Title: QFT2 Lecture - 112923

Speakers: Francois David

Collection: Quantum Field Theory 2 2023/24

Date: November 29, 2023 - 9:00 AM

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SU(2) $\quad A^a_\mu(x) \longrightarrow A_\mu = A^a_\mu \, \underline{t}_a \quad t_a = \frac{1}{2}\sigma_a \quad a = 1,2,3 \; \in$ Adj Repr

D_μ covariant derivative $\quad D_\mu \phi_A = \partial_\mu - i\,[A_\mu, \phi_A]$

Gauge transf $\quad A_\mu \to A_\mu + D_\mu \alpha \qquad \alpha \in$ Adj Rep

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i\,[A_\mu, A_\nu]$$

$$S_{GField}[A] = -\frac{1}{2g^2}\int d^4x \; \mathrm{Tr}\left[F_{\mu\nu}F^{\mu\nu}\right]$$

+ Euclidean

$_a = \frac{1}{2}\sigma_a \quad a = 1,2,3 \in$ Adj 'Repr

$\phi_A = \partial_\mu - i\,[A_\mu, \phi_A]$

$\alpha \in$ Adj Rep

Gauge Fixing

Lorentz-Landau $\quad \partial^\mu A_\mu = 0$

$$\int \mathcal{D}[A] \exp\left(\frac{i}{\hbar} S_{YM}[A]\right)$$

$$\delta[\partial^\mu A_\mu] \times \det[\partial^\mu D_\mu]$$

gauge fixin term, F.P. determinant

$$\det\left[\partial^\mu D_\mu\right] = \int D[\bar{C}, \bar{C}]\, \exp\left(\mathrm{Tr}\int d^4x\, \bar{C}\, \partial^\mu D_\mu C\right)$$

$$\exp\left[i \int d^4x\, \partial^\mu \bar{C}_a(x) \cdot (D_\mu C)^a(x)\right]$$

$$C = C^a(x)\, t_a$$

← Grassman

$$\left.\begin{aligned} \bar{C} &= \bar{C}^a(x)\, t_a \\ C &= C^a(x)\, t_a \end{aligned}\right) \text{Adj Repr}$$

$$\det\left[\partial^\mu D_\mu\right] = \int D[c,\bar{c}]\, \exp\left(\mathrm{Tr}\left[\int d^4x\, \bar{c}\, \partial^\mu D_\mu c\right)\right]$$

$$c = c^a(x)\, t_a$$

Grassman

$$\exp\left[\frac{i}{\hbar}\int d^4x\, \partial^\mu \bar{c}_a(x)\cdot (D_\mu c)^a(x)\right]$$

$$\left.\begin{aligned} \bar{c} &= \bar{c}^a(x)\, t_a \\ c &= c^a(x)\, t_a \end{aligned}\right)$$ Adj Repr

$$\exp\left[\frac{i}{\hbar} S_{FP}[c,\bar{c},A]\right]$$

Faddeev Popov action

$$\det\left[\partial^\mu D_\mu\right] = \int D[\bar{c}, c] \exp\left(\mathrm{Tr}\int d^4x \; \bar{c}\, \partial^\mu D_\mu c\right)$$

$$\exp\left[\frac{i}{\hbar}\int d^4x \; \partial^\mu \bar{c}_a(x) \cdot (D_\mu c)^a(x)\right]$$

$$c = c^a(x)\, t_a$$

Grassman

$$\bar{c} = \bar{c}^a(x)\, t_a$$

$$c = c^a(x)\, t_a$$

Adj Repr

ghost & antighost

$$\exp\left[\frac{i}{\hbar} S_{FP}[c, \bar{c}, A]\right]$$

Faddeev Popov action

S_{FP}

$$\det\left[\partial^\mu D_\mu\right] = \int D[\bar{c}, c] \exp\left(\mathrm{Tr}\int d^4x\, \bar{c}\, \partial^\mu D_\mu c\right)$$

$$c = c^a(x)\, t_a$$

Grassman

$$\exp\left[\frac{i}{\hbar}\int d^4x\, \partial^\mu \bar{c}_a(x) \cdot (D_\mu c)^a(x)\right]$$

$$\bar{c} = \bar{c}^a(x)\, t_a$$

$$c = c^a(x)\, t_a$$

Adj Repr

ghost & antighost

$$\exp\left[\frac{i}{\hbar} S_{FP}[c, \bar{c}, A]\right]$$

Faddeev Popov action

Fermions spin 0 fields $\not\Rightarrow$ CPT theorem ! Unitarity is not satisfied

S_{FP}

struct. cst. of SU(2)

$$S_{FP}[c,\bar{c},A] = \int d^4x \left(\bar{c}_a(x)(-\Delta)\delta_{ab}\, c^b(x) + \epsilon_{abc}\, \partial^\mu \bar{c}^a(x)\, A^b_\mu(x)\, c^c(x) \right)$$

non-abelian group

Feynman $\Rightarrow$ ξ gauge ξ a parameter

cst

$$\delta[\partial^\mu A_\mu - \epsilon]\ \det[\partial^\mu D_\mu]\ \exp\left(-\frac{i}{2\xi}\epsilon^2\right) d\epsilon \left(\frac{1}{\sqrt{2\pi\xi}}\right)$$

Gaussian integration

$$\epsilon = \epsilon^a t_a$$

① $S_{YM}[A] = -\frac{1}{2g^2}\int d^4x\, Tr[F_{\mu\nu}F$

$+$ Euclidean

③ $\int D[A] \exp\left(i S_{YM}[A] + S_{FP}[\bar{c}, c, A] + S_{\xi}[A] \right.$

Perturb Theory $A \Rightarrow g\tilde{A}$

g small parameter

0-order $A, c, \bar{c}$ are free fields

$$\boxed{\begin{aligned} S_{YM} &= \partial\tilde{A}\cdot\partial\tilde{A} \\ S_{FP} &= \partial\bar{c}\;\partial c \\ S_{\xi} &= \frac{1}{2}\left(\partial^{\mu}A_{\mu}\right)^2 \end{aligned}}$$

$+ g\,\tilde{A}\,\tilde{A}\cdot\partial\tilde{A}$

$+ g\,\partial\bar{c}\cdot A\cdot$

$\int dx \cdot Tr[F_{\mu\nu} F^{\mu\nu}]$ g coupling constant | gauge fixing term, F.P. determinant

$S_\xi[A]$)

$S_\xi[A] = \frac{1}{2\xi} \int d^4x \left(\partial^\mu A^a_\mu\right)^2$ indep't of ξ parameter

$\xi \sim g^2 \tilde{\xi}$

$g \tilde{A}\tilde{A} \cdot \partial \tilde{A} + g^2 \tilde{A}\tilde{A}\tilde{A}\tilde{A}$

$g\, \partial \bar{c} \cdot A \cdot c$

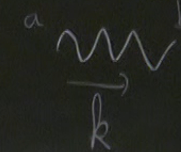
0. order $\langle A^a_\mu A^b_\nu \rangle_0 = G^{ab}_{\mu\nu}$

momentum space

$G^{ab}_{\mu\nu}(k) = \delta^{ab} \frac{-i}{k^2 - i\epsilon_+} \left(h_{\mu\nu} + (\xi - 1) \frac{k_\mu k_\nu}{k^2} \right)$

k 4 momentum $h_{\mu\nu}$ metric tensor.

photon propagator



Ghosts Ghost propagator

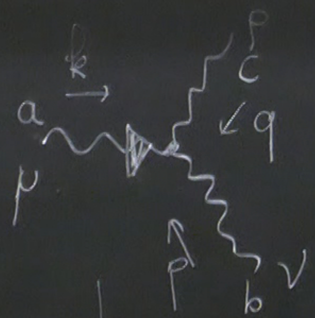
$$\langle C^a(x)\, \bar{C}^b(y) \rangle_0 \longrightarrow C^{ab}(k) = \delta^{ab} \frac{-i}{k^2 - i\epsilon_+}$$

Feyn. Prop for scalar field


ghosts

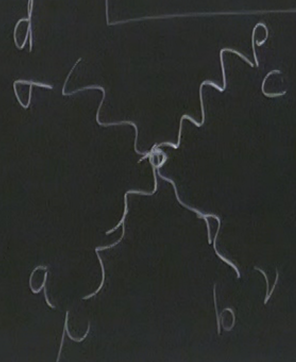
scalar

Interactions



$$g\,\epsilon^{abc}\left(h^{\mu\nu}(k-p)^{\rho} + h^{\nu\rho}(p-q)^{\mu} + h^{\rho\mu}(q-k)^{\nu}\right)$$

$$g\,\epsilon^{abc}\left(k^{\rho}h^{\mu\nu} - k^{\nu}h^{\rho\mu}\right) + \cdots + \cdots\ )$$



$$g^{2}\left(\epsilon^{abe}\epsilon^{cde}\left(h^{\mu\rho}h^{\nu\sigma} - h^{\mu\sigma}h^{\nu\rho}\right) + \text{''} + \text{''}\right)$$

$$\delta_{ab}\delta_{cd} - \delta_{ad}\delta_{bc}$$

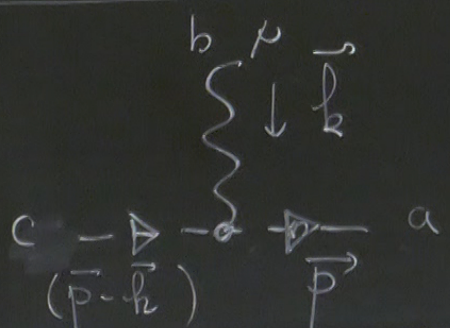
charge conservations

Ghosts   Ghost propagator

$$\langle C^a(x)\,\bar{C}^b(y)\rangle_0 \longrightarrow C^{ab}(k) = \delta^{ab}\,\frac{-i}{k^2 - i\epsilon_+}$$

Feyn. Prop for scalar field

ghosts



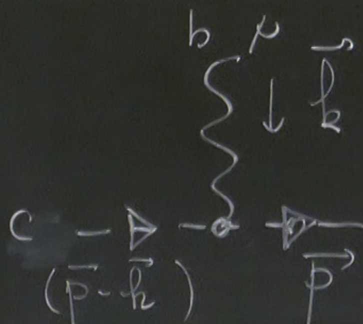
$$-g\,p^\mu\,\epsilon^{abc}$$

Ghosts Ghost propagator

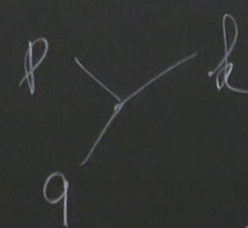
$$\langle C^a(x)\,\bar{C}^b(y)\rangle_0 \longrightarrow C^{ab}(k) = \delta^{ab}\,\underbrace{\frac{-i}{k^2 - i\epsilon_+}}$$

Feyn. Prop for scalar field

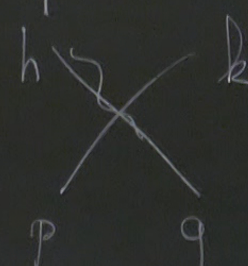
ghosts



$$-g\,p^\mu\,\epsilon^{abc}$$

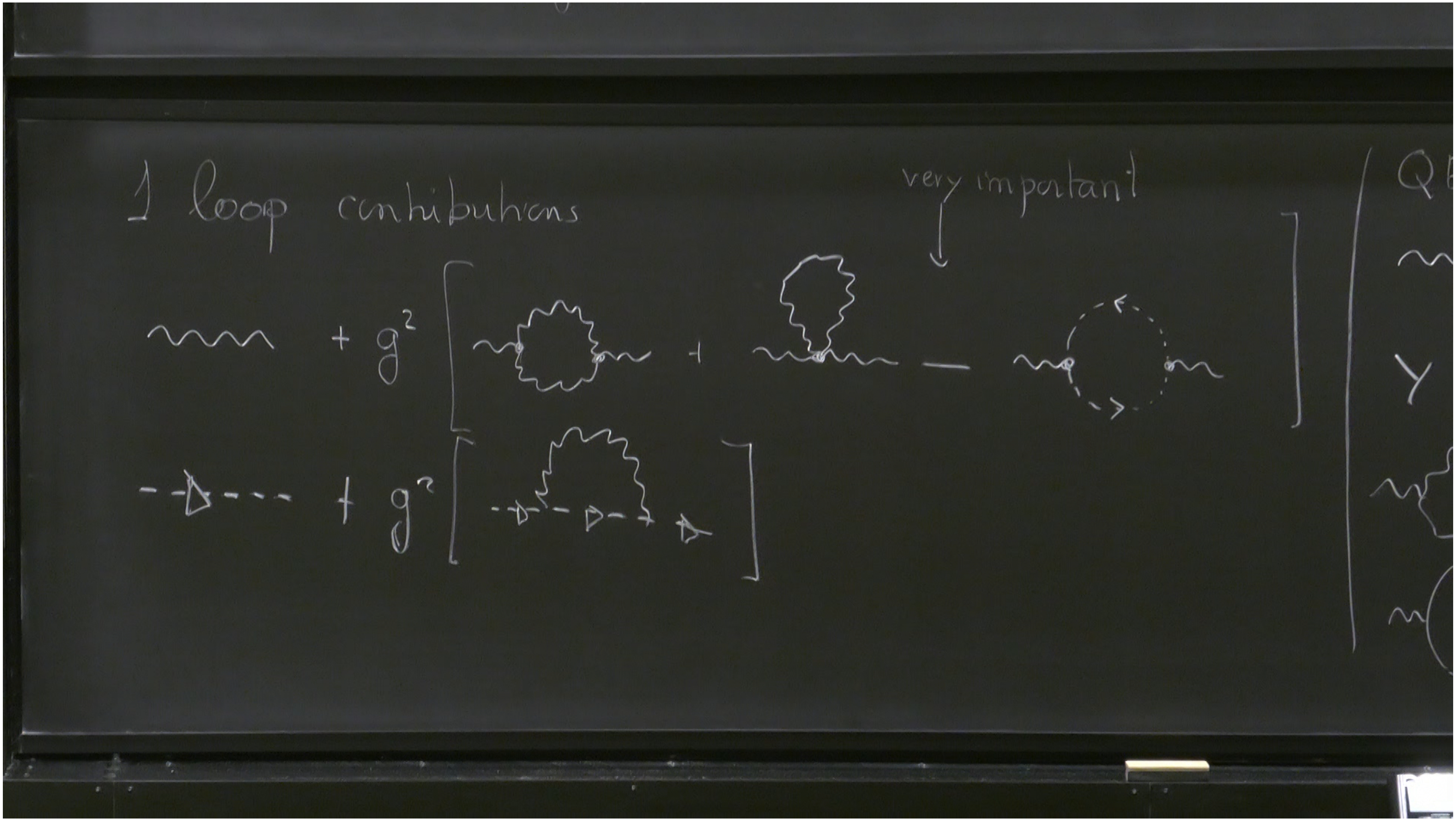


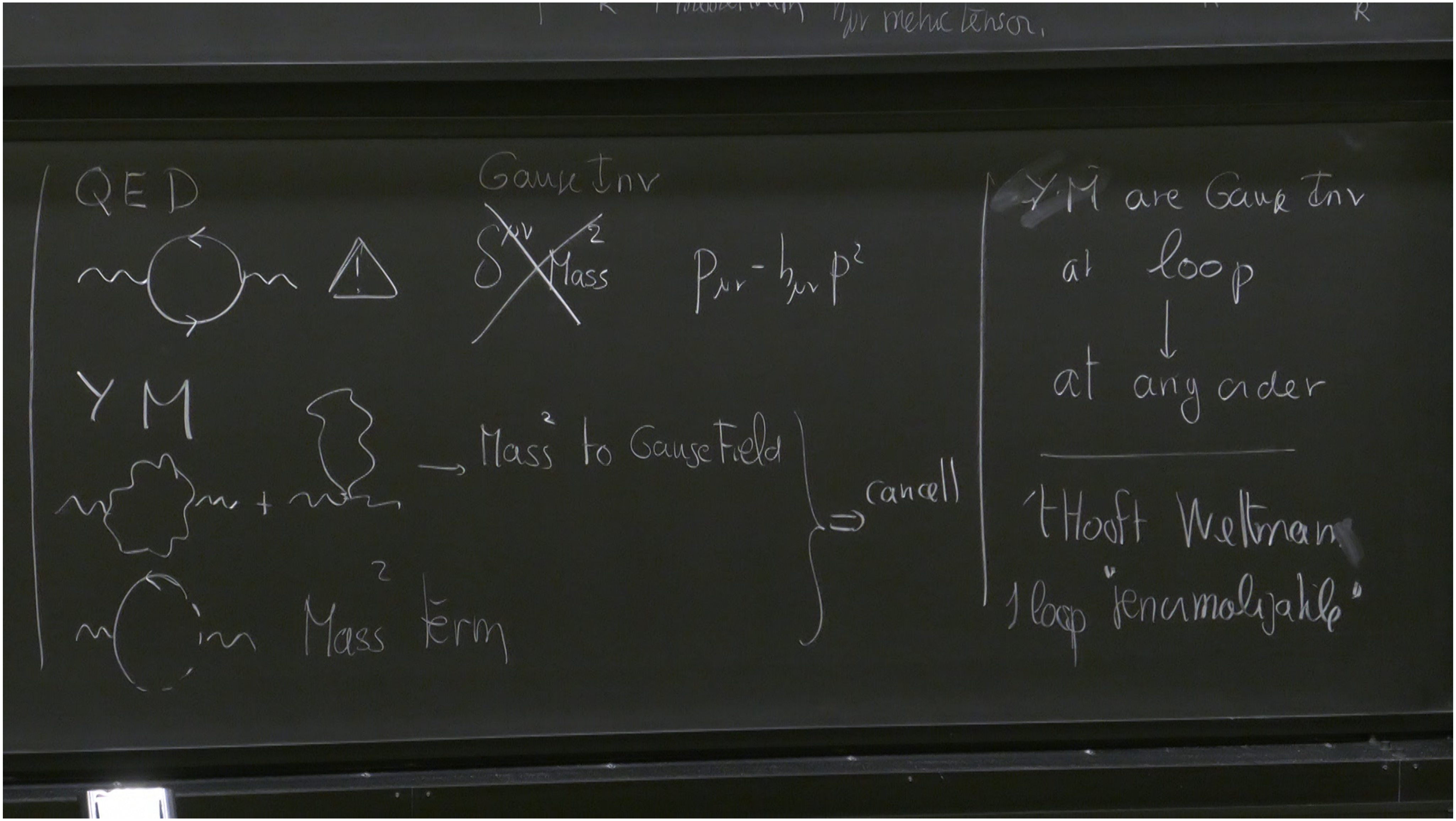
$$\boxed{p+q+k=0}$$



$$\boxed{p+q+k+h=0}$$

conservation of the momenta


1 loop contributions
very important
+ g²
+ g²


ημν metric tensor.
QED
Gauge Inv
δμν Mass2
pμν - ημν p2
YM
Mass2 to Gauge Field
cancell
Mass2 term
YM are Gauge Inv
at loop
at any order
'tHooft Weltman
1 loop "renormalizable"

BRST symmetry ; of the gauge fixed theory

Landau Gauge $\delta[\partial^\mu A_\nu] = \int_{-\infty}^{+\infty} dB \; e^{i B(\partial^\mu A_\mu)}$

B "auxiliary field" $B^a(x)\, t_a \in$ Adj Repr

no dynamics

$S_{YM}[A]$, $S_{ghost}[A]$, $S_{GF}[A] = \int d^4x \, \mathrm{Tr}(i B \cdot \partial^\mu A_\mu)$

$\boxed{\text{BRST Symmetry}}$: of the gauge fixed theory

Landau Gauge $\delta[\partial^\nu A_\nu] = \int_{-\infty}^{+\infty} dB \, e^{iB(\partial^\mu A_\mu)}$

B "auxiliary field" $B^a(x)\, t_a \in$ Adj Repr

no dynamics

$S_{YM}[A]\,,\; S_{ghost}[A]\,,\; S_{GF}[A] = \int d^4x \, \mathrm{Tr}\left(i B\, \partial^\mu A_\mu + \frac{\xi}{2} B^2\right)$

theory

$$S = S_{YM} + S_{ghost} + S_{GF} = S_{\text{TOTAL}}\big[\overbrace{A,\ \underbrace{c,\ \bar{c}}_{\text{Fermions}},\ B}^{\text{Bosons}}\big]$$

depends on the gauge fixing condition

$A_\mu \to A_\mu + D_\mu \alpha$ not a symmetry

replace α by $\epsilon\, c(x)$ with ϵ a "antighost" number

$A_\mu \to A_\mu + \epsilon D_\mu c$ SUSY fields

$$A_\mu \to A_\mu + \epsilon D_\mu c$$
$$c \to c + \epsilon \frac{i}{2} c \cdot c$$
$$\bar{c} \to \bar{c} - i\epsilon B$$
$$B \to B$$

Inf BRST

$$(c \cdot c)^a = \epsilon^a{}_{bc}\, c^b \cdot c^c$$

$$S_{YM}[A] \longrightarrow S_{YM}[A] + \epsilon \cdot 0$$

$$S_{\text{ghost}} + S_{\substack{\text{gauge} \\ \text{Fixing}}} = S_{\substack{\text{TOTAL} \\ \text{G.F}}}[A, c, \bar{c}, B]$$

$$\frac{\quad}{k^2 - i\epsilon_+}\left(\eta_{\mu\nu} + (\xi - 1)\frac{k_\mu k_\nu}{k^2}\right)$$

k 4 momentum $\eta_{\mu\nu}$ metric tensor.

$$S_{\substack{\text{TOTAL}\\ \text{Gauge Fixing}}}[\dots] \longrightarrow S_{\substack{\text{TOTAL}\\ \text{GF}}}[\dots] + \epsilon\, \mathcal{O}$$

also BRST invariant

BRST invariant

BRST the symmetry of any gauge fixing choice
theory

Gauge Inv + Gauge Fixing $\Rightarrow$ BRST sym.

Generator Q

BRST charge

$\text{BRST} \to \epsilon \cdot Q$

$$Q \cdot A_\mu \to D_\mu C$$

$$Q \cdot C \to \frac{i}{2} C \cdot C$$

$$Q\, \bar{C} \to -i\, B$$

$$Q\, B \to 0$$

$$\boxed{Q^2 = Q \circ Q = 0}$$

nilpotent sym.

$$\boxed{R = 0}$$

$$S_{TGF} = \int d^4x \left[\bar{c}\, \partial D c + i B\, \partial A \right] = Q \int d^4x \left(\bar{c}\, \underline{\partial^\mu A_\mu} \right)$$

m.

$$S_{TGF} = Q \int \bar{c} \cdot F[A] \qquad F[A] = \partial^\mu A_\mu$$

$$Q\, S_{TGF} = Q \cdot Q \int \bar{c}\, F[A] = 0$$

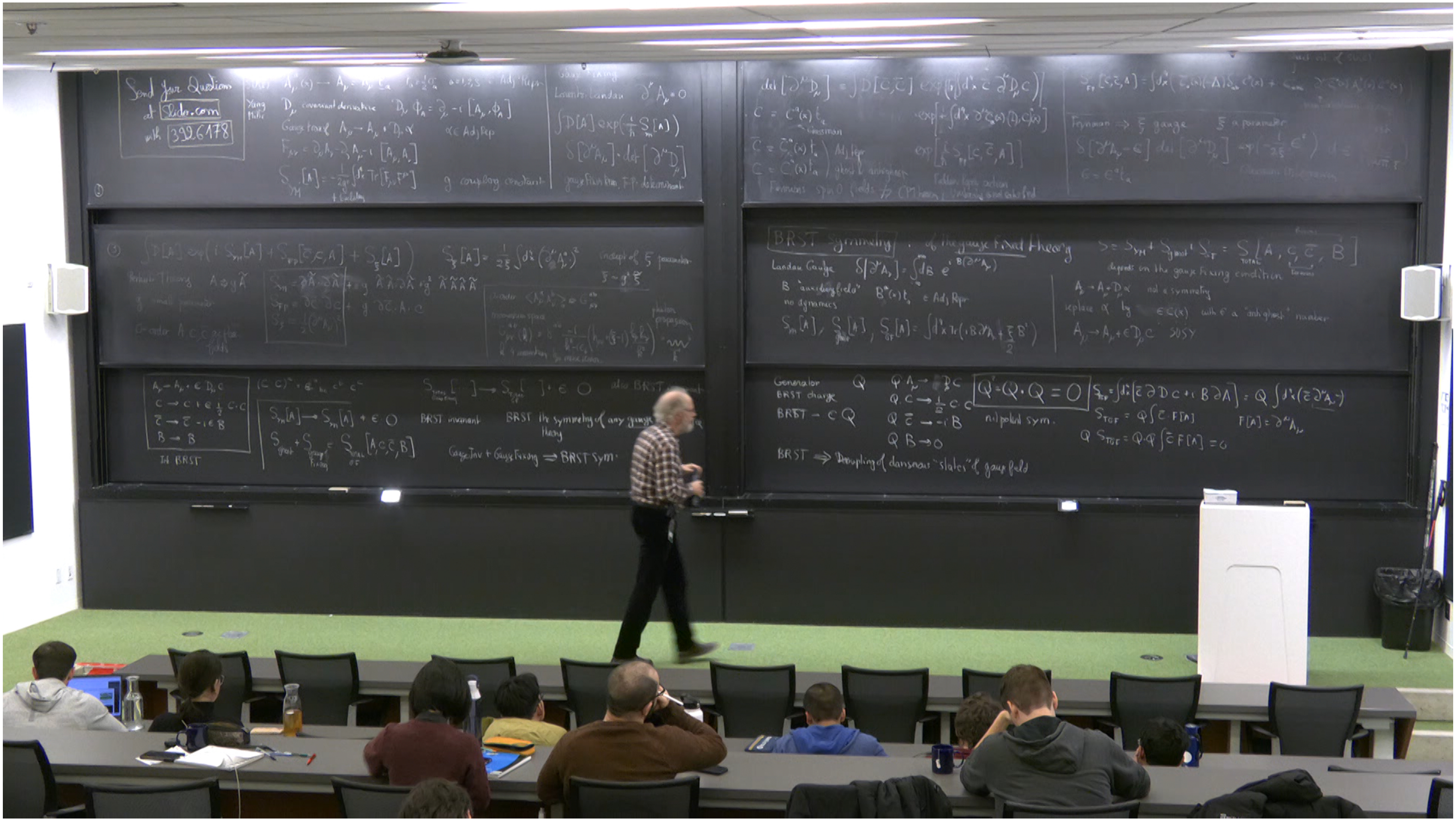
Generator Q $\qquad Q \cdot A_\mu \to D_\mu C$

BRST charge $\qquad Q . C \to \frac{i}{2} C \cdot C$ $\qquad \boxed{Q^2 = Q \cdot Q = 0}$

BRST $\to \epsilon \cdot Q$ $\qquad Q \, \bar{C} \to -i B$ $\qquad$ nilpotent sym.

$Q \, B \to 0$

BRST $\Rightarrow$ Decoupling of dangerous "states" of gauge field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