Title: Statistical Physics Lecture - 112723

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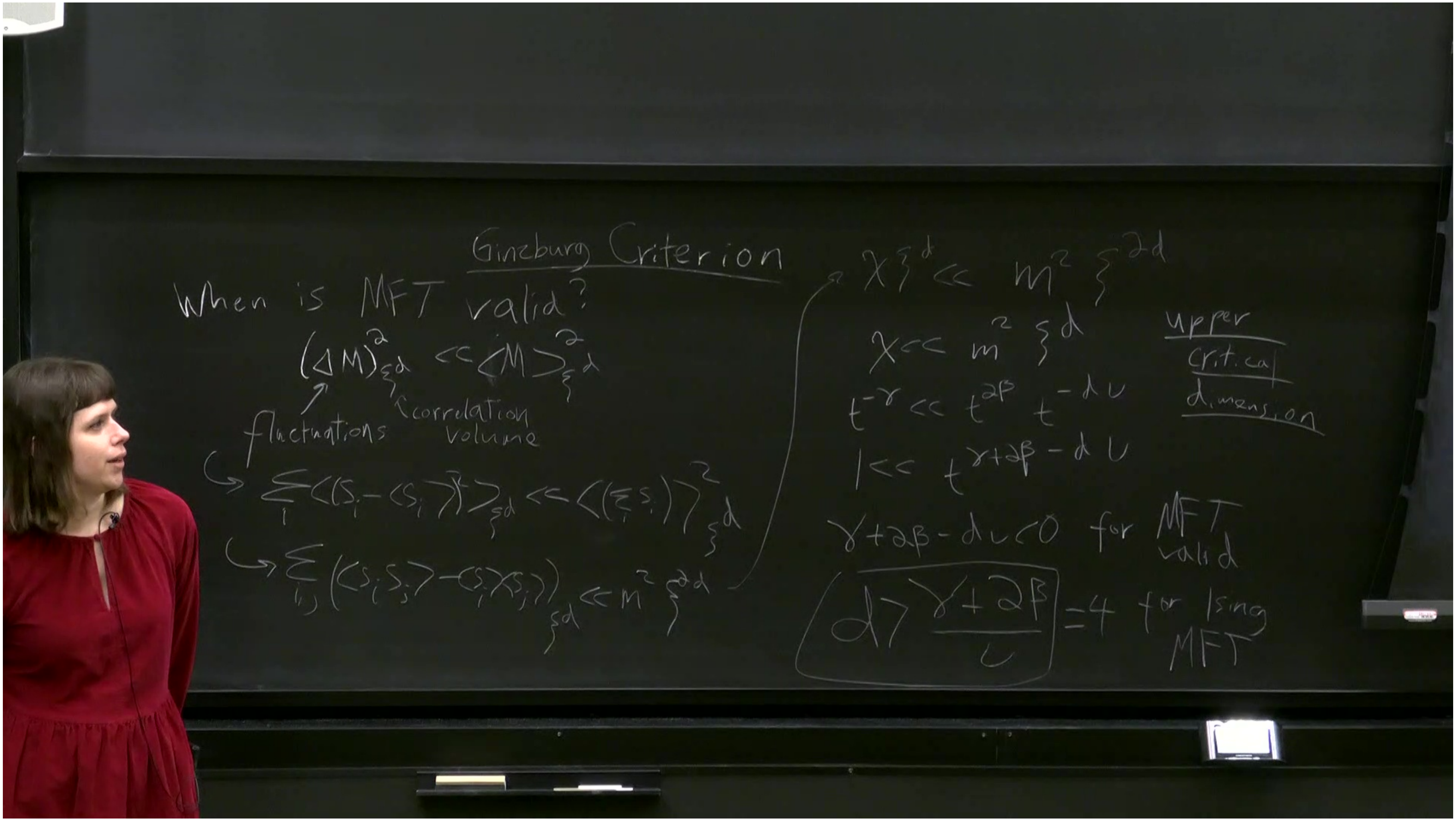
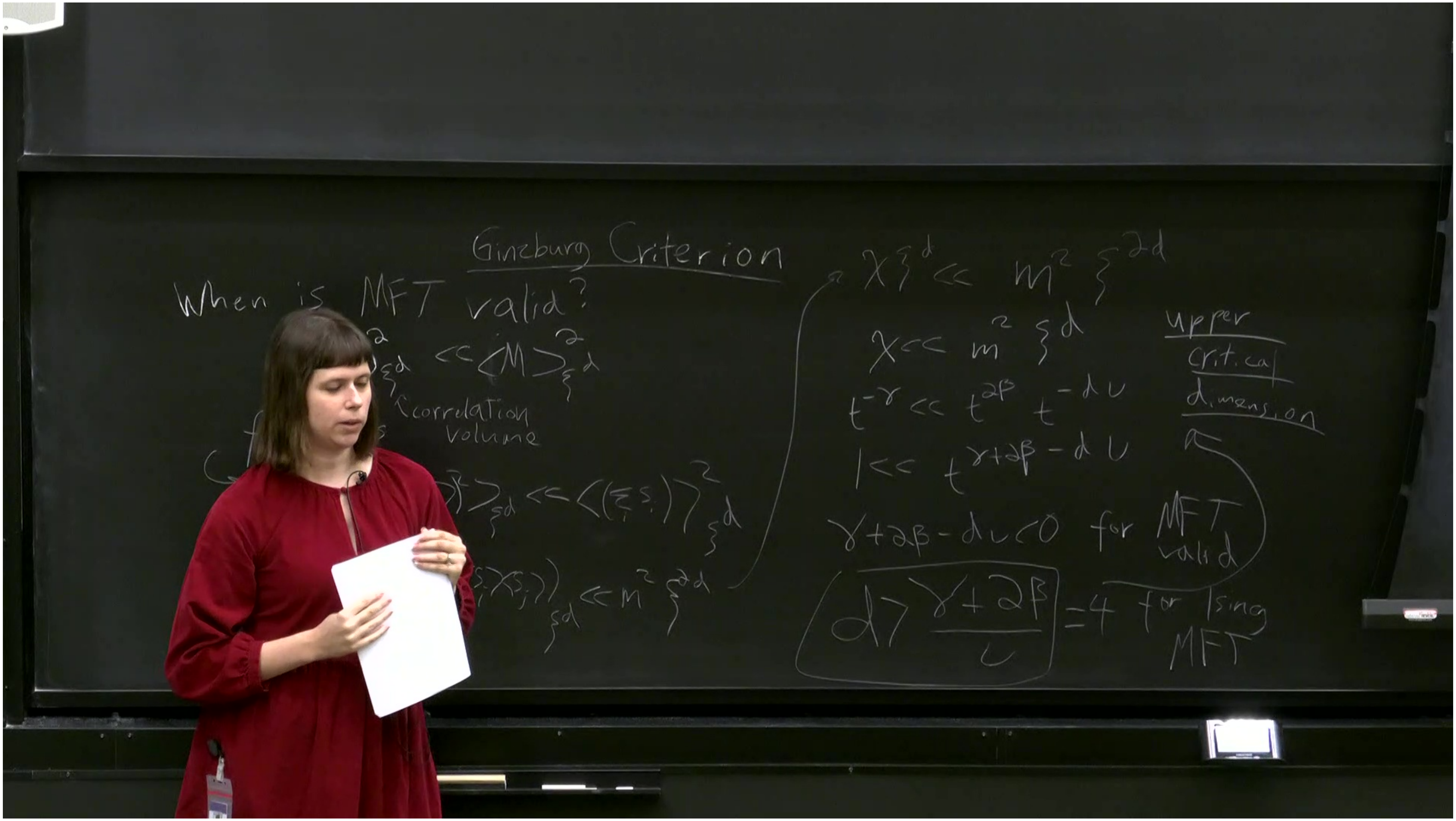

Ginzburg Criterion
When is MFT valid?
$(\Delta M)^2_{\xi^d} \ll \langle M \rangle^2_{\xi^d}$
fluctuations
correlation volume
$\sum_i \langle (S_i - \langle S_i \rangle)^2 \rangle_{\xi^d} \ll \langle (\sum_i S_i) \rangle^2_{\xi^d}$
$\sum_{i,j} (\langle S_i S_j \rangle - \langle S_i \rangle \langle S_j \rangle)_{\xi^d} \ll m^2 \xi^{2d}$
$\chi \xi^d \ll m^2 \xi^{2d}$
$\chi \ll m^2 \xi^d$
$t^{-\gamma} \ll t^{2\beta} t^{-d\nu}$
$1 \ll t^{\gamma + 2\beta - d\nu}$
$\gamma + 2\beta - d\nu < 0$ for MFT valid
$\boxed{d > \frac{\gamma + 2\beta}{\nu}} = 4$ for Ising MFT
upper critical dimension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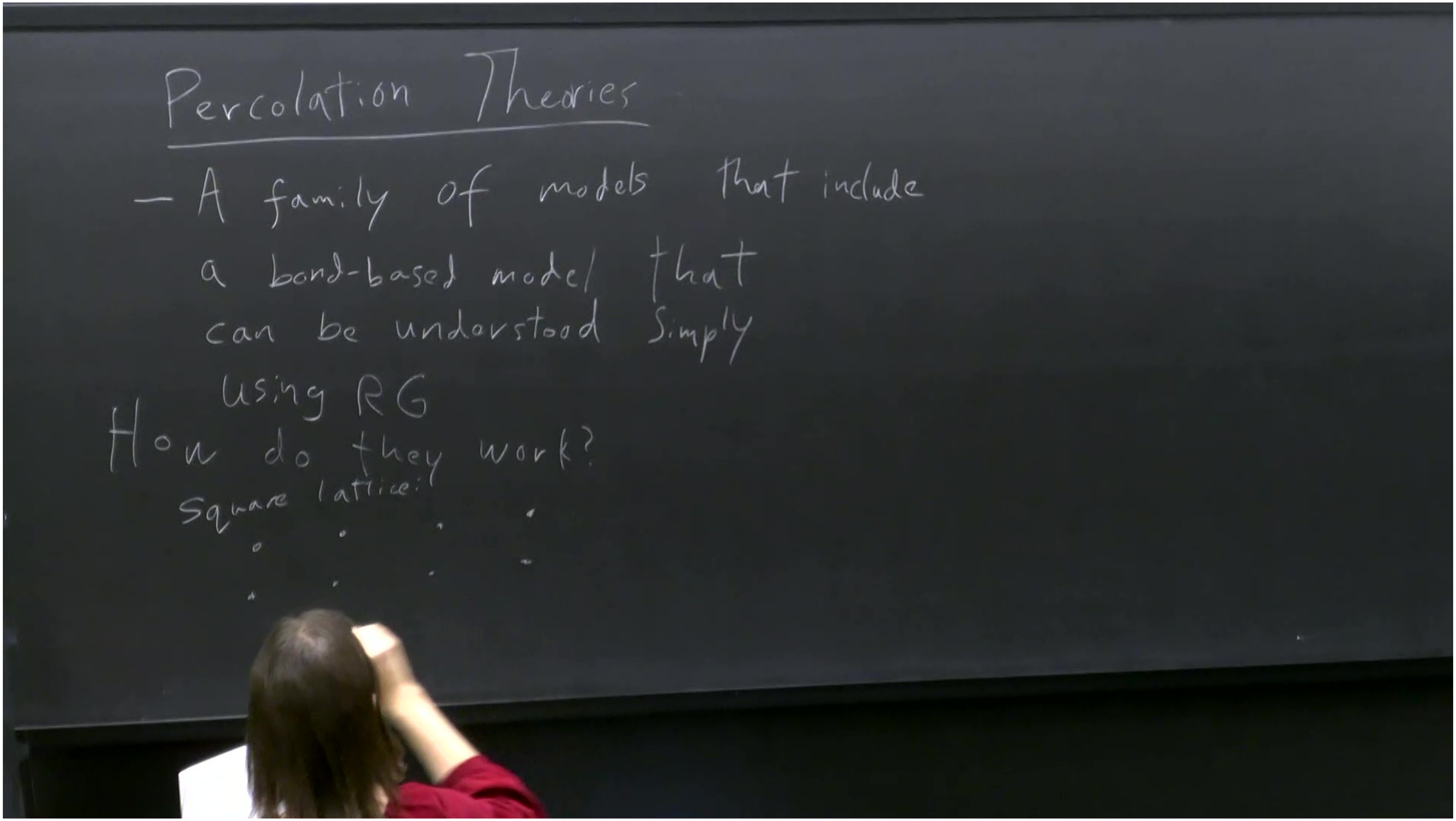
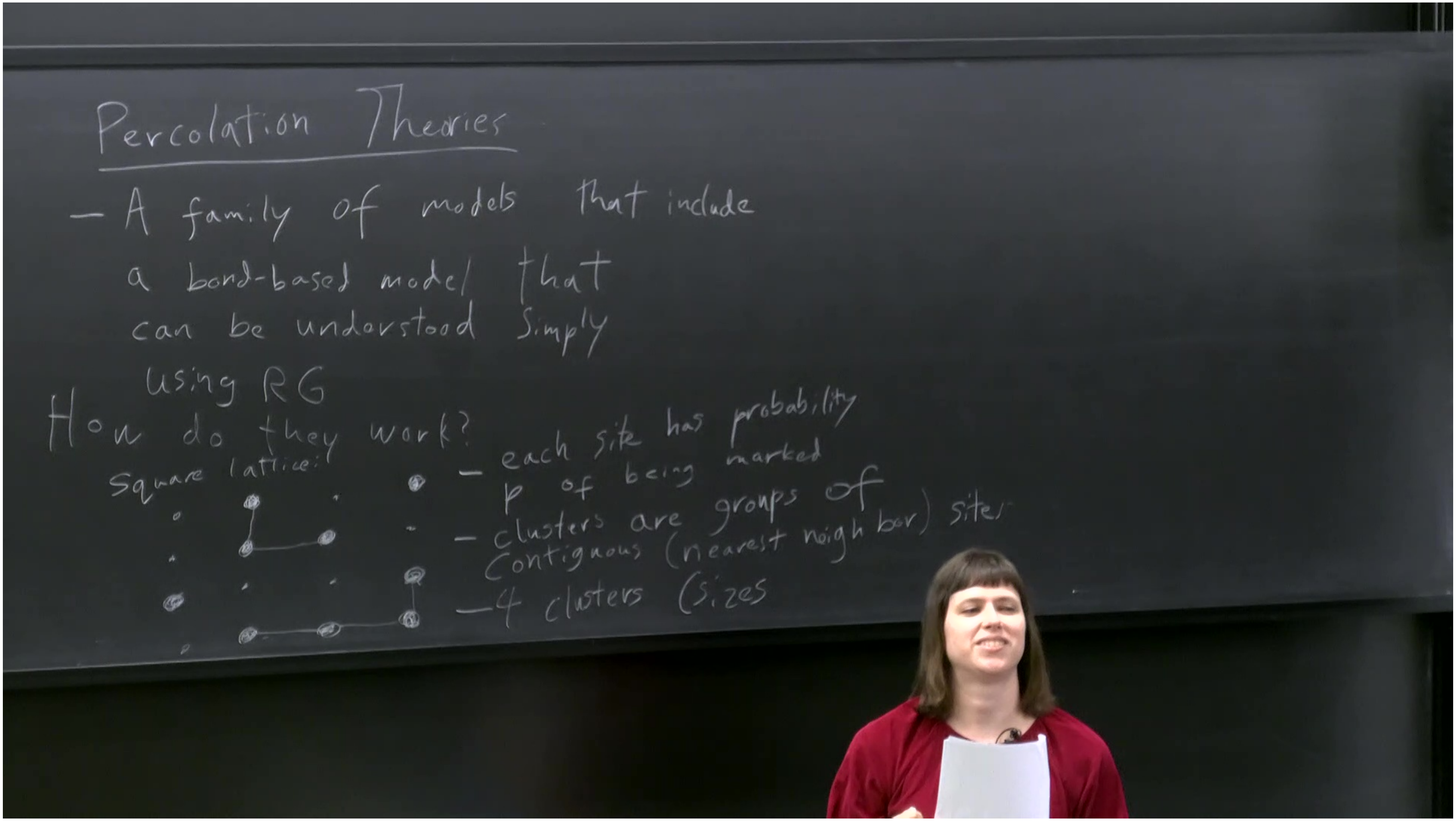
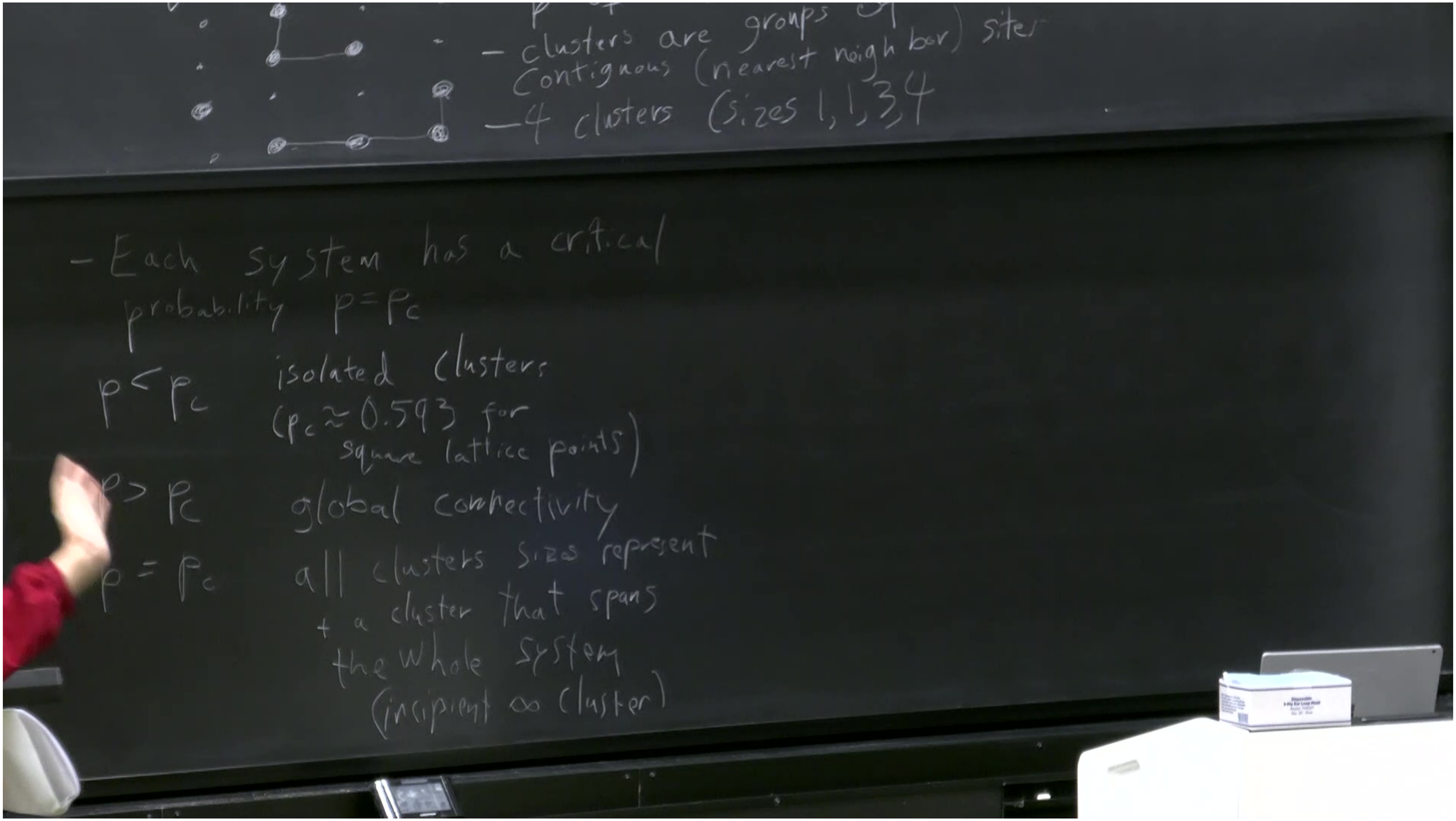
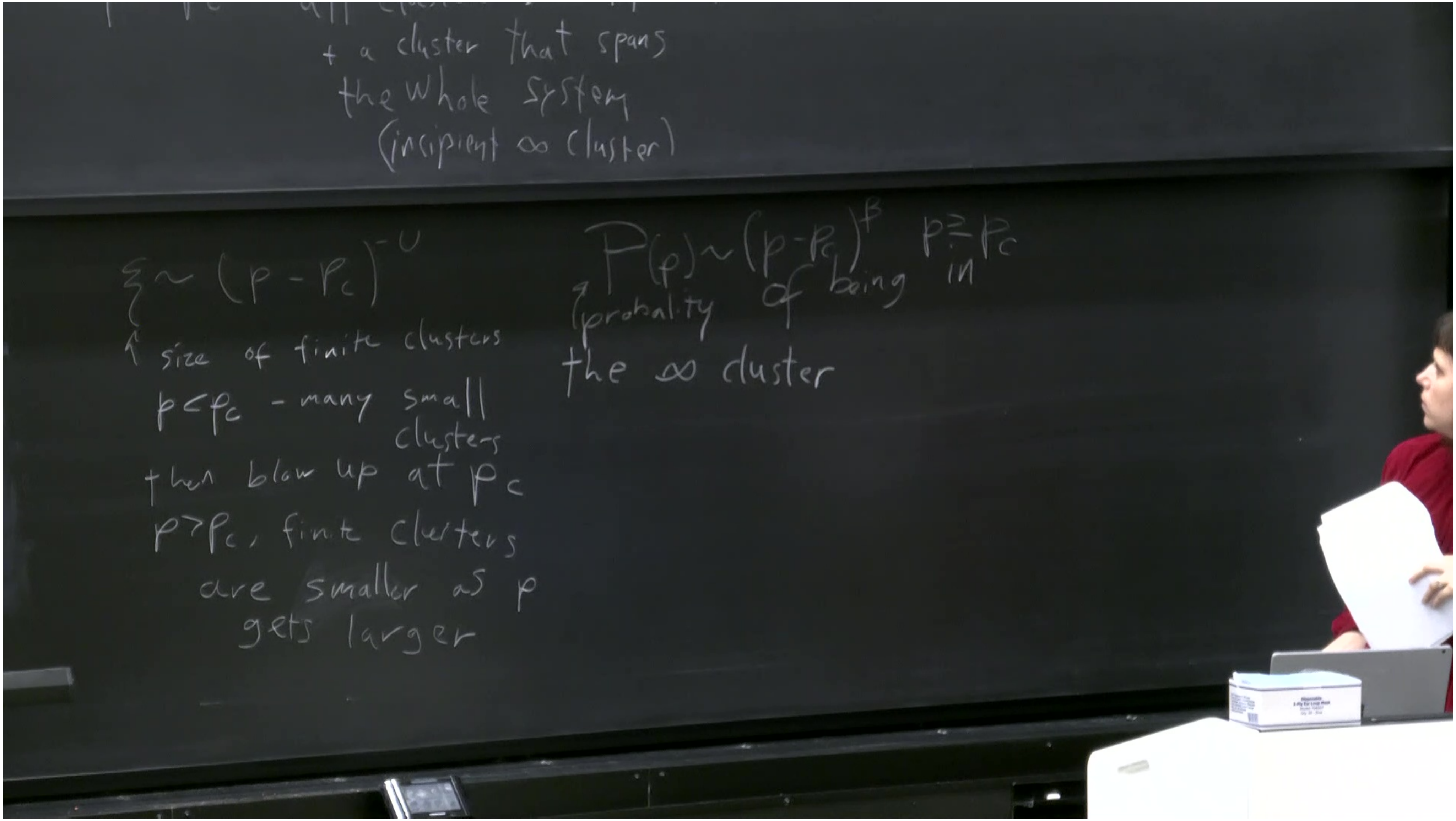

Percolation Theories
– A family of models that include
a bond-based model that
can be understood simply
using RG
How do they work?
square lattice:

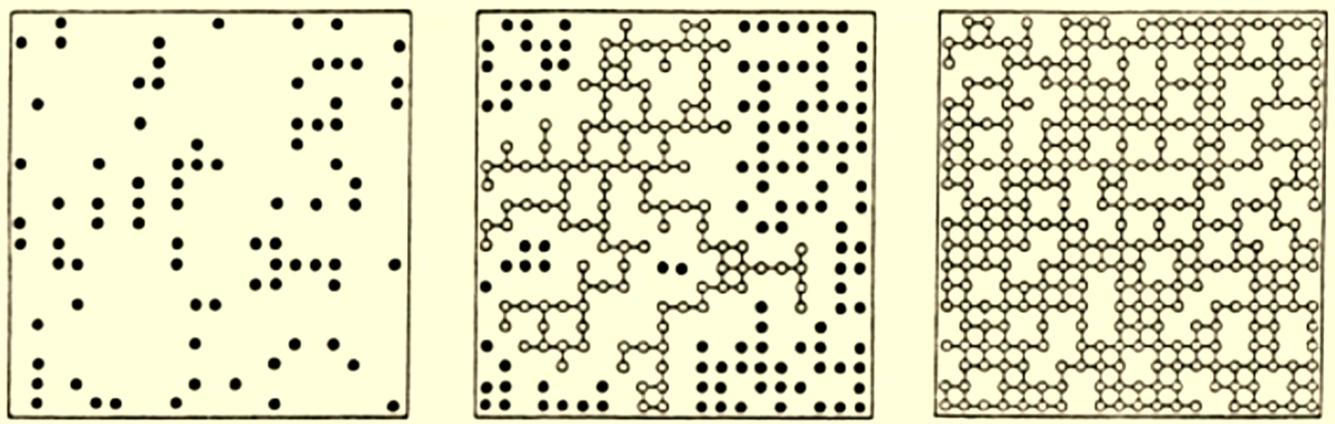

Percolation Theories
– A family of models that include
a bond-based model that
can be understood simply
using RG
How do they work?
square lattice:
– each site has probability
p of being marked
– clusters are groups of
contiguous (nearest neighbor) sites
– 4 clusters (sizes



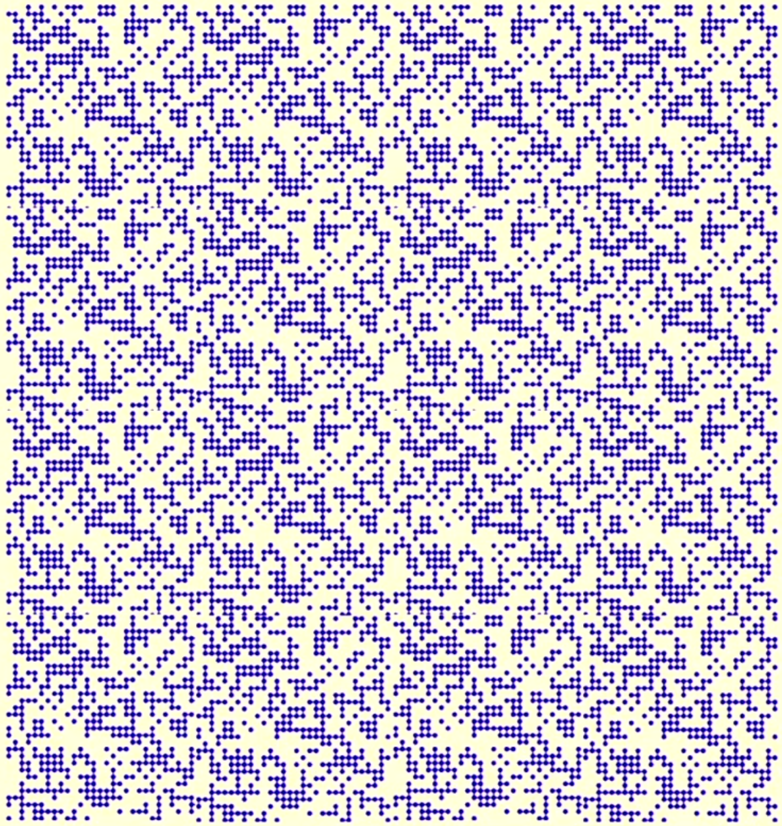
- clusters are groups of
contiguous (nearest neighbor) sites
- 4 clusters (sizes 1, 1, 3, 4
- Each system has a critical
probability p = pc
p < pc isolated clusters
(pc ≈ 0.593 for
square lattice points)
p > pc global connectivity
p = pc all clusters sizes represent
+ a cluster that spans
the whole system
(incipient ∞ cluster)


+ a cluster that spans
the whole system
(incipient ∞ cluster)
ξ ~ (p - p_c)^{-ν}
size of finite clusters
p<p_c - many small clusters
then blow up at p_c
p>p_c, finite clusters
are smaller as p
gets larger
P(p) ~ (p-p_c)^β p ≥ p_c
probability of being in
the ∞ cluster

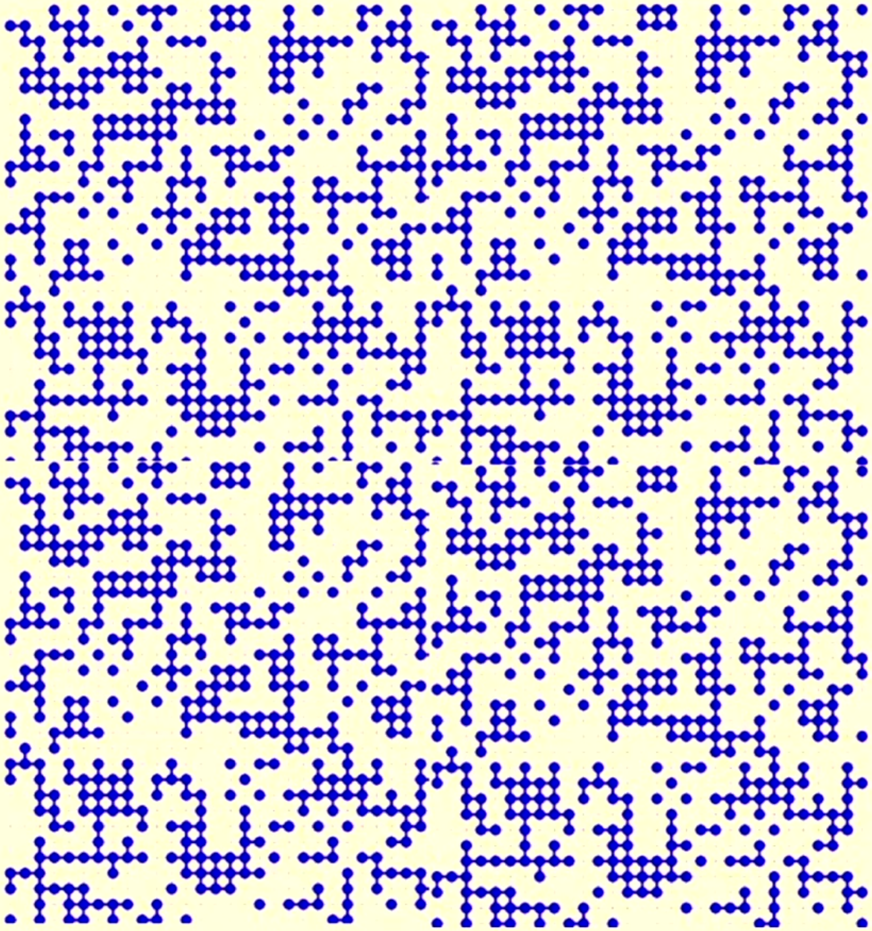
Percolation



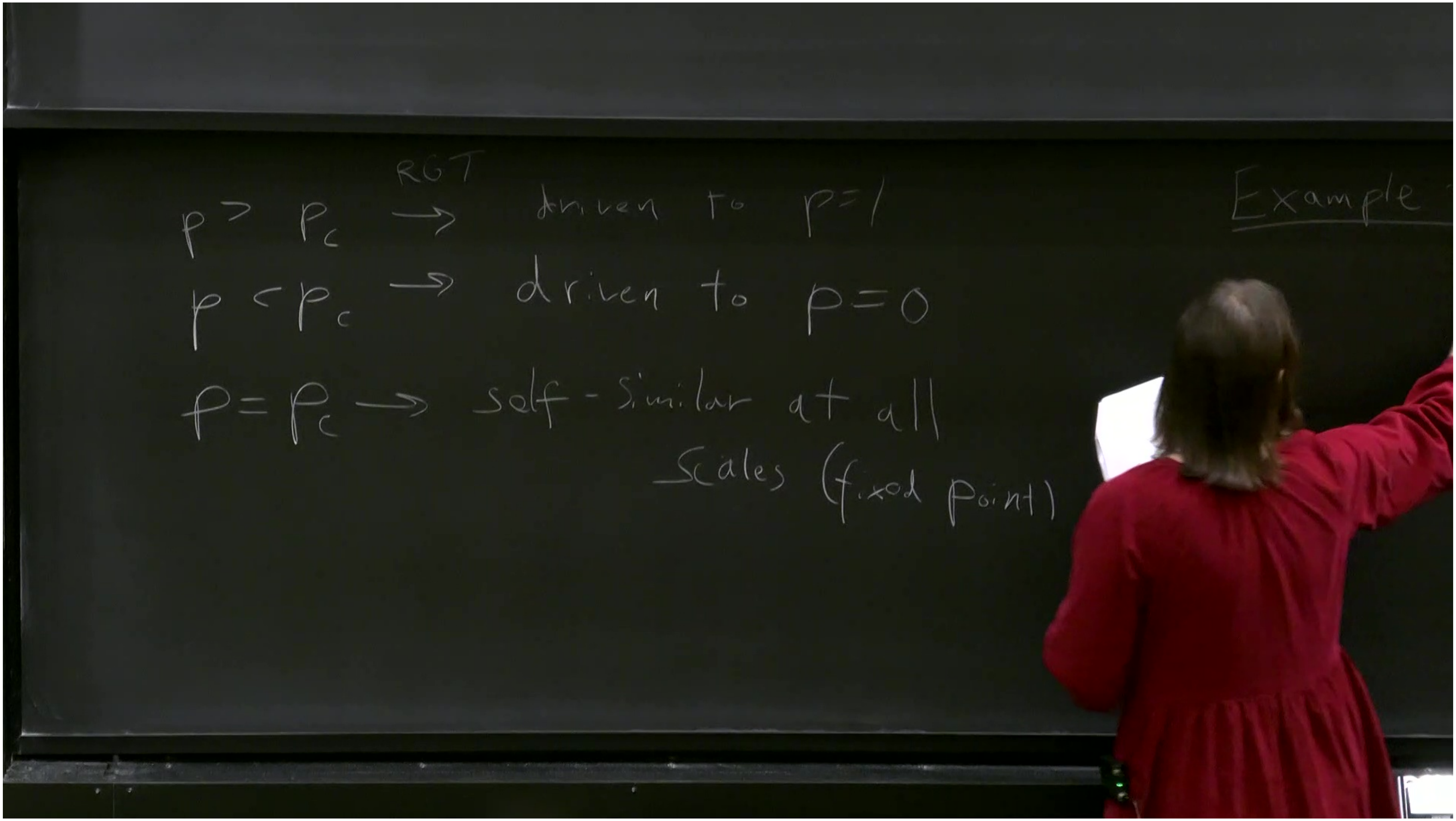
a $p = 0.2$ b $p = 0.59$ c $p = 0.8$

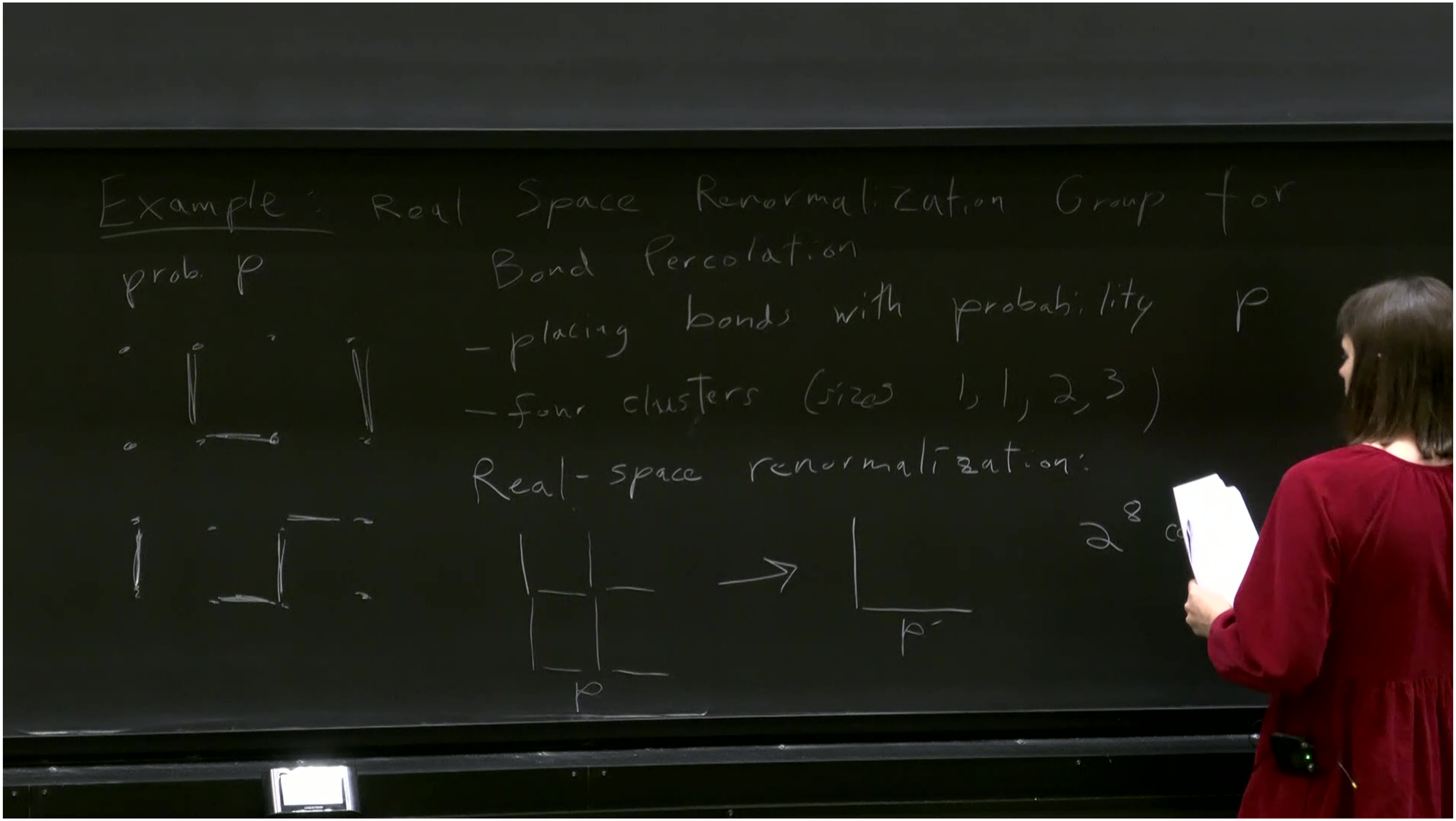


$$p' = R(p)$$

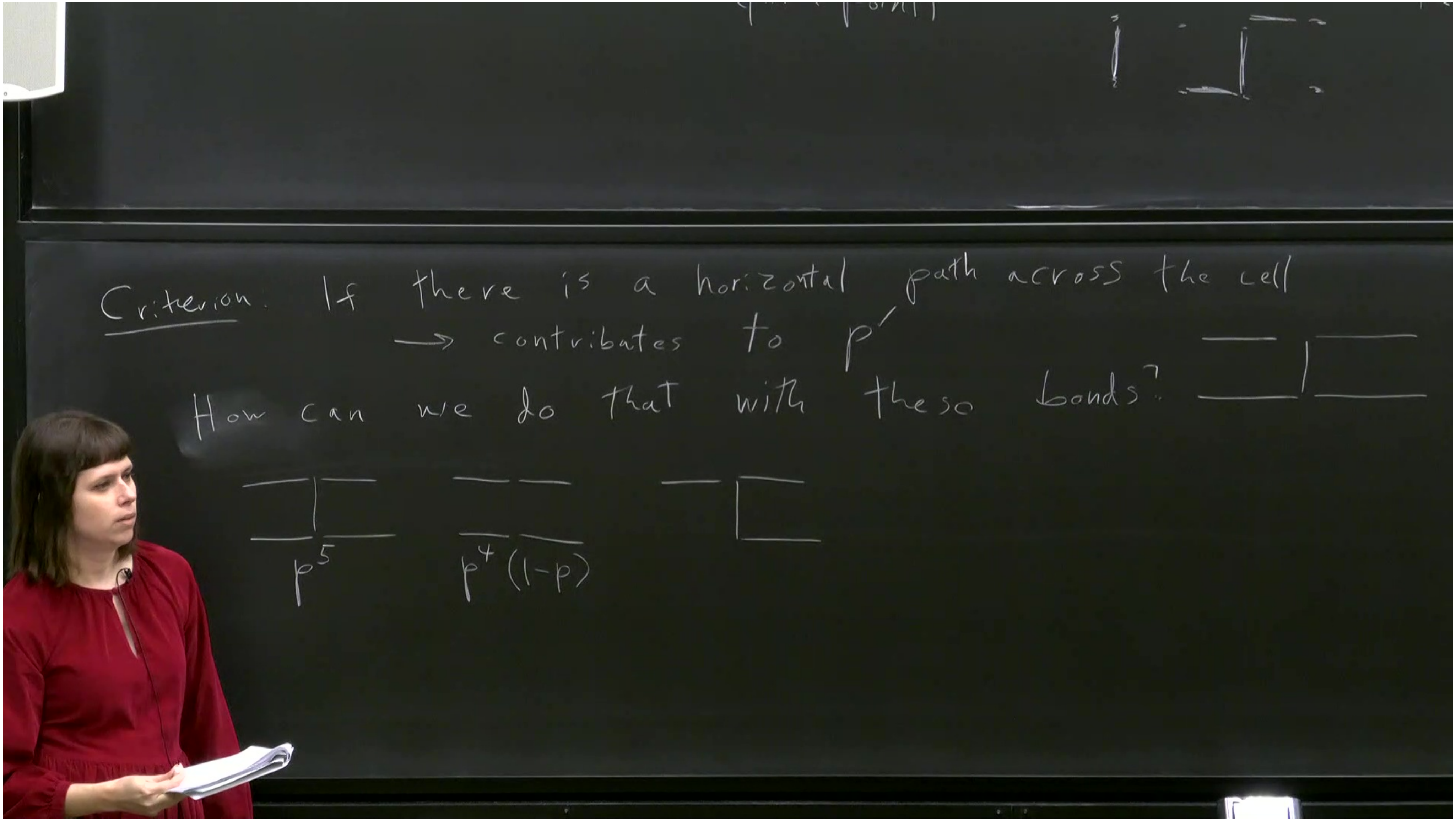


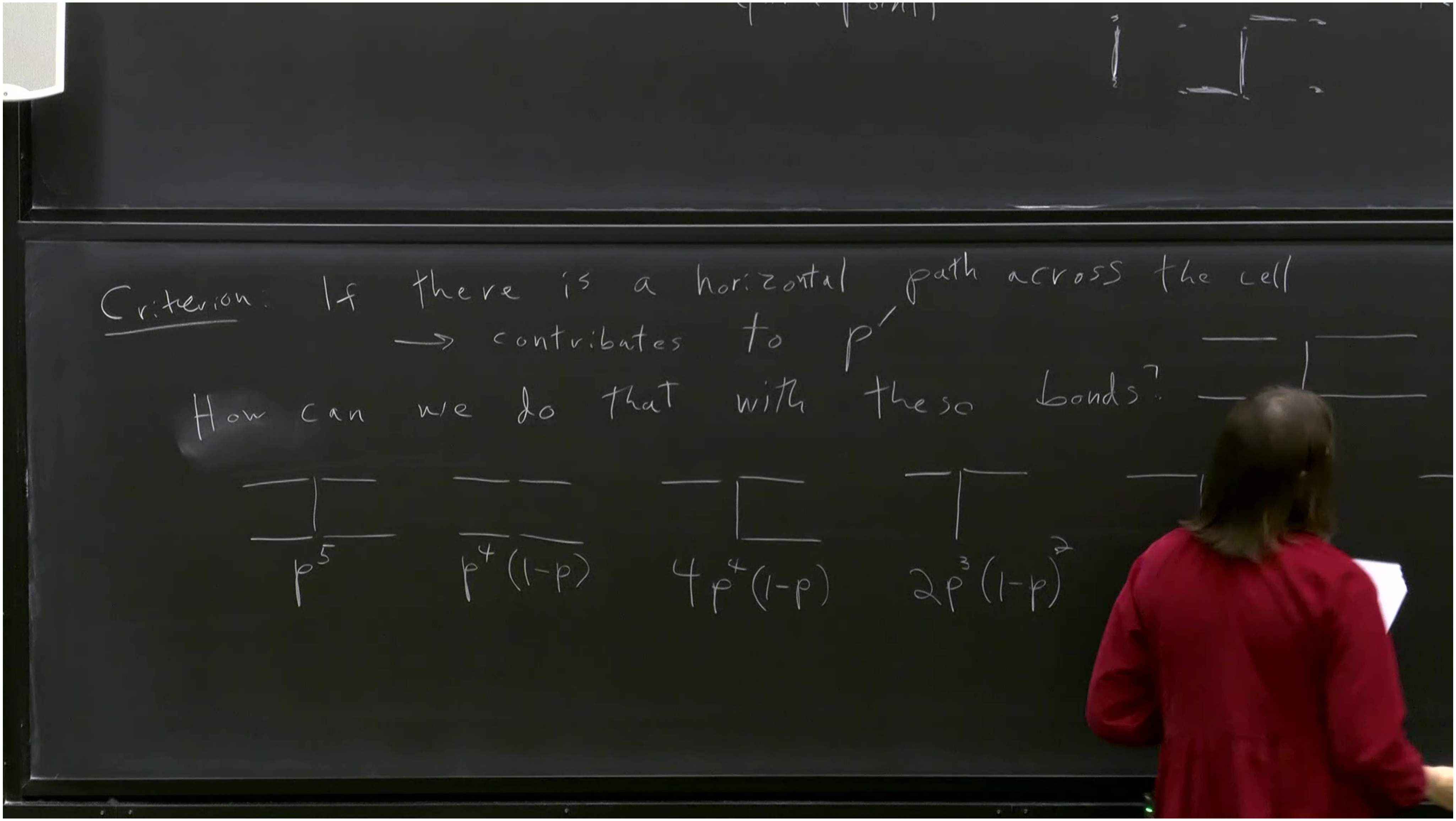
$p > p_c$

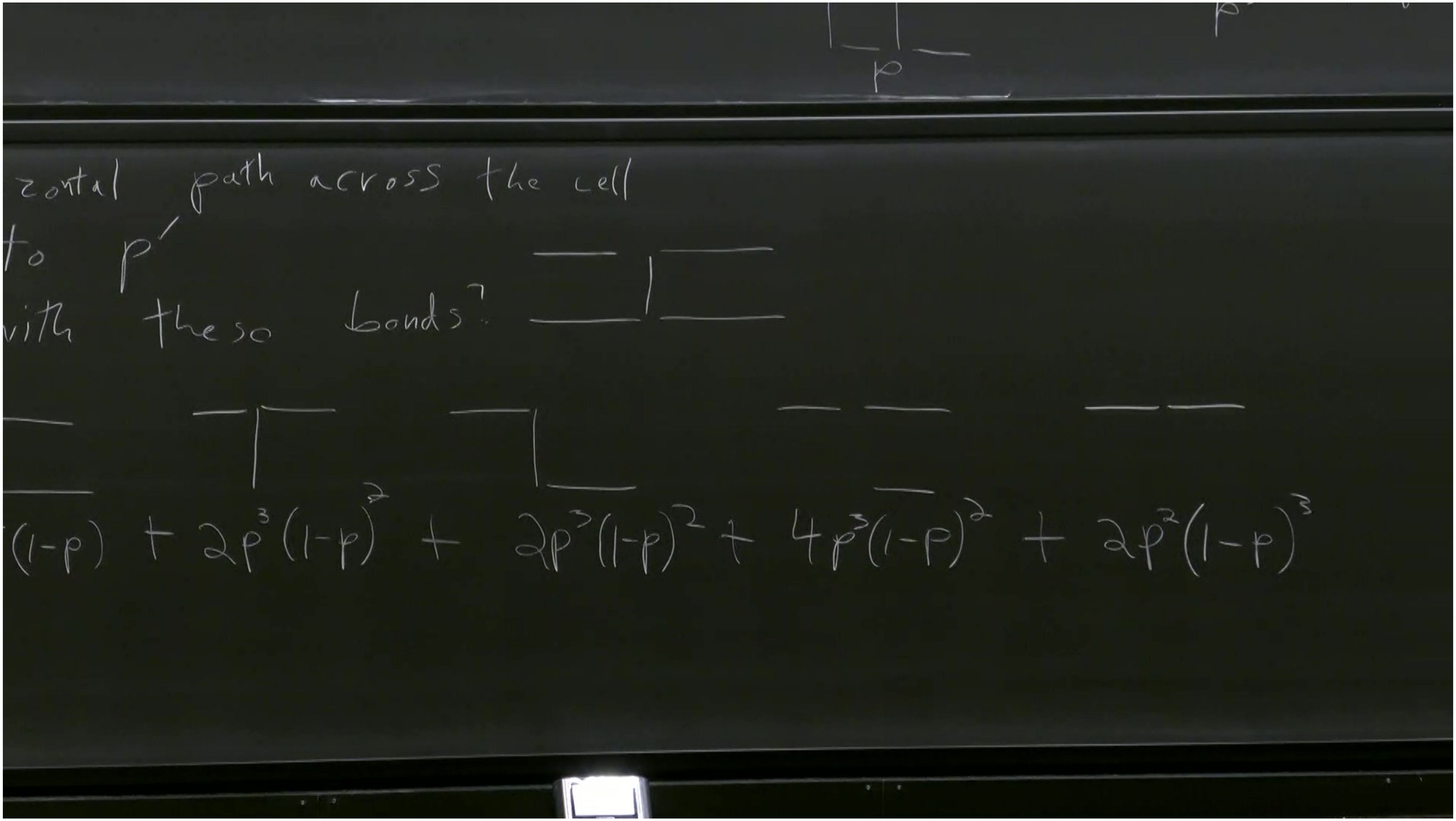

RGT
p > p_c → driven to p=1
p < p_c → driven to p=0
p = p_c → self-similar at all scales (fixed point)
Example

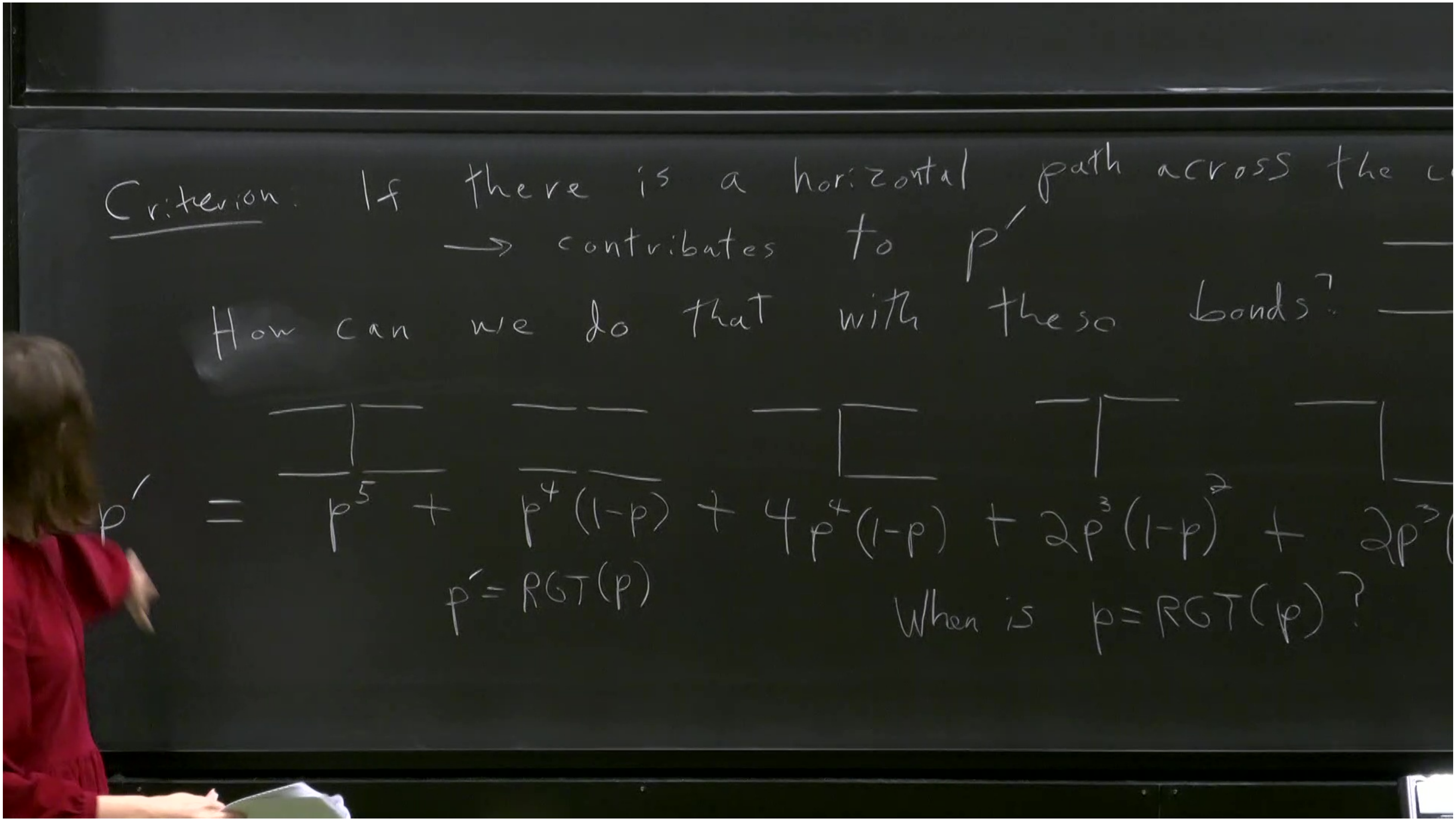


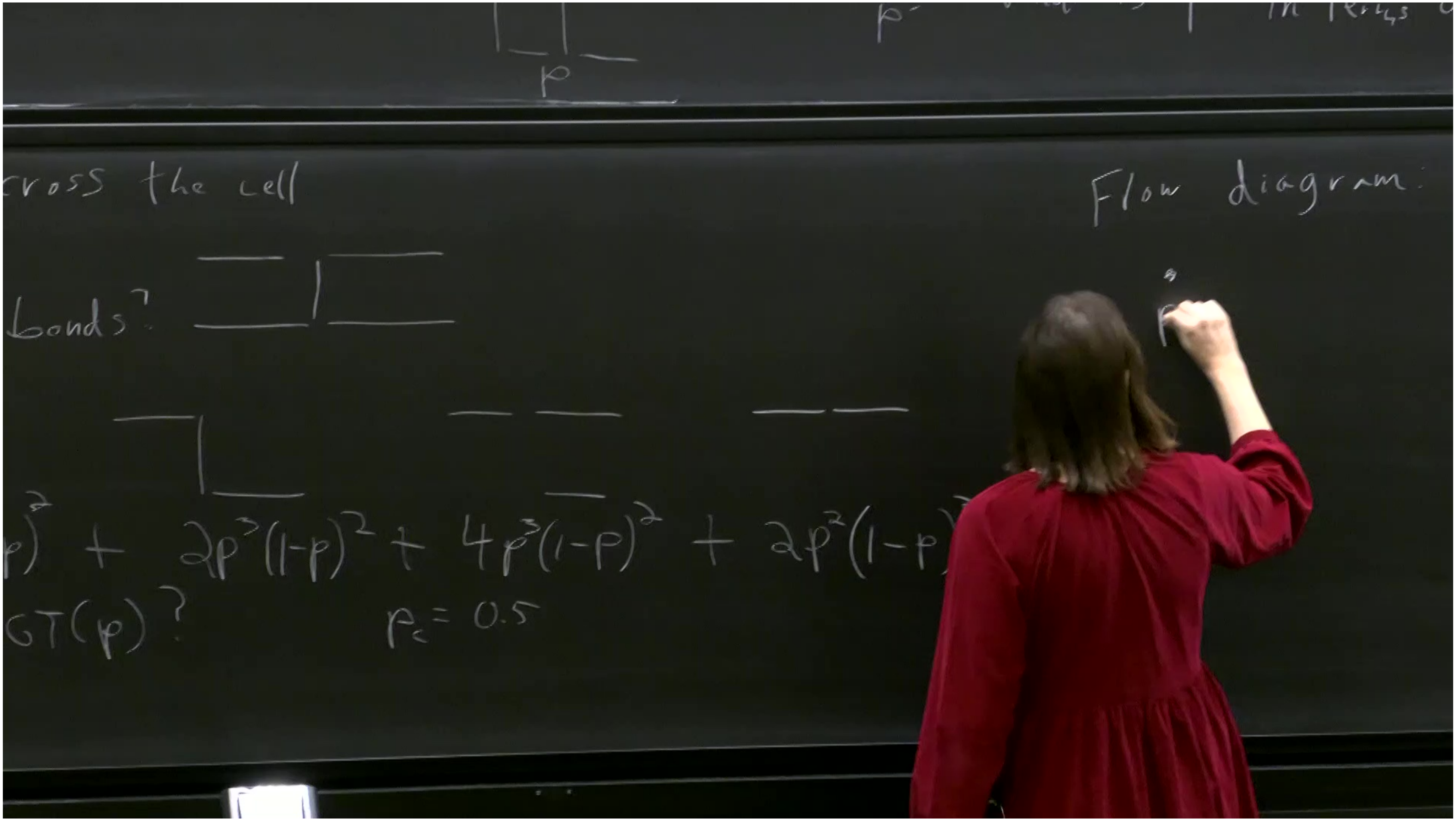
Example: Real Space Renormalization Group for
Bond Percolation
prob. p
- placing bonds with probability p
- four clusters (sizes 1, 1, 2, 3)
Real-space renormalization:
p
p'
2^8


Criterion: If there is a horizontal path across the cell
→ contributes to p′
How can we do that with these bonds?
p^5
p^4(1-p)


Criterion: If there is a horizontal path across the cell
→ contributes to p′
How can we do that with these bonds?
p^5
$p^4(1-p)$
$4p^4(1-p)$
$2p^3(1-p)^2$


zontal path across the cell
to p'
with these bonds?
p
(1-p) + 2p³(1-p)² + 2p³(1-p)² + 4p³(1-p)² + 2p²(1-p)³


Criterion: If there is a horizontal path across the
→ contributes to p'
How can we do that with these bonds?
$p' = p^5 + p^4(1-p) + 4p^4(1-p) + 2p^3(1-p)^2 + 2p^3$
$p' = RGT(p)$
When is $p = RGT(p)$?


the cell
bonds?
Flow diagram:
$+ 2p^3(1-p)^2 + 4p^3(1-p)^2 + 2p^2(1-p)$
$p_c = 0.5$

p' What is p' in terms of p?

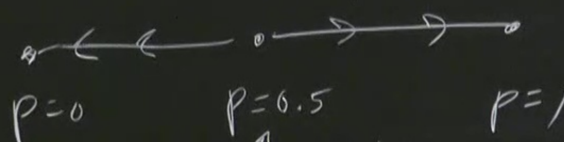
cell

$$\ldots p^3(1-p)^2 + 4p^3(1-p)^2 + 2p^2(1-p)^3$$

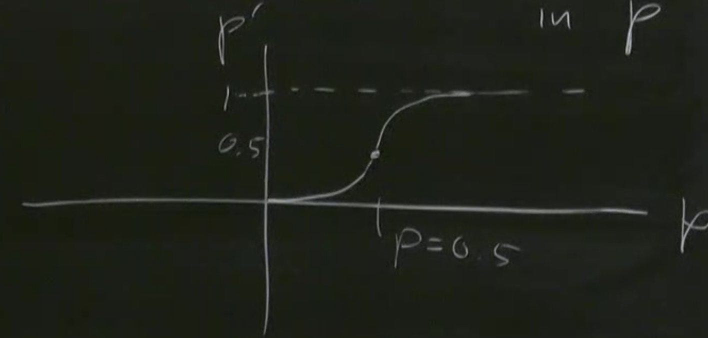
$$p_c = 0.5$$

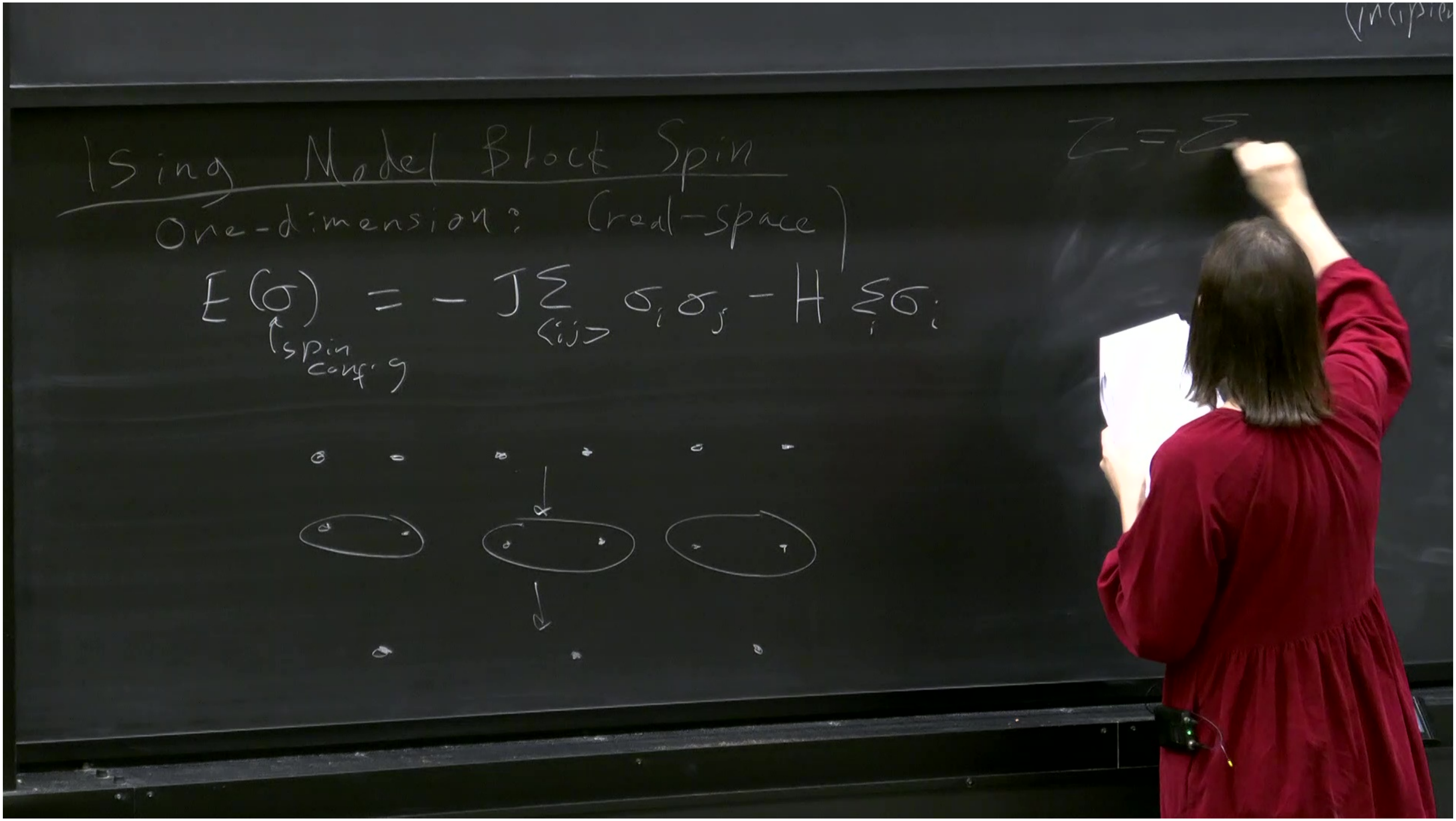
Flow diagram:

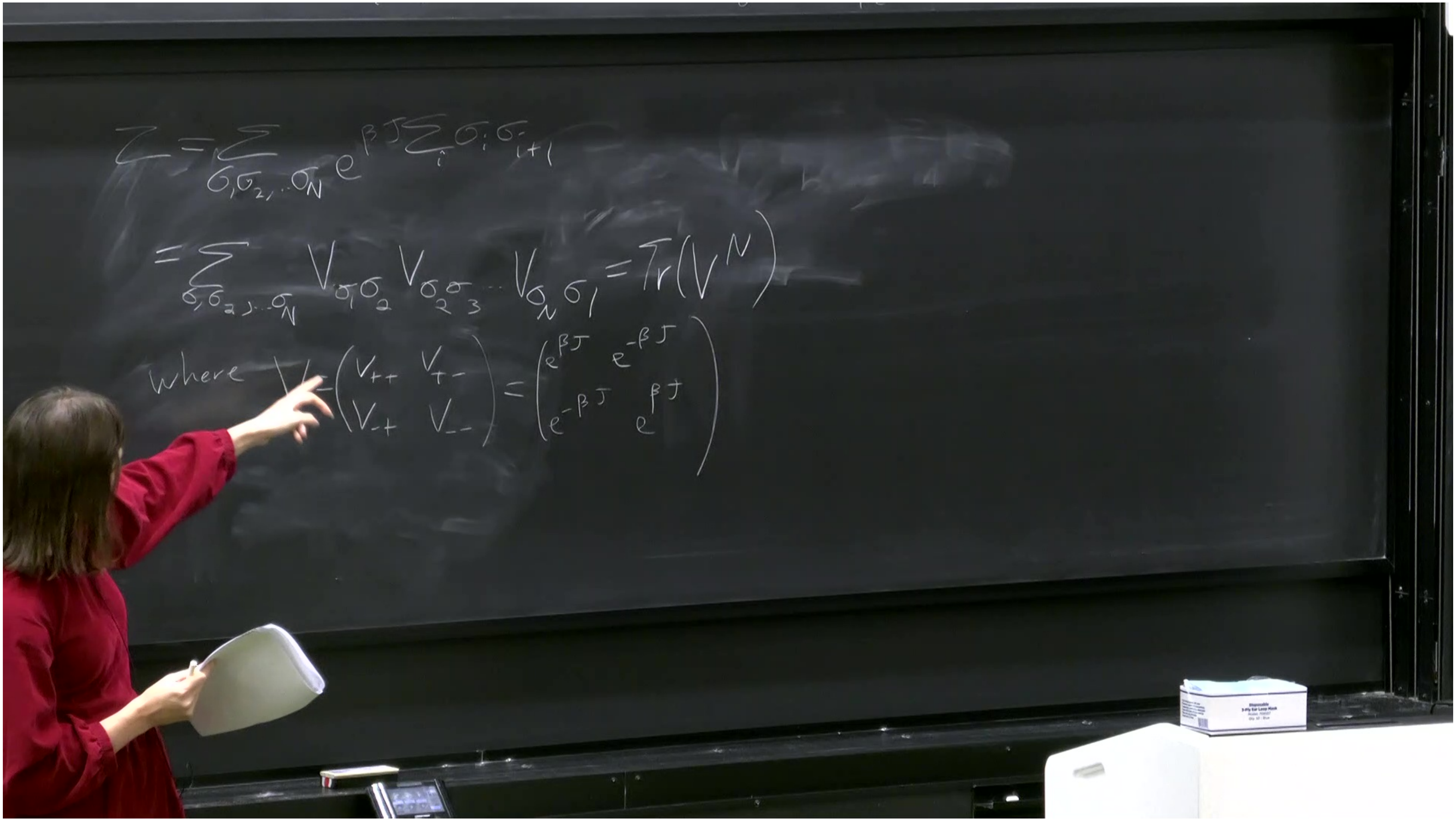
RGT

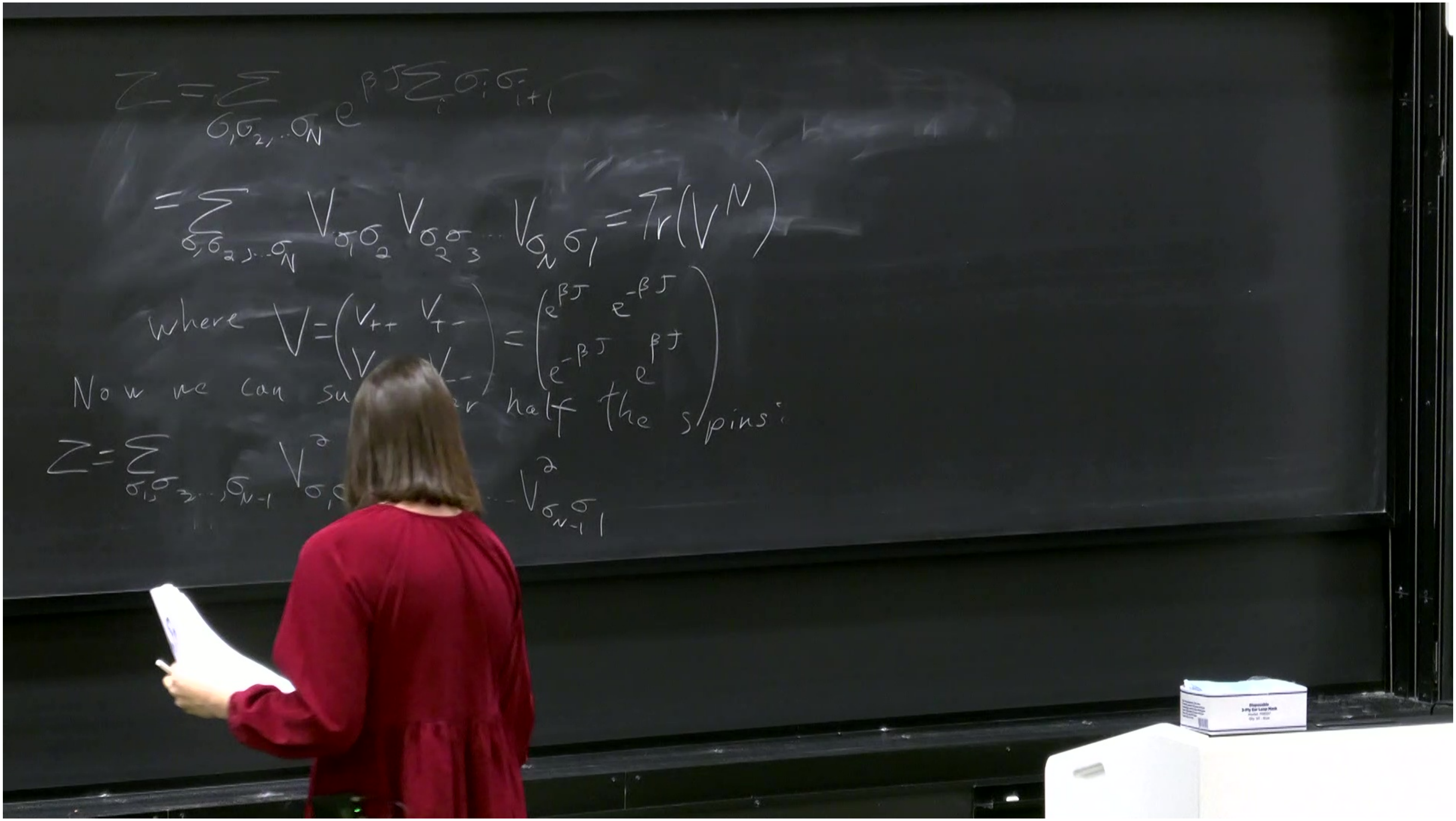


unstable fixed point in p




Ising Model Block Spin
One-dimension: (real-space)
$E(\sigma) = -J\sum_{\langle ij\rangle}\sigma_i\sigma_j - H\sum_i \sigma_i$
spin config
$Z = \sum$


$Z = \sum_{\sigma_1\sigma_2,\ldots\sigma_N} e^{\beta J \sum_i \sigma_i \sigma_{i+1}}$
$= \sum_{\sigma_1\sigma_2,\ldots\sigma_N} V_{\sigma_1\sigma_2} V_{\sigma_2\sigma_3} \ldots V_{\sigma_N\sigma_1} = Tr(V^N)$
where $V = \begin{pmatrix} V_{++} & V_{+-} \\ V_{-+} & V_{--} \end{pmatrix} = \begin{pmatrix} e^{\beta J} & e^{-\beta J} \\ e^{-\beta J} & e^{\beta J} \end{pmatrix}$


$Z = \sum_{\sigma_1, \sigma_2, \ldots \sigma_N} e^{\beta J \sum_i \sigma_i \sigma_{i+1}}$
$= \sum_{\sigma_1, \sigma_2, \ldots \sigma_N} V_{\sigma_1 \sigma_2} V_{\sigma_2 \sigma_3} \cdots V_{\sigma_N \sigma_1} = Tr(V^N)$
where $V = \begin{pmatrix} V_{++} & V_{+-} \\ V & V_{--} \end{pmatrix} = \begin{pmatrix} e^{\beta J} & e^{-\beta J} \\ e^{-\beta J} & e^{\beta J} \end{pmatrix}$
Now we can su r half the spins:
$Z = \sum_{\sigma_1, \sigma_3, \ldots, \sigma_{N-1}} V^2_{\sigma_1 \sigma} \cdots V^2_{\sigma_{N-1} \sigma_1}$

$$Z = \sum_{s_1, s_2, \ldots, s_{N/2}} V^2_{s_1 s_2} V^2_{s_2 s_3} \cdots V^2_{s_{N/2}, s_1}$$

- since βJ always appears as a product, $K = \beta J$

$$V^2 = \begin{pmatrix} e^{2K} + e^{-2K} & 2 \\ 2 & e^{2K} + e^{-2K} \end{pmatrix} = 2 \begin{pmatrix} \cosh 2K & 1 \\ 1 & \cosh 2K \end{pmatrix}$$

$$= \underbrace{2(\cosh 2K)^{\frac{1}{2}}}_{a} \underbrace{\begin{pmatrix} e^{\underbrace{\frac{1}{2} \ln(\cosh 2K)}_{K'}} & e^{-\frac{1}{2} \ln(\cosh 2K)} \\ e^{-\frac{1}{2} \ln(\cosh 2K)} & e^{\frac{1}{2} \ln(\cosh 2K)} \end{pmatrix}}_{V'}$$

Match:

$$V = \begin{pmatrix} e^{K} & e^{-K} \\ e^{-K} & e^{K} \end{pmatrix}$$

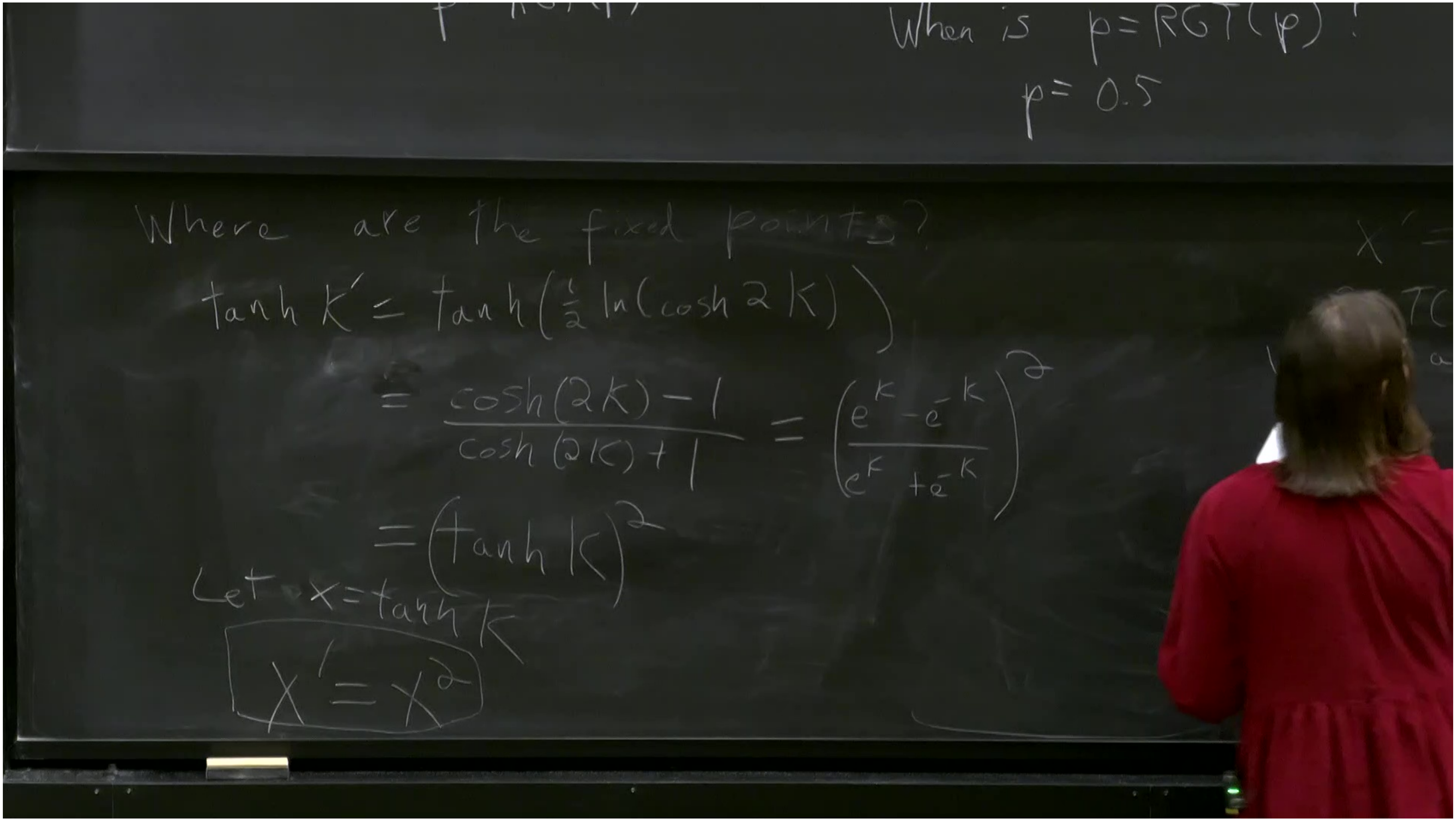
Thus we have:

$$Z(N, K) = a^{N/2} \sum_{s_1, s_2, \ldots s_{N/2}} V'_{s_1 s_2} V'_{s_2 s_3} \cdots V'_{s_{N/2} s_1} = [a(K')]^{N/2} \, \mathrm{Tr}[(V')^{N/2}]$$

N: particle number

K: βJ

$$Z(N,K) = [a(K')]^{N/2} \, Z\left(\frac{N}{2}, K'\right)$$


When is $p = RG_T(p)$?
$p = 0.5$
Where are the fixed points?
$\tanh K' = \tanh\left(\frac{1}{2}\ln(\cosh 2K)\right)$
$= \frac{\cosh(2K)-1}{\cosh(2K)+1} = \left(\frac{e^K - e^{-K}}{e^K + e^{-K}}\right)^2$
$= (\tanh K)^2$
Let $x = \tanh K$
$\boxed{x' = x^2}$

When is $p = RGT(p)$?

$p = 0.5$

$p_c = 0.5$

...d points?

$\cosh 2K)\,)$

$$\frac{\ -1}{\ +1} = \left(\frac{e^{K} - e^{-K}}{e^{K} + e^{-K}}\right)^2$$

$x' = x^2$

$RGT(x) = x^2$

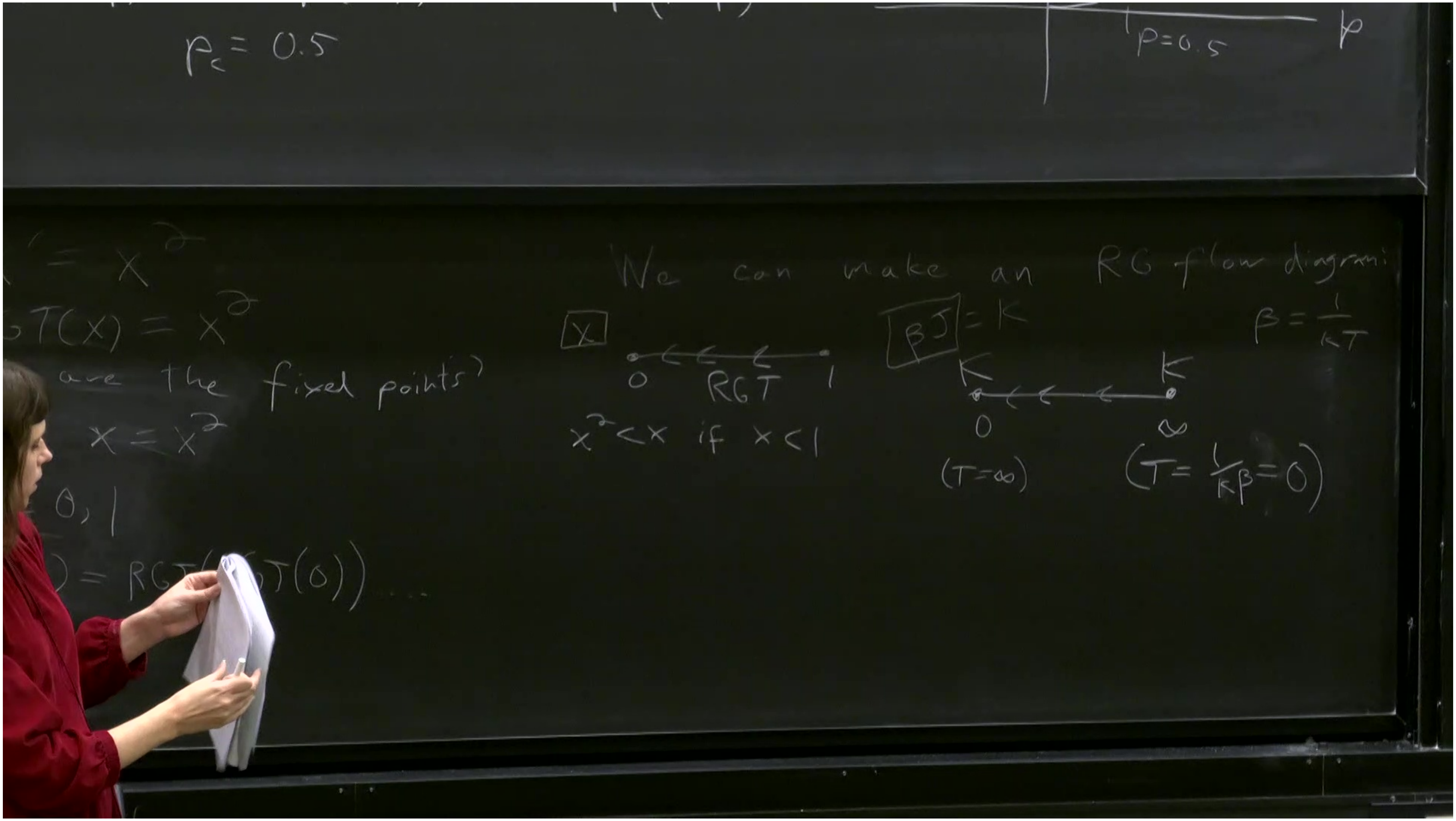
What are the fixed points?

Solve $x = x^2$

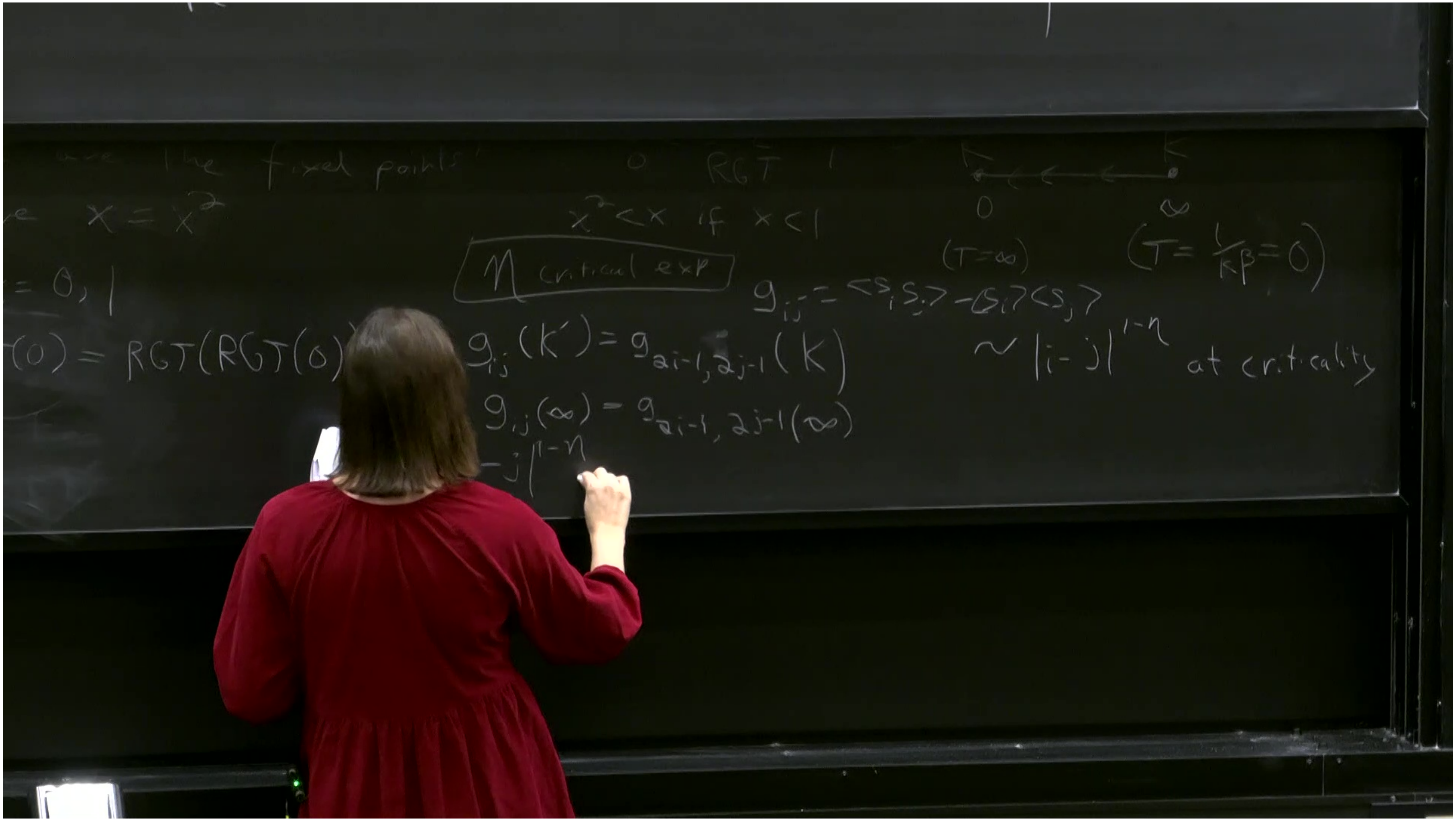
$x = 0, 1$

$0 = RGT(0) = RGT(RGT(0))\ldots$

We



p_c = 0.5
p=0.5
p
We can make an RG flow diagram
x
0 RGT 1
x^2 < x if x < 1
βJ = K
β = 1/kT
K K
0 ∞
(T = ∞)
(T = 1/kβ = 0)
= x^2
T(x) = x^2
are the fixed points?
x = x^2
0, 1
) = RG T(0))


the fixed points
RGT
0
∞
x = x^2
x^2 < x if x < 1
(T=∞)
(T = 1/kβ = 0)
η critical exp
= 0, 1
g_ij = <s_i s_j> - <s_i><s_j>
(0) = RGT(RGT(0))
g_ij(K') = g_2i-1,2j-1(K)
~ |i-j|^(1-η) at criticality
g_ij(∞) = g_2i-1,2j-1(∞)
-j|^(1-η)

are the fixed points 0 RGT 1

$K \quad K$

$0 \leftarrow \leftarrow \leftarrow \leftarrow \infty$

$x = x^2$ $\qquad x^2 < x$ if $x < 1$

$\boxed{\eta \text{ critical exp}}$ $\qquad (T = \infty) \qquad \left(T = \frac{1}{K\beta} = 0\right)$

$= 0, 1$

$g_{ij} = \langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle$

$(0) = RGT(RGT(0)) \ldots$ $\qquad g_{ij}(K') = g_{2i-1,2j-1}(K)$ $\qquad \sim |i-j|^{1-\eta}$ at criticality

$g_{ij}(\infty) = g_{2i-1,2j-1}(\infty)$

$|i-j|^{1-\eta} = 2^{1-\eta}|i-j|^{1-\eta} \rightarrow 1 = 2^{1-\eta} \rightarrow 1-\eta = 0 \rightarrow \boxed{\eta = 1}$