Title: Topological Quantum Field Theories Lecture 20231110

Speakers: Lukas Mueller

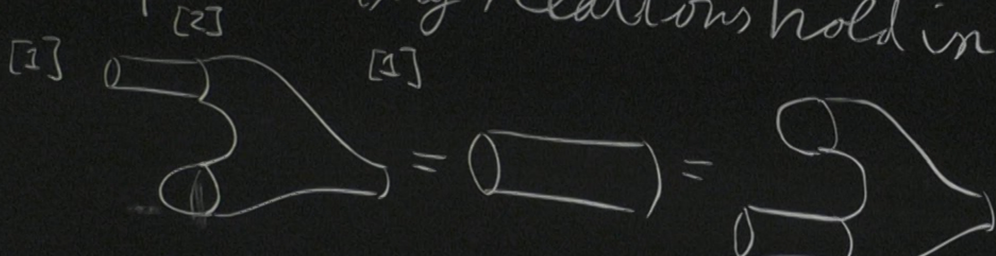
Collection: Topological Quantum Field Theories - mini-course

Date: November 10, 2023 - 2:00 PM

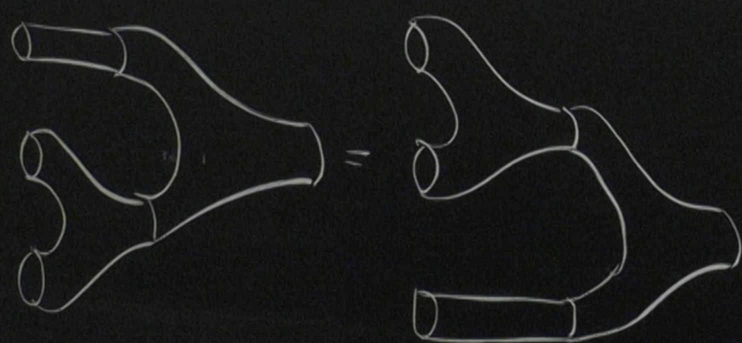
URL: https://pirsa.org/23110035

Prop.

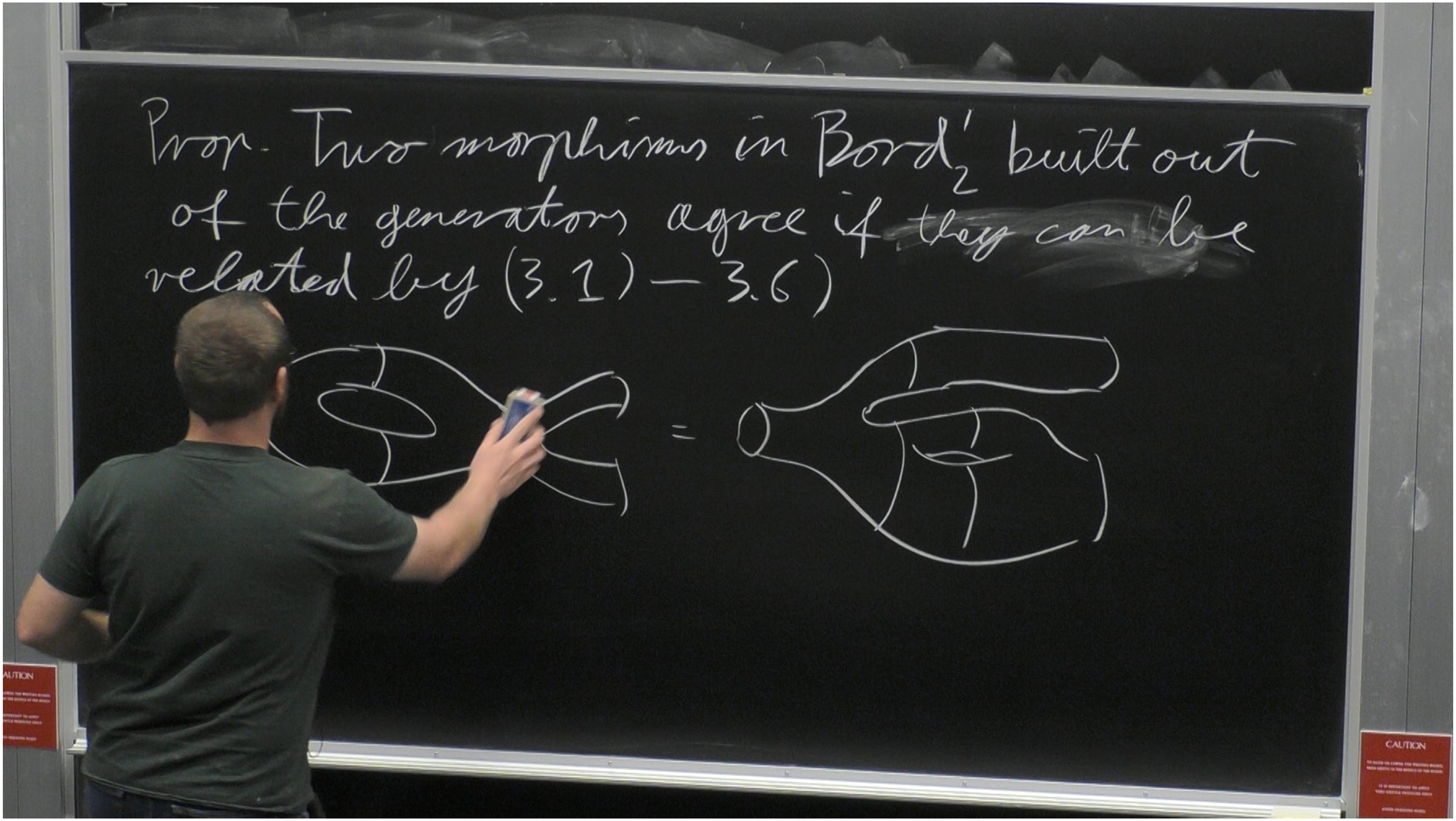
The following relations hold in Bord_2'

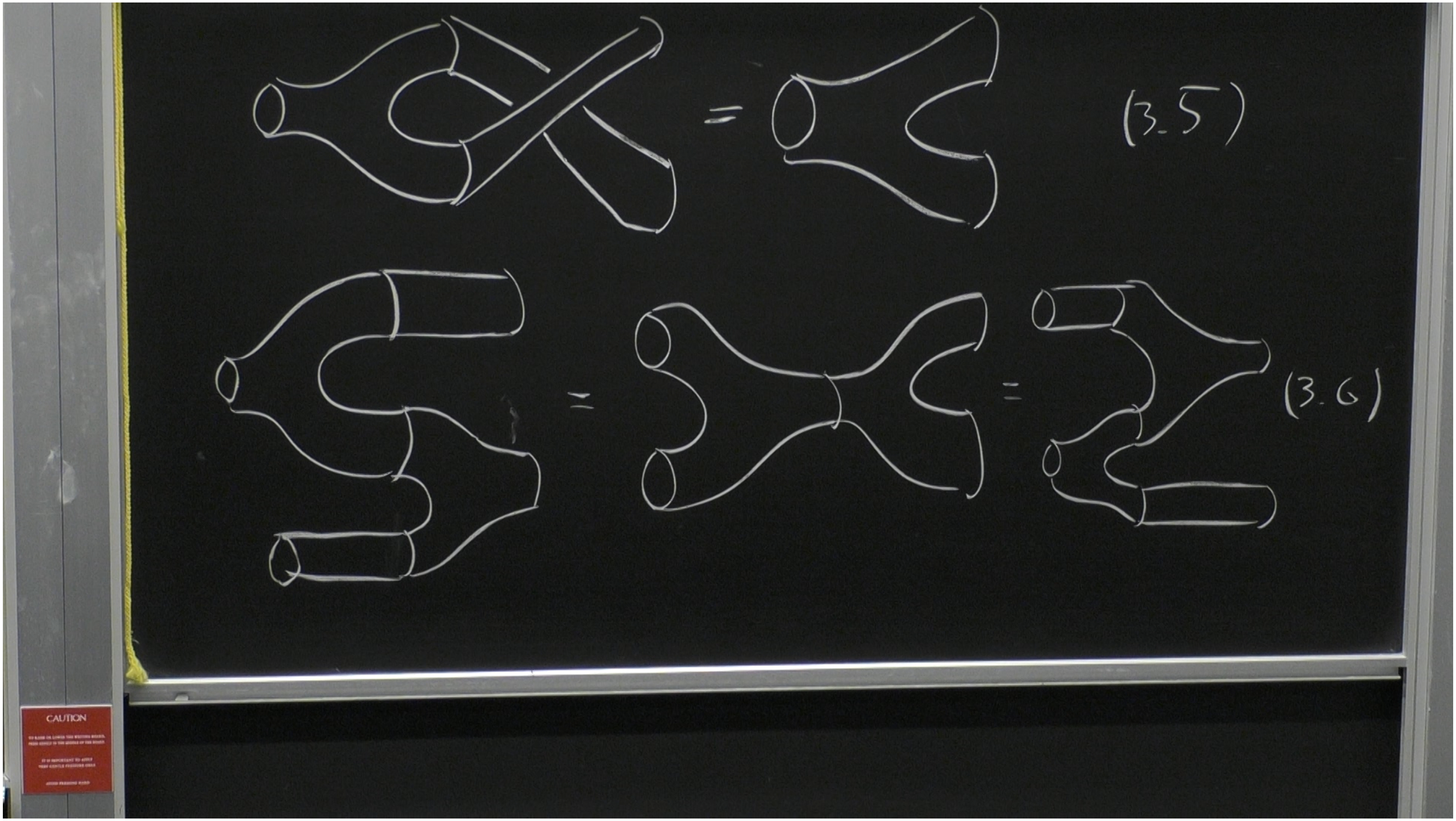


(3.1) + Reflection (3.1)'



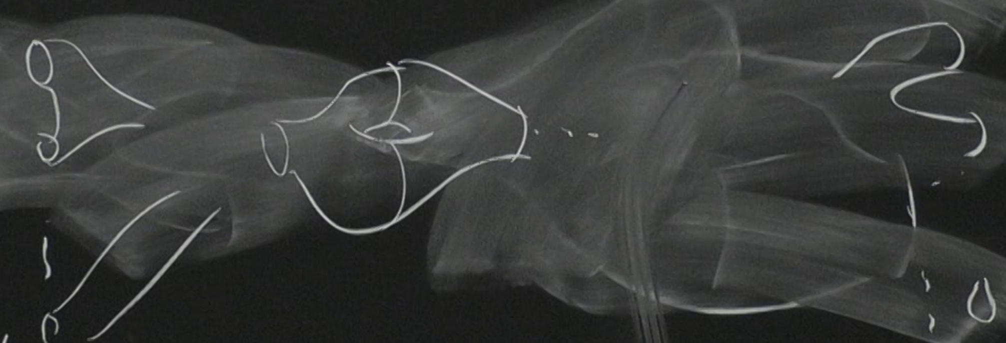
(3.2)

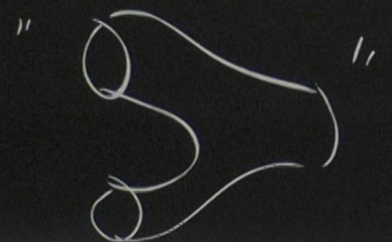

Prop. Two morphisms in Bord'₂ built out
of the generators agree if they can be
related by (3.1) — 3.6)
=

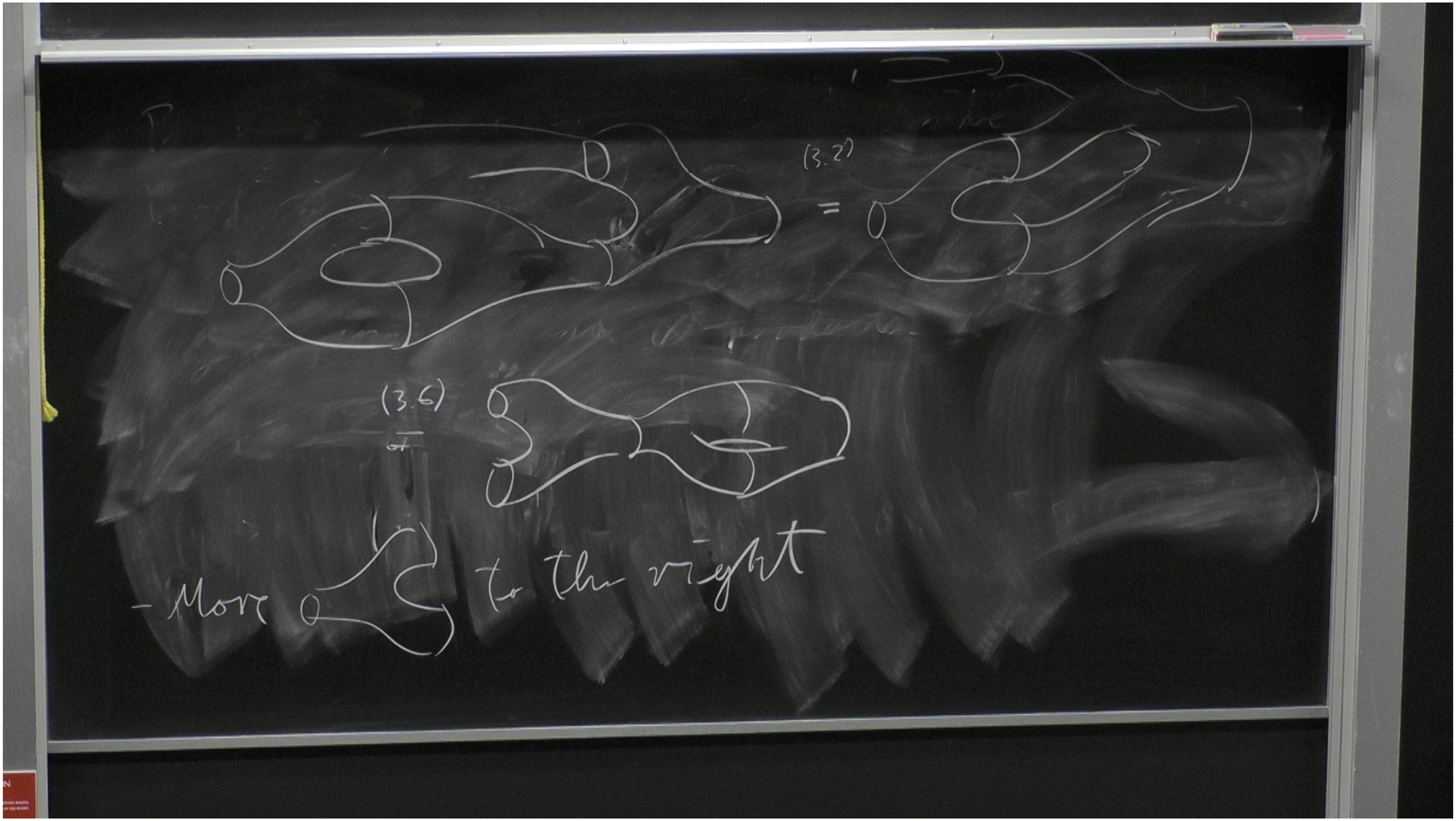

=
(3.5)
=
=
(3.6)
CAUTION

Prop. Two morphisms in Bord_2' built out of the generators agree if ~~they can be~~ related by (3.1) — 3.6)

Proof:
Sketch



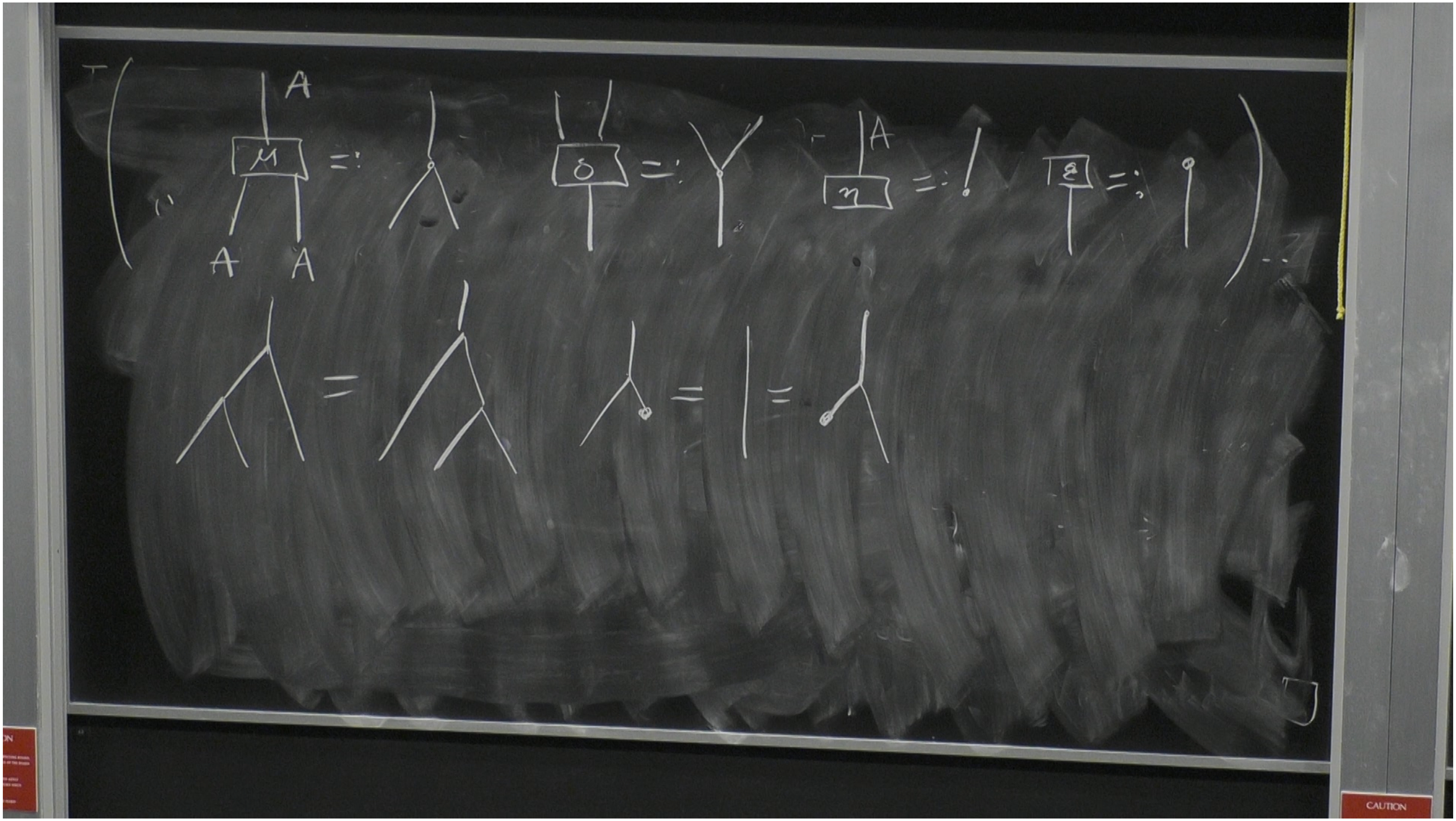
–  move them to the left

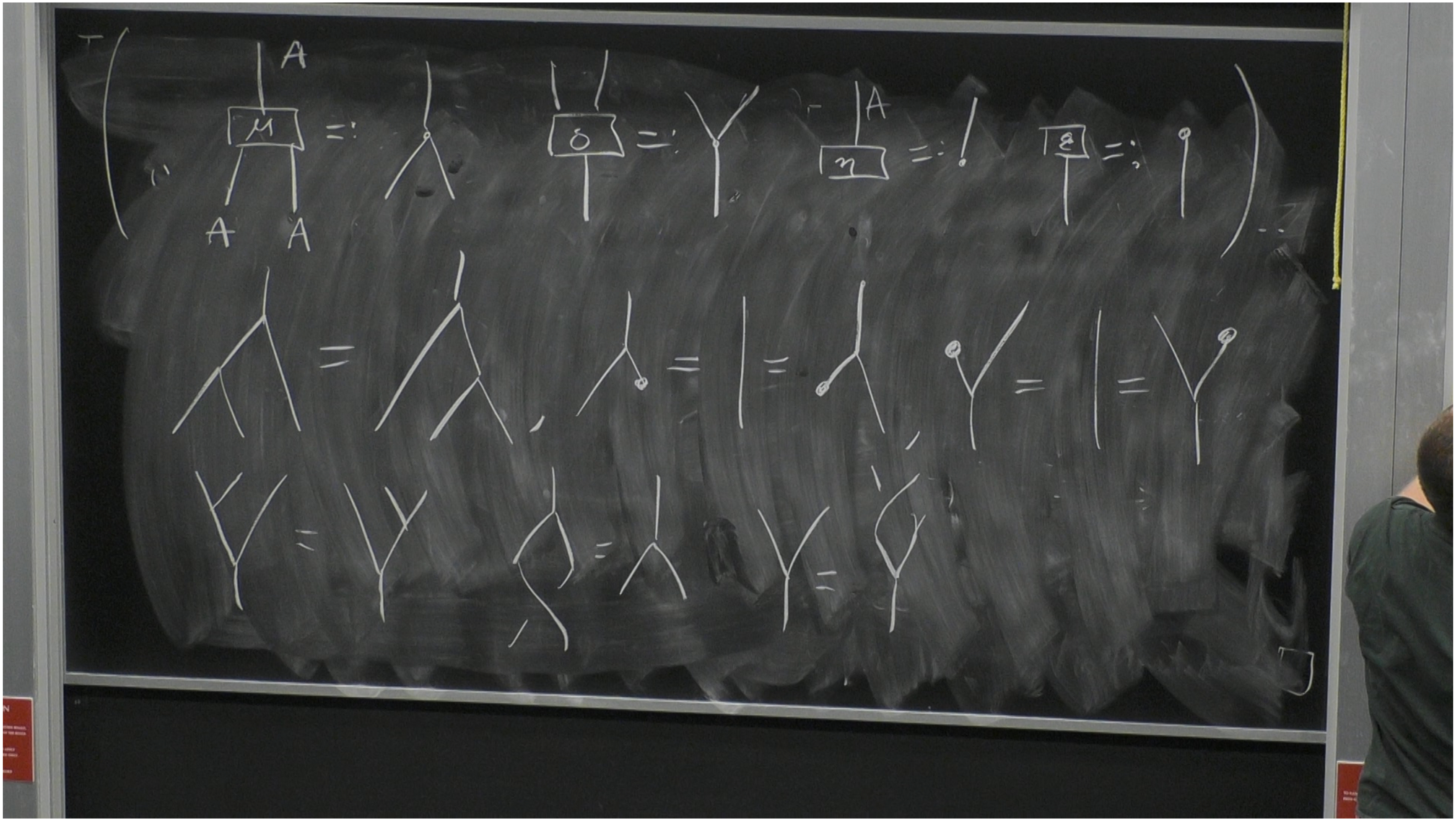

(3.2)
=
(3.6)
=
- Move
to the right

<u>Thm</u>: The groupoid of 2D TQFTs with values in $\mathcal{C}$ is equivalent to the groupoid $c\mathcal{F}A(\mathcal{C})$ with

Obj $\big(A \in \mathcal{C},\ \mu: A \otimes A \longrightarrow A,\ \eta: 1 \longrightarrow A$

$\delta: A \longrightarrow A \otimes A \quad \varepsilon: A \longrightarrow 1$, such that


A
μ
=:
δ
=:
A
η
=:
ε
=:
A A
=
=
=


A
μ
=:
A A
δ
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ε
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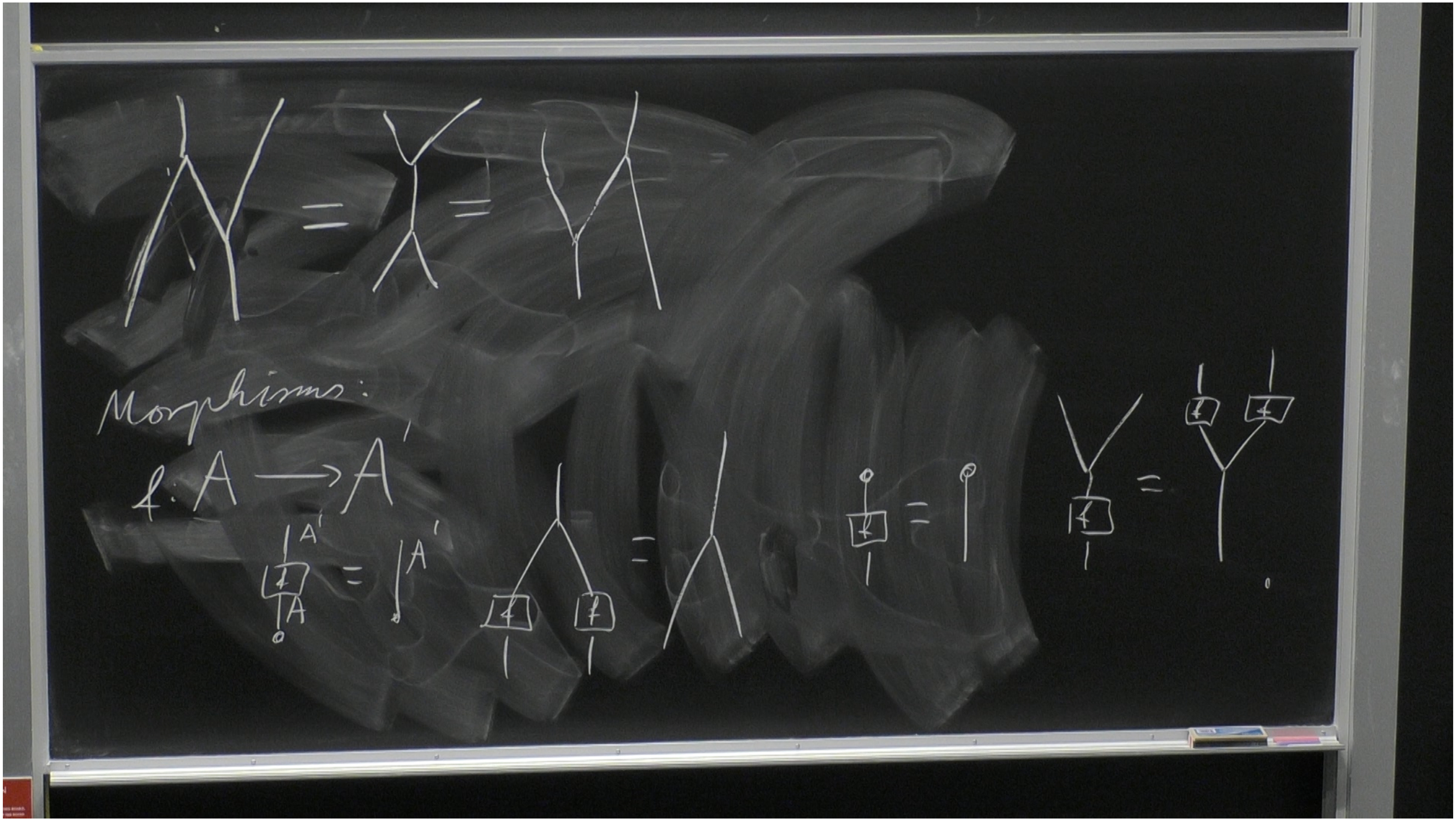
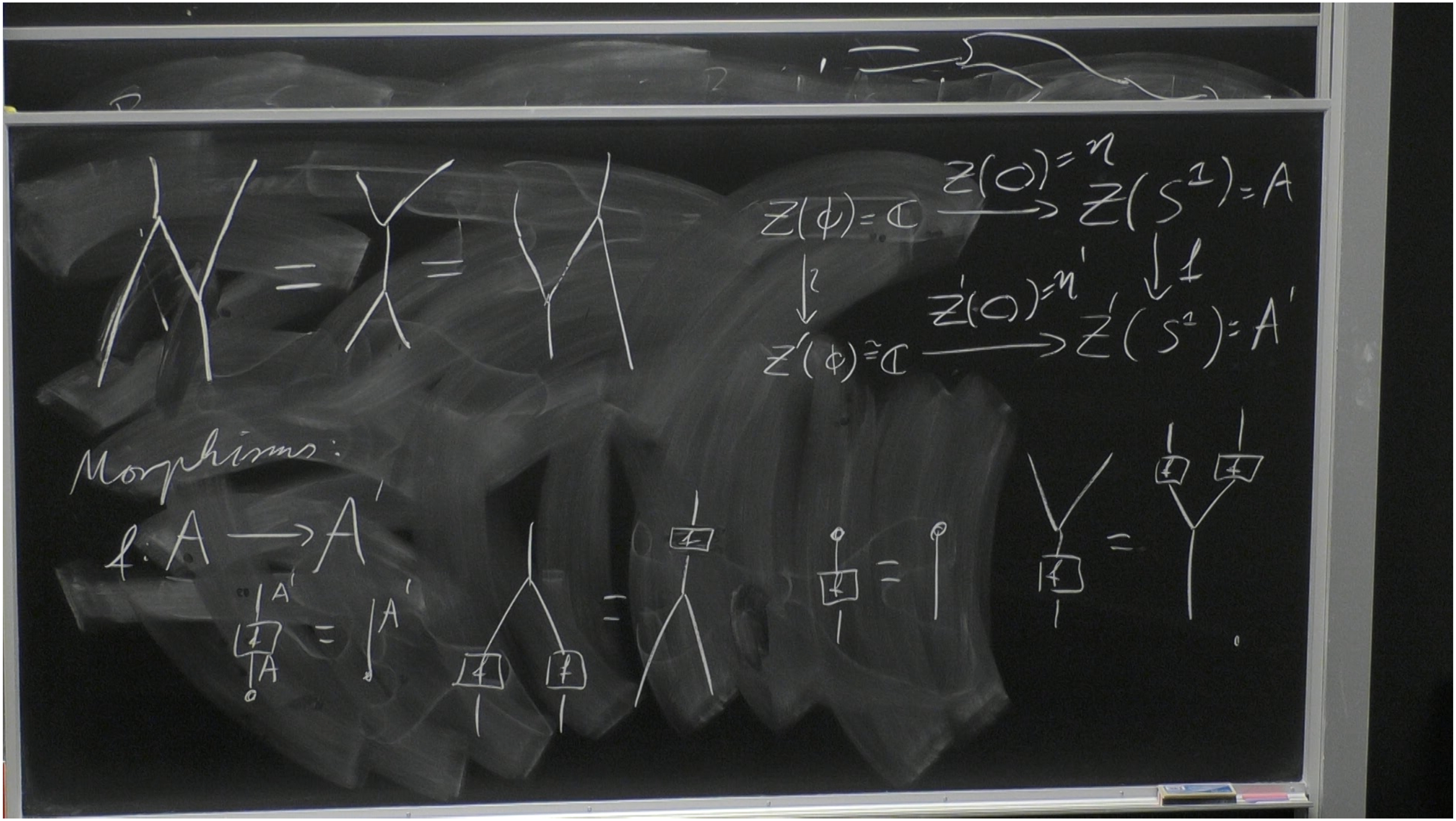

Morphisms:
f: A ⟶ A'

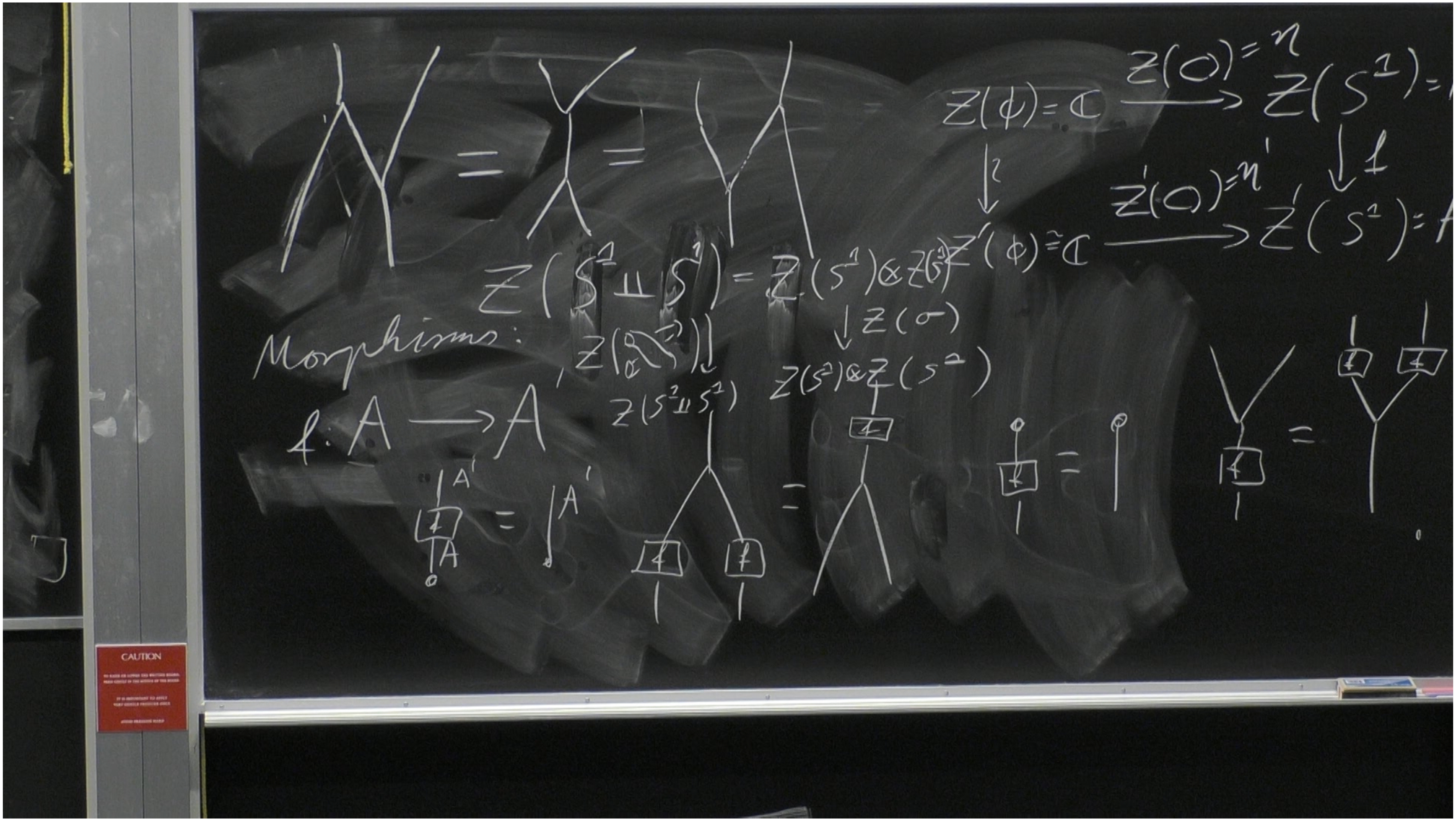

Z(∅) = ℂ
Z(○) = η
Z(S¹) = A
Z'(○) = η'
Z'(∅) ≅ ℂ
Z'(S¹) = A'
Morphisms:
f: A ⟶ A'

Thm: The groupoid of 2D TQFTs with values in $\mathcal{C}$ is equivalent to the groupoid $cFA(\mathcal{C})$ with

Obj $\big(A \in \mathcal{C},\ \mu : A\otimes A \longrightarrow A,\ \eta : 1 \longrightarrow A$

$\delta : A \longrightarrow A\otimes A \quad \varepsilon : A \longrightarrow 1\big)$, such that

$$Z(\ ) : A\otimes A \xrightarrow{\ \sigma\ } A\otimes A, \qquad a\otimes b \mapsto b\otimes a$$


Z(○)=n
Z(φ)=ℂ
Z'(○)=n'
Z'(φ)≅ℂ
Z(S¹⊔S¹)=Z(S¹)⊗Z(S¹)
Z(σ)
Z(S¹)⊗Z(S¹)
Z(S¹⊔S¹)
Morphisms:
f: A ⟶ A
CAUTION

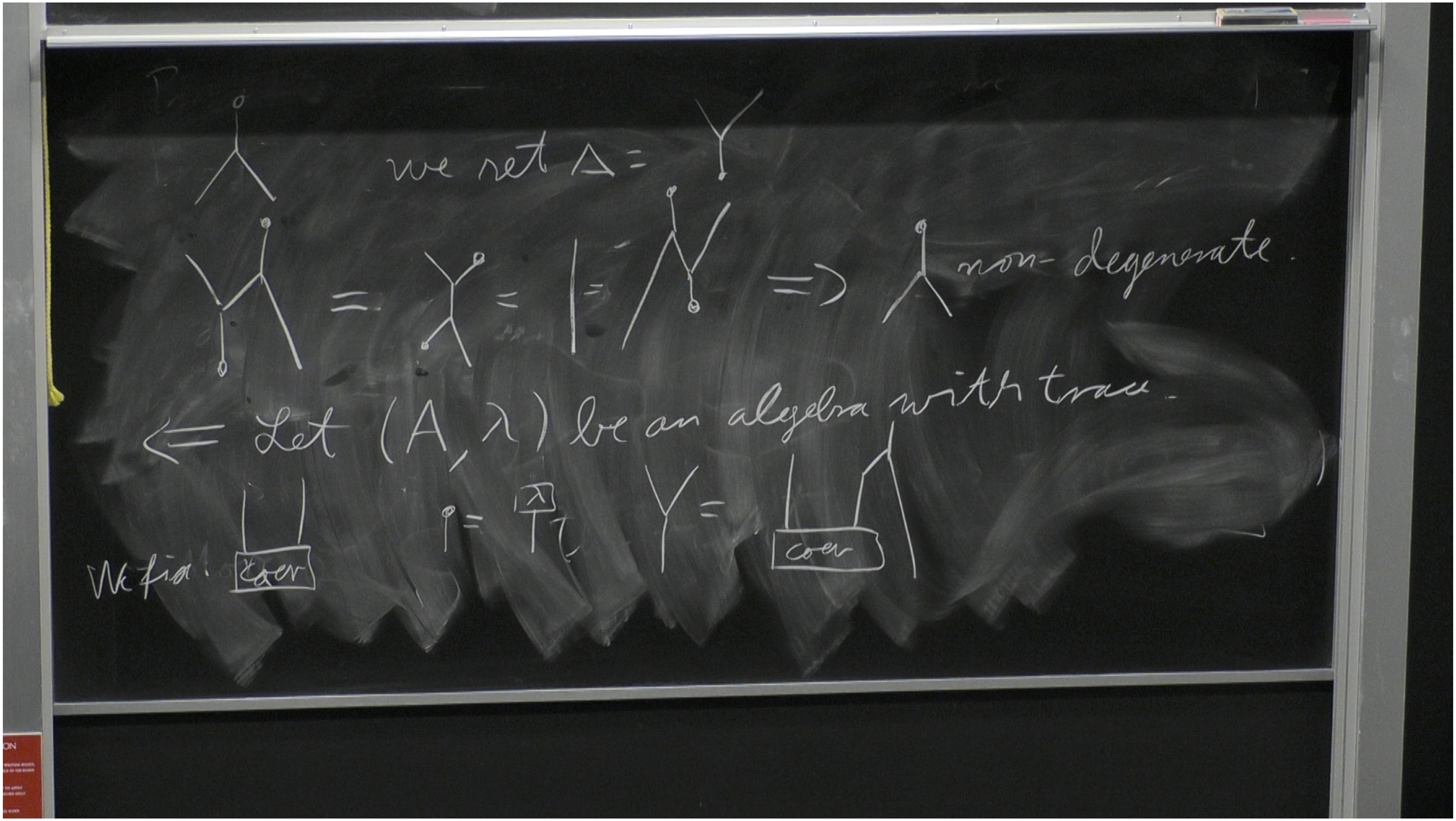
Prop. A FA in Vect is equivalent to an algebra A equipped with a "trace"

$\lambda : A \longrightarrow \mathbb{C}$, s.t. $\mu : A \otimes A \longrightarrow A \xrightarrow{\lambda} \mathbb{C}$

is a non-degenerate pairing.

Proof: Let $(A, \mu, \eta, \varepsilon, \delta)$ be a FA.

$(A, \mu, \eta) + \varepsilon : A \to \mathbb{C}$ is an algebra with trace


we set Δ =
non-degenerate.
⇐ Let (A, λ) be an algebra with trace.
We fix:
coev
λ
coev

Exercise: Show that this defines a FA & the constructions are inverse to each other $\square$

Example:

- Let G be a finite group. The group algebra $\mathbb{C}[G]$ is a FA with $\lambda : \mathbb{C}[G] \longrightarrow \mathbb{C}$

$$\sum_{g \in G} a_g \delta_g \longmapsto a_e$$

$$\left(\sum \lambda_g \delta_g\right) \delta_h = \sum \lambda_g \delta_{gh}$$

Prop. ...
algebr...
$\lambda : A$ ...
is a non...

Proof:

$(A, \mu$ ...

A FA in Vect is equivalent to an

- $\mathrm{Mat}(n \times n)$ $\quad \lambda = \mathrm{tr}(-)$ is a FA
- The centre of any FA is a commutative FA.

Exercise: $Z(\mathbb{C}[G])$ is a commutative FA. Compute the partition function on closed surfaces.

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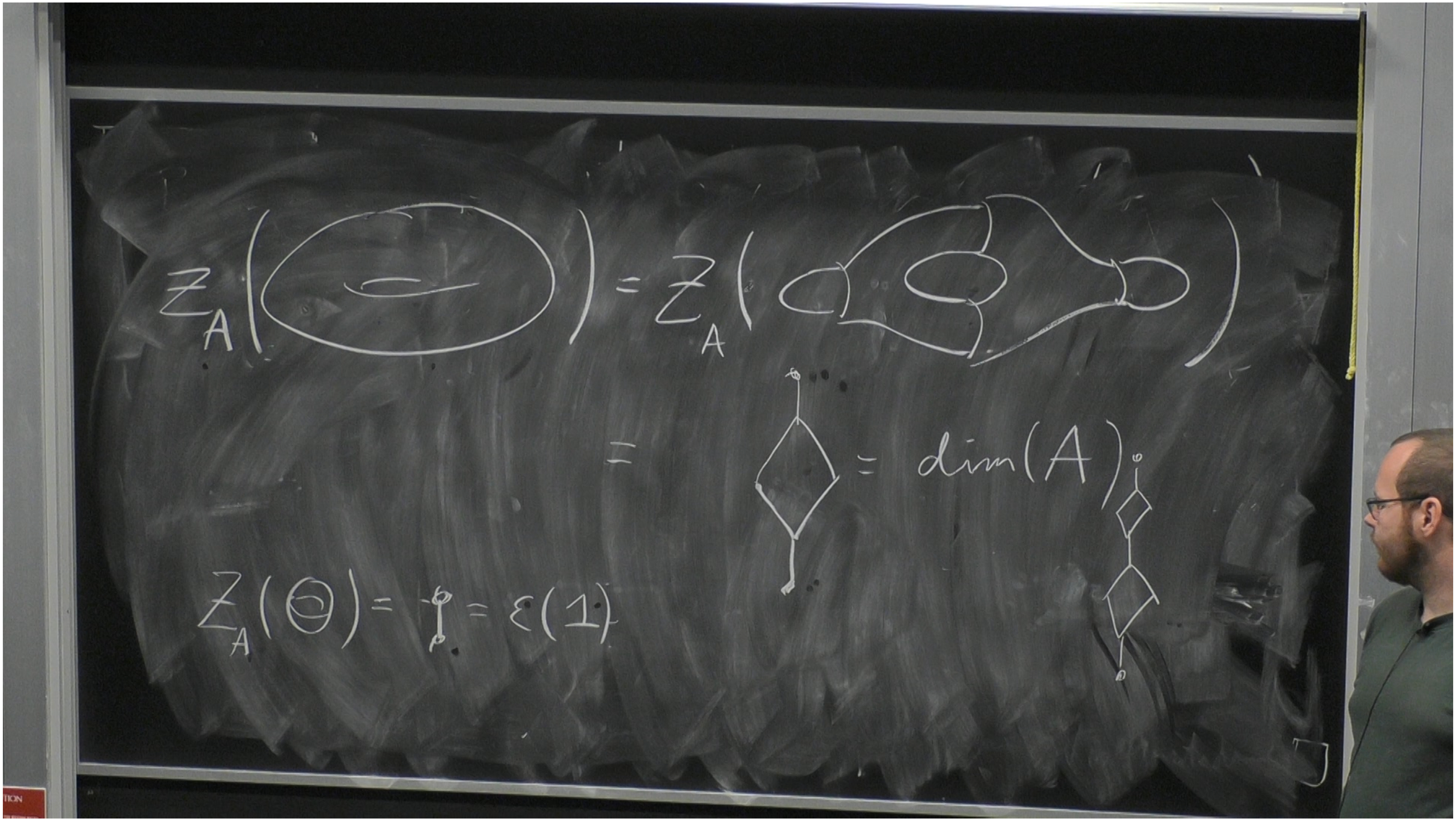

Z_A ( ) = Z_A ( )
= = dim(A)
Z_A ( ) = = ε(1)