Title: Twisted Holography Mini-Course - Lecture 20231102

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Collection: Twisted Holography Mini-Course

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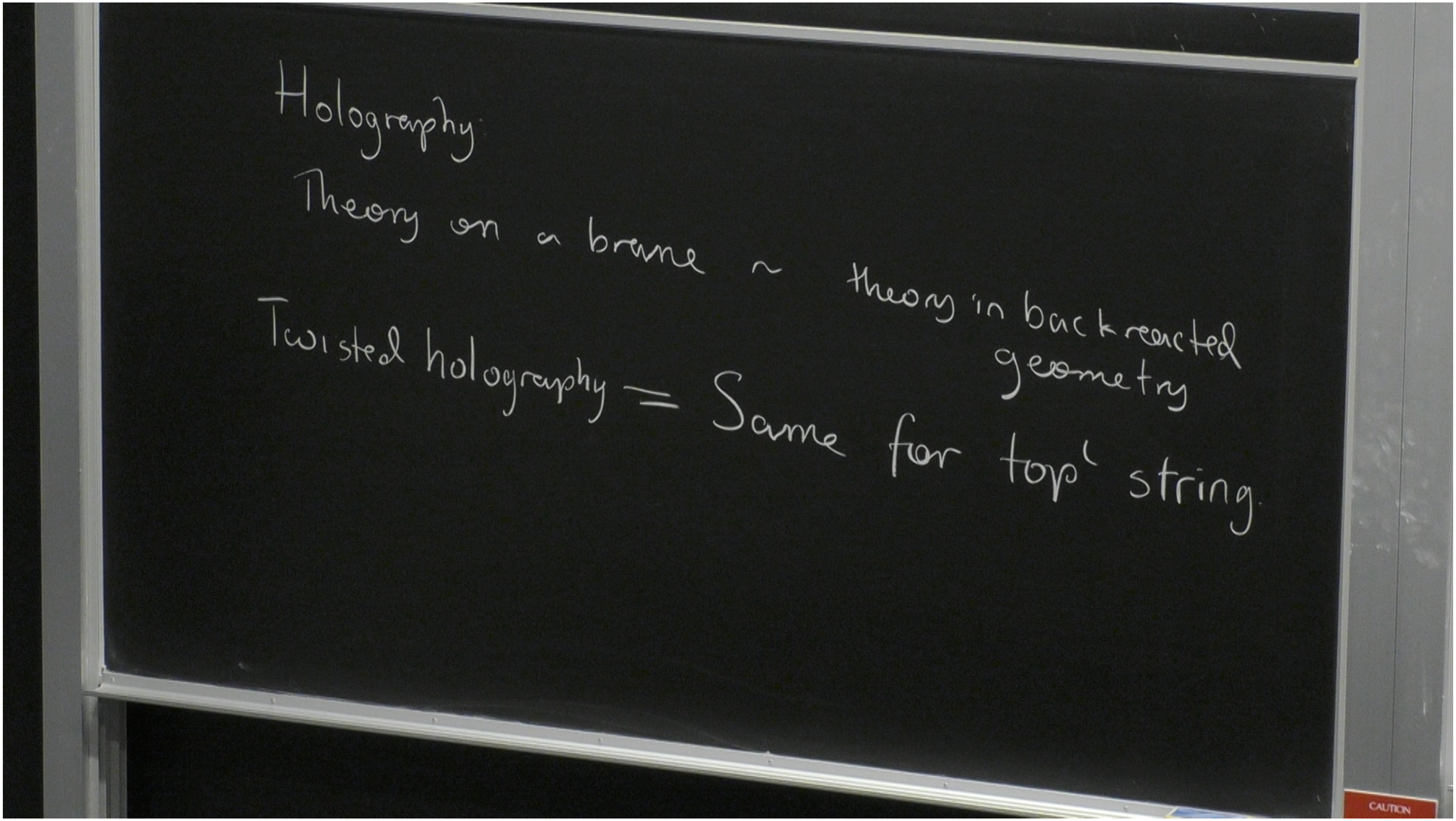
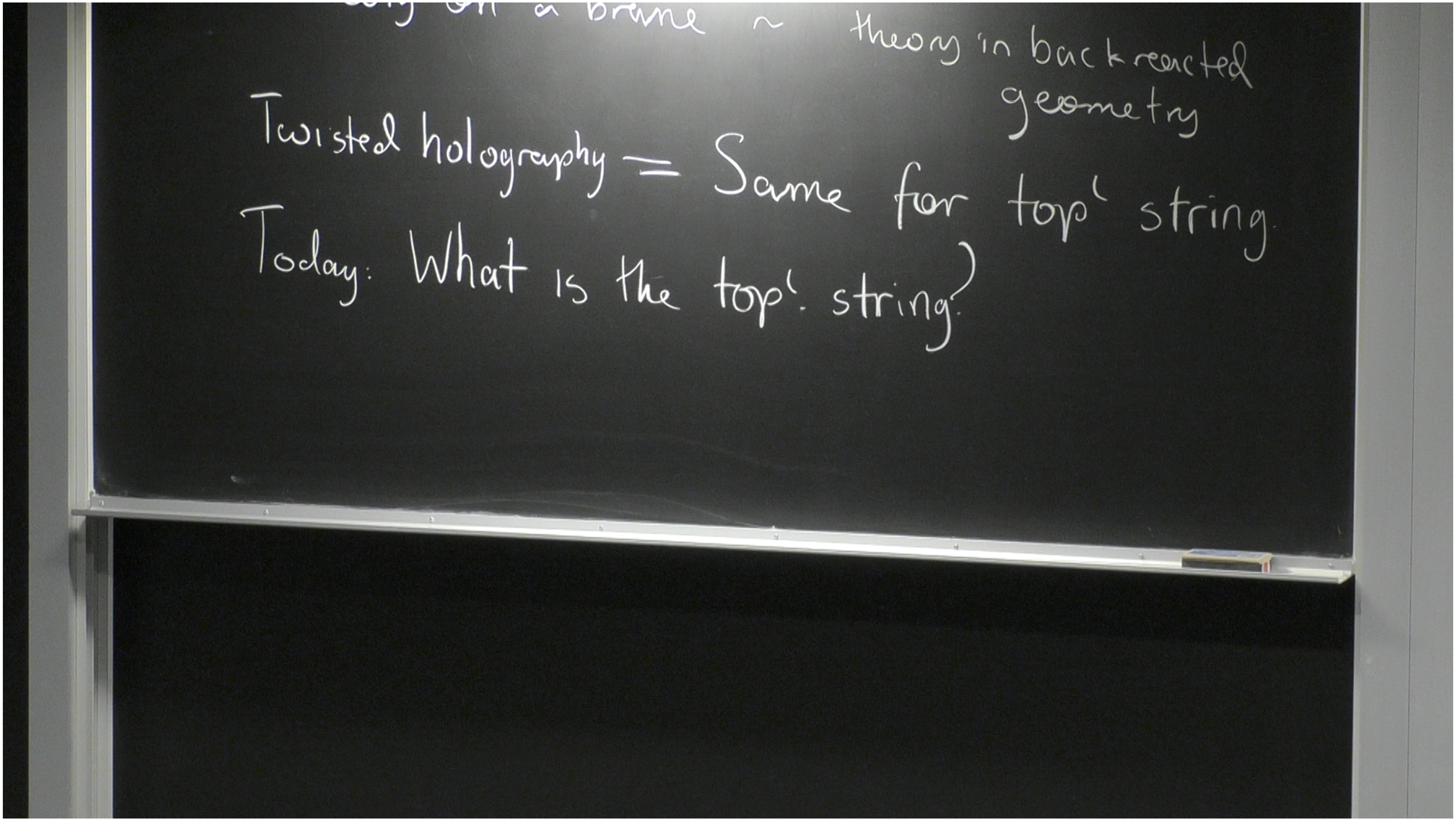

Holography:
Theory on a brane ~ theory in backreacted geometry
Twisted holography = Same for top' string.


brane ~ theory in backreacted geometry
Twisted holography = Same for top' string.
Today: What is the top'. string?

Topological String: X is a Calabi-Yau 3-fold

$(2,2)$ σ-model with target X has

2 top'l. twists: A/B model.

Top'l. string = twisted $(2,2)$ σ-model coupled to 2d topological gravity.

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<u>A-model</u>: All about worldsheet instantons

<u>B-model</u> No worldsheet instantons.

If $f : \Sigma \to X$ satisfies B model EOM $\Rightarrow f$ is constant.

Because of this, string field theory gives a <u>local</u> QFT on X!

This is called Kodaira-Spencer or BCOV theory

Other description: Top' strings $\subseteq$ Physical type II string

On $X \times \mathbb{R}^4$, B-model top' string amplitudes
$= \mathrm{II}B$ BPS amplitudes in a certain background.

90s language:
turn on a graviphoton background on $\mathbb{R}^4$

Modern Turn on an Ω background on
$\mathbb{R}^2_{\varepsilon} \times \mathbb{R}^2_{-\varepsilon}$.

We introduce a b.g. field so that a space-time supercharge Q squares to rotation: $Q^2 = \varepsilon \cdot (\partial_{\theta_1} - \partial_{\theta_2})$

where θ_1, θ_2 are angular coords. on $\mathbb{R}^2 \times \mathbb{R}^2$

B-model $\equiv$ Q-coho. of S^1 invariant states in IIB

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$$\mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^6 \times \underset{x_{11}}{\mathbb{R}} \leftarrow \text{Translations}$$

$$\Downarrow$$

Trans. + rotation

$$(\theta_1, \theta_2 \quad x_{11}) \sim (\theta_1 + \varepsilon, \theta_2 - \varepsilon, x_{11} + 1)$$

Ω-background: <u>localize</u> to fixed point of rotation.

10d IIB $\rightsquigarrow$ 6d B-model

D3 wraps $C \times \mathbb{R}^2_\varepsilon$

$C \subseteq X$ a complex curve

$\rightsquigarrow$ localizes to a 2d CFT on C

$\downarrow$

Beem Rastelli etc. chiral algebra

$\supseteq$ graviton $\supseteq$ a Ricci flat metric on X

BPS states $\supseteq$ metrics on X of $SU(3)$ holonomy

$SU(3)$ holonomy $\Rightarrow$ Kähler

metric is determined by (ω, I)

$\omega \in \Omega^2(X)$

$I: TX \to TX, \; I^2 = -1$

In the B-twist, ω is exact.

Fields of space-time theory for B-model include deforming the complex str. on X, preserving holomorphic volume form.

$$g(v,w) = \omega(v, I w)$$

How to deform the complex structure?

$$\underline{I} : TX \to TX$$

View it as acting on T^*X

$$\underline{I} : \Omega^1(X) \to \Omega^1(X)$$

$\otimes$ this with $\mathbb{C}$

$$\Omega^{1,0}(X) \subseteq \Omega^1(X,\mathbb{C}) \quad +i \text{ eigenspace}$$

$$\Omega^{0,1}(X) \subseteq \Omega^1(X,\mathbb{C}) \quad -i \text{ eigenspace.}$$

of I. Then, in coordinates

$$\bar\partial : \Omega^{0,0} \to \Omega^{0,1}$$

starts as $\sum d\bar{z}_{\bar i} \frac{\partial}{\partial \bar{z}_{\bar i}}$

deforms by adding

$$\sum \mu_j^{\bar i} \, d\bar{z}_{\bar i} \frac{\partial}{\partial z_j}$$

$\mu_j^{\bar i}$ = certain components of I

We introduce a [illegible] field so that

Holomorphic fns in deformed geometry satisfy

$$\bar{\partial}_\mu f = 0$$

i.e

$$\frac{\partial f}{\partial \bar{z}_{\bar{i}}} + \mu^{\bar{i}}_{j} \frac{\partial f}{\partial z_j} = 0$$

To find 3 independent sol^{n}s we need

$$\left[\frac{\partial}{\partial \bar{z}_{\bar{i}}} + \mu^{\bar{i}}_{k}\frac{\partial}{\partial z_k}, \frac{\partial}{\partial \bar{z}_{\bar{j}}} + \mu^{\bar{j}}_{l}\frac{\partial}{\partial z_l}\right] = 0, \text{ or, } \bar{\partial}_\mu^{\,2} = 0$$

$$\frac{\partial f}{\partial \bar{z}_{\bar{i}}} + \mu_{\bar{i}}^{j} \frac{\partial f}{\partial z_j} = 0$$

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This PDE for μ is the equation of motion

Also, we need deformed geometry to be CY.

In coords. z_i, let $z_i + \varepsilon \omega_i$ be hol. coordinates in deformed complex structure

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Constraint on μ is that

$$\frac{\partial}{\partial z_i} \mu^{\bar{i}}_i = 0$$

$\mathrm{Div}\,\mu = 0. \implies \exists$ holomorphic volume form in new cx structure.

Ω original, $\Omega + \varepsilon \mu \vee \Omega$ new volume form

BCOV:

- Look at fields μ satisfying $\mathrm{Div}\,\mu = 0$
- Build a Lagrangian whose EOM enforce integrability, $\bar{\partial}_\mu^2 = 0$

If $\eta \in \Omega^{0,1}(X, \Lambda^2 TX)$

$$\eta = d\bar{z}_{\bar{i}}\, \eta^{\bar{i}}_{jk} \frac{\partial}{\partial z_j} \frac{\partial}{\partial z_k}$$

anti-sym. in j,k

$$\mathrm{Div}\, \eta = d\bar{z}_{\bar{i}} \left(\frac{\partial}{\partial z_j} \eta^{\bar{i}}_{jk} \right) \frac{\partial}{\partial z_k}$$

If μ is s.t. $\mathrm{Div}\, \mu = 0$

then $\exists\ \eta$ so $\mathrm{Div}\, \eta = \mu$

Call this $\mathrm{Div}^{-1} \mu$

KS Action

$$\int \frac{\partial}{\partial \bar{z}_{\bar{k}}} \mu^{\bar{i}}_{l} \, (\mathrm{Div}^{-1}\mu)^{\bar{j}}_{mn} \, \varepsilon_{\bar{i}\bar{j}\bar{k}} \, \varepsilon^{lmn} + \frac{1}{3} \mu^{\bar{i}}_{l} \mu^{\bar{j}}_{m} \mu^{\bar{k}}_{n} \, \varepsilon^{lmn} \, \varepsilon_{\bar{i}\bar{j}\bar{k}}$$

$\bar{\partial}\bar{\partial}^{-1}$

$= \bar{\partial}\partial^{*}\Delta^{-1}$

FACT EOM $\Longrightarrow$ Integrability of μ

$$\bar{\partial}\mu\,(dz_1\, dz_2\, dz_3)$$

$$= d\bar{z}_{\bar{1}}\,\mu^{\bar{1}}{}_{,}\,(dz_1\, dz_2\, dz_3)$$

$$d\bar{z}_{\bar{1}}\,(d\mu^{\bar{1}}{}_{1}\, dz_2\, dz_3 + \dots)$$

$$= 0$$