \Omega^{2,1}(X)$

$\mu_{\bar{j}}^{i}\, d\bar{z}_{\bar{j}}\, \frac{\partial}{\partial z_i}$

Constraint: $\partial \mu = 0$ $\quad \partial \mu \in \Omega^{3,1}(X) = \Omega^{0,1}(X)$

Lagrangian: $\frac{1}{2}\int \bar{\partial}\mu\, \partial^{-1}\mu + \frac{1}{6}\int \mu^3$ $\quad \mu^3 \in \Omega^{0,3}(X, \Lambda^3 T X)$

$= \Omega^{0,3}(X) = \Omega^{3,3}(X)$

$$\partial_{z_i} \vee d^3 z = \varepsilon^{ijk} \, dz_j \, dz_k$$

KS theory has 2 other fields, not as 'important':

$$\alpha_i \in \Pi\Omega^{0,1}(X) \quad \text{fermionic } (0,1) \text{ forms}$$

$i = 1, 2$

$$\int \underbrace{\varepsilon^{ij} \alpha_i \bar{\partial}_\mu \alpha_j}_{\text{Spinors in physical theory}}$$

X CY3, spinors on X

$$S_+ = \Omega^{0,ev} \qquad S_- = \Omega^{0,odd}$$

$$\not{D} = \bar{\partial} + \bar{\partial}^*$$

Gauge symmetry

$$V \in \Omega^{0,0}(X, TX)$$

$$V = V_i \frac{\partial}{\partial z_i}$$

vector field pointing in $\frac{\partial}{\partial z}$ directions

Lie derivative

$$\delta\mu = [V, \mu] + \bar{\partial} V = [V, \bar{\partial}_\mu]$$

generates gauge transformations.

Ex: $X = (\mathbb{C}^\times)^3$ where $\mathbb{C}^\times = \mathbb{C}\setminus 0 = S^1 \times \mathbb{R}$

Then KK reduction to $\mathbb{R}^3$ along $(S^1)^3$

Result is 3d CS theory for group of Volume-preserving diffeos of $(S^1)^3$

$\mu = d\bar{z}_{\bar{j}}\,\mu^{i}_{\bar{j}}\,\partial_{z_i}$

$d\bar{z}_i \rightsquigarrow$ 3 components of 3d gauge field

$V \rightsquigarrow$ CS gauge transformations

Branes

X CY3

"Branes are coherent sheaves on X"

Concretely, brane is

$Y \subseteq X$, Y holomorphic submanifold

E a vector bundle on Y (holomorphic)

Physical String:

$$\mathbb{R}^2_{\varepsilon} \times \underline{\mathbb{R}^2_{-\varepsilon}} \times \underline{\mathbb{C}} \times \underline{\mathbb{C}} \times \underline{\mathbb{C}}$$

	$\mathbb{R}^2_{\varepsilon}$	$\mathbb{R}^2_{-\varepsilon}$	$\mathbb{C}$	$\mathbb{C}$	$\mathbb{C}$
D7					
D5		—		—	—
D3		—			—
D1		—			

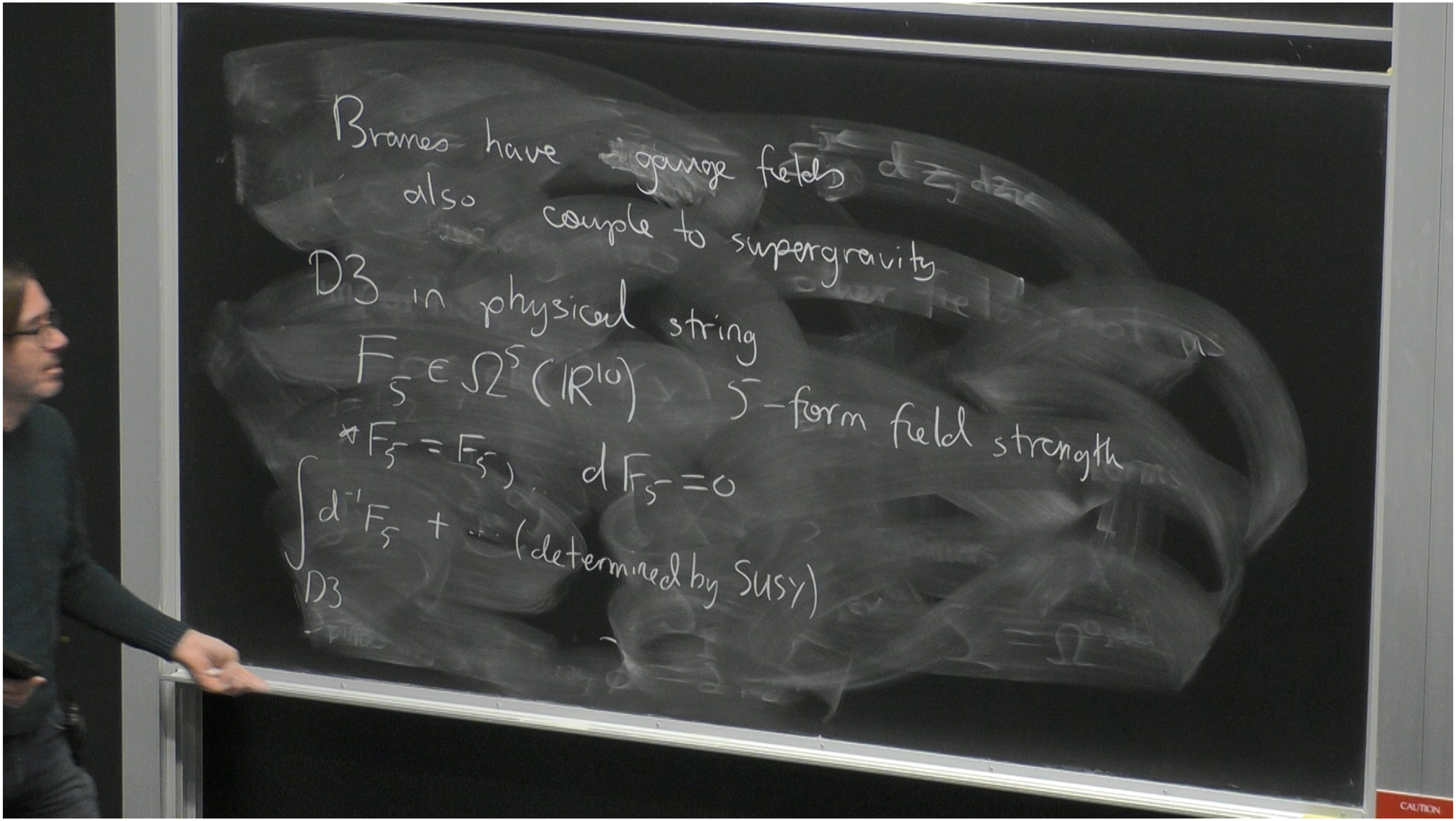
Top' string

Space filling: D5

D3 wraps $\mathbb{C}^2$

D1 wraps $\mathbb{C}$

D-1 wraps 0


Branes have gauge fields
also couple to supergravity
D3 in physical string
$F_5 \in \Omega^5(\mathbb{R}^{10})$ 5-form field strength
$*F_5 = F_5$, $dF_5 = 0$
$\int_{D3} d^{-1}F_5 + \ldots$ (determined by SUSY)

In II B, F_5 and g are related by SUSY

Can think of $\mu \in \Omega^{2,1}$ as coming from F_5 instead

Roughly, $\mu = \text{“}\int_{\mathbb{R}^2_{-\varepsilon}} F_5\text{''}$

Propose (can derive from worldsheet) μ couples to D1 wrapping $C \subseteq X$ $\dim_{\mathbb{C}} C = 1$

by $\int_C \partial^{-1}\mu$ $\quad \partial^{-1}\mu \in \Omega^{1,1}(X)$

Physical String

Backreact by look at field sourced by D3

EOM in presence of N D3's

$$N\int_{D3} d^{-1}F_5 + \int F_5^2$$

EOM are $dF_5 = N\delta_{D3}$

$x_0 \ldots x_3$ D3

$y_4 \ldots y_9$

$$\int_{\|y\|=1} F_5 = N$$

$\rightsquigarrow$ $AdS_5 \times S^5$

5-form F

$$\int_{S^5} F = N$$

Prop Every morphism in Bord_2 can be built from

Top^l String: N D1 branes then the eqn for μ is

$$\bar{\partial}\mu = \delta_{D1}^{N}$$

Solution to this eq^n is Böchner-Martinelli kernel

$z \rightsquigarrow$ D1 worldvolume

$w_1, w_2 \rightsquigarrow$ normal.

$$\mu = N \frac{\varepsilon^{ij}\, \bar{w}_i\, d\bar{w}_j\, dw_1\, dw_2}{\|w\|^4}$$

Check 1) $\bar{\partial}\mu = 0$ except at $w_i = 0$

2) $\displaystyle\int_{\|w\|=1} \mu = N(\pi^2)$

How does geometry change?

$$\mu = N \frac{\varepsilon^{ij} \bar{w}_i d\bar{w}_j}{\|w\|^4} \partial_z$$

Hol. fns in deformed geometry satisfy

$$\bar{\partial} F + N \frac{\varepsilon^{ij} \bar{w}_i d\bar{w}_j}{\|w\|^4} \frac{\partial F}{\partial z} = 0$$

$$\Rightarrow \begin{cases} \dfrac{\partial F}{\partial \bar{w}_2} + N \dfrac{\bar{\omega}_1}{\|w\|^4} \dfrac{\partial F}{\partial z} = 0 \\[2ex] \dfrac{\partial F}{\partial \bar{w}_1} - \dfrac{N \bar{\omega}_2}{\|w\|^4} \dfrac{\partial F}{\partial z} = 0 \end{cases}$$

2 sol'ns are $F = \omega_1$, $F = \omega_2$

2 more are

$$u_1 = \omega_1 z + \frac{N \bar{\omega}_2}{\|w\|^2}$$

$$u_2 = \omega_2 z - \frac{N \bar{\omega}_1}{\|w\|^2}$$

4 coords on a 3d geometry. 1 relation.

$$w_1 u_2 - w_2 u_1 = -N$$

= 2×2 matrices $\begin{pmatrix} w_1 & u_1 \\ w_2 & u_2 \end{pmatrix}$ of determinant $-N$

Physical String

Backreact by look at field sourced by D3

EOM in presence of N D3's

$$N\int_{D3} d^{-1}F_5 + \int F_5^2$$

EOM are $dF_5 = N\delta_{D3}$

$x_0 \ldots x_3$ D3

$y_4 \ldots y_9$

$$\int_{\|y\|=1} F_5 = N$$

$$\int_{\|y\|=1} F = \int_{\|y\|\leq 1} dF = N$$

$\rightsquigarrow$ $AdS_5 \times S^5$

5-form F

$$\int_{S^5} F = N$$

Backreaction gives $SL_2(\mathbb{C}) = SO(3,1)$

$$\cong \mathbb{H}^3 \times S^3 \simeq \text{"} AdS^3 \times S^3)$$

as $SO(3,1) =$ isometries of $\mathbb{H}^3$

S^3 bundle over $\mathbb{H}^3$

$$AdS_3 \times S_3 \subseteq AdS_5 \times S^5$$

B-model | localizes IIB