Title: Statistical Physics Lecture - 111423

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Note: thermodynamics is the process of developing phenomenological laws for many particle systems from macroscopic data

(Boyle's law, Charles' law ...)

- themodynamic relations — appendix

Overview

- Statistical mechanics involves systems of many particles

Scale up to $\sim 10^{23}$ particles!

statistical mechanics

Classical phenom

ideal gases

phase transition (thermal)

quantum phenom.

Bose-Einstein Cond

para magnetism

phase transitions (quantum fluctuations)

- Statistical physics we derive these laws from the ground up
- art of appropriate approximations/asymptotics

We will focus mainly on phase transitions:

- thermodynamic approach
- Statistical approaches
  - mean field
  - renormalization group

This will take us to statistical field theory

QFT: $Z = \int \mathcal{D}\phi \, e^{i/\hbar \int d^d x \, \mathcal{L}(\phi)}$

SFT: $Z = \int \mathcal{D}\phi \, e^{-\beta \int d^d x \, F(\phi)}$

lattice (microscopic) model $\xrightarrow[\text{transformation}]{\text{Hubbard-Stratonovich}}$ scalar field theory

lattice regularization

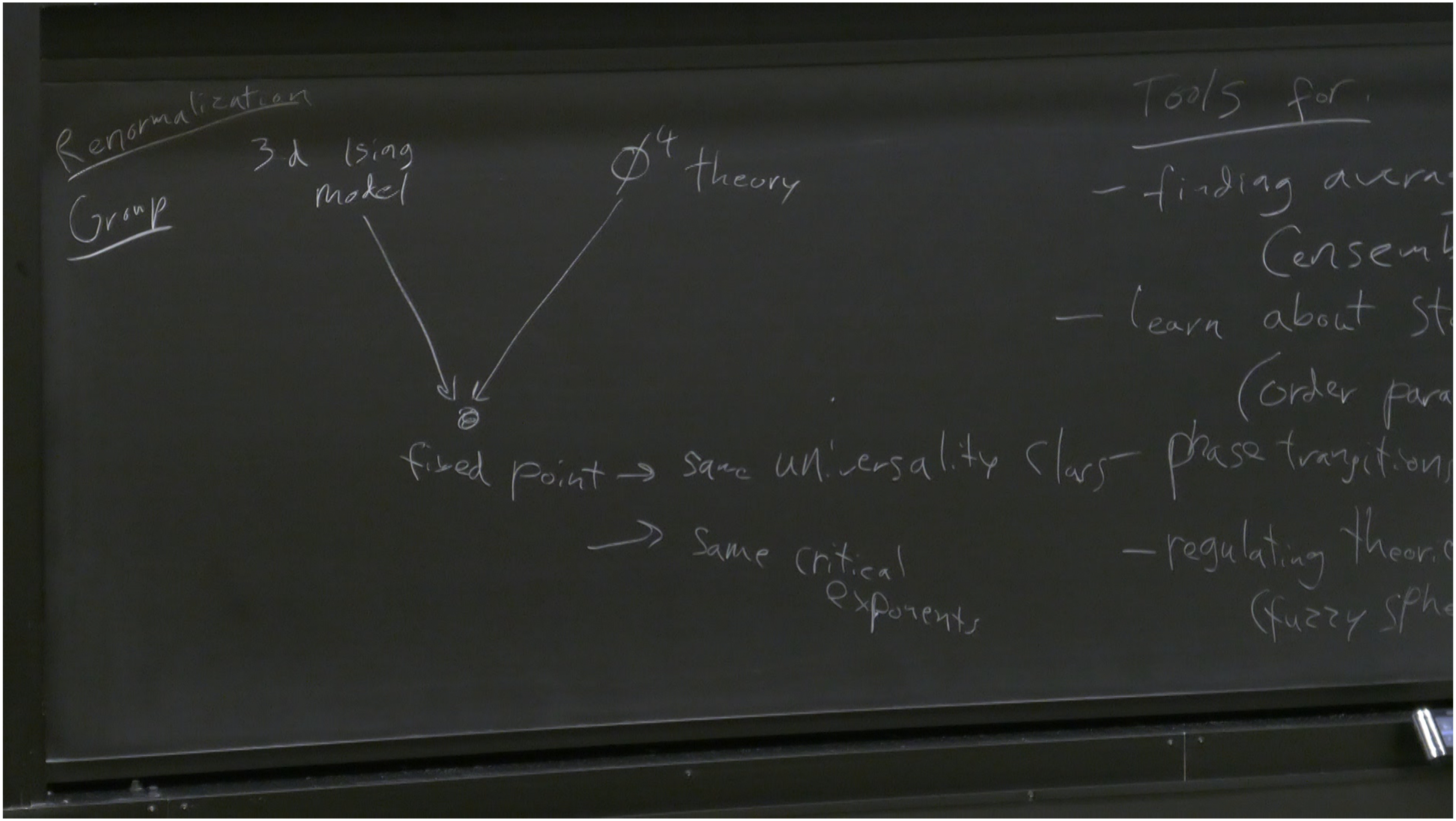
learn from numerics

in the continuum

perturbative expansions

(about $4-\varepsilon$, large-N)

learn analytically


Renormalization Group
3d Ising model
ϕ^4 theory
fixed point → same universality class
→ same critical exponents
Tools for:
– finding
– learn about
(order
phase transitions
– regulating
(fuzzy

Tools for:

ϕ^4 theory

- finding averag observables (ensembles)
- learn about states/phases (order parameter, pure/mixed state)

...oint → same universality class — phase transitions (RG)

→ same critical exponents

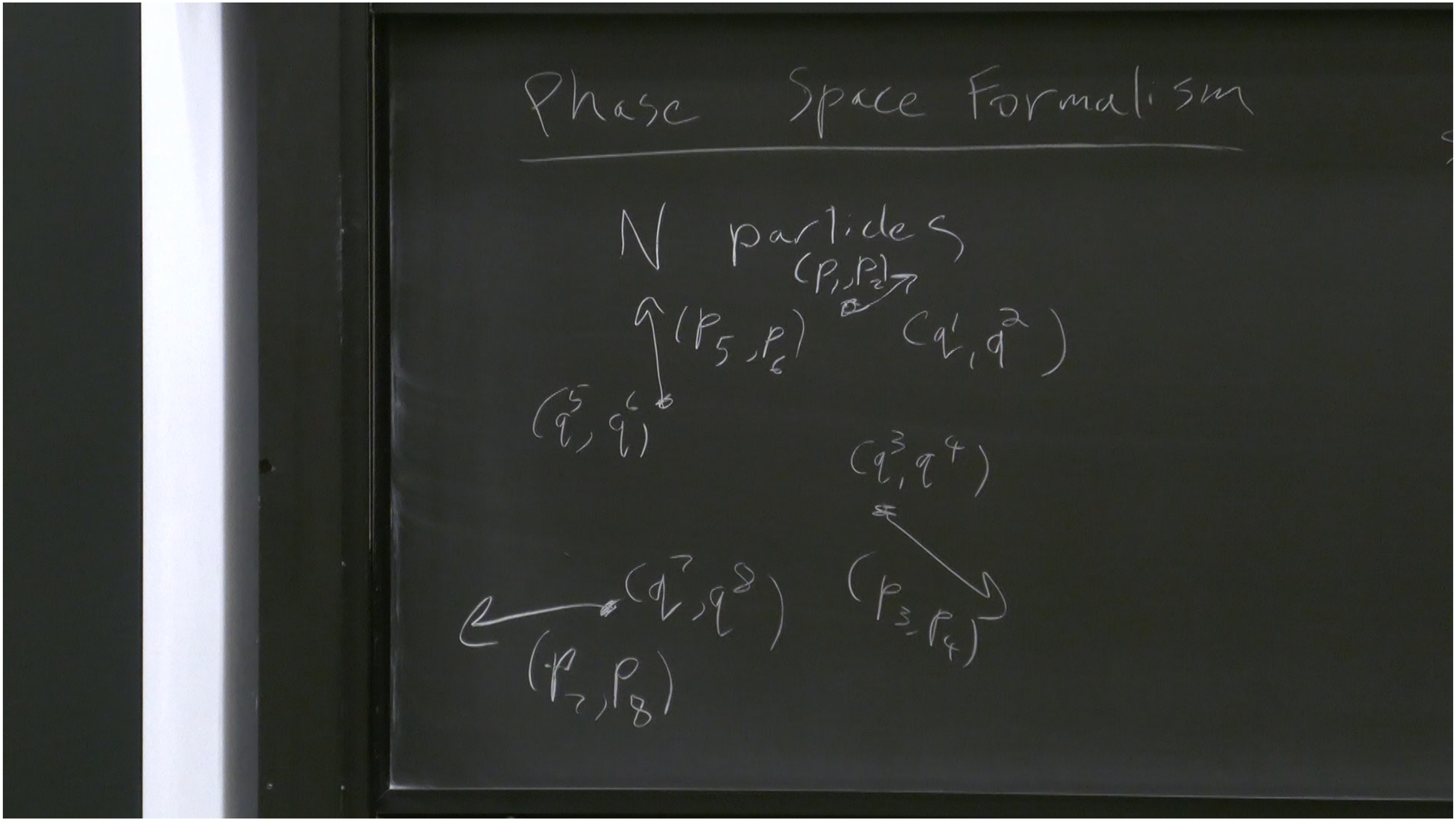
- regulating theories with lattice (fuzzy sphere)

Renormalization Group

3-d Ising model $\longrightarrow$ $\otimes$ $\longleftarrow$ ϕ^4 theory

fixed point $\rightarrow$ same universality class

$\longrightarrow$ same critical exponents


Phase Space Formalism
N particles
(p_1, p_2)
(q^1, q^2)
(p_5, p_6)
(q^5, q^6)
(q^3, q^4)
(q^7, q^8)
(p_3, p_4)
(p_7, p_8)

state (configuration) defined by

$$(q, p) = (q^1, q^2, \dots q^{2N}, p_1, p_2, \dots p_{2N})$$

Dynamics specified by Hamiltonian $H(q, p, t)$

Hamilton's Equations:

$$\dot{q}^i(t) = \frac{\partial H}{\partial p_i}\left(q(t), p(t), t\right)$$

$$\dot{p}_i(t) = -\frac{\partial H}{\partial q^i}\left(q(t), p(t), t\right)$$

i.c.: $q(0) = q^0$, $p(0) = p_0$

$p_1, p_2, \ldots p_{2N})$

...ian $H(q, p, t)$

So what are we interested in at macro level?

- density
- temperate
- pressure
- magnetic?
- does it conduct?

Hamilton's Equations:

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i.c.: $q(0) = q^0, \; p(0) = p_0$

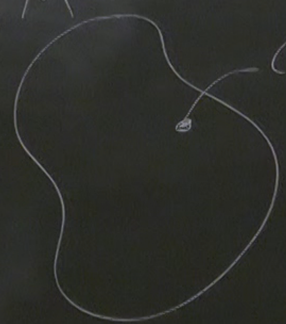
Microstates: (q,p)

phase sp

Macrostates: - temp.
- energy
- mag
- often includes many microstates!

does it conduct

phase space, M



(q,p)

microstate

<u>Macrostate</u>: a way to assign a weight, ρ to each microstate. We use $\rho(q,p)$ for this

density function

Discrete Systems

ε_3 ——— n_3

ε_2 ——— n_2

ε_1 ——— n_1 particles

$$N = \sum_i n_i$$

$$E = \sum_i n_i \varepsilon_i$$

E specifies a macrostate

$\{n_i\}$ specifies a microstate

Example

$\rho(0$

$\rho(\varepsilon$

$\rho(2$

$\rho(3$

$\rho(4$

Example Four coins $H = \varepsilon$ $T = 0$ HHTT

For macrostate 2ε

TTTT How many microstates are there?

six $\frac{4!}{2!2!} = 6$

acrostate

Microstate

$\rho(0) = 1$

$\rho(\varepsilon) = 4$ $\langle E \rangle = \sum_{n=0}^{4} n\varepsilon\,\rho(\varepsilon) \Big/ \sum_{n=0}^{4} \rho(\varepsilon)$

$\rho(2\varepsilon) = 6$

$\rho(3\varepsilon) = 4$ $= \frac{32\varepsilon}{16} = 2\varepsilon$

$\rho(4\varepsilon) = 1$

(p_3, p_4)

(p_7, p_8)

Continuous States

(q, p)

A macrostate can be defined with this measure:

$$d\mu = \rho(q,p)\, d^N q\, d^N p$$

Average value:

$$\langle A \rangle = \frac{\int_M A(q,p)\, d\mu}{\int_M d\mu}$$

Liouville's Theorem

- now we're ready to learn something about ρ from Hamilton's equations

For a region $U \subseteq M$ of the phase space,

$$\int_U \rho(x,t)\, d^{2N}x$$

(p_7, p_8)

Continuous States

(q, p)

A macrostate can be defined with this measure:

$$d\mu = \rho(q,p)\, d^N q\, d^N p$$

Average value:

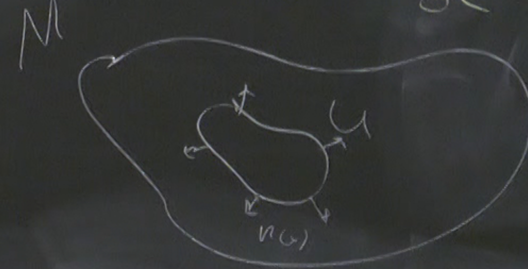
$$\langle A \rangle = \frac{\int_M A(q,p)\, d\mu}{\int_M d\mu}$$

$$X = (q^1, q^2, \ldots, q^N, p_1, p_2, \ldots p_N)$$

Now we examine

$$\frac{d}{dt}\int_U \rho(x,t)\, d^{2N}x$$

equilibrium: $\frac{\partial \rho}{\partial t} = 0$



For trajectories we have:

$$\frac{d}{dt}\int_U \rho(x,t)\, d^{2N}x = -\int_{\partial U} \rho(x)\, v(x,t)\cdot n(x)\, dA$$

$\int_M d\mu$

- Now we have the setup for the divergence theorem:

$$-\int_{\partial U} \rho(x,t)\, v(x,t)\cdot n(x)\, dA$$

$$= -\int_U \nabla\cdot(\rho v)\, d^{2N}x$$

$$\int_U \left(\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho v)\right) d^{2N}x = 0$$

U is arbitrary

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho v) = 0$$

ce theorem:

$\nabla\cdot(\rho v)) \, d^{2N}x = 0$

trary

$\nabla\cdot(\rho v) = 0$

So what is $v(x,t)$?

$$\dot{x}^a = \sum_b J^{ab} \frac{\partial H}{\partial x^b} \quad \text{where} \quad J = \begin{pmatrix} 0 & \mathbb{1}_N \\ -\mathbb{1}_N & 0 \end{pmatrix}$$

$$v^a(x,t) = \sum_b J^{ab} \frac{\partial H}{\partial x^b}(x,t)$$

$J^{ab} = -J^{ba}$

So what is $\nabla \cdot v$

$$\nabla \cdot v = \sum_a \frac{\partial v^a}{\partial x^a} = \sum_{a,b} \frac{\partial}{\partial x^a}\left[J^{ab}\frac{\partial H}{\partial x^b}(x,t)\right] = \sum_{a,b} \frac{\partial}{\partial x^b}\left[-J^{ba}\frac{\partial H}{\partial x^a}(x,t)\right]$$

$$= 0$$

$$\nabla \cdot (\rho v) = \sum_a \frac{\partial}{\partial x^a}(\rho v^a) = \sum_a \frac{\partial \rho}{\partial x^a} v^a$$

$$\frac{\partial \rho}{\partial q^i}\frac{\partial H}{\partial p_i} - \frac{\partial \rho}{\partial p_i}\frac{\partial H}{\partial q^i}$$

$$= \sum_{ab} \frac{\partial \rho}{\partial x^a} J^{ab} \frac{\partial H}{\partial x^b} = \{\rho, H\}$$

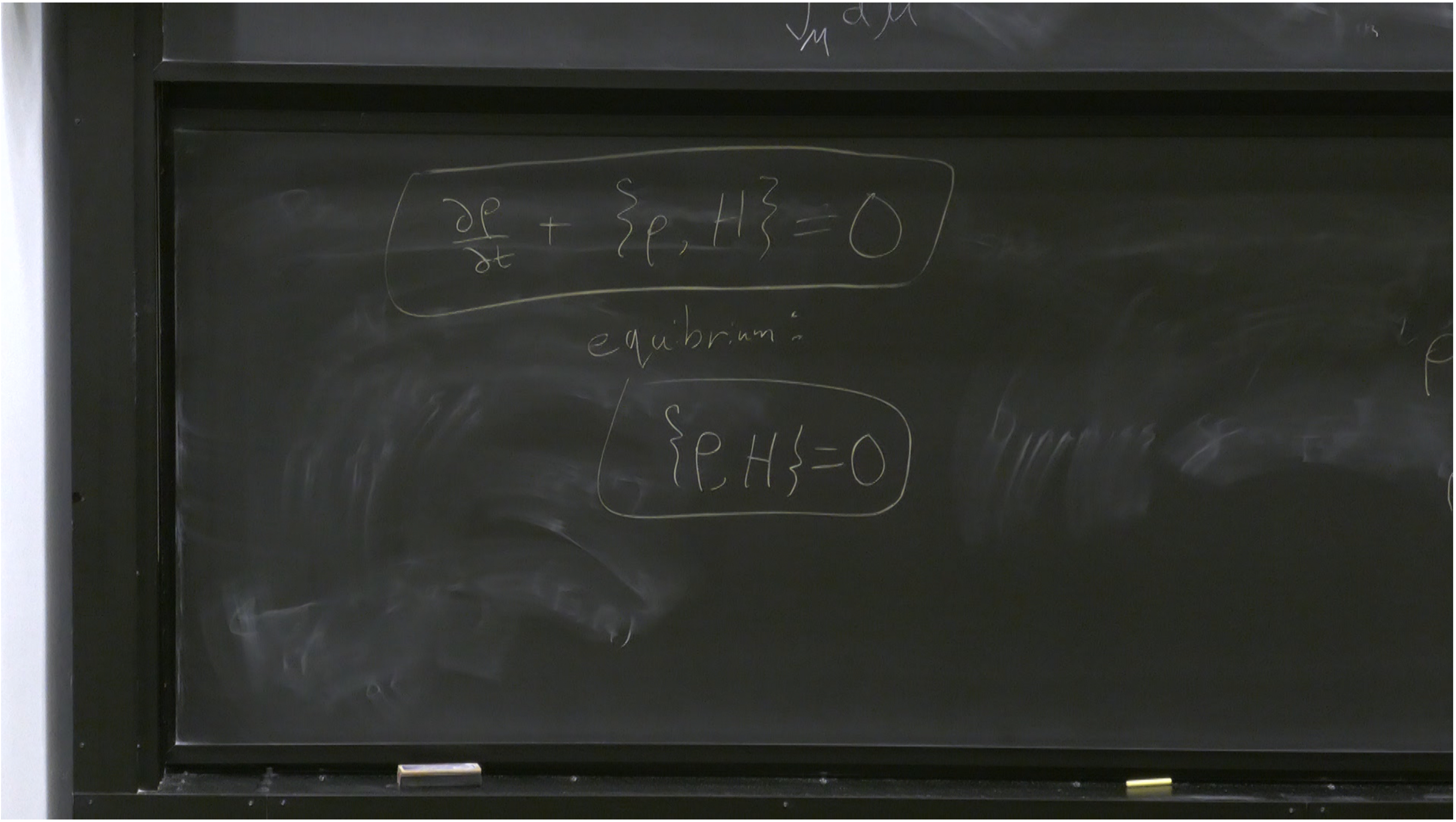
$\rho = \text{const}$ (microcanonical ensemble)

$\rho = f(H)$

$\rho(q,p) \propto \exp(\beta H)$

canonical

grand canonical


$\frac{\partial \rho}{\partial t} + \{\rho, H\} = 0$
equibrium:
$\{\rho, H\} = 0$