Title: QFT2 Lecture - 112723

Speakers: Francois David

Collection: Quantum Field Theory 2 2023/24

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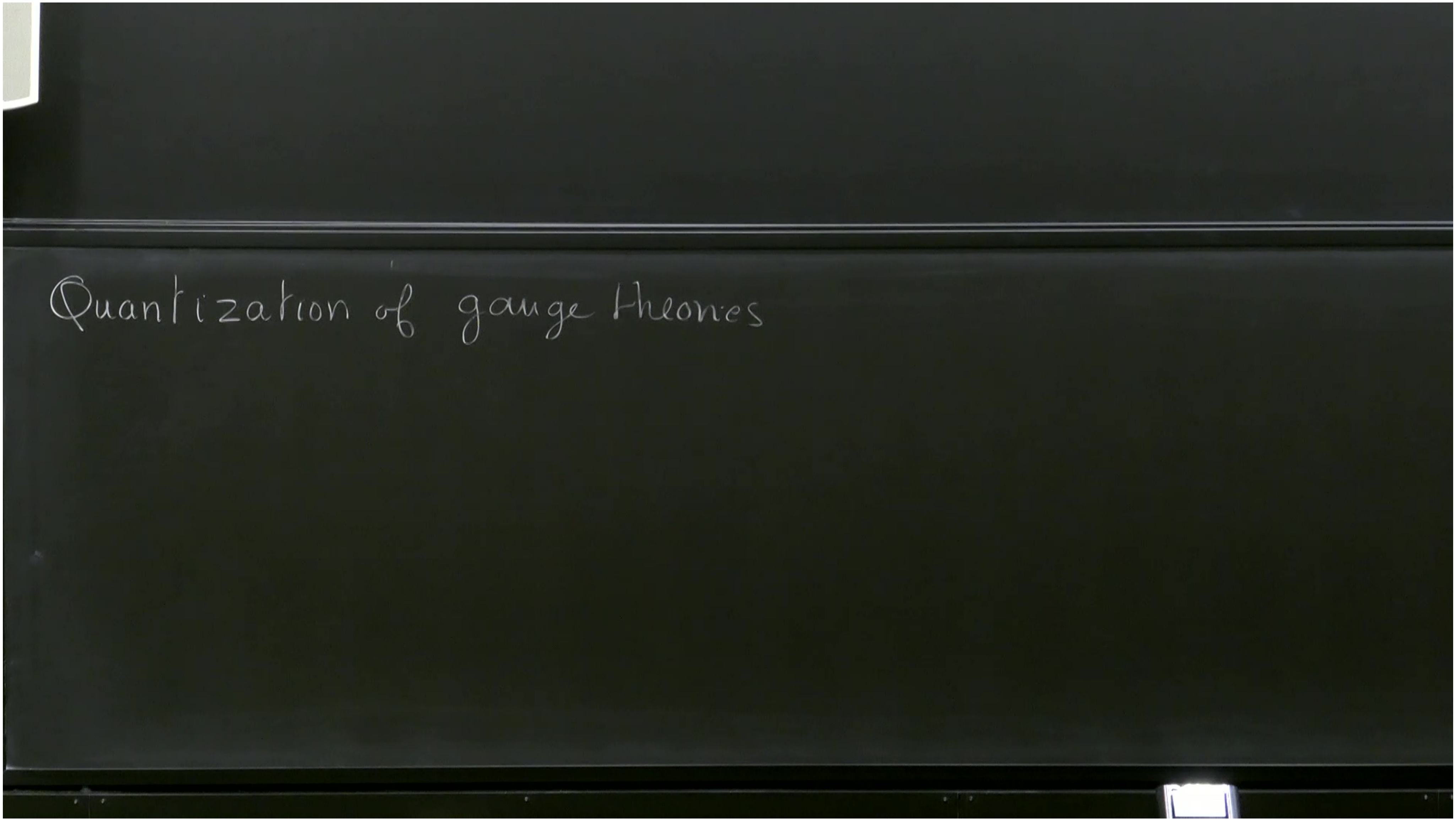
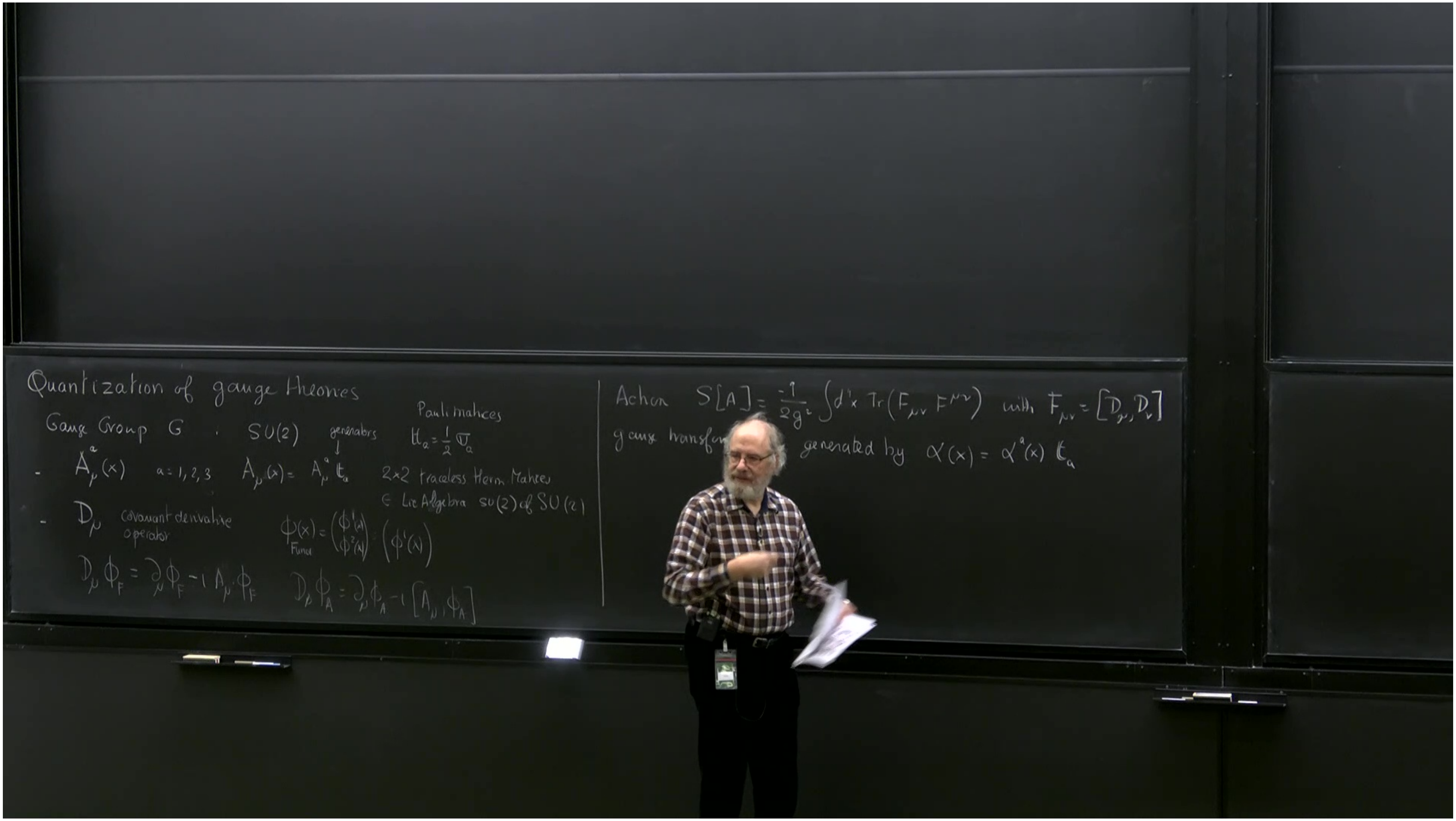


Quantization of gauge theories



Quantization of gauge theories

Gauge Group G : SU(2) generators $\downarrow$ Pauli matrices $t_a = \frac{1}{2}\sigma_a$

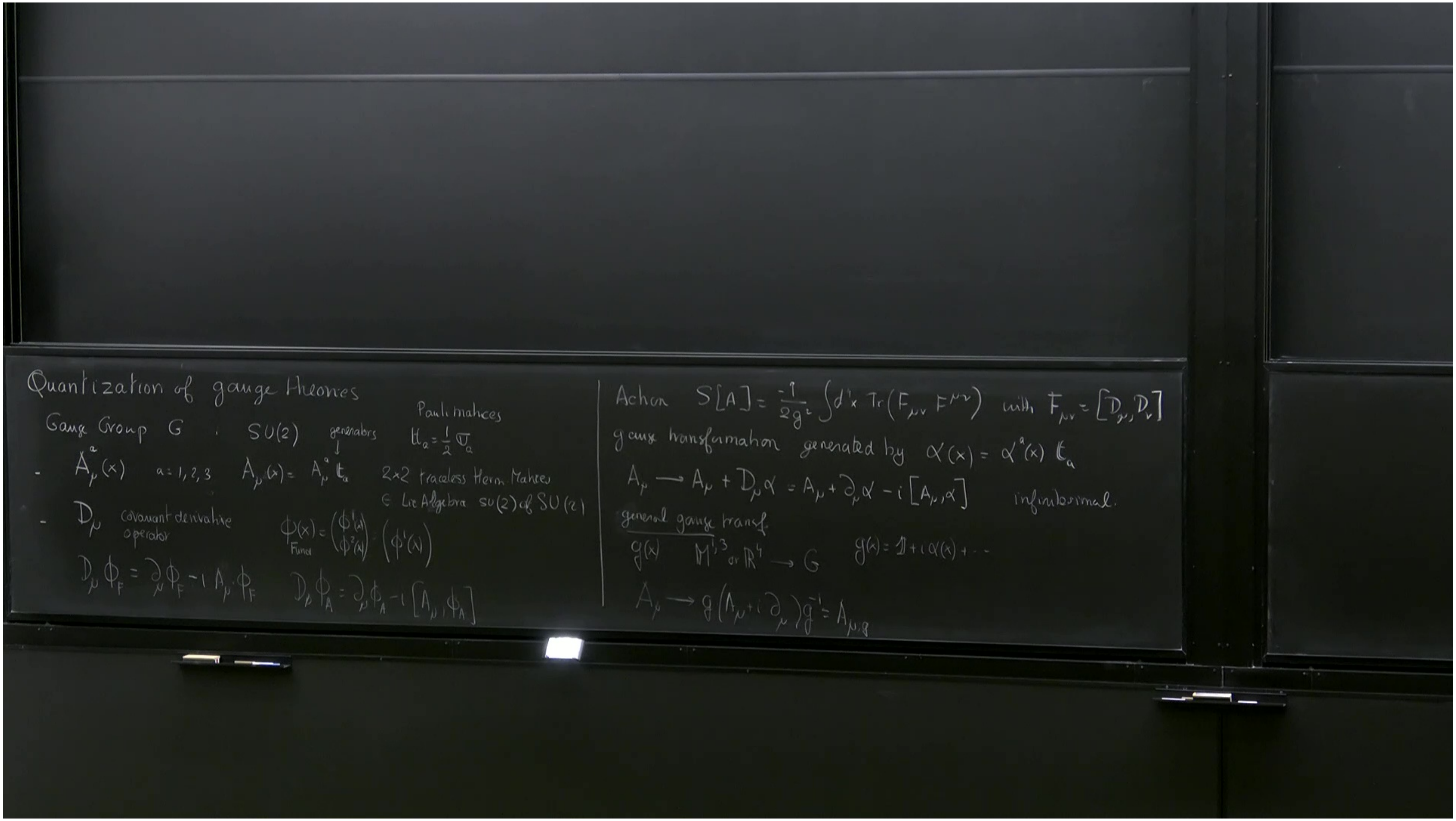
- $A^a_\mu(x)$ $a = 1, 2, 3$ $A_\mu(x) = A^a_\mu t_a$ 2×2 traceless Herm. Matrices $\in$ Lie Algebra su(2) of SU(2)
- D_μ covariant derivative operator

$\phi_{\text{Fund}}(x) = \begin{pmatrix} \phi^1(x) \\ \phi^2(x) \end{pmatrix}$ $\left(\phi^1(x)\right)$

$D_\mu \phi_F = \partial_\mu \phi_F - i A_\mu \cdot \phi_F$ $D_\mu \phi_A = \partial_\mu \phi_A - i \left[A_\mu, \phi_A\right]$

Action $S[A] = \frac{-1}{2g^2} \int d^4x \, \mathrm{Tr}\left(F_{\mu\nu} F^{\mu\nu}\right)$ with $F_{\mu\nu} = \left[D_\mu, D_\nu\right]$

gauge transfor [illegible] generated by $\alpha(x) = \alpha^a(x)\, t_a$



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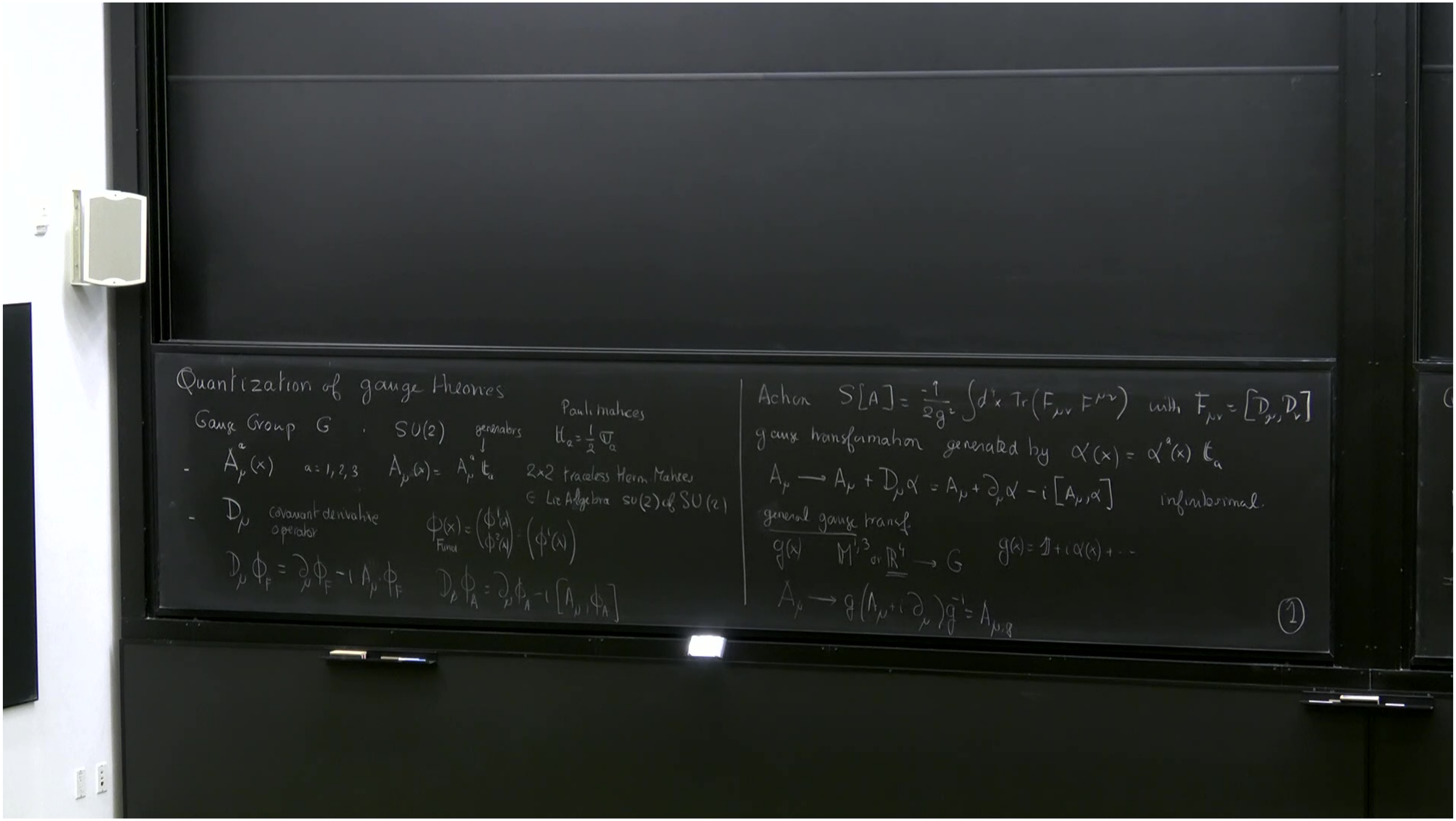
$$A_\mu \longrightarrow A_\mu + D_\mu \alpha = A_\mu + \partial_\mu \alpha - i [A_\mu, \alpha] \qquad \text{infinitesimal.}$$

general gauge transf.

$g(x)$ $M^{1,3}$ or $\mathbb{R}^4 \longrightarrow G$ $g(x) = \mathbb{1} + i\alpha(x) + \dots$

$$A_\mu \longrightarrow g(A_\mu + i\partial_\mu) g^{-1} = A_{\mu;g}$$



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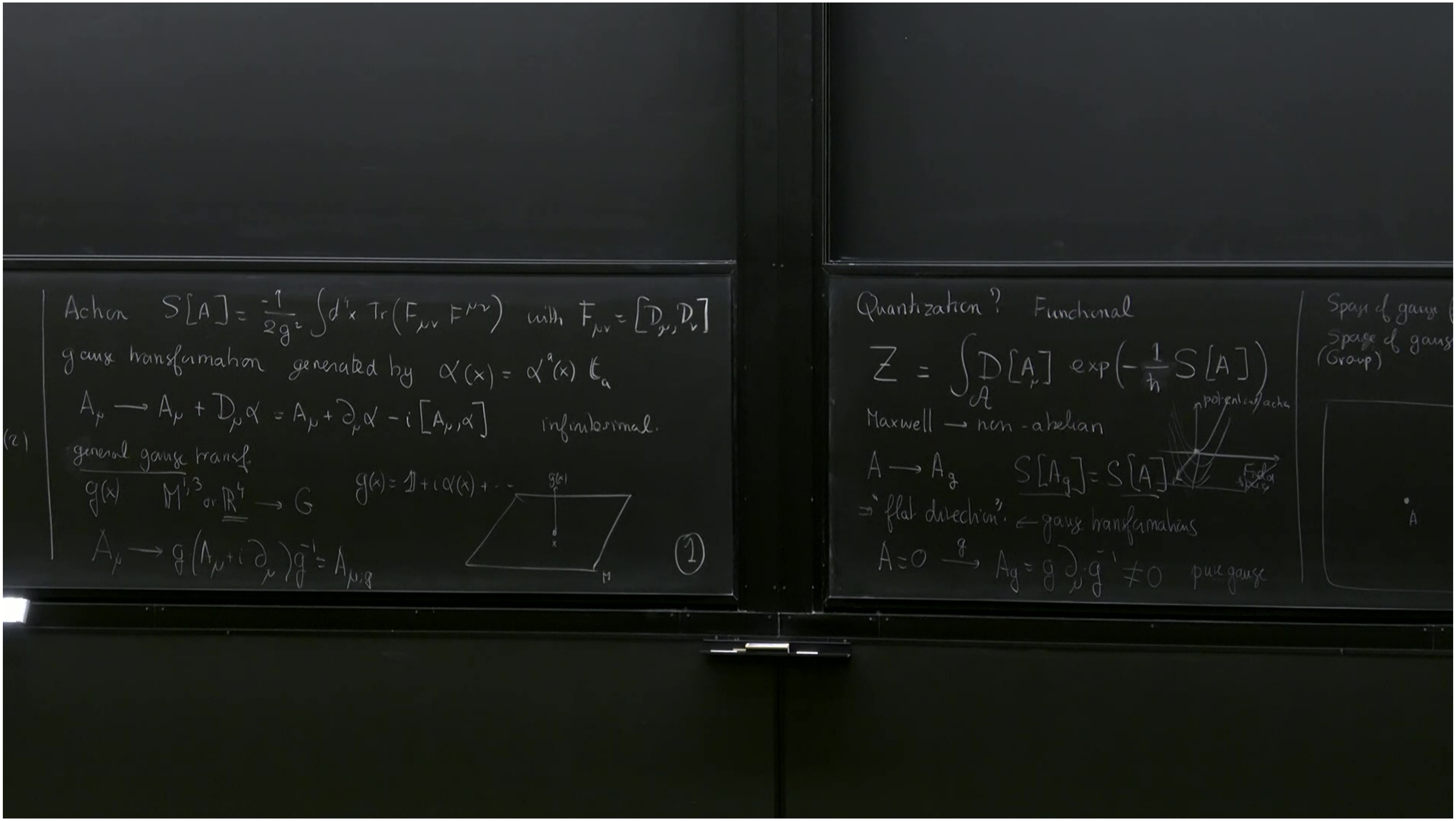
①



(2)

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Quantization? Functional

$$Z = \int_{\mathcal{A}} D[A_\mu] \exp\left(-\frac{1}{\hbar} S[A]\right)$$

Maxwell → non-abelian

$A \longrightarrow A_g \qquad \underline{S[A_g]} = \underline{S[A]}$

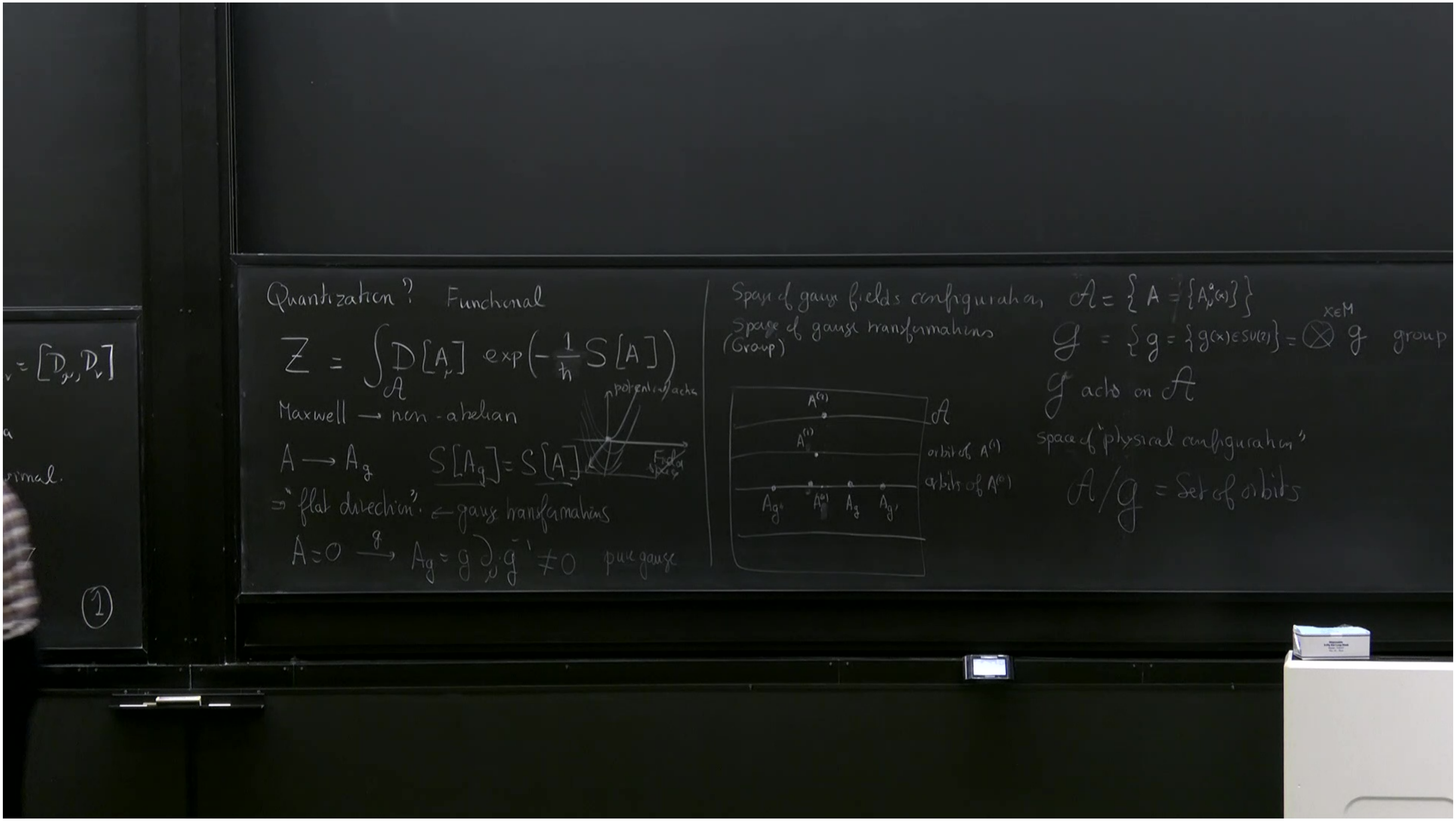
$\Rightarrow$ "flat direction". ← gauge transformations

$A = 0 \xrightarrow{g} A_g = g\,\partial_\mu g^{-1} \neq 0$ pure gauge

Space of gauge (

Space of gauge

(Group)



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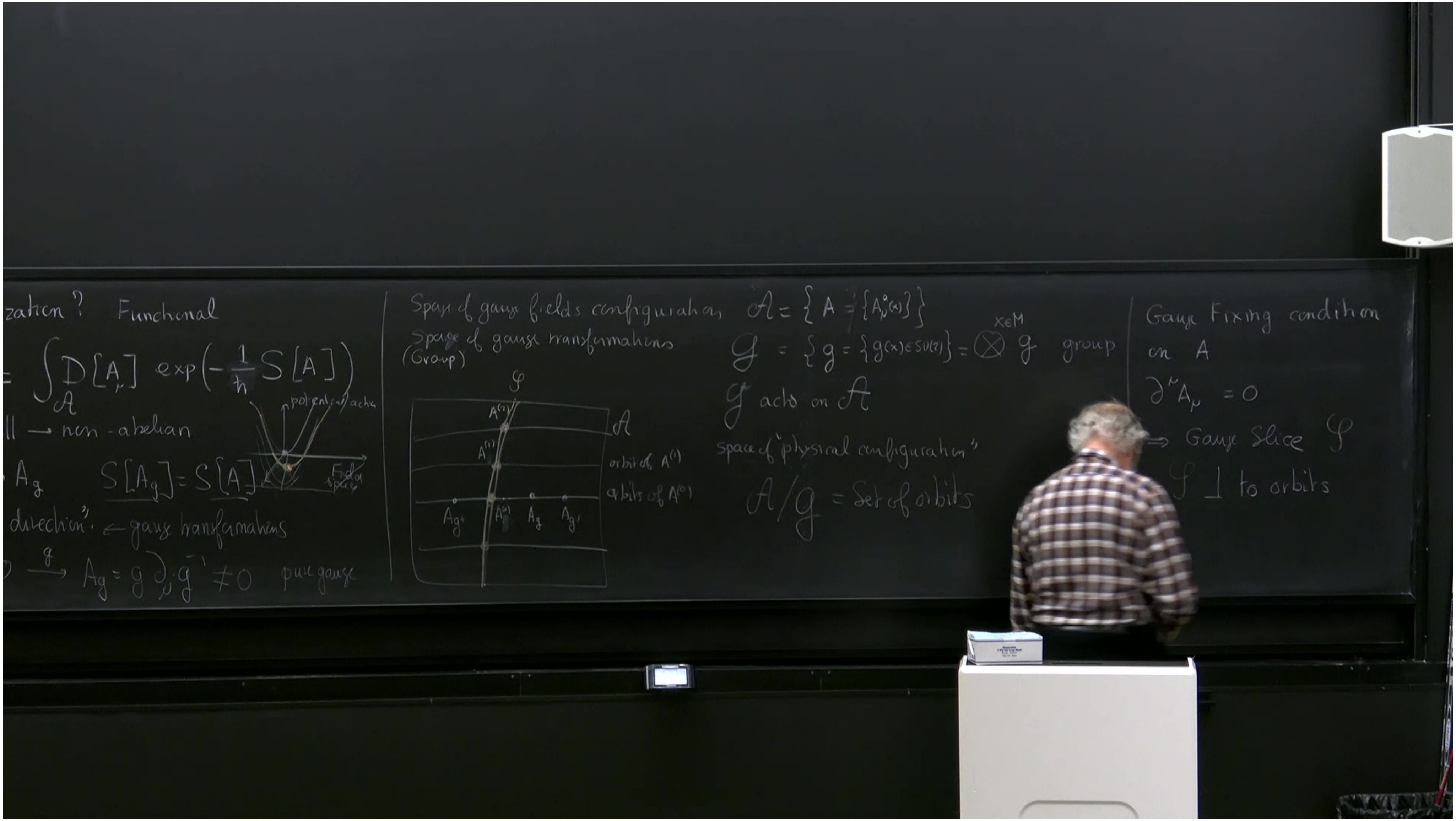
Space of gauge fields configuration $\mathcal{A} = \{ A = \{A^a_\mu(x)\} \}$

Space of gauge transformations (Group) $\mathcal{G} = \{ g = \{ g(x) \in SU(2) \} = \bigotimes_{x \in M} g$ group

$\mathcal{G}$ acts on $\mathcal{A}$

space of "physical configuration"

$\mathcal{A}/\mathcal{G}$ = Set of orbits



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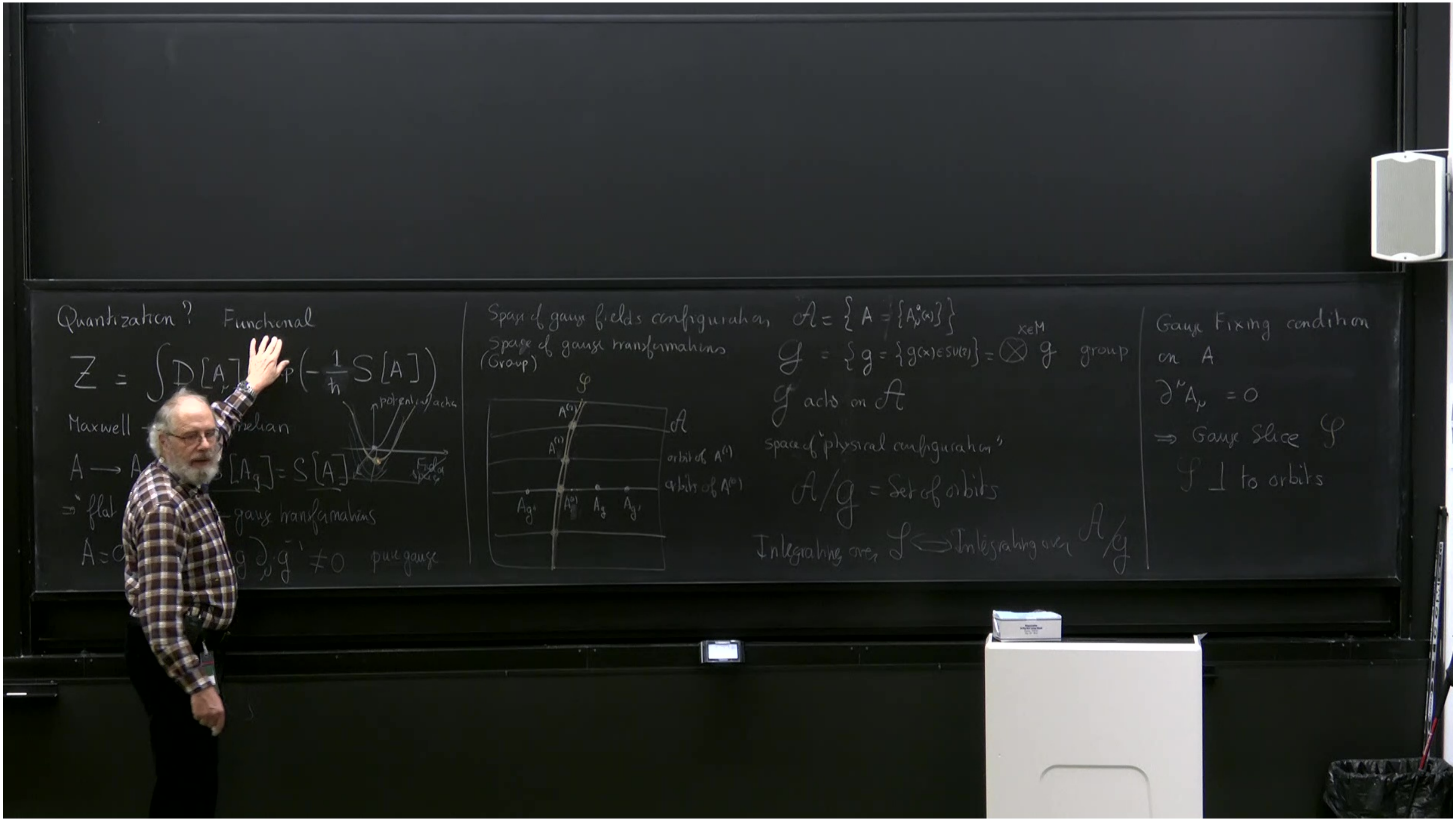
⟹ Gauge Slice $\mathcal{S}$

$\mathcal{S} \perp$ to orbits



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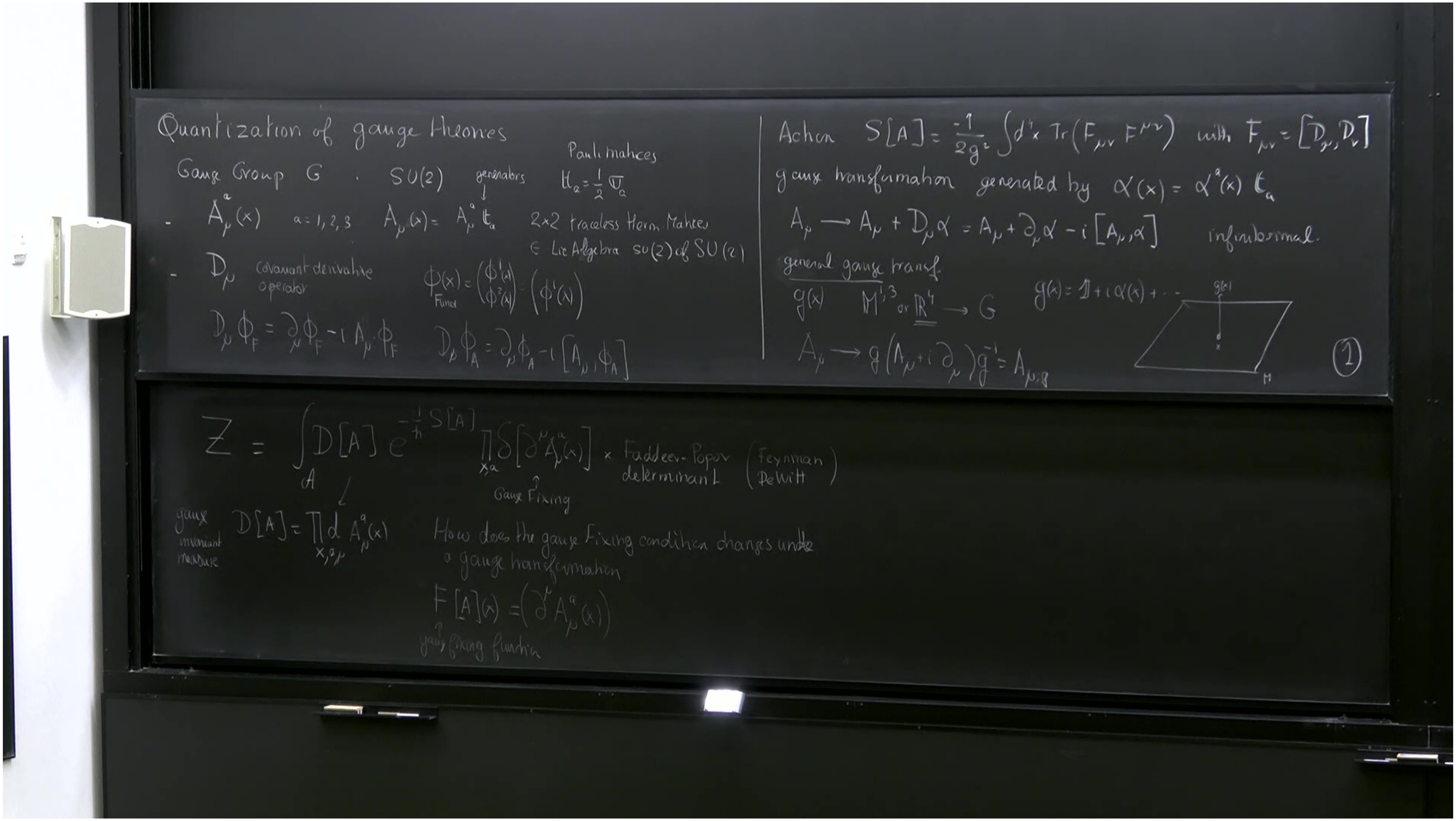

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①

$$Z = \int_A D[A] \, e^{-\frac{1}{\hbar} S[A]} \prod_{x,a} \delta\left[\partial^\mu A^a_\mu(x)\right] \times \text{Faddeev-Popov determinant} \begin{pmatrix} \text{Feynman} \\ \text{DeWitt} \end{pmatrix}$$

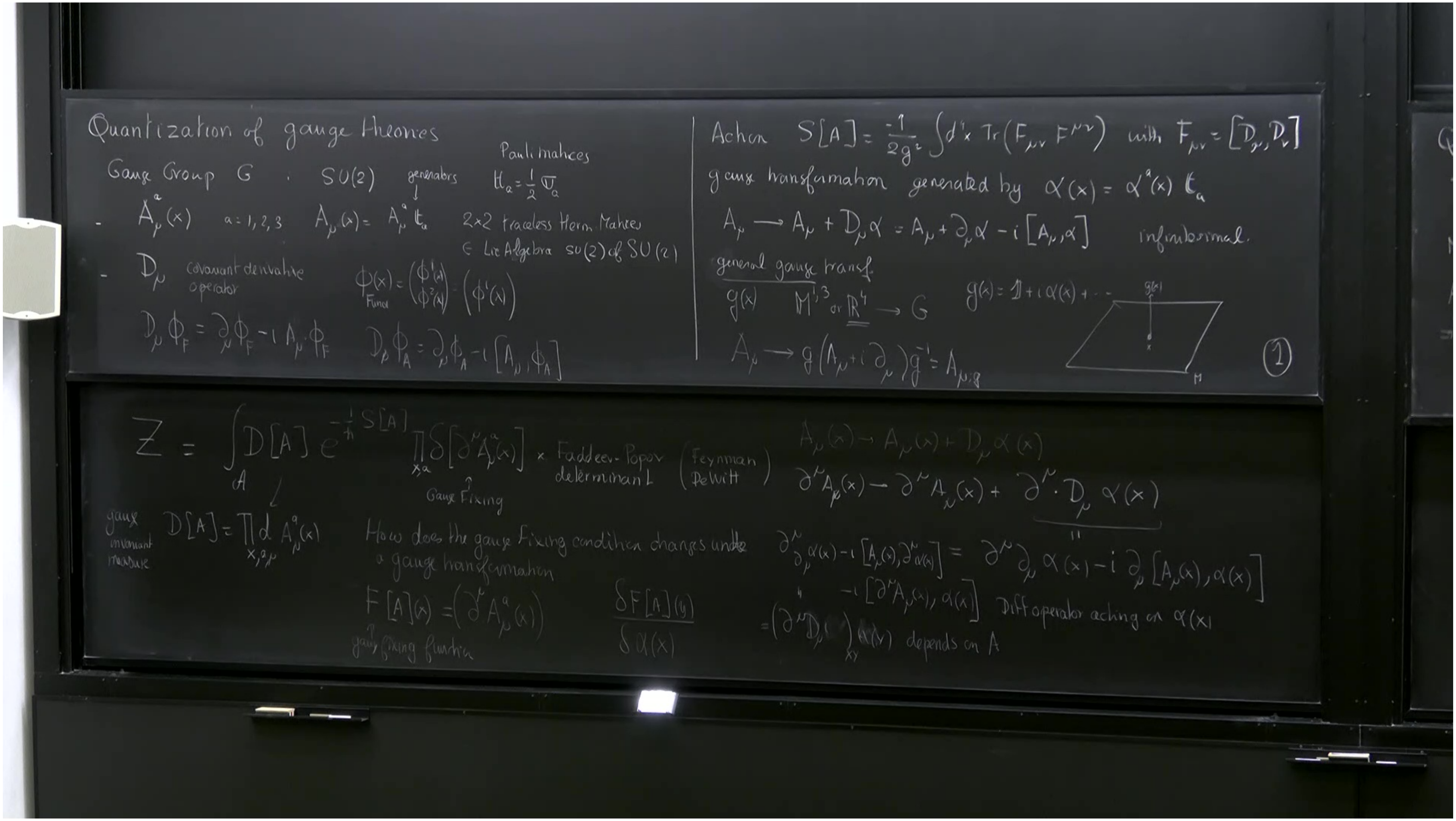
Gauge Fixing

gauge invariant measure $D[A] = \prod_{x,a,\mu} dA^a_\mu(x)$

How does the gauge Fixing condition changes under a gauge transformation

$F[A](x) = \left(\partial^\mu A^a_\mu(x)\right)$

gauge fixing function



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$$A_\mu(x) \to A_\mu(x) + D_\mu \alpha(x)$$

$$\partial^\mu A_\mu(x) \to \partial^\mu A_\mu(x) + \underbrace{\partial^\mu \cdot D_\mu\, \alpha(x)}_{\shortparallel}$$

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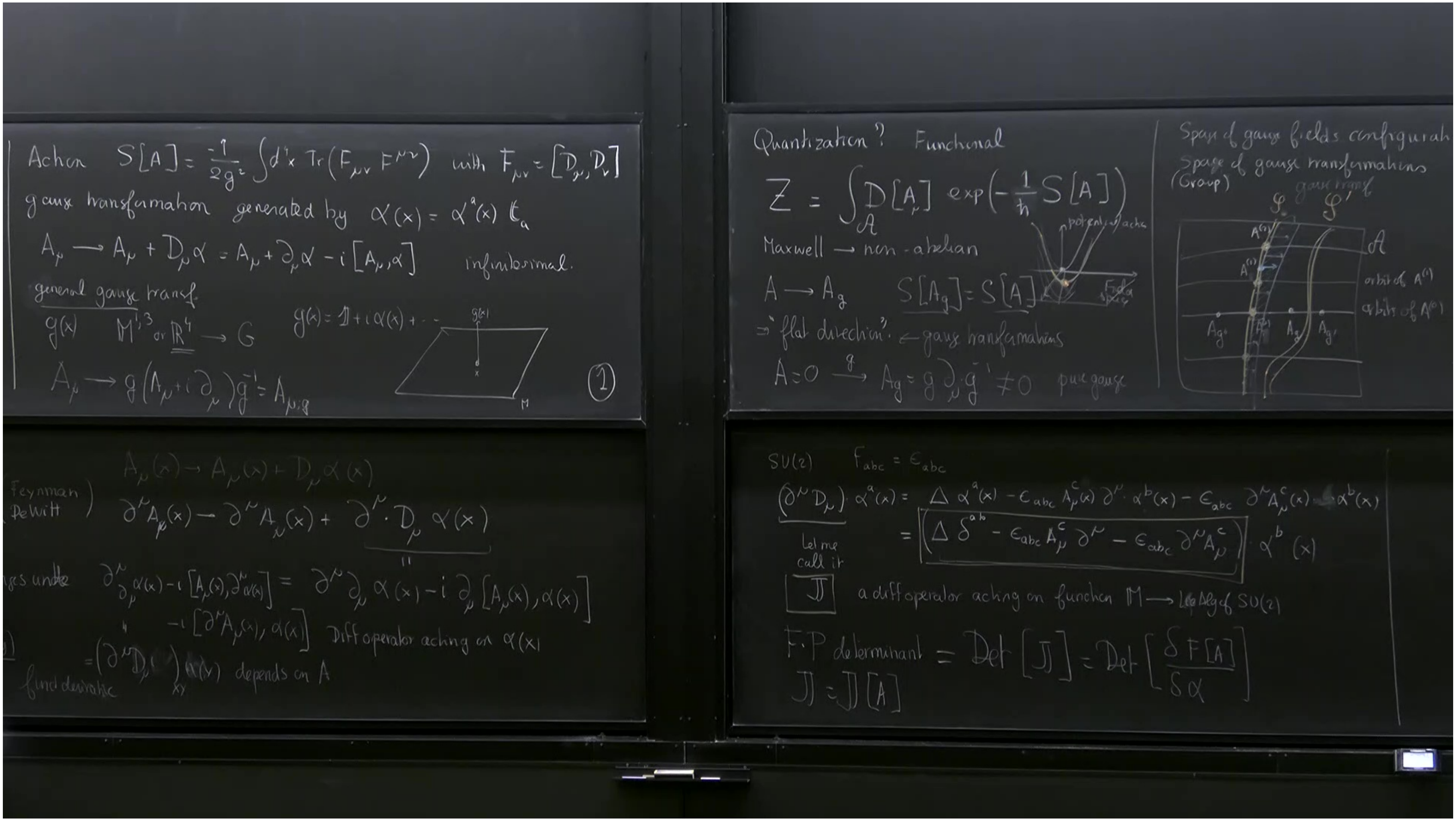
$$\quad + \; - i\left[\partial^\mu A_\mu(x), \alpha(x)\right]$$ Diff operator acting on $\alpha(x)$

$$\underset{\substack{\uparrow \\ \text{gauge fixing function}}}{F}[A](x) = \left(\partial^\mu A^a_\mu(x)\right) \qquad \frac{\delta F[A](y)}{\delta \alpha(x)} = \left(\partial^\mu D_\mu(\;)\right)_{xy} \alpha(y)$$ depends on A



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①

$$A_\mu(x) \to A_\mu(x) + D_\mu\alpha(x)$$

Feynman

DeWitt)

$$\partial^\mu A_\mu(x) \to \partial^\mu A_\mu(x) + \underbrace{\partial^\mu \cdot D_\mu \alpha(x)}$$

ges under

$$\partial^\mu \partial_\mu \alpha(x) - i[A_\mu(x), \partial^\mu \alpha(x)] = \partial^\mu \partial_\mu \alpha(x) - i\,\partial_\mu [A_\mu(x), \alpha(x)]$$

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$$= (\partial^\mu D_\mu(\;))_{xy}\,\alpha(y)$$ depends on A

find derivative

Quantization? Functional

$$Z = \int_{\mathcal{A}} D[A_\mu] \exp\left(-\frac{1}{\hbar} S[A]\right)$$

Maxwell → non-abelian

$A \longrightarrow A_g \qquad \underline{S[A_g]} = \underline{S[A]}$

⇒ "flat direction". ← gauge transformations

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Space of gauge fields configurations

Space of gauge transformations (Group)

SU(2) $\qquad f_{abc} = \epsilon_{abc}$

$$\underline{(\partial^\mu D_\mu)} \cdot \alpha^a(x) = \Delta\alpha^a(x) - \epsilon_{abc} A^c_\mu(x)\,\partial^\mu \cdot \alpha^b(x) - \epsilon_{abc}\,\partial^\mu A^c_\mu(x) \cdot \alpha^b(x)$$

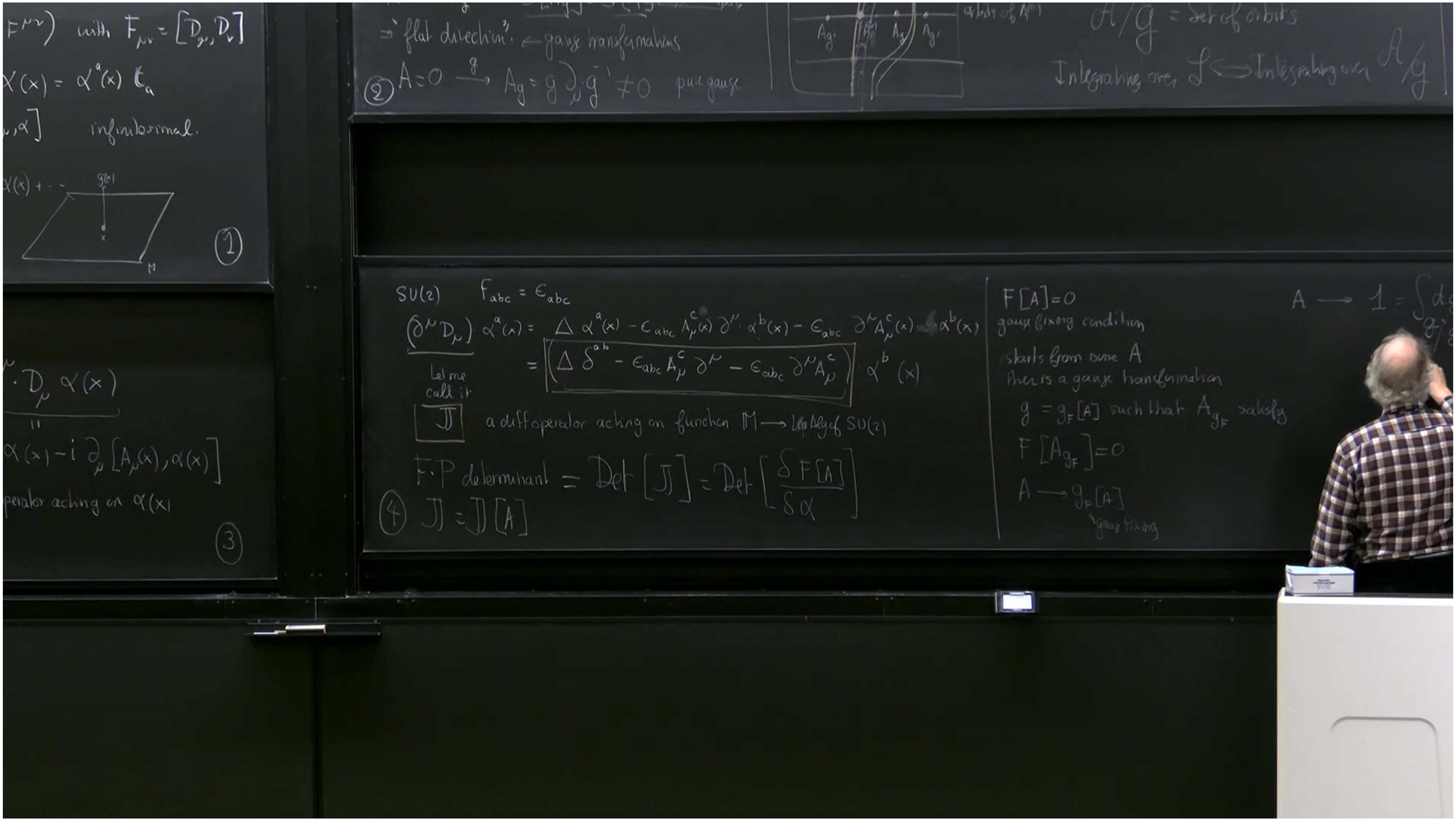
Let me call it

$$= \boxed{\left(\Delta\delta^{ab} - \epsilon_{abc} A^c_\mu \partial^\mu - \epsilon_{abc}\,\partial^\mu A^c_\mu\right)} \cdot \alpha^b(x)$$

$\boxed{\mathbb{J}}$ a diff operator acting on function $\mathbb{M} \longrightarrow$ Lie Alg of SU(2)

$$\text{F.P determinant} = \mathrm{Det}[\mathbb{J}] = \mathrm{Det}\left[\frac{\delta F[A]}{\delta\alpha}\right]$$

$$\mathbb{J} = \mathbb{J}[A]$$



$F^{\mu\nu})$ with $F_{\mu\nu} = [D_\mu, D_\nu]$

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$\alpha(x) + \cdots$

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$\mathcal{A}/\mathcal{G}$ = set of orbits

Integrating over $\mathcal{A} \Longleftrightarrow$ Integrating over $\mathcal{A}/\mathcal{G}$

$\cdot D_\mu\, \alpha(x)$

$\alpha(x) - i\,\partial_\mu [A_\mu(x), \alpha(x)]$

perator acting on $\alpha(x)$

③

SU(2) $\quad f_{abc} = \epsilon_{abc}$

$$\underline{(\partial^\mu D_\mu)}\cdot \alpha^a(x) = \Delta\, \alpha^a(x) - \epsilon_{abc} A^c_\mu(x)\, \partial^\mu \cdot \alpha^b(x) - \epsilon_{abc}\, \partial^\mu A^c_\mu(x)\, \alpha^b(x)$$

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$F[A] = 0$

gauge fixing condition

starts from some A

there is a gauge transformation

$g = g_F[A]$ such that A_{g_F} satisfy

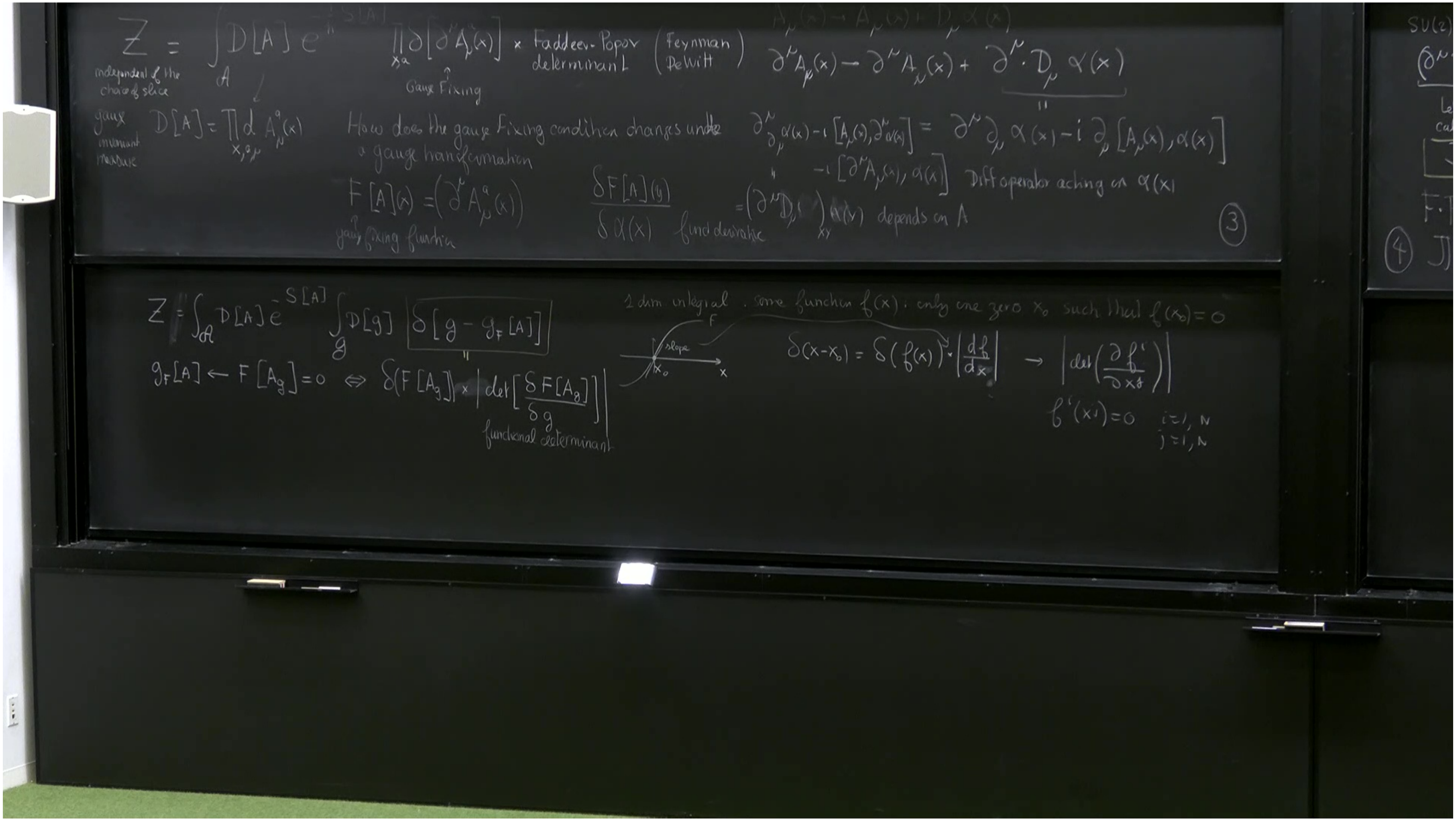
$F[A_{g_F}] = 0$

$A \longrightarrow g_F[A]$

gauge fixing

$A \longrightarrow 1 = \int d$

g



$$Z = \int_{\mathcal{A}} D[A]\, e^{-\frac{1}{\hbar}S[A]} \quad \prod_{x,a} \delta\left[\partial^\mu A^a_\mu(x)\right] \times \text{Faddeev-Popov determinant} \quad \begin{pmatrix}\text{Feynman}\\ \text{DeWitt}\end{pmatrix}$$

independent of the choice of slice

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gauge fixing function

$$\frac{\delta F[A](y)}{\delta \alpha(x)} = \left(\partial^\mu D_\mu\right)_{xy} \alpha(y)$$ find derivative depends on A

③

$$Z = \int_{\mathcal{A}} D[A]\, e^{-S[A]} \int_{\mathcal{G}} D[g] \boxed{\delta\left[g - g_F[A]\right]}$$

$$g_F[A] \leftarrow F[A_g] = 0 \iff \delta(F[A_g]) \times \left|\det\left[\frac{\delta F[A_g]}{\delta g}\right]\right|$$

functional determinant

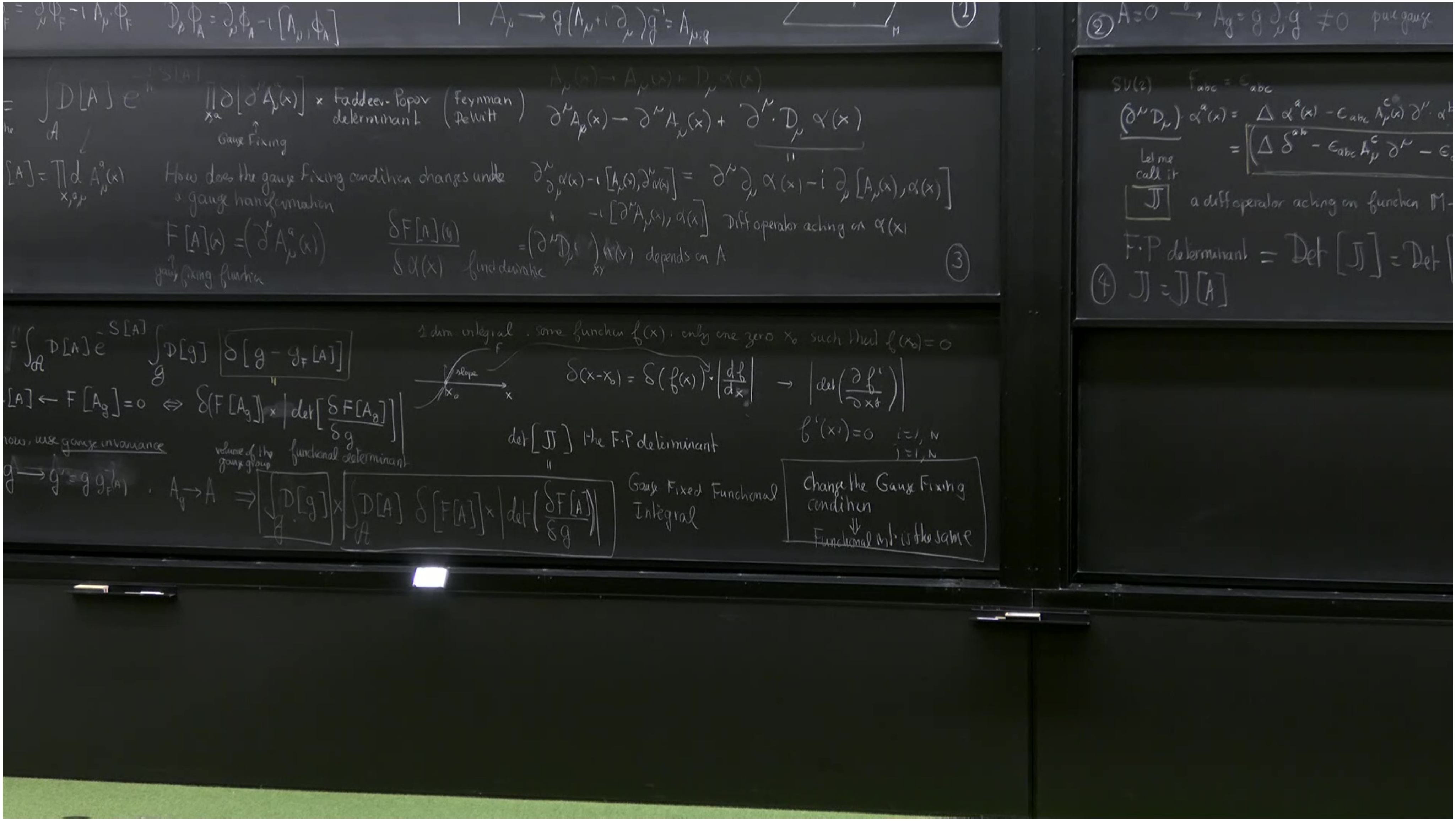
1 dim integral . some function $f(x)$: only one zero x_0 such that $f(x_0) = 0$

$$\delta(x - x_0) = \delta(f(x)) \cdot \left|\frac{df}{dx}\right| \to \left|\det\left(\frac{\partial f^i}{\partial x_j}\right)\right|$$

$f^i(x_i) = 0 \quad i = 1, N$
$j = 1, N$

SU(2)

④



$\partial_\mu \Phi_F - i A_\mu \cdot \Phi_F$ $D_\mu \Phi_A = \partial_\mu \Phi_A - i [A_\mu, \Phi_A]$ $A_\mu \to g(A_\mu + i\partial_\mu) g^{-1} = A_{\mu;g}$ ①

② $A = 0 \xrightarrow{g} A_g = g\,\partial_\mu g^{-1} \neq 0$ pure gauge

$= \int_{\mathcal{A}} D[A]\, e^{-\frac{1}{\hbar}S[A]} \prod_{x,a} \delta[\partial^\mu A^a_\mu(x)] \times$ Faddeev-Popov determinant (Feynman DeWitt)

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$A_\mu(x) \to A_\mu(x) + D_\mu \alpha(x)$

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$= \boxed{(\Delta \delta^{ab} - \epsilon_{abc} A^c_\mu \partial^\mu - \epsilon}$

$\boxed{\mathbb{J}}$ a diff operator acting on function M-

F.P determinant $= \text{Det}[\mathbb{J}] = \text{Det}$

④ $\mathbb{J} = \mathbb{J}[A]$

$= \int_{\mathcal{A}} D[A]\, e^{-S[A]} \int_{g} D[g]\, \boxed{\delta[g - g_F[A]]}$

$[A] \leftarrow F[A_g] = 0 \Leftrightarrow \delta(F[A_g]) \times \left| \det\left[\frac{\delta F[A_g]}{\delta g}\right] \right|$

now, use gauge invariance

volume of the gauge group functional determinant

$g \to g' = g\, g_F^{-1}(A)$, $A_g \to A \Rightarrow \boxed{\int_g D[g]} \times \boxed{\int_{\mathcal{A}} D[A]\, \delta[F[A]] \times \left|\det\left(\frac{\delta F[A]}{\delta g}\right)\right|}$

1 dim integral. some function $f(x)$: only one zero x_0 such that $f(x_0) = 0$

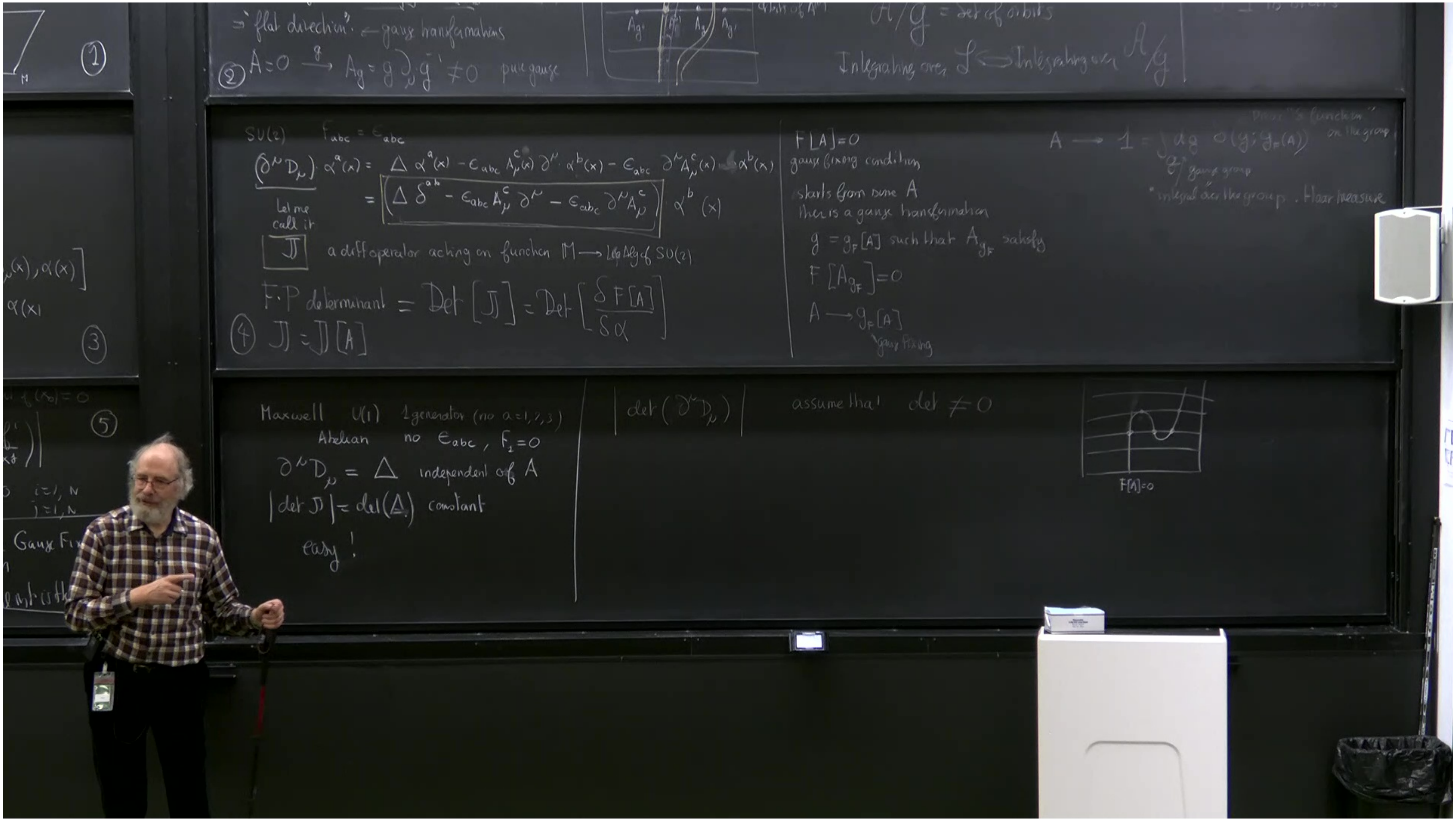
$\delta(x - x_0) = \delta(f(x)) \cdot \left|\frac{df}{dx}\right| \to \left|\det\left(\frac{\partial f^i}{\partial x^j}\right)\right|$

$f^i(x^j) = 0$ $i = 1, N$ $j = 1, N$

$\det[\mathbb{J}]$ the F.P determinant

Gauge Fixed Functional Integral

Change the Gauge Fixing condition
⇓
Functional int is the same



⇒ "flat direction". ← gauge transformations

② $A = 0 \xrightarrow{g} A_g = g\,\partial_\nu g^{-1} \neq 0$ pure gauge

$\mathcal{A}/\mathcal{G}$ = set of orbits

Integrating over $\mathcal{L} \Leftrightarrow$ Integrating over $\mathcal{A}/\mathcal{G}$

$SU(2) \quad f_{abc} = \epsilon_{abc}$

$$\underline{(\partial^\mu D_\mu)}\cdot \alpha^a(x) = \Delta\,\alpha^a(x) - \epsilon_{abc} A^c_\mu(x)\,\partial^\mu \cdot \alpha^b(x) - \epsilon_{abc}\,\partial^\mu A^c_\mu(x)\,\alpha^b(x)$$

$$= \boxed{\left(\Delta\,\delta^{ab} - \epsilon_{abc} A^c_\mu\,\partial^\mu - \epsilon_{abc}\,\partial^\mu A^c_\mu\right)}\,\alpha^b(x)$$

Let me call it

$\boxed{\mathbb{J}}$ a diff operator acting on function $\mathbb{M} \longrightarrow$ Lie Alg of $SU(2)$

$$\text{F.P determinant} = \text{Det}\left[\mathbb{J}\right] = \text{Det}\left[\frac{\delta F[A]}{\delta \alpha}\right]$$

④ $\mathbb{J} = \mathbb{J}[A]$

$F[A] = 0$

gauge fixing condition

starts from some A

there is a gauge transformation

$g = g_F[A]$ such that A_{g_F} satisfy

$F\left[A_{g_F}\right] = 0$

$A \longrightarrow g_F[A]$

gauge fixing

$$A \longrightarrow 1 = \int_{\mathcal{G}} dg\ \delta(g; g_F(A))$$

Dirac "δ function" on the group

gauge group

"integral over the group". Haar measure

Maxwell $U(1)$ 1 generator (no $a = 1, 2, 3$)

Abelian no ϵ_{abc}, $f_1 = 0$

$\partial^\mu D_\mu = \Delta$ independent of A

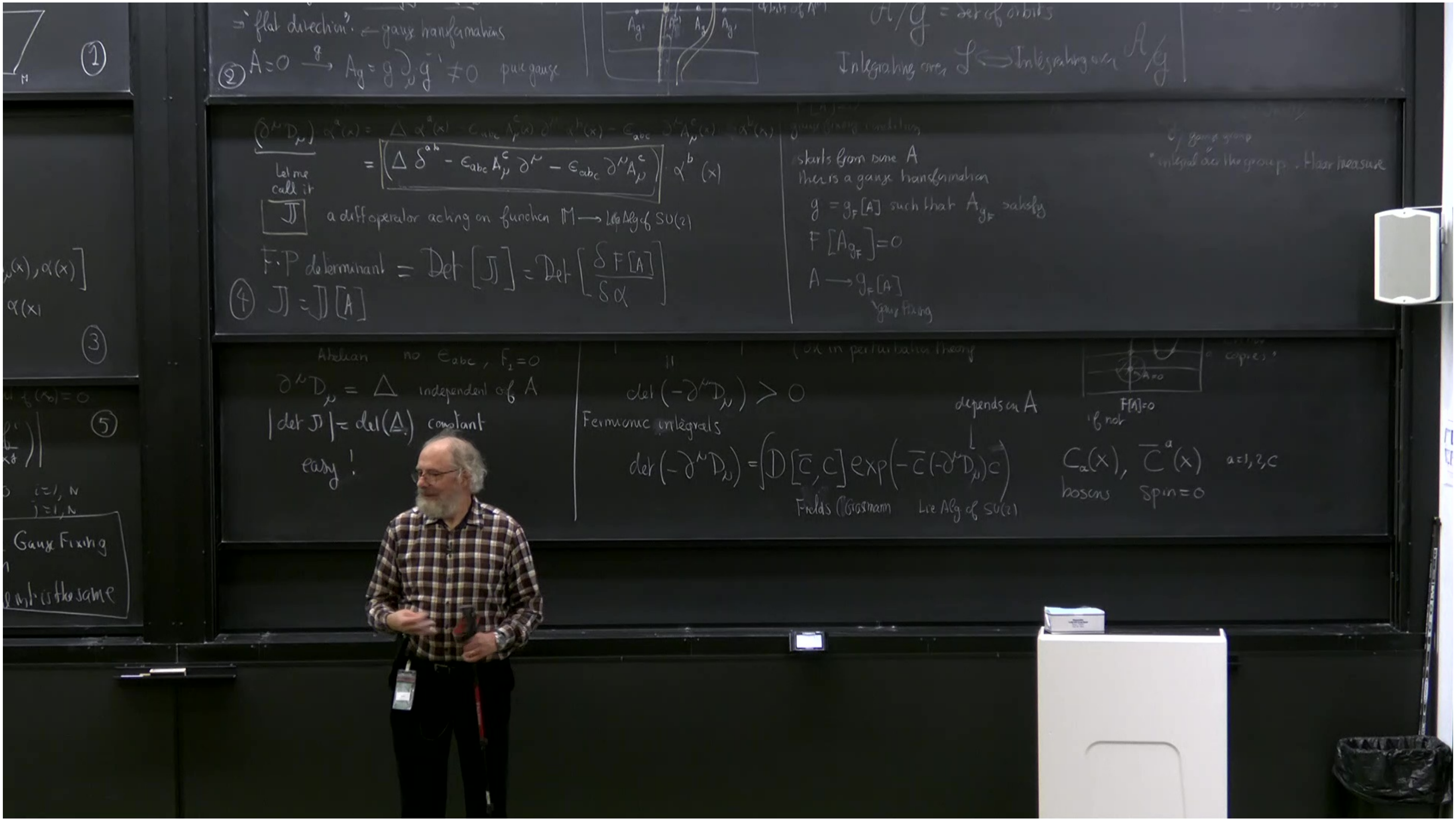
$|\det \mathbb{J}| = \det(\Delta)$ constant

easy !

$|\det(\partial^\mu D_\mu)|$

assume that $\det \neq 0$

$F[A] = 0$



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Integrating over $\mathcal{A} \Longleftrightarrow$ Integrating over $\mathcal{A}/\mathcal{G}$

$\underbrace{(\partial^\mu D_\mu)}_{\text{let me call it}}\,\alpha^a(x) = \Delta\,\alpha^a(x) - \epsilon_{abc} A^c_\mu(x)\,\partial^\mu \alpha^b(x) - \epsilon_{abc}\,\partial^\mu A^c_\mu(x)\,\alpha^b(x)$

$= \boxed{\left(\Delta\,\delta^{ab} - \epsilon_{abc} A^c_\mu\,\partial^\mu - \epsilon_{abc}\,\partial^\mu A^c_\mu\right)}\,\alpha^b(x)$

$\boxed{\mathbb{J}}$ a diff operator acting on function $\mathbb{M} \longrightarrow$ Lie Alg of SU(2)

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$A \longrightarrow \underset{\text{gauge fixing}}{g_F[A]}$

$\mathcal{G}$, gauge group

" integral over the group . Haar measure

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easy !

(OK in perturbation theory

$\det(-\partial^\mu D_\mu) > 0$

Fermionic integrals

depends on A

$\det(-\partial^\mu D_\mu) = \int \mathcal{D}[\bar{C}, C]\,\exp\left(-\bar{C}(-\partial^\mu D_\mu)C\right)$

Fields (Grassmann

Lie Alg of SU(2)

$C_a(x)$, $\bar{C}^a(x)$ a=1,2,C

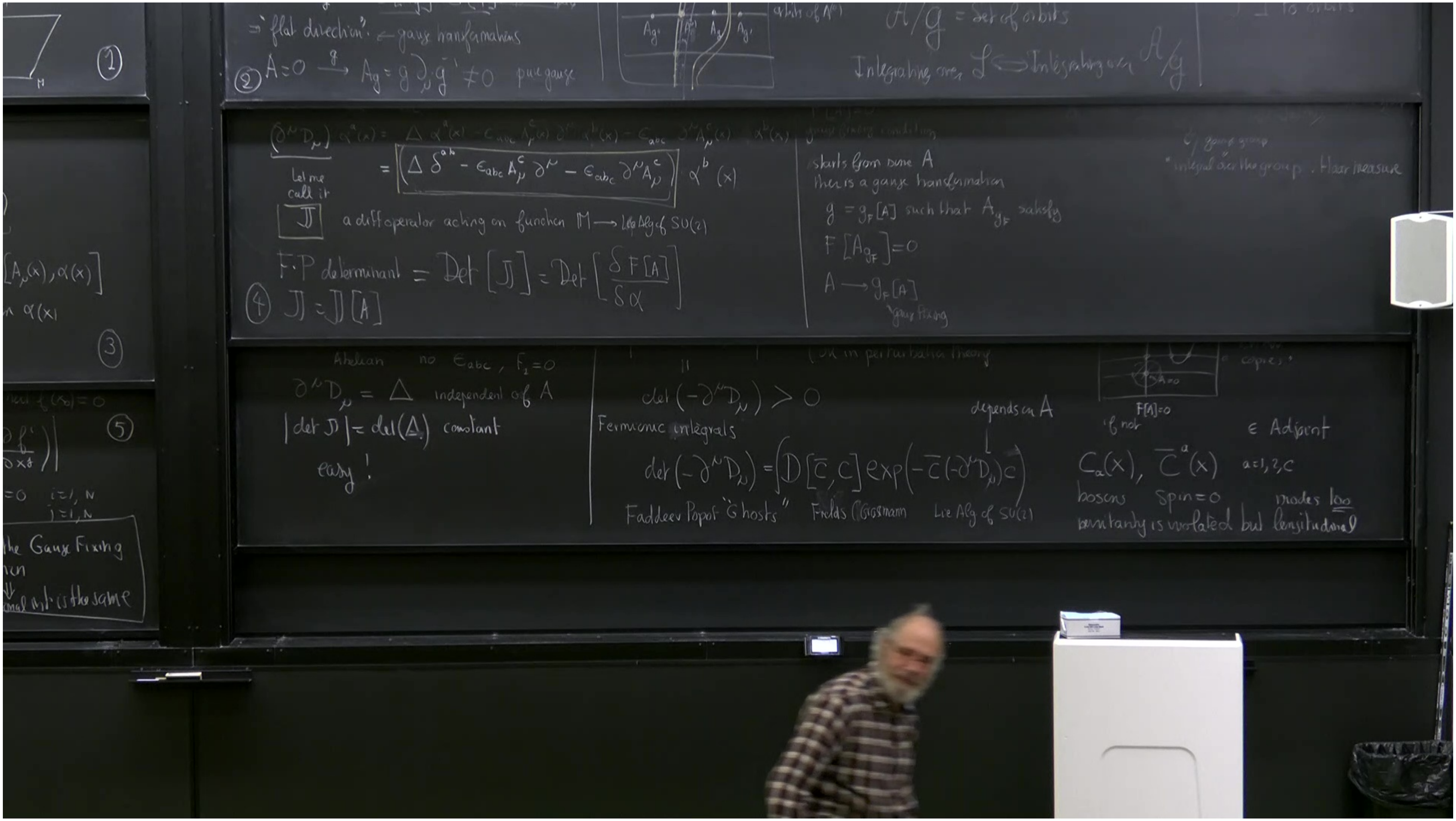
bosons spin=0

$A=0$

$F[A]=0$

if not

copies "



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$$\det(-\partial^\mu D_\mu) = \int \mathcal{D}\left[\bar{c}, c\right] \exp\left(-\bar{c}\,(-\partial^\mu D_\mu)\,c\right)$$

Faddeev Popov "Ghosts" Fields (Grassmann Lie Alg of SU(2)

$F[A]=0$

if not

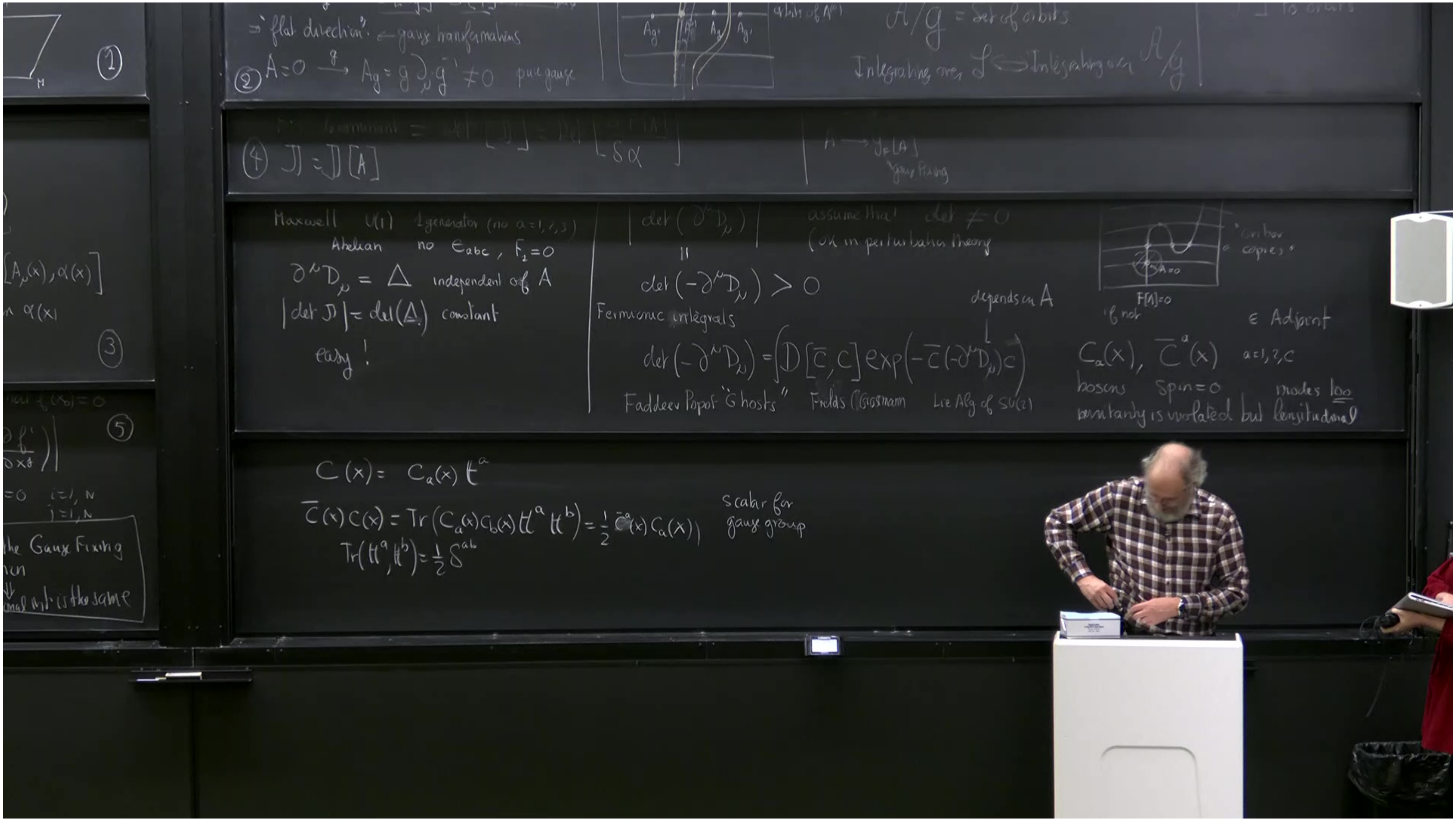
copies?

$\in$ Adjoint

$C_a(x)$, $\bar{C}^a(x)$ $a=1,2,C$

bosons spin=0 modes too

unitarity is violated but longitudinal



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orbits of $A^{(0)}$

$\mathcal{A}/\mathcal{G}$ = Set of orbits

Integrating over $\mathcal{L} \Leftrightarrow$ Integrating over $\mathcal{A}/\mathcal{G}$

④ $J = J[A]$

$\left[\dfrac{\delta F[A]}{\delta\alpha}\right]$

$A \longrightarrow g_F[A]$ gauge fixing

Maxwell $U(1)$ 1 generator (no $a=1,2,3$)

Abelian no ϵ_{abc}, $F_2=0$

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$|\det(\partial^\mu D_\mu)|$

$\parallel$

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Faddeev Popov "Ghosts" Fields (Grassmann Lie Alg of $SU(2)$

"Gribov copies"

$F[A]=0$

$A=0$

if not

$\in$ Adjoint

$C_a(x)$, $\bar{C}^a(x)$ $a=1,2,c$

bosons spin$=0$ modes loo

unitarity is violated but longitudinal

$C(x) = C_a(x)\, t^a$

$\bar{C}(x) C(x) = \mathrm{Tr}\left(C_a(x) C_b(x) t^a t^b\right) = \frac{1}{2}\bar{C}^a(x) C_a(x)$

$\mathrm{Tr}(t^a, t^b) = \frac{1}{2}\delta^{ab}$

scalar for gauge group